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THE GENERAL INSTABILITY OF ECCENTRICALLY STIFFENED CYLINDRICAL SHELLS UNDER AXIAL COMPRESSION AND LATERAL PRESSURE

by John N. Dickson and Richard H. Brolliar

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DYNAMIC RESPONSE OF STRUCTURAL ELEMENTS EXPOSED TO SONIC BOOMS

By David H. Cheng and Jacques E. Benveniste

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ABSTRACT

This report summarizes recent analytical results on the subject of dynamic response of structural elements exposed to sonic booms. The structural elements of interest are uniform beams and plates with various boundary conditions. The disturbances are represented by a variety of boom signatures which approximate those obtained from field measurements.

Responses of structural elements to a unit impulse and to a unit force moving at a constant velocity are first obtained. This enables a comparison to be made of the relative dynamic effects of an N-shaped pressure pulse and an N-shaped traveling wave on a simple structure. It is followed by a study on the effects of boundary restraints using an N-shaped pressure pulse.

Based on the results due to such idealized boom signatures as sine pulse, half-cosine pulse, triangular pulse, N-shaped pulse, and N-shaped pulse with spikes, two simplified methods in evaluating the boom effects on structural elements are proposed: One requires only the knowledge of the peak pressure and the other, the positive impulse. Neither requires the specification of the exact shape of the boom signature.

The above methods are very simple to use, and are applicable to structural elements which are always in contact with the supports. Considerable higher dynamic effects can be expected in cases in which the structural element is loosely bound to supports and may rattle in the wake of boom disturbances. As an illustration, a uniform rattling beam is considered in the Appendix.

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DEFINITION OF SYMBOLS

a_{ij}	Stiffness matrix coefficients
b_r, b_s	Distance from skin centerline to out-of-plane bending shear center of ring and stringer, respectively (radially outward positive)
c_r, c_s	Distance from skin centerline to neutral axis of ring and stringer, respectively (radially outward positive)
e_x, e_y	Normalized strain in shell in x and y directions, respectively
e_{xy}	Normalized shear strain in shell
f	Amplitude of local buckling wave, defined by Figure A1
k, k_r	Et/R^2, $E_r A_r/R^2$
m	Number of longitudinal half waves
$\overline{m}$	$m\ \pi/L$
n	Number of circumferential waves
$\overline{n}$	n/R
p	Lateral pressure on cylinder (internal pressure positive)
p_r	Lateral load per unit area acting on ring
p'	Lateral pressure acting on shell element
p_{ij}	Pressure matrix coefficients
q	Buckling load per unit of length of circumference, measured at skin centerline
q_o	Local buckling load per unit of length of circumference

DEFINITION OF SYMBOLS (Continued)

q'_o	Portion of local buckling load acting on shell element per unit of length of circumference
q_s	Axial load acting on stringer per unit of length of circumference
q'	Axial load acting on shell element per unit of length of circumference
q_{ij}	Buckling load matrix coefficients
s_x, s_y	Normalized stress in shell in x and y directions, respectively
s_{xy}	Normalized shear stress in shell
t	Shell thickness; facesheet thickness for sandwich shells; defined by Figure 8 for corrugated shells
u, v, w	Displacement of cylinder at shell centerline in the x, y, and z directions, respectively
u_r, v_r, w_r	Displacement of ring centroid in the x, y, and z directions, respectively
u_s, v_s, w_s	Displacement of stringer centroid in the x, y, and z directions, respectively
$\bar{w}$	Prebuckling radial displacement of stiffened shell
A	$\bar{k}_r/(\bar{k}L_r + BL_r\bar{k}_r)$
A_r	Area of ring
A_s	Area of stringer; area per pitch of corrugation
D	$(L_s/\lambda)^2$
D_x, D_y	Bending stiffness of shell in longitudinal and circumferential directions, respectively

DEFINITION OF SYMBOLS (Continued)

D_μ	Defined by equation (5)
D_{xx}	Longitudinal flexural rigidity of shell-stringer combination
D_{xs}	$E_s I_{xs}/L_s$
D_{zs}	$E_s I_{zs}/L_s$
D_{yr}	$E_r I_{yr}/L_r$
D_{zr}	$E_r I_{zr}/L_r$
E, E_r, E_s	Moduli of elasticity of shell, ring, and stringer, respectively
$\overline{E}_r$	$E_r A_r/L_r$
$\overline{E}_s$	$E_s A_s/L_s$
$\overline{E}_x$, $\overline{E}_y$	Extensional stiffnesses of the shell in the longitudinal and circumferential directions, respectively
$\overline{E}_\mu$	Defined by equation (5)
G, G_r, G_s	Shear moduli of shell, ring, and stringer, respectively
$\overline{G}$	Gt
H	Distance between facesheet centerlines for sandwich shells
I_{yr}	Moment of inertia of the ring about a longitudinal line through the centroid of the ring cross section
I_{zr}	Moment of inertia of the ring about a radial line through the centroid of the ring cross section
I_{xs}	Moment of inertia of stringer about a tangential line through its centroid; moment of inertia per pitch of corrugation
I_{zs}	Moment of inertia of stringer about a radial line through its centroid

DEFINITION OF SYMBOLS (Continued)

J_r	$1/G_r$ times torsional stiffness of ring
J_s	$1/G_s$ times torsional stiffness of stringer
K	Twisting stiffness of shell
K_r	$G_r J_r / L_r$
K_s	$G_s J_s / L_s$
L	Length of cylinder
L_r	Ring spacing
L_s	Stringer spacing; pitch of corrugation
M_x, M_y	Stress couples acting on shell element in x and y directions, respectively
M_{xy}, M_{yx}	Torsional stress couples acting on skin element
M_{yr}, M_{zr}	Stress couples acting on ring element in y and z directions, respectively
M_{yxr}	Torsional stress couple acting on ring element
M_{xs}, M_{zs}	Stress couples acting on stringer element in x and z directions, respectively
M_{xys}	Torsional stress couple acting on stringer element
N_x, N_y	Stress resultants due to buckling displacements acting on shell element in x and y directions, respectively
N_{xy}, N_{yx}	Shear stress resultants acting on shell element
N_{xs}, N_{xys}	Stress resultants acting on stringer element in x and y directions, respectively

DEFINITION OF SYMBOLS (Continued)

N_{yr}, N_{yxr}	Stress resultants acting on ring element in y and x directions, respectively
$\overline{N}_x$, $\overline{N}_y$	Total stress resultant acting on shell element in x and y directions, respectively
$\overline{N}_{xs}$	Total stress resultant acting on stringer in x direction
$\overline{N}_{yr}$	Total stress resultant acting on ring in y direction
$\overline{N}_{yy}$	Local hoop stress resultant before general instability
Q_x, Q_y	Radial shear stress resultants acting on skin element
Q_{xs}, Q_{yr}	Radial shear stress resultant acting on stringer and ring, respectively
R	Radius to centerline of shell
R_{br}, R_{bs}	$R + b_r$, $R + b_s$, respectively
R_{cr}, R_{cs}	$R + c_r$, $R + c_s$, respectively
T_x, T_y; T_{xr}, T_{yr}; T_{xs}, T_{ys}	Interface moments per unit area acting on skin, ring, and stringer; respectively
U	Amplitude of u
V	Amplitude of v; radial shear force, per unit of length of circumference reacted by the ring
W	Amplitude of w
X, Y, Z; X_r, Y_r, Z_r; X_s, Y_s, Z_s	Interface forces per unit area acting on skin, ring, and stringer, respectively

DEFINITION OF SYMBOLS (Continued)

α	$\sqrt{\lambda^2 + q/(4D_{xx})}$ or buckling wave shape parameter defined in Figure A1
α_p, α_q	Defined by equation (A8) Appendix A
β	$\sqrt{\lambda^2 - q/(4D_{xx})}$
β_x, β_y	Reduced moduli for shell in x and y directions, respectively
β_μ	Reduced modulus for cross stiffness (Poisson's effect)
β_s	Reduced modulus for shear
γ_x	Skin effective width factor
γ_{xy}, γ_{yx}	Shear strain in shell
δ	Number of rings per longitudinal half wave length
ϵ^*	$\frac{\pi^2}{3(1 - \mu^2)} \; {\frac{t}{L_s}}^{2}$
ϵ_x, ϵ_y	Strains in shell in x and y directions, respectively
$\epsilon_{yr}, \epsilon_{xs}$	Circumferential strain in ring and longitudinal strain in stringer, respectively
κ_x, κ_y	Curvature changes in shell in x and y directions, respectively
κ_{yx}	Specific twist of shell element
κ_{xs}, κ_{zs}	Curvature changes of stringer in radial plane and normal to radial plane, respectively
$\kappa_{xys}, \kappa_{yxr}$	Specific twist of stringer and ring, respectively
κ_{yr}, κ_{zr}	Curvature changes in plane and normal to plane of ring, respectively

DEFINITION OF SYMBOLS (Concluded)

λ	Local buckling longitudinal half wave length, see Figure A1
μ	Poisson's ratio of shell
σ_x, σ_y	Stress in shell in x and y directions, respectively
σ_{xy}	Shear stress in shell
$(\quad)_{,x}$	Differentiation with respect to x

THE GENERAL INSTABILITY OF ECCENTRICALLY STIFFENED CYLINDRICAL SHELLS UNDER AXIAL COMPRESSION AND LATERAL PRESSURE

SUMMARY

This report presents a method of analysis to determine the general instability load of an orthogonally stiffened cylindrical shell under axial compression and lateral pressure. The governing equations are derived using small deflection theory; and, consequently, the validity of the method must be restricted to moderately or heavily stiffened cylinders. All the stiffnesses occurring in stiffened shells of this type have been incorporated, and the rings and stringers are considered eccentric with respect to the skin middle surface. Local buckling of the skin between adjacent stringers before general instability is allowed, and the resulting reductions in stiffness properties of the skin are determined as a function of the two principal strains.

Analytical and experimental results are compared for twenty-nine stiffened cylinders loaded in compression and for six stiffened cylinders loaded in bending.

The method has been programmed for use with an IBM 7094 computer. The computer program and detailed instructions for its use are included in this report.

INTRODUCTION

The interest in general instability of stiffened cylindrical shells has increased considerably in the last few years because of the many applications in space vehicle structures, but the basic problem is an old one. For many years, stiffened cylinders were designed almost exclusively by empirical or semi-empirical methods, since they were the most reliable methods available. In more recent years, methods have been developed, using small deflection theory, which are in good agreement with experimental results for all except lightly stiffened cylinders.

A detailed description of the early papers using small deflection theory is given in References 1 and 2. Later papers have shown that the eccentricity of the stiffeners has an appreciable effect on the general instability load. Some of the papers which have considered stiffener eccentricity are discussed in the following paragraph.

Van der Neut showed the importance of ring and stringer eccentricities for cylinders under axial load in Reference 3, published in 1947. Kendricks [4], Bodner and Shaw [5], and Baruch and Singer [6] have shown the same to be true for stiffened cylinders under hydrostatic pressure. Block, Card, and Mikulas [7] investigated a stiffened cylinder under a combination of axial load and lateral pressure. Hedgepeth and Hall [8] and Jones and Card [9] analytically examined stiffened cylinders with fixed ends loaded in compression and made comparisons with the test data given in Reference 10. The authors examined ring stiffened, corrugated cylinders loaded in compression in Reference 11.

A second area of interest is stiffened cylinders in which local buckling of the skin between stringers occurs before general instability. Card [12] presents test data for stiffened cylinders having local buckling before general instability. He uses the general instability method of Reference 13 combined with the buckled skin stiffness properties of Reference 14 to analytically predict the failure loads.

As the available methods for predicting general instability have improved, they have also become more complex, with computer programs generally being needed to implement them.

The purpose of this report is to present, in computerized form, a method to determine the general instability load of an orthogonally stiffened cylindrical shell subjected to axial compression and lateral pressure. The method is developed for the general case in which general instability is preceded by local buckling of the skin between adjacent longitudinal stiffeners.

GENERAL THEORY

Basic Assumptions and Limitations

The equations are developed in this section for buckling caused by general instability of an orthogonally stiffened cylindrical shell (Fig. 1)

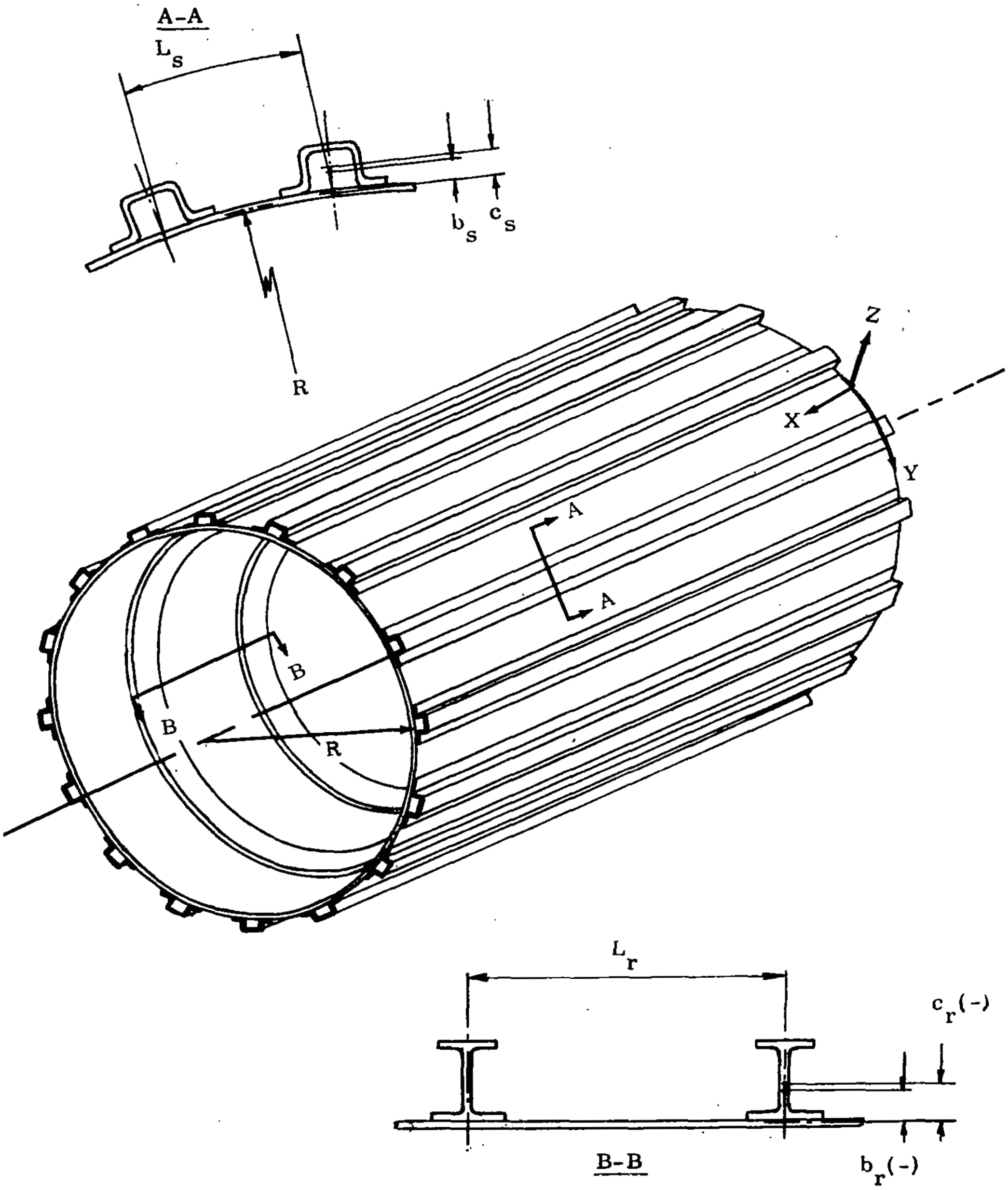


FIGURE 1. STIFFENED CYLINDRICAL SHELL

loaded simultaneously by axial compression and lateral pressure. In the derivation, the following basic assumptions are made:

1. Classical small deflection theory adequately describes the strains in the cylinder in terms of the buckling displacements u, v, and w.

2. The stiffeners are spaced close enough so that their elastic properties may be uniformly distributed.

3. Prebuckling displacements are neglected.

4. The cylinder is simply supported at each end.

The first assumption limits the validity of the method of analysis to cylinders with moderate to heavy stiffening. For monocoque and lightly stiffened cylinders, the general instability load lies below the classical buckling load, and small deflection theory is no longer adequate to describe the state of stress in the cylinder during buckling. Assumption 2 is valid as long as the half wave length of the buckled skin encompasses at least two stiffeners. This condition presents no difficulty as far as the stringers are concerned, but in some cases, the longitudinal half wave length is found to be almost equal to the ring spacing. For these cases the results are unreliable and a discrete ring analysis must be made. The last assumption is not as restrictive as it appears, because if the length of the cylinder is several times larger than the critical longitudinal half wave length, the conditions at the ends of the cylinder should not affect the critical load.

Method of Analysis

For the purpose of formulating the equations of equilibrium, the rings and stringers are assumed to be detached from the cylindrical shell. An element of the shell is shown in Figure 2. This element is acted upon at its edges by the stress resultants and couples caused by the buckling deformations. It is also acted upon by surface forces, which, in addition to the lateral pressure, will include forces and moments applied to the shell by the rings and stringers.

The middle surface of the shell is taken as the load reference surface. Differential equations in terms of the buckling displacements are obtained by considering the equilibrium of a shell element. Similar equations are derived

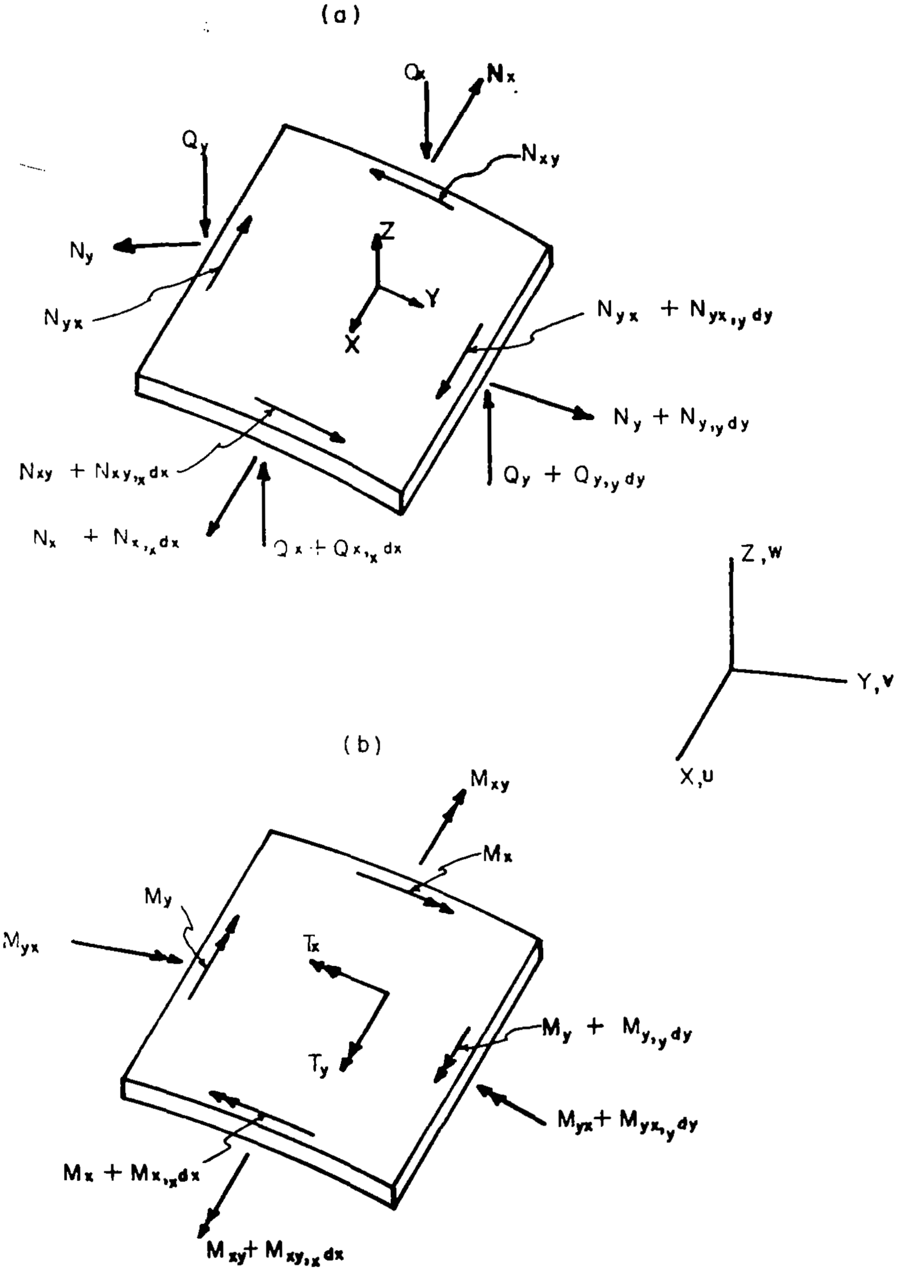


FIGURE 2. FORCES AND MOMENTS ACTING ON SHELL ELEMENT

for corresponding ring and stringer elements and combined with the shell equilibrium equations in a manner that will eliminate the unknown surface forces from the shell equations. The resulting equations contain two types of terms. The first type involves products of the stiffness parameters and derivatives of the displacements; the second type involves products of the external loads and derivatives of the displacements. By assuming periodic functions for the buckling displacements, the three differential equations may be transformed into three linear, homogeneous equations. A nontrivial solution for this set of equations requires that the determinant of its matrix be zero. This condition yields a cubic equation from which the buckling load may be found for a given mode shape.

Kinematic Relations

According to the Love-Kirchhoff postulation of thin shell theory, strains are assumed to vary linearly through the thickness of the shell. The behavior of the shell is therefore completely defined by the strains and curvatures of its middle surface. The strains and curvatures expressed in terms of the displacements of the middle surface may be written

$$
\begin{aligned}
\epsilon_x &= u,_x & \kappa_x &= -w,_{xx} \\
\epsilon_y &= v,_y + \frac{1}{R} w & \kappa_y &= -w,_{yy} + \frac{1}{R} v,_y \\
\gamma_{yx} &= u,_y + v,_x & \kappa_{yx} &= -w,_{xy} + \frac{1}{R} v,_x \quad .
\end{aligned}
\tag{1}
$$

If it is assumed that the stiffeners are rigidly attached to the shell and that their cross sections do not distort, the displacements of the stiffeners at a distance z from the middle surface of the shell are

$$
\begin{aligned}
u_s &= u - zw,_x \\
v_s &= \left(1 + \frac{z}{R}\right) v - zw,_y \\
w_s &= w \; .
\end{aligned}
\tag{2}
$$

The strains and curvature changes in the plane of the stiffener are given at the centroid, and the specific twists and out-of-plane curvatures are given at the shear center of the stiffener cross section. This gives, for a stringer element,

$$\begin{aligned}
\epsilon_{xs} &= u,_{x} - c_{s}\, w,_{xx} \\
\kappa_{xs} &= -\, w,_{xx} \\
\kappa_{zs} &= -\frac{R_{bs}}{R}\, v,_{xx} + b_{s}\, w,_{xxy} \\
\kappa_{xys} &= -\, w,_{xy} + \frac{1}{R}\, v,_{x}
\end{aligned} \qquad (3)$$

and similarly for a ring element,

$$\begin{aligned}
\epsilon_{yr} &= v,_{y} - \frac{c_{r} R}{R_{cr}}\, w,_{yy} + \frac{1}{R_{cr}}\, w \\
\kappa_{yr} &= -\frac{R}{R_{cr}}\, w,_{yy} + \frac{1}{R_{cr}}\, v,_{y} \\
\kappa_{zr} &= -\frac{R^{2}}{R_{br}^{2}}\, u,_{yy} + \frac{1}{R_{br}}\, w,_{x} + \frac{b_{r} R^{2}}{R_{br}^{2}}\, w,_{xyy} \\
\kappa_{yxr} &= -\frac{R^{2}}{R_{br}^{2}}\, w,_{xy} - \frac{R}{R_{br}^{2}}\, u,_{y} \,.
\end{aligned} \qquad (4)$$

The last two of equations (4) are the out-of-plane bending and twisting of the ring as given by Timoshenko and Gere [15].

Constitutive Equations

The stress resultants and stress couples are defined as the forces and moments per unit length, acting at the centroidal surface of the shell or stiffeners. For the shell they may be obtained by integrating the stresses over the thickness of the shell. Denoting the extensional and shear stiffnesses of the shell by $\overline{E}$ and $\overline{G}$ and the bending and torsional stiffnesses by D and K, one may write for the stress resultants and stress couples of the shell

$$\begin{aligned}
N_x &= \overline{E}_x \epsilon_x + \overline{E}_\mu \epsilon_y \\
N_y &= \overline{E}_\mu \epsilon_x + \overline{E}_y \epsilon_y \\
N_{yx} &= \overline{G}\gamma_{yx} \\
M_x &= - D_x \kappa_x - D_\mu \kappa_y \\
M_y &= - D_\mu \kappa_x - D_y \kappa_y \\
M_{xy} + M_{yx} &= - K\kappa_{yx} \quad .
\end{aligned} \tag{5}$$

If there is no local buckling of the skin between stringers and the material is isotropic, one has

$$\overline{E}_x = \overline{E}_y = \frac{Et}{1 - \mu^2} \qquad \overline{E}_\mu = \mu \overline{E}_x$$

$$D_x = D_y = \frac{Et^3}{12(1 - \mu^2)} \qquad D_\mu = \mu D_x .$$

The stress resultants and stress couples for the stringer and ring elements are given by the relations

$$\begin{aligned}
N_{xs} &= \overline{E}_s \epsilon_{xs} & N_{yr} &= \overline{E}_r \epsilon_{yr} \\
M_{xs} &= - D_{xs} \kappa_{xs} & M_{yr} &= - D_{yr} \kappa_{yr} \\
M_{zs} &= - D_{zs} \kappa_{zs} & M_{zr} &= - D_{zr} \kappa_{zr} \\
M_{xys} &= - K_s \kappa_{xys} & M_{yxr} &= - K_r \kappa_{yxr} \; .
\end{aligned} \tag{6}$$

The lateral bending stiffnesses of the stringer and ring elements, D_{zs} and D_{zr}, are usually small when compared to the shear stiffness of the shell, but they will be maintained in the analysis for completeness.

Equilibrium Equations

Consider the equilibrium of an element of the cylindrical shell as shown in Figures 2 and 3. The forces applied to the surface of the shell by the stiffeners are denoted by X, Y, and Z. They are shown in Figure 2a with the stress resultants acting at the edges of the shell. The second figure shows the stress couples and the moments T_x and T_y transferred into the shell by the stiffeners. Six conditions of equilibrium may be written for the shell element, three for the force components and three for the moments. For the total stress resultants $\bar{N}_x$ and $\bar{N}_y$, one may write

$$\begin{aligned} \bar{N}_x &= N_x - q' \\ \bar{N}_y &= N_y + p' R \end{aligned} \tag{7}$$

where q' is the part of the axial load applied to the shell and $p' R$ is the average prebuckling hoop stress resultant in the shell.

When the element deforms, the stress resultant $\bar{N}_x$ will no longer act orthogonal to the y and z axes but will have components parallel to these axes. The components on one side of the element are $\bar{N}_x v,_x$ and $\bar{N}_x w,_x$, respectively, where $v,_x$ and $w,_x$ are the rotations of the shell element. On the opposite side, these components are larger by a differential as shown in Figure 3a and act in the opposite direction. The net contribution by the stress resultant $\bar{N}_x$ to the force components in the y and z directions then becomes

$$\bar{N}_x v,_{xx} \, dx \, dy \quad \text{and} \quad \bar{N}_x w,_{xx} \, dx \, dy ,$$

respectively. Similarly, the components parallel to the x and z axes caused by the stress resultant $\bar{N}_y$ and the pressure force must be considered in the

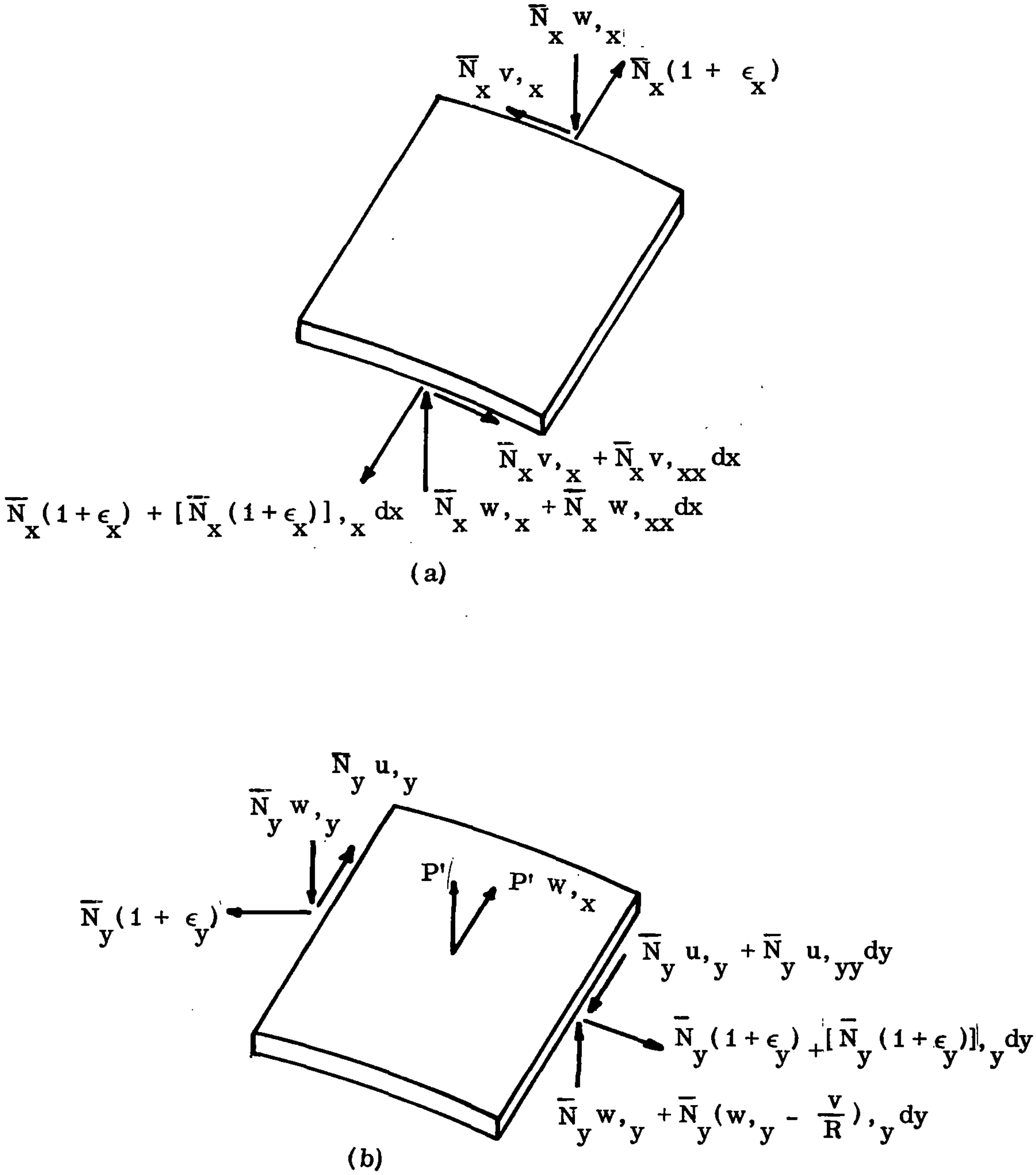


FIGURE 3. EXTERNAL FORCES ACTING ON SHELL ELEMENT

formulation of the equilibrium equations. The rotation $u,_y$ and the tilting $w,_x$ of the element cause a net force in the x direction

$$(\overline{N}_y u,_{yy} - p' w,_x) \, dx \, dy \; .$$

Because of the change of angle between the hoop forces (Fig. 3b), there will be a contribution in the radial direction of magnitude

$$- \overline{N}_y \left(\frac{1}{R} v,_y - w,_{yy}\right) dx \, dy \; .$$

Finally, to take into account the strains at the middle surface of the shell, the method proposed by Flügge [16] will be adopted. In this method, the main stress resultants $\overline{N}_x$ and $\overline{N}_y$ are multiplied by the reference vectors $(1 + \epsilon_x)$ and $(1 + \epsilon_y)$, respectively, and the pressure by the quantity $(1 + \epsilon_x)$ $(1 + \epsilon_y)$. By substituting equations (7) for the total stress resultants and equations (1) for the strains ϵ_x and ϵ_y, the six conditions of equilibrium for the shell element may now be written in the following form:

$$N_{x,x} + N_{yx,y} - q' u,_{xx} + p' (Ru,_{yy} - w,_x) + X = 0$$

$$N_{xy,x} + N_{y,y} + \frac{Q_y}{R} - q' v,_{xx} + p'(Rv,_{yy} + w,_y) + Y = 0$$

$$Q_{x,x} + Q_{y,y} - \frac{N_y}{R} - q' w,_{xx} + p'(Rw,_{yy} + u,_x - v,_y) + Z = 0$$

$$M_{x,x} + M_{yx,y} + Q_x + T_x = 0$$

$$M_{xy,x} + M_{y,y} + Q_y + T_y = 0$$

$$N_{xy} - N_{yx} + \frac{M_{yx}}{R} = 0 \; .$$

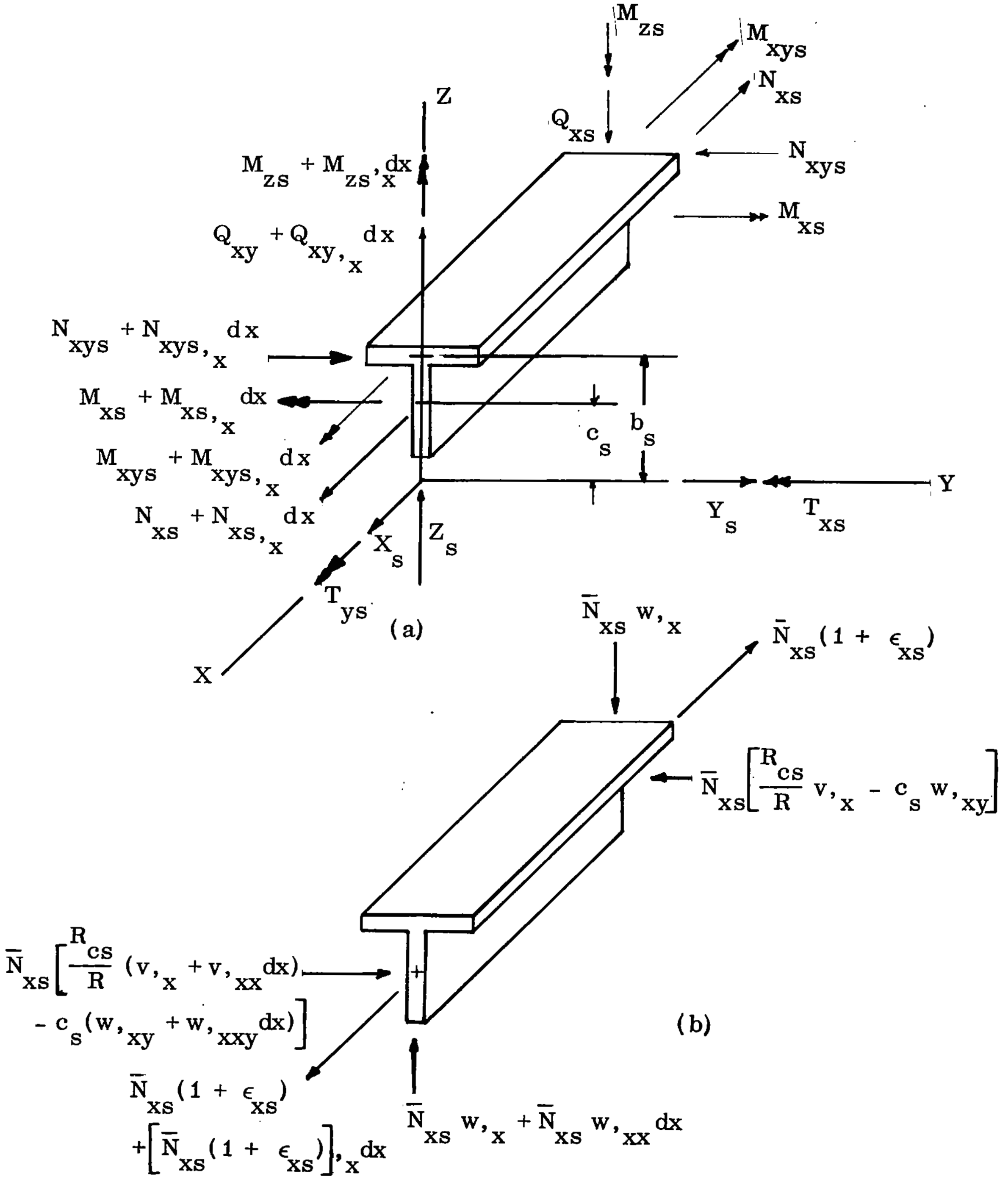


FIGURE 4. FORCES AND MOMENTS ACTING ON STRINGER

Next, the equilibrium of a stringer element will be considered. The stress resultants and couples acting on such an element are shown in Figure 4a. The surface forces X_s, Y_s, and Z_s and the moments T_{xs} and T_{ys} are transferred into the stringer element from the shell. The axial stress resultant $\overline{N}_{xs}$ acts at the centroidal surface of the stringer at a distance c_s from the shell middle surface. Because of this eccentricity, $\overline{N}_{xs}$ yields an important contribution to the moment equation. The lateral shear, N_{xys}, is assumed to act through the shear center of the stringer section at a distance b_s from the shell middle surface. As was done with the shell element, the main stress resultant, $\overline{N}_{xs}$, will be multiplied by the factor $(1 + \epsilon_{xs})$ to take into account the straining of the centroidal surface. Because of the rotations $v_s,_x$ and $w,_x$ of the stringer element, $\overline{N}_{xs}$ will have components in the y and z directions (Figure 4b). The net forces in the y and z directions contributed by these components are

$$\overline{N}_{xs}\left(\frac{R_{cs}}{R} v,_{xx} - c_s w,_{xxy}\right) dx\, dy$$

$$\overline{N}_{xs} w,_{xx}\, dx\, dy \ .$$

By writing for the total stress resultant

$$\overline{N}_{xs} = N_{xs} - q_s$$

where q_s is the part of the axial load q applied to the stringer and substituting for ϵ_{xs} from equation (3), the equilibrium equations for the stringer become

$$N_{xs},_x - q_s(u,_{xx} - c_s w,_{xxx}) + X_s = 0$$

$$N_{xys},_x - q_s\left(\frac{R_{cs}}{R} v,_{xx} - c_s w,_{xxy}\right) + Y_s = 0$$

$$Q_{xs},_x - q_s w,_{xx} + Z_s = 0$$

$$M_{xs,x} - c_s N_{xs,x} + q_s c_s (u,_{xx} - c_s w,_{xxx}) + Q_{xs} + T_{xs} = 0$$

$$M_{xys,x} - b_s N_{xys,x} + q_s c_s \left(\frac{R_{cs}}{R} v,_{xx} - c_s w,_{xxy} \right) + T_{ys} = 0$$

$$M_{zs,x} + N_{xys} = 0 \; . \qquad (9)$$

The stress resultants and couples acting on a ring element are shown in Figure 5a. X_r, Y_r, and Z_r are the forces, and T_{xr} and T_{yr} and the moments applied to the ring element by the shell. The total stress resultant acting on the ring element is

$$\bar{N}_{yr} \doteq N_{yr} + p_r R$$

where

$$p_r = p - p' \; .$$

Figure 5b shows the components caused by rotations of the element and the change in angle between the hoop forces. The net contribution in the x direction becomes

$$\left[\bar{N}_{yr} \frac{R}{R_{cr}} (u,_{yy} - c_r w,_{xyy}) - p_r w,_x \right] dx\, dy$$

and in the z direction

$$-\bar{N}_{yr} \left(\frac{1}{R} v,_y - w,_{yy} \right) dx\, dy \; .$$

By substituting for the hoop strain from equation (4) and using the expression for the total stress resultant, the conditions of equilibrium of the ring element are given by

