CONCERNING THE ORIGIN OF THE MAGNETIC FIELDS ON STARS AND IN INTERSTELLAR SPACE

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Concerning the Origin of the Magnetic Fields on Stars and in Interstellar Space

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ABSTRACT: It is demonstrated that electric currents must flow in a plasma in the case of stationary vortical non-mass-proportional forces, and that the electric currents arising in this manner generate magnetic fields of considerable strength in the interior of stars. The tendency of the isobaric toward the isothermal surfaces thereby required in the case of hydrostatic equilibrium arises, for example, if the total acceleration determining the pressure gradients (gravity and rotation or vortex) possesses a component g^* with no potential. It is found that in the interior of stars g^* values of the order of 1cm/sec^2 cause magnetic fields of 10^3 Gauss. The only case thus far analytically discussed (baroclinic rotation of the entire star) confirms the conclusion strong, but does not lead to an interpretation of the form of the magnetic fields on the sun and on some presumably rapidly rotating stars. Hence two other possibilities are discussed from the qualitative standpoint. If the sunspots are referred to closed hydrodynamic vortex rings, as is done by Bjerknes, these rings lead to $[\nabla P \nabla T] \neq 0$; because of the perturbation of the field of the rotational velocity; the resulting electric currents yield magnetic fields parallel to the turbulence axis, fields which are antiparallel in the two vortex rings of each hemisphere. For rapidly rotating stars, on the other hand, a model is proposed which resembles the relationships observed in terrestrial revolving storms. Relatively rapid meridional circulations are assumed, which lead to acceleration components about the axis of rotation; the latter results in axial magnetic fields. In conclusion a brief sketch is given of the application of these considerations to the occurrence of interstellar magnetic fields.

In the supplement a special model of a star which does not rotate rigidly is calculated; the resulting magnetic fields are indicated.

The discovery of magnetic fields on a number of presumably rapidly rotating stars and the great significance of interstellar electromagnetic fields for the theory of cosmic radiation have again moved the question of the origin of these fields into the foreground. Hence certain reflections are to be given which are to show in a general way how, on the basis of the kinetic gas theory and the electron theory, the existence of strong magnetic fields on rotating stars may be understood, and odd fields are to be expected in interstellar space[1].

1. We proceed from the equation for the velocity $\mathfrak{v} = \mathfrak{v}_i - \mathfrak{v}_e$ of the diffusion of electron gas through ion gas, where $\mathfrak{v}_i$ and $\mathfrak{v}_e$ are the macroscopic velocities of the two components, and immediately consider the case of unnoticeable dissociation $N_i = N_e = N[\mathrm{cm}^{-3}]$ (no charge separation). Let the accelerations $\mathfrak{F}_e$ and $\mathfrak{F}_p$ [cm/sec^2] be caused by external forces (electric fields, radiation pressure) acting on the individual particles. Let $p_i + p_e = p_G = 2NkT$ be the gas pressure. Then

$$\mathfrak{v} = -2D_{12}\left\{\frac{m_p - m_e}{m_p + m_e}\nabla \log p_G - \frac{m_e}{kT}\frac{m_i}{m_i + m_e}(\mathfrak{F}_e - \mathfrak{F}_i)\right\} + D_T\frac{\partial \ln T}{\partial \mathfrak{r}}\left[\frac{\mathrm{cm}}{\mathrm{sec}}\right] \quad (1)$$

$$= \frac{2D_{12}}{kT}\left\{-\frac{1}{2N}\nabla p_G + m_e(\mathfrak{F}_e - \mathfrak{F}_i)\right\} + D_T\frac{\partial \ln T}{\partial \mathfrak{r}} \quad (2)$$

[1]A large part of the considerations described in this communication arose during the years 1939 to 1946 and were summarized in an unpublished report. --For interpretation of the magnetic field of the earth Elsasser proposed thermoelectric currents in the earth's core (Physic. Rev., Vol. 55, p. 489, 1939; Vol. 69, p. 106, 1946; Vol. 70, p. 202, 1946; Vol. 72, p. 821, 1947). --Also see T. G. Cowling, Monthly Notices Roy. Astronom. Soc., Vol. 105, p. 166, 1945 and H. Alfven, Ark. Mat. Astronom. Fysik, Ser. B, Vol. 29, No. 2, 1942; ibid. Ser. A., No. 12, 1943; C. Walen, ibid, Ser. A, Vol. 30, No. 15, 1944.

is valid[1]. In these equations we have now set $m_p + m_e = m_p$. The radiation pressure contained in $\mathfrak{F}$ acts only on the electrons, more strongly by the factor f (dimensionless) than according to the classic theory[2]. Even without non-mass-proportional external forces $m\mathfrak{F}$ and electric current of density $\mathfrak{j}$

$$\mathfrak{j} = +eN\mathfrak{b}\ [\text{el. st. E./cm}^2\ \text{sec}] \tag{3}$$

(e elementary charge in electrostatic units) would arise, it resulting from the circumstance that the pressure gradients exert essentially the same *force* on protons and electrons, so that the acclerations in the reciprocal relationship of the masses are different.

