NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1337

A Parametric Study of Variations in Weight and Performance Characteristics of Large-Area Solar Arrays

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Preface

The work described in this report was performed by the Engineering Mechanics Division of the Jet Propulsion Laboratory.

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Abstract

A parametric study was made to establish relationships between performance characteristics (power-to-weight ratio, structural-member-section dimensions, structural resonant frequencies, deflections due to inertia loads, and structural-member stresses) and changes in design parameters for a large area solar array (LASA) design concept. Variations in design parameters considered in this study include overall geometric scaling of subpanel planform, aspect ratio scaling of subpanel planform, scaling of applied inertial loading, changes in structural material properties, and changes in nonstructural weight.

A computer program was developed to provide results of the parametric study in both tabular and graphical form. The graphical results are presented in a catalogue of plots which can be used to provide "quick look" evaluations of the characteristics to be expected for any new array design which incorporates the basic features of the existing LASA design concept. These plots also illustrate the possible disadvantages (or advantages) associated with alternative (other than beryllium) structural materials.

The parametric study results are not intended as a substitute for a complete and detailed structural analysis which still must be performed for any array to be used in a spaceflight mission. These results can be used, however, as a guide during the preliminary design phase to establish the first estimate of a suitable design to satisfy specified mission requirements.

A Parametric Study of Variations in Weight and Performance Characteristics of Large-Area Solar Arrays

I. Introduction

A program to develop and demonstrate the technology required to produce a large-area solar array (LASA) for spaceflight applications has been pursued for the past two years by the Boeing Company under JPL Contract 951653. The specific objective of this effort is to demonstrate the capability to produce a 50-kW array (at 1 AU) with a power-to-weight ratio of at least 20 W/lb, and capable of meeting performance requirements associated with a hypothetical solar electric propulsion mission to Mars.

Results of this development program to date suggest that its objective can be achieved; in fact, a significant portion of the required technology has been demonstrated. Among the more outstanding items of new technology resulting from this program are a method for fabricating relatively long, low-weight, structural beams made of beryllium, and the development of low-weight substrates composed of stretched fiberglass ribbons.

It has been suggested that all, or part, of the technology developed in the LASA program could be applied in a variety of future spaceflight programs, both unmanned and manned. In order to investigate this possibility, a study program was initiated in the second quarter of FY 1968. One phase of this applications study was a parametric study, which is the subject of this report.

II. Objective

The objective of the present study is to investigate the variations in array performance characteristics resulting from changes in mission requirements, geometric constraints, material properties, and loading conditions.

III. Parametric Study Approach

The basic approach followed in this study was to assume that the weight of a solar array can be associated with either structural (load-carrying) elements or non-structural elements. The structural elements were further classified as beams, whose principal elastic action is associated with bending, and fittings, which transmit loads between beams. It was further assumed that the non-structural elements of an array are uniformly distributed over the structure (whether it is in the stowed or deployed configuration). Since the solar cells, substrate, cover glasses, and electrical leads constitute a large portion of the non-structural weight, and these items are, indeed, quite uniformly distributed, the latter assumption is generally not seriously violated. Design loads for structural members are assumed to be associated with inertial loads (e.g., in ground vibration tests).

An essentially dimensional analysis approach was taken to determine appropriate scale factors which must be applied to the structural elements (beams and fittings) when design conditions are changed. Design conditions which can be varied in the parametric study include material elastic and strength properties, inertial loading, and overall panel dimensions (length or width, or both independently).

IV. Analysis

A. Cases With Overall Scaling

The total weight W_1 of a reference array can be broken down into three components, that is

$$W_1 = W_{b1} + W_{f1} + W_{n1} \tag{1}$$

where

W_{b1} = weight of the structural beams

W_{f1} = weight of the structural fittings

W_{n1} = weight of all non-structural material

and the subscript 1 refers to the reference array.

Consider a second array, denoted by the subscript 2, which differs from the reference array in geometry, material properties, and applied inertial loading. The total weight W_2 of this second array can be decomposed similarly as

$$W_2 = W_{b2} + W_{f2} + W_{n2} \tag{2}$$

and the ratio $W = W_2/W_1$ of the total weights of the two arrays can be written as

$$\begin{aligned} W &= \frac{W_{b2}}{W_1} + \frac{W_{f2}}{W_1} + \frac{W_{n2}}{W_1} \\ &= \frac{W_{b2}}{W_{b1}}\frac{W_{b1}}{W_1} + \frac{W_{f2}}{W_{f1}}\frac{W_{f1}}{W_1} + \frac{W_{n2}}{W_{n1}}\frac{W_{n1}}{W_1} \end{aligned} \tag{3}$$

The beam, fitting, and non-structural weight ratios appearing in Eq. (3) can be expressed in terms of material densities and volumes to yield

$$W = a\frac{\rho_{b2}}{\rho_{b1}}\frac{v_{b2}}{v_{b1}} + b\frac{\rho_{f2}}{\rho_{f1}}\frac{v_{f2}}{v_{f1}} + c\frac{\rho_{n2}}{\rho_{n1}}\frac{v_{n2}}{v_{n1}} \tag{4}$$

where

$a = \dfrac{W_{b1}}{W_1}$ = fraction of the total weight of the reference array contributed by the structural beams,

$b = \dfrac{W_{f1}}{W_1}$ = fraction of the total weight of the reference array contributed by the structural fittings,

$c = \dfrac{W_{n1}}{W_1}$ = fraction of the total weight of the reference array contributed by the non-structural elements,

and

v = volume of the component indicated by subscripts.

Since the non-structural weight is assumed to be uniformly distributed over the panel surfaces, it will be convenient to express the non-structural weight ratio in terms of mass per unit area rather than mass per unit volume; that is

$$c\frac{\rho_{n2}}{\rho_{n1}}\frac{v_{n2}}{v_{n1}} = c\frac{s_2}{s_1}\frac{A_2}{A_1}$$

where

s_i = mass per unit area for array i

and

A_i = area of array i

In order to further simplify the notation, let

$$\rho_b = \frac{\rho_{b2}}{\rho_{b1}}$$

$$\rho_f = \frac{\rho_{f2}}{\rho_{f1}}$$

$$v_b = \frac{v_{b2}}{v_{b1}}$$

$$v_f = \frac{v_{f2}}{v_{f1}}$$

$$s = \frac{s_2}{s_1}$$

and

$$A = \frac{A_2}{A_1}$$

Equation (4) can now be written as

$$W = a\rho_b v_b + b\rho_f v_f + csA \tag{5}$$

Now consider the following geometric scale factors:

λ_{bo} = beam section overall scale factor,

λ_{bt} = beam section material thickness scale factor,

λ_{fo} = fitting section overall scale factor,

λ_{ft} = fitting section material thickness scale factor,

λ_l = panel overall scale factor

Using these scale factors, Eq. (5) can be rewritten as

$$\begin{aligned} W &= a\rho_b\lambda_{bo}\lambda_{bt}\lambda_l + b\rho_f\lambda_{fo}\lambda_{ft}\lambda_l + csA \\ &= a\rho_b\lambda_{bo}\lambda_{bt}\lambda_l + b\rho_f\lambda_{fo}\lambda_{ft}\lambda_l + cs\lambda_l^2 \end{aligned} \tag{6}$$

Relationships can be established between the geometric scale factors appearing in Eq. (6) and the ratios of stresses or deflections of the structures; that is, for inertial loading, beam bending moments are related by

$$\frac{M_2}{M_1} = \lambda_a\lambda_l W$$

where λ_a = ratio of the acceleration loading of array 2 to the acceleration loading of array 1.

The corresponding ratio of beam maximum bending stresses is

$$\frac{\sigma_{b2}}{\sigma_{b1}} = \frac{M_2\, c_2\, I_{b1}}{M_1\, c_1\, I_{b2}}$$

The beam section moments of inertia are related by

$$\frac{I_{b2}}{I_{b1}} = \lambda_{bo}^3\, \lambda_{bt}$$

so that

$$\frac{\sigma_{b2}}{\sigma_{b1}} = \frac{\lambda_a\, \lambda_l}{\lambda_{bo}^2\, \lambda_{bt}}\, W \tag{7}$$

Similarly, the fitting stresses may be related by

$$\frac{\sigma_{f2}}{\sigma_{f1}} = \frac{\lambda_a\, \lambda_l}{\lambda_{fo}^2\, \lambda_{ft}}\, W \tag{8}$$

Array deflections due to beam bending are related by

$$\begin{aligned} \frac{\delta_{b2}}{\delta_{b1}} &= \lambda_a W\, \frac{E_{b1}}{E_{b2}}\, \frac{I_{b1}}{I_{b2}}\, \lambda_l^3 \\ &= \frac{\lambda_a\, \lambda_l^3}{\lambda_{bt}\, \lambda_{bo}^3}\, \frac{E_{b1}}{E_{b2}}\, W \end{aligned} \tag{9}$$

Similarly, the array deflections due to bending of the fittings are related by

$$\frac{\delta_{f2}}{\delta_{f1}} = \frac{\lambda_a\, \lambda_l^2}{\lambda_{ft}\, \lambda_{fo}^3}\, \frac{E_{f1}}{E_{f2}}\, W \tag{10}$$

Critical buckling stresses for the beam sections and the fitting sections are related by

$$\frac{\sigma_{cb2}}{\sigma_{cb1}} = \frac{\lambda_{bt}^2}{\lambda_{bo}^2}\, \frac{E_{b2}}{E_{b1}} \tag{11}$$

and

$$\frac{\sigma_{cf2}}{\sigma_{cf1}} = \frac{\lambda_{ft}^2}{\lambda_{fo}^2}\, \frac{E_{f2}}{E_{f1}} \tag{12}$$

Elastic mode frequencies are proportional to the square root of the ratio of stiffness to mass. Assuming again, as was done in considering deflections, that the stiffness is due to the beam bending stiffness and the fitting bending stiffness, the ratio of elastic mode frequencies could be written, in theory, in terms of ratios of beam bending and fitting bending stiffnesses. An explicit relationship of this type would require an intimate knowledge of the relative effects on overall stiffness due to the beams and the fittings, and would, in general, be quite difficult to obtain. For the purposes of this study it is assumed that two pseudofrequency ratios can be defined as follows:

$$\frac{\omega_{b2}}{\omega_{b1}} = \left(\frac{E_{b2}\, I_{b2}}{E_{b1}\, I_{b1}\, \lambda_l^3\, W}\right)^{1/2}$$

or

$$\frac{\omega_{b2}}{\omega_{b1}} = \left(\frac{E_{b2}}{E_{b1}}\, \frac{\lambda_{bo}^3\, \lambda_{bt}}{\lambda_l^3\, W}\right)^{1/2} \tag{13}$$

and

$$\frac{\omega_{f2}}{\omega_{f1}} = \left(\frac{E_{f2}\, I_{f2}}{E_{f1}\, I_{f1}\, \lambda_l^2\, W}\right)^{1/2}$$

$$= \left(\frac{E_{f2}}{E_{f1}}\, \frac{\lambda_{fo}^3\, \lambda_{ft}}{\lambda_l^2\, W}\right)^{1/2} \qquad (14)$$

Note that the frequency ratio given by Eq. (13) is that which would be obtained by assuming that all stiffness is associated with beam bending, whereas the latter ratio, given by Eq. (14), is associated with fitting stiffness.

