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THEORETICAL SHAPE OF THE MAGNETOSPHERE BOUNDARY TAKING INTO ACCOUNT THE GEOMAGNETIC DIPOLE INCLINATION TO THE GEOGRAPHIC AXIS AND THE NON-DIPOLE PART OF THE GEOMAGNETIC FIELD

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by N. M. Rudneva

SUMMARY

The boundary of the magnetosphere is computed taking into account the inclination of the geomagnetic dipole and the nondipole part of the geomagnetic field. The latter affects little the boundary of the magnetosphere. The asymmetry of the magnetosphere on the daytime side must result in the asymmetry of the tail.

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There exists at present a magnetohydrodynamic theory of Earth's magnetic field interaction with solar wind. The flow of plasma from the Sun encounters in its path the Earth's magnetic field, flows past it with the result that a plasma-free cavity emerges. Electric currents flow in the thin boundary layer of the cavity; these currents hinder the magnetic field penetration into the plasma and result in magnetic field increase inside the cavity [1]. If we assume that on cavity surface particles are specularly reflected at incidence angle κ, the pressure of the plasma, perpendicular to the surface of the cavity is $p_0 \cos^2\kappa$. At normal plasma flux incidence upon the magnetic field, when the velocity direction changes to the inverse, the flux pressure is $p_0 = 2nmv^2$ [2], where m is the mass of an electron-ion pair, n is the concentration of particles in the flux and v is their velocity. It is shown in [3] that the entire plasma pressure must be equilibrated in the course of reflection by the pressure of the total magnetic field on cavity surface. Therefore, the problem of finding the shape of the cavity is formulated as follows [3 - 11]: inside the cavity div $\vec{H} = 0$, rot $\vec{H} = 0$, excluding the point at the place of dipole disposition, while on the surface of the cavity

$$\frac{H_s^2}{8\pi} = p_0 \cos^2 \varkappa.$$

The analytical solution of this problem is obtained only for the two-dimensional case [3, 7 - 9]. The three-dimensional problem is resolved numerically by utilizing the approximate equation [5, 6, 10, 11]

$$\frac{4f^2 H_{ps}^2}{8\pi} = p_0 \cos^2 \varkappa, \qquad f = \frac{H_s}{2H_{ps}}, \tag{1}$$

where f is the position function on the surface; H_s is the total magnetic field on cavity surface; H_p is the geomagnetic field on cavity surface. Bird [5] assumed that f = 1, which is equivalent to the assumption on the equality of H_s to the doubled tangential component of the permanent Earth's magnetic field at any point of the surface of the cavity.

In the current work the problem is being resolved of finding the shape of magnetosphere surface in the case of nondipole magnetic field. We are utilizing Eq.(1), in which we assume f = 1.

The expressions for the normal to the surface and flow velocity, required for the determination of cos κ and H_{ps}, are borrowed from [5]. The utilized system of coordinates is illustrated in Fig.1. The axis y is directed at the Sun. The direction of the dipole moment coincides with neither coordinate axis. $H_{ps} = -\text{grad } U$ is obtained from the expression U of the magnetic field potential determined with the help of the spherical analysis, taking into account the coefficients g_1^0, h_1^1, g_2^1

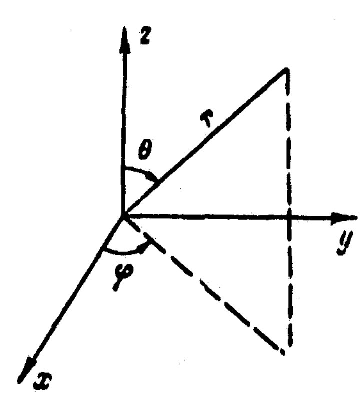

Fig.1

$$U = \frac{R_E^3}{r^2}(g_1^0 \cos\theta + h_1^1 \sin\lambda \sin\theta) + \frac{R_E^4}{r^3}(3g_2^1 \cos\lambda \sin\theta \cos\theta),$$