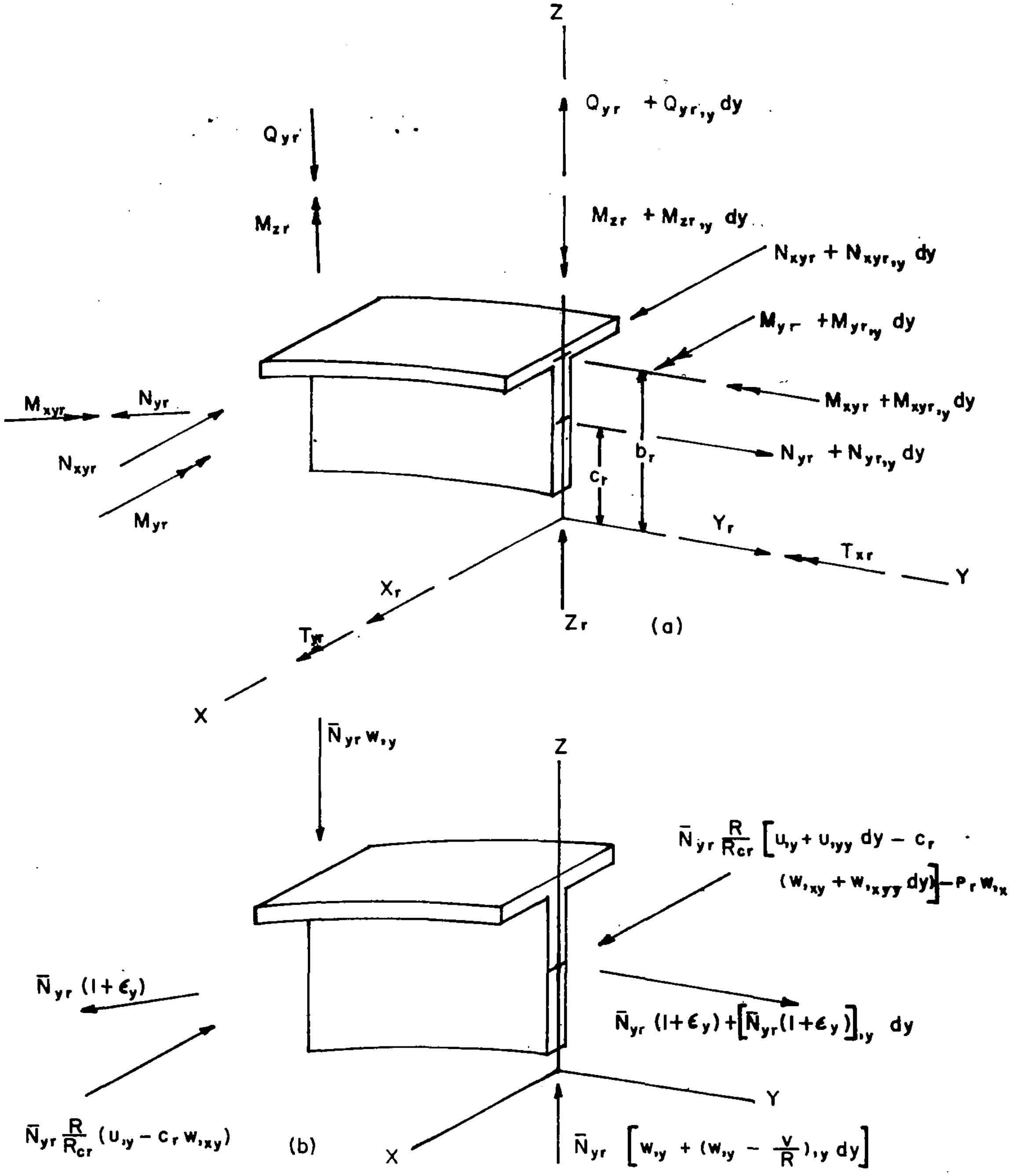


FIGURE 5. FORCES AND MOMENTS ACTING ON THE RING

$$N_{yxr,y} + p_r\left(\frac{R^2}{R_{cr}}u_{,yy} - \frac{c_r R^2}{R_{cr}} w_{,xyy} - w_{,x}\right) + X_r = 0$$

$$N_{yr,y} + \frac{Q_{yr}}{R} + p_r\left(Rv_{,yy} - \frac{c_r R^2}{R_{cr}} w_{,yyy} + \frac{R}{R_{cr}} w_{,y}\right) + Y_r = 0$$

$$Q_{yr,y} - \frac{N_{yr}}{R} + p_r\left[\frac{R}{R_{cr}}\left(R_{cr} + c_r\right) w_{,yy} + u_{,x} - v_{,y} + \frac{c_r}{RR_{cr}} w\right] + Z_r = 0$$

$$\begin{aligned} M_{yxr,y} - b_r N_{yxr,y} + \frac{M_{zr}}{R} - c_r p_r \Bigg(\frac{R^2}{R_{cr}} u_{,yy} - \frac{c_r R^2}{R_{cr}} w_{,xyy} & \\ - w_{,x} \Bigg) + T_{xr} &= 0 \end{aligned}$$

$$\begin{aligned} M_{yr,y} - c_r N_{yr,y} - c_r p_r \left(Rv_{,yy} - \frac{c_r R^2}{R_{cr}} w_{,yyy} + \frac{R}{R_{cr}} w_{,y} \right) & \\ + Q_{yr} + T_{yr} &= 0 \end{aligned}$$

$$\frac{R_{br}}{R} N_{yxr} + M_{zr,y} - \frac{M_{yxr}}{R} = 0. \qquad (10)$$

The forces and moments acting at the interfaces between the shell and the rings and stringer are eliminated by adding equations (8), (9), and (10). The resulting six equations may now be combined to yield the following three equilibrium equations.

$$\begin{aligned} N_{x,x} + N_{xs,x} + N_{yx,y} - \frac{R}{R_{br}} M_{zr,yy} + \frac{1}{R_{br}} M_{yxr,y} - q\left(u_{,xx} - \alpha_q c_s w_{,xxx}\right) & \\ + p\left[R\left(1 - \frac{c_r \alpha_p}{R_{cr}}\right) u_{,yy} - w_{,x} - \frac{c_r R^2 \alpha_p}{R_{cr}} w_{,xyy}\right] &= 0 \end{aligned}$$

$$
\begin{aligned}
&N_{yx,x} - \frac{1}{R}\left(M_{yx,x} + M_{xy,x}\right) - \frac{1}{R} M_{xys,x} - \frac{R_{bs}}{R} M_{zs,xx} + N_{y,y} \\
&\quad + \frac{R_{cr}}{R} N_{yr,y} - \frac{1}{R} M_{y,y} - \frac{1}{R} M_{yr,y} - q\left[\left(1 + \frac{R_{cs}{}^{2} - R^{2}}{R^{2}} \alpha_q\right) v,_{xx}\right. \\
&\quad \left. - \frac{c_s R_{cs}}{R} \alpha_q w,_{xxy}\right] + p\left[R\left(1 + \frac{c_r}{R} \alpha_p\right) v,_{yy} + w,_y - c_r R \alpha_p w,_{yyy}\right] = 0 .
\end{aligned}
$$

$$
\begin{aligned}
&-M_{x,xx} - M_{xs,x} + c_s N_{xs,xx} - M_{y,yy} - M_{yr,yy} + c_r N_{yr,yy} - \frac{1}{R} M_{zr,x} \\
&\quad - \frac{b_r R}{R_{br}} M_{zr,xyy} - M_{xy,xy} - M_{yx,xy} - M_{xys,xy} - \frac{R}{R_{br}} M_{yxr,xy} \\
&\quad - b_s M_{zs,xxy} - \frac{N_y}{R} - \frac{N_{yr}}{R} - q\left(w,_{xx} + c_s \alpha_q u,_{xxx} + \frac{c_s R_{cs}}{R} \alpha_q v,_{xxy}\right. \\
&\quad \left. - c_s{}^{2} \alpha_q w,_{xxxx} - c_s{}^{2} \alpha_q w,_{xxyy}\right) + p\left[u,_x - v,_y + \left(1 + \frac{2c_r}{R_{cr}} \alpha_p\right) R w,_{yy}\right. \\
&\quad + \frac{c_r \alpha_p}{R R_{cr}} w + \frac{c_r R^{2} \alpha_p}{R_{cr}} u,_{xyy} + c_r R \alpha_p v,_{yyy} - c_r \alpha_p w,_{xx} \\
&\quad \left. - \frac{c_r{}^{2} R^{2}}{R_{cr}} \alpha_p w,_{xxyy} - \frac{c_r{}^{2} R^{2}}{R_{cr}} \alpha_p w,_{yyyy}\right] = 0 \qquad (11)
\end{aligned}
$$

where

$$\alpha_p = \frac{p_r}{p}$$

$$\alpha_q = \frac{q_s}{q} \quad .$$

If there is no local buckling of the shell between stringers, p_r and q_s are obtained from equations (17). If q is larger than the local buckling load q_o, α_p and α_q are given by equations (A8) in Appendix A.

The stress resultants and stress couples as defined by equations (5) and (6) may be expressed in terms of the buckling displacements by use of equations (1), (3), and (4), after which substitution of the stress resultants and couples in equation (11) yields

$$
\begin{aligned}
&(\bar{E}_x + \bar{E}_s) u_{,xx} + \left(\bar{G} + \frac{K_r R}{R_{br}^3}\right) u_{,yy} - \frac{D_{zr} R^3}{R_{br}^3} u_{,yyyy} + (\bar{E}_\mu + \bar{G}) v_{,xy} \\
&\quad + \frac{\bar{E}_\mu}{R} w_{,x} + \left(\frac{D_{zr} R}{R_{br}^2} + \frac{K_r R^2}{R_{br}^3}\right) w_{,xyy} - \bar{E}_s c_s w_{,xxx} \\
&\quad + \frac{D_{zr} b_r R^3}{R_{br}^3} w_{,xyyyy} - q(u_{,xx} - \alpha_q c_s w_{,xxx}) \\
&\quad + p\left[R\left(1 - \frac{c_r \alpha_p}{R_{cr}}\right) u_{,yy} - w_{,x} - \frac{c_r R^2 \alpha_p}{R_{cr}} w_{,xyy}\right] \doteq 0 .
\end{aligned}
$$

$$
\begin{aligned}
&(\bar{E}_\mu + \bar{G}) u_{,xy} + \left(\bar{G} + \frac{K + K_s}{R^2}\right) v_{,xx} + \left(\bar{E}_y + \frac{R_{cr}}{R}\bar{E}_r + \frac{D_y}{R^2} + \frac{D_{yr}}{RR_{cr}}\right) v_{,yy} \\
&\quad - \frac{D_{zs} R_{bs}^2}{R^2} v_{,xxxx} + \left(\frac{\bar{E}_y + \bar{E}_r}{R}\right) w_{,y} - \left(\frac{K + K_s + D_\mu}{R}\right) w_{,xxy} \\
&\quad - \left(\frac{D_y}{R} + \frac{D_{yr}}{R_{cr}} + \bar{E}_r c_r\right) w_{,yyy} + \frac{D_{zs} R_{bs} b_s}{R} w_{,xxxxy} \\
&\quad - q\left[\left(1 + \frac{R_{cs}^2 - R^2}{R^2} \alpha_q\right) v_{,xx} - \frac{c_s R_{cs}}{R} \alpha_q w_{,xxy}\right] \\
&\quad + p\left[R\left(1 + \frac{c_r}{R}\alpha_p\right) v_{,yy} + w_{,y} - c_r R \alpha_p w_{,yyy}\right] = 0 .
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{E_{\mu}}{R} u_{,x} + c_s \overline{E}_s u_{,xxx} - \left(\frac{K_r R^2}{R_{br}{}^3} + \frac{D_{zr} R}{R_{br}{}^2} \right) u_{,xyy} - \frac{D_{zr} b_r R^3}{R_{br}{}^3} u_{,xyyyy} \\
&- \left(\frac{\overline{E}_y}{R} + \frac{\overline{E}_r}{R} \right) v_{,y} + \left(\overline{E}_r c_r + \frac{D_y}{R} + \frac{D_{yr}}{R_{cr}} \right) v_{,yyy} \\
&+ \left(\frac{D_{\mu} + K + K_s}{R} \right) v_{,xxy} - \frac{D_{zs} b_s R_{bs}}{R} v_{,xxxxy} - \left(\frac{\overline{E}_y}{R^2} + \frac{\overline{E}_r}{RR_{cr}} \right) w \\
&+ \frac{D_{zr}}{RR_{br}} w_{,xx} + \frac{2\overline{E}_r c_r}{R_{cr}} w_{,yy} - \left(2D_{\mu} + K + K_s + \frac{K_r R^3}{R_{br}{}^3} \right. \\
&\left. - 2D_{zr} \frac{b_r R}{R_{br}{}^2} \right) w_{,xxyy} - (D_x + D_{xs} + c_s{}^2 \overline{E}_s) w_{,xxxx} - \left(D_y + \frac{D_{yr} R}{R_{cr}} \right. \\
&\left. + \frac{\overline{E}_r c_r{}^2 R}{R_{cr}} \right) w_{,yyyy} + D_{zs} b_s{}^2 w_{,xxxxyy} + \frac{D_{zr} b_r{}^2 R^3}{R_{br}{}^3} w_{,xxyyyy} \\
&- q \left(w_{,xx} + c_s \alpha_q u_{,xxx} + \frac{c_s R_{cs} \alpha_q}{R} v_{,xxy} - c_s{}^2 \alpha_q w_{,xxxx} - c_s{}^2 \alpha_q w_{,xxyy} \right) \\
&+ p \left[u_{,x} - v_{,y} + \left(1 + \frac{2c_r \alpha_p}{R_{cr}} \right) Rw_{,yy} + \frac{c_r \alpha_p}{RR_{cr}} w + \frac{c_r R^2 \alpha_p}{R_{cr}} u_{,xyy} \right. \\
&\left. + c_r R \alpha_p v_{,yyy} - c_r \alpha_p w_{,xx} - \frac{c_r{}^2 R^2 \alpha_p}{R_{cr}} w_{,xxyy} - \frac{c_r{}^2 R^2 \alpha_p}{R_{cr}} w_{,yyyy} \right] = 0 \, .
\end{aligned}
\tag{12}
$$

Displacements and Boundary Conditions

The cylinder is in equilibrium under the applied loading just before buckling, and the deformations caused by buckling are measured from this position. A solution to equations (12) is obtained by taking for the buckling displacements the following expressions:

$$
\begin{aligned}
u &= U \cos \bar{m}x \cos \bar{n}y \\
v &= V \sin \bar{m}x \sin \bar{n}y \qquad (13) \\
w &= W \sin \bar{m}x \cos \bar{n}y \quad .
\end{aligned}
$$

This corresponds to the following simply supported boundary conditions at $x = 0, L$.

$$
\begin{aligned}
w &= 0 \qquad N_x = 0 \qquad N_{xs} = 0 \\
v &= 0 \qquad M_x = 0 \qquad M_{xs} = 0 \quad .
\end{aligned}
$$

Thus, at the ends of the cylinder, motion radially and tangentially is prevented, while longitudinal motion is allowed; i.e., $u \neq 0$.

Introducing the expressions for the displacements (13) into the differential equations (12) gives the following three linear equations in matrix form:

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix} + q \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = 0 \qquad (14)
$$

where the A matrix contains all the stiffness terms, the P matrix contains all the pressure terms, and the Q matrix contains all the axial load multipliers. The coefficients of the above matrices are as follows:

$$
a_{11} = -(\bar{E}_x + \bar{E}_s)\bar{m}^2 - \left(\bar{G} + \frac{K_r R}{R_{br}^{\;3}}\right)\bar{n}^2 - \frac{D_{zr} R^3}{R_{br}^{\;3}}\bar{n}^4
$$

$$
a_{12} = a_{21} = (\bar{E}_\mu + \bar{G})\,\bar{m}\,\bar{n}
$$

$$
a_{13} = a_{31} = \frac{\bar{E}_\mu}{R}\bar{m} + \bar{E}_s c_s \bar{m}^3 - \left(D_{zr} + \frac{K_r R}{R_{br}}\right)\frac{R}{R_{br}^{\;2}}\bar{m}\,\bar{n}^2 + \frac{D_{zr} b_r R^3}{R_{br}^{\;3}}\bar{m}\,\bar{n}^4
$$

$$
\begin{aligned}
a_{22} = {} & -\left(\overline{G} + \frac{K + K_s}{R^2}\right)\overline{m}^2 - \left(\overline{E}_y + \frac{R_{cr}}{R}\overline{E}_r + \frac{D_y}{R^2} + \frac{D_{yr}}{RR_{cr}}\right)\overline{n}^2 \\
& - \frac{D_{zs}R_{bs}{}^2\,\overline{m}^4}{R^2} \quad .
\end{aligned}
$$

$$
\begin{aligned}
a_{23} = a_{32} = {} & -\left(\frac{\overline{E}_y + \overline{E}_r}{R}\right)\overline{n} - \left(\frac{K + K_s + D_\mu}{R}\right)\overline{m}^2\overline{n} \\
& - \left(\frac{D_y}{R} + \overline{E}_r c_r + \frac{D_{yr}}{R_{cr}}\right)\overline{n}^3 - \frac{D_{zs}R_{bs}b_s}{R}\,\overline{m}^4\overline{n}
\end{aligned}
$$

$$
\begin{aligned}
a_{33} = {} & -\left(\frac{\overline{E}_y}{R^2} + \frac{\overline{E}_{yr}}{RR_{cr}}\right) - \frac{D_{zr}}{RR_{br}}\overline{m}^2 - \frac{2\overline{E}_r c_r}{R_{cr}}\overline{n}^2 - \Bigg(2D_\mu + K + K_s \\
& + \frac{K_r R^3}{R_{br}{}^3} - \frac{2D_{zr}b_r R}{R_{br}{}^2}\Bigg)\overline{m}^2\overline{n}^2 - (D_x + D_{xs} + c_s{}^2\overline{E}_s)\,\overline{m}^4 - \frac{D_{zr}b_r{}^2R^3}{R_{br}{}^3}\overline{m}^2\overline{n}^4 \\
& - \left(D_y + \frac{D_{yr}R}{R_{cr}} + \frac{\overline{E}_r c_r{}^2 R}{R_{cr}}\right)\overline{n}^4 - D_{zs}b_s{}^2\overline{m}^4\overline{n}^2
\end{aligned}
$$

$$
p_{11} = -\left(1 - \frac{c_r\alpha p}{R_{cr}}\right)R\,\overline{n}^2 p
$$

$$
p_{12} = p_{21} = 0
$$

$$
p_{13} = p_{31} = -\left(1 - \frac{c_r R^2\alpha p}{R_{cr}}\overline{n}^2\right)\overline{m}\,p
$$

$$
p_{22} = -\left(1 + \frac{c_r\alpha p}{R}\right)R\,\overline{n}^2 p
$$

$$p_{23} = p_{32} = -\left(1 + c_r R \alpha_p \bar{n}^2\right) \bar{n} p$$

$$p_{33} = \left[\frac{c_r \alpha_p}{R R_{cr}} + c_r \alpha_p \bar{m}^2 - \left(1 + \frac{2 c_r \alpha_p}{R_{cr}}\right) R \bar{n}^2 - \frac{c_r^{\,2} R^2 \alpha_p}{R_{cr}} \left(\bar{m}^2 + \bar{n}^2\right) \bar{n}^2\right] p$$

$$q_{11} = \bar{m}^2$$

$$q_{12} = q_{21} = 0$$

$$q_{13} = q_{31} = -\alpha_q c_s \bar{m}^3$$

$$q_{22} = \left(1 + \frac{R_{cs}^{\,2} - R^2}{R^2} \alpha_q\right) \bar{m}^2$$

$$q_{23} = q_{32} = \frac{c_s R_{cs}}{R} \alpha_q \bar{m}^2 \bar{n}$$

$$q_{33} = \left[1 + c_s^{\,2} \alpha_q \left(\bar{m}^2 + \bar{n}^2\right)\right] \bar{m}^2 \quad .$$

Determination of Buckling Load

The set of homogeneous equations (14) has nontrivial solutions only when the determinant of its matrix is zero, or

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} = 0$$

where the coefficients of [D] are given by

$$d_{ij} = a_{ij} + p_{ij} + q \cdot q_{ij} \quad .$$

Assuming the pressure to be known, the determinant $|D|$ is a polynomial of third degree in q. Calculating the buckling load for known values of m and n is now reduced to finding the lowest root (eigenvalue) of the characteristic equation

$$q^3 + a\,q^2 + b\,q + c = 0 \tag{15}$$

where a, b, and c are known. The critical buckling load of the cylinder may be found by calculating the lowest root of equation (15) for a wide range of values of m and n, and plotting a family of curves as shown in Figure 6. The critical buckling load will then be the minimum value of q corresponding to integer values of m and n. In the computer program, this minimum value will be indicated for the specified range of m and n.

In the determination of the critical buckling load, as described above, it was tacitly assumed that the quantities α_p and α_q were known so that the coefficients of the P and Q matrices could be determined. If it is assumed that the external loads are distributed uniformly between shell and stiffeners, one has

$$\alpha_p = \frac{p_r}{p} = \frac{\bar{E}_r}{\bar{E}_r + Et}$$

$$\alpha_q = \frac{q_s}{q} = \frac{\bar{E}_s}{\bar{E}_s + Et} \quad . \tag{16}$$

The correct values of α_p and α_q, however, are load dependent and must be calculated from prebuckling stress-strain relations. Using equations (B1) and (B7) and the definition for A given in equation (A2), one may write

$$p_r = \frac{V}{L_r} = \left[p + \frac{\mu}{R}(q - q_s) \right] A$$

and the longitudinal prebuckling strain

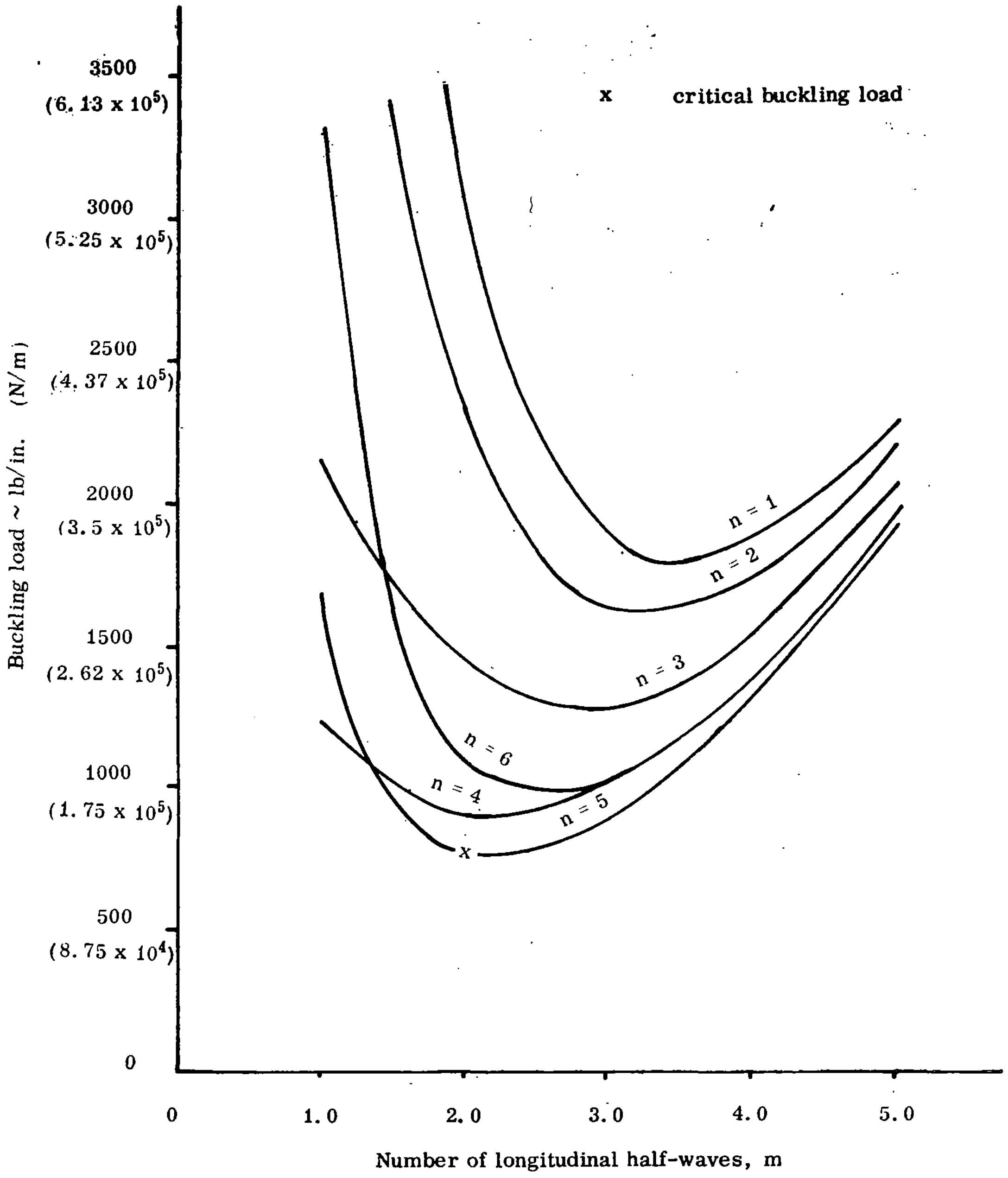


FIGURE 6. BUCKLING LOAD VERSUS MODE SHAPE

$$\bar{\epsilon}_x = -\frac{q_s}{\bar{E}_s} = -\frac{q - q_s}{Et} - \frac{\mu R}{Et}(p - p_r)$$

from which

$$q_s = \frac{[q + \mu R(p - p_r)]\,\bar{E}_s}{Et + \bar{E}_s} .$$

After some manipulation of the above equations, one obtains

$$q_s = \frac{[q(1 - \mu^2 A) + \mu p R(1 - A)]\,\bar{E}_s}{Et + \bar{E}_s(1 - \mu^2 A)}$$

$$p_r = \frac{\left\{p[Et + \bar{E}_s(1 - \mu^2)] + \mu q \frac{Et}{R}\right\} A}{Et + \bar{E}_s(1 - \mu^2 A)} . \qquad (17)$$

Since α_p and α_q appear only in the eccentricity terms of the matrix coefficients, the use of equations (16) for the determination of the critical mode shape should be satisfactory. After the minimum load has been found, however, equations (17) are used to calculate new values of α_p and α_q corresponding to this load. A corrected value for the critical buckling load is now obtained by repeating some of the calculations for the critical mode shape.

COMPARISON WITH TEST RESULTS

The method presented in this report has been compared with three groups of cylinders. Group A contains twenty-three ring-stiffened cylinders, group B contains six ring-stiffened corrugated cylinders, and group C contains six ring-and-stringer stiffened cylinders. Groups A and B were loaded in compression, and group C was loaded in bending.

The testing procedure and test results for the group A cylinders are given in References 17 and 18. The predicted failure load for these cylinders has been calculated using the computer program in Appendix C. Figure 7 shows the cylinder geometry. Table I has the cylinder dimensions, the predicted wave shape, and a comparison of the predicted and actual failure loads. Many of these cylinders had their minimum predicted load for the circumferential mode shape n = 0, which is an axisymmetric buckling mode.

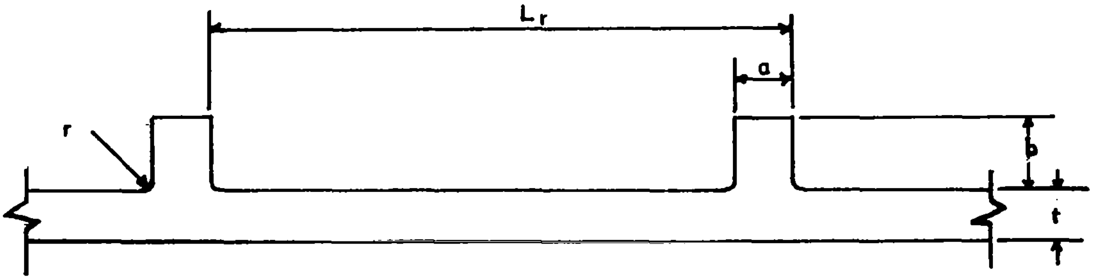


FIGURE 7. LONGITUDINAL CROSS SECTION OF CYLINDER WALL

The agreement between the predicted failure load and the actual failure load is good, particularly for such lightly stiffened cylinders. The problems inherent in lightly stiffened cylinders are further discussed on page 32.

The test results for the group B cylinders provide a comparison with larger, more heavily stiffened cylinders and show the marked effect of stiffener eccentricity. These ring-stiffened corrugated cylinders were tested as part of the Saturn V development program. Figure 8 shows the corrugation cross section. Table II gives the cylinder properties, the predicted wave shape, and a comparison of the predicted and actual failure loads. The actual longitudinal and circumferential buckle wave shapes for these cylinders were evident before the general instability failure, and, in most of the tests, the actual and predicted wave shapes were in agreement (see Reference 11 for a further discussion). The agreement between the predicted and actual failure loads for the group B cylinders is quite good. All of the predicted failure loads agree with the actual failure loads within ±14 percent. One of the specimens, cylinder number 6, is very likely the largest cylinder tested anywhere which has failed in general instability.

TABLE I. RING-STIFFENED CYLINDERS, AXIAL LOAD

Reference Number	Cylinder Number	Cylinder Material	Cylinder Length in. (cm)	Radius in. (cm)	Skin Thickness in. (cm)	Ring Spacing, L_r in. (cm)	Ring Width, a in. (cm)	Ring Height, b in. (cm)	Ring Fillet Radius r in. (cm)	Predicted			Actual Failure Stress ksi (N/cm²)	Percent Error[b]
										Longitudinal Half Wave (m)	Circumferential Full Wave (n)	Failure Stress ksi (N/cm²)		
17	3	2024-T3	2.13 (5.41)	3.81 (9.68)	0.0195 (0.050)	0.0714 (0.181)	0.0345 (0.088)	0.01640 (0.042)	—	5	0	39.28 (27 100)	38.89 (26 100)	1.00
17	4	2014-T6	15.50 (39.4)	12.23 (31.06)	0.0346 (0.088)	0.1334 (0.339)	0.0648 (0.165)	0.03030 (0.077)	—	15	0	21.85 (14 600)	21.66 (14 500)	0.88
17	5	2024-T3	5.76 (14.6)	3.73 (9.47)	0.0106 (0.027)	0.0417 (0.106)	0.0206 (0.052)	0.00948 (0.024)	—	18	0	22.08 (14 800)	21.83 (14 600)	1.15
17	6	↓	1.82 (4.62)	3.73 (9.47)	0.0104 (0.026)	0.0413 (0.105)	0.0192 (0.049)	0.00958 (0.024)	—	6	0	21.54 (14 400)	21.41 (14 300)	0.61
17	7	↓	3.10 (7.37)	3.75 (8.52)	0.0123 (0.031)	0.1224 (0.311)	0.0282 (0.072)	0.02520 (0.064)	—	9	0	25.76 (17 300)	25.49 (17 100)	1.06
17	8	↓	1.62 (4.11)	3.75 (9.52)	0.0122 (0.031)	0.1247 (0.317)	0.0283 (0.072)	0.02530 (0.064)	—	5	0	25.61 (17 200)	25.31 (17 000)	1.19
17	13	↓	5.78 (14.68)	3.74 (9.50)	0.0125 (0.032)	0.2100 (0.533)	0.0302 (0.077)	0.04590 (0.116)	0.0100 (0.025)	17	0	26.85 (17 990)	26.60 (17 830)	0.94
17	14	↓	1.78 (4.52)	3.74 (9.50)	0.0123 (0.031)	0.2112 (0.0536)	0.0300 (0.076)	0.04570 (0.116)	0.0143 (0.036)	5	0	26.79 (17 950)	26.51 (17 760)	1.06
18	5	6061-T6	2.00 (5.08)	3.80 (9.65)	0.0102 (0.026)	0.0111 (0.028)	0.0510 (0.130)	0.00770 (0.020)	—	6	0	19.52 (13 100)	15.40 (10 300)	26.75
18	6	↓	2.00 (5.08)	3.80 (9.65)	0.0048 (0.012)	0.0111 (0.028)	0.0510 (0.130)	0.01000 (0.025)	—	10	0	11.02 (7381)	8.76 (5900)	25.80
18	7	↓	3.00 (7.62)	3.80 (9.65)	0.0097 (0.025)	0.0111 (0.028)	0.0510 (0.130)	0.00780 (0.020)	—	10	0	18.66 (12 500)	15.70 (10 500)	18.85
18	8	↓	3.00 (7.62)	3.80 (9.65)	0.0055 (0.014)	0.0111 (0.028)	0.0510 (0.130)	0.01040 (0.026)	—	14	0	12.33 (8260)	12.40 (8310)	-0.57
18	9	↓	1.77 (4.50)	3.80 (9.65)	0.0121 (0.031)	0.0111 (0.028)	0.0510 (0.130)	0.00460 (0.012)	—	5	0	21.53 (14 400)	18.00 (12 100)	19.61
18	10	↓	3.93 (9.98)	3.80 (9.65)	0.0118 (0.030)	0.0111 (0.028)	0.0510 (0.130)	0.00480 (0.012)	—	11	0	22.08 (14 800)	19.60 (13 100)	12.65
18	11	↓	2.99 (7.59)	3.80 (9.65)	0.0120 (0.030)	0.0111 (0.028)	0.0510 (0.130)	0.00400 (0.010)	—	8	4	21.21 (14 200)	18.00 (12 100)	17.83
18	12	↓	2.10 (5.33)	3.80 (9.65)	0.0053 (0.013)	0.0111 (0.028)	0.0510 (0.130)	0.01060 (0.027)	—	10	0	12.04 (8070)	11.00 (7370)	9.45
18	17	↓	2.40 (6.10)	3.80 (9.65)	0.0049 (0.012)	0.0111 (0.028)	0.0510 (0.130)	0.01030 (0.026)	—	12	0	11.31 (7580)	12.00 (8040)	-5.75
18	18	↓	3.96 (10.06)	3.80 (9.65)	0.0052 (0.028)	0.0111 (0.028)	0.0510 (0.130)	0.01020 (0.026)	—	19	0	11.88 (7960)	10.40 (6970)	14.23
18	19	↓	3.95 (10.06)	3.80 (9.65)	0.0100 (0.025)	0.0111 (0.028)	0.0510 (0.130)	0.00870 (0.022)	—	13	0	19.54 (13 100)	19.70 (13 200)	-0.81
18	20	↓	2.95 (7.49)	3.80 (9.65)	0.0094 (0.024)	0.0111 (0.028)	0.0510 (0.130)	0.00880 (0.022)	—	10	0	18.56 (12 400)	19.90 (13 300)	-6.73
18	32[a]	↓	2.98 (7.57)	3.80 (9.65)	0.0099 (0.025)	0.0111 (0.028)	0.0510 (0.130)	0.00820 (0.021)	—	7	14	16.85 (11 300)	20.10 (13 500)	-17.17
18	33[a]	↓	1.96 (4.98)	3.80 (9.65)	0.0057 (0.014)	0.0111 (0.028)	0.0510 (0.130)	0.01010 (0.026)	—	7	14	11.61 (7780)	10.90 (7300)	6.51
18	34[a]	↓	3.52 (8.94)	3.80 (9.65)	0.0116 (0.029)	0.0111 (0.028)	0.0510 (0.130)	0.00460 (0.012)	—	7	15	19.10 (12 800)	19.90 (13 300)	-4.12

a. These cylinders have internal rings.

b. Percent Error = $100 \left[\frac{\text{predicted stress}}{\text{actual stress}} - 1\right]$

TABLE II. RING-STIFFENED CORRUGATED CYLINDERS, AXIAL LOAD

Cylinder	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
Aluminum Alloy	7075-T6	7075-T6	7075-T6	7075-T6	7075-T6	7075-T6
Cylinder Length (in.)	33.000	33.000	33.000	69.600	69.600	268.600
(cm)	83.800	83.800	83.800	176.800	176.800	682.200
Radius (in.)	24.700	24.700	24.700	49.400	49.400	197.700
(cm)	62.700	62.700	62.700	125.000	125.000	502.200
Corrugation Pitch (L_s), (in.)	1.430	1.430	1.430	2.850	2.850	11.400
(cm)	3.630	3.630	3.630	7.240	7.240	28.950
Corrugation Thickness (t), (in.)	0.020	0.019	0.025	0.041	0.041	0.185
(cm)	0.050	0.048	0.060	0.100	0.100	0.470
Corrugation Depth (d), (in.)	0.440	0.440	0.440	0.870	0.870	3.480
(cm)	1.120	1.120	1.120	2.210	2.210	8.840
Shape of Ring	[	I	I	I	I	I
Ring Spacing (in.)	6.3800	6.3800	6.3800	12.400	12.400	49.500
(cm)	16.2100	16.2100	16.2100	31.500	31.500	125.700
Ring Moment of Inertia ($in.^4$)	0.0050	0.0104	0.0104	0.286	0.286	141.500
(cm^4)	0.2080	0.4330	0.4330	11.900	11.900	5889.000
Ring Area ($in.^2$)	0.0400	0.1210	0.1210	0.180	0.180	3.950
(cm^2)	0.2580	0.7800	0.7800	1.160	1.160	10.030
Ring Eccentricity (in.)	-0.7300	-0.5300	-0.5300	-1.990	-1.990	-8.740
(cm)	-1.8500	-1.3500	-1.3500	-5.050	-5.050	-22.200
Predicted Longitudinal Half Waves (m)	2	3	3	3	3	3
Predicted Circumferential Full Waves (n)	5	5	5	4	4	4
Actual Failure Load (Kips)	131.0	174.0	224.0	659.0	648.0	14 119.0
(N)	5.83×10^5	7.74×10^5	9.96×10^5	2.93×10^6	2.88×10^6	62.90×10^6
Predicted Failure Load (Kips)	119.0	198.0	233.0	659.0	659.0	13 580.0
(N)	5.29×10^5	8.81×10^5	1.04×10^6	2.93×10^6	2.93×10^6	6.05×10^7
Percent Error (%)	-9.2	13.8	4.0	0.0	1.7	-3.80

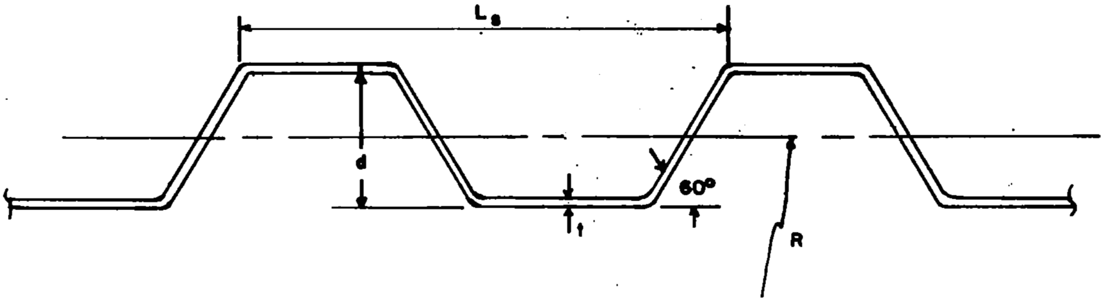


FIGURE 8. CORRUGATION CROSS SECTION

A comparison of Table II with Table I shows that the agreement is better for the group B cylinders than for the group A cylinders. This improvement is probably because the group B cylinders are more heavily stiffened.