Equations (6–14) are a set of nine equations involving the thirteen unknowns: W, λ_{bo}, λ_{bt}, λ_{fo}, λ_{ft}, σ_{b2}/σ_{b1}, σ_{f2}/σ_{f1}, $\sigma_{cb2}/\sigma_{cb1}$, $\sigma_{cf2}/\sigma_{cf1}$, δ_{b2}/δ_{b1}, δ_{f2}/δ_{f1}, ω_{b2}/ω_{b1} and ω_{f2}/ω_{f1}. Two additional equations can be written immediately by requiring equally critical stresses with respect to buckling for the two designs, i.e.

$$\frac{\sigma_{cb2}}{\sigma_{cb1}} = \frac{\sigma_{b2}}{\sigma_{b1}} \qquad (15)$$

and

$$\frac{\sigma_{cf2}}{\sigma_{cf1}} = \frac{\sigma_{f2}}{\sigma_{f1}} \qquad (16)$$

The remaining two equations, required for a complete set, are obtained by considering particular design conditions. Three such design conditions were considered in the study. The conditions and resulting equations follow:

*1. **Deflection-limited design—condition A.*** If array deflections are critical (e.g., due to shroud envelope constraints) one may wish to specify the ratio of deflections, that is

$$\frac{\delta_{b2}}{\delta_{b1}} = \delta \qquad (17)$$

and

$$\frac{\delta_{f2}}{\delta_{f1}} = \delta \qquad (18)$$

Equations (6–18) can now be solved for the above-listed thirteen unknowns. Elimination of all unknowns except W yields

$$W - a\rho_b \delta^{-1/3}\, \lambda_a^{5/9}\, \lambda_l^{20/9} \left(\frac{E_{b2}}{E_{b1}}\right)^{-5/9} W^{5/9}$$

$$- b\, \rho_f \delta^{-1/3}\, \lambda_a^{5/9}\, \lambda_l^{17/9} \left(\frac{E_{f2}}{E_{f1}}\right)^{-5/9} W^{5/9} - cs\lambda_l^2 = 0$$

or

$$W - \delta^{-1/3}\, \lambda_a^{5/9}\, \lambda_l^{17/9} \left[a\rho_b \lambda_l^{1/3} \left(\frac{E_{b1}}{E_{b2}}\right)^{5/9} + b\, \rho_f \left(\frac{E_{f1}}{E_{f2}}\right)^{5/9} \right] W^{5/9} - cs\lambda_l^2 = 0 \qquad (19)$$

Having obtained the weight ratio W from Eq. (19), the other unknowns can be computed from

$$\lambda_{bt} = \left(\lambda_a\, \lambda_l\, W\, \frac{E_{b1}}{E_{b2}}\right)^{1/3} \qquad (20)$$

$$\lambda_{bo} = \left(\frac{\lambda_a\, \lambda_l^3\, W}{\delta\, \lambda_{bt}}\, \frac{E_{b1}}{E_{b2}}\right)^{1/3} \qquad (21)$$

$$\lambda_{ft} = \left(\frac{E_{b2}}{E_{b1}}\, \frac{E_{f1}}{E_{f2}}\right)^{1/3} \lambda_{bt} \qquad (22)$$

$$\lambda_{fo} = \lambda_l^{-1/3} \left(\frac{E_{b2}}{E_{b1}}\, \frac{E_{f1}}{E_{f2}}\right)^{2/9} \lambda_{bo} \qquad (23)$$

$$\frac{\sigma_{b2}}{\sigma_{b1}} = \frac{\lambda_a\, \lambda_l}{\lambda_{bo}^2\, \lambda_{bt}}\, W \qquad (24)$$

$$\frac{\sigma_{f2}}{\sigma_{f1}} = \frac{\lambda_{bo}^2 \lambda_{bt}}{\lambda_{fo}^2\, \lambda_{ft}}\, \frac{\sigma_{b2}}{\sigma_{b1}} \qquad (25)$$

$$\frac{\sigma_{cb2}}{\sigma_{cb1}} = \frac{\sigma_{b2}}{\sigma_{b1}} \qquad (26)$$

$$\frac{\sigma_{cf2}}{\sigma_{cf1}} = \frac{\sigma_{f2}}{\sigma_{f1}} \qquad (27)$$

$$\frac{\omega_{b2}}{\omega_{b1}} = \left(\frac{\lambda_a}{\delta}\right)^{1/2} \qquad (28)$$

and

$$\frac{\omega_{f2}}{\omega_{f1}} = \frac{\omega_{b2}}{\omega_{b1}} \qquad (29)$$

*2. **Stress-limited design—condition B.*** If bending stresses constitute the critical design condition, one may wish to specify relationships between the margins of safety for the two designs in the form

$$\frac{\sigma_{b2}}{\sigma_{b1}} = k_1 \frac{\sigma_{yb2}}{\sigma_{yb1}} \tag{30}$$

and

$$\frac{\sigma_{f2}}{\sigma_{f1}} = k_2 \frac{\sigma_{yf2}}{\sigma_{yf1}} \tag{31}$$

where

σ_{ybi} = yield stress for beam material for array i,

and

σ_{yfi} = yield stress for fitting material for array i

Equations (6–16) and Eqs. (30–31) now consist of thirteen equations in the previously listed thirteen unknowns. Again, the elimination of all unknowns except W yields

$$W - \lambda_a^{2/3} \lambda_l^{5/3} \left[a\rho_b \left(\frac{\sigma_{yb1}}{k_1 \sigma_{yb2}} \right)^{1/2} \left(\frac{E_{b1}}{E_{b2}} \right)^{1/6} + b\,\rho_f \left(\frac{\sigma_{yf1}}{k_2 \sigma_{yf2}} \right)^{1/2} \left(\frac{E_{f1}}{E_{f2}} \right)^{1/6} \right] W^{2/3} - cs\,\lambda_l^2 = 0 \tag{32}$$

The other unknowns are obtained as follows:

$$\lambda_{bt} = \left(\lambda_a \lambda_l W \frac{E_{b1}}{E_{b2}} \right)^{1/3} \tag{33}$$

$$\lambda_{bo} = (\lambda_a \lambda_l W)^{1/3} \left[\frac{1}{k_1} \frac{\sigma_{yb1}}{\sigma_{yb2}} \left(\frac{E_{b2}}{E_{b1}} \right)^{1/3} \right]^{1/2} \tag{34}$$

$$\lambda_{ft} = \left(\frac{E_{b2}}{E_{b1}} \frac{E_{f1}}{E_{f2}} \right)^{1/3} \lambda_{bt} \tag{35}$$

$$\lambda_{fo} = \left[\frac{k_1}{k_2} \frac{\sigma_{yb2}}{\sigma_{yb1}} \frac{\sigma_{yf1}}{\sigma_{yf2}} \left(\frac{E_{b1}}{E_{b2}} \frac{E_{f2}}{E_{f1}} \right)^{1/3} \right]^{1/2} \lambda_{bo} \tag{36}$$

$$\frac{\sigma_{cb2}}{\sigma_{cb1}} = \frac{\sigma_{b2}}{\sigma_{b1}} \tag{37}$$

$$\frac{\sigma_{cf2}}{\sigma_{cf1}} = \frac{\sigma_{f2}}{\sigma_{f1}} \tag{38}$$

$$\frac{\delta_{b2}}{\delta_{b1}} = \frac{\lambda_a \lambda_l^3}{\lambda_{bt} \lambda_{bo}^3} \frac{E_{b1}}{E_{b2}} W \tag{39}$$

$$\frac{\delta_{f2}}{\delta_{f1}} = \frac{\lambda_a \lambda_l^2}{\lambda_{ft} \lambda_{fo}^3} \frac{E_{f1}}{E_{f2}} W \tag{40}$$

$$\frac{\omega_{b2}}{\omega_{b1}} = \left(\lambda_a \frac{\delta_{b1}}{\delta_{b2}} \right)^{1/2} \tag{41}$$

and

$$\frac{\omega_{f2}}{\omega_{f1}} = \left(\lambda_a \frac{\delta_{f1}}{\delta_{f2}} \right)^{1/2} \tag{42}$$

3. ***Frequency-limited design—condition C.*** If the elastic mode frequencies are critical for the contemplated design, one may specify the frequency ratios, that is

$$\frac{\omega_{b2}}{\omega_{b1}} = k_3 \tag{43}$$

and

$$\frac{\omega_{f2}}{\omega_{f1}} = k_3 \tag{44}$$

Equations (6–18) and Eqs. (43) and (44) can be combined to give

$$W - k_3^{2/3} \lambda_a^{2/9} \lambda_l^{17/9} \left[a\rho_b \lambda_l^{1/3} \left(\frac{E_{b1}}{E_{b2}} \right)^{5/9} + b\rho_f \left(\frac{E_{f1}}{E_{f2}} \right)^{5/9} \right] W^{5/9} - cs\lambda_l^2 = 0 \tag{45}$$

and the other unknowns can be obtained from

$$\lambda_{bt} = \left(\lambda_a \lambda_l W \frac{E_{b1}}{E_{b2}} \right)^{1/3} \tag{46}$$

$$\lambda_{bo} = \left(\frac{k_3^2 \lambda_l^3 W}{\lambda_{bt}} \frac{E_{b1}}{E_{b2}} \right)^{1/3} \tag{47}$$

$$\lambda_{ft} = \left(\lambda_a \lambda_l W \frac{E_{f1}}{E_{f2}} \right)^{1/3} \tag{48}$$

$$\lambda_{fo} = \left(\frac{k_3^2 \lambda_l^2 W}{\lambda_{ft}} \frac{E_{f1}}{E_{f2}} \right)^{1/3} \tag{49}$$

$$\frac{\sigma_{b2}}{\sigma_{b1}} = \frac{\lambda_a \lambda_l W}{\lambda_{bo}^2 \lambda_{bt}} \tag{50}$$

$$\frac{\sigma_{f2}}{\sigma_{f1}} = \frac{\lambda_a \lambda_l W}{\lambda_{fo}^2 \lambda_{ft}} \tag{51}$$

$$\frac{\sigma_{cb2}}{\sigma_{cb1}} = \frac{\sigma_{b2}}{\sigma_{b1}} \tag{52}$$

$$\frac{\sigma_{cf2}}{\sigma_{cf1}} = \frac{\sigma_{f2}}{\sigma_{f1}} \tag{53}$$

$$\frac{\delta_{b2}}{\delta_{b1}} = \frac{\lambda_a}{k_3^2} \tag{54}$$

and

$$\frac{\delta_{f2}}{\delta_{f1}} = \frac{\delta_{b2}}{\delta_{b1}} \tag{55}$$

B. Cases With Aspect-Ratio Scaling

The previously developed equations imply an overall geometric scaling factor λ applied in both directions. Geometric scaling by different factors in the two directions requires a slightly modified treatment. For such a development, it is required to divide the total array weight into two parts, each associated with one of the scaling directions. The reference array is broken into components such that the governing equations, analogous to Eq. (1) become

$$\left.\begin{aligned} W_1^H &= W_{b1}^H + W_{f1}^H + W_{n1}^H \\ W_1^V &= W_{b1}^V + W_{f1}^V + W_{n1}^V \\ \text{and} \quad W_1 &= W_1^H + W_1^V \end{aligned}\right\} \tag{56}$$

where the subscripts are the same as defined earlier and the superscripts refer to the two directions to be scaled.

Similarly the weight breakdown of the second array can be defined by:

$$\left.\begin{aligned} W_2^H &= W_{b2}^H + W_{f2}^H + W_{n2}^H \\ W_2^V &= W_{b2}^V + W_{f2}^V + W_{n2}^V \\ \text{and} \quad W_2 &= W_2^H + W_2^V \end{aligned}\right\} \tag{57}$$

Using Eqs. (56) and (57), the ratio $W = \dfrac{W_2}{W_1}$ of the total weights of the two arrays can be developed as follows:

$$W^H = \frac{W_2^H}{W_1^H} = \frac{W_{b2}^H}{W_1^H} + \frac{W_{f2}^H}{W_1^H} + \frac{W_{n2}^H}{W_1^H}$$

$$= \frac{W_{b2}^H}{W_{b1}^H}\frac{W_{b1}^H}{W_1^H} + \frac{W_{f2}^H}{W_{f1}^H}\frac{W_{f1}^H}{W_1^H} + \frac{W_{n2}^H}{W_{n1}^H}\frac{W_{n1}^H}{W_1^H} \tag{58}$$

Similarly,

$$W^V = \frac{W_2^V}{W_1^V} = \frac{W_{b2}^V}{W_1^V} + \frac{W_{f2}^V}{W_1^V} + \frac{W_{n2}^V}{W_1^V}$$

$$= \frac{W_{b2}^V}{W_{b1}^V}\frac{W_{b1}^V}{W_1^V} + \frac{W_{f2}^V}{W_{f1}^V}\frac{W_{f1}^V}{W_1^V} + \frac{W_{n2}^V}{W_{n1}^V}\frac{W_{n1}^V}{W_1^V} \tag{59}$$

and

$$W = \frac{W_2}{W_1} = \frac{W_2^H + W_2^V}{W_1^H + W_1^V}$$

$$= \frac{W^H}{1 + \dfrac{W_1^V}{W_1^H}} + \frac{W^V}{\dfrac{W_1^H}{W_1^V} + 1}$$

or

$$W = \frac{W^H}{1 + A} + \frac{W^V}{1 + \dfrac{1}{A}} \tag{60}$$

where

$$A = \frac{W_1^V}{W_1^H}$$

Expressing Eqs. (58) and (59) in terms of material densities and volumes gives

$$W^H = a^H \frac{\rho_{b2}}{\rho_{b1}}\left(\frac{v_{b2}}{v_{b1}}\right)^H + b^H \frac{\rho_{f2}}{\rho_{f1}}\left(\frac{v_{f2}}{v_{f1}}\right)^H + c\,\frac{\rho_{n2}}{\rho_{n1}}\left(\frac{v_{n2}}{v_{n1}}\right)^H \tag{61}$$

and

$$W^V = a^V \frac{\rho_{b2}}{\rho_{b1}}\left(\frac{v_{b2}}{v_{b1}}\right)^V + b^V \frac{\rho_{f2}}{\rho_{f1}}\left(\frac{v_{f2}}{v_{f1}}\right)^V + c\,\frac{\rho_{n2}}{\rho_{n1}}\left(\frac{v_{n2}}{v_{n1}}\right)^V \tag{62}$$

Where the constants a^H, b^H, c^H, a^V, b^V, and c^V are analogous to those used in Eq. (4), with the superscripts denoting the scaling direction, i.e.,

$a^H = \dfrac{W_{b1}^H}{W_1^H}$ = ratio of the weight of the reference array structural beams along the H-directions to the total reference array, effective weight in the H-direction, etc.

