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APOLLO SPACECRAFT SYSTEMS ANALYSIS PROGRAM
TECHNICAL REPORT
TASK E-34D

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LANDING RADAR ALTIMETER BEAM BANDWIDTH AND DOPPLER EQUATIONS FOR MATHEMATICAL MODEL USE

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TRW SYSTEMS GROUP

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TASK E-34D

25 OCTOBER 1968

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INTRODUCTION

The landing radar acquisition and logic circuitry was designed around a postulated reflectivity curve for the lunar surface. The results of the Surveyor Program have shown that the lunar reflectivity may be as much as 12 dB larger than originally anticipated. This was discussed in a recent TRW report in which a proposed update of the lunar reflectivity model was presented.[1]

When the proposed reflectivity model was employed in the landing radar mathematical models at MSC the acquisition altitude was approximately 50 percent greater than that obtained with the old reflectivity model. This report contains the results of an investigation to determine why the acquisition altitude was not increased more than 50 percent.

The results of the investigation show that the acquisition altitude of the radar is limited primarily by the doppler bandwidth of the altimeter. A second order effect is the spectral broadening due to the terrain scattering. The vehicle attitude is also important, however, this factor is constant for both reflectivity models.

The overall conclusion of this study is that the results of the simulation are correct.

ANALYSIS

A. Signal Bandwidth

The doppler frequency for a particular ray in the antenna beam is given by

$$f_D = \frac{2v}{\lambda} \cos \gamma$$

v = velocity vector

λ = carrier frequency wavelength

γ = angle between the velocity vector and the beam ray direction

[1]Dickerson, E. T., Interim Reflectivity Model, TRW Report Number 11176-H018-T0-00, 9 August 1968, Contract Number NAS9-8166.

Suppose f_{D0} represents the beam center frequency. The upper and lower 3 dB beamwidth frequencies f_{D1} and f_{D2} are given by

$$f_{D1} = \frac{2v}{\lambda} \cos \left(\gamma + \frac{\Delta\gamma}{2}\right)$$

and

$$f_{D2} = \frac{2v}{\lambda} \cos \left(\gamma - \frac{\Delta\gamma}{2}\right).$$

The resulting signal bandwidth is then given by the difference of these two frequencies or

$$BW = f_{D1} - f_{D2}$$

$$= \frac{2v}{\lambda} \sin \theta \sin \frac{\Delta\gamma}{2}$$

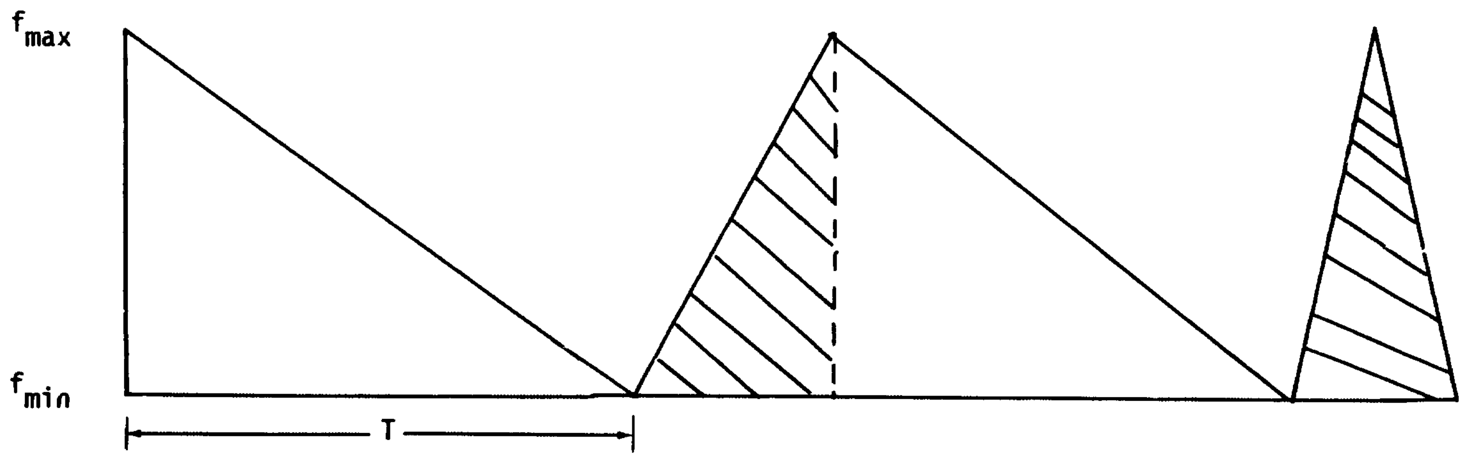
$$= \frac{v \cdot \Delta\gamma}{2} \sin \theta$$