The pressure gradients and the other forces generating diffusion in accordance with equation (1) thus represent an impressed electromotive force in accordance with

$$e\mathfrak{E}^e = \frac{1}{2N}\nabla p_G - \frac{1}{2}\frac{D_T}{D_{12}}k\nabla T, \tag{4}$$

where the radiation force is now ignored.

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[1]Jeans, Dynamical Theory of Gases, Cambridge, 1925, Ch. XIII; Chapman-Cowling, Mathematical Theory of nonuniform Gases, 1939.

[2]A. Sommerfeld, *Atombau und Spektrallinien* [Atomic Structure and Spectral Lines], Vol. II, Chap. 6. The radiation force per electron is in the simplest case (energy transport through radiation); it is thus $(f/N_e)\nabla p_R$, where p_R is the radiation pressure. Factor f has the value 8/5 for the photoelectric effect from the K shell of an atom with $h\nu \gg \chi$, where χ is the binding energy of the electron; in general this value lies between 8/5 and 1.

If $(1/N)\ \nabla p_G$ has a potential, as in the case of gravity and (for example) rigid rotation, the surfaces of equal pressure and equal temperature coincide; hence the radiation pressure also has a potential, and a currentless state is reached through an electrostatic field

$$-e\mathfrak{E} = \frac{1}{2N}\nabla p_S - \frac{1}{2}\frac{D_T}{D_{12}}k\nabla T, \tag{5}$$

which arises automatically as a result of an extremely slight separation of the charges.

However, if the vector field $\mathfrak{E}^e$ also has a component whose rotation does *not* disappear, obviously no compensation by an electrostatic field is possible. Currents generating magnetic fields must flow. In the stationary case

$$\operatorname{div} \mathfrak{j} = \operatorname{div}(\sigma[\mathfrak{E}^e + \mathfrak{E}]) = 0, \tag{6}$$

$$\text{with } \operatorname{rot} \mathfrak{E} = 0, \tag{7}$$

where in accordance with $\operatorname{rot} \mathfrak{H} = \frac{4\pi}{c}\mathfrak{j}$, (8)

$$\sigma = 2\frac{e^2}{kT}D_{12} \tag{9}$$

the conductivity σ [$\sec^{-1}$] has now been introduced in place of D_{12}. $\mathfrak{E}^* = \mathfrak{E}^e + \mathfrak{E}$ is established by means of (7) and (8), as soon as $\mathfrak{E}^e$ and σ are known; the component $\mathfrak{E}^e - \mathfrak{E}^*$ is completely compensated by space charges, but $\mathfrak{E}^*$ only partly and in the case $\nabla\sigma \neq 0$.

2. First we show that deep in the interior of a star of the main series of the Russell diagram, where σ may be assumed to be 10^{17}[1], infinitesimal values

[1]This value corresponds to a temperature of several million degrees.

of $\mathfrak{E}^*$ produce great magnetic field strength. If an acceleration g^* is defined by

$$e\,\mathfrak{E}^* = m\,g^* \quad \left(m = \frac{m_i + m_a}{2} \approx \frac{m_i}{2}\right) \tag{10}$$

and if we set $g^* = 1$ cm/sec^2, it follows that $\mathfrak{E}^* \approx 10^{-15}$ electrostatic units and for rot $\approx 10^{-10.5}$ cm^{-1}

$$H \approx 10^{3}.$$

If we now ask under what conditions vortical accelerations g^* can be present stationarily, the most obvious possibility is found to be that of stationary pure rotation with $\partial\omega/\partial z \neq 0$, , where $\omega(z, \bar{r})$ [sec^{-1}] is the angular velocity of the rotation, the z axis of the axis of rotation, and $\bar{r}$ the distance from it. In this case the rotation of the centrifugal acceleration $\neq 0$, while in the other case ($\partial\omega/\partial z = 0$, $\partial\omega/\partial\bar{r}$ having any desired value) the centrifugal acceleration possesses a potential. If a velocity v_r^* is defined by

$$g^* = \frac{(v_r^*)^2}{r}, \tag{11}$$

we find what velocity values v_r^* are necessary. If we set rot $\approx 1/r$, it follows that

$$H \approx \frac{4\pi}{ec}\,\sigma\,\bar{m}\,(v_r^*)^2 = 10^{+3}\left(\frac{\sigma}{10^{17}}\right)\left(\frac{v_r^*}{10^5}\right)^2, \tag{12}$$

where $4\pi/ec$ numerically $= 10^{-0.1}$.