By introducing the simplified notation used in Eq. (4), Eqs. (61) and (62) can be written as

$$W^H = a^H \rho_b v_b^H + b^H \rho_f v_f^H + c^H sA^H \qquad (63)$$

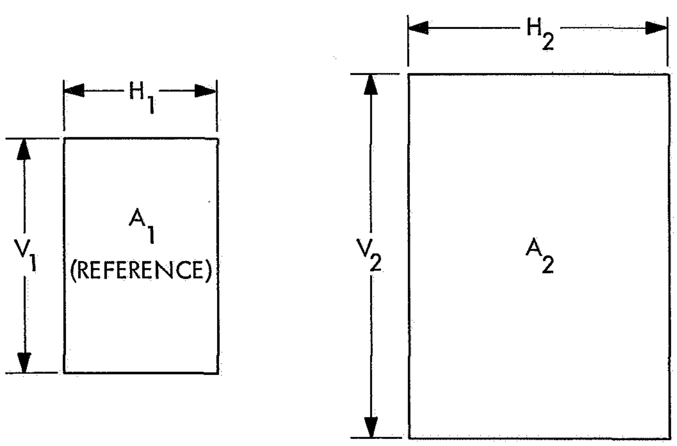
and

$$W^V = a^V \rho_b v_b^V + b^V \rho_f v_f^V + c^V sA^V \qquad (64)$$

Equations (63) and (64) are analogous to Eq. (5).

Examination of the constants a^H, a^V, b^H, b^V, c^H, and c^V used in Eqs. (63) and (64) indicates that the reference array weight must be divided into weights associated with the H-direction and weights associated with the V-direction as indicated by Eq. (56). W_{b1}^H, W_{b1}^V, W_{f1}^H, and W_{f1}^V are determined simply by considering the weights of the beams and their associated fittings along the H and V directions. In order to determine W_1^H and W_1^V, and hence define all the constants of Eqs. (63) and (64), it is required to divide the nonstructural weight of the array into two parts, namely W_{n1}^H and W_{n1}^V. For the purpose of this analysis, it has been assumed that for each panel the nonstructural weight may be divided proportionally to the ratio of the panel dimensions H/V. Since it is assumed that the nonstructural weight is distributed uniformly over each panel, the effective panel areas are divided proportionally.

Consider the two panels:



For each panel

$$\left.\begin{aligned} \frac{A_1^H}{A_1^V} &= \frac{H_1}{V_1} = r_1 \\ \text{and} \qquad & \\ \frac{A_2^H}{A_2^V} &= \frac{H_2}{V_2} \end{aligned}\right\} \qquad (65)$$

Where the superscript on the area terms indicates the direction of the effective area and r_1 is the aspect ratio of the reference panel.

Geometric scale factors may be defined as

$$\left.\begin{aligned} \lambda_H &= \frac{H_2}{H_1} \\ \text{and} \qquad & \\ \lambda_V &= \frac{V_2}{V_1} \end{aligned}\right\} \qquad (66)$$

Then

$$\left.\begin{aligned} A_1 &= A_1^H + A_1^V \\ A_2 &= A_2^H + A_2^V \end{aligned}\right\} \qquad (67)$$

and

$$A_2 = \lambda_H \lambda_V A_1 \qquad (68)$$

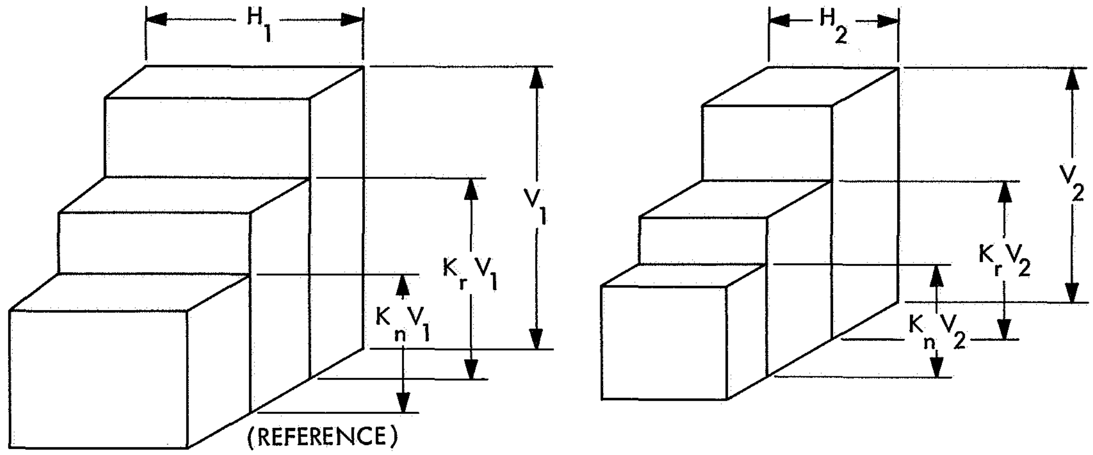
Combining Eqs. (65–68), the following relationships can be derived:

$$\left.\begin{aligned} A_1^H &= \frac{r_1}{1 + r_1} A_1 \\ A_1^V &= \frac{1}{1 + r_1} A_1 \end{aligned}\right\} \qquad (69)$$

$$\left.\begin{aligned} A_2^H &= \frac{\lambda_H r_1}{\lambda_V + \lambda_H r_1} A_2 \\ A_2^V &= \frac{\lambda_V}{\lambda_V + \lambda_H r_1} A_2 \end{aligned}\right\} \qquad (70)$$

$$\left.\begin{aligned} A^H &= \frac{A_2^H}{A_1^H} = \frac{1 + r_1}{\lambda_V + \lambda_H r_1} \lambda_H^2 \lambda_V \\ \text{and} \qquad & \\ A^V &= \frac{A_2^V}{A_1^V} = \frac{1 + r_1}{\lambda_V + \lambda_H r_1} \lambda_H \lambda_r^2 \end{aligned}\right\} \qquad (71)$$

To extend the same type of relationships for an array consisting of a collection of panels having different aspect ratios, consider the following two arrays:



In the reference array,

let there be m panels having dimensions H_1 by V_1
n panels having dimensions H_1 by $K_n V_1$
r panels having dimensions H_1 by $K_r V_1$

consequently there will be

m panels having dimensions H_2 by V_2
n panels having dimensions H_2 by $K_n V_2$
r panels having dimensions H_2 by $K_r V_2$

Proceeding in the same manner as for the single panel, the corresponding relationships are:

$$\left.\begin{aligned} A_1^H &= A_1 r_1 \left(\frac{m}{1 + r_1} + \frac{n K_n}{K_n + r_1} + \frac{r K_r}{K_r + r_1} \right) \\ A_1^V &= A_1 \left(\frac{m}{1 + r_1} + \frac{n K_n^2}{K_n + r_1} + \frac{r K_r^2}{K_r + r_1} \right) \end{aligned}\right\} \tag{72}$$

$$\left.\begin{aligned} A_2^H &= \lambda_H^2 \lambda_V A_1 r_1 \left(\frac{m}{\lambda_V + \lambda_H r_1} + \frac{n K_n}{\lambda_V K_n + \lambda_H r_1} + \frac{r K_r}{\lambda_V K_r + \lambda_H r_1} \right) \\ A_2^V &= \lambda_H \lambda_V^2 A_1 \left(\frac{m}{\lambda_V + \lambda_H r_1} + \frac{n K_n^2}{\lambda_V K_n + \lambda_H r_1} + \frac{r K_r^2}{\lambda_V K_r + \lambda_H r_1} \right) \end{aligned}\right\} \tag{73}$$

$$\left.\begin{aligned} A^H &= \lambda_H^2 \lambda_V \frac{\dfrac{m}{\lambda_V + \lambda_H r_1} + \dfrac{n K_n}{\lambda_V K_n + \lambda_H r_1} + \dfrac{r K_r}{\lambda_V K_r + \lambda_H r_1}}{\dfrac{m}{1 + r_1} + \dfrac{n K_n}{K_n + r_1} + \dfrac{r K_r}{K_r + r_1}} \\ &\text{and} \\ A^V &= \lambda_H \lambda_V^2 \frac{\dfrac{m}{\lambda_V + \lambda_H r_1} + \dfrac{n K_n^2}{\lambda_V K_n + \lambda_H r_1} + \dfrac{r K_r^2}{\lambda_V K_r + \lambda_H r_1}}{\dfrac{m}{1 + r_1} + \dfrac{n K_n^2}{K_n + r_1} + \dfrac{r K_r^2}{K_r + r_1}} \end{aligned}\right\} \tag{74}$$

While Eq. (74) has been derived specifically for the Boeing LASA type configuration, the relationship is general and can easily be extended to other configurations.

On the preceding page r_1 is the aspect ratio of the largest reference panel and λ_V and λ_H are the geometric scale factors for the largest panel.

W^H_{n1} and W^V_{n1} can now be obtained using Eq. (72). Thus, W^H_1 and W^V_1 are determined and a^H, a^V, b^H, b^V, c^H, and c^V can be found.

The following scale factors are now introduced.

λ^H_{bo} = section overall scale factor for the H-direction beams

λ^V_{bo} = section overall scale factor for the V-direction beams

λ^H_{bt} = section material thickness scale factor for the H-direction beams

λ^V_{bt} = section material thickness scale factor for the V-direction beams

λ^H_{fo} = section overall scale factor for the H-direction fittings

λ^V_{fo} = section overall scale factor for the V-direction fittings

λ^H_{ft} = section material thickness scale factor for the H-direction fittings

λ^V_{ft} = section material thickness scale factor for the V-direction fittings.

In terms of the above scale factors, Eqs. (63) and (64) can now be rewritten,

$$W^H = a^H \rho_b \lambda^H_{bo} \lambda^H_{bt} \lambda_H + b^H \rho_f \lambda^H_{fo} \lambda^H_{ft} \lambda_H + c^H s A^H \qquad (75)$$

and

$$W^V = a^V \rho_b \lambda^V_{bo} \lambda^V_{bt} \lambda_V + b^V \rho_f \lambda^V_{fo} \lambda^V_{ft} \lambda_V + c^V s A^V \qquad (76)$$

The derivation of relationships between the geometric scale factors of Eqs. (75) and (76) and the stresses and deflections of the two structures are analogous to those derived in Eqs. (7–16). One set of these relationships is required for the scaling in each direction.