In the GAEC model this is approximated by

$$BW_4 = \sqrt{\left(\frac{2v}{\lambda}\right)^2 - \left(\frac{2v_4}{\lambda}\right)^2} \cdot \Delta\gamma \quad .$$

v_4 = velocity along beam four

This factor accounts only for the normal doppler spreading due to the finite antenna beamwidth in the velocity sensor beams. The bandwidth of the altimeter signal is larger because the altimeter signal is a swept frequency signal as shown in Figure 1. The return from the lower side of the antenna beam is received $2\frac{\Delta R_{B4}}{c}$ seconds before that from the upper side of the beam (ΔR_{B4} is defined in Figure 2). The change in frequency during this time interval is related to the altimeter frequency sweep constant $\frac{df}{dt}$ which in turn can be related to the change in frequency versus range constant $\frac{\Delta f}{\Delta R}$. This constant has two values, one for R < 2500' and another



$$m = \frac{f_{max} - f_{min}}{T} = \frac{\Delta f}{T} \quad \text{cycle/sec}^2$$

Figure 1. Definition of the Altimeter Signal Parameters

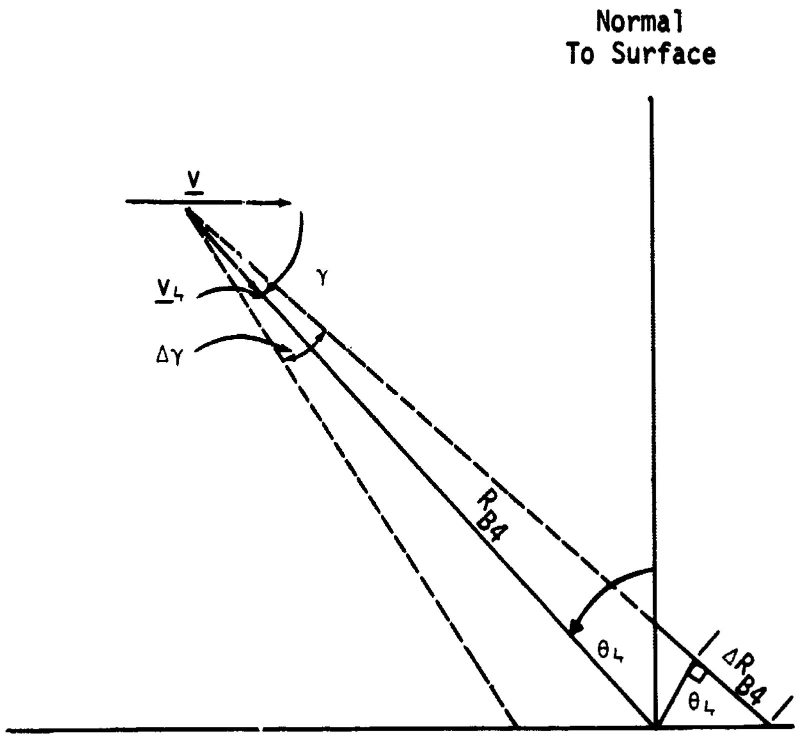


Figure 2. Geometry for Calculating the Altimeter Signal Bandwidth

for R > 2500'. The constant is given by $\frac{2m}{c}$ where the quantity m is defined in Figure 1 and c is the speed of light.