where R_E is the radius of the Earth; r is the distance to the point, at which the potential is determined; λ is the longitude of that point and θ is the polar angle. For the equinoctial period, when the flux velocity is parallel to the plane of the geographic equator, the equation, determining the surface of the magnetosphere with nondipole magnetic field, will be written:

$$\frac{-a}{r^3 b\sqrt{4\pi mnv^2}}\left[g_1\sin\theta - h_1\sin\lambda\cos\theta - \frac{1}{r}g_2\cos\lambda\cos 2\theta + \right.$$
$$\left. + \frac{1}{r}\frac{\partial r}{\partial\theta}\left(2g_1\cos\theta + 2h_1\sin\lambda\sin\theta + \frac{3}{r}g_2\cos\lambda\sin\theta\cos\theta\right)\right] =$$
$$= \pm\ \sin\varphi\sin\theta - \frac{1}{r}\frac{\partial r}{\partial\theta}\sin\varphi\cos\theta - \frac{\cos\varphi}{r\sin\theta}\frac{\partial r}{\partial\varphi}\Big), \tag{2}$$

where

$$a^{-2} = 1 + \left(\frac{1}{r}\frac{\partial r}{\partial\theta}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial r}{\partial\varphi}\right)^2, \quad b^{-2} = 1 + \left(\frac{1}{r}\frac{\partial r}{\partial\theta}\right)^2$$

are determined in [5]; ϕ is the angle in the equatorial plane ($\phi = \pi/2$ at the subsolar point). Following are the denotations used: $g_1 = R_E^3 g_1^0$, $h_1 = R_E^3 h_1^1$, $g_2 = R_E^4\cdot 3g_2^1$. Eq.(2) is a nonlinear differential equation in partial derivatives with two unknowns, θ and ϕ, whose solution is unknown in literature.

We shall write Eq.(2) in the assumption that at $\phi = \pm\frac{\pi}{2}$, $\frac{\partial r}{\partial\phi} = 0$, i. e., we shall obtain the equation for the cross-section of magnetosphere surface by a meridional plane. If $\partial r/\partial\phi = 0$, we have a = b and

$$-\frac{1}{\sqrt{4\pi mnv^2}\,r^3}\left[g_1\sin\theta - h_1\sin\lambda\cos\theta - \frac{1}{r}g_2\cos\lambda\cos 2\theta + \right.$$
$$\left. + \frac{1}{r}\frac{dr}{d\theta}\left(2g_1\cos\theta + 2h_1\sin\lambda\sin\theta + \frac{3}{r}g_2\cos\lambda\sin\theta\cos\theta\right)\right] =$$
$$= \pm\left[\sin\left(\pm\frac{\pi}{2}\right)\sin\theta - \sin\left(\pm\frac{\pi}{2}\right)\frac{1}{r}\frac{dr}{d\theta}\cos\theta\right]. \tag{3}$$

The expression $\frac{dr}{d\phi} = 0$ is valid only for a surface, symmetrical with respect to the meridional plane. Generally speaking, no such symmetry can be expected in the case of a nondipole field. However, in order to simplify the solution of the problem we shall consider the surface of the magnetosphere of a nondipole field as symmetrical in the direct neighborhood of the meridional plane. Then, we may assume $\partial r/\partial\phi = 0$

In a form resolved relative to the derivative and after substituting in it the numerical values of coefficients g_1^0, h_1^1, g_2^1 borrowed, for example, from [12], Eq.3 assumes the form

$$\frac{dR}{d\theta}=R\,\frac{\pm R^3\sin\theta\mp\left(\sin\theta+0{,}195\sin\lambda\cos\theta+0{,}3\frac{R_E'}{R}\cos\lambda\cos 2\theta\right)}{\pm R^3\cos\theta\pm\left(2\cos\theta-0{,}39\sin\lambda\sin\theta-0{,}45\frac{R_E'}{R}\cos\lambda\sin 2\theta\right)} \tag{4}$$

where

$$R=\frac{r}{r_0},\quad r_0=\frac{g^{1/3}}{(4\pi mnv^2)^{1/6}}=\frac{M^{1/3}}{(4\pi mnv^2)^{1/6}},\quad R_E'=\frac{R_E}{r_0},$$