1. Deflection-limited design—condition A. It will be assumed that the deflection ratio will be the same for each of the two directions, for both the beams and fittings, i.e.

$$\left(\frac{\delta_{b2}}{\delta_{b1}}\right)^H = \left(\frac{\delta_{b2}}{\delta_{b1}}\right)^V = \left(\frac{\delta_{f2}}{\delta_{f1}}\right)^H = \left(\frac{\delta_{f2}}{\delta_{f1}}\right)^V = \delta \qquad (77)$$

The following two equations in W^H and W^V result:

$$W^H - \delta^{-1/3} \lambda_a^{5/9} \lambda_H^{17/9} \left[a^H \rho_b \lambda_H^{1/3} \left(\frac{E_{b1}}{E_{b2}}\right)^{5/9} + b^H \rho_f \left(\frac{E_{f1}}{E_{f2}}\right)^{5/9} \right] (W^H)^{5/9} - c^H s A^H = 0 \qquad (78)$$

and

$$W^V - \delta^{-1/3} \lambda_a^{5/9} \lambda_V^{17/9} \left[a^V \rho_b \lambda_V^{1/3} \left(\frac{E_{b1}}{E_{b2}}\right)^{5/9} + b^H \rho_f \left(\frac{E_{f1}}{E_{f2}}\right)^{5/9} \right] (W^V)^{5/9} - c^V s A^V = 0 \qquad (79)$$

Having solved the above equations for W^H and W^V, W can be obtained using Eq. (60). The other pertinent factors can now be obtained from:

$$\lambda^H_{bt} = \left(\lambda_a \lambda_H W^H \frac{E_{b1}}{E_{b2}}\right)^{1/3} \qquad (80)$$

$$\lambda^V_{bt} = \left(\lambda_a \lambda_V W^V \frac{E_{b1}}{E_{b2}}\right)^{1/3} \qquad (81)$$

$$\lambda^H_{bo} = \left(\frac{\lambda_a \lambda_H^3 W^H}{\delta \lambda^H_{bt}} \frac{E_{b1}}{E_{b2}}\right)^{1/3} \qquad (82)$$

$$\lambda^V_{bo} = \left(\frac{\lambda_a \lambda_V^3 W^V}{\delta \lambda^V_{bt}} \frac{E_{b1}}{E_{b2}}\right)^{1/3} \qquad (83)$$

$$\lambda^H_{ft} = \left(\frac{E_{b2}}{E_{b1}} \frac{E_{f1}}{E_{f2}}\right)^{1/3} \lambda^H_{bt} \qquad (84)$$

$$\lambda^V_{ft} = \left(\frac{E_{b2}}{E_{b1}} \frac{E_{f1}}{E_{f2}}\right)^{1/3} \lambda^V_{bt} \qquad (85)$$

$$\lambda^H_{fo} = \lambda_H^{-1/3} \left(\frac{E_{b2}}{E_{b1}} \frac{E_{f1}}{E_{f2}}\right)^{2/9} \lambda^H_{bo} \qquad (86)$$

$$\lambda^V_{fo} = \lambda_V^{-1/3} \left(\frac{E_{b2}}{E_{b1}} \frac{E_{f1}}{E_{f2}}\right)^{2/9} \lambda^V_{bo} \qquad (87)$$

$$\left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^H = \frac{\lambda_a \lambda_H}{(\lambda^H_{bo})^2 \lambda^H_{bt}} W^H \qquad (88)$$

$$\left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^V = \frac{\lambda_a \lambda_V}{(\lambda_{bo}^V)^2 \lambda_{bt}^V} W^V \tag{89}$$

$$\left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^H = \frac{(\lambda_{bo}^H)^2 \lambda_{bt}^H}{(\lambda_{fo}^H)^2 \lambda_{ft}^H} \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^H \tag{90}$$

$$\left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^V = \frac{(\lambda_{bo}^V)^2 \lambda_{bt}^V}{(\lambda_{fo}^V)^2 \lambda_{ft}^V} \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^V \tag{91}$$

$$\left(\frac{\sigma_{cb2}}{\sigma_{cb1}}\right)^H = \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^H \tag{92}$$

$$\left(\frac{\sigma_{cb2}}{\sigma_{cb1}}\right)^V = \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^V \tag{93}$$

$$\left(\frac{\sigma_{cf2}}{\sigma_{cf1}}\right)^H = \left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^H \tag{94}$$

$$\left(\frac{\sigma_{cf2}}{\sigma_{cf1}}\right)^V = \left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^V \tag{95}$$

$$\left(\frac{\omega_{b2}}{\omega_{b1}}\right)^H = \left(\frac{\omega_{b2}}{\omega_{b1}}\right)^V = \left(\frac{\lambda_a}{\delta}\right)^{1/2} \tag{96}$$

and

$$\left(\frac{\omega_{f2}}{\omega_{f1}}\right)^H = \left(\frac{\omega_{f2}}{\omega_{f1}}\right)^V = \left(\frac{\lambda_a}{\delta}\right)^{1/2} \tag{97}$$

2. Stress-limited design—condition B. The margins of safety for both directions will be assumed to be equal for the beams as well as the fittings. Hence,

$$\left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^H = \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^V = K_1 \frac{\sigma_{yb2}}{\sigma_{yb1}} \tag{98}$$

and

$$\left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^H = \left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^V = K_2 \frac{\sigma_{yf2}}{\sigma_{yf1}} \tag{99}$$

The two equations analogous to Eq. (32) now become

$$W^H - \lambda_a^{2/3} \lambda_H^{5/3} \left[a^H \rho_b \left(\frac{\sigma_{yb1}}{K_1 \sigma_{yb2}}\right)^{1/2} \left(\frac{E_{b1}}{E_{b2}}\right)^{1/6} + b^H \rho_f \left(\frac{\sigma_{yf1}}{K_2 \sigma_{yf2}}\right)^{1/2} \left(\frac{E_{f1}}{E_{f2}}\right)^{1/6} \right] (W^H)^{2/3} - c^H s A^H = 0 \tag{100}$$

and

$$W^V - \lambda_a^{2/3} \lambda_H^{5/3} \left[a^V \rho_b \left(\frac{\sigma_{yb1}}{K_1 \sigma_{yb2}}\right)^{1/2} \left(\frac{E_{b1}}{E_{b2}}\right)^{1/6} + b^V \rho_f \left(\frac{\sigma_{yf1}}{K_2 \sigma_{yf2}}\right)^{1/2} \left(\frac{E_{f1}}{E_{f2}}\right)^{1/6} \right] (W^V)^{2/3} - c^V s A^V = 0 \tag{101}$$

Here again W can be obtained using Eq. (60). The pertinent factors can now be determined from

$$\lambda_{bt}^H = \left(\lambda_a \lambda_H W^H \frac{E_{b1}}{E_{b2}}\right)^{1/3} \tag{102}$$

$$\lambda_{bt}^V = \left(\lambda_a \lambda_V W^V \frac{E_{b1}}{E_{b2}}\right)^{1/3} \tag{103}$$

$$\lambda_{bo}^H = \left(\lambda_a \lambda_H W^H\right)^{1/3} \left[\frac{1}{K_1} \frac{\sigma_{yb1}}{\sigma_{yb2}} \left(\frac{E_{b2}}{E_{b1}}\right)^{1/3}\right]^{1/2} \tag{104}$$

$$\lambda_{bo}^V = \left(\frac{\lambda_V}{\lambda_H} \frac{W^V}{W^H}\right)^{1/3} \lambda_{bo}^H \tag{105}$$

$$\lambda_{ft}^H = \left(\frac{E_{b2}}{E_{b1}} \frac{E_{f1}}{E_{f2}}\right)^{1/3} \lambda_{bt}^H \tag{106}$$

$$\lambda_{ft}^V = \left(\frac{E_{b2}}{E_{b1}} \frac{E_{f1}}{E_{f2}}\right)^{1/3} \lambda_{bt}^V \tag{107}$$

$$\lambda_{fo}^H = \left[\frac{K_1}{K_2} \frac{\sigma_{yb2}}{\sigma_{yb1}} \frac{\sigma_{yf1}}{\sigma_{yf2}} \left(\frac{E_{b1}}{E_{b2}} \frac{E_{f2}}{E_{f1}}\right)^{1/3}\right]^{1/2} \lambda_{bo}^H \tag{108}$$

$$\lambda_{fo}^V = \left(\frac{\lambda_V}{\lambda_H} \frac{W^V}{W^H}\right)^{1/3} \lambda_{fo}^H \tag{109}$$

$$\left(\frac{\sigma_{cb2}}{\sigma_{cb1}}\right)^H = \left(\frac{\sigma_{cb2}}{\sigma_{cb1}}\right)^V = K_1 \frac{\sigma_{yb2}}{\sigma_{yb1}} \tag{110}$$

$$\left(\frac{\sigma_{cf2}}{\sigma_{cf1}}\right)^H = \left(\frac{\sigma_{cf2}}{\sigma_{cf1}}\right)^V = K_2 \frac{\sigma_{yf2}}{\sigma_{yf1}} \tag{111}$$

$$\left(\frac{\delta_{b2}}{\delta_{b1}}\right)^H = \frac{\lambda_a \lambda_H^3}{\lambda_{bt}^H (\lambda_{bo}^H)^3} \frac{E_{b1}}{E_{b2}} W^H \tag{112}$$

$$\left(\frac{\delta_{b2}}{\delta_{b1}}\right)^V = \frac{\lambda_a \lambda_V^3}{\lambda_{bt}^V (\lambda_{bo}^V)^3} \frac{E_{b1}}{E_{b2}} W^V \tag{113}$$

$$\left(\frac{\delta_{f2}}{\delta_{f1}}\right)^H = \frac{\lambda_a \lambda_H^2}{\lambda_{ft}^H (\lambda_{fo}^H)^3} \frac{E_{f1}}{E_{f2}} W^H \tag{114}$$

$$\left(\frac{\delta_{f2}}{\delta_{f1}}\right)^V = \frac{\lambda_a \lambda_V^2}{\lambda_{ft}^V (\lambda_{fo}^V)^3} \frac{E_{f1}}{E_{f2}} W^V \tag{115}$$

$$\left(\frac{\omega_{b2}}{\omega_{b1}}\right)^H = \left[\lambda_a \left(\frac{\delta_{b1}}{\delta_{b2}}\right)^H\right]^{1/2} \tag{116}$$

$$\left(\frac{\omega_{b2}}{\omega_{b1}}\right)^V = \left[\lambda_a \left(\frac{\delta_{b1}}{\delta_{b2}}\right)^V\right]^{1/2} \tag{117}$$

$$\left(\frac{\omega_{f2}}{\omega_{f1}}\right)^H = \left[\lambda_a \left(\frac{\delta_{f1}}{\delta_{f2}}\right)^H\right]^{1/2} \tag{118}$$

and

$$\left(\frac{\omega_{f2}}{\omega_{f1}}\right)^V = \left[\lambda_a \left(\frac{\delta_{f1}}{\delta_{f2}}\right)^V\right]^{1/2} \tag{119}$$

Note that there are four pseudofrequencies for the stress limited design.

3. ***Frequency-limited design—condition C.*** It is assumed that the frequency ratios for all four pseudofrequencies will be the same, i.e.

$$\left(\frac{\omega_{b2}}{\omega_{b1}}\right)^H = \left(\frac{\omega_{b2}}{\omega_{b1}}\right)^V = \left(\frac{\omega_{f2}}{\omega_{f1}}\right)^H = \left(\frac{\omega_{f2}}{\omega_{f1}}\right)^V = K_3$$

The two equations analogous to Eq. (45) are,

$$W^H - K_3^{2/3} \lambda_a^{2/9} \lambda_H^{17/9} \left[a^H \rho_b \lambda_H^{1/3} \left(\frac{E_{b1}}{E_{b2}}\right)^{5/9} + b^H \rho_f \left(\frac{E_{f1}}{E_{f2}}\right)^{5/9}\right] (W^H)^{5/9} - c^H s A^H = 0 \tag{120}$$

and

$$W^V - K_3^{2/3} \lambda_a^{2/9} \lambda_V^{17/9} \left[a^V \rho_b \lambda_V^{1/3} \left(\frac{E_{b1}}{E_{b2}}\right)^{5/9} + b^V \rho_f \left(\frac{E_{f1}}{E_{f2}}\right)^{5/9}\right] (W^V)^{5/9} - c^V s A^V = 0 \tag{121}$$

