Final Report

for

INVESTIGATION AND ANALYSIS OF TIME CODES

January 7, 1968

Contract NAS 5-10540

Prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

ADVANCED COMMUNICATIONS INFORMATION MANAGEMENT

RESEARCH • DEVELOPMENT • ENGINEERING

ADCOM

A TELEDYNE COMPANY

AD/COM

Final Report

for

INVESTIGATION AND ANALYSIS OF TIME CODES

January 7, 1968

Contract NAS 5-10540

Prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

Approved by Elie J Baghdady
Elie J. Baghdady
Technical Director

ADCOM, Inc.
808 Memorial Drive
Cambridge, Massachusetts

G-110

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF TABLES (cont.)

1. INTRODUCTION

This document constitutes the Final Report by ADCOM, a Teledyne Company, to the NASA Goddard Space Flight Center on Contract NAS5-10540.

With the First Quarterly Report dated October 6, 1967, on the above contract, as background, the present report is aimed principally at developing the concept of a practicable optimal time code system, and at a comparison of the performance of the optimum system with the performance of the existing NASA and IRIG systems in order to determine the margin of improvement possible over the existing systems.

The topical organization of this report is as follows:

The time code signal design requirements for a practicable optimal system are investigated in Section 2. The system concept is based on the approach advanced in Section 4.2 of the First Quarterly Report, which advocates capitalization on the optimality of correlation detection as the technique for extracting time marker location information. Accordingly, the use of binary-sequence waveforms that utilize the available time frame in various interesting ways is examined, with emphasis on the autocorrelation properties of each waveform. In this way a practical basis for the optimal frame utilization strategy and time-code signal design is established. The performance achievable with the resulting frame synchronization sequences is then analyzed and a comparison is made with sequences typical of current NASA and IRIG systems.

In Section 3, a summary is given of conclusions and recommendations. The highlights of the performance comparison of the optimal and the current systems are restated.

2. OPTIMAL TIME CODE SYSTEM CONCEPT

A system for the transmission and distribution of any kind of information is determined principally by its signal design and detection techniques. The fundamental signal design requirements for an optimal -- but practicable -- system for timing information are the major topic of the present section. The prescribed detection technique is a cross-correlation process.

The importance of the autocorrelation properties of the time marker waveform in the design of an optimal system was stressed in Section 4.2 of the First Quarterly Report. It was also pointed out that maximal length linear sequences, when represented by appropriate waveforms, could provide the most desirable type of autocorrelation properties. Consequently, the major signal design consideration in the present discussion is the autocorrelation function of the waveform representation of the binary code sequence. The carrier modulation design for transmission over wireline, tropo and HF are also discussed. The performance achievable with the type of code waveforms recommended here is computed and illustrated.

2.1 Characterization of Time Information Signal

a) Time Markers -- Accuracy, Precision, and Unambiguity

Accuracy is a measure of the error in the position of the marker. It is the difference between the indicated and actual times. Hence, accuracy is a measure of the delay uncertainties in the transmission link, including propagation medium and systematic errors due to imperfections of terminal equipment.

Precision is a measure of the limit on exactness to which marker position can be read as a result of random errors from all sources. Thus, precision is a measure of the "width" of the probability density function of the marker indicator plus the disturbances present. In the current NASA and IRIG codes, once bit and frame sync have been acquired, precision is determined by the S/N ratio in accordance with Fig. 4.31, and by low-frequency cutoff in accordance with Tables 4.4 and 4.5 of the First Quarterly Report.

For a time code to convey useful information, each marker must be unambiguous -- that is, uniquely identifiable. Ambiguity results if more than one marker position indication can result from examination of the marker code signal. For example, a marker code signal whose autocorrelation function possesses side-peaks, or "sidelobes", that are comparable with the main peak, may be falsely acquired at the position of a side peak, causing an erroneous indication of the position of the time marker in question.

b) Acquirability -- Acquisition Process, and Processing Time for Unambiguous Acquisition

Acquisition of a time code is the process of deriving the time markers from the incoming signal plus noise, and hence the unambiguous establishment of the time instants conveyed by the marker positions. The acquisition procedure depends upon the code structure and its processing requirements.

Optimally, the acquisition process is a cross-correlation operation wherein a locally generated replica of the marker code "word" is cross-correlated in various time positions with the incoming code signal until the absolute peak of correlation is obtained. If the autocorrelation function of the code sequence waveform possesses side peaks that are not well below the main (absolute) peak, false acquisition may result on one of the side peaks. The location of the marker will then be indicated incorrectly. Such code sequence waveforms are therefore potentially <u>ambiguous</u> indicators of the time <u>position</u> of a marker relative to the occurrences for which the marker sequence is intended to provide the time. The time label serves merely to "tag" a marked time position.

For unambiguous marker acquisition, and hence time marking, a clear indication of the absolute maximum of the correlation function is necessary. Such an indication can be secured if the in-phase correlation peak towers prominently over the "sidelobes". What structure is optimal for the sidelobes relative to the main lobe depends on what occupies the time intervals on both sides of the interval covered by the code sequence waveform for each marker.

The processing time required for establishing unambiguous acquisition is of importance only in real-time (or quick look) examination of the transmitted time information. In nonreal-time operation, no acquisition speed-up operations are necessary.

c) Susceptibility to Interruption

The term interruption as applied to a time code frame shall mean a noise-induced event occurring some time after correct frame acquisition which looks like an indication of loss of frame synchronization and which therefore throws the system back into the acquisition mode.

There are two basic types of disturbance that may cause frame interruption: additive and multiplicative. An additive disturbance would be some level of background noise, causing occasional frame interruptions whose effect may be reduced to a predictably low value by merely designing the system to insure an adequate signal-to-noise level at the receiver.

When a multiplicative disturbance (such as signal fading) is present the received signal strength will fluctuate widely, giving rise to a fluctuating signal-to-noise ratio at the receiver. In order to analyze susceptibility to interruption for this case, one should realize that if the additive noise is of the gaussian variety, then the probability of interruption is highly sensitive to the signal-to-noise ratio, i.e., a change of one to two dB in SNR may cause a change of several orders of magnitude in the probability of interruption. Consequently, the whole range of possible signal-to-noise ratios may be divided into three regions: a "high" region in which the probability of interruption is negligible, a "low" region in which the probability of interruption is unacceptably high, and a transition region of one or two dB in width. The fraction-of-time that an interruption in the reception of the code signal will be encountered is determined by the probability that the SNR will fall below the threshold of the code tracking circuits.

d) Labeling Requirements -- Basic Redundancy Characteristics

In the labeling of the markers, each must be unambiguous, but only when viewed within a sequence. It is not necessary to label each marker completely within its own frame. For example, if a given marker is completely and uniquely identified, the label for the next marker is completely redundant except for the least significant bit (the bit that changed). With the knowledge that the two markers are consecutive and therefore separated by exactly one bit, this last bit is also redundant.

It is therefore not necessary to provide any label for many of the markers, and many of those with labels need not have complete identification. In this manner, the frame interval may be left free for an optimum PR code for very rapid and accurate synchronization of the marker. For example, we might insert a partial label every tenth frame, consisting only of the digit that changed (4 bits in BCD). Then for every ten partial labels (or once every one hundred frames) we might insert a complete label. This scheme would appear to work quite well for any of the shorter frame times, but will have to be modified for the larger ones. (Clearly, one does not wish to wait one hundred hours for a complete label if the frame time is one hour.)

2.2 Correlation Properties of Binary Codes and Binary Code Waveform Representation

2.2.1 Properties of Codes Typical of Current NASA and IRIG Usage

Both the NASA and IRIG codes obtain subframe and frame synchronization by inserting additional "sync" bits at appropriate intervals throughout the frame. Three different frame synchronization patterns are used. These are utilized respectively by 1) the NASA pulse duration modulated (PDM) binary-coded decimal (BCD) codes, 2) the NASA serial decimal code, and 3) the IRIG PDM BCD codes.

In all codes, the frames are divided into subframes, with 6, 10, or 12 subframes per frame. The number is picked to make the subframe a convenient length. For example, if the frame width is one minute, 12 subframes are used, so that each subframe is 5 seconds wide. In one hour frames there are 6 subframes of 10 minutes each. Most of the other frame widths are a convenient decimal number in length of time, for example, 0.1 sec., 1 sec., or 10 sec., and 10 subframes are used. In all codes, each subframe consists of 10 bits or pulses. One bit width is defined as the distance between leading edges of consecutive pulses.

There are three NASA PDM BCD codes - the one sec., the one minute and the one hour. A mark is conveyed by a pulse equal to 60 per cent of the bit width while a space is given by a 20 per cent of bit-width pulse. Subframe synchronization is accomplished by an additional pulse of "mark" width at the beginning of each subframe. Frame synchronization is attained by five consecutive bits (1/2 subframe) at the end of each frame. The autocorrelation function of this frame synchronization sequence is given in Fig. 2.2 in the next section.

There is one NASA Serial Decimal Code with a frame time of 10 sec. Data is not binary coded but is displayed in decimal form, and recovered by counting the number of pulses in each subframe. Each subframe corresponds to a particular digit in the data. All pulses are of equal width, approximately 50 per cent of the bit width. Subframe synchronization is accomplished by one extra pulse added at the beginning of each subframe. Frame synchronization is attained by two additional pulses at the beginning of each frame.

There are six IRIG PDM BCD codes - 0.1 sec., 1 sec., 10 sec., 1 min., 5 min., and 1 hour. The pulses are ternary (3-level) pulse duration modulated. Sync pulses are 80 per cent of bit width, marks are 50 per cent, and spaces are 20 per cent. Subframe sync is by one sync pulse, the last bit of each subframe. Frame sync is accomplished by means of two pulses, the first and last bits of each frame.

2.2.2 Autocorrelation Properties of Various Waveform Representations of a Pseudorandom Time Code Sequence

Any coded message requires synchronization for proper decoding. In particular, a timing code requires synchronization for proper identification of the location of the timing markers. Present timing codes use a pulse train of five pulses for synchronization. Improvements that may be attained by using a pseudo-random (PR) code sequence will be considered. The shortest pulse width in a code determines the bandwidth of the code. To maintain the same bandwidth, the shortest pulse in the present timing code will be used as a guide to determining the bit width of the PR code.

Given a train of timing markers as in Fig. 2.1, the time between markers, designated a frame, is divided between synchronization and a label for the preceding marker. Three possible ratios of synchronization time to label or data time within the frame will be discussed. These are a) only a minor subinterval, say 5 percent, used for synchronization, as is the case in present codes, b) half of frame used for synchronization, c) full frame used for synchronization.

A measure of the suitability for synchronization purposes of any particular code is given by its autocorrelation function, $R(k)$. Indication of proper acquisition is simplified if the amplitude of the principal peak, $R(0)$, towers prominently over secondary peaks in $R(k)$ for $k \neq 0$.

2.2.2.1 Minor Subinterval Used for Synchronization

In the current timing codes, only a minor subinterval is used for synchronization of the marker. If this constraint is maintained, it is of interest to compare what improvement can be attained over the current system by using a pseudo-random code approach. The present five pulse code and its autocorrelation function is given in Fig. 2.2. Note the large secondary peaks which increase the probability of false acquisition in the presence of noise.