M being the magnetic moment of the Earth. The signs $\pm$ ahead of R^3 correspond to two half-planes $\phi = \pm\frac{\pi}{2}$. The signs $\pm$ before the parentheses in the numerator and the denominator depend on the relation of signs of H_{ps} and $\cos\kappa$. At the same time, $\cos\kappa = (\vec{n}_s\vec{v})$, where $\vec{n}_s$ is a unitary vector of the inner normal to the surface, $\vec{v}$ is a unitary vector of flux velocity. To impart Eq.(1) a physical sense it is necessary that $1 \geqslant \cos\kappa \geqslant 0$. As to the sign of H_{ps}, the sign change on the daytime side can be apriori taken for granted somewhere near the poles, in neutral point. We shall assume H_{ps} to be positive on the daytime side from the equator to neutral points, and on the night side, and negative from neutral points to the poles. Then the meridional cross-section of the daytime surface of the magnetosphere from equator to neutral points will be given by the equation

$$\frac{dR}{d\theta}=R\,\frac{R^3\sin\theta-\left(\sin\theta+0{,}195\sin\lambda\cos\theta+0{,}3\frac{R_E'}{R}\cos\lambda\cos 2\theta\right)}{R^3\cos\theta+\left(2\cos\theta-0{,}39\sin\lambda\sin\theta-0{,}45\frac{R_E'}{R}\cos\lambda\sin 2\theta\right)}, \tag{5}$$

and the remaining part of the meridional cross-section of the surface – by the equation

$$\frac{dR}{d\theta}=R\,\frac{-R^3\sin\theta+\left(\sin\theta+0{,}195\sin\lambda\cos\theta+0{,}3\frac{R_E'}{R}\cos\lambda\cos 2\theta\right)}{-R^3\cos\theta-\left(2\cos\theta-0{,}39\sin\lambda\sin\theta-0{,}45\frac{R_E'}{R}\cos\lambda\sin 2\theta\right)}. \tag{6}$$

In case of nondipole magnetic field the shape of the cavity will be dependent on the UT. Eqs.(5) and (6) assume their simplest form for the time 0600 h.UT, when $\lambda = \pi/2$

$$\frac{dR}{d\theta} = R\frac{R^3\sin\theta - \sin\theta - 0{,}195\cos\theta}{R^3\cos\theta + 2\cos\theta - 0{,}39\sin\theta}, \tag{7}$$

$$\frac{dR}{d\theta} = R\frac{R^3\sin\theta + \sin\theta + 0{,}195\cos\theta}{R^3\cos\theta - 2\cos\theta + 0{,}39\sin\theta}. \tag{8}$$

For the moment of time 1800 h.UT, when $\lambda = -\pi/2$,

$$\frac{dR}{d\theta} = R\frac{R^3\sin\theta \mp (\sin\theta - 0{,}195\cos\theta)}{R^3\cos\theta \pm (2\cos\theta + 0{,}39\sin\theta)}. \tag{9}$$

and for the moment of time 1200 h.UT when $\lambda = 0$,

$$\frac{dR}{d\theta} = R\frac{R^3\sin\theta \mp \left(\sin\theta + 0{,}3\,\frac{R_E'}{R}\cos 2\theta\right)}{R^3\cos\theta \pm \left(2\cos\theta - 0{,}45\,\frac{R_E'}{R}\sin 2\theta\right)}. \tag{10}$$

The approximate Runge-Kutta method was applied for the integration of Eqs.(7) and (8). Formulas of third order were utilized. The computations yield an error in the third sign of values R. The computation step is 0.1 rad. The initial conditions $R = 1$, $\theta = 78°$ and $R = 1$, $\theta = \pi/2$ were chosen from the following considerations: a) the soltuion of the corresponding equations for the dipole field [5,6] yields $R = 1$ on the daytime side near the equator; b) $dR/d\theta$ drifts to infinity at $\theta = 83°$, $R = 1$, as a consequence of which one may not take advantage of the approximate method in the vicinity of that point. The drifting of $dR/d\theta$ to infinity at $\theta = 83°$ corresponds to the forward singular point $\theta = \pi/2$, $R = 1$ in [6]. From the physical standpoint this point is apparently the stagnation point of the flux.