The other factors can now be determined from

$$\lambda_{bt}^H = \left(\lambda_a \lambda_H W^H \frac{E_{b1}}{E_{b2}}\right)^{1/3} \tag{122}$$

$$\lambda_{bt}^V = \left(\lambda_a \lambda_V W^V \frac{E_{b1}}{E_{b2}}\right)^{1/3} \tag{123}$$

$$\lambda_{bo}^H = \left(\frac{K_3^2 \lambda_H^3 W^H}{\lambda_{bt}^H} \frac{E_{b1}}{E_{b2}}\right)^{1/3} \tag{124}$$

$$\lambda_{bo}^V = \frac{\lambda_V}{\lambda_H} \left(\frac{\lambda_{bt}^H}{\lambda_{bt}^V} \frac{W^V}{W^H}\right)^{1/3} \lambda_{bo}^H \tag{125}$$

$$\lambda_{ft}^H = \left(\lambda_a \lambda^H W^H \frac{E_{f1}}{E_{f2}}\right)^{1/3} \tag{126}$$

$$\lambda_{ft}^V = \left(\lambda_a \lambda^V W^V \frac{E_{f1}}{E_{f2}}\right)^{1/3} \tag{127}$$

$$\lambda_{fo}^H = \left(\frac{K_3^2 \lambda_H^2 W^H}{\lambda_{ft}^H} \frac{E_{f1}}{E_{f2}}\right)^{1/3} \tag{128}$$

$$\lambda_{fo}^V = \left(\frac{K_3^2 \lambda_V^2 W^V}{\lambda_{ft}^V} \frac{E_{f1}}{E_{f2}}\right)^{1/3} \tag{129}$$

$$\left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^H = \frac{\lambda_a \lambda_H W^H}{(\lambda_{bo}^H)^2 \lambda_{bt}^H} \tag{130}$$

$$\left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^V = \frac{\lambda_a \lambda_V W^V}{(\lambda_{bo}^V)^2 \lambda_{bt}^V} \tag{131}$$

$$\left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^H = \frac{\lambda_a \lambda_H W^H}{(\lambda_{fo}^H)^2 \lambda_{ft}^H} \tag{132}$$

$$\left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^V = \frac{\lambda_a \lambda_V W^V}{(\lambda_{fo}^V)^2 \lambda_{ft}^V} \tag{133}$$

$$\left(\frac{\sigma_{cb2}}{\sigma_{cb1}}\right)^H = \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^H \tag{134}$$

$$\left(\frac{\sigma_{cb2}}{\sigma_{cb1}}\right)^V = \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^V \tag{135}$$

$$\left(\frac{\sigma_{cf2}}{\sigma_{cf1}}\right)^H = \left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^H \tag{136}$$

$$\left(\frac{\sigma_{cf2}}{\sigma_{cf1}}\right)^V = \left(\frac{\sigma_{f2}}{\sigma_{f1}}\right)^V \tag{137}$$

and

$$\left(\frac{\delta_{b2}}{\delta_{b1}}\right)^H = \left(\frac{\delta_{b2}}{\delta_{b1}}\right)^V = \left(\frac{\delta_{f2}}{\delta_{f1}}\right)^H = \left(\frac{\delta_{f2}}{\delta_{f1}}\right)^V = \frac{\lambda_a}{K_3^2} \tag{138}$$

In most prospective applications of large area solar arrays, weight is of critical concern. The power-to-weight ratio P_R then, is one of the most significant parameters. The power-to-weight ratio P_{R2} for array 2 can be written

$$P_{R2} = \frac{P_{A2}}{P_{A1}} \frac{A_2}{A_1} \frac{P_{R1}}{W}$$

$$= \frac{P_{A2}}{P_{A1}} \frac{\lambda_H \lambda_V}{W} P_{R1} \tag{139}$$

where

P_{A2} = power per unit area for array 2

P_{A1} = power per unit area for array 1

and

P_{R1} = power-to-weight ratio for the reference array

If the same type solar cells are used on both arrays, the ratio P_{A2}/P_{A1} becomes unity, and

$$P_{R2} = \frac{\lambda_H \lambda_V}{W} P_{R1} \tag{140}$$

V. Parametric Data

The pertinent equations for each of the three design conditions were programmed for the IBM 1620. The program is described in detail in Appendix A. Dimensionless parameters were established for each of the design conditions. These parameters differ for each condition since the applicable equations differ in their form.

The program calculates all pertinent parameters which are developed in the analysis. The more important of these, the power-to-weight ratio P_{R2}, the stress parameter S, the beam scale parameter B, and the frequency parameter F, are plotted as functions of the material and load parameter H for each of the three design conditions. Material property values for the independent parameter H were chosen to include most currently available candidate materials. The parametric data is presented for the condition of identical materials used for both beams and fittings.

For purposes of this study, for the deflection limited case, the deflection ratio δ was assumed to be unity. For the stress limited design, the allowable stress ratios K_1 and K_2 were each assumed to be unity. For the frequency limited design, the frequency ratio K_3 was assumed to be unity. or all design conditions the ratio s of nonstructural weights per unit area is unity.

The data is presented for a range of both the lateral scale factor λ_H and the longitudinal scale factor λ_V from 0.4 to 1.6 in increments of 0.2. The data is arranged such that for each design condition there is one set of plots for each lateral scale factor λ_H. The plots for the deflection limited, stress limited, and frequency limited conditions are presented, in that order, in Appendix B.

The power-to-weight ratio, identical for all design conditions, is given by

$$P_{R2} = \frac{\lambda_H \lambda_V P_{R1}}{W}$$

For each of the three conditions, the pertinent parameters are defined in the following subsections.

A. Deflection-Limited Design — Condition A

The material and load parameters are defined as

$$H = \left(\frac{E_{b2}}{E_{b1}}\right)^{5/9} \lambda_a^{-5/9} \frac{\rho_{b1}}{\rho_{b2}}$$

The stress parameter is defined as

$$S = \left(\frac{E_{b1}}{E_{b2}}\right)^{7/9} \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^j$$

Where the superscript j is either H or V such as to give the larger value of S.

The beam scale parameter is defined as

$$B = \left(\frac{E_{b2}}{E_{b1}}\right)^{2/9} \lambda_{bo}^j$$

where the superscript j is either H or V such as to give the larger value of B.

B. Stress-Limited Design — Condition B

The material and load parameter is defined as,

$$H = \left(\frac{E_{b2}}{E_{b1}}\right)^{1/6} \left(\frac{\sigma_{yb2}}{\sigma_{yb1}}\right)^{1/2} \lambda_a^{-2/3} \frac{\rho_{b1}}{\rho_{b2}}$$

The deflection parameter is defined as

$$D = \left(\frac{E_{b2}}{E_{b1}}\right)^{7/6} \left(\frac{\sigma_{yb1}}{\sigma_{yb2}}\right)^{3/2} \left(\frac{\delta_{b2}}{\delta_{b1}}\right)^j$$

where j is either H or V such as to give the larger absolute deflection in the new design.

The beam scale parameter is defined as

$$B = \left(\frac{E_{b1}}{E_{b2}}\right)^{1/6} \left(\frac{\sigma_{yb2}}{\sigma_{yb1}}\right)^{1/2} \lambda_{bo}^{j}$$

where j is either H or V such as to give the larger value of B.

The frequency ratio is defined as

$$F = \lambda_a^{1/2} \left[\left(\frac{\delta_{b1}}{\delta_{b2}}\right)^{j}\right]^{1/2}$$

where j is either H or V such as to give the lower absolute frequency of the new design.

C. Frequency-Limited Design — Condition C

The material and load parameter is defined as

$$H = \left(\frac{E_{b2}}{E_{b1}}\right)^{5/9} \lambda_a^{-2/9} \frac{\rho_{b1}}{\rho_{b2}}$$

The stress parameter is defined as

$$S = \left(\frac{E_{b1}}{E_{b2}}\right)^{7/9} \left(\frac{\sigma_{b2}}{\sigma_{b1}}\right)^{j}$$

where j is either H or V such as to give the larger value of S.

The beam scale parameter is given as

$$B = \left(\frac{E_{b2}}{E_{b1}}\right)^{2/9} \lambda_{bo}^{j}$$

where j is either H or V such as to give the larger value of B.

VI. Conclusion

The parametric study computer program can be used in two fundamental modes. The printout mode can be used to provide performance and scaling parameters for a specific configuration (or a number of specific configurations) which are considered as candidates for a particular mission application. This mode might be used, for instance, when the assumptions made to generate the plots (Appendix B) are seriously violated, or when a drastically different design is to be used as the reference design.

The plotting mode, which was used to generate the parametric curves, is particularly useful in providing quick-look information concerning the characteristics of a contemplated new design. If the power-to-weight ratio, vertical scale factor λ_V, and the lateral scale factor λ_H are specified, the appropriate P_{R2} vs H curve will indicate the minimum value of the material and load parameter H which can be used. Conversely, if the material and load factors are specified, this curve will show the resulting power-to-weight ratio which one can expect of the new design.

As indicated in Appendix B, the parameters H, S, B, D, and F appearing in the plots are functions of material properties (elastic modulus, yield stress, and mass density) and the load factor λ_a. In order to facilitate use of the plotted results, these parameters have been evaluated for a large number (49) of materials which may be considered for array structures, for a unit value of the load factor λ_a, and unit values of the pertinent ratios (stress ratio, beam scale factor, deflection ratio, or frequency ratio). These "standard values" are presented in Table 1.

Use of the plotted parametric study results is illustrated in Fig. 1, in which abscissas corresponding to array structures entirely composed of titanium 6AL-4V, 7075 aluminum, boron-epoxy composite, and beryllium, are indicated. Figure 1, which is for the deflection limited case and for a lateral scale factor of 1.00, illustrates the potential large weight advantage of the beryllium structure over, say, aluminum. The all beryllium structure provides a power-to-weight ratio of 24.3 W/lb compared to 8.6 W/lb for the aluminum structure (all for a vertical scale factor of 1.00). Figure 2, for the stress limited case, shows a much smaller weight advantage of beryllium over aluminum. Also of interest in Fig. 2 is the significant weight advantage of boron-epoxy over beryllium, for this case.

When one considers the use of alternative structural materials, the implications of the beam section overall scale factor, stress ratio and frequency ratio should also be investigated. Figure 3, for instance, indicates that if the beryllium and titanium were replaced by 7075 aluminum alloy in the Boeing LASA design, a beam scale parameter B of 1.26 would be required to maintain the same maximum deflection. This parameter translates into a beam depth physical scale factor (see Table 1) of