Table III has the calculated failure load for these six cylinders with three different ring locations to show the effect of ring eccentricity. The cases in columns A and C of Table III have the same amount of eccentricity; only the direction of eccentricity is different. The cases in column B have no ring eccentricity. As Table III shows, the effect of ring eccentricity is appreciable, and if it had been ignored in calculating the failure load for the cylinders tested, the calculations would have been very unconservative.

TABLE III. EFFECT OF RING ECCENTRICITY

Corrugated Cylinder No.	Predicted Failure Load (KIPS)		
	Col. A Rings Inside	Col. B Rings at Corrugation ℄	Col. C Rings Outside
1	119	254	295
2	198	351	492
3	233	417	563
4	659	1226	1254
6	13 580	23 840	24 250

The testing procedure and test results for the group C cylinders, which were loaded in bending, are given in Reference 12. These cylinders had local skin buckling before the overall general instability failure of the cylinder. The cylinder stiffening elements are shown in Figure 9. The predicted failure load for these cylinders was obtained by equating the average load per inch around the circumference of the cylinder caused by an axial load to the maximum load per inch caused by a bending moment.

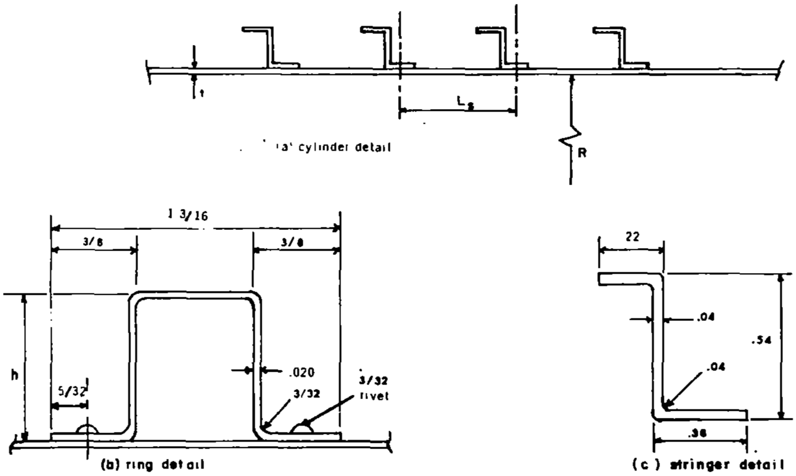


FIGURE 9. GROUP C CYLINDER STIFFENING ELEMENTS

Table IV has the cylinder properties, the predicted general instability wave shape, the computed failure load without the skin stiffness being reduced by local buckling, the predicted failure load, and the actual failure load. In general, the computed failure load was reduced 35 to 45 percent when the reductions in the skin stiffnesses caused by local buckling were considered. For the group C cylinders, the predicted failure loads agree with the actual failure loads within ±14 percent.

Some of these cylinders had a small number of longitudinal half waves. This often indicates that the cylinder end conditions should be carefully considered. A closer examination of these cylinders shows that the predicted failure load did not change greatly as the number of longitudinal waves was increased. Thus, it appears that the cylinder end condition did not appreciably affect the failure load.

TABLE IV. RING AND STRINGER STIFFENED CYLINDERS, BENDING LOAD

Type	Cylinder[d]	Cylinder Material	L_r in. (cm)	L_s in. (cm)	h in. (cm)	t in. (cm)	A_s in.^{2} (cm^2)	Computed[a] Failure Load lb/in. (N/m)	Predicted[b]			Actual[c] Failure Load lb/in. (N/m)	Percent Error
									Longitudinal Half Waves (m)	Circumferential Full Waves (n)	Failure Load lb/in. (N/m)		
I	1	7075-T6	6 (15.24)	2.48 (6.299)	0.54 (1.372)	0.0199 (0.0505)	0.0380 (0.2451)	1860.0 (325 550.)	3	6	1192.0 (208 600.)	1136.0 (198 800.)	+ 4.9
	2	7075-T6	9 (22.86)	2.48 (6.299)	0.54 (1.372)	0.0199 (0.0505)	0.0379 (0.2574)	1746.0 (305 550.)	3	7	1127.0 (197 200.)	1000.0 (175 000.)	+12.7
	3	7075-T6	12 (30.48)	2.48 (6.299)	0.54 (1.372)	0.0199 (0.0505)	0.0381 (0.2457)	1642.0 (287 350.)	3	7	1083.0 (189 500.)	948.0 (165 900.)	+14.2
II	1	7075-T6	6 (15.24)	4.04 (10.26)	0.30 (0.762)	0.0197 (0.050)	0.0396 (0.2516)	1184.0 (207 200.)	1	5	665.0 (116 400.)	726.0 (127 050.)	- 8.4
	2	7075-T6	9 (22.86)	4.04 (10.26)	0.30 (0.762)	0.0197 (0.050)	0.0381 (0.2457)	1019.0 (178 330.)	2	7	605.0 (105 900.)	652.0 (114 100.)	- 7.2
	3	7075-T6	12 (30.48)	4.04 (10.26)	0.30 (0.762)	0.0197 (0.050)	0.0389 (0.2509)	928.0 (162 400.)	1	6	553.0 (96 780.)	615.0 (107 600.)	-10.1

a. Without local skin buckling

b. With local skin buckling

c. Maximum q (lb/in.) caused by the applied bending moment

d. For all cylinders: radius to skin midplane, R, 38.6 in.; test section lengths, L, 72. in.

DISCUSSION

The method of analysis given in this report shows that stiffener eccentricity has a marked effect on the general instability buckling load of a cylinder. As shown in Table III, moving the rings from the inside to the outside of a corrugated cylinder increased the general instability buckling load 90 to 150 percent. Certainly this eccentricity effect must be included in any general instability calculation. The computer program given in Appendix C makes this inclusion relatively simple for the stress analyst.

When a cylinder has local skin buckling between the stringers before the general instability failure, the skin stiffnesses that are used in the general instability calculations must be reduced. The procedure used to reduce these stiffnesses is developed in Appendix A. To calculate the reduced skin stiffnesses the average hoop stress in the skin must be known. The procedure used to calculate the average hoop stress resultant is developed in Appendix B. The general instability load can be reduced significantly by local skin buckling. For the cylinders listed in Table IV, the load reduction varied from 30 to 45 percent.

The method given here is based on the assumption that the average number of rings (δ) in each longitudinal half wave is sufficient so that the rings can be considered to be uniformly distributed along the cylinder. Van der Neut [19] performed a study to determine what error was produced by using a "smeared" ring approach when δ was low. He states that for stiffened cylinders, the error is about 4 percent for $\delta = 2.0$ and 6 percent for $\delta = 1.6$, the exact error being dependent upon the stiffness properties. The test data examined here support this conclusion. Cylinders 2 and 3 in Table II have a δ of 1.7 and their percent errors are not out of line with the remainder of the data, which have higher δ's.

A second assumption used in the derivation is the application of small deflection theory. This theory is adequate for moderately stiffened and heavily stiffened cylinders, as the test results show. It may be used for lightly stiffened cylinders, so long as the cylinder imperfections do not appreciably affect the failure load. Unfortunately, at present there is no well tested method for determining when imperfections must be considered in lightly stiffened cylinders. Almroth [20] has proposed a method for analyzing lightly stiffened cylinders. This method uses a reduction factor, which is based on the cylinder stiffnesses, to determine the buckling load.

The method given in this report was developed using the approach proposed by Flügge [16] for handling the coupling between the in-plane extensions in the shell and the applied loads. Because of this, the method given here is valid both for cylinders which buckle in the axisymmetric mode ($n = 0$) and for cylinders which buckle as a column ($m = 1$, $n = 1$). Methods based on the Donnell assumptions are not this flexible. In general, using the Flügge [16] technique gives a lower and more accurate buckling load than that obtained using the Donnell assumptions when the number of circumferential waves (n) is low (0, 1, 2, or 3).

The computer program given in Appendix C has been written as generally as possible. It can be used to examine cylinders both for general instability and panel instability. The instructions for operating the program are given in Appendix D.

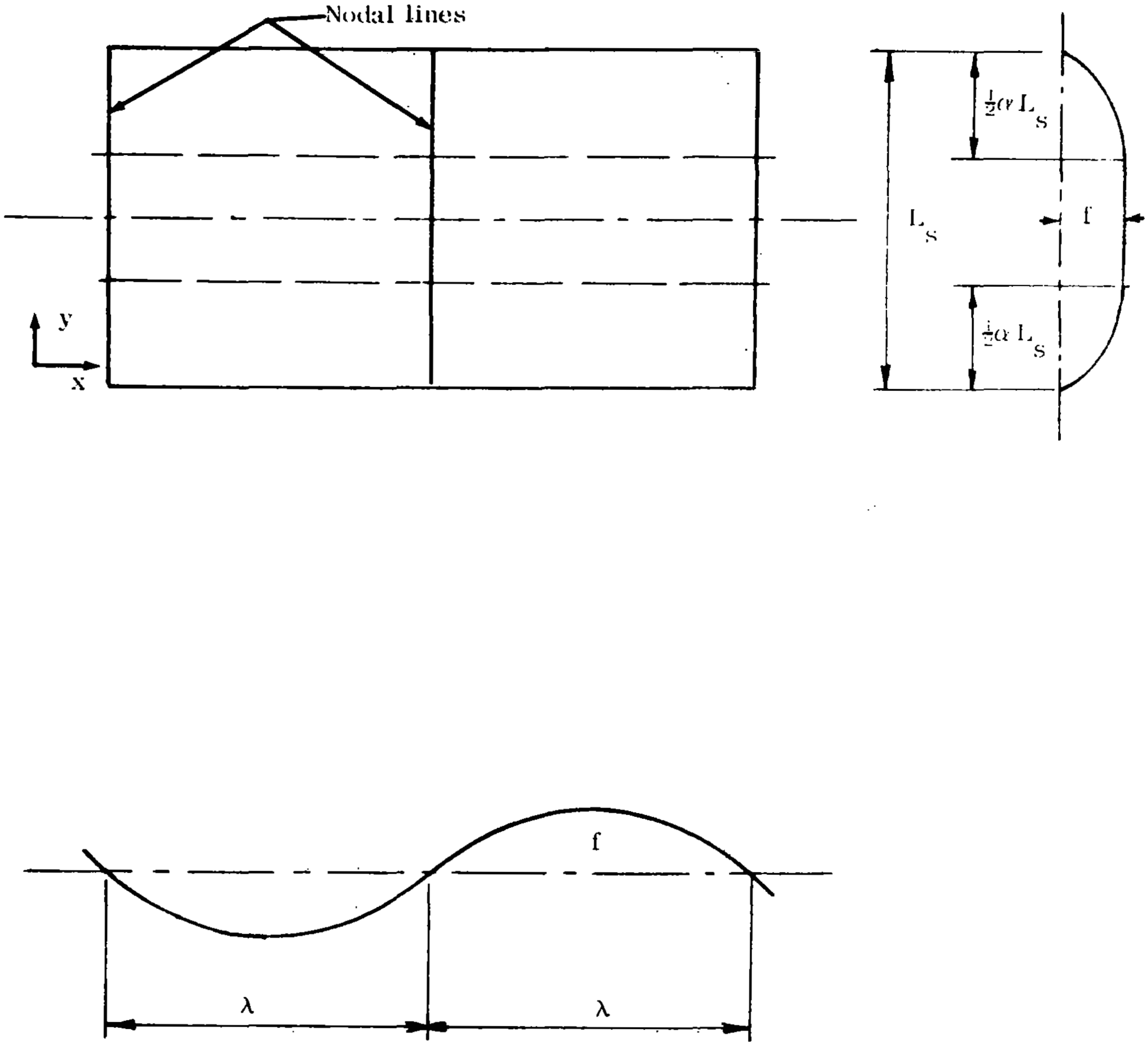
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama, June 28, 1968
933-31-01-00-62

APPENDIX A. LOCAL BUCKLING OF SHELL

In this report local buckling will be defined as buckling of the shell between two adjacent stringers caused by a combination of axial compression and lateral tension or compression. Lateral compression, if present, is assumed to contribute to, but not to be the primary cause of, the local buckling. When local buckling occurs, the longitudinal extensional stiffness of the shell is drastically reduced, and as a result, the shell will no longer carry its full share of the axial load. The lateral stiffness, the cross stiffness (Poisson's effect), and the shear stiffness are also affected. Since all the parameters above enter into the general instability analysis, the complete stiffness matrix of the buckled shell with respect to incremental deformations must be known.

An analysis to determine the elements of such a stiffness matrix was performed by Van der Neut [21] for rectangular, simply supported, flat plates. In the analysis it was assumed that the panel was sufficiently long so that the only geometric properties affecting local buckling were the panel width and thickness. Van der Neut presented his results in graphical form, giving average stresses and stiffness reduction factors in terms of the normalized strains. His graphs, however, do not cover the range of strain to critical strain ratios required to check the available experimental results. In addition, a small error was discovered in one of the equations of Reference 21. This small error was apparently introduced when the manuscript was written and retained in the programming of the numerical computations. It was, therefore, decided to generate a new set of numerical data using the procedure suggested by Van der Neut and to incorporate it into the computer program as semi-permanent data. A brief description of this procedure is given in the next paragraph.

Van der Neut established his data on the basis of Koiter's shear field theory [22] using the first of several wave forms considered by Koiter. As shown in Figure A1, this wave form is sinusoidal in the longitudinal direction. To account for large strain to critical strain ratios, the amplitude of the sine wave is held constant for part of the panel width around the center of the panel and then decreases to zero at the edges; hence only the edge strips are double curved. The potential energy of the buckled panel is determined in accordance with the assumed deflection pattern and minimized with respect to the four parameters, f, λ, m, and α. This yields four simultaneous equations from which these parameters may be determined in terms of the strain components. Expressions may now be derived for the average panel stresses and differentiated with respect to the strains to obtain the reduced moduli.



$0 < y < \frac{1}{2}\alpha L_s \qquad z = f \sin\frac{\pi y}{\alpha L_s} \sin\frac{\pi x}{\lambda}$

$\frac{1}{2}\alpha L_s < y < (1 - \frac{1}{2}\alpha) L_s \qquad z = f \sin\frac{\pi x}{\lambda}$

FIGURE A1. LOCAL BUCKLING PATTERN

By defining the normalized stresses and strains

$$e_x = \frac{\epsilon_x}{\epsilon^*} \qquad e_y = \frac{\epsilon_y}{\epsilon^*} \qquad e_{xy} = \frac{\gamma_{xy}}{\epsilon^*}$$

$$s_x = \frac{\sigma_x}{E\epsilon^*} \qquad s_y = \frac{\sigma_y}{E\epsilon^*} \qquad s_{xy} = \frac{\sigma_{xy}}{E\epsilon^*}$$

where

$$\epsilon^* = \frac{\pi}{3(1 - \mu^2)} \left(\frac{t}{L_s}\right)^2$$

is the critical compressive longitudinal strain, the reduced moduli may be expressed by the following partial differentials:

$$\beta_x = \frac{\partial s_x}{\partial e_x} \qquad \beta_y = \frac{\partial s_y}{\partial e_y} \qquad \beta_s = \frac{E}{G} \frac{\partial s_{xy}}{\partial \gamma_{xy}}$$

$$\beta_\mu = \frac{\partial s_x}{\partial e_y} = \frac{\partial s_y}{\partial e_x} \quad .$$

The average normalized stresses, s_x and s_y, are plotted in Figures A2 and A3; and the reduced moduli β_x, β_μ, β_y, and β_s are plotted in Figures A4 through A7, respectively, as a function of the normalized strains, e_x and e_y.

The values of s_x and s_y that define the point of initial buckling are given by:

$$s_{xo} = -\frac{1}{2}(D + 1)$$

$$s_{yo} = -\frac{1}{4}(1 - D^2) \tag{A1}$$

where $D = (L_s/\lambda)^2$. With the use of equations (B1) and (B7) the average hoop stress resultant in the shell may be written in the form:

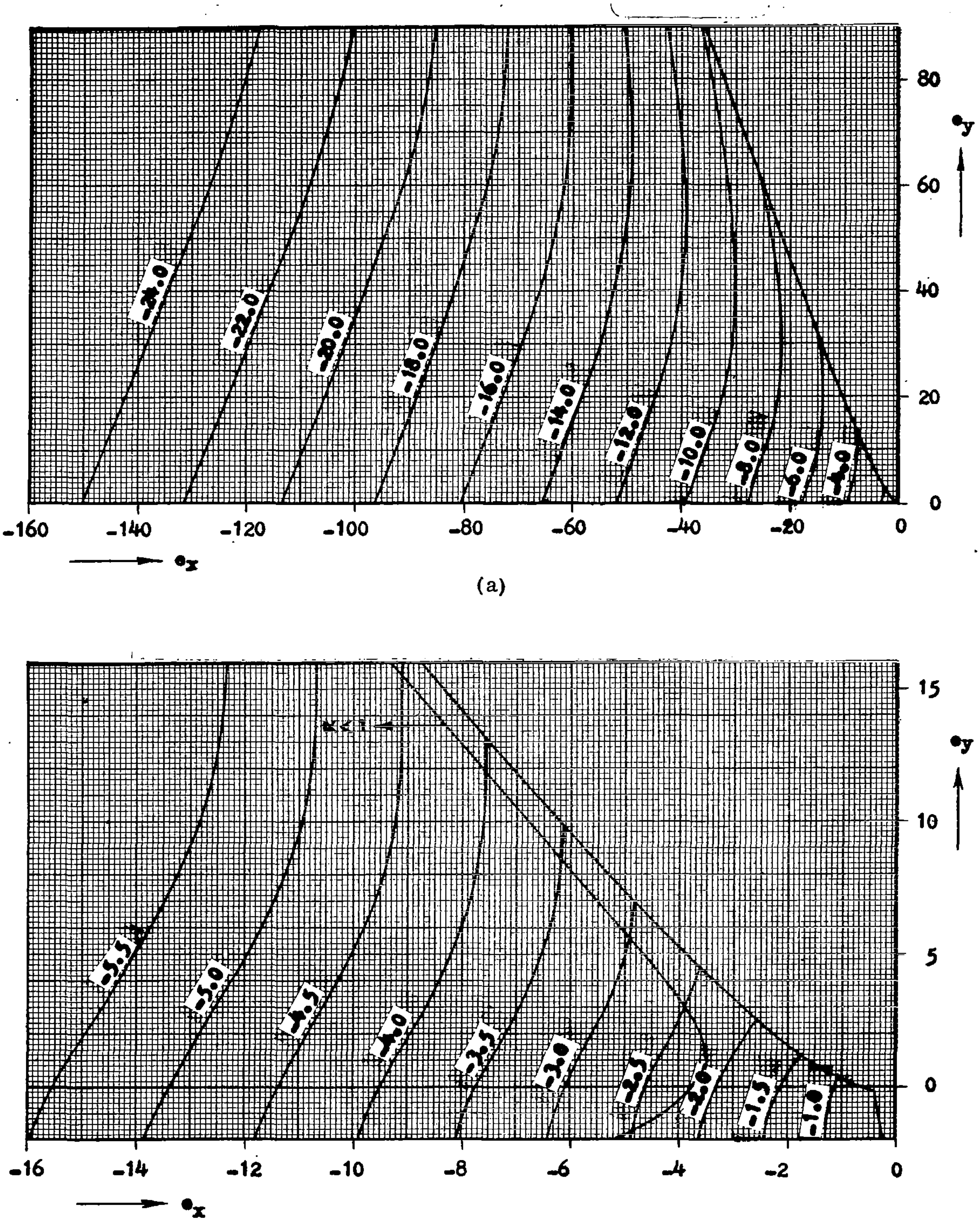


FIGURE A2. s_x VERSUS e_x, e_y FOR LARGE STRAINS (a) AND SMALL STRAINS (b)

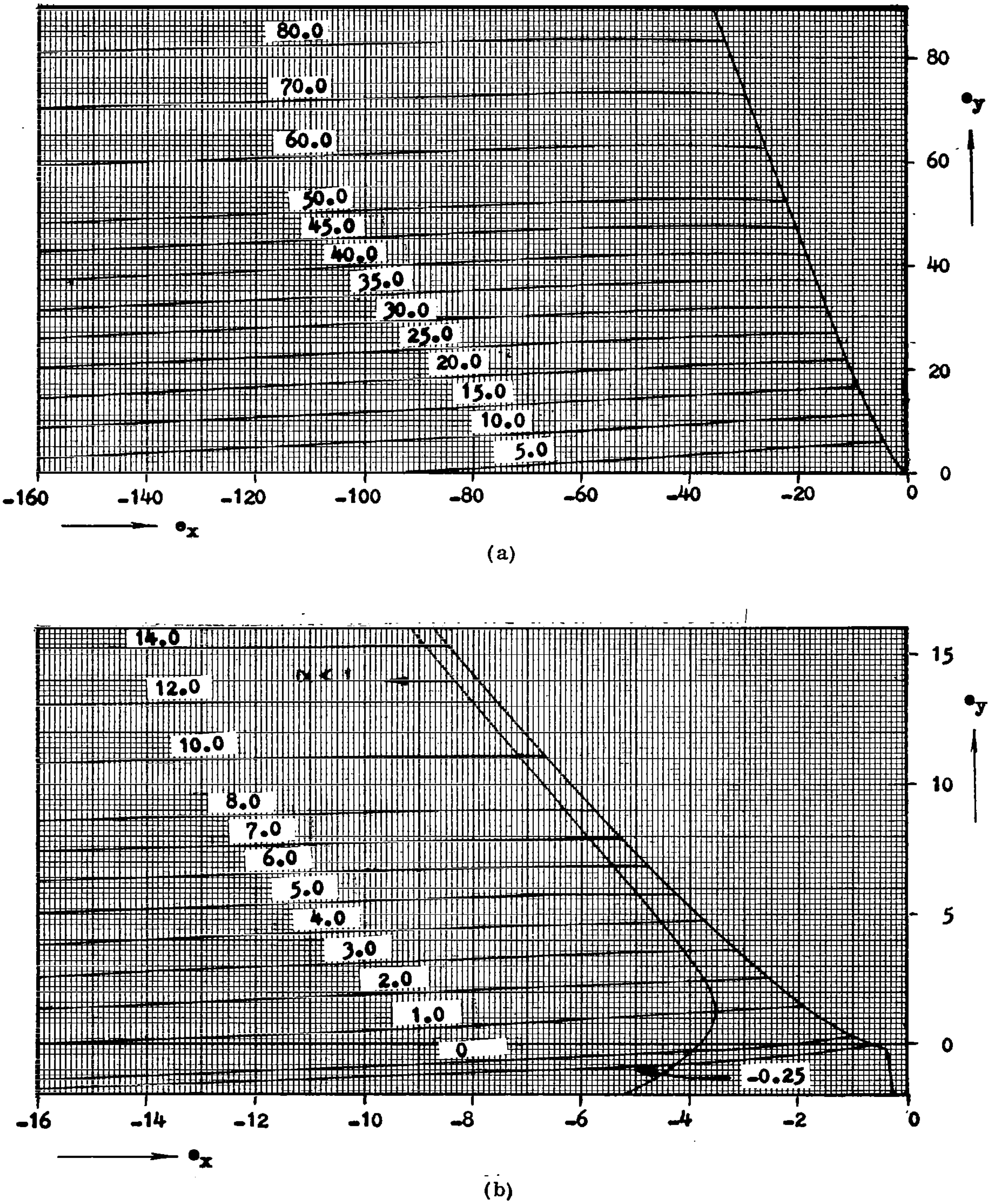


FIGURE A3. s_y VERSUS e_x, e_y FOR LARGE STRAINS (a) AND SMALL STRAINS (b)

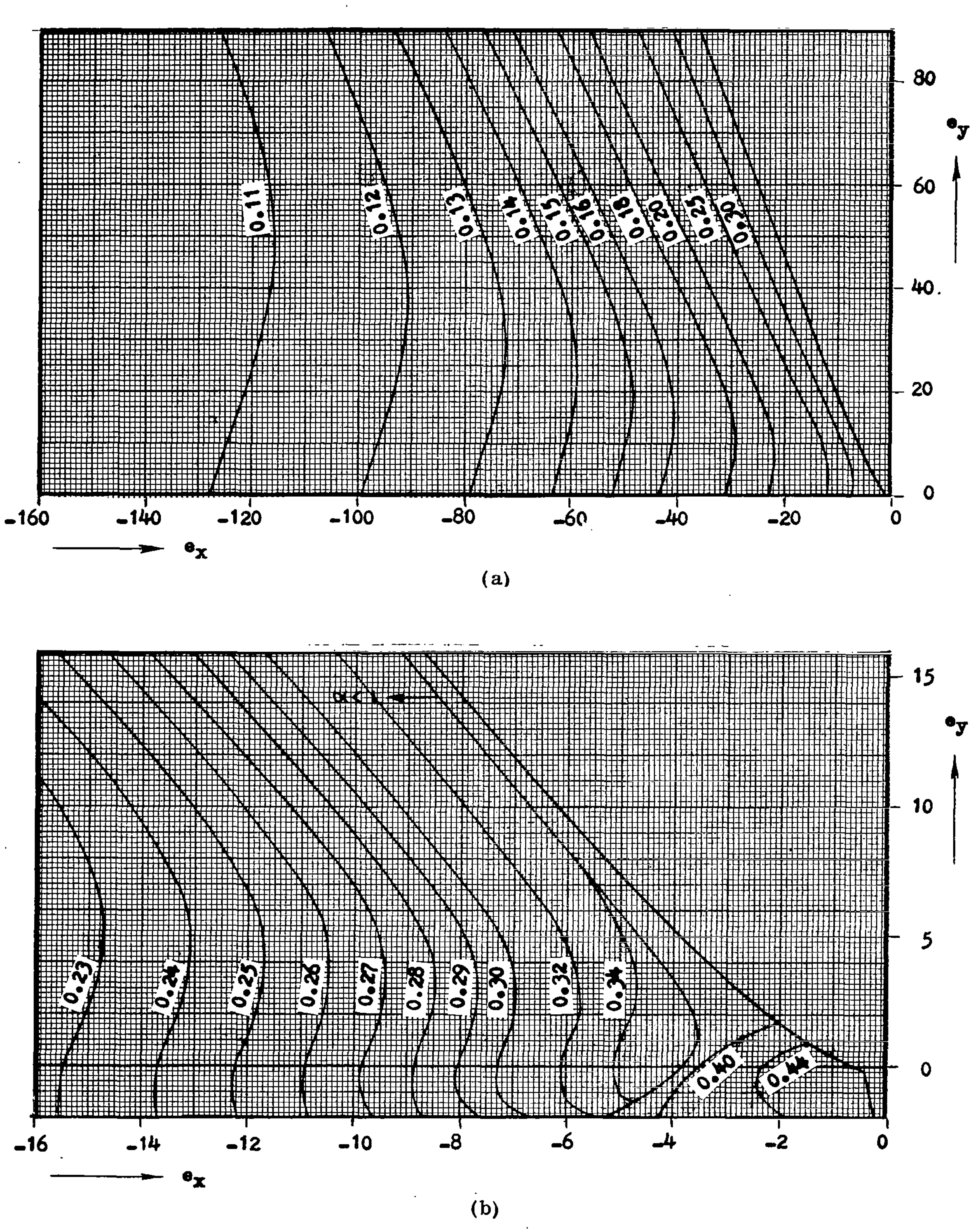


FIGURE A4. β_x VERSUS e_x, e_y FOR LARGE STRAINS (a) AND SMALL STRAINS (b)

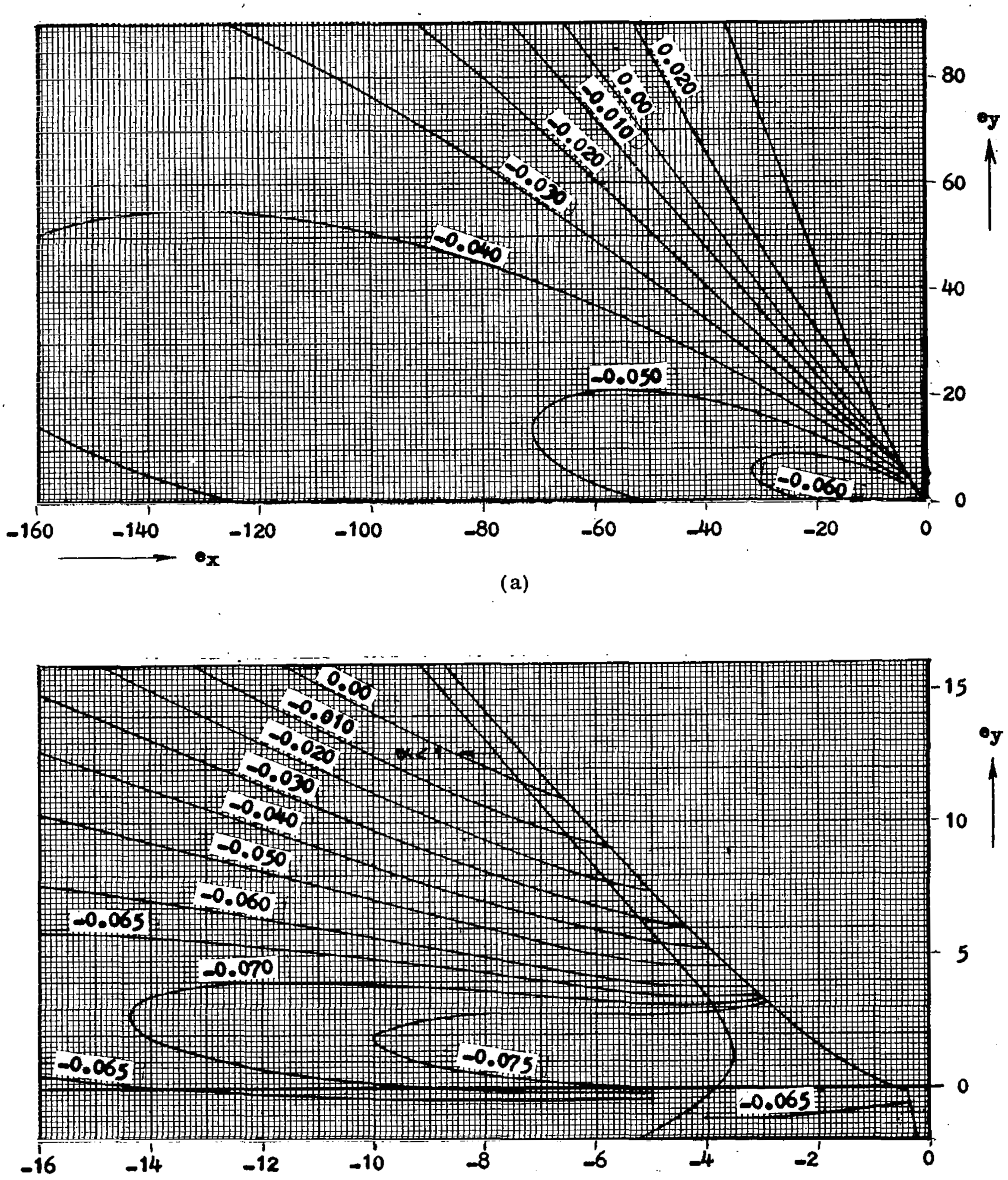
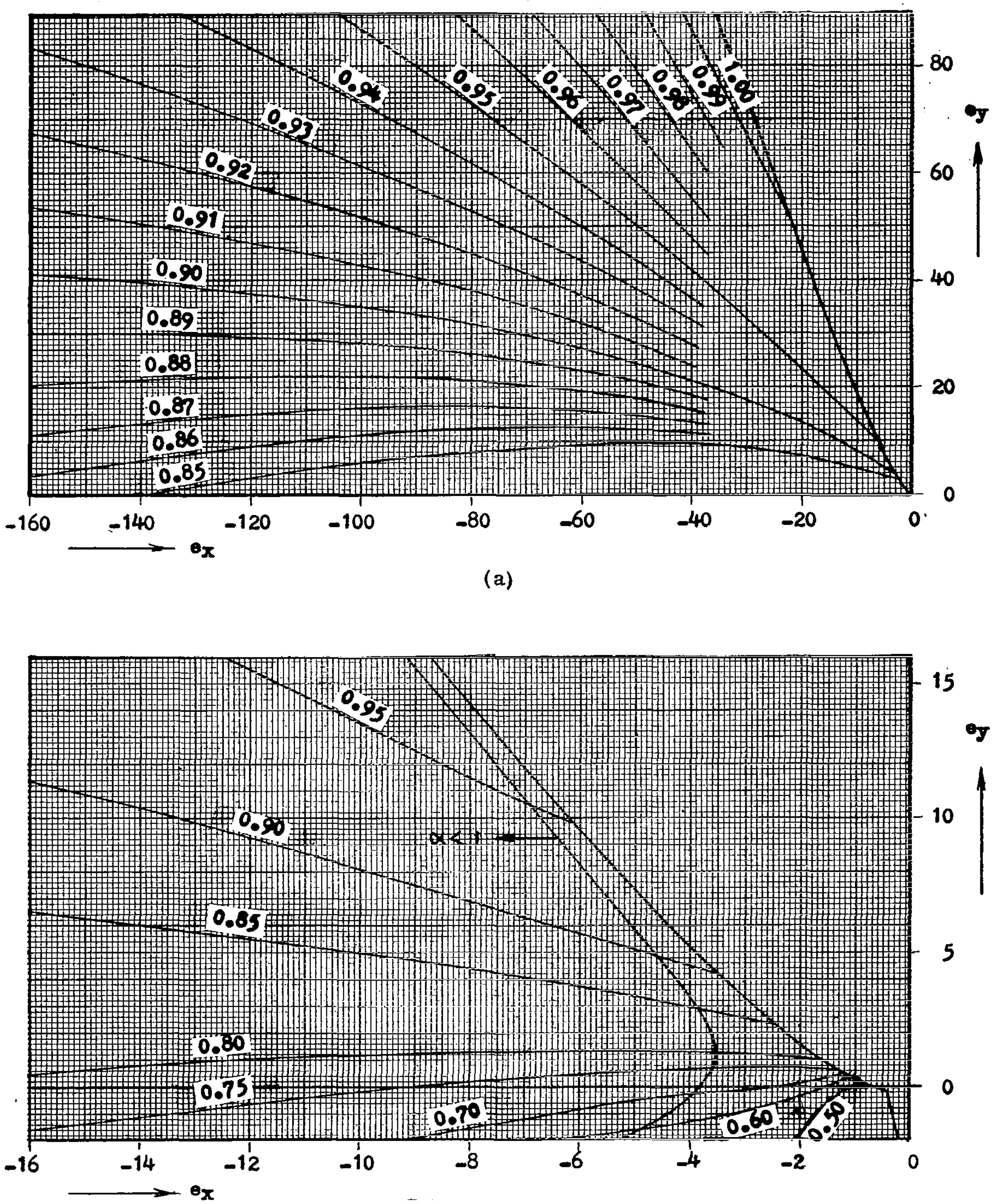