The interaction of the magnetic field with the matter of a star yields the force density

$$k = \frac{1}{c}[\mathfrak{j}\,\mathfrak{H}] = \frac{-1}{4\pi}[\mathfrak{H}\ \mathrm{rot}\ \mathfrak{H}]$$
$$= \frac{-1}{8\pi}\nabla\mathfrak{H}^2 + \frac{1}{4\pi}(\mathfrak{H}\ \mathrm{grad})\,\mathfrak{H}. \tag{13}$$

If k is resolved into an irrotational component k_1 and a solenoidal component k_2, the following is valid:

$$\mathrm{rot}\,k_1 = 0, \quad 4\pi\,\mathrm{div}\,k_1 = -(\mathfrak{H}\,\Delta\,\mathfrak{H}) - (\mathrm{rot}\,\mathfrak{H})^2, \tag{14}$$

$$4\pi\,\mathrm{rot}\,k_2 = \mathrm{rot}\,(\mathfrak{H}\ \mathrm{grad})\,\mathfrak{H}. \tag{15}$$

Hence k makes a contribution to pressure distribution; in the stationary case, of course, k_2 must be conpensated by acceleration fields exciting circulation.

In the case of cylindrical symmetry (coordinates $\bar{r}$, ϕ, z) and (for example) the existence of only a ϕ component of $\mathfrak{H}$,, the completely irrotational component $(-1/8\,\pi)\,\Delta\mathfrak{H}^2$ is everywhere $\perp\phi$, while the other component is everywhere $\parallel\bar{r}$ and has the value $\mathfrak{H}^2/4\pi\bar{r}$. The latter has a vortical component, but one which is generally small relative to ρg^* deep in the interior of stars. The following is valid:

$$\frac{1}{4\pi}\frac{\mathfrak{H}^2/\bar{r}}{\rho g^*} \approx \frac{4\pi}{c^2}\left(\frac{\sigma^2}{\rho}\right)\overline{\mu}^2\left(\frac{v^*}{c}\right)^2$$
$$\approx 10^{-6}\frac{1}{\rho}\left(\frac{\sigma}{10^{17}}\right)^2\left(\frac{v^*}{10^5}\right)^2. \tag{16}$$

In this equation $\sigma^2/\rho \sim p_R/p_G$.

In the most general case of stationary movements in hydrostatic equilibrium the hydrodynamic equation of motion in the Eulerian form, friction being ignored (Φ is the gravitational potential)

$$(\mathfrak{v}\ \mathrm{grad})\,\mathfrak{v} = \nabla\frac{\mathfrak{v}^2}{2} - [\mathfrak{v}\ \mathrm{rot}\ \mathfrak{v}] = -\nabla\phi - \frac{1}{\varrho}\nabla P. \qquad (17)$$

Hence the condition for the existence of eddy currents yields for the velocity fields the relation

$$\mathrm{rot}\left(\frac{1}{\varrho}\nabla P\right) = \left[\nabla\frac{1}{\varrho}\nabla P\right] = \mathrm{rot}\,[\mathfrak{v}\ \mathrm{rot}\ \mathfrak{v}] \neq 0. \qquad (18)$$

This more general form is of interest if meridional circulations or stationary vortices overlap the rotation, ones in which the changes of state are not adiabatic. Along the "fluid" lines carried along, on which the changes of state are adiabatic,

$$\oint\frac{1}{\varrho}\,dP = \iint\left[\nabla\frac{1}{\varrho}\nabla P\right]dF = 0. \qquad (19)$$

3. Let us now apply the considerations of sections 1 and 2 to the interior of stars. The theory of internal structure shows that the (convective) energy transport is impeded in elongated convection zones (e.g., determined in the central convection zone, in which the carbon cycle takes place $\perp z$ by rotation, but not $\parallel z$[1]. This gives rise to slow (non-adiabatic) meridional circulations, but ones which then transport rotational moment in addition to thermal energy and thereby cause a deviation of the rotation law from the state of rigid rotation, with (in the general case) $\frac{\partial\omega}{\partial z} \neq 0$.