Table 1. Standard parameter values for typical structural materials

Material	E, lb/in.^{2}	ρ, lb/in.^{3}	σ_y, lb/in.^{2}	Deflection-limited			Stress-limited				Frequency-limited		
				H	B	S	$\dot{H}$	D	B	F	H	S	B
PH15-7MO STEEL	29000000.	.2770	200000.	.191	.915	1.360	.470	.067	2.252	3.856	.191	1.360	.915
17-7 PH STEEL	29000000.	.2760	170000.	.191	.915	1.360	.435	.085	2.076	3.414	.191	1.360	.915
17-4 PH STEEL	28500000.	.2800	180000.	.187	.912	1.379	.440	.077	2.142	3.600	.187	1.379	.912
300 MARAGING STEEL	26500000.	.2900	290000.	.173	.897	1.459	.532	.034	2.752	5.371	.173	1.459	.897
250 MARAGING STEEL	26500000.	.2900	255000.	.173	.897	1.459	.499	.042	2.581	4.877	.173	1.459	.897
9NI-4CO-.45C STEEL	28900000.	.2830	390000.	.186	.915	1.364	.642	.024	3.146	6.377	.186	1.364	.915
9NI-4CO-.45C STEEL	28900000.	.2830	260000.	.186	.915	1.364	.524	.045	2.569	4.705	.186	1.364	.915
9NI-4CO-.45C STEEL	28900000.	.2830	220000.	.186	.915	1.364	.482	.058	2.363	4.151	.186	1.364	.915
AFC-77 STAINLESS	29000000.	.2800	182000.	.189	.915	1.360	.443	.077	2.148	3.593	.189	1.360	.915
D-6AC STEEL	30000000.	.2830	260000.	.190	.922	1.325	.527	.047	2.553	4.603	.190	1.325	.922
D-6AC STEEL	30000000.	.2830	220000.	.190	.922	1.325	.485	.060	2.348	4.061	.190	1.325	.922
4340 STEEL	30000000.	.2830	270000.	.190	.922	1.325	.537	.044	2.601	4.735	.190	1.325	.922
4340 STEEL	30000000.	.2830	220000.	.190	.922	1.325	.485	.060	2.348	4.061	.190	1.325	.922
4340 STEEL	30000000.	.2830	200000.	.190	.922	1.325	.462	.069	2.239	3.781	.190	1.325	.922
.40 BORON - .6 ALUM	27200000.	.0940	80000.	.543	.902	1.430	.867	.246	1.439	2.013	.543	1.430	.902
.60 BORON - .4 EPOX	35000000.	.0600	180000.	.979	.954	1.175	2.124	.098	2.070	3.193	.979	1.175	.954
.68 GLASS - EPOXY	7000000.	.0700	165000.	.343	.667	4.111	1.333	.017	2.592	7.650	.343	4.111	.667
.65 SIC - EPOXY	38000000.	.0880	310000.	.699	.972	1.102	1.927	.047	2.680	4.576	.699	1.102	.972
.61 CARBON - EPOXY	36000000.	.0660	200000.	.904	.960	1.150	2.045	.086	2.172	3.399	.904	1.150	.960
.63 BERYLL - EPOXY	26900000.	.0580	78200.	.875	.900	1.442	1.386	.251	1.425	1.992	.875	1.442	.900
.44 AL203 - EPOXY	24000000.	.0630	70000.	.756	.878	1.576	1.185	.260	1.375	1.959	.756	1.576	.878
.65 GLASS - PI	32300000.	.0630	57000.	.892	.937	1.251	1.123	.501	1.180	1.412	.892	1.251	.937
2014-T6 ALUMINUM	10600000.	.1010	59000.	.299	.732	2.977	.592	.129	1.446	2.777	.299	2.977	.732
2024-T81ALUMINUM	10600000.	.1000	65000.	.302	.732	2.977	.627	.112	1.518	2.986	.302	2.977	.732
2219-T81 ALUMINUM	10600000.	.1020	50000.	.296	.732	2.977	.539	.166	1.331	2.452	.296	2.977	.732
6061-T6 ALUMINUM	10000000.	.0980	40000.	.299	.722	3.115	.497	.217	1.202	2.146	.299	3.115	.722
7075-T6 ALUMINUM	10400000.	.1010	73000.	.296	.729	3.021	.656	.092	1.614	3.294	.296	3.021	.729
7079-T6 ALUMINUM	10400000.	.0990	68000.	.302	.729	3.021	.646	.102	1.557	3.123	.302	3.021	.729
7078-T6 ALUMINUM	10400000.	.1020	78000.	.293	.729	3.021	.672	.083	1.668	3.462	.293	3.021	.729
2024-T351 ALUMINUM	10600000.	.1000	47000.	.302	.732	2.977	.533	.182	1.291	2.341	.302	2.977	.732
7075-T7351 ALUMINUM	10400000.	.1010	63000.	.296	.729	3.021	.610	.114	1.499	2.949	.296	3.021	.729
7039-T6351 ALUMINUM	10400000.	.0990	56000.	.302	.729	3.021	.586	.137	1.413	2.700	.302	3.021	.729
AZ313-H24 MAGNESIUM	6500000.	.0640	32000.	.360	.656	4.355	.634	.183	1.155	2.334	.360	4.355	.656
AZ61A	6500000.	.0650	33000.	.354	.656	4.355	.634	.175	1.173	2.389	.354	4.355	.656
AZ80A-T5 MAGNESIUM	6500000.	.0650	40000.	.354	.656	4.355	.698	.131	1.292	2.759	.354	4.355	.656
ZK60A-T5 MAGNESIUM	6500000.	.0660	44000.	.349	.656	4.355	.721	.113	1.355	2.964	.349	4.355	.656
LA141A-T7 MAGNESIUM	6200000.	.0490	15000.	.458	.649	4.518	.562	.541	.797	1.359	.458	4.518	.649
62BE-38AL LOCKALLOY	29200000.	.0760	37000.	.699	.917	1.353	.737	.851	.967	1.083	.699	1.353	.917
CR INGT SHT BERYLL	42000000.	.0660	40000.	.985	.994	1.020	.938	1.157	.946	.929	.985	1.020	.994
CR PWDR SHT BERYLL	43100000.	.0660	45000.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
TI-6AL-4V	16500000.	.1600	160000.	.241	.807	2.110	.662	.048	2.212	4.533	.241	2.110	.807
TI-6AL-4V	16500000.	.1600	150000.	.241	.807	2.110	.641	.053	2.142	4.319	.241	2.110	.807
TI-6AL-4V	16500000.	.1600	140000.	.241	.807	2.110	.619	.059	2.069	4.101	.241	2.110	.807
TI-6AL-4V	16500000.	.1600	130000.	.241	.807	2.110	.597	.066	1.994	3.879	.241	2.110	.807
TI-BV-11CR-3AL	15000000.	.1760	170000.	.208	.790	2.272	.611	.039	2.317	5.015	.208	2.272	.790
TI-5AL-2.5SN(ELI)	16000000.	.1610	110000.	.236	.802	2.161	.543	.082	1.844	3.484	.236	2.161	.802
TI-6AL-4V BSTA-1000	16500000.	.1600	148000.	.241	.807	2.110	.637	.054	2.128	4.275	.241	2.110	.807
TI-6AL-4V BSTA-1250	16500000.	.1600	140000.	.241	.807	2.110	.619	.059	2.069	4.101	.241	2.110	.807
INCONEL 718	29000000.	.2960	197000.	.178	.915	1.360	.436	.068	2.235	3.813	.178	1.360	.915

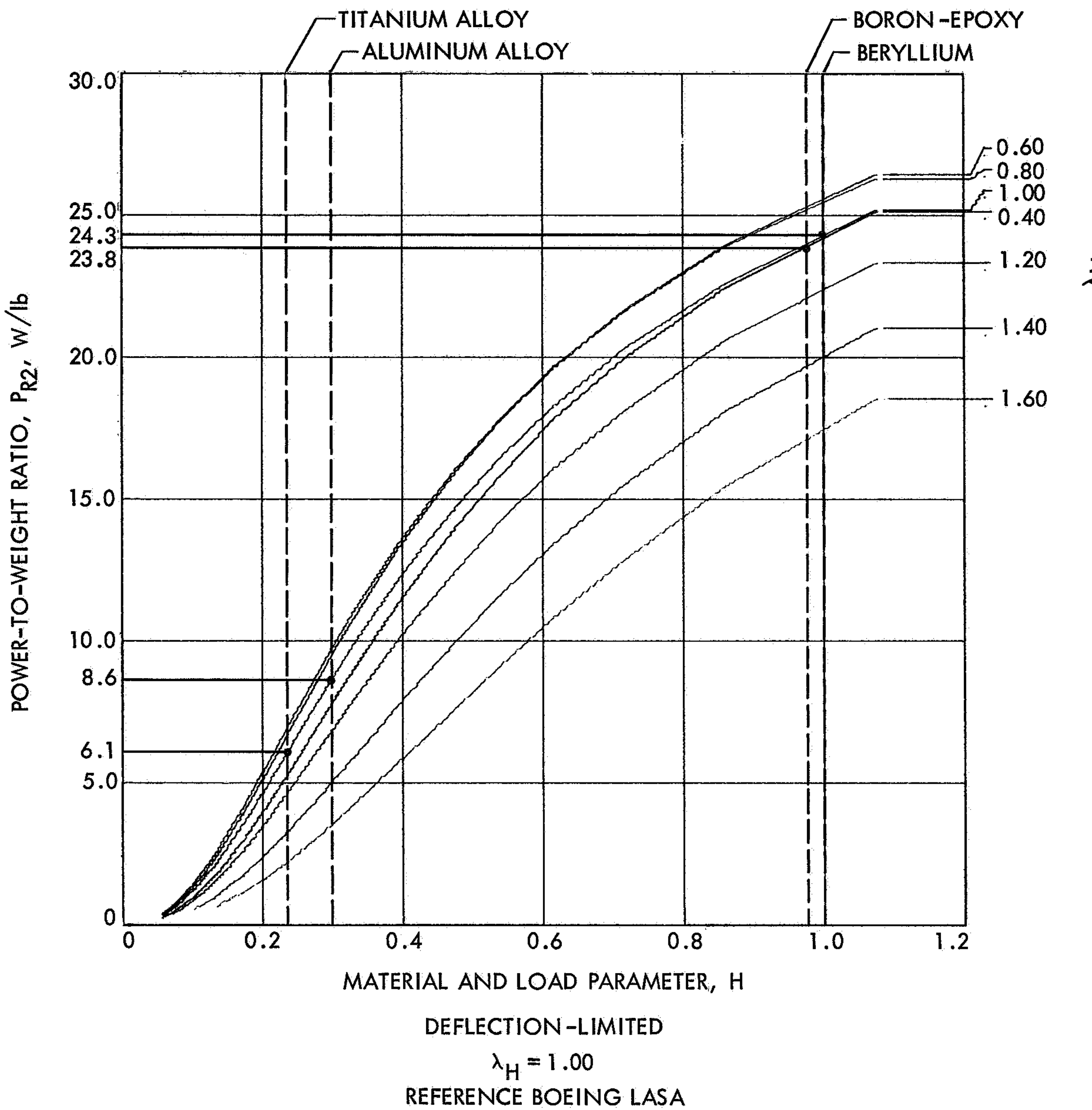
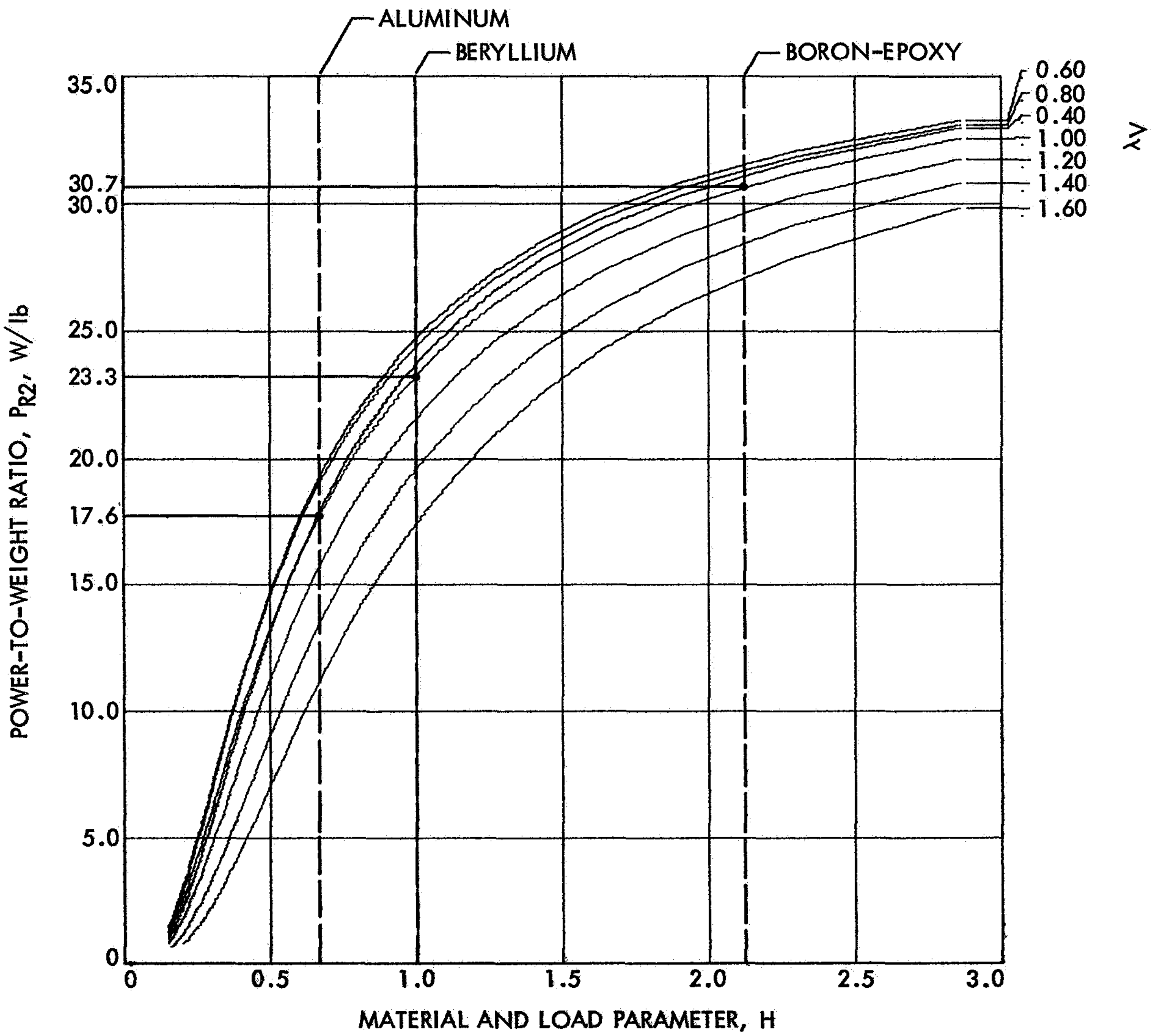


Fig. 1. Typical variation of power-to-weight ratio—deflection-limited case

1.26/.729 = 1.73; that is, the aluminum beams would have to be 73% deeper than the presently designed beryllium beams. Thus, the aluminum design would result in a much more voluminous stowed configuration.

The catalogue of parametric plots, together with the computer program developed in this study, provide a ready means of evaluating the performance and stowed volume efficiencies of contemplated new designs based on the Boeing LASA design. The computer program itself is sufficiently general that it can readily be applied to other array design concepts to determine the implications of overall geometric scaling, aspect ratio scaling, inertial loads scaling, and changes in structural materials.