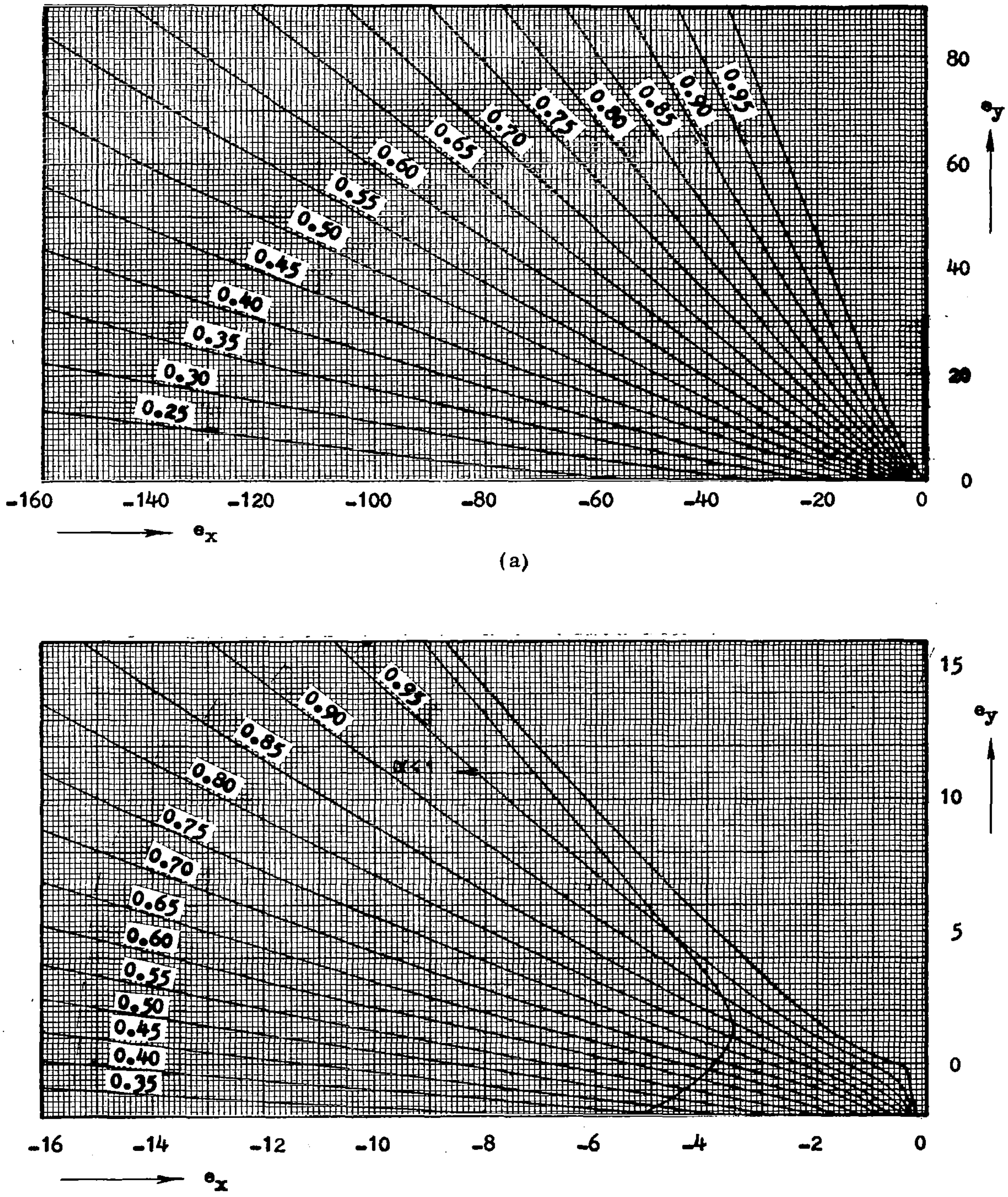
FIGURE A5. β_{μ} VERSUS e_x, e_y FOR LARGE STRAINS (a) AND SMALL STRAINS (b)



(a)

(b)

FIGURE A6. β_y VERSUS e_x, e_y FOR LARGE STRAINS (a) AND SMALL STRAINS (b)



(a)

(b)

FIGURE A7. β_s VERSUS e_x, e_y FOR LARGE STRAINS (a) AND SMALL STRAINS (b)

$$p'R = pR(1 - A) - \mu A q'_o \tag{A2}$$

where

$$A = \frac{\bar{k}_r}{\bar{k}L_r + BL_r\bar{k}_r} .$$

Equation (A2) may be normalized by dividing through by $Et\epsilon^*$, after which substitution of (A1) yields the quadratic equation

$$D^2 + 2\mu AD + 2\mu A - 1 - 4(1 - A)\frac{pR}{Et\epsilon^*} = 0 , \tag{A3}$$

and since D must be a positive quantity, the only valid solution is:

$$D = -\mu A + \sqrt{(\mu A - 1)^2 + 4(1 - A)\frac{pR}{Et\epsilon^*}} . \tag{A4}$$

The total axial stress resultant is found by equating the strains in the shell and stringers, or:

$$\epsilon_x = \frac{1}{Et}(-q_0' - \mu p'R) = -\frac{1}{\bar{E}_s}(q_0 - q_0') \quad ;$$

therefore,

$$q_0 = q_0'\left(1 + \frac{\bar{E}_s}{Et}\right) + \mu p'R\,\frac{\bar{E}_s}{Et} .$$

Normalization of the equation above and substitution of (A1) and (A2) gives:

$$q_0 = \frac{1}{2}(D + 1)\left(1 - \mu^2 A + \frac{Et}{\bar{E}_s}\right)\bar{E}_s\epsilon^* + \mu\,\bar{E}_s(1 - A)\frac{pR}{Et} . \tag{A5}$$

Since A is a function of q_0 for a given value of p, the correct value of q_0 must be found by iteration.

If the general instability load lies above the load calculated from equation (A5), general instability is preceded by local buckling and the shell stiffnesses must be multiplied by appropriate reduction factors as obtained from Figures A4 through A7. Since the average stresses and the reduced moduli are given in terms of strains, the magnitude of the strains for a given combination of axial load and lateral pressure must first be determined.

For the total axial load, one may write:

$$q = q' + q_s = -s_x E t \epsilon^* - e_x \overline{E}_s \epsilon^*$$

and solving for the longitudinal strain gives:

$$e_x = -\frac{q}{\overline{E}_s \epsilon^*} - \frac{s_x E t}{\overline{E}_s} . \qquad \text{(A6)}$$

The average hoop strain is obtained by substituting equations (B11) and (B1) into equation (B8) to give:

$$\epsilon_y = \frac{1}{E t} \left[(pR + \mu q')(1 - A) - A \Delta N \right] \quad ;$$

and, after normalizing and substituting for ΔN from equation (B10), one has:

$$e_y = \left(\frac{pR}{E t \epsilon^*} - \mu s_x \right)(1 - A) - A(s_y - \mu s_x - e_y) . \qquad \text{(A7)}$$

Since the average stresses and, to a lesser degree, the value of A are strain dependent, the strains as given by equations (A6) and (A7) must be obtained by iteration. This is done as follows. A value for A is calculated by taking γ_x in equation (B5) equal to 1.0. Next, by setting

$$s_y - \mu s_x - e_y = 0$$

in equation (A7) and

$$s_x = -\frac{q}{(E t + \overline{E}_s) \epsilon^*}$$

in equations (A6) and (A7), initial values for the strains may be calculated. The average stresses corresponding to these strains are obtained from Figures A2 and A3 and substituted in equations (A6) and (A7) to yield a new set of strains. This procedure is repeated until the magnitudes of the average stresses are within 1 percent of those obtained from the previous iteration. The value of A must now be recalculated by using γ_x as obtained from equation (B5).

This will result in a new set of stresses and, hence, a new value for A. Iterations must therefore be continued until a value for A is obtained, which is within 1 percent of that obtained previously. Since A is usually not very sensitive to changes in γ_x, this part of the iterative procedure converges quickly.

Using the moduli obtained from Figures A4 through A7, the reduced stiffnesses of the shell may not be calculated. In addition, the following quantities are needed for the determination of the general instability load:

$$\alpha_p = 1 - \frac{s_y E t \epsilon^*}{pR} = \frac{p_r}{p}$$

$$\alpha_q = 1 + \frac{s_x E t \epsilon^*}{q} = \frac{q_s}{q} \quad . \qquad \text{(A8)}$$

APPENDIX B. DETERMINATION OF AVERAGE HOOP STRESS RESULTANT

When a stiffened cylindrical shell is subjected to uniform axial compression and/or lateral pressure, the resulting radial deformation will be approximately uniform only when the stiffener spacing is very small. In most practical applications, this holds true for the longitudinal stiffeners or stringers, but not for the rings; and radial expansion will vary along the length of the cylinder as shown in Figure B1. If the cylinder is loaded in axial compression only, the restraining effect of the ring will produce hoop compression stresses in the shell and the rings will be in tension. Internal pressure, on the other hand, produces tensile stresses in the shell as well as in the rings. Under combined loading, the hoop stresses in the shell may be either tension or compression depending on the relative magnitude of the axial load and internal pressure. If the local hoop stress resultant before general instability is denoted by $\overline{N}_{yy}$, the average hoop stress resultant becomes:

$$p'R = \frac{1}{L_r} \int_0^{L_r} \overline{N}_{yy} \, dx$$

which may also be written in the form

$$p'R = pR - \frac{VR}{L_r} , \qquad (B1)$$

where V is the radial shear force per unit length reacted by the ring, L_r is the ring spacing, and pR is the total hoop stress resultant.

The average hoop stress resultant is required to calculate the quantities α_p and α_q used in the general instability analysis and to determine the point of initial buckling of the shell. It is also needed in the calculation of the reduced moduli for those cylinders in which general instability is preceded by local buckling.

The radial shear force V will now be determined for the general case of a ring-and-stringer stiffened cylinder under uniform axial compression and lateral pressure. With the assumption of small stringer spacing, the following differential equation is obtained by considering the equilibrium of a small element of the shell (Fig. B2):

$$M,_{xx} + q\bar{w},_{xx} + \frac{\bar{N}_{yy}}{R} = p \ ,$$

and since

$$M = D_{xx}\bar{w},_{xx}$$

$$\bar{N}_{yy} = Et\,\frac{\bar{w}}{R} - \mu q' \ , \tag{B2}$$

this may be written

$$D_{xx}\bar{w},_{xxxx} + q\bar{w},_{xx} + \bar{k}\bar{w} = p + \frac{\mu}{R}q' \tag{B3}$$

where $\bar{w}$ is the prebuckling radial displacement of the shell, $\bar{k} = Et/R^2$, and D_{xx} is the flexural rigidity of the shell-stringer combination given by the equation

$$D_{xx} = D_x + D_{xs} + c_s^2\bar{E}_s\,\frac{\gamma_x Et}{\bar{E}_s(1-\mu^2) + \gamma_x Et} \ . \tag{B4}$$

The effective width factor γ_x is equal to unity if there is no local buckling of the shell. If the shell buckles before general instability, one has for

$$\gamma_x = \frac{s_x(1-\mu^2)}{e_x + \mu e_y} \ . \tag{B5}$$

With the definitions

$$\lambda = \sqrt[4]{\frac{\bar{k}}{4D_{xx}}}$$

$$\alpha = \sqrt{\lambda^2 + \frac{q}{4D_{xx}}}$$

$$\beta = \sqrt{\lambda^2 - \frac{q}{4D_{xx}}}$$

the solution to the differential equation (B3) for the case $q < 2\sqrt{\bar{k} D_{xx}}$ may be written in the form

$$\bar{w} = \frac{1}{\bar{k}}\left(p + \frac{\mu}{R} q'\right) + (C_1 \sinh\beta x + C_2 \cosh\beta x) \cos\alpha x$$

$$+ (C_3 \sinh\beta x + C_4 \cosh\beta x) \sin\alpha x \quad . \tag{B6}$$

The integration constants may be determined from the boundary conditions at the rings, viz.,

$$\bar{w} = \frac{V}{\bar{k}_r} \quad \text{and} \quad \bar{w},_x = 0$$

at $x = 0$ and $x = L_r$. The spring constant of the ring is $k_r = E_r A_r / R^2$. Substitution of the boundary conditions in equation (B6) gives

$$C_1 = - C\alpha \left(\frac{\cosh\beta L_r - \cos\alpha L_r}{\alpha \sinh\beta L_r + \beta \sin\alpha L_r} \right)$$

$$C_2 = C = \frac{V}{\bar{k}_r} - \frac{1}{\bar{k}}\left(p + \frac{\mu}{R} q'\right)$$

$$C_3 = C \left(\frac{-\beta \sinh\beta L_r + \alpha \sin\alpha L_r}{\alpha \sinh\beta L_r + \beta \sin\alpha L_r} \right)$$

$$C_4 = C\beta \left(\frac{\cosh\beta L_r - \cos\alpha L_r}{\alpha \sinh\beta L_r + \beta \sin\alpha L_r} \right) .$$

The radial shear force V may now be found from the relation

$$V = -2D_{xx} \bar{w}_{,xxx} ,$$

which gives

$$V = \frac{p + \frac{\mu}{R} q'}{\frac{\bar{k}}{\bar{k}_r} + B} \qquad \text{(B7)}$$

where

$$B = \frac{\lambda^2}{2\alpha\beta} \; \frac{\alpha \sinh\beta L_r + \beta \sin\alpha L_r}{\cosh\beta L_r - \cos\alpha L_r} .$$

Although the solution above for the shear force was obtained for the $q < 2\sqrt{\bar{k} D_{xx}}$, equation (B7) is valid also when $q \geq 2\sqrt{\bar{k} D_{xx}}$, provided the following values for B are used.

$$B = \frac{\lambda^2}{2\alpha} \; \frac{\alpha L_r + \sin\alpha L_r}{1 - \cos\alpha L_r} \qquad q = 2\sqrt{\bar{k} D_{xx}}$$

$$B = \frac{\lambda^2}{2\alpha\bar{\beta}} \; \frac{\bar{\beta} \sin\alpha L_r + \alpha \sin\bar{\beta} L_r}{\cos\bar{\beta} L_r - \cos\alpha L_r} \qquad q > 2\sqrt{\bar{k} D_{xx}}$$

In the last expression, β has been replaced by

$$\bar{\beta} = \sqrt{\frac{q}{4D_{xx}} - \lambda^2} .$$

The average hoop stress resultant in the shell may be found by substitution of (B7) into equation (B1), and the average prebuckling hoop strain becomes

$$\bar{\epsilon}_y = \frac{1}{Et} (p'R + \mu q') \quad . \tag{B8}$$

When general instability of the stiffened cylinder is preceded by local buckling of the shell between stringers, a rigorous determination of the radial shear force V is not possible. A satisfactory approximation may be obtained, however, by writing the second of equations (B2) in the form

$$\bar{N}_{yy} = Et \frac{\bar{w}}{R} - \mu q' + \Delta N \quad . \tag{B9}$$

The term ΔN has been added on the right side of equation (B9) to account for the nonlinear portion of the strain caused by buckling of the shell. This term must be consistent with the wave shape assumed in Reference 22. It is therefore a function of the post-buckling stresses in the shell and is given by the expression

$$\Delta N = Et\epsilon^* (s_y - \mu s_x - e_y) \quad . \tag{B10}$$

With the above modification, equation (B7) becomes

$$V = \frac{p + \frac{\mu}{R} q' + \frac{1}{R} \Delta N}{\frac{\bar{k}}{\bar{k}_r} + B} \tag{B11}$$

where B must be calculated with a reduced flexural rigidity according to equations (B4) and (B5).

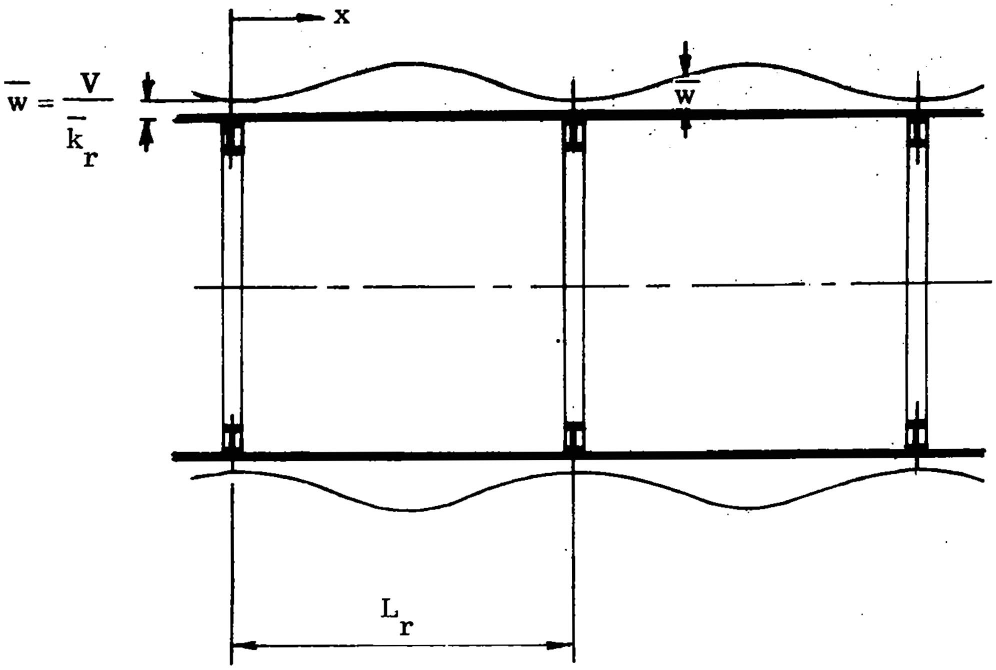


FIGURE B1. DEFLECTION OF CYLINDER BETWEEN RINGS

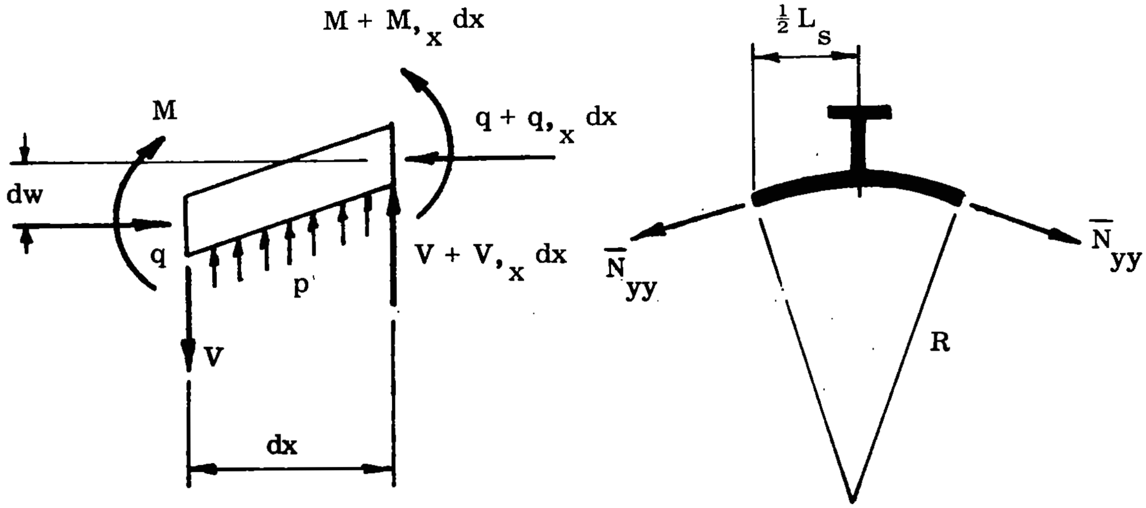


FIGURE B2. EQUILIBRIUM OF SMALL ELEMENT

APPENDIX C. COMPUTER PROGRAM

A computer program to determine the general instability or panel instability load of several types of stiffened cylinders is presented in this appendix. The program is written in FORTRAN IV for use with an IBM 7094 computer. Input instructions and a sample problem are given in Appendix D. To obtain maximum efficiency for each of the types of cylinders considered and to avoid unnecessary computations, the program has been subdivided into a number of subroutines. A list of these subroutines, together with a brief description of their function, is given in Table CI. Flow charts of the main program and its subroutines are presented in Figure C1. Table CII shows a comparison between the notation used in the program and that used in the text.

The following types of cylinders are considered in the program:

1. Cylinders with rings and stringers
2. Cylinders with stringers only
3. Cylinders with rings only
4. Isotropic core sandwich cylinders
5. Isotropic core sandwich cylinders with rings
6. Open corrugated cylinders
7. Open corrugated cylinders with rings.

In the computer program, loads are calculated for all mode shapes under investigation assuming that there is no local buckling of the skin and no effect caused by ring restraint. For the first two types of cylinders, these loads are compared with the skin local buckling load q_0. Reduced stiffness moduli are calculated for all loads that exceed q_0 and that are within a certain percentage of the minimum load. This percentage has been set equal to 20 percent in the present program and is read in as part of the semi-permanent data. The reason for not just re-calculating the load corresponding to the critical wave shape is that quite frequently another mode shape becomes critical when the reduced stiffness moduli are used. The effect of ring restraint is accounted for in cylinder types 1, 3, 5, and 7. The core of the sandwich cylinders is assumed to be infinitely rigid in shear; therefore, the analysis does not apply for cylinders with weak cores.

Cylinders may be checked for either general or panel instability (buckling between rings), the latter mode of failure being of interest only for cylinder types 1, 3, 5, and 7. When panel instability is specified, the ring stiffness matrix is set equal to zero and the cylinder length is made equal to the ring spacing.

The remainder of Appendix C is organized in the following manner: Figure C1 followed by Tables CI, CII, and the computer program (Table CIII) which begins on page 67.

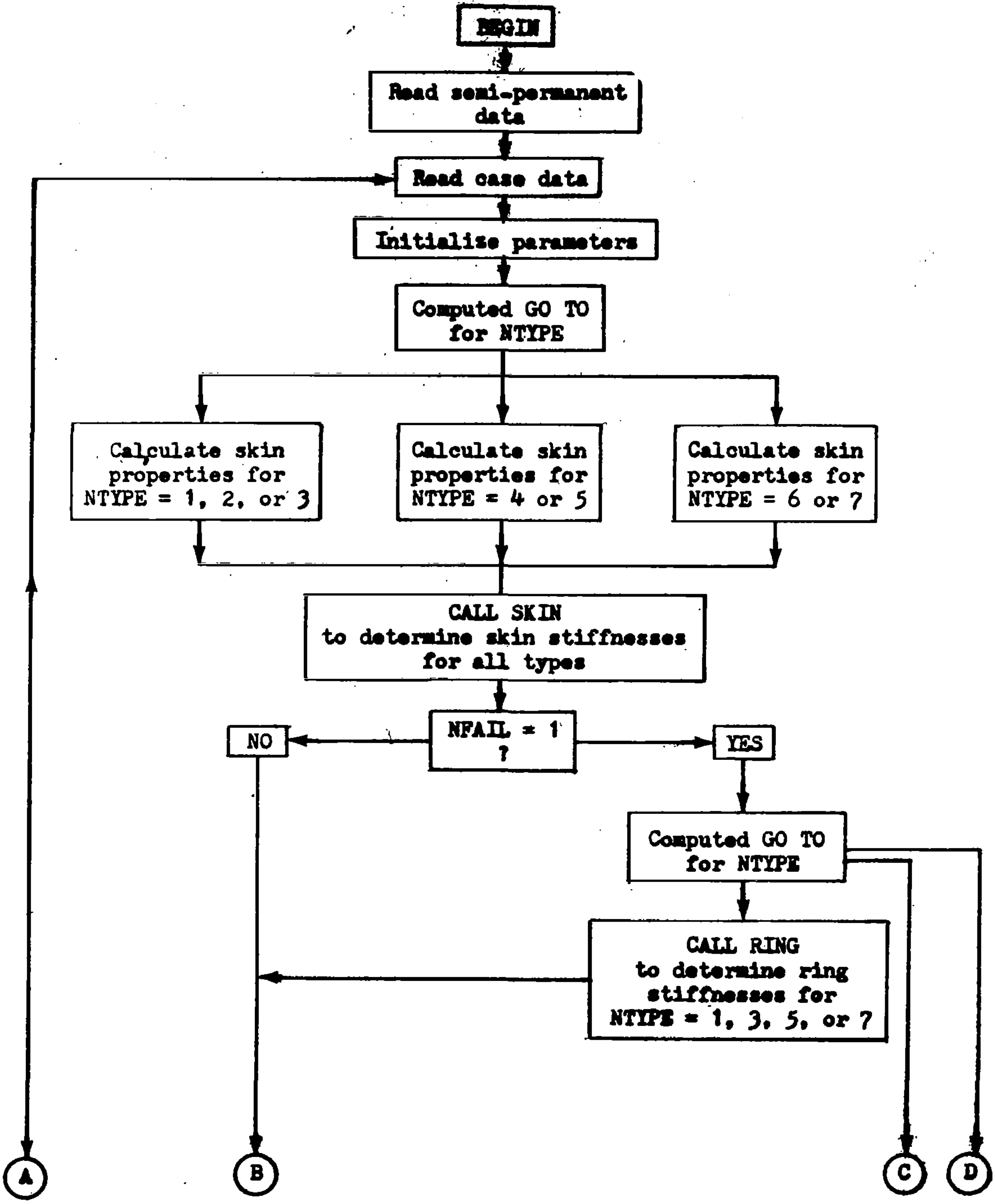


FIGURE C1. COMPUTER PROGRAM FLOW CHART

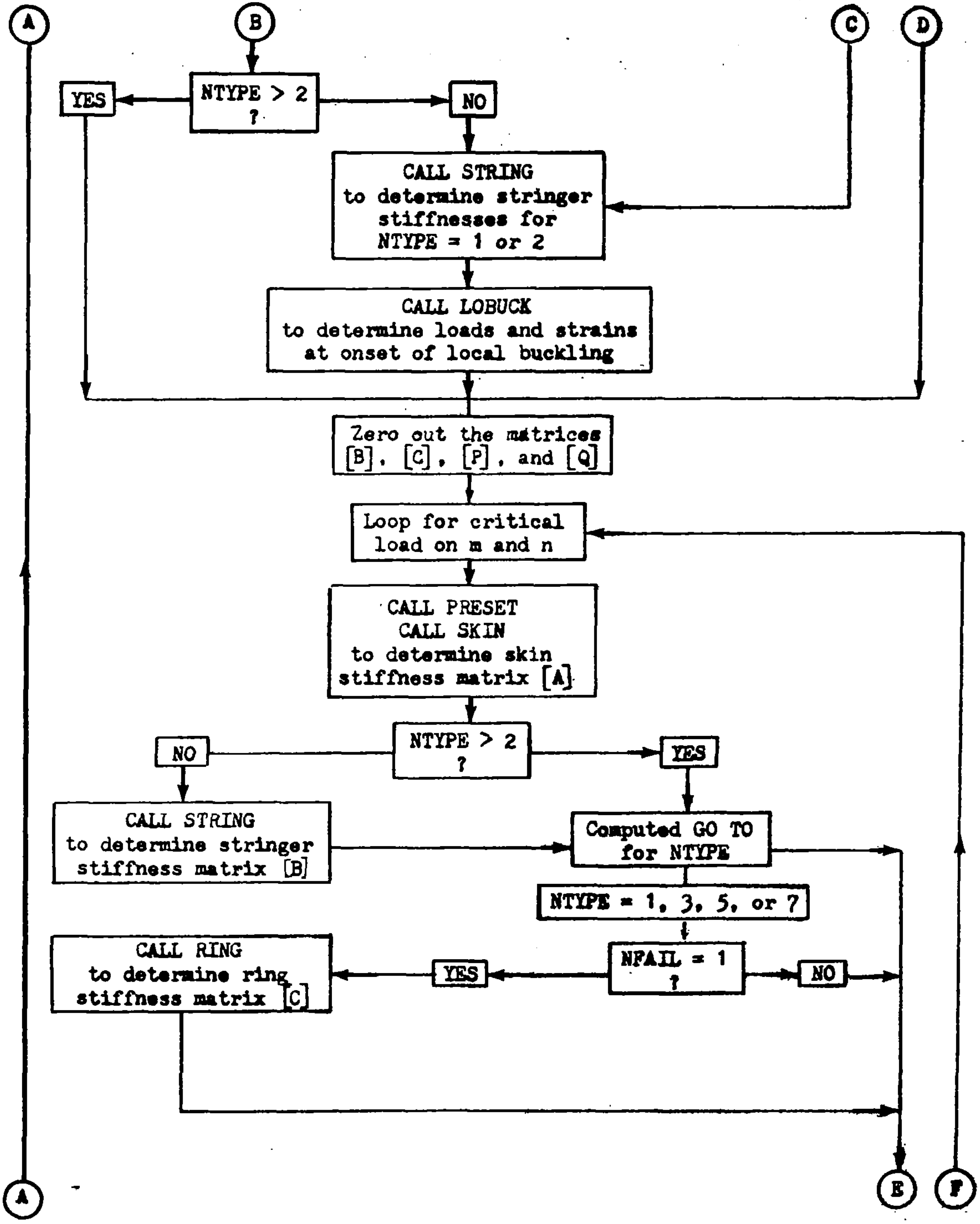


FIGURE C1. (Continued)

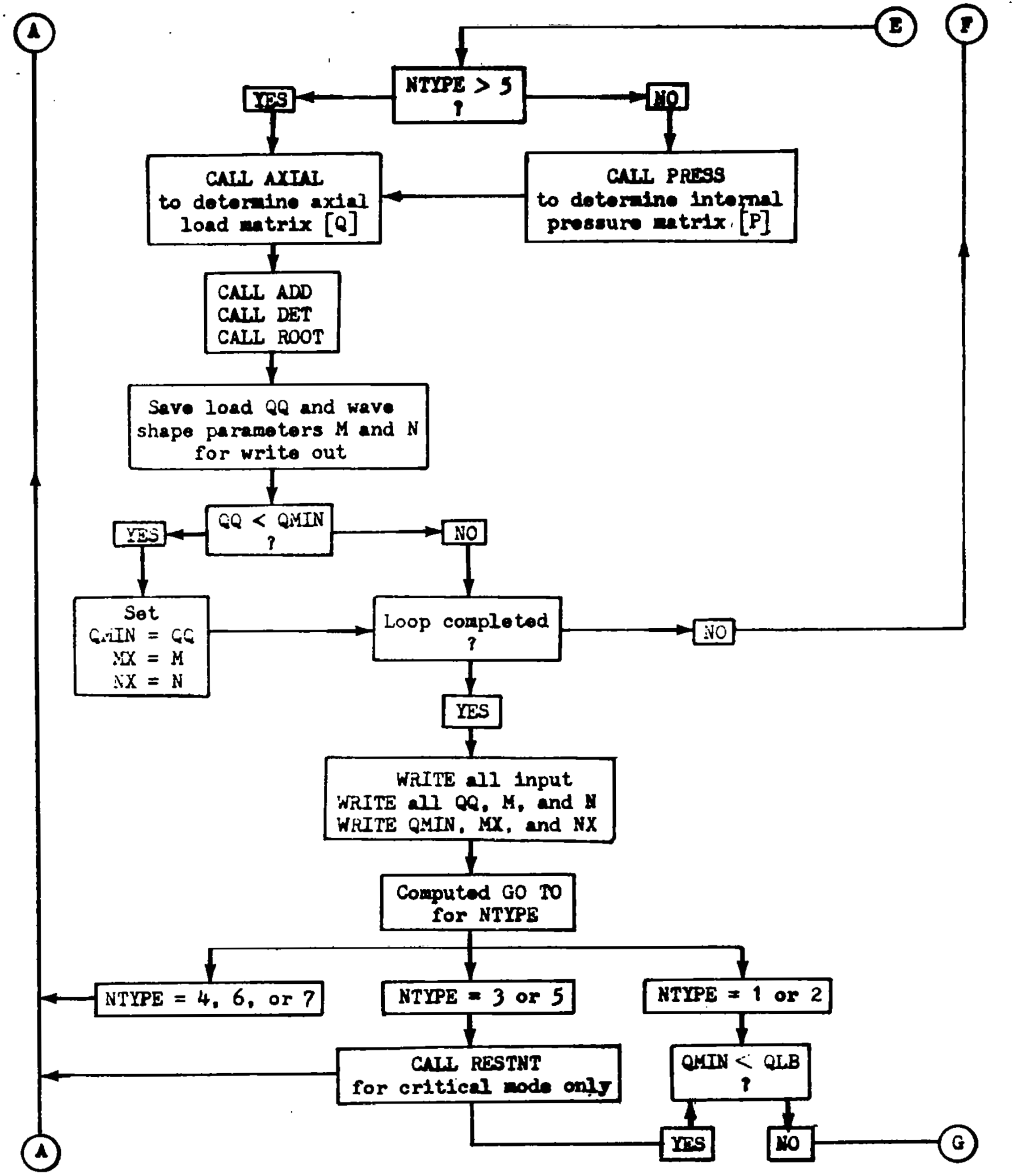
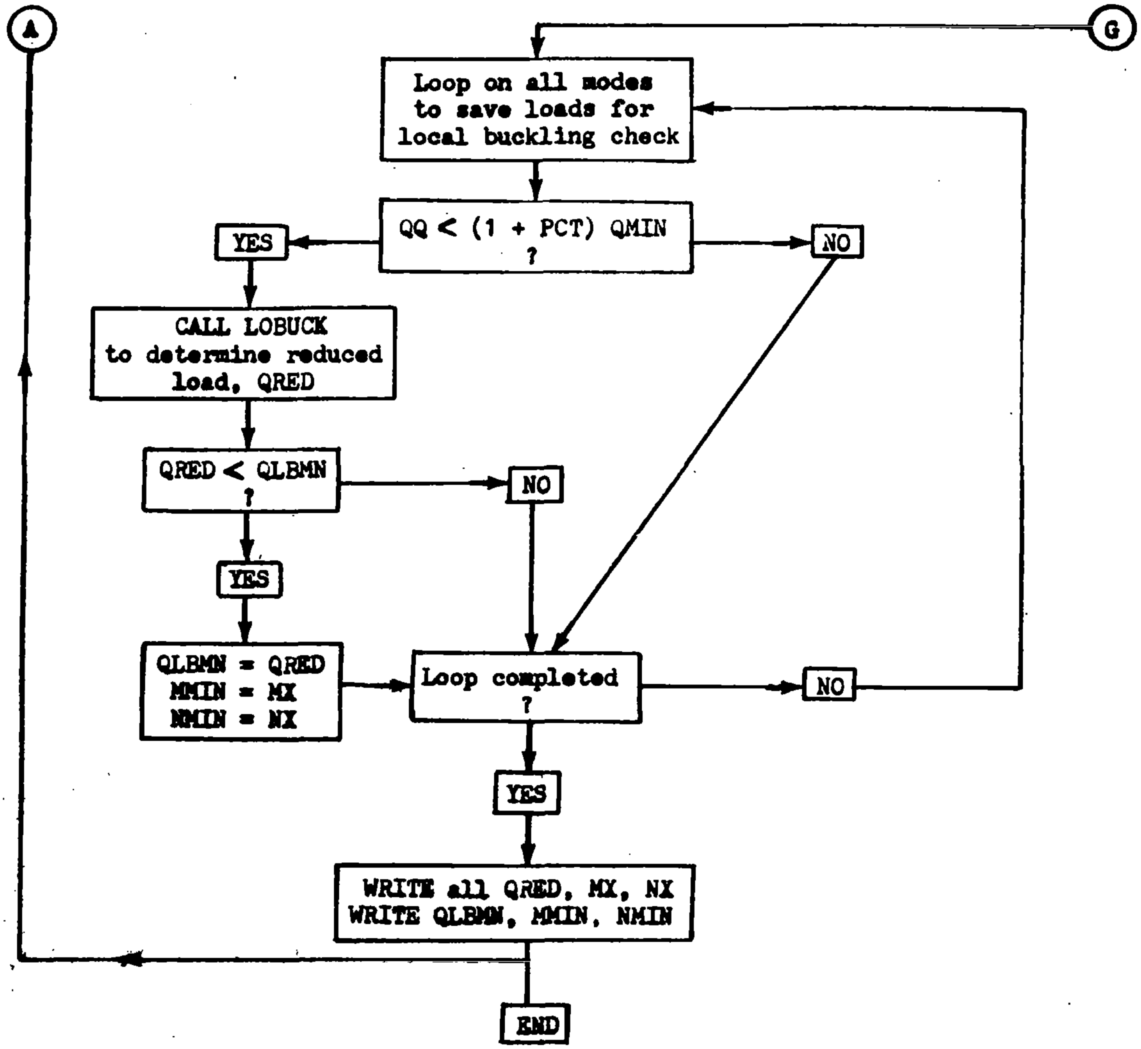
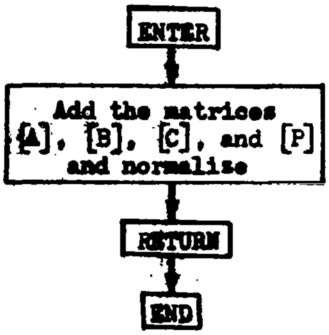