[1]In connection with the following discussion, cfr. Biermann, Z. Astrophysik, Vol. 25, p. 135, 1948.

According to the theorem of Bjerknes, which is derived from equation (18), if the rotational velocity is substituted for v, this leads to a stationary tendency of the isothermal toward the isobaric surfaces, for the order of magnitude of which $\frac{v_r^2}{rg}$ is obtained; in this case the meridional circulations serve to maintain the proper rotational state and the thermal equilibrium. Hence in stars with subatomic energy sources the case of rotation with $\partial\omega/\partial z \neq 0$ is the one generally realized. Indeed, observations of the rotational law of the solar surface and the long-lived focal area of sunspots indicated this type of rotation. The special model of solar rotation by M. Schwarzschild[1], of course one calculated without allowance for the circulation, also shows $\partial\omega/\partial z \neq 0$.

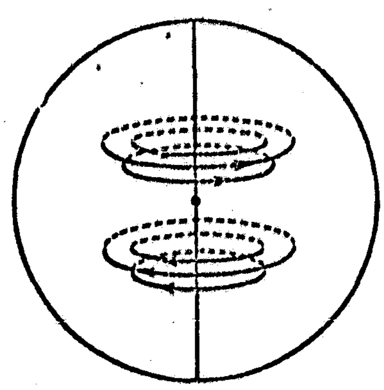

Figure 1. Model Distribution of Magnetic Field Strength. More Rapid Rotation of the Zones of the Star Near the Center are Assumed.

A model distribution of magnetic field strength, on the assumption of a specific rotational law, one based on calculations by A. Schluter, is given in the supplement. It is assumed that the central parts of the sun rotate more rapidly than the outer ones. Figure 1 gives a (qualitative) picture of the pattern of the field lines.

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[1] M. Schwarzschild, Astrophysic. J., Vol. 106, p. 427, 1947.

Table 1. Angular Velocity and Magnetic Field in the Case of Non-rigid Rotation of the Sun.

($\lambda = d\log \omega^2/d\log r$; cfr. Supplement)

	Angular Velocity ω [°/d] for		Magnetic Field H for ±45° in Gauss	
λ	$r/R = 0{,}2$	$r/R = 0{,}5$	$r/R = 0{,}2$	$r/R = 0{,}5$
−3	1750	112	−4220	−1230
−2	350	56	−462	−352
−1	70	28	−56	−88
0*	14,0	14,0	0*	0*
+1	2,8	7,0	+ 7,0	+ 20
+2	0,56	3,5	+ 7,8	+ 38

*Rigid Rotation

The field lines of the magnetic field which come about in this manner run parallel to the circles of latitude under the surface, forming two anti-parallel packets in the two hemispheres; hence they can be brought to the surface only by local dynamic disturbances[1]. In this case the magnetic pressure $\mathfrak{H}^2/8\pi$ must be overcome, but only on the surface itself is it comparable to the gas pressure or the kinetic energy head (for plausible velocity values).

4. Hence it is found that the electric currents caused by diffusion effects of the type described are, with quite modest assumptions as to the acting potentialless accelerations or the velocities causing them, sufficient to produce and maintain magnetic fields of the order of magnitude observed on stars. On the other hand, the special model discussed, specifically, rapid rotation of the central regions of a star, still does not yield the geometric position of the field lines permitting understanding of the observations of sunspots or the magnetic stars. Thus the question arises of whether other

[1]Cfr. T. G. Cowling, Monthly Notices Roy. astronom. Soc., Vol. 94, p. 39, 1934.

plausible models may be indicated which are better suited to the magnetic fields observed with respect to their form.