STRESS-LIMITED

$\lambda_H = 1.00$

REFERENCE BOEING LASA

Fig. 2. Typical variation of power-to-weight ratio—stress-limited case

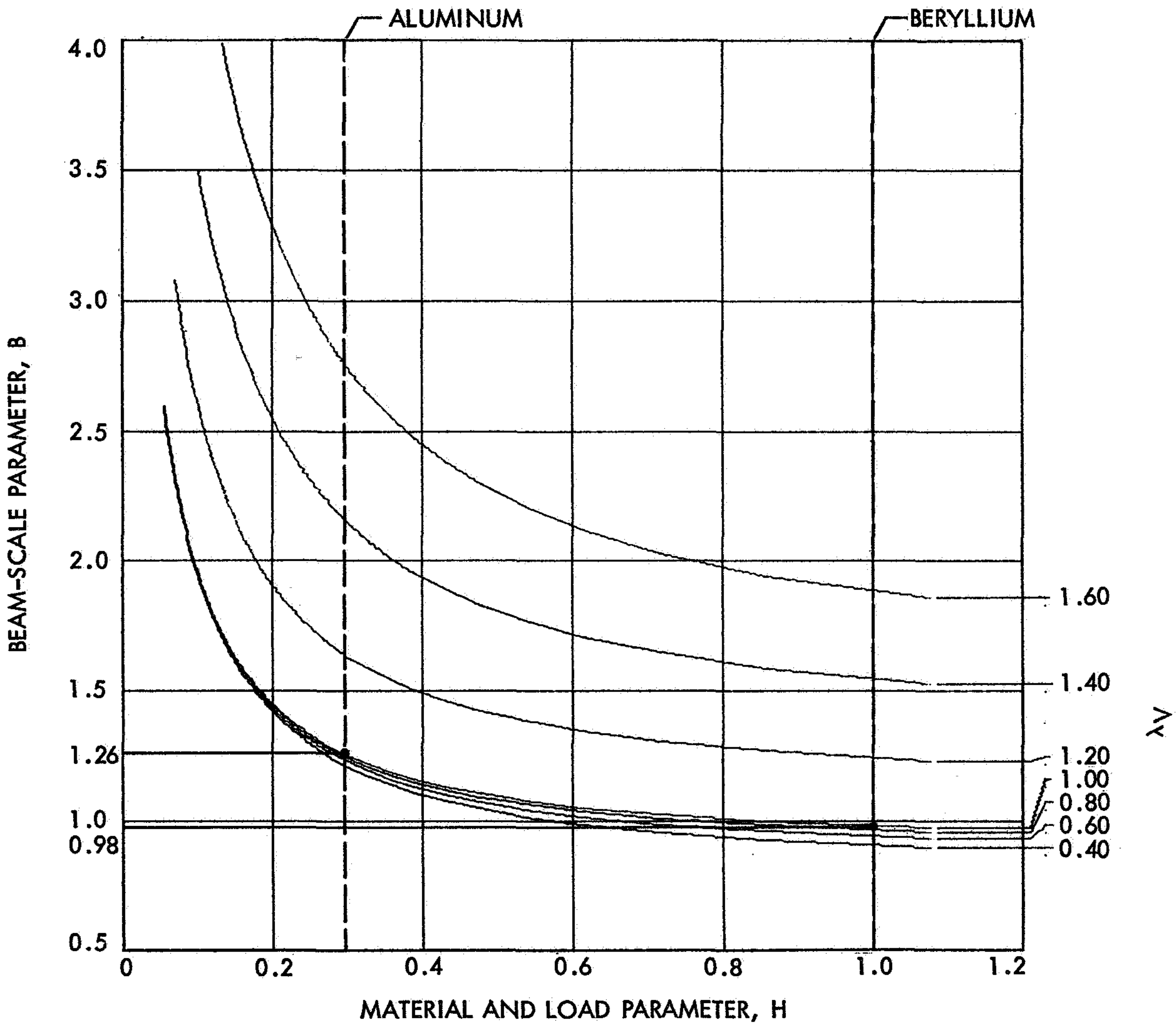


Fig. 3. Typical variation of beam-scale ratio—deflection-limited case

Appendix A
Computer Program

I. Description

The program is written in FORTRAN II-D, and structured to operate on the IBM 1620, Model 2 computer, equipped with 1311 disk drive, 1443 printer, 1627 plotter, 1622 card reader/punch, and 40,000 character (numeric) core storage capacity.

The program consists of three linked mainline programs and 13 subroutines plus library subroutines. The mainline programs perform, for the most part, as control functions only. Virtually all computations are done in subroutines. This technique greatly simplifies program modification, expansion, or adaptation to other computers.

Options are included in the program to allow maximum user control over mode of operation and parameter selection. For example, studies may be made using discrete (real) materials, or may be based on generalized materials whose structural properties may be varied as desired.

The program has been written using the equations derived in the main body of the report.

The critical stresses of the two designs are allowed to differ by a constant factor, to increase the versatility of the program. Thus Eqs. (15) and (16) of Section IV are programmed as

$$\frac{\sigma_{cb2}}{\sigma_{cb1}} = K \frac{\sigma_{b2}}{\sigma_{b1}}$$

and

$$\frac{\sigma_{cf2}}{\sigma_{cf1}} = K \frac{\sigma_{f2}}{\sigma_{f1}}$$

where the constant K is input data. For the purpose of this parametric study, K was assumed to be unity.

A brief description of each routine follows:

Name	Description
Study 1	Initialization link. Reads basic operating constants, establishes storage requirements common to entire program, and initializes plotter. Calls subroutines Read 1, Ex, Plot Calls link Study 2
Study 2	Computation link. Increments variables, calls input, output, and computational subroutines. Calls subroutines Header, Out 1, Read 1 Solve 1, Case A1, Case B1, Case C1 Calls link Study 3
Study 3	Plotting link. Calls scaling, labelling and plotting subroutines. Calls subroutines, Scale 1, Label 1, Label 2, Label 3, Plot Calls link Study 2
Read 1	Input subroutine. Reads and initializes variables.
Header	Heading subroutine. Prints appropriate titling, listing of input parameters, and output matrix form, dependent on mode and case under consideration.
Out 1	Output subroutine. Prints results, stores plotting parameters on disk, and determines maxima and minima for plotting purposes.
Ex	Generates fractional exponents for use in computations. (Allows refinement of exponents throughout the program by changing this routine only.)
Label 1	Labelling subroutines.
Label 2	Label plots with appropriate scales, titles, and data.
Label 3	Calls subroutines Plot, Char
Solve 1	Computes basic parameters used in solution of exponential equation, calls subroutine to solve equation, and tests suitability of results. (This subroutine contains the coded equivalent of Eq. (74) derived specifically for use with the Boeing LASA as a reference. Considering one panel of the total array, the reference consists of four panels of dimension 97.7 by 159.4 in., six panels 97.7 by 136.7 in., and three panels

Name	Description
	97.7 by 113.2 in. Thus,

$$r_1 = 0.613,\ m = 4,\ n = 3,\ r = 6$$

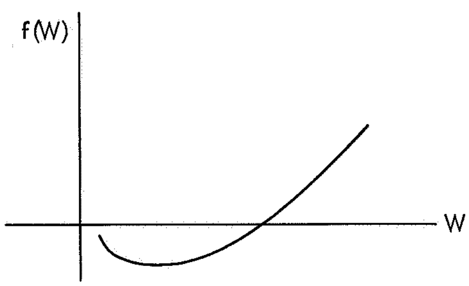
$$K_n = 0.710,\ K_r = 0.858$$

for which Eq. (74) simplifies to

$$\left.\begin{aligned} A^H &= \lambda_H^2\,\lambda_V\left(\frac{0.527}{\lambda_V + 0.613\,\lambda_H} + \frac{0.281}{0.710\,\lambda_V + 0.613\,\lambda_H} + \frac{0.678}{0.858\,\lambda_V + 0.613\,\lambda_H}\right) \\ A^V &= \lambda_H\lambda_V^2\left(\frac{0.604}{\lambda_V + 0.613\,\lambda_H} + \frac{0.228}{0.710\,\lambda_V + 0.613\,\lambda_H} + \frac{0.667}{0.858\,\lambda_V + 0.613\,\lambda_H}\right) \end{aligned}\right\} \quad (141)$$

Eq. (141) is coded in the program. Appropriate changes must be made to these equations if a different reference design is considered.)

Solve 2 — Subroutine to solve exponential equation of the form of Eqs. (19), (32), and (45) for W by the Newton-Raphson method. These equations have the general characteristics indicated below for the data considered.



An initial estimate of 20 for W yields acceptable solutions for most materials under consideration with a reasonably small number of iterations needed. Iteration is terminated when

$$\left|\frac{W_{n+1} - W_n}{W_{n+1}}\right| \leq 10^{-5}$$

Unacceptable conditions result when the computed value of W exceeds the range of practical interest.

Case A1 — Computes Condition A (Deflection-limited design) parameters

Case B1 — Computes Condition B (Stress-limited design) parameters.

Case C1 — Computes Condition C (Frequency-limited design) parameters

II. Input Format

Input to the program consists of constants followed by data pertinent to the mode selected. Data cards 1–5 are common to both modes and are formatted as follows:

Card 1 — Mode Selection

Format—I1 (Single-digit, fixed-point field)

Allowable Data—0 (zero) or 1 corresponding to mode desired:

Mode 0—based on generalized materials and iterated by incrementing various material and dimensional parameters. Results are printed and plotted.

Mode 1—Based on discrete material combinations. Results are printed only.

Note: (Card columns 2-80 are not used by the program and may be used to identify the data set or to supply other pertinent information.)

Card 2 — Reference Design Identifier

Format—20A2 (40 alphameric characters)

Allowable Data—Up to 40 alphameric characters in card columns 1 through 40, left-justified. Special characters (/, =, −, +, *, (,), $, @, Period, Comma) are permitted.

Cards 3, 4, and 5 — Basic Constants

Format—8E10.0 (8 10-digit exponential fields per card)

Allowable Data—Numeric data including decimal point. If exponents are specified, they must be right-justified in the field. Exponents may be in any of the FORTRAN II Forms, i.e., $E \pm 07$, $E \pm 7$, $E7$, ± 7.

Field Assignment

Card 3—a^H, b^H, c^H, a^V, b^V, c^V, A, r_1

Card 4—P_{A1}, E_{b1}, σ_{yb1}, ρ_{b1}, E_{f1}, σ_{yf1}, ρ_{f1}, δ

Card 5—K, K_3, s, K_1, K_2, λ_a

Mode 0 requires 1 additional data card. Mode 1 requires 3 additional cards for each material combination.

Mode 0, Card 6—Iteration Parameters

Format—4 I1, 6X, 2E10.0, 9E5.0

Field Assignment (CC is used to indicate card columns)

CC 1-4—Case Selection, a single digit code identifies the limiting parameter:

Case	Limiting Parameter
1	Deflection-limited
2	Stress-limited
3	Frequency-limited

From one to three case numbers may be specified, beginning in CC 1, in any sequence as well as individually or paired (e.g. 123, 312, 2 (only), 21, 31, etc.) Two control digits are available for flexibility:

0—(zero or blank)—Detection of a zero or blank will cause execution to terminate at the conclusion of processing of the previous non-zero case. No re-start is allowed.

4—Detection of a 4 in CC 2, 3, or 4 will allow a new set of iteration parameters (card 6 only) to be read in and processed.

(Note: Detection of a blank (zero) in CC 1 will cause execution to terminate immediately. A digit other than blank (zero) or 4 in CC 4 will be treated as a blank and execution will be terminated.)

CC 11-20—E (Young's Modulus of Beam Material)

CC 21-30—σ_b (Allowable Stress for Beam Material)

CC 31-35—$\lambda_{H_{min}}$ (Lower Limit of λ_H, Geometric Scale Factor—See Eq. (66))

CC 36-40—$\lambda_{H_{max}}$ (Upper Limit of λ_H, Geometric Scale Factor)

CC 41-45—$\Delta\lambda_H$ (Increment of λ_H)

CC 46-50—$\lambda_{V_{min}}$ (Lower Limit of λ_V, Geometric Scale Factor—See Eq. (66))

CC 51-55—$\lambda_{V_{max}}$ (Upper Limit of λ_V, Geometric Scale Factor)

CC 56-60—$\Delta\lambda_V$ (Increment of λ_V)

CC 61-65—$\rho_{b_{min}}$ (Lower Limit of ρ_b)

CC 66-70—$\rho_{b_{max}}$ (Upper Limit of ρ_b)

CC 71-75—$\Delta\rho_b$ (Increment of ρ_b)

Note: The parameter is initially set to the minimum value and increased by the increment until it exceeds the maximum. The minimum value must be specified. If the maximum value and increment are not supplied, the program will execute the computational sequence for the minimum value and then transfer and increment the next parameter or case selector, as appropriate. The case selector, λ_H, λ_V, and ρ_b operations are "nested" such that ρ_b cycles most rapidly and the case selection least rapidly.)