Present time codes use a synchronization time of five bit widths. The shortest pulse width used in the code, a space, occupies 20 per cent of a bit

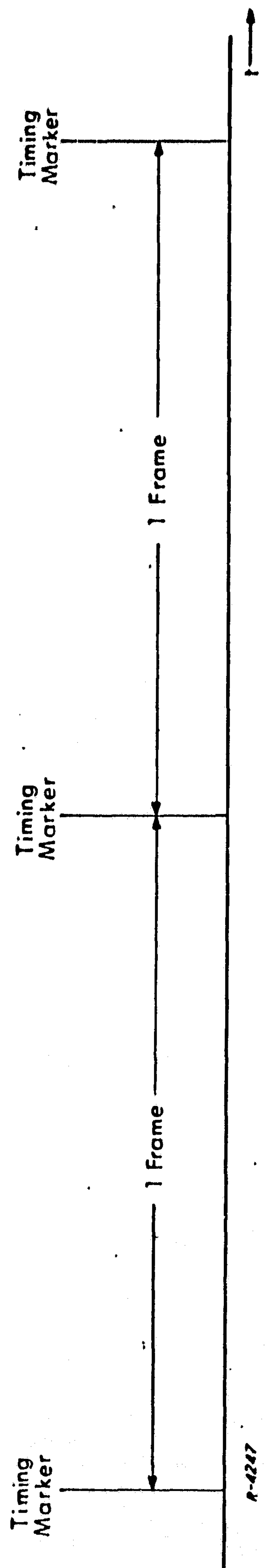

Fig. 2.1 Relationship Between Timing Markers and Frames.

width. Using the width of this pulse as a guide, a maximum of 25 bits may be used in the PR code. The PR codes with the most desirable properties are the so-called Maximal Length Codes. Recurring maximal-length codes are available in lengths (or number of bits) per period, n, given by

$$n = 2^{\ell} - 1 \qquad \ell \text{ an integer} \tag{2.1}$$

Therefore the 15 bit maximal-length PR code is the longest that may be used without increasing the bandwidth of the timing code. The 15 bit maximal length PR code is given by the following binary sequence

111100010011010

This code and its autocorrelation function is presented in Fig. 2.3 in the ±1 NRZ waveform. Note the improvement over the current code with a single large peak in the autocorrelation function at the exact point of synchronization, and very little correlation elsewhere.

Binary sequences may be represented by a number of different waveforms, and it is of interest to determine which of these waveforms yields the best autocorrelation function. The waveform and the corresponding autocorrelation function for the 15 bit maximal length PR code is presented in Figs. 2.3 through 2.10 for the following waveforms: ±1 NRZ (Fig. 2.3), RZ-Unipolar (Fig. 2.4), RZ-Bipolar (Fig. 2.5), Bipolar (Fig. 2.6), Split Phase (Fig. 2.7), Transition Encoded (Fig. 2.8), Dicode (Fig. 2.9) and Duobinary (Fig. 2.10). Each figure is drawn to the same scale for ease of comparison of the shapes of the autocorrelation functions. Clearly, the ±1 NRZ and Split Phase waveform have the most desirable properties (high primary peak and lowest secondary peaks of the autocorrelation function).

2.2.2.2 Half of Frame Used for Synchronization

Let us now examine what improvements may be attained in synchronization if we are not restricted to the same percentage of the frame used in the current codes, but are allowed half the frame for synchronization. Taking a nominal 100 bits/frame for the current codes, and again the narrowest

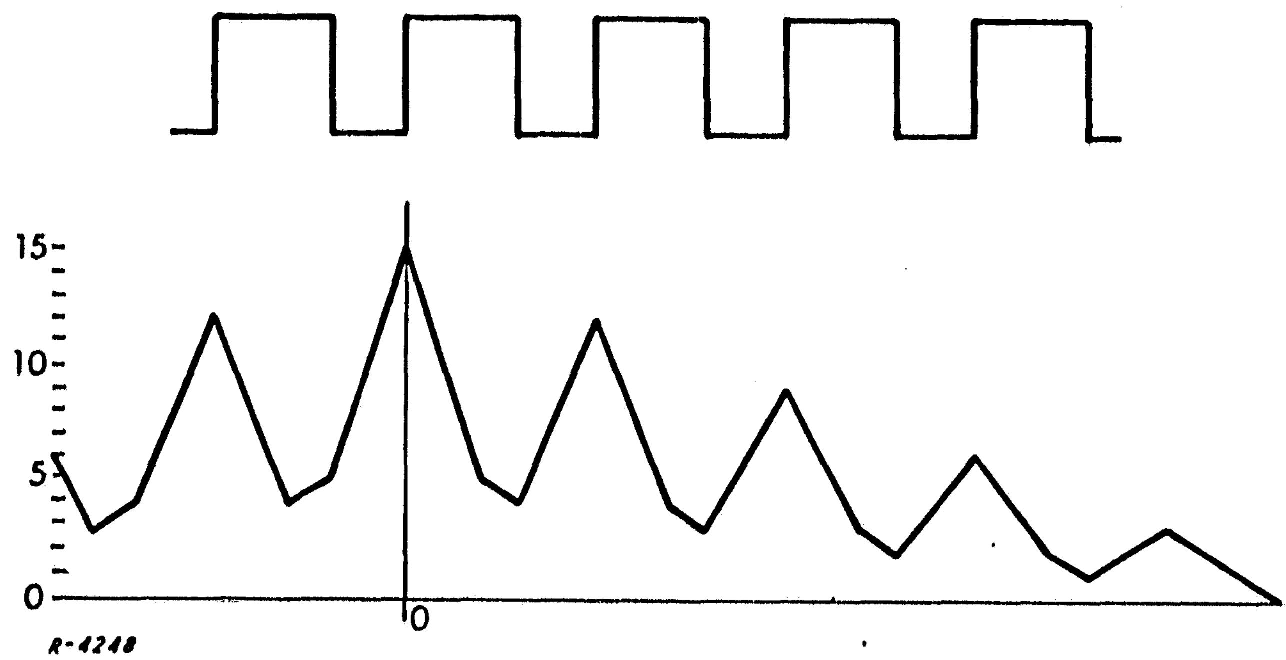

Fig. 2.2 Present Synchronization Code and Autocorrelation Function

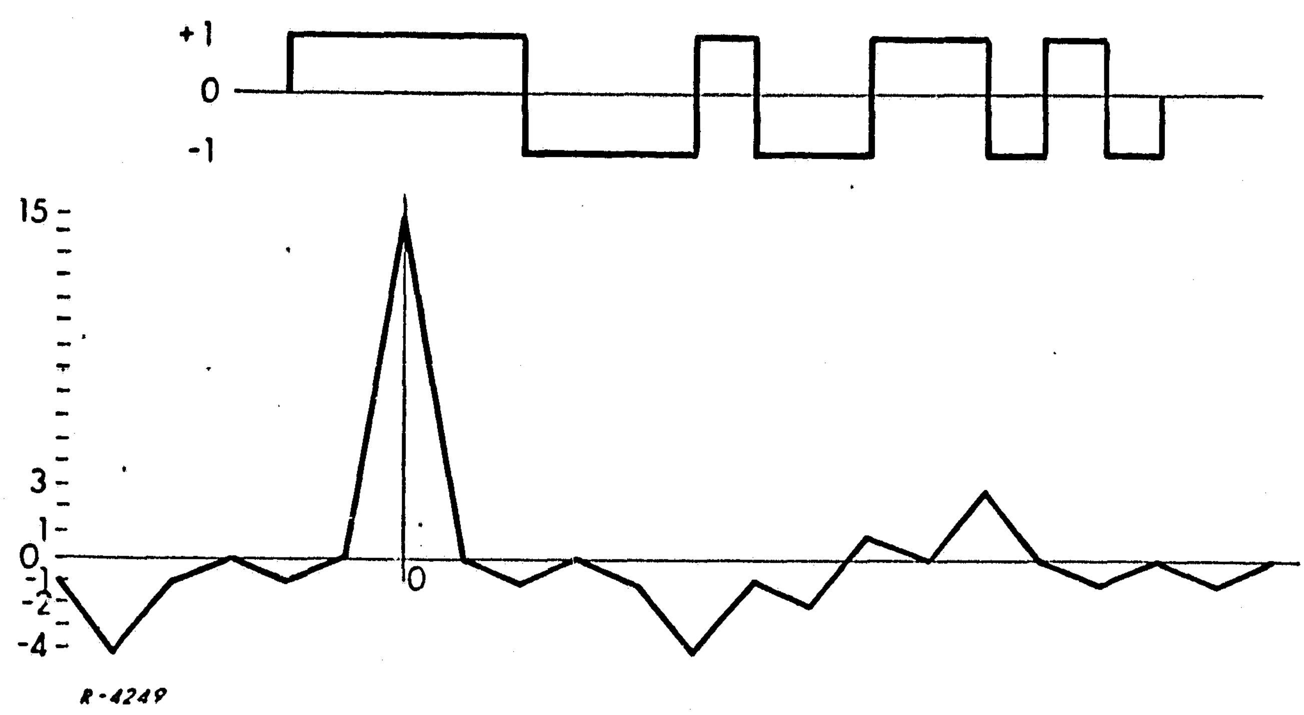

Fig. 2.3 15 Bit Maximal Length PR Code and Corresponding Autocorrelation Function ±1 NRZ Waveform