According to the numerical solution of Eq.(7), the distance to the magnetosphere boundary increases from $\theta = 78°$ in the direction toward the North pole and decreases from $\theta = 90°$ in the direction to the South pole. Eq.(8) is resolved with initial conditions $R = 1$ at $\theta = 19°$ and $R = 1$ at $\theta = 161°$, which are the solutions of the corresponding equations for the dipole field [6]. The points of identical numerical solutions of Eqs.(7) and (8) may be considered as neutral points, at which the magnetic field changes sign. The coordinates of of the northern neutral point are $R = 1.06$, $\theta = 16°$, those of the southern are $R = 0.96$, $\theta = 22°$. The coordinates of neutral points in [6] are $R = 1$ at $\theta = 19.1°$ and $R = 1$ at $\theta = 160.9°$.

The numerical solution of Eq.(8) yields a sharp increase of R from neutral points to the poles. When $R \approx 1.16$, $\theta \approx 7°$, $dR/d\theta = \infty$, which does not allow us to resolve Eq.(8) by the approximate method for $\theta < 7°$. In the southern hemisphere ($\pi/2 < \theta < \pi$) the right-hand part of Eq.(8) is continuous. Its solution with the initial condition determined above is a closed line, symmetrical relative to axis Z and with a small minimum above the poles, $dR/d\theta = 0$ at $\theta = 173°$. This integral curve cannot represent the cross section of the magnetosphere surface in the region of poles and on the night side, for here $\cos \kappa < 0$. The physical conditions in the vicinity of the southern and northern poles must not differ too much. We shall consider that in the circumpolar regions Eq,(1) cannot describe the surface of the magnetosphere and it remains undetermined when the problem is stated that way. The insufficiency of the approximation (1) in the region of poles is recognized in well known works on the boundary of the magnetosphere. Another statement of the problem is required for the mathematical assignment of magnetosphere surface, which would take into account the pressure of thermal motion of particles and of the interplanetary magnetic field (possibly other effects also) and could not be neglected in the circumpolar regions.

As soon as the surface of the magnetosphere ceases to be parallel to solar wind velocity, so that $\cos \kappa > 0$, Eq.(1) can again be used for the determination of the surface of the magnetosphere, considering the dynamic pressure of the wind as prevailing over other effects. However, because of the uncertainty of the surface in the region of poles, initial condition of Eq.(8) can not be found for the night side. One may think that if R attains various values at poles on the daytime side, these values of R will also be different on the night side. This is why for the night side Eq.(8) should be resolved with different initial conditions for the two hemispheres. The integral curves of Eq.(8) for $R > 1.24$ in the northern hemisphere and for $R > 1.27$ in the southern hemisphere are lines for which R increases with the increase of θ. These integral curves are analogous to the curves in [6], determining the night magnetosphere.

The view of the meridional cross section of the magnetosphere surface of inclined dipole field for 0600 hours U.T. is represented in Figure 2 (next page).