For this purpose we proceed from the idea proposed by Bjerknes for interpretation of sunspots and ask what rotation law $\omega(z, \bar{r})$ in the hemisphere would lead to two antiparallel closed rings of *magnetic field lines* such as would have to exist according to Bjerknes' model of solar activity. If we first consider an activity minimum, at a certain depth beneath the surface the one ring would have to be at a latitude of approximately 30° and the other very near the plane of the equator. According to Bjerknes, these rings are vortical rings rotating in opposite directions. Thus *between* them some matter must constantly be sucked from the interior to the surface (Figure 2) or pumped from the surface into the interior. In the first case matter with a relatively smaller rotational moment and relatively higher temperature is brought to the surface, so that on the surface under stationary conditions there should be a relative minimum of the rotational velocity and a maximum of temperature, while in the second case the reverse would have to obtain. The opposite apparently occurs toward the poles of the vortical ring at a latitude of 30° and directly at the equator. Both effects--change in the distribution of the rotation impulse and the temperature--act in the same direction and must enter into equilibrium over the Bjerknes relation. Thus the vortical rings postulated by Bjerknes lead precisely to a tendency of the isobaric and the isothermal surfaces in the vicinity of the vortical rings such that two antiparallel magnetic rings result in each hemisphere. If the vortex tubes move about each other in the course of the cycle, after 11 years the magnetic relationships are also reversed[1].

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[1] Induction effects, which are assigned the main law in the works of Alfven[1] and Walen, come into play in this case.

Of course, this by no means clarifies the dynamics and genetics of the vortical system postulated. The purpose was merely to show that the kinematics of the sunspots proposed by Bjerknes influences the rotation law or the pressure-density distribution precisely as would be necessary for generation of two antiparallel magnetic rings in each hemisphere. The circulation of 23 years would obviously have to lead the magnetic fields around as well. To permit explanation of the observed field strengths of several thousand Gauss without the need for postulated too high velocities v_r^* (i.e., too great local velocity differences in rotation about the solar axis), the rings would for the most part have to run at a very great depth (≈ 100,000 km), where the conductivity is high enough. This assumption is also necessary for the reason that otherwise the rotational law observed on the surface would have to exhibit more distinct traces of these fluctuations deep in the interior.

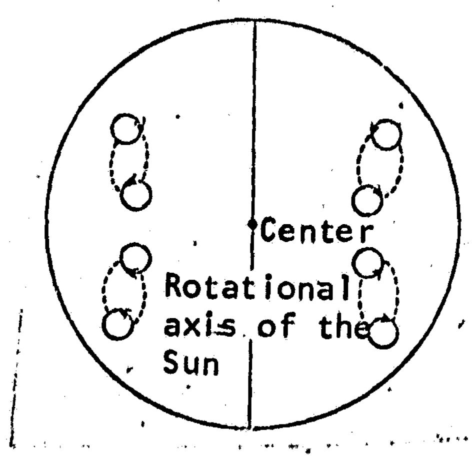

Figure 2. Illustration of the Kinematics of Sunspots. Instantaneous Position of the Vortex Tubes (According to Bjerknes) in a Section Through the Axis of Rotation. The Slow Meridional Circulation Postulated, Which Leads the Vortex Tubes About Each Other in 2 X 11 Years, is Shown by Broken Lines.

It is natural to ask the question of the accuracy with which the Bjerknes relation must be applied.

$$\mathrm{rot}\,\dot{\mathfrak{v}} = \left[\nabla \frac{1}{\varrho}\,\nabla P\right] \tag{20}$$

holds for the circulation accelerations which arise. In the state of stationary rotation it must be precisely equal to rot $[\mathfrak{w}[\mathfrak{w}\mathfrak{r}]]$, where $[\mathfrak{w}[\mathfrak{w}\mathfrak{r}]]$ is the centrifugal acceleration. If this is not the case, the relation gives the acceleration of the circulations which arise. The order of magnitude of this acceleration in relation to $(1/\rho)\nabla P = g$ is obviously given by the angle of the two gradients. An angle of, for example, $10^{-4.5} = 6''$ again leads to $|\dot{\mathfrak{v}}| \approx 1$ cm/sec^2, i.e., in one day two velocities of the order of 1 km/sec and paths of the order of 100,000 km, so that in a few days each disturbance of the pressure-density distribution of the order of magnitude under discussion here must lead to a state in which the Bjerknes relation is valid.

For reasons of energy, insofar as the rotational velocity does not increase sharply toward the interior, the action of the vortex rings can occur under stationary conditions only in a region which is slightly hyperdiabatically stratified. Should the hydrogen zone be of the depth proposed by the author[1], approximately the correct relationship would occur perhaps in their deeper parts. An excess above the adiabatic gradients of 10^{-9}°/cm would lead to relative temperature differences of the order of 10^{-5} over stretches of the order of 10^{10} cm; it appears to be appropriate to the assumed $g^*:g$ ratio.

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[1] L. Biermann, Astronom. Nachr., Vol. 264, p. 395, 1938; Z. Astrophysik, Vol. 21, p. 320, 1942.