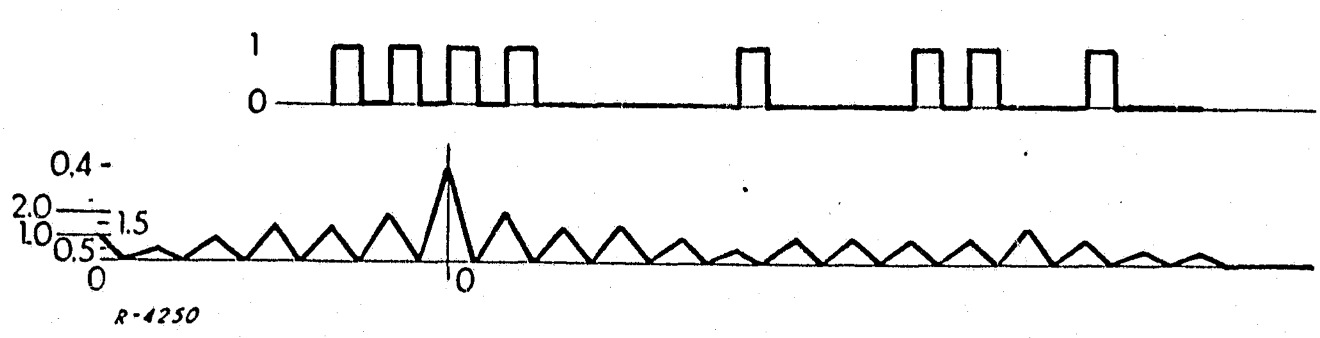

Fig. 2.4 RZ - Unipolar

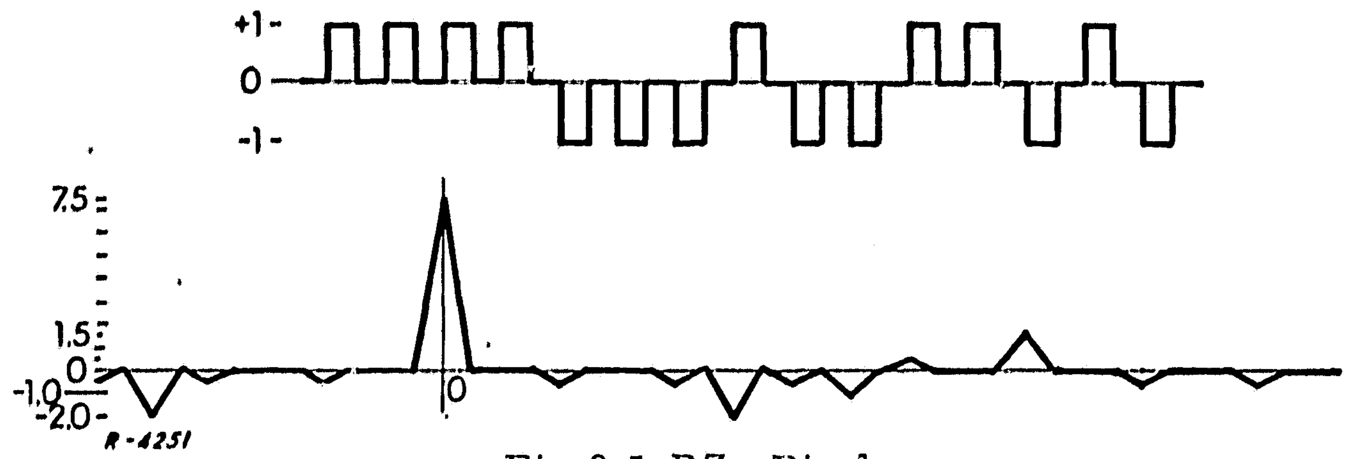

Fig. 2.5 RZ - Bipolar

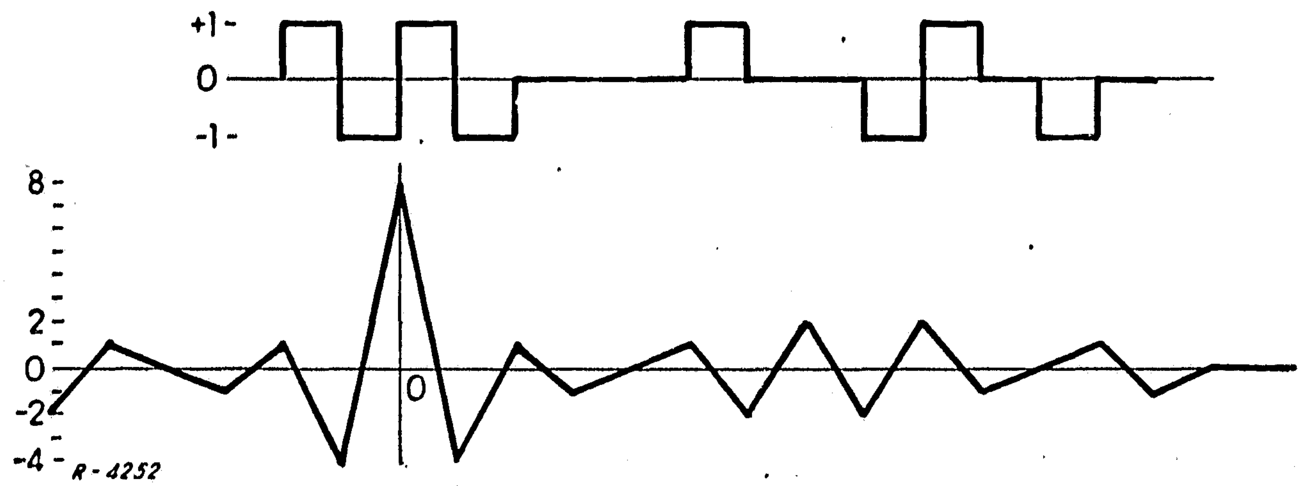

Fig. 2.6 Bipolar

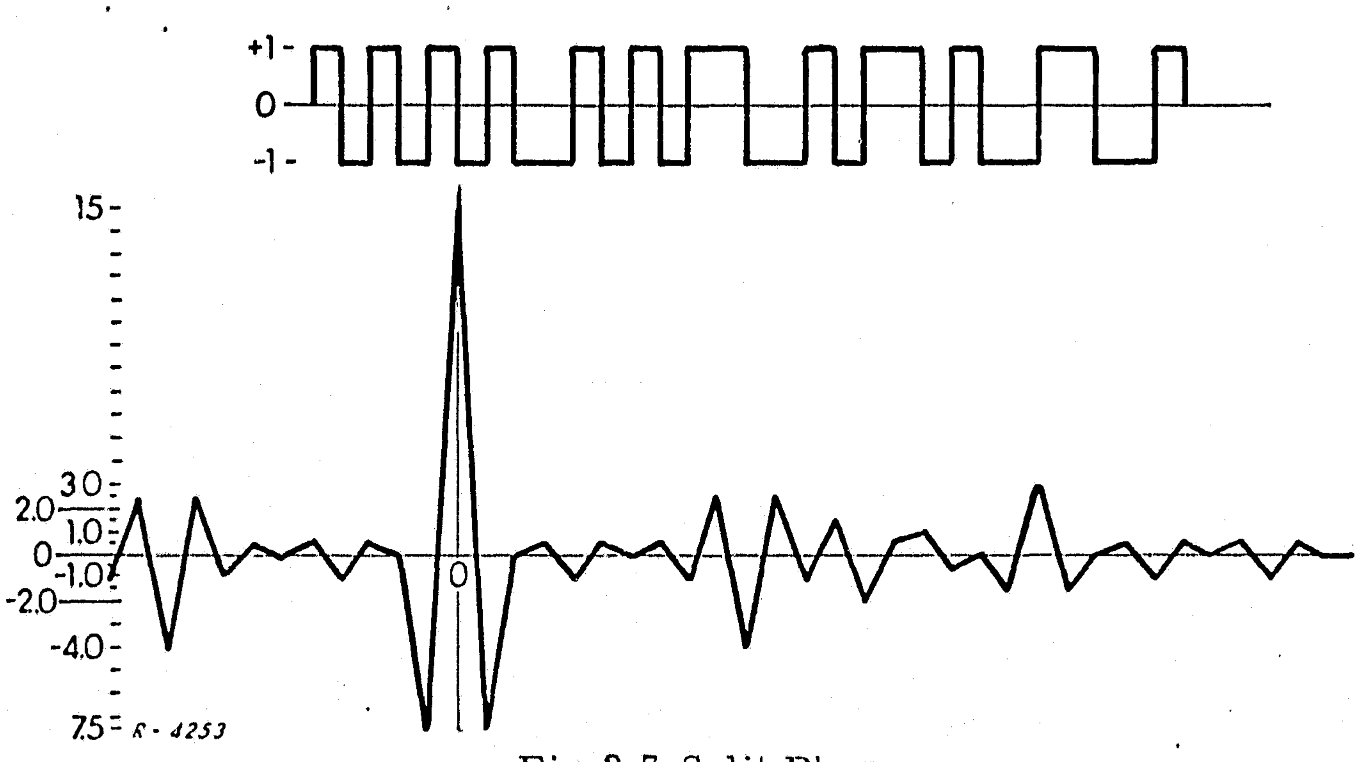

Fig. 2.7 Split Phase

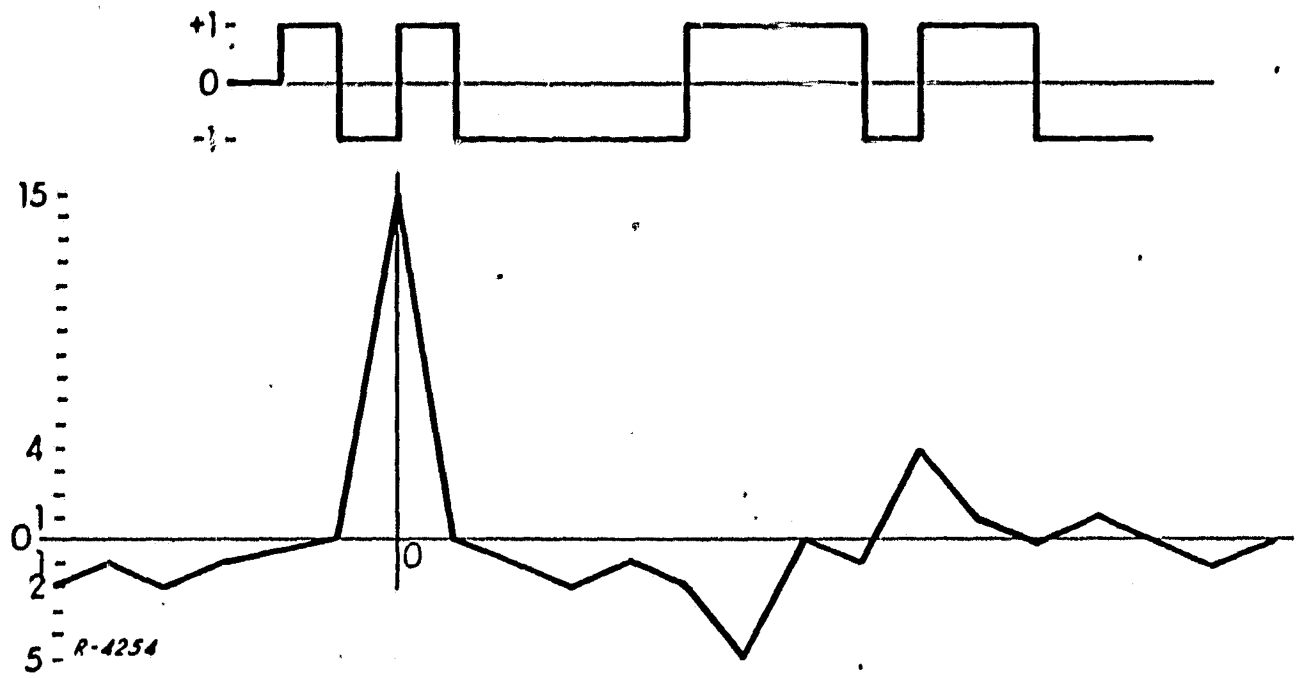

Fig. 2.8 Transition Encoding

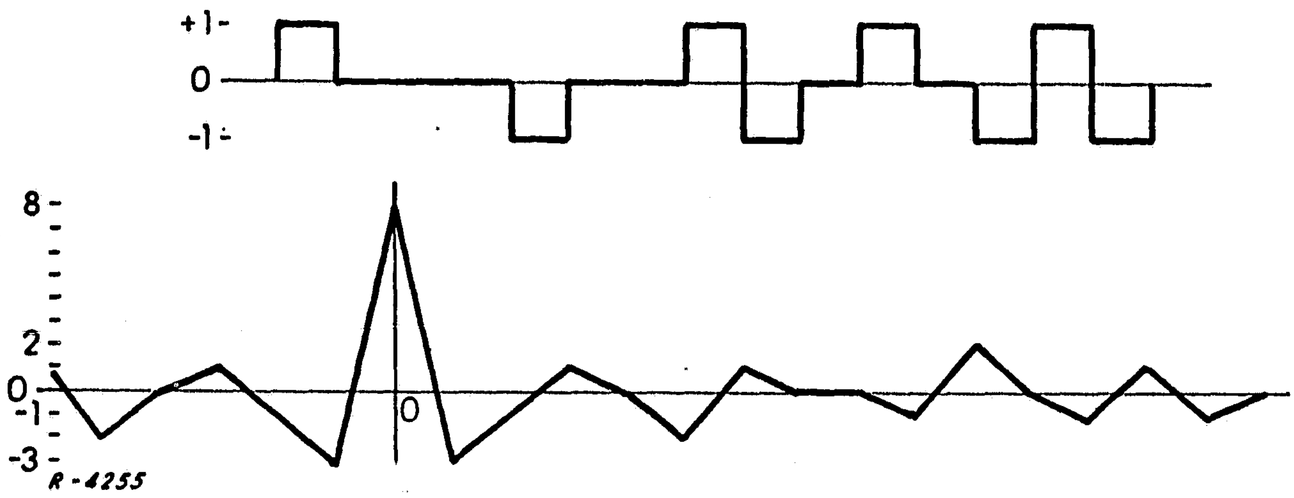

Fig. 2.9 Dicode

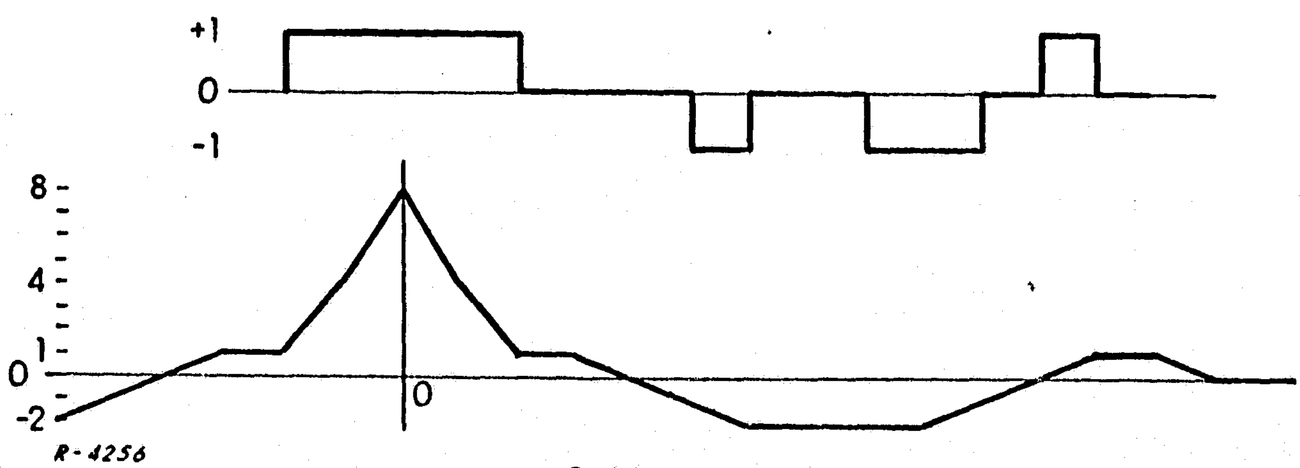

Fig. 2.10 Duobinary

pulse (a space) of 20 percent of a bit, our PR code may have about 250 pulses. From Eq.(2.1), n = 255 for the closest maximal-length code. The sequence is given in Table 2.1.