FIGURE C1. (Continued)



SUBROUTINE ADD



SUBROUTINE AXIAL

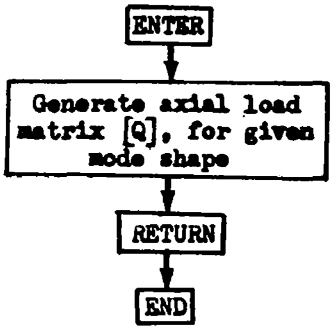
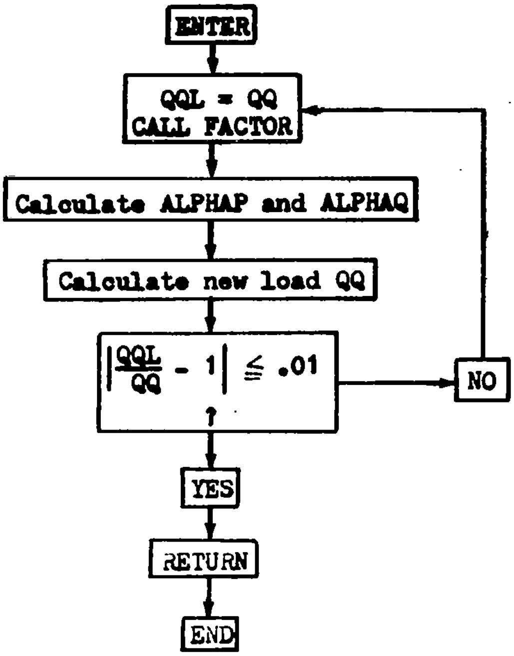
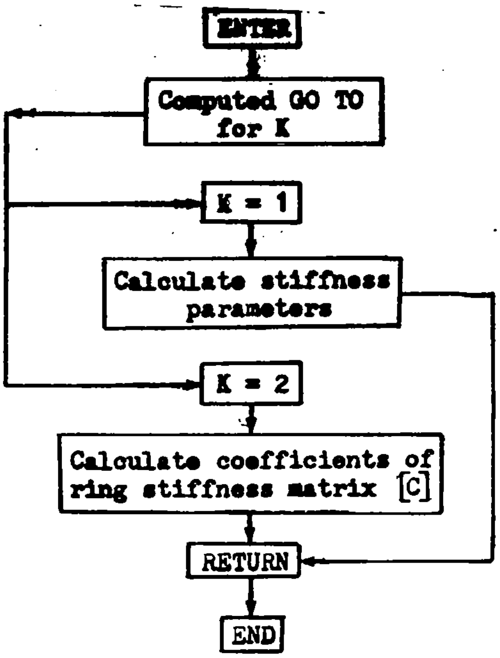
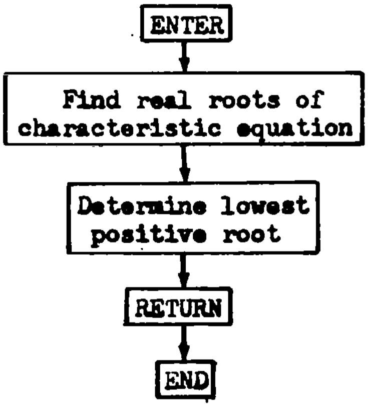


FIGURE C1. (Continued)





SUBROUTINE ROOT



SUBROUTINE STRING

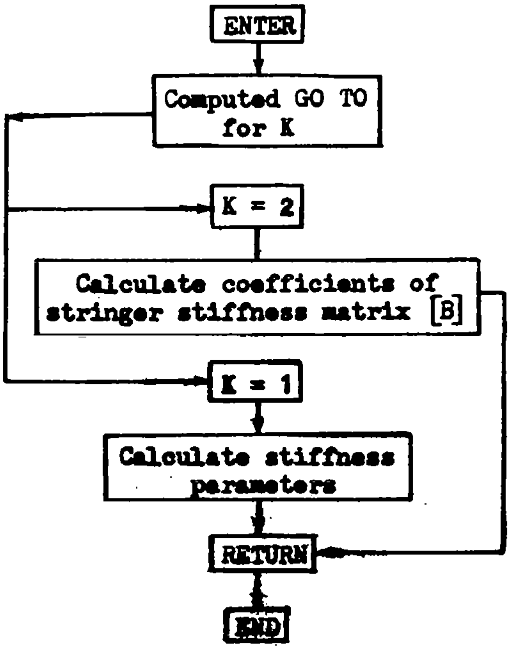


FIGURE C1. (Continued)

SUBROUTINE LOBUCK

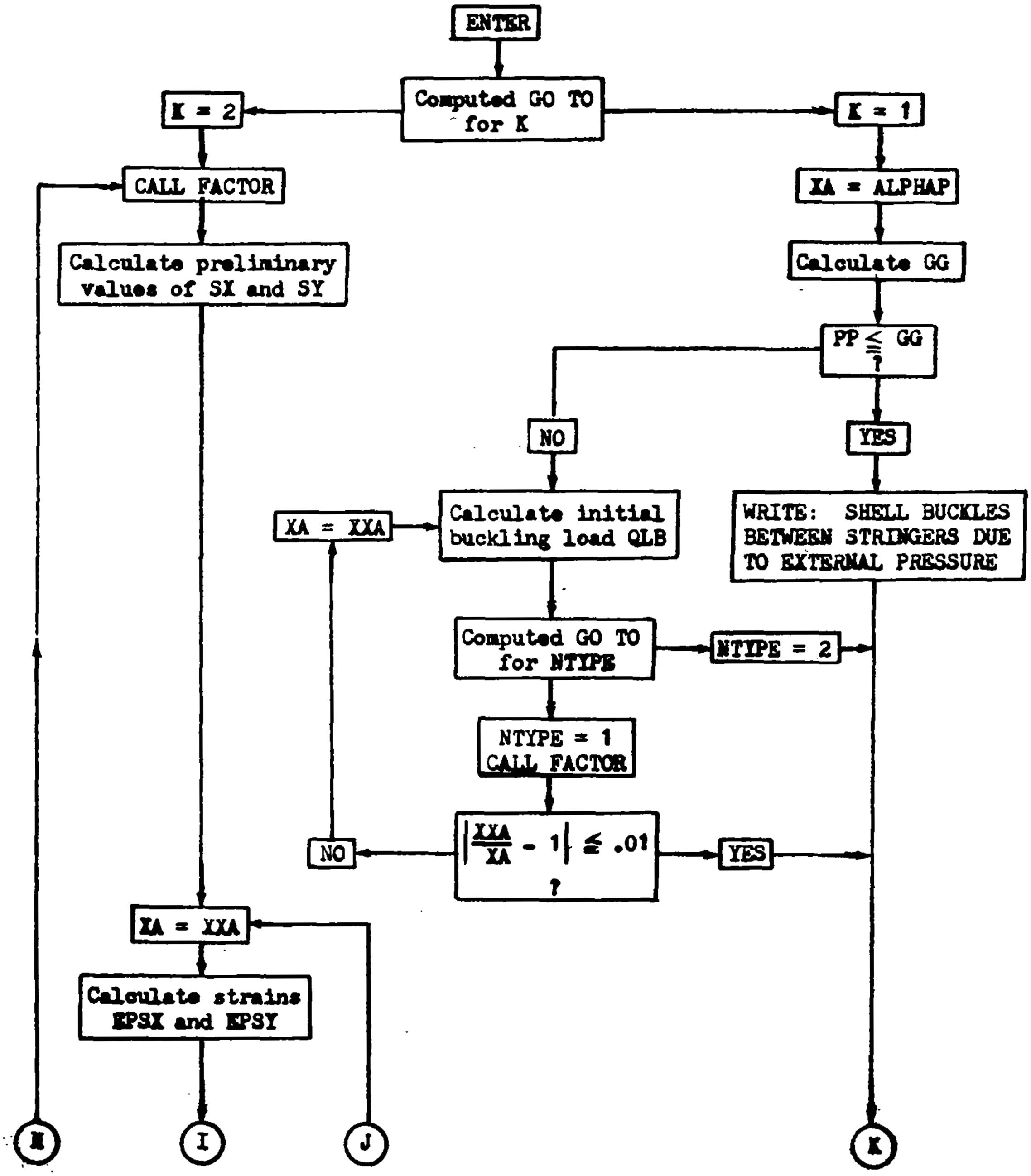


FIGURE C1. (Continued)

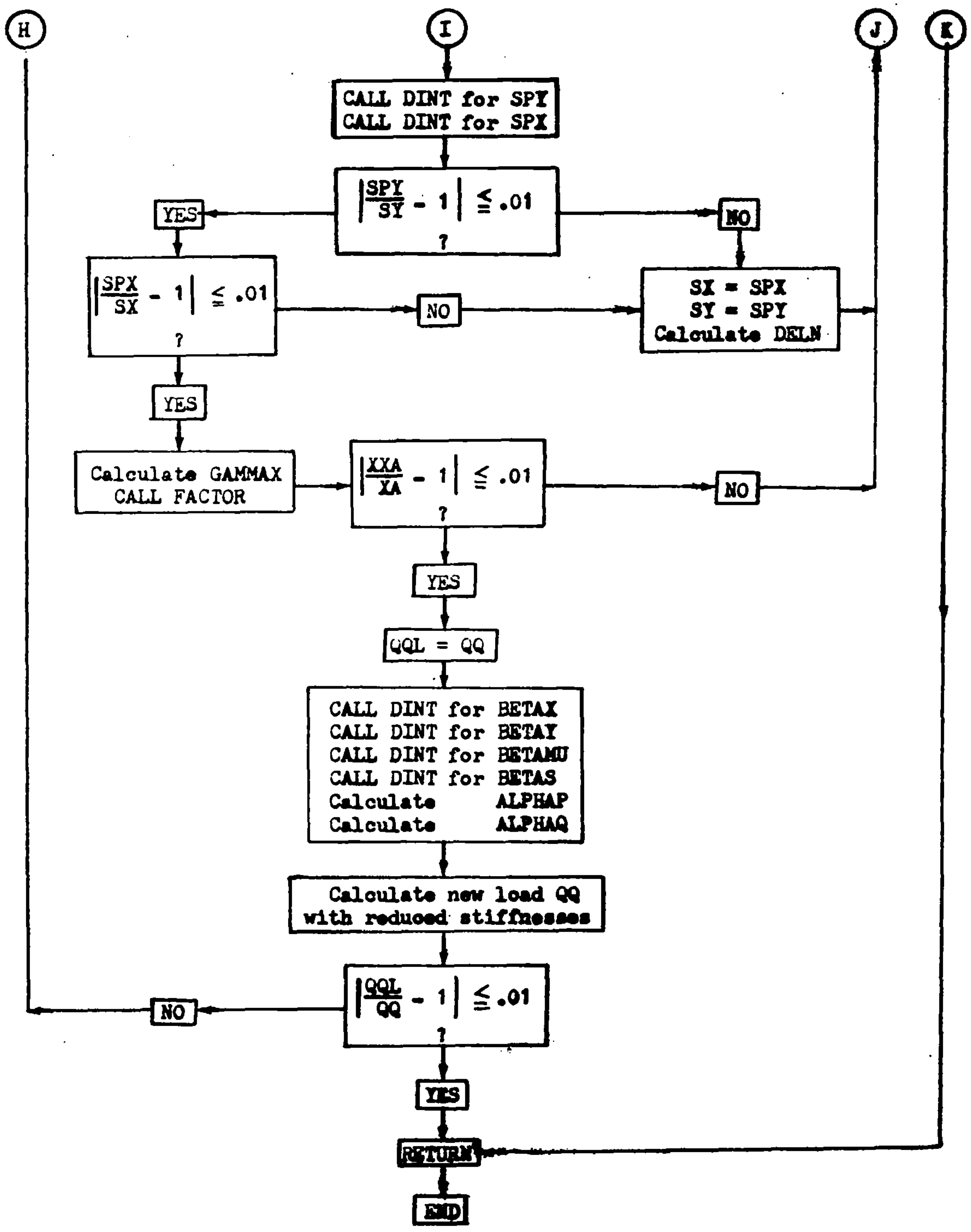
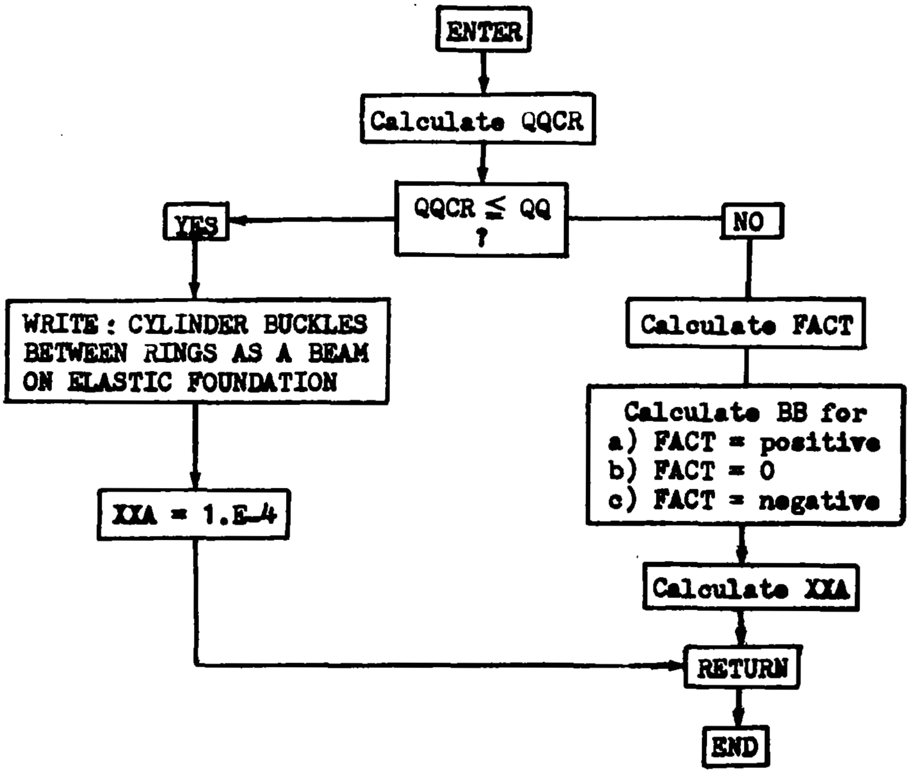


FIGURE C1. (Continued)

SUBROUTINE FACTOR



SUBROUTINE SKIN

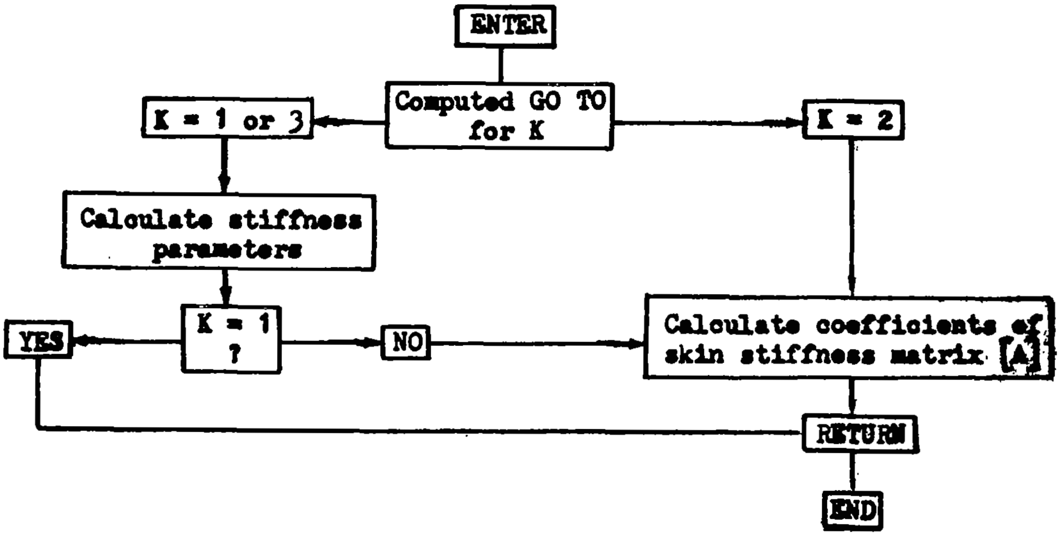


FIGURE C1. (Continued)

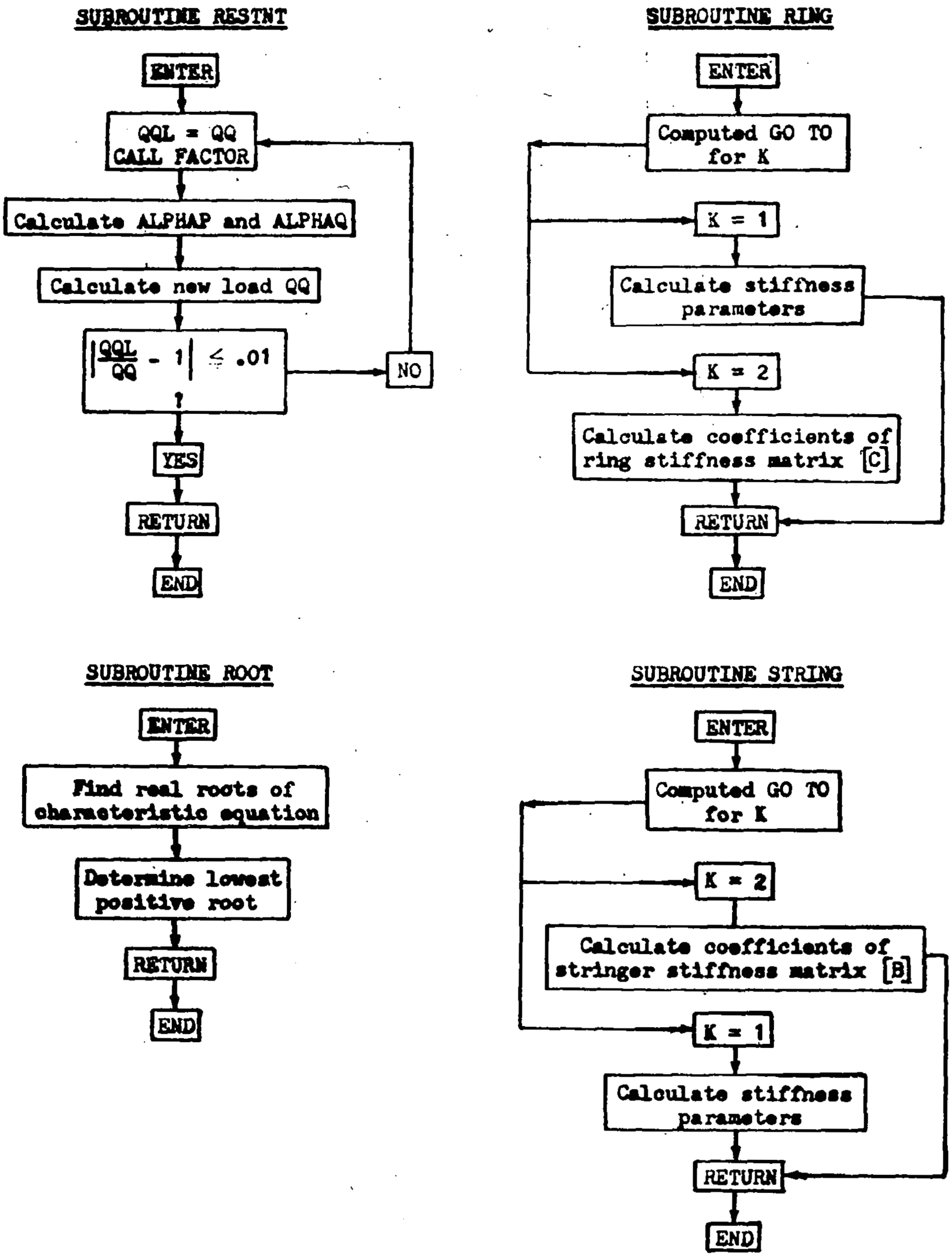


FIGURE C1. (Concluded)

TABLE CI. LIST OF PROGRAM SUBROUTINES

Name	Description of Function
SKIN	Calculates skin stiffness properties $(k = 1)$ and the stiffness matrix $[A]$ for unbuckled $(k = 2)$ and buckled skin $(k = 3)$.
STRING	Calculates stringer stiffness properties $(k = 1)$ and stiffness matrix $[B]$ $(k = 2)$.
RING	Calculates ring stiffness properties $(k = 1)$ and stiffness matrix $[C]$ $(k = 2)$.
PRESS	Computes the internal pressure matrix $[P]$.
AXIAL	Computes axial load matrix $[Q]$.
ADD	Computes normalized $[D]$ matrix, $[D] = ([A] + [B] + [C] + [P])/EAX$
DET	This subroutine calculates the polynomial coefficients of the characteristic equation: $\lvert Q \rvert q^3 + (QD) q^2 + (DQ) q + \lvert D \rvert = 0$
ROOT	This subroutine finds the lowest root (eigenvalue) of the characteristic equation (see DET) and, hence, the buckling load for a given mode shape.
FACTOR	This is essentially the beam on elastic foundation analysis given in Appendix B. The factor A computed is required to calculate the average hoop stress resultant in the shell.

TABLE CI. (Concluded)

Name	Description of Function
LOBUCK	This subroutine calculates the axial load q_0 at which the skin buckles between adjacent stringers and returns it to the main program. For cases in which the general instability load exceeds the local buckling load q_0, the normalized strains ϵ_x and ϵ_y are determined by iteration according to the procedure outlined in Appendix A. After the correct strains are obtained, the average stresses and reduced moduli are found by interpolating between known values (see DINT). Finally the quantities α_p and α_q, required in the calculation of the matrices [P] and [Q], are determined.
DINT	Determines average stresses and reduced moduli of buckled skin for given ϵ_x and ϵ_y by interpolating linearly between the constant strain values given in the tables.
PRESET	This subroutine calculates certain wave shape parameters.
RESTNT	Re-calculates the values of α_p and α_q to account for ring restraint after the critical mode shape has been determined. Cases in which there is no local buckling of the skin only are considered. See equation (17).

TABLE CII. COMPARISON BETWEEN TEXT AND PROGRAM NOTATION

Text	Program	Text	Program
a_{ij}	A(I, J)	$\bar{E}_r$, $\bar{E}_s$	EBARR, EBARS
b_{ij}	B(I, J)	G, G_r, G_s	G, GR, GS
b_r, b_s	BR, BS	$\bar{G}$	GBAR
c_{ij}	C(I, J)	H	H
c_r, c_s	CR, CS	I_{xs}, I_{zs}	QIXS, QIZS
m, $\bar{m}$	M, QMBAR	I_{yr}, I_{zr}	QIYR, QIZR
n, $\bar{n}$	N, QNBAR	J_r, J_s	QJR, QJS
p	PP	K, K_r, K_s	QK, QKR, QKS
p_{ij}	P(I, J)	L, L_r, L_s	QL, QLR, QLS
q, q_o	QQ, QLB	R	R
q_{ij}	Q(I, J)	α_p, α_q	ALPHAP, ALPHAQ
t	T	β_x, β_y	BETAX, BETAY
A	XXA	β_μ, β_s	BETAMU, BETAS
A_r, A_s	AR, AS	ϵ_x, ϵ_y	EPSX, EPSY
D_x, D_y	DX, DY	ϵ^*	EPSTAR
D_{xs}, D_{zs}	DXS, DZS	γ_x	GAMMAX
D_{yr}, D_{zr}	DYR, DZR	μ	QMU
E, E_r, E_s	E, ER, ES	$\bar{E}_x + \bar{E}_s$	EAX
$\bar{E}_x$, $\bar{E}_y$	EBARX, EBARY	$\bar{E}_\mu$	EBARMU

TABLE CIII. COMPUTER PROGRAM

```
$IBFTC MAIN    DECK
C     THE GENERAL INSTABILITY OF ECCENTRICALLY STIFFENED CYLINDRICAL    MAIN0000
C     SHELLS UNDER AXIAL COMPRESSION AND LATERAL PRESSURE               MAIN0010
      DIMENSION QRED(50),MAT(18)                                        MAIN0020
      DIMENSION QQSAV(400),MSAV(400),NSAV(400),QQSV(50),MSV(50),NSV(50) MAIN0030
      DIMENSION A(3,3),B(3,3),C(3,3),D(3,3),P(3,3),Q(3,3)               MAIN0040
      COMMON /BLOCKA/EPX(30),EPY(30),VAR(6,30,30),NEPX,NEPY             MAIN0050
      COMMON /BLOCKB/EAX                                                MAIN0060
      COMMON /BLOCKC/EPSXSV,EPSYSV,T                                    MAIN0070
      COMMON /BLOCKD/AS,BS,GS,QIXS,QIZS,QJS                             MAIN0080
      COMMON /BLOCKE/AR,BR,ER,GR,QIYR,QIZR,QJR                          MAIN0090
      COMMON /BLOCKF/BETAMU,BETAS,BETAX,BETAY,GT,ES                     MAIN0100
      COMMON /BLOCKG/DY                                                 MAIN0110
      COMMON /BLOCKH/CS,CS2,EBARS,ET,GMB,QMBAR,QNB,QNBAR                MAIN0120
      COMMON /BLOCKI/EPSX,EPSY                                          MAIN0130
      COMMON /BLOCKJ/DX,DXS,EBARR,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         MAIN0140
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     MAIN0150
      COMMON /BLOCKL/CR,QL,QNBMB,QNBMB2,QNBR,QR,RQNB                    MAIN0160
      COMMON /BLOCKM/RCR,RRCR                                           MAIN0170
      COMMON /BLOCKN/IDOWRT,A,B,C,D,P,Q                                 MAIN0180
C                                                                       MAIN0190
C     NTYPE = 1  CYLINDER WITH RINGS AND STRINGERS                      MAIN0200
C     NTYPE = 2  CYLINDER WITH STRINGERS ONLY                           MAIN0210
C     NTYPE = 3  CYLINDER WITH RINGS ONLY                               MAIN0220
C     NTYPE = 4  ISOTROPIC CORE SANDWICH CYLINDER                       MAIN0230
C     NTYPE = 5  ISOTROPIC CORE SANDWICH CYLINDER WITH RINGS            MAIN0240
C     NTYPE = 6  OPEN CORRUGATED CYLINDER                               MAIN0250
C     NTYPE = 7  OPEN CORRUGATED CYLINDER WITH RINGS                    MAIN0260
C     NFAIL = 1  GENERAL INSTABILITY                                    MAIN0270
C     NFAIL = 2  PANEL INSTABILITY                                      MAIN0280
C     ***  IF IDOWRT = 1, INTERMEDIATE DATA IS NOT WRITTEN OUT          MAIN0290
C     ***  IF IDOWRT = 2, INTERMEDIATE DATA IS WRITTEN OUT              MAIN0300
      PI=3.14159                                                        MAIN0310
      KTOT=6                                                            MAIN0320
C     SEMI-PERMANENT TABLES                                             MAIN0330
      EPSXSV=0.                                                         MAIN0340
      EPSYSV=0.                                                         MAIN0350
      READ (5,1004) IDOWRT                                              MAIN0360
      READ (5,1002) PCT                                                 MAIN0370
      PCTH=1.+PCT                                                       MAIN0380
      READ (5,1000) NEPX,NEPY                                           MAIN0390
      WRITE (6,1100) NEPX,NEPY                                          MAIN0400
      READ (5,1002) (EPX(I),I=1,NEPX)                                   MAIN0410
      WRITE (6,1002) (EPX(I),I=1,NEPX)                                  MAIN0420
      READ (5,1002) (EPY(J),J=1,NEPY)                                   MAIN0430
      WRITE (6,1002) (EPY(J),J=1,NEPY)                                  MAIN0440
      DO 58 K=1,KTOT                                                    MAIN0450
      DO 58 I=1,NEPX                                                    MAIN0460
      READ (5,1002) (VAR(K,I,J),J=1,NEPY)                               MAIN0470
 58   WRITE (6,1002) (VAR(K,I,J),J=1,NEPY)                              MAIN0480
C                                                                       MAIN0490
```

TABLE CIII. (Continued)

```
C     CASE DATA                                                          MAIN0500
 100  READ (5,1001) MAT,E,ES,ER,G,GS,GR,QMU,BS,CS,QLS,AS,QIXS,QIZS,      MAIN0510
     1QJS,H,BR,CR,QLR,AR,QIYR,QIZR,QJR,R,QL,T,PP,M1,MM,N1,               MAIN0520
     2NN,NTYPE,NFAIL                                                     MAIN0530
      EBARR=0.                                                           MAIN0540
      EBARS=0.                                                           MAIN0550
      DXS=0.                                                             MAIN0551
      QMIN=10.E30                                                        MAIN0560
      QLBMN=10.E30                                                       MAIN0570
      ALPHAP=0.                                                          MAIN0580
      ALPHAQ=0.                                                          MAIN0590
      GAMMAX=1.                                                          MAIN0600
      MTOT=MM-M1+1                                                       MAIN0610
      NTOT=NN-N1+1                                                       MAIN0620
      JTOT=MTOT*NTOT                                                     MAIN0630
      PPR=PP*R                                                           MAIN0640
      TWOPIR=2.*PI*R                                                     MAIN0650
      ONEQM2=1.-QMU**2                                                   MAIN0660
      ET=E*T                                                             MAIN0670
      GT=G*T                                                             MAIN0680
      R2=R**2                                                            MAIN0690
      T3=T**3                                                            MAIN0700
      NNSAV = NN                                                         MAIN0701
      N1SAV = N1                                                         MAIN0702
      QLSAV = QL                                                         MAIN0703
      RCR=R+CR                                                           MAIN0704
      RRCR=R/RCR                                                         MAIN0705
      GO TO (101,101,101,102,102,103,103),NTYPE                          MAIN0710
  101 BETAX=1./ONEQM2                                                    MAIN0720
      BETAY = BETAX                                                      MAIN0730
      BETAMU = QMU*BETAX                                                 MAIN0740
      BETAS = 1.                                                         MAIN0750
      DX=E*T3/(12.*ONEQM2)                                               MAIN0760
      DY=DX                                                              MAIN0770
      QK=G*T3/3.                                                         MAIN0780
      GO TO 110                                                          MAIN0790
  102 BETAX=2./ONEQM2                                                    MAIN0800
      BETAY=BETAX                                                        MAIN0810
      BETAMU=QMU*BETAX                                                   MAIN0820
      BETAS=2.                                                           MAIN0830
      QI=T3/6.+T*H**2/2.                                                 MAIN0840
      DX=E*QI/ONEQM2                                                     MAIN0850
      DY=DX                                                              MAIN0860
      QK=4.*G*QI                                                         MAIN0870
      GO TO 110                                                          MAIN0880
  103 DEVFAC=AS/(T*QLS)                                                  MAIN0890
      BETAX=DEVFAC                                                       MAIN0900
      BETAY=0.                                                           MAIN0910
      BETAMU=0.                                                          MAIN0920
      BETAS=1./DEVFAC                                                    MAIN0930
      DX=E*QIXS/QLS                                                      MAIN0940
      DY=0.                                                              MAIN0950
      QK=0.                                                              MAIN0960
C     SKIN PROPERTIES FOR ALL TYPES                                      MAIN0970
```

TABLE CIII. (Continued)

```
  110  CALL SKIN(1,DUMMY1)                                              MAIN0980
C                                                                       MAIN0990
   112 GO TO (114,116,114,130,114,130,114),NTYPE                        MAIN1000
C                                                                       MAIN1010
C      RING PROPERTIES FOR TYPES 1, 3, 5, AND 7                         MAIN1020
  114  CALL RING(1,DUMMY2)                                              MAIN1030
  115  IF (NTYPE-2)116,116,130                                          MAIN1040
C                                                                       MAIN1050
C      STRINGER PROPERTIES FOR TYPES 1 AND 2                            MAIN1060
  116  CALL STRING(1,DUMMY3)                                            MAIN1070
       ALPHAQ = EBARS/(EBARS+ET)                                        MAIN1071
C                                                                       MAIN1080
C      STRAINS AND LOADS AT ONSET OF LOCAL BUCKLING                     MAIN1090
       CALL LOBUCK(1,DUMMY4,DUMMY5)                                     MAIN1100
C                                                                       MAIN1110
  130  DO 132 I = 1,3                                                   MAIN1120
       DO 132 J = 1,3                                                   MAIN1130
       B(I,J)=0.                                                        MAIN1140
       C(I,J)=0.                                                        MAIN1150
       P(I,J)=0.                                                        MAIN1160
   132 Q(I,J)=0.                                                        MAIN1170
       N1=N1+1                                                          MAIN1180
       NN=NN+1                                                          MAIN1190
       J=0                                                              MAIN1200
C                        ***  CRITICAL MODE LOOP  ***                   MAIN1210
   134 DO 180 N = N1,NN                                                 MAIN1220
       DO 180 M = M1,MM                                                 MAIN1230
       NIN=N-1                                                          MAIN1240
       CALL PRESET(M,NIN)                                               MAIN1250
       CALL SKIN(2,A)                                                   MAIN1260
       IF(NTYPE-2)144,144,146                                           MAIN1270
  144  CALL STRING(2,B)                                                 MAIN1280
   146 GO TO (148,154,148,154,148,154,148),NTYPE                        MAIN1290
   148 GO TO (150,154),NFAIL                                            MAIN1300
  150  CALL RING(2,C)                                                   MAIN1310
   154 IF(NTYPE-6)156,162,162                                           MAIN1320
  156  CALL PRESS(P)                                                    MAIN1330
  162  CALL AXIAL(Q)                                                    MAIN1340
       CALL ADD(A,B,C,D,P)                                              MAIN1350
       CALL DET(D,Q,DDD,DQ,QQQ,QD)                                      MAIN1360
       CALL ROOT(DDD,QQQ,DQ,QD)                                         MAIN1370
       J=J+1                                                            MAIN1380
       QQSAV(J)=QQ                                                      MAIN1390
       MSAV(J)=M                                                        MAIN1400
       NSAV(J)=NIN                                                      MAIN1410
       IF(QQ-QMIN)178,180,180                                           MAIN1420
  178  QMIN=QQ                                                          MAIN1430
       MX=M                                                             MAIN1440
       NX=NIN                                                           MAIN1450
  180  CONTINUE                                                         MAIN1460
C      ***  WRITE INPUT                                                 MAIN1470
       WRITE (6,1112) MAT,E,ES,ER,G,GS,GR,QMU,BS,CS,QLS,AS,QIXS,QIZS,   MAIN1480
     1QJS,H,BR,CR,QLR   ,AR,QIYR,QIZR,QJR,R,QLSAV,T,PP,M1,MM,N1SAV,     MAIN1490
     2NNSAV,NTYPE,NFAIL                                                 MAIN1500
```

TABLE CIII. (Continued)

```
C     ***  WRITE OUTPUT  (LOOP FOR QQ ARRAY)                            MAIN1510
      WRITE (6,1110)                                                    MAIN1520
      II=0                                                              MAIN1530
      JJ=0                                                              MAIN1540
      IF(MTOT-1)810,811,810                                             MAIN1550
  811 IF(NTOT-1)810,812,810                                             MAIN1560
  812 II=1                                                              MAIN1570
      GO TO 809                                                         MAIN1580
  810 IQUIT=1                                                           MAIN1590
      NWRT=NTOT/2                                                       MAIN1600
      IF(N1-1)813,814,813                                               MAIN1610
  813 NWRT=NWRT+1                                                       MAIN1620
  814 DO 800 I=1,NWRT                                                   MAIN1630
      DO 801 J=1,MTOT                                                   MAIN1640
      II=II+1                                                           MAIN1650
      JJ=II+MTOT                                                        MAIN1660
      IF(NSAV(II)-NTOT)805,804,805                                      MAIN1670
  804 IF(MSAV(II)-MTOT)808,809,808                                      MAIN1680
  809 IQUIT=2                                                           MAIN1690
  808 WRITE (6,1105) MSAV(II),NSAV(II),QQSAV(II)                        MAIN1700
      GO TO (801,803),IQUIT                                             MAIN1710
  805 IF(NSAV(JJ)-NTOT)806,807,807                                      MAIN1720
  807 IQUIT=2                                                           MAIN1730
  806 WRITE (6,1105) MSAV(II),NSAV(II),QQSAV(II),MSAV(JJ),NSAV(JJ),     MAIN1740
     1QQSAV(JJ)                                                         MAIN1750
      GO TO (801,803),IQUIT                                             MAIN1760
  801 CONTINUE                                                          MAIN1770
  802 II=II+MTOT                                                        MAIN1780
      WRITE (6,1107)                                                    MAIN1790
  800 MSTRT=MSTRT+MTOT                                                  MAIN1800
  803 QQ=QMIN                                                           MAIN1810
      WRITE (6,1101) QQ,MX,NX                                           MAIN1820
C     TYPES 4, 6, AND 7 ARE FINISHED, AND PROGRAM GOES TO NEXT CASE     MAIN1830
      GO TO (202,202,220,100,220,100,100),NTYPE                         MAIN1840
  202 IF(QQ-QLB)75,75,76                                                MAIN1850
   75 GO TO (220,100),NTYPE                                             MAIN1860
C                                                                       MAIN1870
C                                     ***  RING RESTRAINT  ***          MAIN1880
  220 CALL RESTNT(MX,NX)                                                MAIN1890
      WRITE (6,1102) QQ                                                 MAIN1900
      QTOT=QQ*TWOPIR                                                    MAIN1910
      WRITE (6,1111) QTOT                                               MAIN1920
      GO TO 87                                                          MAIN1930
C                       ***  LOCAL BUCKLING (TYPES 1 AND 2)  ***        MAIN1940
   76 QQHI=QQ*PCTH                                                      MAIN1950
      L=0                                                               MAIN1960
      WRITE (6,1103)                                                    MAIN1970
C     LOOP TO SAVE LOADS FOR LOCAL BUCKLING                             MAIN1980
      DO 70 J=1,JTOT                                                    MAIN1990
      IF(QQSAV(J)-QQHI)71,71,70                                         MAIN2000
   71 L=L+1                                                             MAIN2010
      QQSV(L)=QQSAV(J)                                                  MAIN2020
      MSV(L)=MSAV(J)                                                    MAIN2030
      NSV(L)=NSAV(J)                                                    MAIN2040
```