It is thus found that one can probably understand the magnetic fields of the sunspots on the basis of this diffusion effect of free electrons as soon as one has understood the dynamics of the spots.

For the magnetic fields observed on stars of the type of 78 Virginis, the Bjerknes concept presumably yields no suitable model. Thus let us discuss qualitatively another possibility, one which is suggested by the relationships observed in terrestrial revolving storms. In these storms one finds at the outside (at a great distance from the axis) a constant rotational moment, in the interior constant angle of velocity (as a result of the predominance of friction), and in between a transitional region in which the matter sucked outward apparently is subject to delays possessing a substantial component about the axis (i.e., $\parallel \phi$). In the stationary case accelerations of this kind thus lead in a plasma to *axial* magnetic fields[1]. Let us attempt to transfer this concept.

For this purpose let us consider a rapidly rotating star. Let v_r^2 be of the order of $rg_{\mathrm{eff}} \approx kT/m$; that is, let the star be directly at the boundary of rotational instability. In this case the velocity of the meridional circulations will be a not inconsiderable fraction of the turbulence velocity and the rotation velocity, and all the velocities in question are in rough approximation of the same order. On the other hand, the adiabatic relationships no longer provide a good approximation for the interior, because the turbulent energy transport in these stars is greatly impeded by the rotation. The

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[1]Vertical acceleration fields also produce electric currents in plasma, as may be derived from equation (1); in this connection Cfr. A. Schluter, Z. Naturforschg., Vol. 5a, p. 72, 1950.

meridional circulation itself will be turbulent, at least in large areas of the interior of the star[1]. A stationary state will result from the interaction of circulation and turbulent friction, in such as way that, despite a certain circulation of the angular momentum as well, the rotational law $\omega(z, r)$ undergoes only secular change.

If these concepts of the structure and the rotational state of rapidly rotating stars are true, substantial acceleration components about the axis of rotation must obviously arise in certain zones of these stars, components which, as already remarked, lead to axial magnetic fields. Since the circulations (apparently) carry the magnetic field along with them, quantitative discussion is highly complicated, the more so as the theories outlined for the structure of such stars also actually have to be carried through. With the formulas already given one can only estimate that magnetic field strength of the order of 10^3 to 10^4 Gauss would be attained with circulation velocities of the order of 1 to 10 km/sec. These do not appear to be implausible for rapidly rotating stars.

For the sake of comparison let it be noted that the wind speed in terrestrial revolving storms can reach a considerable percentage of the velocity of sound.

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[1] General considerations of the theory of turbulence indicate that such stars exhibit turbulence of a much greater scale than do slowly rotating stars. I am indebted to v. Weizsacker for observations on this point.

5. In this section we discuss the question of the time scale[1] and indicate the extension of this approach to interstellar space. The period of persistence of a magnetic field constructed in matter at rest by impressed electric forces, as well as the decay time of a given magnetic field (energy consumption by joulean heat), amount from the standpoint of order of magnitude to $\sigma Q/c^2$, where Q is the cross-section (cm^2). But if electric forces are now induced by movements of the conducting matter, energy of motion can hereby be converted into magnetic energy. If the medium is a very good conductor, the energy losses due to joulean heat are very small, and the mechanical energy is converted quantitatively into magnetic energy. The time scale mentioned above is then misleading. In this case the magnetic flux through a closed line flowing along with the matter remains constant in the first approximation for $\mathfrak{E}^* = 0$. The magnetic flux (Gauss X cm^2) can be changed only by non-vortex-free and non-mass-proportional forces. Thus as soon as one must reckon with the fact that the resulting magnetic field strengths and energy densities are considerably increased by induced mass movements and resulting induction effects, one must ask whether the ring integral of the impressed electromotive forces suffices for production of the magnetic flux $\int \mathfrak{H}_n dF$ (Gauss cm^2) observed. This leads to the relation

$$-\frac{d}{dt}\int \mathfrak{H}_n \, dF = c \oint \mathfrak{E}^* \, d\sigma \approx \frac{c}{e} \overline{m} \, \mathfrak{v}^{*2} \approx 10^6 \, (\mathfrak{v}^*/\mathrm{km/sec})^2 . \tag{21}$$

[1]In this connection cfr. T. G. Cowling[1].

On the sun one observes in the spots fluxes of 10^{20} to 10^{22} Gauss/cm^2, and on stars of the type of 78 Virginis approximately 10^{25} Gauss/cm^2. If one assumes that both arose in $3 \cdot 10^8$ years ($= 10^{16}$ sec), 10^6 of 10^9 Gauss cm^2/sec follows for the flux change, which would require velocities v_r* of the order of 1 km/sec or 30 km/sec.