In Sec. 2.2.2.1 it was determined that the ±1 NRZ and the Split Phase waveform have the best autocorrelation properties, and hence are the most suitable for synchronization. If k is the shift in number of bits, the autocorrelation of split phase and NRZ are identical for integer values of k. However, split phase, being a double-frequency waveform with respect to the bit rate, will have an additional autocorrelation value at each odd multiple of k/2. The 255 bit maximal length PR code autocorrelation function, R(k), in ±1 NRZ and Split Phase is given in Table 2.2. For ±1 NRZ, use only integer values of k; for Split Phase, use all values. For non-integer values of k for NRZ, connect the integer values with straight lines. For Split Phase, connect the half-integer values. Note that R(-k) = R(+k), so only positive values of k are tabulated.

From Table 2.2 it is seen that all secondary peaks are down at least 25 dB from the primary peak, R(0) = +255. There is one secondary peak of 13 at k = ±148, 25.9 dB down, and two values of 12 at ±23 and ±191, 26.6 dB down. In addition, there are five values of 11, 27.3 dB down; five 10's, 28.1 dB down, and six 9's, 29.0 dB down. All remaining values are over 30 dB down from the primary peak.

2.2.2.3 Full Frame Utilization

Most desirable of the alternatives is full frame utilization for synchronization purposes. Under these circumstances, the marker labels must, of course, be multiplexed or sent by some other means. The labels might, for example, be frequency-division multiplexed on a subcarrier, time division multiplexed by sending a label every five or six frames, or sent over another channel. If full frame synchronization is available, optimum use may be made of the PR code. In the preceding sections, it was necessary to calculate the isolated autocorrelation function because successive synchronization sequences were not contiguous. In the isolated autocorrelation function, the sequence is assumed to exist in a single interval of n bits only, and to be zero outside that interval. Since in full frame utilization successive sequences are contiguous, we use the repetitive autocorrelation. In repetitive autocorrelation the sequence is assumed to repeat every n bits. It is in the repetitive case that the maximal length sequence displays its optimum qualities. Using the same criteria as before, and Eq. (2.1), we have for full frame synchronization, of a sequence of length 255.

$$n = 255$$

$$R(k) = 255 \quad k = 0 \text{ or a multiple of } 255$$

$$R(k) = -1 \quad \text{for } k = 0, 1/2, 1, 3/2, 2, \ldots$$

The above result applies to both ±1 NRZ and Split Phase Modulations.

Table 2.1

255 BIT MAXIMAL LENGTH BINARY SEQUENCE

11111111001000001010011111010101011100000
1100010101100110010111111011110011011101
1100101010010100010010110100011001110011
1100011011000010001011101011110110111110
0001101001101011011010100000100111011001
0010011000000111010010001110001000000010
110001111010000

Table 2.2

AUTOCORRELATION FUNCTION, R(k), (ISOLATED) OF 255 BIT MAXIMAL LENGTH BINARY SEQUENCE FOR SPLIT PHASE AND ±1 NRZ WAVEFORMS

For Split Phase Waveform Use All Values of k.
For ±1 NRZ Waveform Use Only Integer Values of k.

k	R(k)	k	R(k)	k	R(k)	k	R(k)
0	255	20	- 3	40	- 7	60	- 5
0.5	-127.5	20.5	- 1.5	40.5	5.5	60.5	- 3.5
1	0	21	6	41	- 4	61	2
1.5	- 0.5	21.5	- 4.5	41.5	2.5	61.5	2.5
2	1	22	3	42	- 1	62	- 7
2.5	- 1.5	22.5	- 7.5	42.5	- 2.5	62.5	7.5
3	2	23	12	43	6	63	- 8
3.5	- 2.5	23.5	- 8.5	43.5	- 7.5	63.5	10.5
4	3	24	5	44	9	64	-13
4.5	- 2.5	24.5	- 4.5	44.5	- 2.5	64.5	4.5
5	2	25	4	45	- 4	65	4
5.5	- 2.5	25.5	- 7.5	45.5	- 0.5	65.5	- 1.5
6	3	26	11	46	5	66	- 1
6.5	- 2.5	26.5	- 7.5	46.5	- 2.5	66.5	- 2.5
7	2	27	4	47	0	67	6
7.5	- 1.5	27.5	- 5.5	47.5	- 0.5	67.5	- 4.5
8	1	28	7	48	1	68	3
8.5	0.5	28.5	- 2.5	48.5	- 3.5	68.5	- 3.5
9.0	- 2	29	- 2	49	6	69	4
9.5	3.5	29.5	3.5	49.5	- 3.5	69.5	1.5
10	- 5	30	- 5	50	1	70	- 7
10.5	4.5	30.5	7.5	50.5	- 3.5	70.5	3.5
11	- 4	31	-10	51	6	71	0
11.5	4.5	31.5	8.5	51.5	- 5.5	71.5	- 3.5
12	- 5	32	- 7	52	5	72	7
12.5	3.5	32.5	2.5	52.5	- 5.5	72.5	- 8.5
13	- 2	33	2	53	6	73	10
13.5	3.5	33.5	- 2.5	53.5	- 6.5	73.5	- 2.5
14	- 5	34	3	54	7	74	- 5
14.5	6.5	34.5	1.5	54.5	2.5	74.5	- 1.5
15	- 8	35	- 6	55	-12	75	8
15.5	3.5	35.5	2.5	55.5	- 3.5	75.5	- 4.5
16	1	36	1	56	5	76	1
16.5	0.5	36.5	1.5	56.5	1.5	76.5	- 2.5
17	- 2	37	- 4	57	- 8	77	4
17.5	- 0.5	37.5	3.5	57.5	3.5	77.5	- 5.5
18	3	38	- 3	58	1	78	7
18.5	- 2.5	38.5	1.5	58.5	4.5	78.5	- 4.5
19	2	39	0	59	-10	79	2
19.5	0.5	39.5	3.5	59.5	2.5	79.5	- 1.5

Table 2.2 (cont.)

k	R(k)	k	R(k)	k	R(k)	k	R(k)
80	1	104	3	128	- 9	152	11
80.5	- 2.5	104.5	- 2.5	128.5	1.5	152.5	- 4.5
81	4	105	2	129	6	153	- 2
81.5	3.5	105.5	0.5	129.5	2.5	153.5	- 3.5
82	-11	106	- 3	130	-11	154	9
82.5	10.5	106.5	8.5	130.5	7.5	154.5	- 0.5
83	-10	107	-14	131	- 4	155	- 8
83.5	8.5	107.5	7.5	131.5	4.5	155.5	9.5
84	- 7	108	- 1	132	- 5	156	-11
84.5	3.5	108.5	6.5	132.5	1.5	156.5	4.5
85	0	109	-12	133	2	157	2
85.5	- 2.5	109.5	11.5	133.5	1.5	157.5	2.5
86	5	110	-11	134	- 5	158	- 7
86.5	- 3.5	110.5	3.5	134.5	7.5	158.5	6.5
87	2	111	4	135	-10	159	- 6
87.5	- 1.5	111.5	- 5.5	135.5	5.5	159.5	4.5
88	1	112	7	136	- 1	160	- 3
88.5	- 2.5	112.5	2.5	136.5	0.5	160.5	5.5
89	4	113	-12	137	0	161	- 8
89.5	2.5	113.5	6.5	137.5	- 3.5	161.5	5.5
90	- 9	114	- 1	138	7	162	- 3
90.5	2.5	114.5	- 2.5	138.5	2.5	162.5	5.5
91	4	115	6	139	-12	163	- 8
91.5	- 5.5	115.5	- 8.5	139.5	9.5	163.5	6.5
92	7	116	11	140	- 7	164	- 5
92.5	- 4.5	116.5	- 1.5	140.5	3.5	164.5	- 1.5
93	2	117	- 8	141	0	165	8
93.5	- 4.5	117.5	4.5	141.5	- 5.5	165.5	- 1.5
94	7	118	- 1	142	11	166	- 5
94.5	- 4.5	118.5	0.5	142.5	- 1.5	166.5	3.5
95	2	119	0	143	- 8	167	- 2
95.5	- 3.5	119.5	- 4.5	143.5	6.5	167.5	2.5
96	5	120	9	144	- 5	168	- 3
96.5	- 5.5	120.5	- 6.5	144.5	- 2.5	168.5	4.5
97	6	121	4	145	10	169	- 6
97.5	- 1.5	121.5	- 0.5	145.5	-10.5	169.5	3.5
98	- 3	122	- 3	146	11	170	- 1
98.5	- 3.5	122.5	- 0.5	146.5	- 5.5	170.5	- 2.5
99	10	123	4	147	0	171	6
99.5	- 8.5	123.5	- 3.5	147.5	- 6.5	171.5	- 7.5
100	7	124	3	148	13	172	9
100.5	1.5	124.5	- 6.5	148.5	- 7.5	172.5	- 9.5
101	-10	125	10	149	2	173	10
101.5	4.5	125.5	- 1.5	149.5	0.5	173.5	- 2.5
102	1	126	- 7	150	- 3	174	- 5
102.5	5.5	126.5	- 0.5	150.5	3.5	174.5	3.5
103	-12	127	8	151	- 4	175	- 2
103.5	4.5	127.5	0.5	151.5	- 3.5	175.5	2.5

Table 2.2 (cont.)

k	R(k)	k	R(k)	k	R(k)	k	R(k)
176	- 3	200	11	224	9	248	- 3
176.5	5.5	200.5	- 1.5	224.5	- 6.5	248.5	3.5
177	- 8	201	- 8	225	4	249	- 4
177.5	6.5	201.5	7.5	225.5	- 2.5	249.5	3.5
178	- 5	202	- 7	226	[illegible]	250	- 3
178.5	3.5	202.5	6.5	226.5	3.5	250.5	3.5
179	- 2	203	- 6	227	- 8	251	- 4
179.5	5.5	203.5	6.5	227.5	6.5	251.5	3.5
180	- 9	204	- 7	228	- 5	252	- 3
180.5	2.5	204.5	4.5	228.5	8.5	252.5	2.5
181	4	205	- 2	229	-12	253	- 2
181.5	3.5	205.5	4.5	229.5	8.5	253.5	1.5
182	-11	206	- 7	230	- 5	254	- 1
182.5	9.5	206.5	4.5	230.5	5.5	254.5	0.5
183	- 8	207	- 2	231	- 6	255	0
183.5	4.5	207.5	1.5	231.5	9.5		
184	- 1	208	- 1	232	-13		
184.5	- 2.5	208.5	3.5	232.5	8.5		
185	6	209	- 6	233	- 4		
185.5	- 0.5	209.5	1.5	233.5	5.5		
186	- 5	210	3	234	- 7		
186.5	4.5	210.5	3.5	234.5	2.5		
187	- 4	211	-10	235	2		
187.5	5.5	211.5	8.5	235.5	0.5		
188	- 7	212	- 7	236	- 3		
188.5	3.5	212.5	3.5	236.5	3.5		
189	0	213	0	237	- 4		
189.5	2.5	213.5	- 1.5	237.5	1.5		
190	- 5	214	3	238	1		
190.5	- 3.5	214.5	- 4.5	238.5	0.5		
191	12	215	6	239	- 2		
191.5	- 9.5	215.5	- 2.5	239.5	- 2.5		
192	7	216	- 1	240	7		
192.5	- 6.5	216.5	- 0.5	240.5	- 5.5		
193	6	217	2	241	4		
193.5	- 1.5	217.5	- 2.5	241.5	- 2.5		
194	- 3	218	3	242	1		
194.5	4.5	218.5	- 0.5	242.5	- 2.5		
195	- 6	219	- 2	243	4		
195.5	- 1.5	219.5	- 1.5	243.5	- 3.5		
196	9	220	5	244	3		
196.5	- 3.5	220.5	- 0.5	244.5	- 3.5		
197	- 2	221	- 4	245	4		
197.5	- 2.5	221.5	3.5	245.5	- 2.5		
198	7	222	- 3	246	1		
198.5	- 0.5	222.5	- 1.5	246.5	0.5		
199	- 6	223	6	247	- 2		
199.5	- 2.5	223.5	- 7.5	247.5	2.5		