TABLE CIII. (Continued)

```
 70    CONTINUE                                                         MAIN2050
       DO 211 II=1,L                                                    MAIN2060
       QQ=QQSV(II)                                                      MAIN2070
       MX=MSV(II)                                                       MAIN2080
       NX=NSV(II)                                                       MAIN2090
       IF(QQ-QLB)211,211,401                                            MAIN2100
 401   CALL LOBUCK(2,MX,NX)                                             MAIN2110
       QRED(II)=QQ                                                      MAIN2120
       IF(QRED(II)-QQSV(II)) 212,212,213                                MAIN2130
  213  QRED(II) = QQSV(II)                                              MAIN2140
  212  IF(QRED(II)-QLBMN)80,211,211                                     MAIN2141
 80    QLBMN=QRED(II)                                                   MAIN2142
       MMIN=MX                                                          MAIN2150
       NMIN=NX                                                          MAIN2160
 211   CONTINUE                                                         MAIN2170
       WRITE (6,1104) (MSV(I),NSV(I),QQSV(I),QRED(I),I=1,L)             MAIN2180
       WRITE (6,1101) QLBMN,MMIN,NMIN                                   MAIN2190
       QTOT=QLBMN*TWOPIR                                                MAIN2200
       WRITE (6,1111) QTOT                                              MAIN2210
C                                                                       MAIN2220
C      EPSX AND EPSY PRINT OUT                                          MAIN2230
 87    IF(EPSXSV)81,82,81                                               MAIN2240
 81    WRITE (6,1106) EPSXSV                                            MAIN2250
 82    IF(EPSYSV)84,100,84                                              MAIN2260
 84    WRITE (6,1108) EPSYSV                                            MAIN2270
       GO TO 100                                                        MAIN2280
C                        ***  FORMAT STATEMENTS  ***                    MAIN2290
 1000 FORMAT(2I2)                                                       MAIN2300
 1001 FORMAT(18A4/8E10.0/8E10.0/8E10.0/2E10.0,6I2)                      MAIN2310
 1002 FORMAT(8F10.4)                                                    MAIN2320
 1004 FORMAT(I2)                                                        MAIN2330
 1100 FORMAT(2I5)                                                       MAIN2340
 1101 FORMAT(///4X,42HTHE MINIMUM AXIAL LOAD IN THE ABOVE RANGE ,       MAIN2350
     13HIS ,F9.1,7H LBS/IN/6X,7HAT M = ,I2,9H AND N = ,I2)              MAIN2360
 1102 FORMAT(///4X,42HTHE MINIMUM AXIAL LOAD IN THE ABOVE RANGE ,       MAIN2370
     116HAFTER CORRECTION/6X,21HFOR RING RESTRAINT IS,F7.1,             MAIN2380
     27H LBS/IN)                                                        MAIN2390
 1103 FORMAT(////4X,42HTHE FOLLOWING CASES HAVE BEEN CHECKED FOR ,      MAIN2400
     114HLOCAL BUCKLING//3X,1HM,14X,1HN,7X,15HAXIAL LOAD/INCH,          MAIN2410
     24X,23HREDUCED AXIAL LOAD/INCH/)                                   MAIN2420
 1104 FORMAT(I4,I15,F17.1,F23.1)                                        MAIN2430
 1105 FORMAT(I4,I5,F12.1,I15,I5,F12.1)                                  MAIN2440
 1106 FORMAT(///4X,7HEPSX = ,F8.2,1X,22HIS NOT IN CURVE RANGE.,         MAIN2450
     121H EPSX = -150. IS USED/,20X,28HFOR CALCULATIONS OF MINIMUM ,    MAIN2460
     219HLOCAL BUCKLING LOAD)                                           MAIN2470
 1107 FORMAT(//)                                                        MAIN2480
 1108 FORMAT(///4X,7HEPSY = ,F8.2,1X,22HIS NOT IN CURVE RANGE.,         MAIN2490
     121H EPSY = +100. IS USED/,20X,28HFOR CALCULATIONS OF MINIMUM ,    MAIN2500
     219HLOCAL BUCKLING LOAD)                                           MAIN2510
 1110 FORMAT((1H1,///21X,11HOUTPUT DATA)/(21X,11H///////////)/////(3X,  MAIN2520
     11HM,4X,1HN,2X,15HAXIAL LOAD/INCH,9X,1HM,4X,1HN,2X,                MAIN2530
     215HAXIAL LOAD/INCH/))                                             MAIN2540
 1111 FORMAT(///4X,24HTHE TOTAL AXIAL LOAD IS ,E12.4,4H LBS)            MAIN2550
 1112 FORMAT(1H1///(11X,37HGENERAL INSTABILITY OF ECCENTRICALLY ,       MAIN2560
```

TABLE CIII. (Continued)

```
     121HSTIFFENED CYLINDRICAL)/(14X,31HSHELLS UNDER AXIAL COMPRESSION ,MAIN2570
     220HAND LATERAL PRESSURE)//(18A4)//(35X,10HINPUT DATA)/(35X,       MAIN2580
     310H//////////)///                                                 MAIN2590
     47X,4HE = ,2PE10.2,9X,5HES = ,2PE10.2,7X,5HER = ,2PE10.2//         MAIN2600
     57X,4HG = ,1PE10.2,9X,5HGS = ,1PE10.2,7X,5HGR = ,1PE10.2//         MAIN2610
     67X,6HQMU = ,E12.3,5X,5HBS = ,E12.3,5X,5HCS = ,E12.3//             MAIN2620
     77X,6HQLS = ,E12.3,5X,5HAS = ,E12.3,5X,7HQIXS = ,2PE12.4//         MAIN2630
     87X,7HQIZS = ,2PE12.4,4X,6HQJS = ,2PE12.4,4X,4HH = ,E12.3//        MAIN2640
     97X,5HBR = ,E12.3,6X,5HCR = ,E12.3,5X,6HQLR = ,E12.3//             MAIN2650
     A7X,5HAR = ,E12.3,6X,7HQIYR = ,2PE12.4,3X,7HQIZR = ,2PE12.4//      MAIN2660
     B7X,6HQJR = ,2PE12.4,5X,4HR = ,E12.3,6X,5HQL = ,E12.3//            MAIN2670
     C7X,4HT = ,E12.3,7X,5HPP = ,E12.3///7X,5HM1 = ,I2,6X,              MAIN2680
     D5HMM = ,I2,6X,5HN1 = ,I2,6X,5HNN = ,I2//7X,8HNTYPE = ,            MAIN2690
     EI1,4X,8HNFAIL = ,I1)                                              MAIN2700
      END                                                               MAIN2710
$IBFTC AD       DECK
      SUBROUTINE ADD(A,B,C,D,P)                                         ADD 0000
      DIMENSION A(3,3),B(3,3),C(3,3),D(3,3),P(3,3)                      ADD 0010
      COMMON /BLOCKB/EAX                                                ADD 0020
      DO 1 I=1,3                                                        ADD 0030
      DO 1 J=1,3                                                        ADD 0040
 1    D(I,J)=(A(I,J)+B(I,J)+C(I,J)+P(I,J))/EAX                          ADD 0050
      RETURN                                                            ADD 0060
      END                                                               ADD 0070
$IBFTC AX       DECK
      SUBROUTINE AXIAL(Q)                                               AXL 0000
      DIMENSION Q(3,3)                                                  AXL 0010
      COMMON /BLOCKD/AS,BS,GS,QIXS,QIZS,QJS                             AXL 0020
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                AXL 0030
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     AXL 0040
C                                                                       AXL 0050
      RCS = R + CS                                                      AXL 0060
      RCSR = RCS/R                                                      AXL 0070
C                                                                       AXL 0080
      Q(1,1) = QMB                                                      AXL 0090
      Q(2,1)=0.                                                         AXL 0100
      Q(3,1) = -ALPHAQ*CS*QMB*QMBAR                                     AXL 0110
      Q(1,2)=0.                                                         AXL 0120
      Q(2,2) = (1.+ALPHAQ*(RCSR**2-1.))*QMB                             AXL 0130
      Q(3,2) = ALPHAQ*CS*RCSR*QMB*QNBAR                                 AXL 0140
      Q(1,3) = Q(3,1)                                                   AXL 0150
      Q(2,3) = Q(3,2)                                                   AXL 0160
      Q(3,3) = (1.+ALPHAQ*CS2*(QMB+QNB))*QMB                            AXL 0170
      RETURN                                                            AXL 0180
      END                                                               AXL 0190
$IBFTC DETER    DECK
      SUBROUTINE DET(X,Y,XXS1,XYS1,XXS2,XYS2)                           DET 0000
      DIMENSION X(3,3),Y(3,3),YS(3,3)                                   DET 0010
      K=1                                                               DET 0020
 3    CONTINUE                                                          DET 0030
      XX = X(1,1)*X(2,2)*X(3,3) + X(1,3)*X(2,1)*X(3,2) +                DET 0040
     1   X(1,2)*X(2,3)*X(3,1) - X(1,3)*X(2,2)*X(3,1) -                  DET 0050
     2   X(1,2)*X(2,1)*X(3,3) - X(1,1)*X(2,3)*X(3,2)                    DET 0060
      XY1 = Y(1,1)*(X(2,2)*X(3,3) - X(2,3)*X(3,2)) +                    DET 0070
```

TABLE CIII. (Continued)

```
     1   Y(1,2)*(X(2,3)*X(3,1) - X(2,1)*X(3,3)) +                       DET 0080
     2   Y(1,3)*(X(2,1)*X(3,2) - X(2,2)*X(3,1))                         DET 0090
      XY2 = Y(2,1)*(X(1,3)*X(3,2) - X(1,2)*X(3,3)) +                    DET 0100
     1   Y(2,2)*(X(1,1)*X(3,3) - X(1,3)*X(3,1)) +                       DET 0110
     2   Y(2,3)*(X(1,2)*X(3,1) - X(1,1)*X(3,2))                         DET 0120
      XY3 = Y(3,1)*(X(1,2)*X(2,3) - X(1,3)*X(2,2)) +                    DET 0130
     1   Y(3,2)*(X(1,3)*X(2,1) - X(1,1)*X(2,3)) +                       DET 0140
     2   Y(3,3)*(X(1,1)*X(2,2) - X(1,2)*X(2,1))                         DET 0150
      XY = XY1 + XY2 + XY3                                              DET 0160
      GO TO (1,2),K                                                     DET 0170
 1    XXS1=XX                                                           DET 0180
      XYS1=XY                                                           DET 0190
      DO 4 I=1,3                                                        DET 0200
      DO 4 J=1,3                                                        DET 0210
      YS(I,J)=Y(I,J)                                                    DET 0220
      Y(I,J)=X(I,J)                                                     DET 0230
 4    X(I,J)=YS(I,J)                                                    DET 0240
      K=2                                                               DET 0250
      GO TO 3                                                           DET 0260
 2    XXS2=XX                                                           DET 0270
      XYS2=XY                                                           DET 0280
      DO 5 I=1,3                                                        DET 0290
      DO 5 J=1,3                                                        DET 0300
      X(I,J)=0.                                                         DET 0310
      Y(I,J)=0.                                                         DET 0320
 5    YS(I,J)=0.                                                        DET 0330
      RETURN                                                            DET 0340
      END                                                               DET 0350
$IBFTC DINTR    DECK
      SUBROUTINE DINT(NTBL,VAROUT)                                      DINT0000
      DIMENSION VR(20)                                                  DINT0010
      COMMON /BLOCKA/EPX(30),EPY(30),VAR(6,30,30),NEPX,NEPY             DINT0020
      COMMON /BLOCKI/EPSX,EPSY                                          DINT0030
      VAROUT=0.                                                         DINT0040
      NYM1=NEPY-1                                                       DINT0050
      IF(EPY(1)-EPSY)11,11,6                                            DINT0060
 11   IF(EPX(1)-EPSX)6,12,12                                            DINT0070
 12   DO 3 J=1,NYM1                                                     DINT0080
      IF(EPY(J)-EPSY)4,5,3                                              DINT0090
 4    IF(EPSY-EPY(J+1))5,5,3                                            DINT0100
 3    CONTINUE                                                          DINT0110
      GO TO 6                                                           DINT0120
 5    JL=J                                                              DINT0130
      JT=J+1                                                            DINT0140
      DO 7 I=1,NEPX                                                     DINT0150
      IF(EPX(I)-EPSX)8,8,7                                              DINT0160
 7    CONTINUE                                                          DINT0170
      GO TO 6                                                           DINT0180
 8    IT=I                                                              DINT0190
      IL=I-1                                                            DINT0200
      DO 9 J=JL,JT                                                      DINT0210
      DEN1=EPX(IT)-EPX(IL)                                              DINT0220
      IF(DEN1)9,2,9                                                     DINT0230
 9    VR(J)=VAR(NTBL,IT,J)-(VAR(NTBL,IT,J)-VAR(NTBL,IL,J))*(EPX(IT)     DINT0240
```

TABLE CIII. (Continued)

```
     1-EPSX)/DEN1                                                       DINT0250
      DEN2=EPY(JT)-EPY(JL)                                              DINT0260
      IF(DEN2)10,2,10                                                   DINT0270
 10   VAROUT=VR(JT)-(VR(JT)-VR(JL))*(EPY(JT)-EPSY)/DEN2                 DINT0280
 6    IF(VAROUT)1,2,1                                                   DINT0290
 2    VAROUT=1.E-20                                                     DINT0300
 1    CONTINUE                                                          DINT0310
      RETURN                                                            DINT0320
      END                                                               DINT0330
$IBFTC FCTR    DECK
      SUBROUTINE FACTOR(XXA,QQCR,IBBR)                                  FCTR0000
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                FCTR0010
      COMMON /BLOCKJ/DX,DXS,EBARK,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         FCTR0020
      GET = GAMMAX*ET                                                   FCTR0030
      DXX = DX+ DXS + CS2*EBARS*GET/(EBARS*ONEQM2+GET)                  FCTR0040
      ZK = ET/R2                                                        FCTR0050
      N=(ZK/DXX)**.25*QLR/PI+.5                                         FCTR0060
      IF(N)7,8,7                                                        FCTR0070
 8    XN=1.                                                             FCTR0080
      GO TO 9                                                           FCTR0090
 7    XNDN                                                              FCTR0100
 9    XN2=(XN*PI/QLR)**2                                                FCTR0110
      QQCR=DXX*XN2+ZK/XN2                                               FCTR0120
      IF(QQ-QQCR)1,6,6                                                  FCTR0130
6     XXA=1.E-4                                                         FCTR0150
      IBBR=2                                                            FCTR0151
      RETURN                                                            FCTR0160
 1    D4=4.*DXX                                                         FCTR0170
      QLAM = (ZK/D4)**.25                                               FCTR0180
      Q2 = QLAM**2                                                      FCTR0190
      QD4 = QQ/D4                                                       FCTR0200
      ALFA = SQRT(Q2+QD4)                                               FCTR0210
      BETA = SQRT(ABS(Q2-QD4))                                          FCTR0220
      ALFAL = ALFA*QLR                                                  FCTR0230
      BETAL = BETA*QLR                                                  FCTR0240
      EBTL=EXP(BETAL)                                                   FCTR0250
      SINHBL=(EBTL-1./EBTL)/2.                                          FCTR0260
      COSHBL=(EBTL+1./EBTL)/2.                                          FCTR0270
      SALFA = SIN(ALFAL)                                                FCTR0280
      CALFA = COS(ALFAL)                                                FCTR0290
      SBETA = SIN(BETAL)                                                FCTR0300
      BESAL = BETA*SALFA                                                FCTR0310
      A2 = 2.*ALFA                                                      FCTR0320
      A2B = A2*BETA                                                     FCTR0330
      FACT = QQ-2.*SQRT(ZK*DXX)                                         FCTR0340
      IF (FACT) 2,3,4                                                   FCTR0350
    2 DENOM = A2B*(COSHBL-CALFA)                                        FCTR0360
      BB = Q2*(ALFA*SINHBL+BESAL)/DENOM                                 FCTR0370
      GO TO 5                                                           FCTR0380
    3 DENOM = A2*(1.-CALFA)                                             FCTR0390
      BB = Q2*(ALFAL+SALFA)/DENOM                                       FCTR0400
      GO TO 5                                                           FCTR0410
    4 DENOM = A2B*(COS(BETAL)-CALFA)                                    FCTR0420
      BB = Q2*(BESAL+ALFA*SBETA)/DENOM                                  FCTR0430
```

TABLE CIII. (Continued)

```
    5 XXA = EBARR/(ET+BB*QLR*EBARR)                                     FCTR0440
      IBBR=1                                                            FCTR0441
      RETURN                                                            FCTR0450
      END                                                               FCTR0460
$IBFTC LOCBUC  DECK
      SUBROUTINE LOBUCK(K,MX,NX)                                        LOBK0000
      DIMENSION A(3,3),B(3,3),C(3,3),D(3,3),P(3,3),Q(3,3)               LOBK0010
      COMMON /BLOCKA/EPX(30),EPY(30),VAR(6,30,30),NEPX,NEPY             LOBK0020
      COMMON /BLOCKB/EAX                                                LOBK0030
      COMMON /BLOCKC/EPSXSV,EPSYSV,T                                    LOBK0040
      COMMON /BLOCKD/AS,BS,GS,QIXS,QIZS,QJS                             LOBK0050
      COMMON /BLOCKE/AR,BR,ER,GR,QIYR,QIZR,QJR                          LOBK0060
      COMMON /BLOCKF/BETAMU,BETAS,BETAX,BETAY,GT,ES                     LOBK0070
      COMMON /BLOCKG/DY                                                 LOBK0080
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                LOBK0090
      COMMON /BLOCKI/EPSX,EPSY                                          LOBK0100
      COMMON /BLOCKJ/DX,DXS,EBARR,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         LOBK0110
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     LOBK0120
      COMMON /BLOCKL/CR,QL,QNBMB,QNBMB2,QNBR,QR,RQNB                    LOBK0130
      COMMON /BLOCKM/RCR,RRCR                                           LOBK0140
      COMMON /BLOCKN/IDOWRT,A,B,C,D,P,Q,                                LOBK0150
      IGO=1                                                             LOBK0160
      GO TO (1,2),K                                                     LOBK0170
 1    EPSTAR=(PI*T/QLS)**2/(3.*ONEQM2)                                  LOBK0180
      IBBR=1                                                            LOBK0181
      XA=ALPHAP                                                         LOBK0190
      ETEP=ET*EPSTAR                                                    LOBK0200
      ESEP=EBARS*EPSTAR                                                 LOBK0210
C                                                                       LOBK0220
C     LOOP FOR LOCAL BUCKLING LOAD ACCOUNTING FOR RING RESTRAINT  TYPE 1LOBK0230
 3    GG=ETEP*(2.*QMU*XA-1.)/(4.*(1.-XA)*R)                             LOBK0240
      IF(PP-GG)4,5,5                                                    LOBK0250
 4    WRITE (6,1100)                                                    LOBK0260
      RETURN                                                            LOBK0270
 5    DD=-QMU*XA + SQRT((QMU*XA-1.)**2 + 4.*(1.-XA)*PPR/ETEP)           LOBK0280
      QLB=.5*(DD+1.)*(1.-XA*QMU**2+ET/EBARS)*ESEP + QMU*EBARS*(1.-XA)*  LOBK0290
     1PPR/ET                                                            LOBK0300
      GO TO (40,41),IDOWRT                                              LOBK0310
 41   WRITE (6,1200) QLB                                                LOBK0320
 40   GO TO (6,7),NTYPE                                                 LOBK0330
 6    QQ=QLB                                                            LOBK0340
      CALL FACTOR(XXA,QQCR,IBBR)                                        LOBK0350
      IF(ABS(XXA/XA-1.)-.01)7,7,8                                       LOBK0360
 8    XA=XXA                                                            LOBK0370
      GO TO 3                                                           LOBK0380
C     LOOP FOR LOCAL BUCKLING IF QLB LOAD ) AXIAL LOAD QQ [K > 2]       LOBK0390
 2    IDOGO=1                                                           LOBK0400
      GO TO (38,37),NTYPE                                               LOBK0410
   37 XXA = 1.E-20                                                      LOBK0420
      GO TO 39                                                          LOBK0421
38    CALL FACTOR(XXA,QQCR,IBBR)                                        LOBK0422
   39 SX=-QQ/(ETEP+ESEP)                                                LOBK0423
      SY=PPR*(1.-XXA)/ETEP+QMU*SX*XXA                                   LOBK0430
C                   ***  CHECK POINT 1  ***                             LOBK0440
```

TABLE CIII. (Continued)

```
      COU = 1.                                                          LOBK0441
      GO TO (42,43),IDOWRT                                              LOBK0450
   43 WRITE (6,1201) MX,NX,SX,SY                                        LOBK0460
   42 DELN=0.                                                           LOBK0470
C                                                                       LOBK0480
C     LOOP FOR SX AND SY                                                LOBK0490
    9 XA=XXA                                                            LOBK0500
      EPSXSV=0.                                                         LOBK0510
      EPSYSV=0.                                                         LOBK0520
      EPSX=-(QQ+SX*ETEP)/ESEP                                           LOBK0530
      EPSY=(PPR/ETEP-QMU*SX)*(1.-XA) - XA*DELN/ETEP                     LOBK0540
      IF(EPSX+150.)10,11,11                                             LOBK0550
   10 EPSXSV=EPSX                                                       LOBK0560
      EPSX=-150.                                                        LOBK0570
   11 IF(EPSY-100.)12,13,14                                             LOBK0580
   12 IF(EPSY+2.)15,13,13                                               LOBK0590
   14 EPSYSV=EPSY                                                       LOBK0600
      EPSY=100.                                                         LOBK0610
      GO TO 13                                                          LOBK0620
   15 EPSY=-2.                                                          LOBK0630
   13 CALL DINT(1,SPX)                                                  LOBK0640
      CALL DINT(2,SPY)                                                  LOBK0650
C                        ***  CHECK POINT 2  ***                        LOBK0660
      GO TO (44,45),IDOWRT                                              LOBK0670
   45 WRITE (6,1202) MX,NX,SPX,SPY,EPSX,EPSY                            LOBK0680
   44 IF(ABS(SPY/SY-1.)-.01)16,16,17                                    LOBK0690
   16 IF(ABS(SPX/SX-1.)-.01)18,18,17                                    LOBK0700
   17 SX=SPX                                                            LOBK0710
      SY=SPY                                                            LOBK0720
      DELN=(SY-QMU*SX-EPSY)*ETEP                                        LOBK0730
      GO TO 9                                                           LOBK0740
   18 IF(SY+.25)23,23,24                                                LOBK0750
   23 WRITE (6,1100)                                                    LOBK0760
   24 GAMMAX=SX*ONEQM2/(EPSX+QMU*EPSY)                                  LOBK0770
      COU = COU + 1.                                                    LOBK0771
C                        ***  CHECK POINT 3  ***                        LOBK0780
      GO TO (62,47),IDOWRT                                              LOBK0790
   47 WRITE (6,1203) MX,NX,GAMMAX                                       LOBK0800
  62  GO TO (46,54),NTYPE                                               LOBK0801
  46  CALL FACTOR(XXA,QQCR,IBBR)                                        LOBK0810
      IF ( COU -10.) 54,55,55                                           LOBK0811
  55  IF (XXA-1.E-4) 54,25,54                                           LOBK0812
   54 IF(ABS(XA/XXA-1.)-.01)25,25,9                                     LOBK0820
C                                                                       LOBK0830
   25 QQL=QQ                                                            LOBK0840
      CALL DINT(3,BETAX)                                                LOBK0850
      CALL DINT(4,BETAMU)                                               LOBK0860
      CALL DINT(5,BETAY)                                                LOBK0870
      CALL DINT(6,BETAS)                                                LOBK0880
C                        ***  CHECK POINT 4  ***                        LOBK0890
      GO TO (48,49),IDOWRT                                              LOBK0900
   49 WRITE (6,1204) MX,NX,BETAX,BETAY,BETAMU,BETAS                     LOBK0910
   48 IF(PPR)32,31,32                                                   LOBK0920
   32 ALPHAP=1.-SY*ETEP/PPR                                             LOBK0930
```

TABLE CIII. (Continued)

```
 31   ALPHAQ=1.+SX*ETEP/QQ                                              LOBK0940
      CALL PRESET(MX,NX)                                                LOBK0950
      CALL SKIN(3,A)                                                    LOBK0960
      GO TO (26,27),IGO                                                 LOBK0970
 26   IGO=2                                                             LOBK0980
      CALL STRING(2,B)                                                  LOBK0990
      IF(NTYPE-2)28,27,27                                               LOBK1000
 28   GO TO (29,27),NFAIL                                               LOBK1010
 29   CALL RING(2,C)                                                    LOBK1020
 27   CALL PRESS(P)                                                     LOBK1030
      CALL AXIAL(Q)                                                     LOBK1040
      CALL ADD(A,B,C,D,P)                                               LOBK1050
      CALL DET(D,Q,DDD,DQ,QQQ,QD)                                       LOBK1060
      CALL ROOT(DDD,QQQ,DQ,QD)                                          LOBK1070
C                   ***  CHECK POINT 5  ***                             LOBK1080
      GO TO (50,51),IDOWRT                                              LOBK1090
 51   WRITE (6,1205) MX,NX,ALPHAP,ALPHAQ,QQ                             LOBK1100
 50   IF(QQ-QLB)33,33,34                                                LOBK1110
34    IF (ABS(QQL/QQ-1.)-.01) 60,60,2                                   LOBK1120
60    GO TO (7,61), IBBR                                                LOBK1121
61    WRITE (6,1207) QQ,QQCR                                            LOBK1122
      GO TO 7                                                           LOBK1123
 33   GO TO (35,36),IDOGO                                               LOBK1130
 35   IDOGO=2                                                           LOBK1140
      QQ=QLB                                                            LOBK1150
      SX=-(DD+1.)/2.                                                    LOBK1160
      SY=(DD**2-1.)/4.                                                  LOBK1170
      EPSX=SX-QMU*SY                                                    LOBK1180
      EPSY=SY-QMU*SX                                                    LOBK1190
C                   ***  CHECK POINT 6  ***                             LOBK1200
      GO TO (25,53),IDOWRT                                              LOBK1210
 53   WRITE (6,1206) MX,NX,SX,SY,EPSX,EPSY                              LOBK1220
      GO TO 25                                                          LOBK1230
 36   QQ=QLB                                                            LOBK1240
 7    RETURN                                                            LOBK1250
 1100 FORMAT (//                                                        LOBK1260
    1 12X56HSHELL BUCKLES BETWEEN STRINGERS DUE TO EXTERNAL PRESSURE)   LOBK1270
 1200 FORMAT(///20X,6HQLB = ,F10.3//)                                   LOBK1280
 1201 FORMAT(2X,13HCHECK POINT 1,4X,4HM = ,I2,4X,4HN = ,I2,             LOBK1290
     14X,5HSX = ,F9.2,4X,5HSY = ,F9.2)                                  LOBK1300
 1202 FORMAT(2X,13HCHECK POINT 2,4X,4HM = ,I2,4X,4HN □ ,I2,             LOBK1310
     14X,6HSPX = ,F9.2,4X,6HSPY = ,F9.2,4X,7HEPSX = ,F9.3,4X,           LOBK1320
     27HEPSY = ,F9.3)                                                   LOBK1330
 1203 FORMAT(2X,13HCHECK POINT 3,4X,4HM = ,I2,4X,4HN = ,I2,             LOBK1340
     14X,9HGAMMAX = ,F9.5)                                              LOBK1350
 1204 FORMAT(2X,13HCHECK POINT 4,4X,4HM = ,I2,4X,4HN = ,I2,             LOBK1360
     14X,8HBETAX = ,F9.5,4X,8HBETAY = ,F9.5,4X,9HBETAMU = ,F9.5,        LOBK1370
     24X,8HBETAS = ,F9.5)                                               LOBK1380
 1205 FORMAT(2X,13HCHECK POINT 5,4X,4HM = ,I2,4X,4HN = ,I2,             LOBK1390
     14X,9HALPHAP = ,F9.6,4X,9HALPHAQ = ,F9.6,4X,5HQQ = ,F9.2)          LOBK1400
 1206 FORMAT(2X,13HCHECK POINT 6,4X,4HM = ,I2,4X,4HN □ ,I2,             LOBK1410
     14X,5HSX = ,F9.2,4X,5HSY = ,F9.2,4X,7HEPSX = ,F9.3,4X,             LOBK1420
     27HEPSY = ,F9.3)                                                   LOBK1430
1207  FORMAT(//2X,63HCYLINDER BUCKLES BETWEEN RINGS AS A BEAM ON ELASTICLOBK1431
```

TABLE CIII. (Continued)

```
     1 FOUNDATION.,/2X,35HTHE STABILITY LOAD BEING CHECKED IS,E12.5,7H LLOBK1432
     2BS/IN,43H AND THE BEAM ON ELASTIC FOUNDATION. LOAD IS,E12.5,8H LBS/LOBK1433
     3IN.)                                                              LOBK1434
      END                                                               LOBK1440
$IBFTC PRST    DECK
      SUBROUTINE PRESET(M,N)                                            PSET0000
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                PSET0010
      COMMON /BLOCKJ/DX,DXS,EBARR,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         PSET0020
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     PSET0030
      COMMON /BLOCKL/CR,QL,QNBMB,QNBMB2,QNBR,QR,RQNB                    PSET0040
      QM=M                                                              PSET0050
      QN=N                                                              PSET0060
      GO TO (1,2,1,2,1,2,1),NTYPE                                       PSET0070
 1    GO TO (2,3),NFAIL                                                 PSET0080
 3    QL=QLR                                                            PSET0090
 2    QMBAR=QM*PI/QL                                                    PSET0100
      QNBAR = QN/R                                                      PSET0110
      QMB=QMBAR**2                                                      PSET0120
      QNB=QNBAR**2                                                      PSET0130
      QNBMB=QNBAR*QMBAR                                                 PSET0140
      QNBMB2=QNBMB**2                                                   PSET0150
      QNBR=QNBAR/R                                                      PSET0160
      RQNB=R*QNB                                                        PSET0170
      QR=CR*RQNB+1.                                                     PSET0180
      RETURN                                                            PSET0190
      END                                                               PSET0200
$IBFTC PRSS    DECK
      SUBROUTINE PRESS(P)                                               PRSS0000
      DIMENSION P(3,3)                                                  PRSS0010
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                PRSS0020
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     PRSS0030
      COMMON /BLOCKL/CR,QL,QNBMB,QNBMB2,QNBR,QR,RQNB                    PRSS0040
      COMMON /BLOCKM/RCR,RRCR                                           PRSS0050
      IF(PP)1,2,1                                                       PRSS0060
 1    QRCR=-CR*RQNB+1./RRCR                                             PRSS0070
      P(1,1) = -(1.-ALPHAP*CR/RCR)*RQNB*PP                              PRSS0080
      P(3,1) = -(1.-ALPHAP*CR*RRCR*RQNB)*QMBAR*PP                       PRSS0090
      P(2,2) = -(1.+ALPHAP*CR/R)*RQNB*PP                                PRSS0100
      P(3,2) = -(1.+ALPHAP*CR*RQNB)*QNBAR*PP                            PRSS0110
      P(1,3) = P(3,1)                                                   PRSS0120
      P(2,3) = P(3,2)                                                   PRSS0130
      P(3,3) = -(RQNB+ALPHAP*(-1./R+QR**2/RCR+CR*RRCR*QRCR*QMB))*PP     PRSS0140
 2    RETURN                                                            PRSS0150
      END                                                               PRSS0160
$IBFTC REST    DECK
      SUBROUTINE RESTNT(MX,NX)                                          REST0000
      DIMENSION A(3,3),B(3,3),C(3,3),D(3,3),P(3,3),Q(3,3)               REST0010
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                REST0020
      COMMON /BLOCKJ/DX,DXS,EBARR,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         REST0030
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     REST0040
      COMMON /BLOCKN/IDOWRT,A,B,C,D,P,Q                                 REST0041
      IGO=1                                                             REST0050
      GAMMAX=1.                                                         REST0060
 1    QQL=QQ                                                            REST0070
```

TABLE CIII. (Continued)

```
      CALL FACTOR(XA,QQCR,IBBR)                                         REST0080
      ONEA=1.-XA*QMU**2                                                 REST0090
      DENO=ET+EBARS*ONEA                                                REST0100
      IF(PPR)9,10,9                                                     REST0110
    9 ALPHAP=XA*(ET+EBARS*ONEQM2 + QMU*QQ*ET/PPR)/DENO                   REST0120
   10 ALPHAQ=EBARS*(ONEA + QMU*PPR*(1.-XA)/QQ)/DENO                     REST0130
      CALL PRESET(MX,NX)                                                REST0140
      GO TO (2,8),IGO                                                   REST0150
    2 IGO=2                                                             REST0160
      CALL SKIN(2,A)                                                    REST0170
      IF(NTYPE-2)3,3,4                                                  REST0180
    3 CALL STRING(2,B)                                                  REST0190
    4 GO TO (5,6),NFAIL                                                 REST0200
    5 CALL RING(2,C)                                                    REST0210
    6 CALL PRESS(P)                                                     REST0220
      CALL AXIAL(Q)                                                     REST0230
      CALL ADD(A,B,C,D,P)                                               REST0240
      CALL DET(D,Q,DDD,DQ,QQQ,QD)                                       REST0250
      CALL ROOT(DDD,QQQ,DQ,QD)                                          REST0260
      IF (ABS(QQL/QQ-1.)-.01) 11,11,1                                   REST0270
   11 GO TO (8,12), IBBR                                                REST0271
   12 WRITE (6,1200) QQ,QQCR                                            REST0272
    8 RETURN                                                            REST0280
 1200 FORMAT(//2X,63HCYLINDER BUCKLES BETWEEN RINGS AS A BEAM ON ELASTICREST0290
     1 FOUNDATION.,/2X,35HTHE STABILITY LOAD BEING CHECKED IS,E12.5,7H LREST0300
     2BS/IN,43H AND THE BEAM ON ELASTIC FOUNDATION LOAD IS,E12.5,8H LBS/REST0310
     3IN.)                                                              REST0320
      END                                                               REST0330
$IBFTC RINGER  DECK
      SUBROUTINE RING(K,C)                                              RING0000
      DIMENSION C(3,3)                                                  RING0010
      COMMON /BLOCKE/AR,BR,ER,GR,QIYR,QIZR,QJR                          RING0020
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                RING0030
      COMMON /BLOCKJ/DX,DXS,EBARR,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         RING0040
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     RING0050
      COMMON /BLOCKL/CR,QL,QNBMB,QNBMB2,QNBR,QR,RQNB                    RING0060
      COMMON /BLOCKM/RCR,RRCR                                           RING0070
      GO TO (1,2),K                                                     RING0080
    1 ERQLR=ER/QLR                                                      RING0090
      EBARR=ERQLR*AR                                                    RING0100
      DYR=ERQLR*QIYR                                                    RING0110
      DZR=ERQLR*QIZR                                                    RING0120
      QKR = GR*QJR/QLR                                                  RING0130
      RBR □ R + BR                                                      RING0140
      RCR □ R + CR                                                      RING0150
      RRBR = R/RBR                                                      RING0160
      RRCR = R/RCR                                                      RING0170
      ALPHAP=EBARR/(EBARR+ET)                                           RING0180
      RETURN                                                            RING0190
    2 QRBR=-BR*RQNB+1./RRBR                                             RING0200
      QRCR=-CR*RQNB+1./RRCR                                             RING0210
      RRBR3=RRBR**3                                                     RING0220
      C(1,1)=-(DZR*QNB**2+QKR*QNBR**2)*RRBR3                            RING0230
      C(3,1) = -(DZR*QRBR + QKR)*QMBAR*(QNBAR*RRBR)**2/RBR              RING0240
```