The treatment set forth for the interior of stars may be transferred to interstellar space; when this is done, the different conductivity properties of this highly rarified matter (especially in the presence of magnetic fields) must be taken into account[1]. If we assume on $[10^{18}$ cm$]^2$ a vortex mechanism which generates a magnetic flux and calculate with v_r = 10 km/sec, in a period of $3 \cdot 10^4$ years (10^{12} sec) we obtain a flux of again 10^{20} Gauss cm^2, i.e., at first 10^{-16} Gauss. However, similar field strengths could be reached also by magnetic fields given off by strongly magnetic stars. The magnetic energy density could and should be elevated to the vicinity of the turbulent energy density, as a result of induction effects and the constant occurrence of new turbulence elements. However, these matters will be discussed in another place[2].

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[1]This extension of the considerations was suggested to me by discussions with C. F. v. Weizsacker and O. Haxel. The precise equations describing the electromagnetic behavior of such matter in the presence of external forces are discussed by A. Schluter, Z. Naturforschg., Vol. 5a, p. 72, 1950.

[2]A. Schluter and L. Biermann, Interstellar Magnetic Fields, Z. Naturforschg., Vol. 5a, 1950.

Supplement

Model Solution

As a completely integrable model for the occurrence of magnetic fields through non-rigid rotation let us consider a star (radius R) all of the elements of the volume of which rotate about the same axis, at an angular velocity $\vec{\omega}$, which depends only on the central distance r of the point in question. We make for $\vec{\omega}$ the special assumption:

$$\vec{\omega} = \mathfrak{a}\left(\frac{r}{R}\right)^{\lambda/2}; \quad \lambda > -5. \tag{22}$$

λ is here an arbitrary parameter for which the only requirement is that it be greater than -5 in order that the rotational energy will remain finite.

If we ignore the effects of radiation pressure and thermal diffusion, in hydrostatic equilibrium the pressure gradient of gravitational acceleration plus centrifugal acceleration must maintain the equilibrium:

$$\begin{aligned} \frac{1}{N(m_1+m_e)} \nabla p_G &= \mathfrak{g} + [\vec{\omega}\,[\vec{\omega}\,\mathfrak{r}]] \\ &= \mathfrak{g} + \left(\frac{r}{R}\right)^{\lambda} [\mathfrak{a}\,[\mathfrak{a}\,\mathfrak{r}]]. \end{aligned}$$

the system of equations to be solved according to (4), (6), and (7) hereby become

$$\begin{aligned} \mathfrak{j} &= \sigma(\mathfrak{E} + \mathfrak{E}^e); \\ e\,\mathfrak{E}^e &= \frac{1}{2N} \nabla p_G = \bar{m}\,\mathfrak{g} + \bar{m}\left(\frac{r}{R}\right)^{\lambda} [\mathfrak{a}\,[\mathfrak{a}\,\mathfrak{r}]] \\ \operatorname{div} \mathfrak{j} &= 0; \quad \operatorname{rot} \mathfrak{E} = 0. \end{aligned} \tag{23}$$

Let us now make the simplifying assumption that the conductivity σ in the interior of the star is constant:

$$\sigma = \begin{cases} \sigma_0 \text{ for } r < R, \\ 0 \text{ for } r > R \end{cases}$$

We now resolve $\mathfrak{E}^e$ into the vortical and potential components

$$\mathfrak{E}^e = \mathfrak{E}^* + \mathfrak{E}'; \quad \operatorname{div} \mathfrak{E}^* = 0; \quad \operatorname{rot} \mathfrak{E}' = 0,$$

so that

$$\operatorname{rot} \mathfrak{E}^* = \frac{m}{e} \operatorname{rot} [\vec{\omega} [\vec{\omega} \mathfrak{r}]]; \quad \operatorname{rot} \mathfrak{g} = 0.$$

As may be verified by differentiation, the solution for $\mathfrak{E}^*$ unequivocal except for one homogenous term proves to be

$$\mathfrak{E}^* = \frac{m}{e} \frac{1}{\lambda + 5} \left(\frac{r}{R}\right)^{\lambda} \left\{ (\lambda + 3)\, \mathfrak{a} (\mathfrak{a} \mathfrak{r}) - \lambda \frac{\mathfrak{r} (\mathfrak{a} \mathfrak{r})^2}{r^2} - \mathfrak{a}^2 \mathfrak{r} \right\}. \tag{24}$$