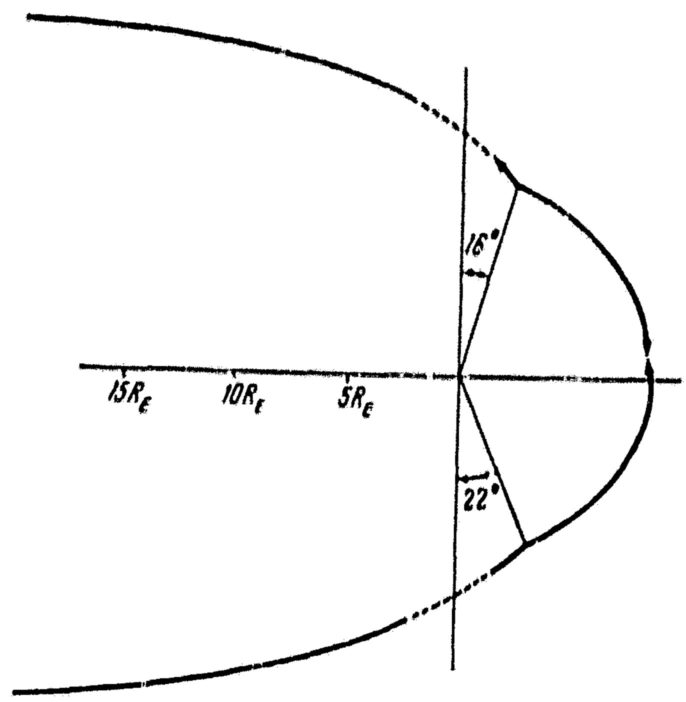

Fig.2

The solid lines show the computed cross section and the dashed lines – the assumed one. Arrows indicate the discontinuity of the right-hand parts of Eqs.(7) and (8). The radius of the cavity is given in Earth's radii. The construction was made for the following solar wind parameters: v = 500 km/sec, n = = 10 cm^{-3}, with R = 8.5 R_E. The solar wind velocity is parallel to the plane of the geographic equator. The night magnetosphere is determined by (8) with initial conditions: R = 1.3, θ = 170° and R = 1.5, θ = 10°. For the assumed initial conditions the half-width of the tail is by ~ 2R_E greater in the northern than in the southern hemispheres. With other initial conditions the tail's asymmetry may increase to several Earth's radii.

In Eq.(10) the terms arising out of the nondipole state of the geomagnetic field, are by one order smaller than the terms conditioned by dipole inclination in Eqs.(7) and (8). The distance to the boundary of the magnetosphere at dipole orientation corresponding to 1200 hrs UT may differ from the distance to magnetosphere boundary of the dipole field by ~ 0.01 R in all. At increase of the dynamic pressure of the wind the differences in magnetosphere boundary, induced by nondipole terms, increase. In this case the dipole orientation of the magnetosphere boundary in circumpolar regions is not determined by Eq.(1) either. At $\theta = \pi/2$. $dR/d\theta = \infty$. The region of uncertainty near poles at such an orientation of the magnetic dipole will be smaller thrn at dipole orientation corresponding to 0600 hours UT. Note that for the determination of the boundary of the magnetosphere it is possible to select from the family of integral curves of Fig.123σ of [6] a curve close to that selected by the authors of [6], but with finite region of uncertainty near the poles.

We laid at the basis of the calculation performed the equality of the dynamic pressure of solar wind to the transverse pressure of the magnetic field at the boundary of the magnetosphere. The solar wind plasma is considered as cold and devoid of proper magnetic field. Such a representation is apparently valid for low and middle latitudes; in order to determine the boundary of the manetosphere in the region of poles, it is necessary to take also into account other effects, for here the dynamic pressure may result to be near zero.

The arbitrariness of initial conditions for approximate calculations creates a certain indefiniteness in the relation of the obtained R with the assumed θ. However, the quantitaive results are quite trustworthy.

In this way, the inclination of the geomagnetic dipole may change the distance to the magnetosphere boundary by about 1 R_E, whereupon this change takes place as a function of latitude. The dependence on longitude, conditioned by geomagnetic dipole inclination is absent in the near-equatorial regions by mere physical considerations, while at other latitudes it has a form different from that obtained experimentally in [13, 14]. The field of universal anomalies may affect the distance to magnetosphere boundary by several fractions of the terrestiral radius, which cannot explain the results obtained in [13, 14] either. There must exist a mechanism enhancing the geomagnetic anomalies to a significant effect at magnetosphere boundary.

The position of neutral points at various moment of UT may differ by 3°. The magnetosphere tail's half-width for two hemispheres - northern and southern-may fluctuate within the limits of a few R_E as a function of UT.

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T H E   E N D