TABLE CIII. (Continued)

```
      C(2,2)=-(EBARR*RCR+DYR/RCR)*QNB/R                                 RING0250
      C(3,2)=-QNBR*(EBARR*QR +DYR*RRCR*QNB)                             RING0260
      C(1,3)=C(3,1)                                                     RING0270
      C(2,3)=C(3,2)                                                     RING0280
      C(3,3)=-(EBARR*QR**2/R2+DYR*QNB**2)*RRCR-QMB*RRBR3*               RING0290
     1(DZR*QRBR**2/R2+QKR*QNB)                                          RING0300
      RETURN                                                            RING0310
      END                                                               RING0320
$IBFTC ROOTER  DECK
      SUBROUTINE ROOT(DDD,QQQ,DQ,QD)                                    ROOT0000
      COMMON /BLOCKB/EAX                                                ROOT0010
      COMMON /BLOCKJ/DX,DXS,EBARR,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         ROOT0020
      Q1=-DDD/DQ                                                        ROOT0030
 1    E1=QQQ*Q1**3+QD*Q1**2+DQ*Q1+DDD                                   ROOT0040
      DEDQ = 3.*QQQ*Q1**2 + 2.*QD*Q1 + DQ                               ROOT0050
      IF(ABS(DEDQ)-(1.E-20))2,2,3                                       ROOT0051
    2 SDEDQ=DEDQ/ABS(DEDQ)                                              ROOT0060
      DEDQ=1.E-20*SDEDQ                                                 ROOT0061
      IF(E1 -1.E+17)3,3,8                                               ROOT0070
    8 Q2=1.10*Q1                                                        ROOT0071
      GO TO 9                                                           ROOT0080
    3 Q2=Q1-E1/DEDQ                                                     ROOT0081
    9 IF(ABS(Q2/Q1-1.)-.01)4,4,5                                        ROOT0090
 5    Q1=Q2                                                             ROOT0100
      GO TO 1                                                           ROOT0110
 4    RAD = -3.*(QQQ*Q2)**2 - 2.*QQQ*QD*Q2 + QD**2 - 4.*QQQ*DQ          ROOT0120
      IF(RAD)6,7,7                                                      ROOT0130
 7    Q3=-(QD+QQQ*Q2+SQRT(RAD))/(2.*QQQ)                                ROOT0140
      Q4=-(QD+QQQ*Q2-SQRT(RAD))/(2.*QQQ)                                ROOT0150
      Q2=AMIN1(Q2,Q3,Q4)                                                ROOT0160
      QQ=Q2*EAX                                                         ROOT0170
 6    RETURN                                                            ROOT0180
      END                                                               ROOT0190
$IBFTC SKINER  DECK
      SUBROUTINE SKIN(K,A)                                              SKIN0000
      DIMENSION A(3,3)                                                  SKIN0010
      COMMON /BLOCKB/EAX                                                SKIN0020
      COMMON /BLOCKF/BETAMU,BETAS,BETAX,BETAY,GT,ES                     SKIN0030
      COMMON /BLOCKG/DY                                                 SKIN0040
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                SKIN0050
      COMMON /BLOCKJ/DX,DXS,EBARR,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         SKIN0060
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     SKIN0070
      COMMON /BLOCKL/CR,QL,QNBMB,QNBMB2,QNBR,QR,RQNB                    SKIN0080
      GO TO (1,2,1),K                                                   SKIN0090
 1    EBARX=ET*BETAX                                                    SKIN0100
      EBARY=ET*BETAY                                                    SKIN0110
      EBARMU=ET*BETAMU                                                  SKIN0120
      GBAR=GT*BETAS                                                     SKIN0130
      GO TO (3,2,2),K                                                   SKIN0140
 2    A(1,1)=-EBARX*QMB-GBAR*QNB                                        SKIN0150
      A(2,1)=QNBMB*(EBARMU+GBAR)                                        SKIN0160
      A(3,1) = EBARMU*QMBAR/R                                           SKIN0170
      A(1,2) = A(2,1)                                                   SKIN0180
      A(2,2)=-(EBARY+DY/R2)*QNB-(GBAR+QK/R2)*QMB                        SKIN0190
```

TABLE CIII. (Continued)

```
      A(3,2)=-(EBARY+(QK+QMU*DY)*QMB+DY*QNB)*QNBR                       SKIN0200
      A(1,3) = A(3,1)                                                   SKIN0210
      A(2,3) = A(3,2)                                                   SKIN0220
      A(3,3)=-(EBARY/R2+DY*(QNB**2+2.*QMU*QNBMB2)+QK*QNBMB2+DX*QMB**2)  SKIN0230
      EAX=EBARX+EBARS                                                   SKIN0240
 3    RETURN                                                            SKIN0250
      END                                                               SKIN0260
$IBFTC STRNG   DECK
      SUBROUTINE STRING(K,B)                                            STRN0000
      DIMENSION B(3,3)                                                  STRN0010
      COMMON /BLOCKD/AS,BS,GS,QIXS,QIZS,QJS                             STRN0020
      COMMON /BLOCKF/BETAMU,BETAS,BETAX,BETAY,GT,ES                     STRN0030
      COMMON /BLOCKH/CS,CS2,EBARS,ET,QMB,QMBAR,QNB,QNBAR                STRN0040
      COMMON /BLOCKJ/DX,DXS,EBARR,GAMMAX,ONEQM2,PI,QK,QLR,QQ,R2         STRN0050
      COMMON /BLOCKK/ALPHAP,NFAIL,NTYPE,PP,PPR,QLS,QMU,R,QLB,ALPHAQ     STRN0060
      COMMON /BLOCKL/CR,QL,QNBMB,QNBMB2,QNBR,QR,RQNB                    STRN0070
      GO TO (1,2),K                                                     STRN0080
 1    ESQLS=ES/QLS                                                      STRN0090
      CS2=CS*CS                                                         STRN0100
      EBARS=ESQLS*AS                                                    STRN0110
      DXS=ESQLS*QIXS                                                    STRN0120
      DZS=ESQLS*QIZS                                                    STRN0130
      QKS = GS*QJS/QLS                                                  STRN0140
      RETURN                                                            STRN0150
 2    QKSQMB=QKS*QMB                                                    STRN0160
      RBS = R + BS                                                      STRN0161
      DZSMB4=DZS*QMB**2                                                 STRN0170
      B(1,1)=-EBARS*QMB                                                 STRN0180
      B(3,1)=-B(1,1)*CS*QMBAR                                           STRN0190
      B(2,2)=-(QKSQMB+DZSMB4*RBS**2)/R2                                 STRN0200
      B(3,2)=-(QKSQMB+DZSMB4*RBS*BS)*QNBR                               STRN0210
      B(1,3) □ B(3,1)                                                   STRN0220
      B(2,3)=B(3,2)                                                     STRN0230
      B(3,3)=-((EBARS*CS2+DXS)*QMB**2+(QKS+DZS*CS2*QMB)*QNBMB2)         STRN0240
      RETURN                                                            STRN0250
      END                                                               STRN0260
$DATA
01
.25
2929
.0001     -1.       -2.       -3.       -4.       -5.       -6.       -7.
-8.       -9.       -10.      -12.      -14.      -16.      -18.      -20.
-25.      -30.      -35.      -40.      -45.      -50.      -60.      -70.
-80.      -90.      -100.     -125.     -150.
-2.       -1.       .0001     1.        2.        3.        4.        5.
6.        7.        8.        9.        10.       12.       14.       16.
18.       20.       25.       30.       35.       40.       45.       50.
60.       70.       80.       90.       100.
   -0.3961   -0.3398   -0.3723   -0.7095    0.0       0.0       0.0       0.0
    0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
    0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
    0.0       0.0       0.0       0.0       0.0
   -0.8669   -0.8714   -0.9483   -1.1596   -1.2797    0.0       0.0       0.0
    0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
```

TABLE CIII. (Continued)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-1.3152	-1.3559	-1.4513	-1.5862	-1.6986	-1.8045	-1.8650	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-1.7410	-1.7934	-1.8814	-1.9893	-2.0924	-2.1798	-2.2429	-2.2922
-2.3317	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-2.1505	-2.2054	-2.2821	-2.3689	-2.4611	-2.5397	-2.6043	-2.6555
-2.6941	-2.7165	-2.7190	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-2.5473	-2.5759	-2.6381	-2.7197	-2.8047	-2.8842	-2.9492	-3.0025
-3.0426	-3.0708	-3.0872	-3.0957	-3.1127	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-2.8776	-2.9067	-2.9703	-3.0479	-3.1318	-3.2090	-3.2776	-3.3332
-3.3772	-3.4101	-3.4366	-3.4491	-3.4615	-3.4343	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-3.1801	-3.2169	-3.2824	-3.3601	-3.4392	-3.5171	-3.5864	-3.6465
-3.6979	-3.7344	-3.7672	-3.7863	-3.7986	-3.7949	-3.7882	-3.7203
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-3.4691	-3.5147	-3.5796	-3.6536	-3.7301	-3.8080	-3.8793	-3.9434
-3.9976	-4.0431	-4.0790	-4.1073	-4.1240	-4.1380	-4.1300	-4.0890
-4.0428	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-3.7514	-3.7986	-3.8659	-3.9362	-4.0150	-4.0887	-4.1597	-4.2270
-4.2850	-4.3354	-4.3780	-4.4086	-4.4377	-4.4636	-4.4624	-4.4413
-4.4015	-4.3491	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-4.0221	-4.0743	-4.1355	-4.2097	-4.2832	-4.3558	-4.4269	-4.4960
-4.5595	-4.6134	-4.6584	-4.6989	-4.7326	-4.7717	-4.7854	-4.7772
-4.7459	-4.7023	-4.4901	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-4.5369	-4.5943	-4.6543	-4.7243	-4.7954	-4.8672	-4.9375	-5.0084
-5.0742	-5.1316	-5.1852	-5.2349	-5.2790	-5.3426	-5.3836	-5.3998
-5.3919	-5.3682	-5.2231	-5.0486	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-5.0282	-5.0861	-5.1473	-5.2113	-5.2797	-5.3533	-5.4205	-5.4909
-5.5551	-5.6167	-5.6743	-5.7312	-5.7811	-5.8577	-5.9237	-5.9619
-5.9808	-5.9801	-5.8992	-5.7456	-5.5110	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-5.4920	-5.5516	-5.6088	-5.6743	-5.7433	-5.8121	-5.8774	-5.9448
-6.0149	-6.0754	-6.1360	-6.1957	-6.2475	-6.3404	-6.4143	-6.4760
-6.5183	-6.5380	-6.5184	-6.4012	-6.2139	-5.9683	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-5.9417	-5.9952	-6.0536	-6.1212	-6.1874	-6.2479	-6.3175	-6.3844
-6.4488	-6.5136	-6.5718	-6.6369	-6.6916	-6.7938	-6.8811	-6.9521
-7.0095	-7.0531	-7.0807	-7.0154	-6.8735	-6.6639	-6.3820	-6.1177
0.0	0.0	0.0	0.0	0.0			

TABLE CIII. (Continued)

-6.3728	-6.4245	-6.4833	-6.5465	-6.6115	-6.6754	-6.7374	-6.8032
-6.8714	-6.9331	-6.9934	-7.0528	-7.1122	-7.2190	-7.3178	-7.4005
-7.4701	-7.5235	-7.6012	-7.5882	-7.4897	-7.3195	-7.0890	-6.8393
-6.2224	0.0	0.0	0.0	0.0			
-7.3826	-7.4305	-7.4869	-7.5495	-7.6139	-7.6744	-7.7332	-7.8016
-7.8615	-7.9184	-7.9824	-8.0432	-8.1004	-8.2191	-8.3243	-8.4243
-8.5103	-8.5900	-8.7401	-8.8229	-8.8405	-8.7840	-8.6677	-8.4842
-8.0117	-7.2792	-6.5580	0.0	0.0			
-8.3193	-8.3699	-8.4342	-8.4889	-8.5398	-8.6030	-8.6639	-8.7194
-8.7857	-8.8455	-8.9107	-8.9627	-9.0270	-9.1396	-9.2437	-9.3493
-9.4515	-9.5413	-9.7378	-9.8819	-9.9703	-9.9991	-9.9766	-9.9017
-9.5910	-9.0787	-8.4549	-7.5810	0.0			
-9.2161	-9.2669	-9.3132	-9.3724	-9.4257	-9.4883	-9.5463	-9.5953
-9.6522	-9.7175	-9.7734	-9.8347	-9.8876	-10.0074	-10.1148	-10.2238
-10.3275	-10.4278	-10.6386	-10.8201	-10.9599	-11.0501	-11.0940	-11.0918
-10.9603	-10.6410	-10.1495	-9.4728	-8.7652			
-10.0613	-10.1109	-10.1514	-10.2154	-10.2646	-10.3153	-10.3748	-10.4370
-10.4947	-10.5420	-10.6086	-10.6644	-10.7110	-10.8309	-10.9334	-11.0514
-11.1470	-11.2530	-11.4870	-11.6834	-11.8518	-11.9845	-12.0759	-12.1267
-12.1196	-11.9661	-11.6418	-11.1482	-10.5550			
-10.8667	-10.9145	-10.9701	-11.0163	-11.0690	-11.1280	-11.1764	-11.2309
-11.2908	-11.3361	-11.4017	-11.4448	-11.5062	-11.6221	-11.7316	-11.8288
-11.9439	-12.0403	-12.2851	-12.4935	-12.6839	-12.8394	-12.9653	-13.0593
-13.1518	-13.1104	-12.9318	-12.6072	-12.1512			
-11.6446	-11.6900	-11.7443	-11.7964	-11.8461	-11.8980	-11.9520	-12.0032
-12.0492	-12.1022	-12.1607	-12.2207	-12.2687	-12.3726	-12.4902	-12.5975
-12.7010	-12.7946	-13.0503	-13.2740	-13.4720	-13.6430	-13.7933	-13.9093
-14.0756	-14.1168	-14.0444	-13.8498	-13.5538			
-13.1262	-13.1690	-13.2161	-13.2697	-13.3172	-13.3650	-13.4059	-13.4677
-13.5049	-13.5615	-13.6228	-13.6598	-13.7246	-13.8239	-13.9335	-14.0296
-14.1349	-14.2279	-14.4712	-14.7146	-14.9326	-15.1261	-15.3205	-15.4791
-15.7255	-15.8933	-15.9676	-15.9498	-15.8549			
-14.5130	-14.5652	-14.6123	-14.6575	-14.7043	-14.7382	-14.7993	-14.8503
-14.8980	-14.9367	-14.9972	-15.0474	-15.0980	-15.1906	-15.2814	-15.3850
-15.4970	-15.5955	-15.8367	-16.0672	-16.2927	-16.5088	-16.7119	-16.8843
-17.2072	-17.4509	-17.6141	-17.7115	-17.7432			
-15.8346	-15.8920	-15.9371	-15.9742	-16.0161	-16.0623	-16.1133	-16.1634
-16.2099	-16.2475	-16.3054	-16.3423	-16.4035	-16.4899	-16.5784	-16.6774
-16.7875	-16.8698	-17.1278	-17.3624	-17.5757	-17.8083	-17.9994	-18.2042
-18.5457	-18.8460	-19.0775	-19.2514	-19.3591			
-17.1106	-17.1605	-17.2049	-17.2344	-17.2917	-17.3338	-17.3802	-17.4247
-17.4729	-17.5163	-17.5480	-17.6060	-17.6534	-17.7478	-17.8406	-17.9383
-18.0307	-18.1126	-18.3563	-18.5924	-18.8084	-19.0354	-19.2298	-19.4352
-19.8069	-20.1432	-20.4275	-20.6522	-20.8041			
-18.3376	-18.3787	-18.4237	-18.4457	-18.5084	-18.5536	-18.5857	-18.6220
-18.6821	-18.7287	-18.7735	-18.7969	-18.8473	-18.9515	-19.0263	-19.1343
-19.2266	-19.3185	-19.5426	-19.7744	-19.9839	-20.2154	-20.4176	-20.6162
-21.0151	-21.3686	-21.6813	-21.9373	-22.1474			
-21.2195	-21.2445	-21.2995	-21.3168	-21.3838	-21.4003	-21.4653	-21.4941
-21.5445	-21.5840	-21.6190	-21.6706	-21.7103	-21.7982	-21.8668	-21.9454
-22.0315	-22.1231	-22.3534	-22.5675	-22.7780	-23.0065	-23.2170	-23.4140
-23.8189	-24.1996	-24.5315	-24.8455	-25.1384			
-23.9100	-23.9489	-23.9820	-24.0254	-24.0385	-24.1027	-24.1363	-24.1799
-24.2178	-24.2582	-24.2990	-24.3350	-24.3493	-24.4454	-24.5408	-24.6209

TABLE CIII. (Continued)

```
-24.6976  -24.7818  -24.9676  -25.1999  -25.4057  -25.5874  -25.8178  -25.9963
-26.4112  -26.8043  -27.1651  -27.4844  -27.8342
 -0.9335   -0.6083   -0.3214    0.3297    0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.9472   -0.5773   -0.1598    0.4947    1.3314    0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.9395   -0.5271   -0.0230    0.6380    1.4366    2.2984    3.2064    0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.9104   -0.4577    0.0890    0.7596    1.5345    2.3775    3.2633    4.1831
  5.1421    0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.8706   -0.3874    0.1826    0.8595    1.6251    2.4537    3.3215    4.2195
  5.1558    6.0721    7.0340    0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.8287   -0.3360    0.2602    0.9486    1.7084    2.5270    3.3810    4.2632
  5.1780    6.0946    7.0373    7.9850    8.9760    0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.8255   -0.2905    0.3353    1.0329    1.7924    2.5982    3.4418    4.3142
  5.2087    6.1209    7.0493    7.9866    8.9462   10.8214    0.0       0.0
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.8122   -0.2382    0.4083    1.1146    1.8714    2.6707    3.5053    4.3658
  5.2479    6.1510    7.0700    7.9970    8.9375   10.8319   12.8228   14.7110
  0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.7652   -0.1826    0.4798    1.1925    1.9490    2.7457    3.5693    4.4241
  5.2981    6.1904    7.0994    8.0162    8.9499   10.8405   12.7791   14.6926
 16.6463    0.0       0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.7492   -0.1256    0.5505    1.2688    2.0269    2.8179    3.6380    4.4820
  5.3489    6.2335    7.1352    8.0471    8.9834   10.8472   12.7549   14.6774
 16.6177   18.5333    0.0       0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.7087   -0.0664    0.6185    1.3439    2.1025    2.8928    3.7063    4.5481
  5.4072    6.2799    7.1826    8.0787    9.0027   10.8520   12.7502   14.6654
 16.5950   18.5191   23.6156    0.0       0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.6182    0.0504    0.7532    1.4874    2.2490    3.0393    3.8471    4.6804
  5.5284    6.3876    7.2689    8.1610    9.0635   10.9025   12.7707   14.6510
 16.5067   18.4964   23.4546   28.1460    0.0       0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
 -0.5184    0.1685    0.8852    1.6262    2.3942    3.1834    3.9885    4.8124
  5.6513    6.5054    7.3765    8.2649    9.1468   10.9600   12.8178   14.6752
 16.5614   18.4814   23.3424   28.1688   33.2423    0.0       0.0       0.0
  0.0       0.0       0.0       0.0       0.0
```

TABLE CIII. (Continued)

-0.4186	0.2854	1.0121	1.7611	2.5306	3.3193	4.1245	4.9513
5.7846	6.6325	7.4912	8.3627	9.2436	11.0351	12.8627	14.7097
16.6094	18.4741	23.2790	28.1678	33.1166	38.1997	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.3132	0.4015	1.1370	1.8932	2.6688	3.4532	4.2598	5.0773
5.9132	6.7548	7.6073	8.4784	9.3505	11.1351	12.9372	14.7676
16.6275	18.5001	23.2644	28.1431	33.0214	38.0455	42.9476	48.0653
0.0	0.0	0.0	0.0	0.0			
-0.2077	0.5155	1.2599	2.0207	2.7981	3.5877	4.3941	5.2105
6.0399	6.8833	7.7317	8.5929	9.4748	11.2233	13.0209	14.8401
16.7096	18.5556	23.2709	28.0947	32.9567	37.9210	42.8472	47.8892
57.9853	0.0	0.0	0.0	0.0			
0.0510	0.7952	1.5546	2.3274	3.1125	3.9113	4.7154	5.5339
6.3562	7.1920	8.0355	8.8883	9.7514	11.5025	13.2787	15.0517
16.8677	18.6959	23.3607	28.0784	32.9284	37.7393	42.6507	47.5675
57.5687	67.9993	77.8708	0.0	0.0			
0.3074	1.0670	1.8413	2.6217	3.4121	4.2114	5.0251	5.8365
6.6684	7.5002	8.3453	9.1966	10.0421	11.7762	13.5234	15.2969
17.0854	18.8953	23.4893	28.1834	32.9378	37.7428	42.5321	47.4152
57.2839	67.3588	77.3674	86.6851	0.0			
0.5634	1.3353	2.1138	2.9034	3.7007	4.5099	5.3211	6.1390
6.9624	7.8010	8.6405	9.4837	10.3356	12.0585	13.7949	15.5628
17.3351	19.1300	23.6715	28.3360	33.0303	37.7716	42.5260	47.4323
57.1309	67.0040	76.9901	86.8148	97.2397			
0.6123	1.5934	2.3786	3.1772	3.9775	4.7845	5.6011	6.4280
7.2577	8.0871	8.9283	9.7708	10.6210	12.3386	14.0653	15.8299
17.5845	19.3694	23.9030	28.5100	33.1581	37.8222	42.6040	47.3689
57.1097	66.9349	76.7390	86.8028	96.8470			
1.0564	1.8444	2.6414	3.4407	4.2467	5.0622	5.8786	6.7083
7.5360	8.3677	9.2084	10.0529	10.9019	12.6110	14.3401	16.0971
17.8478	19.6176	24.1090	28.7043	33.3211	37.9548	42.6656	47.4184
57.0324	66.7958	76.6141	86.6491	96.5257			
1.2964	2.0904	2.8934	3.6998	4.5104	5.3262	6.1492	6.9799
7.8020	8.6377	9.4805	10.3362	11.1738	12.8855	14.6098	16.3563
18.1101	19.8687	24.3757	28.9025	33.5135	38.1083	42.8452	47.5361
57.1052	66.7463	76.5253	86.3537	96.2759			
1.7678	2.5698	3.3806	4.1969	5.0148	5.8405	6.6633	7.4976
8.3298	9.1676	10.0127	10.8536	11.7125	13.4126	15.1405	16.8760
18.6155	20.3744	24.8107	29.3489	33.8678	38.5135	43.1741	47.8678
57.2517	66.7959	76.5638	86.1700	95.9772			
2.2184	3.0332	3.8516	4.6728	5.4964	6.3200	7.1602	7.9975
8.8315	9.6704	10.5219	11.3703	12.2215	13.9351	15.6443	17.3755
19.1492	20.8958	25.3167	29.8035	34.2937	38.9016	43.5385	48.1682
57.5706	67.0831	76.6007	86.3462	95.9814			
2.6538	3.4801	4.3054	5.1296	5.9584	6.7914	7.6321	8.4692
9.3139	10.1524	11.0051	11.8484	12.7125	14.4146	16.1360	17.8675
19.6228	21.3639	25.7796	30.2449	34.7830	39.3463	43.8810	48.4687
57.8695	67.2567	76.7466	86.4371	96.0581			
3.0846	3.9149	4.7452	5.5736	6.4118	7.2487	8.0939	8.9322
9.7769	10.6313	11.4719	12.3285	13.1817	14.8954	16.6217	18.3521
20.1031	21.8351	26.2513	30.6960	35.1969	39.7124	44.2674	48.8511
58.1758	67.5316	77.0749	86.5969	96.0419			
3.5051	4.3363	5.1726	6.0025	6.8483	7.6929	8.5288	9.3766
10.2328	11.0788	11.9296	12.7828	13.6387	15.3633	17.0839	18.8220

TABLE CIII. (Continued)

```
20.5589    22.3087    26.7118    31.1490    35.6243    40.1946    44.6836    49.2938
58.5806    67.8299    77.3423    86.7796    96.2442
 4.5079     5.3461     6.1965     7.0345     7.8911     8.7351     9.5913    10.4463
11.3033    12.1569    13.0071    13.8772    14.7335    16.4717    18.1860    19.9259
21.6732    23.4180    27.8259    32.3022    36.7440    41.2444    45.7278    50.3337
59.4323    68.7558    78.0443    87.4061    96.8346
 5.4663     6.3177     7.1686     8.0241     8.8685     9.7377    10.5917    11.4510
12.3104    13.1788    14.0323    14.8973    15.7524    17.4905    19.2358    20.9893
22.7257    24.4810    28.9161    33.3410    37.7846    42.2569    46.8369    51.3266
60.4701    69.7196    78.9512    88.1826    97.6496
 0.4931     0.6182     0.8345     0.4374     0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.4614     0.5211     0.5926     0.4303     0.4048     0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.4359     0.4558     0.4523     0.4156     0.3912     0.3624     0.3854     0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.4166     0.4223     0.4136     0.3933     0.3750     0.3627     0.3698     0.3506
 0.3842     0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.4027     0.4031     0.3713     0.3634     0.3562     0.3532     0.3533     0.3490
 0.3645     0.3480     0.3660     0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.3924     0.3397     0.3431     0.3388     0.3348     0.3339     0.3359     0.3394
 0.3455     0.3435     0.3525     0.3489     0.3609     0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.3049     0.3204     0.3216     0.3187     0.3159     0.3154     0.3176     0.3218
 0.3272     0.3328     0.3379     0.3408     0.3481     0.3581     0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.2965     0.3042     0.3045     0.3021     0.3000     0.2993     0.3009     0.3045
 0.3096     0.3159     0.3222     0.3285     0.3341     0.3451     0.3428     0.3557
 0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.2860     0.2904     0.2904     0.2885     0.2866     0.2858     0.2866     0.2892
 0.2935     0.2990     0.3054     0.3120     0.3189     0.3311     0.3381     0.3476
 0.3594     0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.2759     0.2788     0.2784     0.2769     0.2751     0.2742     0.2745     0.2762
 0.2794     0.2838     0.2893     0.2958     0.3025     0.3161     0.3284     0.3376
 0.3477     0.3662     0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
 0.2668     0.2686     0.2683     0.2668     0.2653     0.2644     0.2643     0.2653
 0.2674     0.2708     0.2755     0.2807     0.2868     0.3001     0.3137     0.3257
 0.3355     0.3506     0.3775     0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0
```

TABLE CIII. (Continued)

0.2513	0.2519	0.2515	0.2503	0.2491	0.2481	0.2476	0.2477
0.2487	0.2506	0.2533	0.2567	0.2608	0.2712	0.2833	0.2962
0.3092	0.3203	0.3489	0.3697	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.2383	0.2386	0.2381	0.2372	0.2361	0.2351	0.2345	0.2342
0.2345	0.2355	0.2370	0.2391	0.2417	0.2490	0.2581	0.2687
0.2805	0.2928	0.3214	0.3456	0.3714	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.2277	0.2276	0.2272	0.2263	0.2253	0.2244	0.2238	0.2234
0.2233	0.2237	0.2245	0.2257	0.2274	0.2322	0.2389	0.2468
0.2562	0.2666	0.2950	0.3213	0.3456	0.3757	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.2184	0.2184	0.2179	0.2171	0.2163	0.2156	0.2148	0.2143
0.2141	0.2141	0.2145	0.2151	0.2162	0.2194	0.2240	0.2299
0.2370	0.2451	0.2697	0.2968	0.3210	0.3471	0.3579	0.4032
0.0	0.0	0.0	0.0	0.0			
0.2105	0.2104	0.2099	0.2093	0.2085	0.2078	0.2072	0.2066
0.2062	0.2061	0.2062	0.2066	0.2072	0.2093	0.2124	0.2168
0.2222	0.2286	0.2482	0.2721	0.2976	0.3208	0.3372	0.3735
0.4276	0.0	0.0	0.0	0.0			
0.1947	0.1946	0.1942	0.1936	0.1930	0.1924	0.1919	0.1913
0.1909	0.1906	0.1903	0.1903	0.1904	0.1909	0.1922	0.1941
0.1969	0.2002	0.2114	0.2262	0.2444	0.2651	0.2873	0.3089
0.3562	0.3793	0.4240	0.0	0.0			
0.1827	0.1825	0.1820	0.1816	0.1812	0.1807	0.1802	0.1798
0.1793	0.1789	0.1786	0.1784	0.1782	0.1782	0.1787	0.1796
0.1608	0.1826	0.1890	0.1980	0.2097	0.2238	0.2400	0.2581
0.2975	0.3315	0.3695	0.4150	0.0			
0.1730	0.1728	0.1725	0.1721	0.1718	0.1713	0.1709	0.1706
0.1702	0.1698	0.1695	0.1691	0.1690	0.1686	0.1687	0.1690
0.1695	0.1704	0.1742	0.1800	0.1875	0.1969	0.2080	0.2211
0.2515	0.2869	0.3206	0.3591	0.3695			
0.1651	0.1649	0.1647	0.1643	0.1640	0.1637	0.1633	0.1629
0.1625	0.1623	0.1619	0.1616	0.1614	0.1610	0.1608	0.1607
0.1610	0.1614	0.1636	0.1673	0.1723	0.1787	0.1866	0.1957
0.2182	0.2455	0.2773	0.3104	0.3356			
0.1584	0.1582	0.1580	0.1577	0.1574	0.1571	0.1568	0.1565
0.1561	0.1559	0.1555	0.1554	0.1551	0.1546	0.1543	0.1542
0.1541	0.1543	0.1554	0.1578	0.1613	0.1658	0.1714	0.1780
0.1943	0.2148	0.2396	0.2689	0.2992			
0.1527	0.1525	0.1523	0.1520	0.1518	0.1515	0.1512	0.1509
0.1507	0.1504	0.1501	0.1498	0.1496	0.1492	0.1488	0.1485
0.1484	0.1485	0.1490	0.1504	0.1528	0.1560	0.1602	0.1651
0.1774	0.1929	0.2119	0.2346	0.2603			
0.1432	0.1431	0.1429	0.1427	0.1425	0.1423	0.1421	0.1418
0.1417	0.1414	0.1411	0.1410	0.1407	0.1403	0.1399	0.1396
0.1394	0.1392	0.1392	0.1396	0.1406	0.1423	0.1445	0.1473
0.1546	0.1640	0.1758	0.1895	0.2055			
0.1358	0.1356	0.1354	0.1353	0.1351	0.1350	0.1347	0.1345
0.1344	0.1342	0.1340	0.1338	0.1336	0.1332	0.1329	0.1326
0.1323	0.1320	0.1317	0.1317	0.1321	0.1329	0.1341	0.1358
0.1402	0.1462	0.1538	0.1630	0.1734			
0.1296	0.1294	0.1293	0.1292	0.1291	0.1289	0.1287	0.1286
0.1284	0.1283	0.1281	0.1279	0.1277	0.1274	0.1271	0.1268

TABLE CIII. (Continued)

0.1265	0.1263	0.1258	0.1255	0.1256	0.1259	0.1266	0.1275
0.1304	0.1343	0.1394	0.1457	0.1531			
0.1244	0.1243	0.1241	0.1241	0.1239	0.1238	0.1236	0.1235
0.1233	0.1232	0.1231	0.1229	0.1228	0.1225	0.1222	0.1219
0.1216	0.1214	0.1209	0.1205	0.1204	0.1204	0.1208	0.1213
0.1231	0.1257	0.1292	0.1336	0.1390			
0.1199	0.1198	0.1197	0.1197	0.1195	0.1193	0.1193	0.1192
0.1190	0.1188	0.1187	0.1187	0.1185	0.1182	0.1180	0.1177
0.1175	0.1172	0.1167	0.1163	0.1161	0.1160	0.1161	0.1164
0.1174	0.1191	0.1216	0.1248	0.1287			
0.1110	0.1109	0.1108	0.1108	0.1106	0.1106	0.1104	0.1104
0.1103	0.1102	0.1101	0.1100	0.1099	0.1097	0.1095	0.1093
0.1091	0.1069	0.1084	0.1080	0.1077	0.1074	0.1072	0.1072
0.1073	0.1079	0.1090	0.1105	0.1123			
0.1042	0.1041	0.1041	0.1040	0.1040	0.1038	0.1038	0.1037
0.1036	0.1035	0.1035	0.1034	0.1034	0.1032	0.1030	0.1028
0.1027	0.1025	0.1022	0.1017	0.1014	0.1012	0.1009	0.1007
0.1005	0.1005	0.1009	0.1016	0.1025			
0.0574	0.0473	-0.1789	-0.2307	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.0101	-0.0258	-0.1485	-0.1776	-0.1331	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0209	-0.0651	-0.1230	-0.1370	-0.1116	-0.0915	-0.0578	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0356	-0.0706	-0.1024	-0.1089	-0.0961	-0.0787	-0.0567	-0.0493
-0.0219	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0419	-0.0683	-0.0796	-0.0933	-0.0866	-0.0723	-0.0571	-0.0456
-0.0275	-0.0258	-0.0099	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0439	-0.0413	-0.0762	-0.0867	-0.0831	-0.0723	-0.0590	-0.0460
-0.0332	-0.0255	-0.0141	-0.0095	0.0011	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0015	-0.0501	-0.0740	-0.0826	-0.0810	-0.0732	-0.0624	-0.0505
-0.0390	-0.0284	-0.0191	-0.0118	-0.0033	0.0095	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0219	-0.0543	-0.0723	-0.0796	-0.0793	-0.0739	-0.0654	-0.0554
-0.0449	-0.0345	-0.0249	-0.0162	-0.0085	0.0048	0.0109	0.0212
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0321	-0.0565	-0.0708	-0.0773	-0.0778	-0.0741	-0.0676	-0.0594
-0.0502	-0.0408	-0.0315	-0.0227	-0.0145	-0.0006	0.0094	0.0186
0.0281	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			

TABLE CIII. (Continued)

-0.0386	-0.0578	-0.0695	-0.0753	-0.0763	-0.0739	-0.0690	-0.0623
-0.0546	-0.0463	-0.0378	-0.0294	-0.0213	-0.0067	0.0056	0.0150
0.0238	0.0355	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0427	-0.0585	-0.0684	-0.0736	-0.0750	-0.0734	-0.0697	-0.0643
-0.0579	-0.0509	-0.0432	-0.0356	-0.0280	-0.0135	-0.0005	0.0103
0.0191	0.0294	0.0480	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0476	-0.0589	-0.0664	-0.0707	-0.0724	-0.0720	-0.0699	-0.0665
-0.0621	-0.0569	-0.0512	-0.0452	-0.0390	-0.0262	-0.0138	-0.0023
0.0083	0.0173	0.0369	0.0500	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0501	-0.0587	-0.0647	-0.0684	-0.0702	-0.0704	-0.0693	-0.0671
-0.0640	-0.0603	-0.0560	-0.0513	-0.0465	-0.0359	-0.0252	-0.0146
-0.0043	0.0054	0.0258	0.0412	0.0555	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0514	-0.0582	-0.0631	-0.0664	-0.0682	-0.0687	-0.0682	-0.0668
-0.0647	-0.0620	-0.0588	-0.0552	-0.0513	-0.0429	-0.0338	-0.0247
-0.0153	-0.0063	0.0147	0.0318	0.0459	0.0618	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
-0.0520	-0.0576	-0.0618	-0.0646	-0.0664	-0.0671	-0.0670	-0.0662
-0.0647	-0.0628	-0.0603	-0.0576	-0.0545	-0.0476	-0.0401	-0.0323
-0.0242	-0.0162	0.0036	0.0218	0.0364	0.0506	0.0572	0.0780
0.0	0.0	0.0	0.0	0.0			
-0.0522	-0.0570	-0.0606	-0.0631	-0.0648	-0.0656	-0.0658	-0.0653
-0.0644	-0.0629	-0.0611	-0.0589	-0.0564	-0.0508	-0.0446	-0.0380
-0.0309	-0.0239	-0.0062	0.0112	0.0270	0.0402	0.0496	0.0667
0.0918	0.0	0.0	0.0	0.0			
-0.0519	-0.0553	-0.0580	-0.0600	-0.0614	-0.0624	-0.0629	-0.0629
-0.0626	-0.0620	-0.0611	-0.0599	-0.0585	-0.0550	-0.0510	-0.0466
-0.0417	-0.0367	-0.0233	-0.0098	0.0040	0.0173	0.0301	0.0415
0.0645	0.0746	0.0923	0.0	0.0			
-0.0512	-0.0537	-0.0558	-0.0575	-0.0588	-0.0597	-0.0603	-0.0606
-0.0606	-0.0604	-0.0599	-0.0593	-0.0585	-0.0564	-0.0537	-0.0506
-0.0472	-0.0435	-0.0336	-0.0230	-0.0121	-0.0010	0.0098	0.0208
0.0415	0.0577	0.0738	0.0921	0.0			
-0.0503	-0.0523	-0.0540	-0.0554	-0.0565	-0.0574	-0.0580	-0.0584
-0.0587	-0.0587	-0.0585	-0.0582	-0.0577	-0.0564	-0.0546	-0.0525
-0.0500	-0.0473	-0.0397	-0.0313	-0.0226	-0.0137	-0.0047	0.0046
0.0228	0.0409	0.0564	0.0726	0.0776			
-0.0493	-0.0510	-0.0525	-0.0537	-0.0547	-0.0555	-0.0561	-0.0565
-0.0568	-0.0570	-0.0570	-0.0568	-0.0566	-0.0558	-0.0546	-0.0531
-0.0513	-0.0493	-0.0434	-0.0367	-0.0296	-0.0223	-0.0148	-0.0072
0.0084	0.0242	0.0401	0.0550	0.0660			
-0.0484	-0.0499	-0.0511	-0.0522	-0.0530	-0.0538	-0.0544	-0.0549
-0.0552	-0.0554	-0.0555	-0.0555	-0.0554	-0.0550	-0.0542	-0.0531
-0.0518	-0.0503	-0.0457	-0.0403	-0.0345	-0.0284	-0.0220	-0.0156
-0.0024	0.0110	0.0249	0.0393	0.0528			
-0.0476	-0.0488	-0.0499	-0.0508	-0.0516	-0.0523	-0.0529	-0.0533
-0.0537	-0.0540	-0.0541	-0.0542	-0.0542	-0.0540	-0.0535	-0.0527
-0.0518	-0.0506	-0.0470	-0.0427	-0.0378	-0.0327	-0.0272	-0.0217
-0.0104	0.0012	0.0131	0.0255	0.0380			
-0.0460	-0.0469	-0.0478	-0.0486	-0.0492	-0.0498	-0.0504	-0.0508
-0.0512	-0.0514	-0.0517	-0.0518	-0.0519	-0.0520	-0.0518	-0.0515