On the assumption that $\mathfrak{E} = -\mathfrak{E}'$ we could satisfy the condition $\operatorname{div} \mathfrak{j}' = 0$, but not on the surface. Because of this condition, on the surface the requirement must be set that the normal component of $(\mathfrak{E} + \mathfrak{E}^e)$ will vanish. Hence we assume:

$$\mathfrak{E} = \mathfrak{E}'' - \mathfrak{E}', \text{ d. h. } \mathfrak{E} + \mathfrak{E}^e = \mathfrak{E}'' + \mathfrak{E}^*,$$

$$\operatorname{rot} \mathfrak{E}'' = 0, \quad \operatorname{div} \mathfrak{E}'' = 0 \text{ for } r \leq R, \tag{25}$$

$$(\mathfrak{r}\, \mathfrak{E}'') + (\mathfrak{r}\, \mathfrak{E}^*) = 0 \text{ for } r = R.$$

The only solution of system (25) with the value of (24) for $\mathfrak{E}^e$ is in the interior of the star

$$\mathfrak{E}'' = \frac{m}{e}\,\frac{1}{\lambda+5}\,\{3\,\mathfrak{a}\,(\mathfrak{a}\,\mathfrak{r}) - \mathfrak{a}^2\,\mathfrak{r}\}; \quad r < R.$$

It thus follows for the current density in the interior that

$$\begin{aligned} \mathfrak{j} &= \sigma_0\,(\mathfrak{E} + \mathfrak{E}'') = \sigma_0\,(\mathfrak{E}'' + \mathfrak{E}) \\ &= \frac{\sigma_0\,m}{e}\,\frac{1}{\lambda+5}\left(1-\left(\frac{r}{R}\right)^{\lambda}\right)\{3\,\mathfrak{a}\,(\mathfrak{a}\,\mathfrak{r}) - \mathfrak{a}^2\,\mathfrak{r}\} \\ &\quad - \frac{\sigma_0\,m}{e}\,\frac{\lambda}{\lambda+5}\left(\frac{r}{R}\right)^{\lambda}\left\{\mathfrak{a}\,(\mathfrak{a}\,\mathfrak{r}) - \frac{\mathfrak{r}\,(\mathfrak{a}\,\mathfrak{r})^2}{r^2}\right\}. \end{aligned} \tag{26}$$

The magnetic field strength is obtained as the solution of the equations

$$\operatorname{div}\mathfrak{H} = 0, \quad c\,\operatorname{rot}\mathfrak{H} = \begin{cases} 4\pi\,\mathfrak{j} & r < R \\ 0 & r > R \end{cases}; \quad \mathfrak{H} \to 0 \text{ for } r \to \infty$$

unequivocally as

$$\mathfrak{H} = \begin{cases} 4\pi\,\sigma_0\,\frac{m}{e\,c}\,\frac{\lambda+5}{\lambda}\left(1-\left(\frac{r}{R}\right)^{\lambda}\right)[\mathfrak{a}\,\mathfrak{r}]\,(\mathfrak{a}\,\mathfrak{r}) & \text{for } r < R \\ 0 & \text{for } r > R, \end{cases} \tag{27}$$

as may again be confirmed by differentiation.

The magnetic field lines run entirely in the interior as circles parallel to the equator about the axis of rotation. Since they thus coincide with the stream lines of the motion of the elements of volume of the star, we are additionally justified in not having taken the electrodynamic effects of the motion of conducting matter in a magnetic field into account.

For the sake of illustration let us calculate the field strengths which might possibly arise on the sun by way of this mechanism.

We assume

$e/\bar{m} = 5.74 \cdot 10^{14}$ el. stat. cgs. (one-half proton mass),
$\sigma = 10^{17}$ el. stat. cgs. (correct approximately for $r/R = 0.4$),
$|\alpha| = 14°/d = 2.83 \cdot 10^{-6}$ sec^{-1} (equitorial angular velocity of the sun),
$R = R_{\odot} = 6.95 \cdot 10^{10}$ cm (solar radius).

There thus follows for the value H of the magnetic field (ϕ = heliographic width):

$$H = 1{,}41 \cdot 10^{3} \cdot \frac{(r/R)^2}{\lambda + 5} \left(1 - \left(\frac{r}{R}\right)^{\lambda}\right) \sin 2\varphi \,[\text{Gauß}].$$

The resulting numerical values are given in Table 1.