Employing the above quantities the additional bandwidth due to the sweeping function is given by

$$\Delta BW_L = \left(\frac{4m}{c}\right) \Delta R_{B4}$$

where ΔR_{B4} is defined in Figure 2. ΔBW_L may be related to the actual range through trigonometric relationships and the angle of incidence θ_L. The results are

$$\Delta BW = \frac{4m}{c} R_{B4} \sin\left(\frac{\Delta\gamma}{2}\right) \tan \theta_L \, .$$

GAEC has made a further approximation to this expression by replacing $\sin \frac{\Delta\gamma}{2}$ by $\frac{\Delta\gamma}{2}$. Employing this approximation

$$\Delta BW = \frac{4m}{c} \cdot \frac{\Delta\gamma}{2} R_{B4} \tan \theta_L = \frac{2m \cdot \Delta\gamma}{c} R_{B4} \tan \theta_L$$

The total bandwidth is given by the sum of the two functions or

$$BW_L' = |BW_L \pm \Delta BW_L| .$$

Since the modulator curve is not exactly linear GAEC has chosen to account for this factor by adding it to the above factors in an RMS fashion. Since the error due to the nonlinearity is approximately 3 percent of range, the resulting equation is

$$BW_L'' = (BW_L')^2 + .0009(R_{B4})^2$$

The term BW_L in the above equations is preceded by a $\pm$ sign hence a mathematical test must be performed to determine the proper sign to employ in the mathematical model.

The quantity tested in the present model is

$$\frac{?}{\sin \varepsilon}\left(\frac{V_4}{V} - \cos \theta_4 \cos \varepsilon\right) \qquad \begin{matrix} \le 0 \\ \ge 0 \end{matrix}$$

If this quantity is <0 use -, if >0 use + in the formation of BW_4'.

The purpose of this test is to determine when the altimeter beam doppler is positive and when it is negative. An examination of the equation and Figure 3 reveals that the quantity is negative for negative doppler, zero for no doppler and positive for positive doppler. The test may be reduced to testing the sign $u_4 \cdot \underline{v}$ where u_4 is a unit vector along beam 4 and $\underline{v}$ is the velocity vector.

The results of this test is to decrease the altimeter signal bandwidth when the doppler is negative and increase it when the doppler is positive.

The signal bandwidth is employed in the equation for the return power

$$W_{s4} = c_{35} \frac{P_{r4}}{B_{W4}} e^{-c_{45}\left(\frac{f - f_{D4}}{BW_4}\right)^2}$$

The effect of the smaller bandwidth for the negative doppler case is to increase the coefficient of the above expression while compressing the spectrum. When positive doppler is present the coefficient is reduced and the spectrum is broadened.

The net result of the above is that the signal power within the tracker bandwidth varies according to + or - doppler.