2.2.3 Optimal Correlation Properties of Time Code Frame Synchronization Sequences

2.2.3.1 General

The most basic and commonly used method of search for a binary frame sync sequence is the Digital Correlator Technique (Fig. 2.11). An n-bit long window which has been 'fed' with the n-bit sync sequence is filled at any moment with the portion of time code waveform received in the n most recent bit periods. The correlator executes a comparison of each bit in the window with the corresponding bit in the sequence to see whether they agree or disagree in binary value. The number of agreements is totalled up and made available as a digital correlation score. If the threshold is exceeded, i.e., if there are $n - \epsilon$ or more agreements in the n positions, frame sync is announced. Otherwise, the sliding search is performed again during the next bit interval. For purposes of analysis, it is convenient to consider a binary sequence to be a string of +1's and - 1's, and to consider that the correlator performs bit-by-bit multiplications. The correlator output therefore ranges from +n to -n, and the "threshold" is $n - 2\epsilon$. Correlation scores based upon the difference between the number of agreements and the number of disagreements are obviously equivalent to scores based upon the agreements only.

It is necessary to distinguish between the two different types of frame sequences (in terms of frame occupancy) which we shall analyze. These are:

i) Recurring Sequences - the frame sync sequence occupies the entire time frame, and thus repeats itself steadily without interruption.

ii) Nonrecurring Sequences - half or less of the frame time is devoted to the sequence, the remainder being filled with time labeling data bits.

Before proceeding to the detailed analysis of these two sequence classes, a few general comments are in order. When recurring sequences are employed the correlator is always filled with bits of the sequence. At precisely the on-time

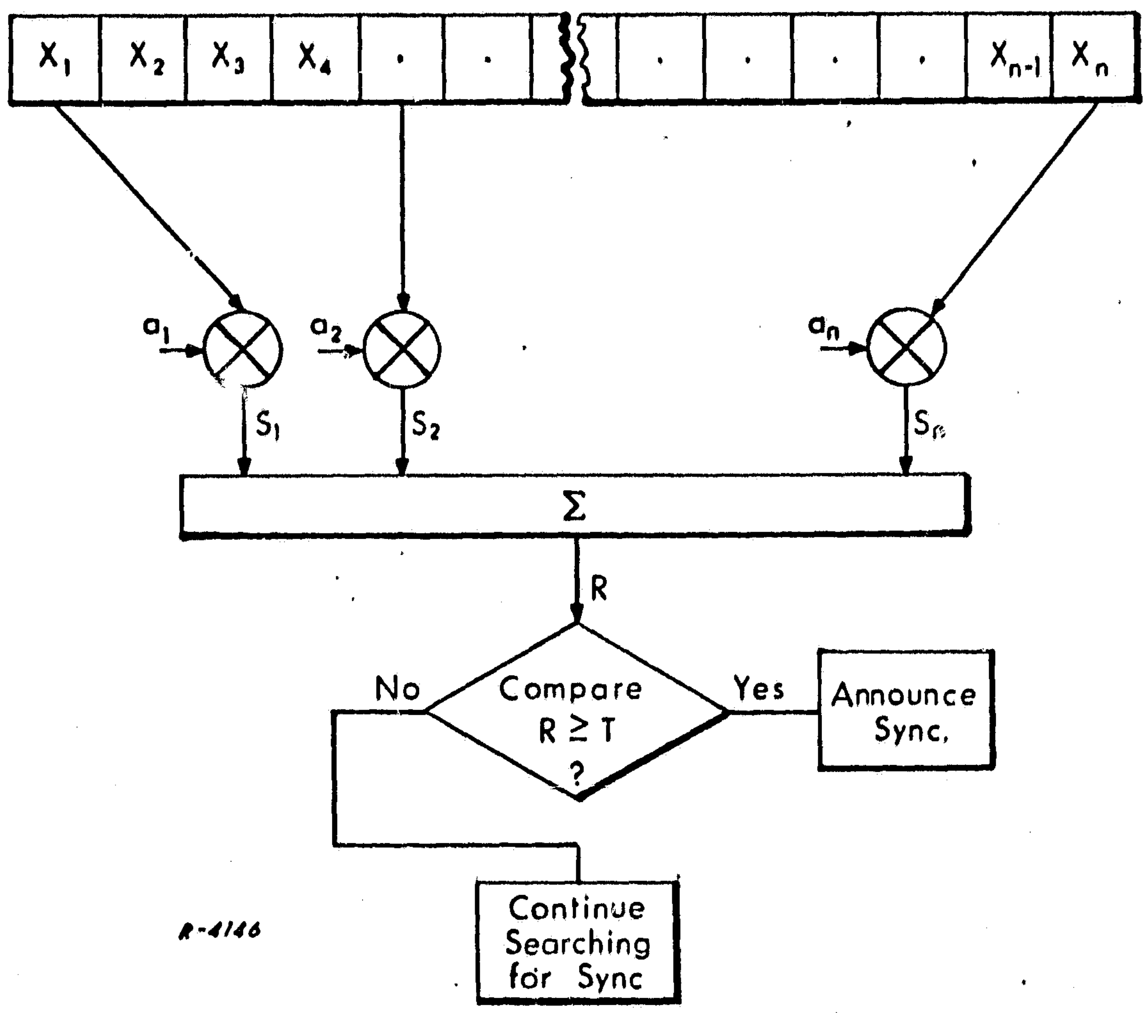

Fig. 2.11 Basic Digital Correlation for Frame Acquisition

a_1, $a_2 \ldots a_n$ are "known" sync sequence elements to be compared with x_1, $x_2, \ldots x_n$ present in register.

$$a_i \cdot x_i = S_i \equiv \begin{cases} +1 \text{ for "Agreement"} \\ -1 \text{ for "Disagreement"} \end{cases}$$

interval, the bits will be stored in proper order in the correlator, whose response in the absence of bit errors will be n. During any other time, the bits in the correlator will be a cyclically permuted version of the sequence. The correlator response will therefore be the so-called cyclic autocorrelation function of the sequence.

In the case of nonrecurring sequences, there are three categories of bit patterns that may be present in the correlator. The first is the "random" region in which no part of the sequence is in the correlator. The bits in the correlator are the labeling data bits, presumed for purposes of analysis to be completely random. The next region is the "overlap" region that develops as the sequence moves into the correlator. The correlator response in this region depends upon the sum of the "isolated" autocorrelation function and a random number. The final category is the on-time position when the sync sequence fills the correlator, and the full response of n units will be produced in the absence of bit errors.

Frame sync sequence acquisition performance is best measured in terms of two quantities, and the tradeoffs between them as the acceptance threshold is varied. P_{FA} is the probability of a frame sync announcement being falsely generated at some "wrong" bit position over a frame interval. P_M is the probability of missed detection of the pattern at the one on-time bit position.

The channel noise representation that will be used is that of the binary symmetric channel in which bit errors occur at random with probability p. In order to relate probabilities of system failure analytically to signal-to-noise ratios, it is assumed that the bits fed to the correlator have been coherently detected. For signal power S, bit duration T, and spectral (wideband) gaussian noise density of N_o watts/Hz, the bit error probability for optimum coherent detection of Binary Antipodal signals is:

$$p = \Phi\left(\sqrt{\frac{2ST}{N_o}}\right) = \Phi(\sqrt{2\rho}) \qquad (2.2)$$

where ρ is the signal-to-noise ratio, i.e., the ratio of signal power to noise power contained in the nominal bit bandwidth of $1/T$ Hz, and

$$\Phi(x) \equiv \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \tag{2.3}$$

2.2.3.2 Recurring Sequences

The recurring sequences we shall analyze are the pseudorandom sequences of maximal-length type. Such sequences are of length $n = 2^{\ell}-1$, where ℓ is any integer. The cyclic autocorrelation function of a sequence $x_1, x_2 \ldots x_n$, for a shift of j bit positions from the true on-time position is:

$$R_c(j) = \sum_{i=1}^{n-j} x_i \cdot x_{i+j} + \sum_{i=1}^{j} x_i \cdot x_{n-j+i} \tag{2.4}$$

From Table 4. 2 of the First Quarterly Report, we recall that for a maximal-length, linear recurring sequence of +1 and -1 elements and period n,

$$R_c(j) = \begin{cases} n & j = 0 \\ -1 & j = 1, 2, \ldots n-1 \end{cases} \tag{2.5}$$

Alternately stated, for any cycled shift of the sequence, there will be $(n + 1)/2$ disagreements and $(n - 1)/2$ agreements in the correlator. Equation (2.5) would give the correlator response if ther were no bit errors to contend with. Observe that each time channel noise produces a bit error that changes a disagreement into an agreement, the value of R in Fig. 2.11 is increased by 2, and that each time an agreement is changed to a disagreement R is reduced by 2. The total noise perturbation effect upon the n bits of sequence in the correlator may be described as a ν-bit error pattern, of which ν_+ bits are of the first type and ν_- are of the second type. Thus, the 'noisy' correlator response may be described as:

$$R(j) = \begin{cases} n - 2\nu_- & j = 0 \\ -1 + 2(\nu_+ - \nu_-) & j = 1, 2, \ldots n-1 \end{cases} \tag{2.6}$$

The threshold value of R(j) is:

$$T = n - 2\epsilon \tag{2.7}$$

where ϵ is the maximum number of disagreements allowed for positive recognition to be announced. At any overlap position specified by a given j, the probability of a false recognition is

$$P_F(j) = P_r\{2(\nu_+ - \nu_-) \geq T + 1\} \tag{2.8}$$

It is most reasonable to treat the individual $P_F(j)$ as events of very small probability. The total probability of false sync is closely (and conservatively) approximated by the sum of these probabilities:

$$P_{FA} = \sum_{j=1}^{n-1} P_F(j) \tag{2.9}$$

In the calculation of P_{FA}, the following relationships are employed:

$$P_r\{\nu_+ - \nu_- = q\} =$$

$$\sum_{i=0}^{i=n/2+1/2-q} \binom{n/2+1/2}{q+i}\binom{n/2-1/2}{i} p^{q+2i}(1-p)^{n-q-2i}$$

$$1 \leq q \leq n/2 + 1/2 \tag{2.10}$$

$$P_i\{\nu_+ - \nu_- = 0\} =$$

$$\sum_{i=0}^{i=n/2-1/2} \binom{n/2+1/2}{i}\binom{n/2-1/2}{i} p^{2i}(1-p)^{n-2i} \tag{2.11}$$

Equations (2.8) - (2.11) may be used to find P_{FA} as a funtion of p for any allowable sequence length and threshold.