TABLE CIII. (Continued)

-0.0510	-0.0503	-0.0481	-0.0452	-0.0418	-0.0380	-0.0341	-0.0299
-0.0212	-0.0123	-0.0031	0.0061	0.0156			
-0.0446	-0.0454	-0.0461	-0.0467	-0.0473	-0.0478	-0.0483	-0.0487
-0.0490	-0.0493	-0.0495	-0.0498	-0.0499	-0.0501	-0.0502	-0.0500
-0.0498	-0.0494	-0.0480	-0.0461	-0.0437	-0.0409	-0.0379	-0.0347
-0.0279	-0.0208	-0.0135	-0.0060	0.0014			
-0.0434	-0.0440	-0.0446	-0.0451	-0.0456	-0.0461	-0.0465	-0.0469
-0.0472	-0.0475	-0.0477	-0.0480	-0.0481	-0.0484	-0.0486	-0.0486
-0.0485	-0.0483	-0.0475	-0.0461	-0.0444	-0.0424	-0.0401	-0.0376
-0.0321	-0.0264	-0.0204	-0.0142	-0.0080			
-0.0422	-0.0428	-0.0433	-0.0438	-0.0442	-0.0446	-0.0450	-0.0453
-0.0456	-0.0459	-0.0462	-0.0464	-0.0466	-0.0469	-0.0471	-0.0472
-0.0472	-0.0471	-0.0467	-0.0458	-0.0445	-0.0430	-0.0413	-0.0393
-0.0349	-0.0302	-0.0251	-0.0200	-0.0148			
-0.0412	-0.0417	-0.0422	-0.0426	-0.0430	-0.0434	-0.0437	-0.0440
-0.0443	-0.0445	-0.0448	-0.0450	-0.0452	-0.0455	-0.0458	-0.0459
-0.0460	-0.0460	-0.0458	-0.0452	-0.0443	-0.0431	-0.0418	-0.0402
-0.0367	-0.0328	-0.0285	-0.0242	-0.0197			
-0.0391	-0.0395	-0.0398	-0.0402	-0.0405	-0.0408	-0.0410	-0.0413
-0.0415	-0.0417	-0.0420	-0.0421	-0.0423	-0.0426	-0.0429	-0.0431
-0.0433	-0.0434	-0.0435	-0.0434	-0.0430	-0.0425	-0.0417	-0.0409
-0.0388	-0.0362	-0.0334	-0.0304	-0.0272			
-0.0374	-0.0377	-0.0380	-0.0382	-0.0385	-0.0387	-0.0390	-0.0392
-0.0394	-0.0396	-0.0397	-0.0399	-0.0401	-0.0404	-0.0406	-0.0409
-0.0410	-0.0412	-0.0415	-0.0415	-0.0414	-0.0412	-0.0408	-0.0404
-0.0391	-0.0374	-0.0355	-0.0334	-0.0311			
0.3405	0.2959	0.3499	0.7727	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.3637	0.3791	0.4913	0.7525	0.8575	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.3910	0.4442	0.5772	0.7381	0.8340	0.8846	0.9180	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.4224	0.4912	0.6076	0.7295	0.8150	0.8672	0.9014	0.9205
0.9414	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.4532	0.5224	0.6222	0.7267	0.8005	0.8515	0.8861	0.9093
0.9291	0.9384	0.9514	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.4731	0.5414	0.6469	0.7281	0.7905	0.8375	0.8721	0.8976
0.9172	0.9305	0.9428	0.9504	0.9593	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.4803	0.5834	0.6645	0.7306	0.7840	0.8263	0.8594	0.8854
0.9057	0.9215	0.9340	0.9437	0.9524	0.9647	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			

TABLE CIII. (Continued)

0.5336	0.6119	0.6780	0.7335	0.7797	0.8177	0.8489	0.8741
0.8946	0.9114	0.9250	0.9361	0.9452	0.9592	0.9678	0.9760
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.5688	0.6334	0.6890	0.7367	0.7772	0.8116	0.8403	0.8645
0.8847	0.9016	0.9158	0.9276	0.9377	0.9532	0.9639	0.9724
0.9793	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.5957	0.6503	0.6983	0.7399	0.7759	0.8070	0.8336	0.8564
0.8759	0.8926	0.9070	0.9193	0.9299	0.9467	0.9591	0.9683
0.9756	0.9830	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.6168	0.6643	0.7061	0.7431	0.7754	0.8038	0.8284	0.8499
0.8685	0.8847	0.8991	0.9113	0.9221	0.9397	0.9534	0.9637
0.9716	0.9789	0.9920	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.6485	0.6859	0.7192	0.7491	0.7759	0.7998	0.8211	0.8401
0.8570	0.8720	0.8855	0.8974	0.9081	0.9264	0.9411	0.9530
0.9627	0.9704	0.9851	0.9936	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.6718	0.7022	0.7298	0.7547	0.7775	0.7981	0.8167	0.8335
0.8488	0.8626	0.8752	0.8866	0.8967	0.9148	0.9297	0.9421
0.9526	0.9615	0.9779	0.9886	0.9973	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.6895	0.7152	0.7385	0.7599	0.7796	0.7976	0.8141	0.8293
0.8430	0.8557	0.8673	0.8779	0.8877	0.9050	0.9197	0.9322
0.9430	0.9522	0.9704	0.9830	0.9921	1.0009	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.7039	0.7258	0.7460	0.7647	0.7820	0.7979	0.8126	0.8262
0.8389	0.8505	0.8613	0.8713	0.8805	0.8971	0.9112	0.9236
0.9343	0.9436	0.9626	0.9768	0.9868	0.9954	0.9996	1.0097
0.0	0.0	0.0	0.0	0.0			
0.7158	0.7348	0.7526	0.7691	0.7844	0.7987	0.8120	0.8244
0.8359	0.8467	0.8567	0.8661	0.8749	0.8904	0.9041	0.9161
0.9269	0.9361	0.9552	0.9700	0.9813	0.9899	0.9958	1.0044
1.0168	0.0	0.0	0.0	0.0			
0.7380	0.7523	0.7658	0.7786	0.7905	0.8019	0.8124	0.8225
0.8319	0.8408	0.8492	0.8572	0.8648	0.8786	0.8910	0.9019
0.9119	0.9208	0.9397	0.9546	0.9668	0.9769	0.9853	0.9921
1.0045	1.0096	1.0168	0.0	0.0			
0.7540	0.7654	0.7763	0.7865	0.7962	0.8055	0.8143	0.8226
0.8306	0.8382	0.8454	0.8523	0.8587	0.8710	0.8821	0.8922
0.9013	0.9098	0.9278	0.9424	0.9546	0.9648	0.9734	0.9810
0.9935	1.0022	1.0096	1.0178	0.0			
0.7665	0.7758	0.7847	0.7933	0.8015	0.8093	0.8167	0.8239
0.8307	0.8372	0.8435	0.8495	0.8553	0.8661	0.8761	0.8853
0.8937	0.9015	0.9187	0.9329	0.9447	0.9548	0.9634	0.9711
0.9838	0.9943	1.0023	1.0099	1.0122			
0.7764	0.7843	0.7918	0.7992	0.8062	0.8128	0.8193	0.8256
0.8315	0.8372	0.8428	0.8481	0.8532	0.8629	0.8719	0.8803
0.8881	0.8954	0.9115	0.9252	0.9367	0.9465	0.9551	0.9627
0.9754	0.9859	0.9949	1.0024	1.0077			
0.7846	0.7914	0.7981	0.8044	0.8104	0.8164	0.8220	0.8275
0.8328	0.8379	0.8428	0.8476	0.8521	0.8609	0.8691	0.8768

TABLE CIII. (Continued)

0.8839	0.8907	0.9058	0.9190	0.9301	0.9397	0.9481	0.9556
0.9681	0.9785	0.9874	0.9953	1.0021			
0.7916	0.7976	0.8034	0.8090	0.8144	0.8196	0.8246	0.8295
0.8342	0.8388	0.8432	0.8476	0.8517	0.8597	0.8671	0.8742
0.8809	0.8872	0.9014	0.9138	0.9246	0.9339	0.9423	0.9496
0.9620	0.9722	0.9809	0.9886	0.9954			
0.8030	0.8078	0.8124	0.8169	0.8212	0.8255	0.8296	0.8336
0.8374	0.8412	0.8449	0.8485	0.8520	0.8587	0.8650	0.8711
0.8768	0.8823	0.8949	0.9061	0.9160	0.9249	0.9328	0.9399
0.9519	0.9619	0.9705	0.9777	0.9842			
0.8120	0.8159	0.8198	0.8235	0.8271	0.8306	0.8341	0.8375
0.8407	0.8439	0.8471	0.8501	0.8531	0.8589	0.8644	0.8696
0.8747	0.8795	0.8907	0.9009	0.9099	0.9182	0.9256	0.9324
0.9441	0.9539	0.9622	0.9694	0.9756			
0.8193	0.8227	0.8259	0.8291	0.8322	0.8352	0.8382	0.8411
0.8439	0.8466	0.8494	0.8520	0.8546	0.8597	0.8645	0.8691
0.8736	0.8779	0.8879	0.8971	0.9055	0.9131	0.9201	0.9264
0.9378	0.9473	0.9554	0.9626	0.9687			
0.8255	0.8284	0.8313	0.8340	0.8367	0.8393	0.8419	0.8444
0.8469	0.8494	0.8517	0.8541	0.8563	0.8608	0.8651	0.8692
0.8733	0.8771	0.8862	0.8945	0.9022	0.9093	0.9159	0.9219
0.9327	0.9419	0.9499	0.9569	0.9630			
0.8309	0.8334	0.8359	0.8382	0.8407	0.8430	0.8453	0.8475
0.8497	0.8519	0.8540	0.8561	0.8581	0.8621	0.8660	0.8697
0.8733	0.8768	0.8851	0.8927	0.8998	0.9065	0.9125	0.9183
0.9286	0.9374	0.9452	0.9520	0.9580			
0.8416	0.8434	0.8454	0.8471	0.8490	0.8508	0.8526	0.8543
0.8560	0.8577	0.8593	0.8610	0.8626	0.8657	0.8687	0.8717
0.8746	0.8774	0.8841	0.8905	0.8963	0.9019	0.9070	0.9120
0.9210	0.9291	0.9363	0.9427	0.9484			
0.8498	0.8513	0.8528	0.8543	0.8557	0.8572	0.8586	0.8600
0.8613	0.8627	0.8640	0.8654	0.8666	0.8692	0.8717	0.8742
0.8766	0.8789	0.8845	0.8898	0.8948	0.8996	0.9041	0.9084
0.9164	0.9236	0.9301	0.9361	0.9415			
0.9057	1.0152	1.1135	1.2193	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.6893	0.8312	0.9578	1.0717	1.1274	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.5456	0.6934	0.8342	0.9479	1.0202	1.0719	1.1000	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.4746	0.6168	0.7427	0.8479	0.9276	0.9853	1.0245	1.0570
1.0773	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.4299	0.5589	0.6735	0.7717	0.8496	0.9103	0.9564	0.9924
1.0198	1.0424	1.0610	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			

TABLE CIII. (Continued)

0.3971	0.5135	0.6198	0.7110	0.7862	0.8469	0.8957	0.9346
0.9666	0.9925	1.0138	1.0319	1.0455	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.3689	0.4794	0.5777	0.6623	0.7335	0.7931	0.8424	0.8836
0.9177	0.9463	0.9699	0.9905	1.0073	1.0362	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.3513	0.4537	0.5439	0.6221	0.6896	0.7470	0.7962	0.8379
0.8731	0.9038	0.9293	0.9517	0.9710	1.0029	1.0272	1.0473
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.3389	0.4329	0.5160	0.5890	0.6527	0.7080	0.7557	0.7974
0.8335	0.8647	0.8920	0.9155	0.9366	0.9711	0.9983	1.0203
1.0372	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.3292	0.4157	0.4924	0.5607	0.6204	0.6735	0.7202	0.7610
0.7972	0.8291	0.8571	0.8822	0.9041	0.9408	0.9704	0.9942
1.0135	1.0288	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.3211	0.4010	0.4725	0.5361	0.5931	0.6440	0.6890	0.7291
0.7646	0.7964	0.8256	0.8506	0.8733	0.9120	0.9435	0.9691
0.9904	1.0076	1.0443	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.3081	0.3769	0.4397	0.4962	0.5475	0.5942	0.6362	0.6742
0.7086	0.7400	0.7687	0.7945	0.8178	0.8592	0.8933	0.9218
0.9462	0.9666	1.0074	1.0317	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.2975	0.3579	0.4136	0.4646	0.5114	0.5540	0.5933	0.6291
0.6622	0.6925	0.7206	0.7463	0.7696	0.8122	0.8478	0.8784
0.9046	0.9275	0.9723	1.0033	1.0285	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.2884	0.3424	0.3926	0.4388	0.4814	0.5210	0.5579	0.5921
0.6230	0.6524	0.6794	0.7043	0.7278	0.7699	0.8067	0.8382
0.8662	0.8903	0.9390	0.9751	1.0024	1.0249	0.0	0.0
0.0	0.0	0.0	0.0	0.0			
0.2806	0.3295	0.3750	0.4171	0.4567	0.4936	0.5278	0.5597
0.5899	0.6176	0.6439	0.6680	0.6909	0.7327	0.7692	0.8016
0.8302	0.8553	0.9075	0.9471	0.9769	1.0013	1.0202	1.0378
0.0	0.0	0.0	0.0	0.0			
0.2737	0.3182	0.3599	0.3989	0.4355	0.4698	0.5024	0.5326
0.5607	0.5877	0.6129	0.6366	0.6592	0.6996	0.7358	0.7682
0.7976	0.8235	0.8773	0.9193	0.9521	0.9783	0.9993	1.0179
1.0461	0.0	0.0	0.0	0.0			
0.2591	0.2958	0.3304	0.3631	0.3940	0.4237	0.4518	0.4780
0.5033	0.5276	0.5502	0.5719	0.5927	0.6310	0.6661	0.6973
0.7264	0.7527	0.8097	0.8554	0.8930	0.9236	0.9489	0.9701
1.0039	1.0281	1.0455	0.0	0.0			
0.2475	0.2786	0.3081	0.3364	0.3637	0.3893	0.4141	0.4377
0.4602	0.4818	0.5024	0.5228	0.5413	0.5773	0.6106	0.6409
0.6687	0.6948	0.7516	0.7992	0.8390	0.8728	0.9010	0.9252
0.9638	0.9929	1.0148	1.0332	0.0			
0.2378	0.2648	0.2908	0.3156	0.3396	0.3625	0.3845	0.4061
0.4266	0.4461	0.4652	0.4833	0.5011	0.5342	0.5654	0.5943

TABLE CIII. (Concluded)

0.6211	0.6462	0.7026	0.7505	0.7910	0.8261	0.8563	0.8832
0.9258	0.9589	0.9846	1.0057	1.0212			
0.2296	0.2535	0.2767	0.2988	0.3203	0.3411	0.3610	0.3803
0.3990	0.4173	0.4344	0.4513	0.4681	0.4989	0.5284	0.5555
0.5815	0.6055	0.6602	0.7078	0.7487	0.7840	0.8157	0.8434
0.8899	0.9261	0.9549	0.9786	0.9971			
0.2225	0.2440	0.2647	0.2849	0.3043	0.3231	0.3415	0.3593
0.3763	0.3933	0.4091	0.4253	0.4403	0.4691	0.4966	0.5233
0.5472	0.5706	0.6233	0.6704	0.7109	0.7466	0.7785	0.8070
0.8551	0.8941	0.9257	0.9519	0.9729			
0.2162	0.2357	0.2546	0.2729	0.2908	0.3081	0.3249	0.3414
0.3574	0.3729	0.3878	0.4025	0.4168	0.4443	0.4699	0.4946
0.5180	0.5405	0.5915	0.6366	0.6771	0.7127	0.7452	0.7740
0.8234	0.8640	0.8977	0.9256	0.9486			
0.2057	0.2221	0.2382	0.2538	0.2691	0.2841	0.2987	0.3127
0.3269	0.3403	0.3533	0.3666	0.3790	0.4034	0.4266	0.4491
0.4702	0.4910	0.5384	0.5812	0.6193	0.6550	0.6863	0.7154
0.7658	0.8083	0.8448	0.8753	0.9014			
0.1970	0.2112	0.2252	0.2388	0.2522	0.2655	0.2782	0.2907
0.3030	0.3152	0.3268	0.3384	0.3497	0.3719	0.3931	0.4133
0.4329	0.4516	0.4957	0.5361	0.5725	0.6063	0.6372	0.6659
0.7162	0.7595	0.7968	0.8295	0.8573			
0.1896	0.2022	0.2146	0.2267	0.2387	0.2504	0.2618	0.2730
0.2841	0.2951	0.3056	0.3162	0.3263	0.3464	0.3659	0.3845
0.4023	0.4200	0.4606	0.4984	0.5340	0.5658	0.5958	0.6230
0.6735	0.7163	0.7542	0.7878	0.8171			
0.1833	0.1946	0.2057	0.2167	0.2273	0.2379	0.2483	0.2585
0.2685	0.2785	0.2883	0.2977	0.3071	0.3254	0.3432	0.3603
0.3771	0.3934	0.4317	0.4671	0.5005	0.5310	0.5602	0.5869
0.6359	0.6783	0.7164	0.7499	0.7799			
0.1778	0.1880	0.1981	0.2081	0.2178	0.2274	0.2370	0.2464
0.2555	0.2645	0.2734	0.2825	0.2911	0.3079	0.3246	0.3403
0.3558	0.3709	0.4070	0.4405	0.4722	0.5018	0.5293	0.5556
0.6029	0.6444	0.6823	0.7159	0.7461			
0.1665	0.1749	0.1830	0.1912	0.1991	0.2072	0.2148	0.2227
0.2302	0.2377	0.2452	0.2525	0.2597	0.2740	0.2880	0.3017
0.3150	0.3279	0.3588	0.3886	0.4164	0.4425	0.4672	0.4915
0.5349	0.5749	0.6113	0.6442	0.6740			
0.1577	0.1647	0.1716	0.1785	0.1854	0.1920	0.1987	0.2052
0.2117	0.2182	0.2245	0.2308	0.2373	0.2494	0.2614	0.2732
0.2848	0.2961	0.3240	0.3497	0.3745	0.3986	0.4213	0.4432
0.4836	0.5209	0.5550	0.5867	0.6156			

APPENDIX D. COMPUTER PROGRAM OPERATING INSTRUCTIONS

INPUT

The order of input into the computer is shown in Figure D1.

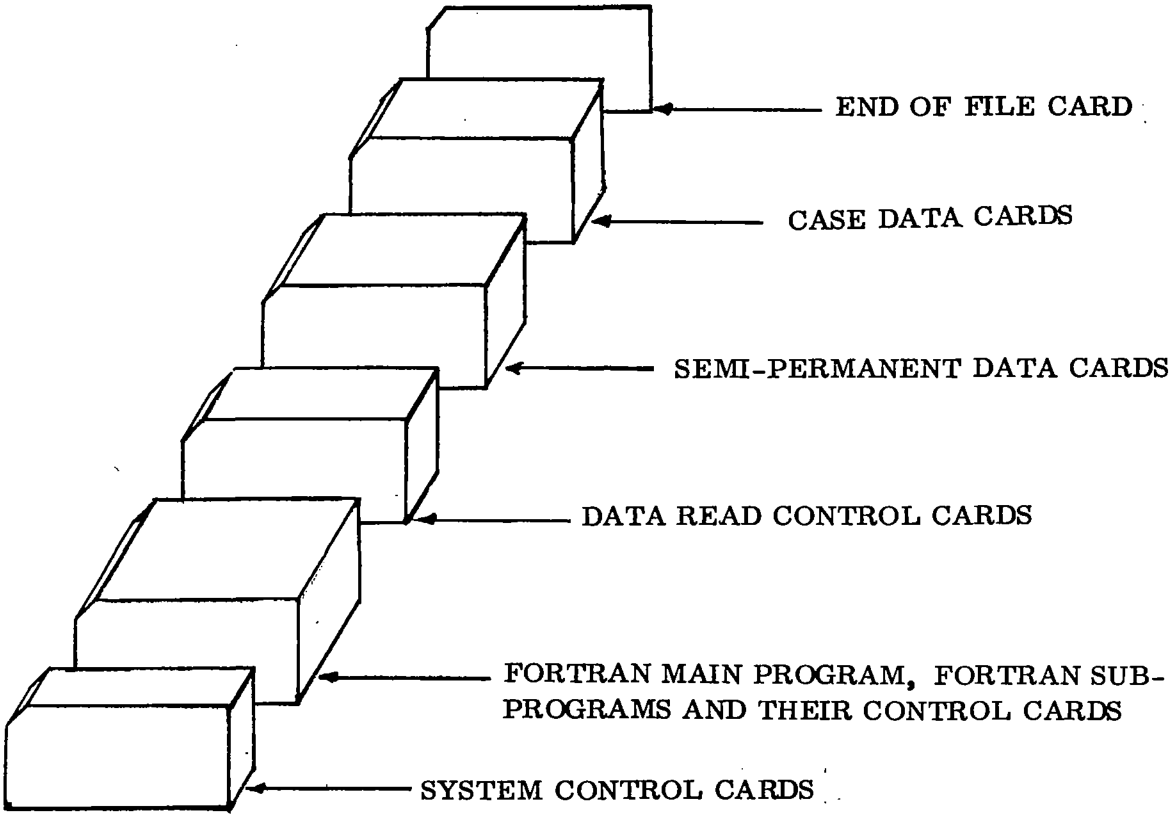


FIGURE D1. ORDER OF INPUT

The system control cards, Fortran main program and subprograms, data read control cards, and semi-permanent data are listed in Appendix C. The semi-permanent data are data arrays used to determine the reduced shell moduli for cylinders with local buckling.

The input format for the case data cards is shown in Table DI.

TABLE DI. INPUT FORMAT

Card No.	Format	Data to be Read In
1	18A4	MAT
2	8E10.0	E, ES, ER, G, GS, GR, QMU, BS
3	8E10.0	CS, QLS, AS, QIXS, QIZS, QJS, H, BR
4.	8E10.0	CR, QLR, AR, QIYR, QIZR, QJR, R, QL
5	2E10.0, 6I2	T, PP, M1, MM, N1, NN, NTYPE, NFAIL

All of the data input terms are defined by Table CII, the Definition of Symbols or Table DII.

TABLE DII. COMPUTER PROGRAM INPUT SYMBOL TABLE

Symbol	Definition
MAT	Case title
M1	Lowest value of (m) considered
MM	Highest value of (m) considered
N1	Lowest value of (n) considered
NN	Highest value of (n) considered
NTYPE	Type of cylinder NTYPE = 1; cylinder with rings and stringers NTYPE = 2; cylinder with stringers only NTYPE = 3; cylinder with rings only NTYPE = 4; isotropic core sandwich cylinder NTYPE = 5; isotropic core sandwich cylinder with rings NTYPE = 6; open corrugated cylinder NTYPE = 7; open corrugated cylinder with rings
NFAIL	Type of failure examined NFAIL = 1; general instability NFAIL = 2; panel instability

The minimum value of M1 is (M1 = 1), and the minimum value of N1 is (N1 = 0), axisymmetric buckling. The range of wave shapes considered should be large enough so that the lowest buckling load is definitely within the range. Any number of case data sets may run one after the other. The end of file card is used to end the program.

OUTPUT

For all NTYPE's the program prints the input data and the buckling load for each mode shape considered. The program then gives the minimum buckling load mode shape, of those considered, and the buckling load for this mode shape. This completes the output for NTYPE's 4, 6, and 7, which have no rings or stringers.

For NTYPE's 3 and 5, which have rings only, the program recalculates and prints the buckling load at the minimum load mode shape. This recalculation is performed using the more exact ring restraint terms. This completes the output for NTYPE's 3 and 5.

For 1 and 2, which have stringers, the program checks to see if the minimum buckling load is above or below the local buckling load. If it is below, the program recalculates the minimum buckling load for NTYPE 1 and ends for NTYPE 2. If, for NTYPE's 1 and 2, the minimum buckling load is above the local buckling load, the program recalculates and prints the buckling load for all mode shapes having buckling loads within 20 percent of the minimum buckling load. In this recalculation the shell stiffnesses are reduced to account for local buckling, and the more exact ring restraint terms are used. The program then prints the minimum buckling load of those re-examined and its mode shape. This completes the output for NTYPE's 1 and 2.

Figures D2 and D3 show the program output for two sample cases. Case 1 has rings, stringers, and local buckling. Case 2 has rings only.

```
     GENERAL INSTABILITY ØF ECCENTRICALLY STIFFENED CYLINDRICAL
        SHELLS UNDER AXIAL CØMPRESSIØN AND LATERAL PRESSURE

                          CYLINDER NØ. 1

                                 INPUT DATA
                                 //////////

E =   10.50E 06          ES =   10.50E 06          ER =   10.50E 06

G =    4.00E 06          GS =    4.00E 06          GR =    4.00E 06

QMU =     3.200E-01      BS =     0.000E-39        CS =     2.420E-01

QLS =     2.480E 00      AS =     3.800E-02        QIXS =  17.7000E-04

QIZS =  00.0000E-40      QJS =  22.6000E-06        H =    00.000E-40

BR =  -29.000E-02        CR =  -24.800E-02         QLR =   60.000E-01

AR =   44.600E-03        QIYR =  20.3000E-04       QIZR =  38.8000E-04

QJR =  24.5000E-04       R =   38.600E 00          QL =   72.000E 00

T =   19.900E-03         PP =   00.000E-40

M1 =  1       MM = 10       N1 =  0       NN = 15

NTYPE = 1     NFAIL = 1
```

FIGURE D2. SAMPLE CASE NO. 1

```
                    ØUTPUT DATA
                    ///////////

 M    N   AXIAL LØAD/INCH          M    N   AXIAL LØAD/INCH
 1    0      79166.8               1    1      32181.6
 2    0      25830.9               2    1      18679.8
 3    0      11471.2               3    1      10032.8
 4    0       6562.6               4    1       6122.8
 5    0       4431.6               5    1       4260.6
 6    0       3425.3               6    1       3348.2
 7    0       2976.4               7    1       2938.2
 8    0       2847.7               8    1       2827.7
 9    0       2925.8               9    1       2914.9
10    0       3150.8              10    1       3144.9


 1    2      12043.3               1    3       5131.2
 2    2      10626.5               2    3       6099.6
 3    2       7353.3               3    3       5118.6
 4    2       5119.6               4    3       4057.6
 5    2       3831.0               5    3       3309.0
 6    2       3144.3               6    3       2875.2
 7    2       2834.2               7    3       2689.7
 8    2       2772.0               8    3       2692.2
 9    2       2884.4               9    3       2839.8
10    2       3128.1              10    3       3103.5


 1    4       2781.6               1    5       2293.9
 2    4       3749.0               2    5       2581.6
 3    4       3613.1               3    5       2680.7
 4    4       3189.9               4    5       2563.5
 5    4       2821.5               5    5       2427.1
 6    4       2600.4               6    5       2359.0
 7    4       2533.1               7    5       2387.6
 8    4       2602.5               8    5       2516.1
 9    4       2788.7               9    5       2738.8
10    4       3075.1              10    5       3047.7


 1    6       2906.4               1    7       4504.2
 2    6       2114.9               2    7       2141.1
 3    6       2156.4               3    7       1937.1
 4    6       2153.3               4    7       1926.3
 5    6       2140.9               5    7       1961.2
 6    6       2170.6               6    7       2043.4
 7    6       2268.4               7    7       2184.9
 8    6       2443.6               8    7       2392.7
 9    6       2697.0               9    7       2669.2
10    6       3025.9              10    7       3013.9
```

FIGURE D2. (Continued)

```
 1     8       7220.3                1     9      11304.6
 2     8       2591.1                2     9       3469.7
 3     8       1972.0                3     9       2245.4
 4     8       1860.3                4     9       1946.4
 5     8       1883.8                5     9       1907.1
 6     8       1981.2                6     9       1986.7
 7     8       2143.2                7     9       2147.8
 8     8       2369.5                8     9       2378.8
 9     8       2660.6                9     9       2675.5
10     8       3015.9               10     9       3035.6

 1    10      17078.4                1    11      24916.4
 2    10       4824.0                2    11       6728.1
 3    10       2764.8                3    11       3553.6
 4    10       2186.3                4    11       2590.6
 5    10       2033.7                5    11       2270.7
 6    10       2064.2                6    11       2219.6
 7    10       2203.8                7    11       2316.5
 8    10       2425.6                8    11       2514.7
 9    10       2718.4                9    11       2793.8
10    10       3076.9               10    11       3143.7

 1    12      35239.4                1    13      48510.1
 2    12       9275.8                2    13      12575.4
 3    12       4646.6                3    13       6087.2
 4    12       3176.4                4    13       3966.1
 5    12       2628.8                5    13       3121.8
 6    12       2460.8                6    13       2797.7
 7    12       2492.4                7    13       2739.3
 8    12       2651.9                8    13       2843.3
 9    12       2906.6                9    13       3061.9
10    12       3240.1               10    13       3370.8

 1    14      65231.9                1    15      85947.5
 2    14      16743.6                2    15      21928.1
 3    14       7926.1                3    15      10220.0
 4    14       4986.7                4    15       6268.8
 5    14       3766.5                5    15       4582.1
 6    14       3242.0                6    15       3806.7
 7    14       3065.9                7    15       3482.1
 8    14       3096.0                8    15       3417.7
 9    14       3265.7                9    15       3524.0
10    14       3540.6               10    15       3754.6
```

FIGURE D2. (Continued)

```
   THE MINIMUM AXIAL LOAD IN THE ABOVE RANGE IS     1860.3 LBS/IN
     AT M =   4 AND N =   8

   THE FOLLOWING CASES HAVE BEEN CHECKED FOR LOCAL BUCKLING

  M               N          AXIAL LOAD/INCH    REDUCED AXIAL LOAD/INCH
  1               5              2293.9                 1482.6
  2               6              2114.9                 1244.4
  3               6              2156.4                 1191.9
  4               6              2153.3                 1238.8
  5               6              2140.9                 1323.2
  6               6              2170.6                 1444.7
  7               6              2268.4                 1599.2
  2               7              2141.1                 1494.3
  3               7              1937,1                 1213.2
  4               7              1926.3                 1206.5
  5               7              1961.2                 1279.1
  6               7              2043.4                 1398.9
  7               7              2184.9                 1560.9
  3               8              1972.0                 1394.3
  4               8              1860.3                 1271.3
  5               8              1883.8                 1302.5
  6               8              1981,2                 1409.2
  7               8              2143.2                 1565.5
  3               9              2245.4                 1756.2
  4               9              1946.4                 1446.7
  5               9              1907.1                 1403.0
  6               9              1986.7                 1473.8
  7               9              2147.8                 1612.9
  4              10              2186.3                 1745.7
  5              10              2033.7                 1584.4
  6              10              2064.2                 1597.5
  7              10              2203.8                 1705.4
  5              11              2270.7                 1861.1
  6              11              2219.6                 1788.5
  7              11              2316.5                 1348.4


   THE MINIMUM AXIAL LOAD IN THE ABOVE RANGE IS     1191.9 LBS/IN
     AT M =   3 AND N =   6


   THE TOTAL AXIAL LOAD IS    0.2891E 06 LBS
```

FIGURE D2. (Concluded)

```
GENERAL INSTABILITY ØF ECCENTRICALLY STIFFENED CYLINDRICAL
     SHELLS UNDER AXIAL CØMPRESSIØN AND LATERAL PRESSURE

                      CYLINDER NØ. 2

                                INPUT DATA
                                //////////

E =  10.60E 06          ES =  10.60E 06          ER =  10.60E 06

G =   4.00E 06          GS =   4.00E 06          GR =   4.00E 06

QMU =     3.250E-01     BS =    0.000E-39        CS =     0.000E-39

QLS =     1.000E 02     AS =    0.000E-39        QIXS =  00.0000E-40

QIZS =  00.0000E-40     QJS =  00.0000E-40       H =    00.000E-40

BR =    32.450E-03      CR =   32.450E-03        QLR =   13.300E-02

AR =    19.634E-04      QIYR =  15.0000E-08      QIZR =  00.0000E-40

QJR =  00.0000E-40      R =   12.230E 00         QL =   15.500E 00

T =   34.600E-03        PP =   00.000E-40

M1 =  1        MM = 20        N1 =  0        NN =  9

NTYPE = 3      NFAIL = 1
```

FIGURE D3. SAMPLE CASE NO. 2

```
                    ØUTPUT DATA
                    ///////////


  M      N   AXIAL LØAD/INCH            M      N   AXIAL LØAD/INCH
  1      0       83293.0                1      1       46133.3
  2      0       21182.6                2      1       18488.3
  3      0        9450.9                3      1        8926.4
  4      0        5339.1                4      1        5177.5
  5      0        3443.2                5      1        3379.3
  6      0        2422.9                6      1        2393.6
  7      0        1818.2                7      1        1803.4
  8      0        1436.7                8      1        1428.7
  9      0        1186.4                9      1        1181.9
 10      0        1018.8               10      1        1016.3
 11      0         906.4               11      1         905.1
 12      0         832.8               12      1         832.2
 13      0         787.4               13      1         787.2
 14      0         763.5               14      1         763.6
 15      0         756.3               15      1         756.5
 16      0         762.6               16      1         763.0
 17      0         780.1               17      1         780.6
 18      0         807.1               18      1         807.6
 19      0         842.4               19      1         842.9
 20      0         884.9               20      1         885.5


  1      2       20212.2                1      3        9070.1
  2      2       13308.0                2      3        8848.0
  3      2        7635.4                3      3        6110.4
  4      2        4743.4                4      3        4153.6
  5      2        3200.8                5      3        2940.7
  6      2        2309.8                6      3        2182.8
  7      2        1760.4                7      3        1693.8
  8      2        1405.4                8      3        1368.8
  9      2        1169.0                9      3        1148.4
 10      2        1009.1               10      3         997.6
 11      2         901.3               11      3         895.1
 12      2         830.4               12      3         827.5
 13      2         786.7               13      3         786.0
 14      2         763.9               14      3         764.5
 15      2         757.4               15      3         758.9
 16      2         764.2               16      3         766.3
 17      2         782.0               17      3         784.5
 18      2         809.3               18      3         812.1
 19      2         844.7               19      3         847.7
 20      2         887.4               20      3         890.5
```

FIGURE D3. (Continued)

1	4	4366.1	1	5	2311.0
2	4	5775.1	2	5	3805.5
3	4	4717.6	3	5	3591.2
4	4	3525.2	4	5	2937.3
5	4	2638.5	5	5	2329.1
6	4	2027.6	6	5	1859.3
7	4	1609.8	7	5	1515.4
8	4	1321.8	8	5	1267.6
9	4	1121.6	9	5	1090.4
10	4	982.5	10	5	964.9
11	4	887.1	11	5	877.8
12	4	823.9	12	5	819.8
13	4	785.1	13	5	784.4
14	4	765.5	14	5	767.0
15	4	761.1	15	5	764.1
16	4	769.3	16	5	773.3
17	4	788.1	17	5	792.7
18	4	816.0	18	5	821.1
19	4	851.8	19	5	857.3
20	4	894.8	20	5	900.4

1	6	1410.4	1	7	1082.3
2	6	2573.0	2	7	1814.0
3	6	2735.0	3	7	2106.9
4	6	2428.2	4	7	2008.6
5	6	2036.6	5	7	1775.1
6	6	1690.7	6	7	1531.1
7	6	1416.9	7	7	1320.1
8	6	1209.7	8	7	1151.3
9	6	1056.5	9	7	1021.9
10	6	945.7	10	7	926.0
11	6	867.8	11	7	857.7
12	6	815.6	12	7	811.8
13	6	784.1	13	7	784.4
14	6	769.2	14	7	772.3
15	6	768.0	15	7	772.9
16	6	778.3	16	7	784.6
17	6	798.6	17	7	805.7
18	6	827.5	18	7	835.2
19	6	864.0	19	7	872.1
20	6	907.4	20	7	915.8

FIGURE D3. (Continued)

```
  1     8        1100.0                1     9        1382.0
  2     8        1364.0                2     9        1126.4
  3     8        1659.1                3     9        1352.0
  4     8        1675.1                4     9        1419.2
  5     8        1550.9                5     9        1365.7
  6     8        1386.7                6     9        1261.1
  7     8        1229.1                7     9        1147.2
  8     8        1095.1                8     9        1043.4
  9     8         988.2                9     9         957.0
 10     8         907.0               10     9         889.6
 11     8         848.3               11     9         840.2
 12     8         808.8               12     9         807.1
 13     8         785.7               13     9         788.3
 14     8         776.5               14     9         782.1
 15     8         779.2               15     9         786.8
 16     8         792.2               16     9         801.2
 17     8         814.2               17     9         824.2
 18     8         844.3               18     9         855.0
 19     8         881.7               19     9         892.8
 20     8         925.7               20     9         937.0


   THE MINIMUM AXIAL LØAD IN THE ABØVE RANGE IS    756.3 LBS/IN
     AT  M = 15 AND N =   0



   THE MINIMUM AXIAL LØAD IN THE ABØVE RANGE AFTER CØRRECTIØN
     FØR RING RESTRAINT IS  756.3 LBS/IN


   THE TØTAL AXIAL LØAD IS    58113. LBS
```

FIGURE D3. (Concluded)

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AIR FORCE WEAPONS LABORATORY/AFWL/
KIRTLAND AIR FORCE BASE, NEW MEXICO 87117

ATT E. LOU BOWMAN, ACTING CHIEF TECH. LI

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