Since the total signal power is given by

$$S_4 = \int_{f_{d4} - B_{e4}}^{f_{d4} + B_{e4}} W_{s4} G_p G_t \, df$$

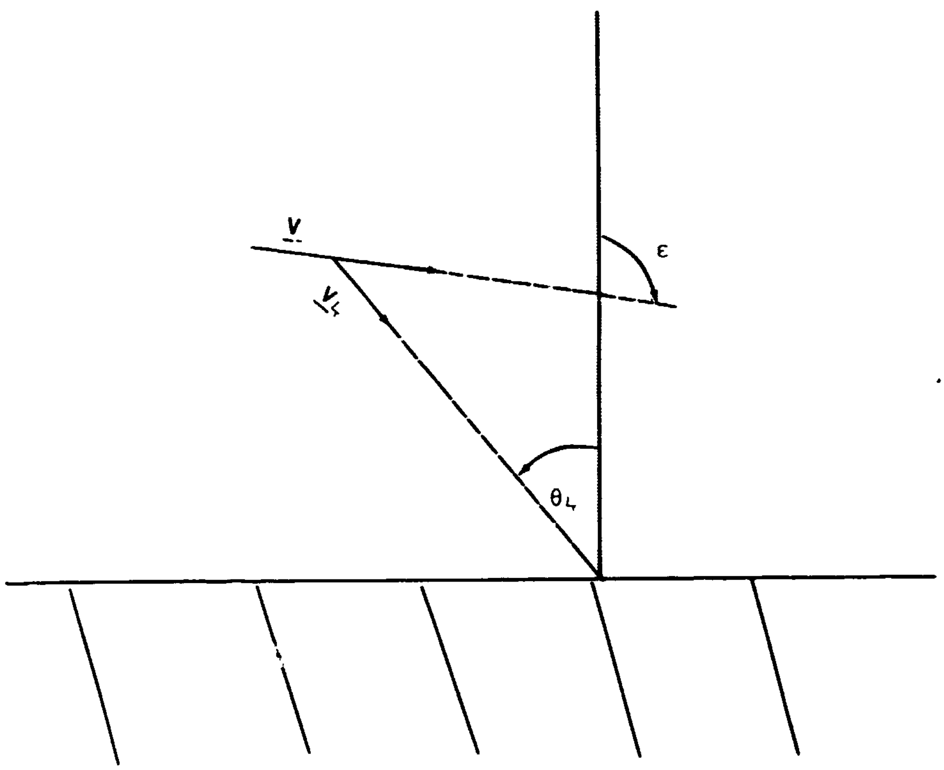


Figure 3. Geometry For Doppler Sign Test

B. Altimeter Doppler Frequency

The instantaneous frequency in the altimeter channel is given by the sum of f_D and f_r where f_r is the result of the linear sweep illustrated in Figure 1 and f_r is given by

$$f_r = m \cdot \Delta t$$

m has been defined previously, however, Δt is the time required for the altimeter signal to propogate down to the lunar surface and return. That is

$$\Delta t = \frac{2R_{B4}}{c}$$

hence

$$f_r = \frac{m \cdot 2R_{B4}}{c} .$$

The total doppler shift is given by

$$f_{B4} = f_D + f_r$$

$$= \frac{2v}{\lambda_4} \cos \gamma + \frac{2\Delta f}{TC} R_{B4}$$

The landing radar altimeter channel was designed to accept a total doppler of approximately 133 kHz. It is thus evident that if f_{B4} exceeds this value the altimeter frequency tracker will not lock up. The time T and frequency excursion Δf in the altimeter were designed such that R_{B4} could be as large as 49,000 feet. This maximum altitude could be obtained only under low velocity conditions along Beam 4. The constant $\frac{2\Delta f}{TC}$ is 2.32 for altitudes greater than 2500 feet. Hence the frequency shift f_r is 112 kHz at maximum altitude. The velocity must then be less than

$$v = \frac{(135 - 112) \cdot 10^3 \cdot \lambda}{2}$$

$$= 1130 \text{ ft/sec.}$$

This example shows that in general the altimeter will not acquire at large altitudes due to the higher velocities during that phase of the mission.

CONCLUSIONS

The landing radar altimeter will not acquire at significantly higher altitudes when the higher level reflectivity curve is employed for several reasons. First the spacecraft velocity is much larger during the high level portion of the descent and the resulting doppler shifts the signal outside the tracker bandwidth. Second the increased signal bandwidth will reduce the signal-to-noise ratio in the altimeter frequency tracker preventing lock-up until a lower altitude.

It is estimated that the altimeter channel may acquire at altitudes up to 50 percent higher than those obtained for the old reflectivity model. Definite values can be obtained only by a detailed analysis of the descent trajectory.

ACKNOWLEDGEMENT ADDENDUM

Report Number 11176-H044-R0-00

Range Coverage for the CSM Rendezvous Radar
Transponder Antenna Raised 4" and Tilted Forward
15°

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As indicated on Figure 2 the antenna pattern matrix was prepared by Mr. D. Cubley of MSC/SESD.

ERRATA

Landing Radar Altimeter Beam Bandwidth and Doppler Equations for Mathematical Model Use

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ERRATA

PAGE 5

The square root symbol was omitted from the equation for BW_4''. This equation should read:

$$BW_4'' = \sqrt{(BW_4')^2 + .0009(R_{B4})^2}$$

PAGE 6

A set of parenthesis were omitted from the equation for W_{s4}. This equation should read:

$$W_{s4} = c_{35} \frac{P_{r4}}{B_{W4}} e^{-c_{45} \frac{\left(f - f_{D4}\right)^2}{BW_4}}$$