The probability of missed detection when the sequence fills the correlator is merely the probability that the n bits will contain more than ϵ errors:

$$P_M = \sum_{i=\epsilon+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \qquad (2.12)$$

Expressions obtained for P_{FA} and P_M will be polynomials in p. For the smaller values of p usually of interest, these expressions are well approximated by the term involving the lowest power of p. The dominant terms of the expressions for P_{FA} and P_M are shown in Table 2.3 for several combinations of sequence length and acceptance threshold. These values can all be shown to be reasonable, somewhat conservative, approximations for $p < 10^{-2}$ ($\rho > 4.3$ dB). Corresponding values describing P_{FA} and P_M determined by using the above formulas for the present NASA 36 bit time code format (the frame sync is, of course, not of the recurring type) are also shown in Table 2.3. The values for the NASA 36 bit code depend upon successful recognition of 5 consecutive 1's. The quantity P_{FA} is given by Eq. (4.32) of the First Quarterly Report for the 36-bit code. Equation (4.35) of the same report can be used to determine P_M for the 36-bit code by substitution of P_M for N_F. (The quantity N_F is used in the First Quarterly Report to denote the probability of false initiation of reaquisition caused by failure to detect correctly the 5 bits used for frame sync, whereas P_M in the present discussion denotes the probability of missed detection independently of whether or not correct acquisition has been achieved. While the two quantities are conceptually different, the outcome is the same in both cases since in each case a functional failure is considered to occur if we fail to detect correctly all 5 consecutive bits.)

Figures 2.12 and 2.13 use the data of Table 2.3 to plot P_M and P_{FA} as functions of the signal-to-noise ratio ρ. Table 2.3 does not go above n = 31. For vary large values of n (e.g., 63, 127, 255) and reasonable thresholds, P_{FA} and P_M become extremely small for values of ρ corresponding to

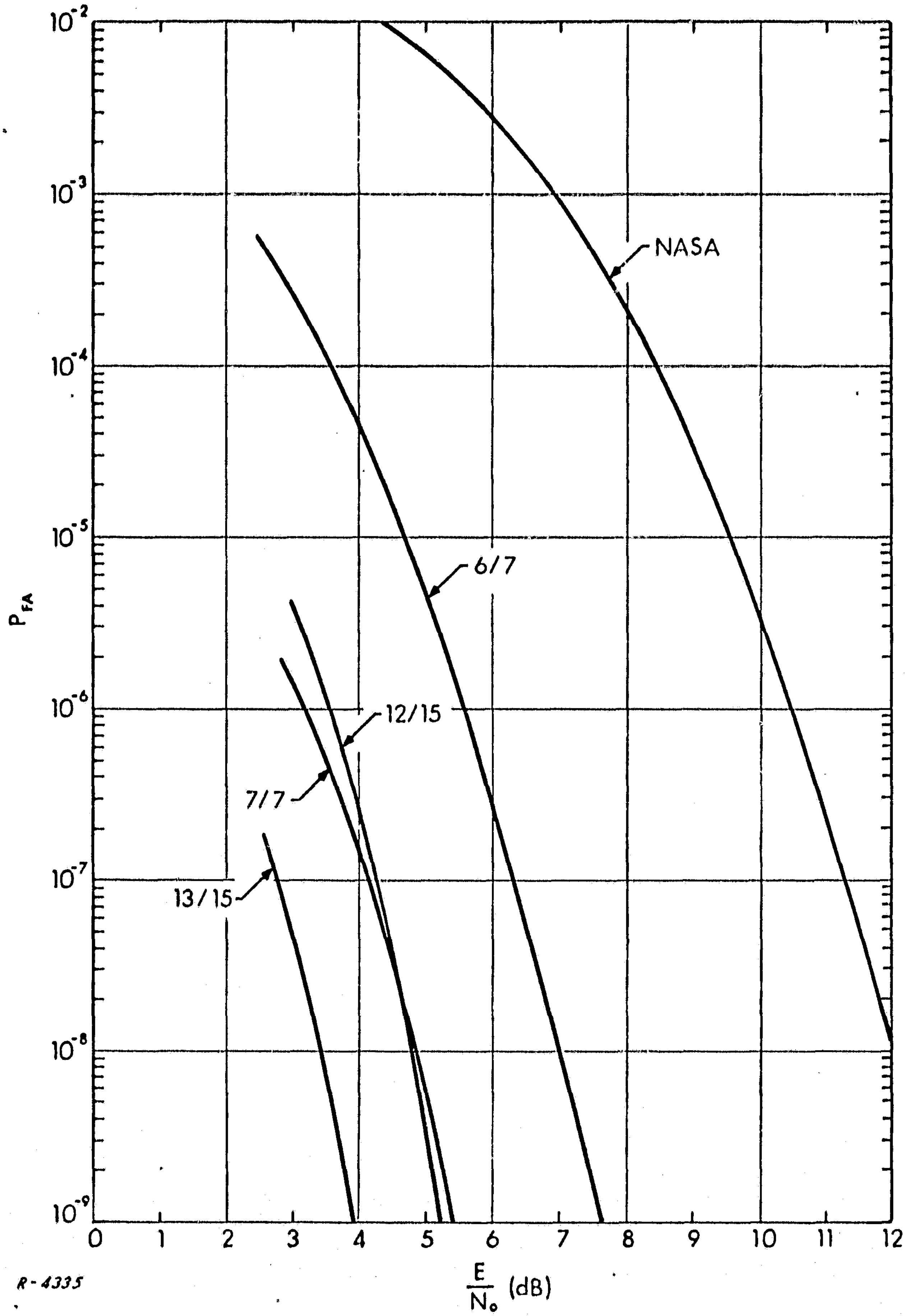

Fig. 2.12 P_{FA} as a Function of E/N_o, NASA 36 Bit Code, and Recurring Sequences of Lengths 7 and 15 With Various Thresholds

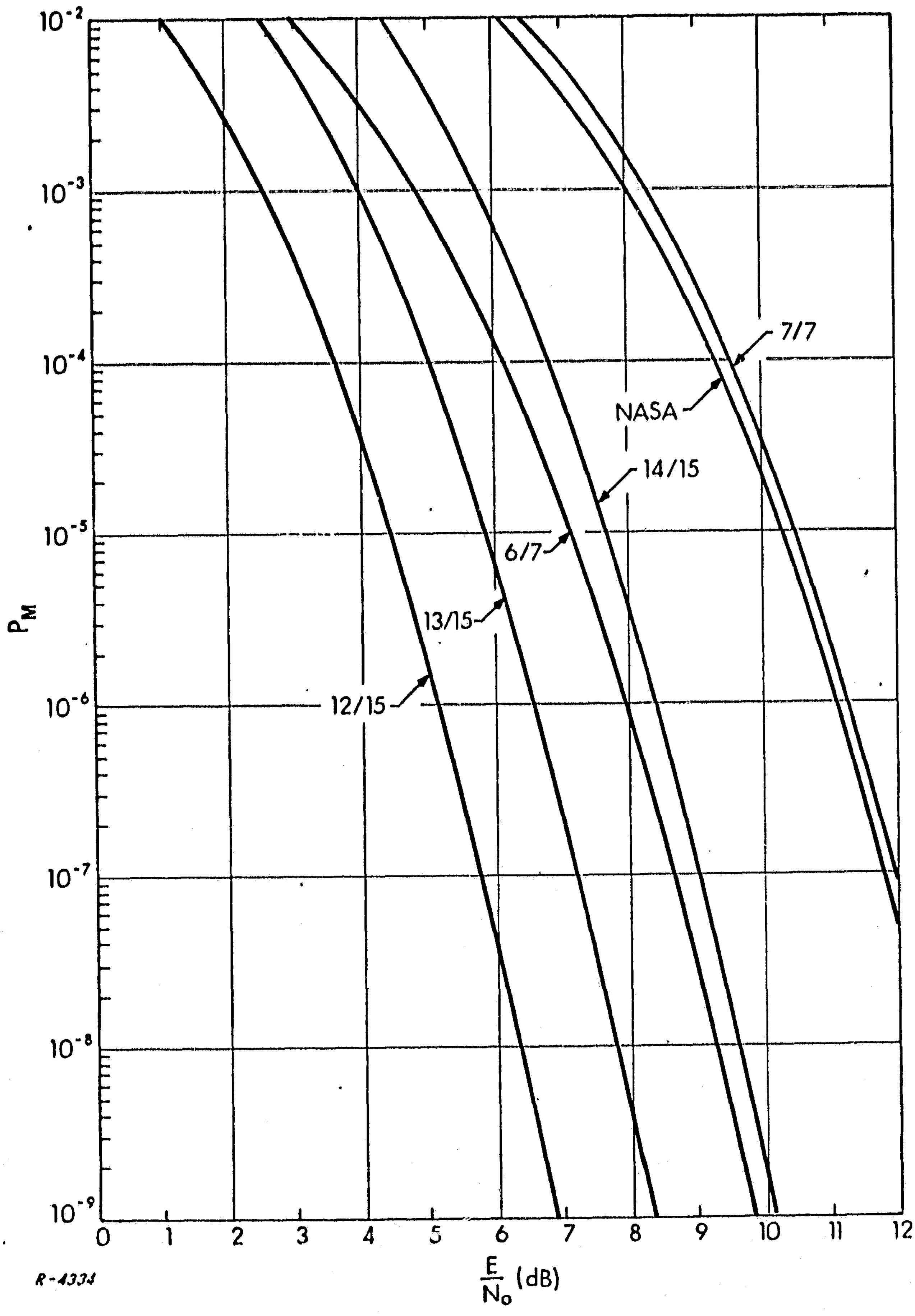

Fig. 2.13 P_M as a Function of E/N_o, NASA 36 Bit Code, and Sequences of Lengths 7 and 15 With Various Thresholds

$E/N_o > 0$ dB, and assume significant values mainly for $E/N_o < 0$ dB. Probability curves for $E/N_o < 0$ dB correspond to situations of questionable practical value, and require numerical computations far too laborious for their practical significance. P_{FA} curves for the 14/15 and 15/15 cases as well as those cases for sequence lengths of 31 fall off-scale to the left.

With the aid of Figs. 2.12 and 2.13 the analyst can evaluate the 'advantage' (for equal energy per bit) of the long sequences whose characteristics have been plotted over the frame sync pattern of the present NASA 36 bit BCD code. For example, if the values $P_{FA} = 10^{-6}$, $P_M = 10^{-6}$ are selected as desirable performance objectives, then it is found that:

i) The sequence of length 15, with a threshold requirement of 12 agreements requires 6.0 dB less signal-to-noise ratio than the NASA code to meet the P_M specification, and 6.9 dB less to meet the P_{FA} specification. With the threshold changed to 13 out of 15, the P_M advantage is 4.5 dB and the P_{FA} advantage is 8.8 dB.

ii) The sequence of length 7, with a threshold requirement of 6 agreements, needs 3.2 dB less signal-to-noise ratio to meet the P_M specification, and 5.0 dB less to meet the P_{FA} specification.

It is interesting to observe in conclusion that for a given sequence length the obtained P_{FA} and P_M values are traded against each other through the raising and lowering of the threshold. However, it can easily be seen that, no matter what the actual threshold, the product of P_{FA} and P_M will be proportional to a fixed power of p. This proportionality is:

$$P_{FA} \cdot P_M \propto p^{n/2 + 3/2} \tag{2.13}$$

For the NASA codes this same product is proportional to p^2.

Table 2.3

SUMMARY OF FRAME ACQUISITION PROPERTIES
OF SEVERAL RECURRING MAXIMAL LENGTH
SEQUENCES, FOR VARIOUS ACCEPTANCE THRESHOLD

Sequence Length n	Threshold: Min. No. of agreements/n	Probability of Missed Detection, P_M	Probability of False Acquisition, P_{FA}
7	7/7	$7p$	$6p^4$
7	6/7	$21p^2$	$24p^3$
15	15/15	$15p$	$14p^8$
15	14/15	$105p^2$	$112p^7$
15	13/15	$455p^3$	$392p^6$
15	12/15	$1365p^4$	$784p^5$
31	31/31	$31p$	$30p^{16}$
31	30/31	$465p^2$	$480p^{15}$
31	29/31	$4,495p^3$	$360p^{14}$
31	28/31	$31,465p^4$	$16,800p^{13}$
NASA 36 Bit Code		$\approx 5p$	$\approx p$

p = bit error probability

It is of interest to point out that the curves in Fig. 2.12 and 2.13 can be used for system comparisons on bases other than equal energy per bit, because of their general form. For example, suppose that the duration of a sequence is considered to be fixed. The signal bandwidth then increases with sequence length. But for a given noise background and signal energy, the E/N_o per bit (abscissa in Figs. 2.12 and 2.13) decreases as sequence length increases.

One can therefore construct a comparison of performance of recurring sequences, having equal energies but varying lengths (and bandwidths). For a given value of E/N_o for the signal (total signal energy divided by background noise power (density) one must divide by sequence length n to obtain the appropriate per bit value with which points may be read from Fig. 2.12 and 2.13. In this way the performance curves linking P_{FA} and P_M to the total signal E/N_o are traced out.

2.2.3.3 Nonrecurring Sequences

We next concern ourselves with the analysis of nonrecurring sequences which fill no more than half the frame interval. Such sequences are surrounded on both sides either by a blank guard interval or by 'random' data bits conveying digital time information for the frame. The random data bits constitute the greater hazard to false synchronization, since an unfortunate combination of bits might produce a strong correlator response at some 'wrong' bit interval. The total probability of false synchronization over an entire frame interval is the sum of false synchronization probabilities over the 'random' and the 'overlap' regions:

$$P_{FA} = P_O + P_R \tag{2.14}$$

In the overlap region, at an instant j bit intervals before (or after) the on-time instant, there will be n - j bits of the frame synchronization sequence in the correlator, additively contributing $R_c(j)$ to the correlator output. $R_c(j)$ is the isolated autocorrelation function of the sequence for a displacement of j bits:

$$R_c(j) = \sum_{i=1}^{n-j} x_i \cdot x_{i+j} \tag{2.15}$$

The total correlator output in the absence of bit errors would be $R_c(j)$ plus the sum of j random numbers due to the presence of the data bits:

$$R(j) = R_c(j) + \sum_{k=1}^{j} Z_k \tag{2.16}$$

The Z_k's are considered to be statistically independent, equiprobably assuming the values +1 and -1.

The introduction of bit errors as a variable greatly complicates the analysis; the term $2(\nu_+ - \nu_-)$ must be added to R(j) in Eq. (2.16). Of all the n-j bits of the sequence in the correlator $[n-j-R_c(j)]/2$ will form disagreements; out of these ν_+ will be changed to agreements by bit errors. Similarly ν_- of the $[n-j+R_c(j)]/2$ agreements will be changed to disagreements. The presence of ν_+, ν_- and the Z_k's as random variables in the expression for R(j) will lead to an involved and complicated calculation to evaluate the probability of R(j) exceeding some given threshold.

To make matters even more complicated, it is necessary to extend our summations over the entire overlap region, n-1 bit intervals on each side of the on-time interval:

$$P_O = 2 \cdot \sum_{j=1}^{n-1} P_r \{R(j) \geq T\} \tag{2.17}$$

The exact calculation of P_O for several sequence lengths and thresholds would be a formidable task to undertake. In order to reduce the computational effort to a manageable level, the following assertion (later substantiated) is made.

P_O consists of a constant term (generated by the random bits) plus terms in powers of p. For all but very large values of p, the expression for P_O is well approximated by a constant. Further, it should be noted that the greatest false acquisition hazard in the overlap region is during times of small overlap, i.e., when most of the bits in the correlator are random. When the overlap is more considerable, a properly designed frame synchronization sequence will introduce multiple disagreements into the correlator and thereby prevent false sync acquisition.

Since

$$R_c(n-\alpha) \geq -\alpha \tag{2.18}$$

i.e., when there are α bits of overlap, there may be no more than α disagreements, one may reason:

$$P \geq 2 \cdot \sum_{\alpha=1}^{\epsilon} \left\{ \sum_{i=0}^{\epsilon-\alpha} \binom{n-\alpha}{i} \right\} 2^{-(n-\alpha)} \qquad \qquad p \to 0 \tag{2.19}$$

The summation of Eq. (2.19) above expresses the fact that when there are α bits of overlap, there may be α disagreements caused by this, and if $\epsilon - \alpha$ or fewer of the $n - \alpha$ random bits in the correlator produce disagreements then a false acquisition of the frame will occur. Table 2.4 displays this optimum value of P_O for various sequence lengths and thresholds.

The assertion that P_O is insensitive to p is best sustained by making use of calculations executed by Maury and Styles*. These authors performed an exhaustive computer search to determine 'optimum' frame sync sequences. These sequences achieved the minimum possible value of P_O for a threshold level allowing two disagreements and a bit error probability of 0.10. Consequently they are 'good', but not optimum, sequences for other thresholds and bit error probabilities. Values of P_O as a function of sequence length and threshold were computed in this reference, from which the entries in the fourth column of Table 2.4 were obtained.

The comparison of the 'optimum' value for $p = 0$ with an actual calculated (larger than optimum) value for $p = 0.10$ clearly shows that there is little sensitivity of P_O to p, except for the 7/7 and 15/15 cases where no disagreements are allowed and bit errors are necessary to produce false acquisitions.

If the frame synchronization sequence is exactly half of a frame duration in length, the 'random' interval during which the correlator contains only data bits will be one bit interval. In general if the frame consists of N bits, and the frame synchronization sequence of n bits the random interval is of

---

* Maury, Jr., J. L. and Styles, F. J., "Development of Optimum Frame Synchronization Codes for Goddard Space Flight Center PCM Telemetry Standards", presented at National Telemetry Conference, 1964.

Table 2.4

PROBABILITIES OF FALSE ALARM IN THE OVERLAP REGION FOR NONRECURRING SEQUENCES WITH VARIOUS SEQUENCE LENGTHS AND THRESHOLDS

Sequence Length n	Threshold: Min. No. of agreements/n	Optimum P_O $p \to 0$	P_O from Ref. 1 $p = 0.10$
7	7/7	0	$7.0 \cdot 10^{-3}$
7	6/7	0.0312	0.120
15	15/15	0	$2.0 \cdot 10^{-5}$
15	14/15	$1.220 \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$
15	13/15	$2.08 \cdot 10^{-3}$	$7.8 \cdot 10^{-3}$
15	12/15	0.0168	0.065
31	29/31	$6.03 \cdot 10^{-8}$	$1.6 \cdot 10^{-7}$
31	28/31	$9.83 \cdot 10^{-7}$	$2.0 \cdot 10^{-6}$

duration N - 2n + 1 bit intervals. Thus, the probability of a false acquisition in the random region is:

$$P_R = [N - 2n + 1] \cdot 2^{-n} \sum_{i=0}^{\epsilon} \binom{n}{i} \tag{2.20}$$

Table 2.5 exhibits total P_{FA} for sequences of half interval duration; the table is generated by adding values of P_O from Table 2.4 to values of P_R calculated for $n = N/2$. We see clearly the insensitivity of P_{FA} to the bit error probability. For all p nominally less than 10^{-2}, it is safe to consider that P_{FA} is constant and given by the optimum value for $p \to 0$.

The system analyst studying potential acquisition performance using nonrecurring sequences would select a sequence length and threshold combination providing the desired P_{FA}. For dB comparisons the 'minimum' values of

Table 2.5

PROBABILITIES OF FALSE ACQUISITION, P_{FA}, FOR NONRECURRING HALF INTERVAL SEQUENCES WITH VARIOUS SEQUENCE LENGTHS AND THRESHOLDS

Sequence Length n	Threshold: Min. No. of agreements/n	Optimum P_{FA} $p \to 0$	P_{FA}, using Ref. 1 $p = 0.10$
7	7/7	$7.81 \cdot 10^{-3}$	0.0148
7	6/7	0.0938	0.182
15	15/15	$3.05 \cdot 10^{-5}$	$5.0 \cdot 10^{-5}$
15	14/15	$6.10 \cdot 10^{-4}$	$1.04 \cdot 10^{-3}$
15	13/15	$5.78 \cdot 10^{-3}$	0.0115
15	12/15	0.0344	0.083
31	29/31	$2.88 \cdot 10^{-7}$	$3.9 \cdot 10^{-7}$
31	28/31	$4.04 \cdot 10^{-6}$	$4.3 \cdot 10^{-6}$

P_{FA} from Eqs.(2.19) and (2.20) or Table 2.5 are considered to apply at p = 4.3 dB. Equations (2.7) - (2.11) or Table 2.3 may be utilized to find P_M which is the same for nonrecurring and recurring sequences with the same bit length and threshold.

The poorer performance of nonrecurring sequences as compared to recurring sequences of the same length can best be seen from the following two calculated illustrative examples on length 15 sequences.

i) On a 13 of 15 threshold a nonrecurring sequence will have $P_{FA} = 5.8 \cdot 10^{-3}$, which, as we have noted, will obtain down to p = 4.3 dB. The recurring sequence could, with the same threshold, obtain that P_{FA} at $\rho \approx -4.5$ dB. The present NASA 36-bit format requires 5.1 dB. Both sequences will have the same performance in P_M relative to the NASA format: for $P_M = 5.8 \cdot 10^{-3}$, 3.7 dB less than the 6.7 dB required by the NASA format are needed.

ii) With a 14 of 15 threshold $P_{FA} = 6.10 \cdot 10^{-4}$ for the nonrecurring sequence. The recurring sequence achieves this at -6.1 dB; 7.3 dB is required by the NASA format. Values of 6.1 dB for the sequences to achieve $P_M = 6.10 \cdot 10^{-4}$ compares with 8.4 dB for the NASA format.

The observations made regarding half interval sequences generally apply to sequences occupying a smaller fraction of an interval, except that P_{FA} will be somewhat larger due to the increased number of bit positions in the random region.

2.2.3.4 Additional Comments and Evaluation

It has so far been seen that nonrecurring half-interval sequences offer considerably poorer frame acquisition performance than recurring sequences of the same bit length and total energy. (A half interval sequence by definition uses half the available signal energy in a frame interval for frame synchronization; the comparisons in Sec. 2.2.3.3 above implicitly assume that the system using the recurring sequence likewise uses only half the available frame energy for this purpose, so that performance differences reflect only differences in sequence performance capability.)

There is an important negative consideration regarding the utility of nonrecurring sequences for frame synchronization of time codes. Note that binary frame time data is not truly a string of random bits. Over the course of an entire labeling cycle (which might be as long as a year) the data sequence may, in fact, assume all possible values. But over somewhat shorter intervals the labeling data remains highly correlated to preceding data. This leads to a very undesirable situation: the particular configuration of data bits causing false acquisition on one frame will persist for the next several frames, except for changes in a few of the least significant bits.

There is only one reasonable way to circumvent the unhappy condition of having high correlation between labeling bits and frame synchronization sequence for several consecutive frames. That is to intersperse idle, unchanging bits with the data bits (many of the index marker 0's in the NASA and IRIG codes fall in this category). The occurrence of undesirable data bit configurations is thereby reduced, but inefficient use has been made of part of the available waveform energy to attain this objective. The inefficient use of waveform energy is precisely what one hopes to avoid through time code optimization. The same observation would apply to schemes where the sequence is surrounded on both sides by a blank guard space. In conjunction with a digital correlator recognizing the three states +1, -1, and "null", undesirable false response is suppressed by the inefficient technique of having signal turned off part of the time.

Under the restriction that a certain fraction of the frame time is devoted to the synchronization sequence, it would not appear that any advantage is to be gained by allocating part of this time to guard space or to idle bits instead of applying it totally and directly to form the best possible sequence of contiguous bits. Thus, our assertion, intuitively acceptable though unverified by rigorous proof, is that optimal frame acquisition properties for nonrecurring sequences are achieved when all energy not required for frame labeling is devoted to the sequence.

The analyses executed in this section have demonstrated the relative desirability of recurring sequences over nonrecurring sequences for frame synchronization purposes. It is important to affirm that the comparison has been carried out under conditions which permit of the making of general conclusions. First of all, the assumption of coherent detection upon bits encoded other than antipodally, e. g., PPM or PDM, would not change any of the dB differences between schemes discussed in the analysis. The absolute values of p in dB at which given values of P_{FA} or P_M obtain would be shifted in accordance with a degradation factor expressing the suboptimality of the particular bit encoding (see, for example, Section 4. 4. 2 of the First Quarterly Report where these factors are worked out for PAM, PDM and PPM).

Neither does the assumption of a digital correlator rather than an analog correlator matched to the sequence make a major difference in the comparison. Digital correlation is a quantized, sectionalized approximation to analog correlation. For long sequences and high signal-to-noise ratios, the dB difference between the two approaches zero. There is a significant difference in absolute performance for low signal-to-noise ratios, i.e., when bit decision errors become frequent. However, comparative performance differences between sequences should change far less than absolute performance. Analog correlation does, however, permit the use of a continuously variable threshold level.

2.2.4 Applicable Frame Utilization Strategies

2.2.4.1 Full Occupancy by Marker Code - Modulated Occasionally by Label and Other Data

If occupancy of the full frame is to be allowed by the marker code, the label or data for the mark must be conveyed by other means. One possibility is to modulate occasionally the marker code by modulo 2 addition of the binary coded label information. Alternatively, the desired label information may be conveyed on a sinusoidal subcarrier situated in frequency perhaps at 2/T or higher, if T is the bit duration of the code marker waveform.

2.2.4.2 Full Occupancy by Marker Code - No Modulation but Occasional Skipping of the Marker and Substitution of Label or Other Information

It may be elected to convey the label information by periodically interrupting the code and inserting the labeling information. Each interruption will degrade the detection of only one marker that precedes it and another marker that immediately follows it. In view of the fact that markers are expected to be regularly spaced and their spacings are exactly known, a few markers can occasionally be skipped, especially if they are spaced by relative short-time intervals, without destroying the ability to identify properly the full conveyed time scale.

2.2.4.3 Partial Occupancy by Marker Code

If only partial occupancy of the frame is allowed for the marker, no problem exists in conveying the label for the marker as it may be transmitted during the off time of the marker code. The cost is substantial degradation in the performance of the marker. This is exhibited both in the lowering of the primary peak value, R(0), and in the raising of the secondary peaks. For example, if a half frame is utilized for marker purposes, the primary peak of the correlation will be only half that of a full frame utilization scheme. If five per cent of the frame is used, the value will be five per cent of that of a full frame scheme.

In any case, part or the whole of the label (seconds, minutes, days of year) may be transmitted with each marker. The partial label might consist of only the least significant bits of the label; that is, the bits that change with every frame.

2.3 Transmission Channel Signal Design

2.3.1 Wireline Channel

The conventional (so-called 4 kHz) wireline channel has a frequency response with a passband from about 300 to 3300 Hz. The loss of the DC content of the waveform below 300 Hz precludes sending any of the codes at baseband, except possibly the IRIG format A code with a 0.1 sec frame time.

Any of the codes may, however, be transmitted over the wireline channel by use of a suitable carrier. Three of the codes have a relatively high bandwidth. These are the NASA 36 bit and IRIG B (one sec frame time) and the IRIG A (0.1 sec frame time). These wider bandwidth codes should be modulated onto a carrier that places them in the center of the bandpass of the wireline channel, i.e., 1.8 kHz. The remaining codes have relatively narrow bandwidths, and any desired carrier between about 400 and 3000 Hz is acceptable.

2.3.2 LOS and BH Tropo Channel

The coherence bandwidths of LOS and BH tropo channels are sufficiently high that no significant limitations need be expected from this source. The fading rate, however, is sufficiently rapid that degradation may be incurred on the codes with longer pulse widths. These are the NASA 20 bit and IRIG D with one-hour frame times and IRIG F (5 min. frame).

The modulation to be used with the LOS and BH tropo channels should be either FM or PSK. Linear modulation (AM) may not be seriously considered because of its inefficiency of available power utilization, because available power amplification elements are peak power limited and do not provide adequate average power levels in linear operation.

Signal "outage" will result over a fading medium whenever the signal level drops below the threshold of proper maintenance of code acquisition. A simple but effective estimate of probability of interruption over a link suffering fading-induced outages would be to calculate the probability of the signal-to-noise ratio dropping below some critical value (or threshold) in the transition region. A workable estimate of this probability can be obtained by integrating the probability density function of the signal-to-noise ratio over the range below the threshold.

As an illustration, consider the particularly useful model of deep fading often closely approximated in BH links as a Rayleigh fading signal with a stationary additive gaussian noise at the receiver. The signal-to-noise ratio is then governed by the probability density function:

$$p(\rho) = \frac{1}{\overline{\rho}} e^{-\rho/\overline{\rho}} \tag{2.21}$$

where $\overline{\rho}$ is the average value of the signal-to-noise ratio. The probability that ρ will be less than some value ρ_o is then

$$\begin{aligned} P_r\left\{\rho \le \rho_o\right\} &= 1 - e^{-\rho_o/\overline{\rho}} \\ &\approx \rho_o/\overline{\rho} \quad \text{for } \rho_o/\overline{\rho} \ll 1 \end{aligned} \tag{2.22}$$

As a clarifying example of the application of this calculation, consider the length 15 recurring maximal length sequence using a 12-out-of-15 threshold logic (see Section 2.3.3). In a fixed signal-to-noise ratio environment P_i will be near 10^{-6} at 5 dB and near 10^{-2} at 2 dB. Suppose 10^{-5} to 10^{-6} is considered to be a desired range of values for P_i. To achieve this safely in Rayleigh fading, one would like the chance of the signal-to-noise ratio dropping below 2 dB to be in the 10^{-4} to 10^{-5} range. From Eq. (2.21) it is seen that an average signal-to-noise ratio in the 42 to 52 dB range is necessary. It is characteristic of deep fading channels that extremely high average signal-to-noise ratios are required to assure reliable transmission without the use of diversity transmission.

2.3.3 Space Channel

Space links are generally operated at frequencies in excess of 100 MHz under peak-power limited conditions. Therefore, FM or ϕM is normally the preferred type of carrier modulation. Phase modulation is often favored because it will provide a carrier component at phase deviations of about 1 radian or less for purposes of spatial acquisition of a spacecraft and autotrack by ground antennas.

2.4 Summary of Performance Comparison of the Optimal Code vs. Current NASA and IRIG Codes

A meaningful way to compare the relative performance of competitive approaches to the attainment of a system objective is as follows: First set a significant measure of acceptable system performance, such as some desired value of the probability of false frame acquisition. Then derive a measure of needed signal strength or signal-to-noise ratio required by each of the competitive approaches. Performance differences are then directly shown in terms of dB advantage. Such relative dB advantages may be translated into reduced transmitter power requirements for a given communication range, capability to function satisfactorily in noisier environments, or other significant functionally descriptive advantages.

The following illustrative example was executed to compare the frame acquisition performance of existing duration-modulated NASA and IRIG time codes with implementations using pseudorandom acquisition sequences of length 15. The pseudorandom sequences, both of recurring and nonrecurring type, are assumed to employ antipodal bit modulations, of which split-phase modulation is a commonly employed example.

The presence of additive white gaussian noise with single-sided spectral density N_o constitutes the disturbance in the presence of which a probability of false acquisition of 10^{-6} is to be maintained.

Figure 2.12 is employed in the construction of the Table 2.6. The computational values of E/N_o for the NASA codes is carried out as follows. First, note that the "NASA" curve in Fig. 2.12 is plotted on the basis of a binary antipodal waveform. Accordingly, a modification factor must be introduced which describes the poorer energy utilization of PDM signalling. The NASA BCD codes, having "space" length = 0.2T, "mark" length = 0.6T are equivalent in terms of bit error probabilities in noise to signals having durations of 0.3T and 0.7T respectively for "space" and "mark" (see Appendix B, First Quarterly Report). This is a "symmetrical" PDM case, as analyzed in Sec. 4.4.2.1 of the First Quarterly Report with the parameter $\alpha = 0.3$. The relative detectability for symmetrical PDM as noted in Table 4.3 of the QPR is $10\log_{10}[(1-2\alpha)'/4] = -10$ dB.

It should be emphasized that the principal advantage of the optimal system lies in the fact that the recurrent maximal-length code requires considerably less signal power to realize a specified degree of performance than do the current NASA and IRIG codes under otherwise identical conditions. Consequently, if efficient utilization of available signal power is an important consideration, the use of the optimal code will result in substantially less drain on available signal power, so that more of this power can be diverted to the transmission of the data and information that are timed by the time-code. Moreover, in time distribution systems, the ability to accomplish the desired quality of time transmission with less signal power means that the optimal time distribution system would present less of an interference threat to other co-situated systems.

From the viewpoint of equipment realization and complexity, the basic digital correlator technique (Fig. 2.11) for the recognition of frame synchronism in the optimal system poses no complexity problems. The storing of the n most recently received bits and their correlation with the n-bit sequence is easily accomplished with an n-bit shift register, an equal number of simple logic units (such as 'OR' circuits) and an adder.

Table 2.6

Comparison of NASA BCD codes frame acquisition properties (FDM bit encodings) vs. recurring and nonrecurring sequences of length 15 with thresholds of 12 and 13.

Sequence Type	E/N_o per bit for $P_{FA} = 10^{-6}$	dB, Relative to 13/15 Recurring
NASA	20.5 dB	-19.0 dB
Recurring 13/15	1.5 dB	-17.1 dB
Recurring 12/15	3.6 dB	0
Nonrecurring 13/15	*	-
Nonrecurring 12/15	**	-

* P_{FA} is insensitive to E/N_o. For any 'high' value of E/N_o, $P_{FA} \approx 5.8 \cdot 10^{-3}$.

** P_{FA} is insensitive to E/N_o. For any 'high' value of E/N_o, $P_{FA} \approx 0.03$.

3. SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

The optimal time code system is one that employs a recurring maximal-length sequence with period equal to a full time frame to convey (after cross-correlation detection at the receiving end) the frame time markers. The most advantageous autocorrelation properties for the time marker sequence waveform result if rectangular ±1 or rectangular split-phase waveforms are used. The basic digital correlator technique (Fig. 2.11) is used for the recognition of frame synchronism and the generation of isolated time frame markers. The storing of the n most recently received bits and their correlation with the n-bit sequence is easily accomplished with an n-bit shift register, an equal number of simple logic units (such as 'OR' circuits) and an adder.

The recurrent maximal-length code requires considerably less signal power to realize a specified degree of performance than do the current NASA and IRIG codes under otherwise identical conditions. Consequently, if efficient utilization of available signal power is an important consideration, the use of the optimal code will result in substantially less drain on available signal power, so that more of this power can be diverted to the transmission of the data and information that are timed by the time-code.

Sequences occupying only a part of the frame length, with time label and other coded information signals in the remainder of the frame, will generally be undesirable unless circumstances permit of the long sequence lengths necessary to achieve desirable probabilities of false alarm.

With full frame occupancy by the marker code, the label or data for the mark can be conveyed by

(a) occasional modulation of the marker code with the coded label information;

(b) a sinusoidal subcarrier, the modulated subcarrier being multiplexed in a suitable manner with the code marker waveform; or

(c) periodically interrupting the code and inserting the labeling information.

The simplest of these is (c). Each interruption will degrade the detection of only one marker that precedes it and another marker that immediately follows it. But in view of the fact that markers are expected to be regularly spaced and their spacings are exactly known, a few markers can even be skipped occasionally (especially if they are spaced by relative short-time intervals) without destroying the ability to identify properly the full conveyed time scale.