Mode 1, Card 6*—Beam Material Parameters

Mode 1, Card 7*—Fitting Material Parameters

(*These data cards have identical formats)

Format 20A2, 3 E10.0

CC 1-40 —Material description. Up to 40 alphameric characters. Special characters (see card 2) are permissible.

CC 41-50—E_b or E_f (Young's modulus for beam or fitting material as appropriate, psi)

CC 51-60—σ_b or σ_f (Allowable stress, psi)

CC 61-70—ρ_b or ρ_f (Density, lb/in.^{3})

Mode 1, Card 8—Geometric Scale Factors

Format 2E10.0

Field Assignment

CC 1-10 —λ_H
CC 11-20—λ_V See Equations (66)

The program will read and process the data sequentially through the deflection limited, stress limited, and frequency limited cases. Data sets (cards 6, 7, and 8) will continue to be read-in and processed as long as supplied. Execution may be terminated by causing a monitor control record (e.g., "End-of-Job" card) to be read in place of data cards, or by standard console procedures.

Appendix B
Numerical Values

The numerical values used in this parametric study were obtained from the results of the Boeing LASA program.

The Boeing LASA configuration employs beryllium beams and titanium fittings for its main structural elements. The substrate is composed of stretched fiberglass ribbon.

The pertinent information is contained in Boeing Report D2-113355-4, of October 1967: *Large Area Solar Array*. The data follows:

Total weight	$W_1 = 2101$ lb
Horizontal beams	$W^H_{b1} = 315$ lb
Vertical beams	$W^V_{b1} = 361$ lb
Horizontal fittings	$W^H_{f1} = 69$ lb
Vertical fittings	$W^V_{f1} = 168$ lb
Non-structural weight	$W^H_{n1} + W^V_{n1} = 1188$ lb

From Eq. (72), with

$$m = 4 \qquad r_1 = 0.613$$

$$n = 3 \qquad K_n = 0.710$$

$$r = 6 \qquad K_r = 0.858$$

$$A^H_1 = 4.653\ A_1$$

$$A^V_1 = 6.626\ A_1$$

$$\frac{A^H_1}{A^V_1} = 0.702$$

then

$$\frac{W^H_{n1}}{W^V_{n1}} = \frac{A^H_1}{A^V_1} = 0.702$$

$$W^H_{n1} = 490 \text{ lb}$$

$$W^V_{n1} = 698 \text{ lb}$$

Using Eq. (56)

$$W^H_1 = (315 + 69 + 490) \text{ lb} = 874 \text{ lb}$$

$$W^V_1 = (361 + 168 + 698) \text{ lb} = 1227 \text{ lb}$$

then

$$A = \frac{W^V_1}{W^H_1} = \frac{1227}{874} = 1.404$$

and

$$a^H = \frac{W^H_{b1}}{W^H_1} = \frac{315}{874} = 0.360$$

$$b^H = \frac{W^H_{f1}}{W^H_1} = \frac{69}{874} = 0.079$$

$$c^H = \frac{W^H_{n1}}{W^H_1} = \frac{490}{874} = 0.561$$

$$a^V = \frac{W^V_{b1}}{W^V_1} = \frac{361}{1227} = 0.294$$

$$b^V = \frac{W^V_{f1}}{W^V_1} = \frac{168}{1227} = 0.137$$

$$c^V = \frac{W^V_{n1}}{W^V_1} = \frac{698}{1227} = 0.569$$

The following material properties were used for the Boeing LASA design

$$E_{b1} = 4.31 \times 10^7 \text{ psi}$$

$$\sigma_{yb1} = 5.50 \times 10^4 \text{ psi}$$

$$\rho_{b1} = 0.066 \text{ lb/in.}^3$$

$$E_{f1} = 1.65 \times 10^7 \text{ psi}$$

$$\sigma_{yf1} = 1.30 \times 10^5 \text{ psi}$$

$$\rho_{f1} = 0.160 \text{ lb/in.}^3$$

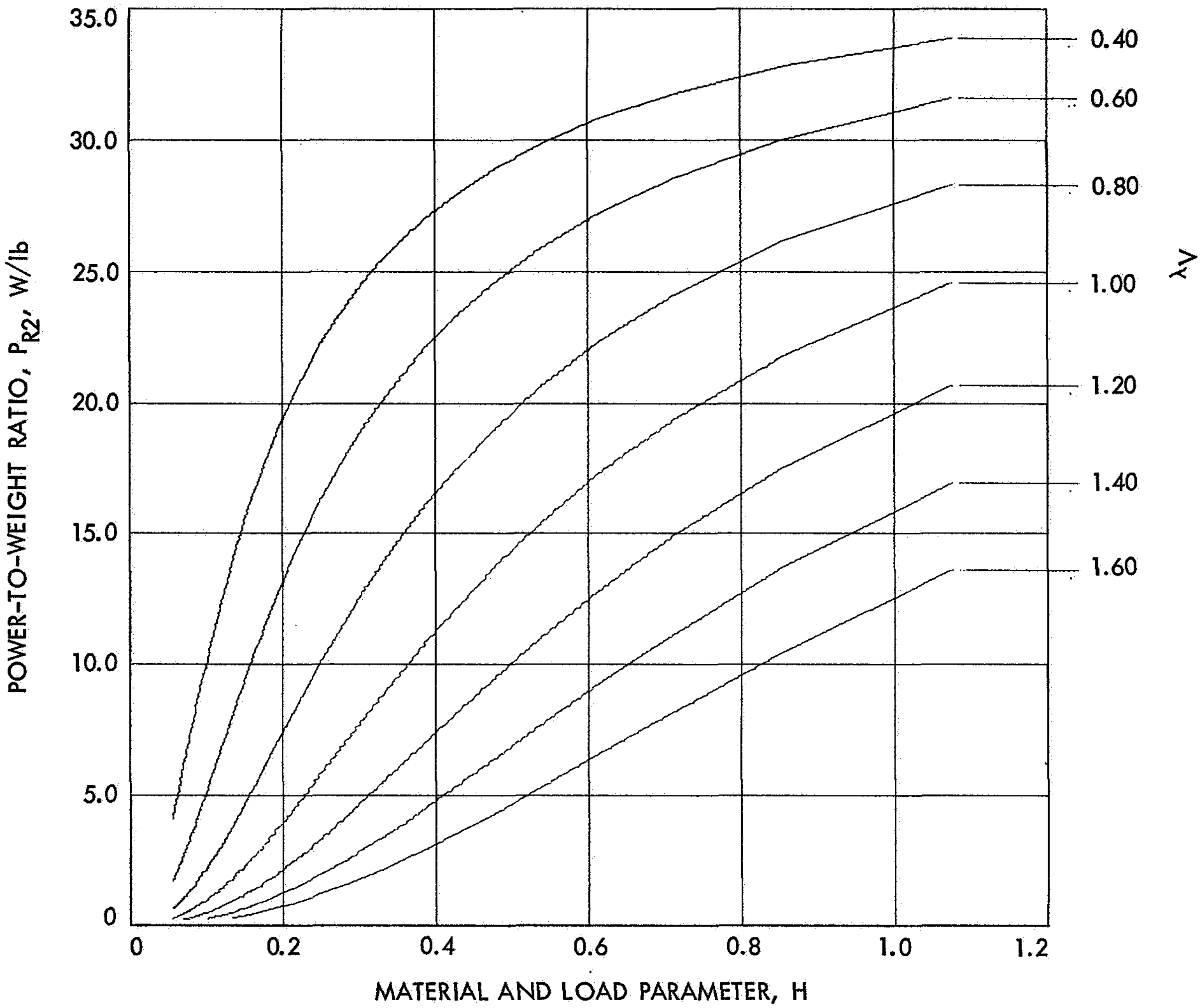
The power-to-weight ratio for the Boeing design was taken as 21.8 W/lb.

Appendix C
Parametric Plots

Design data, in the form of plots, are presented using parameters defined in Section V. In the plots, both λ_H and λ_V vary from 0.4 to 1.6 in increments of 0.2.

I. Deflection-Limited Design—Condition A

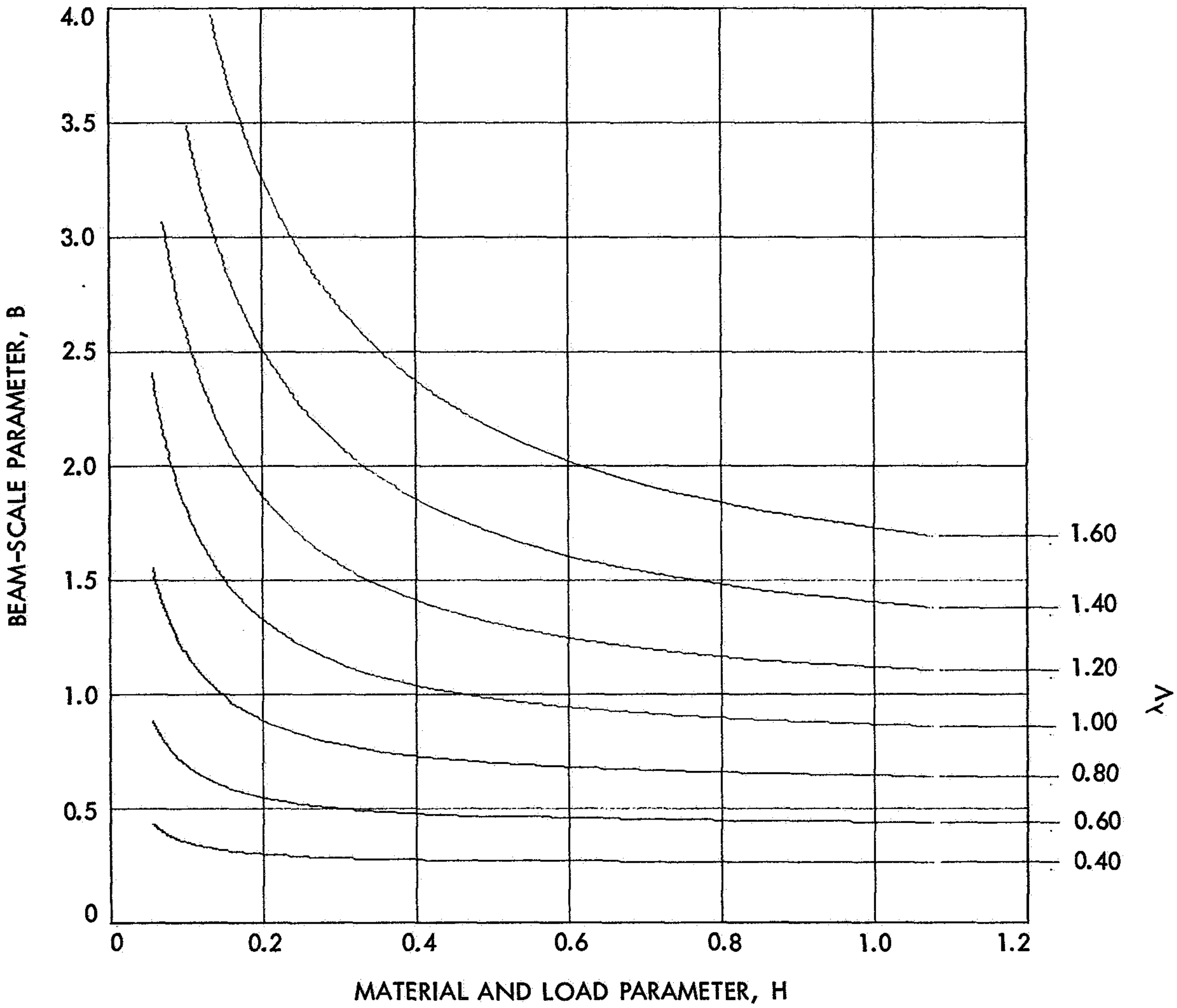
Each set of constant values of λ_H contains three graphs plotted for the dependent parameters P_{R2}, B, and S, defined in Section V-A; these are plotted vs the independent parameter H for constant values of λ_V.



DEFLECTION - LIMITED

$\lambda_H = 0.40$

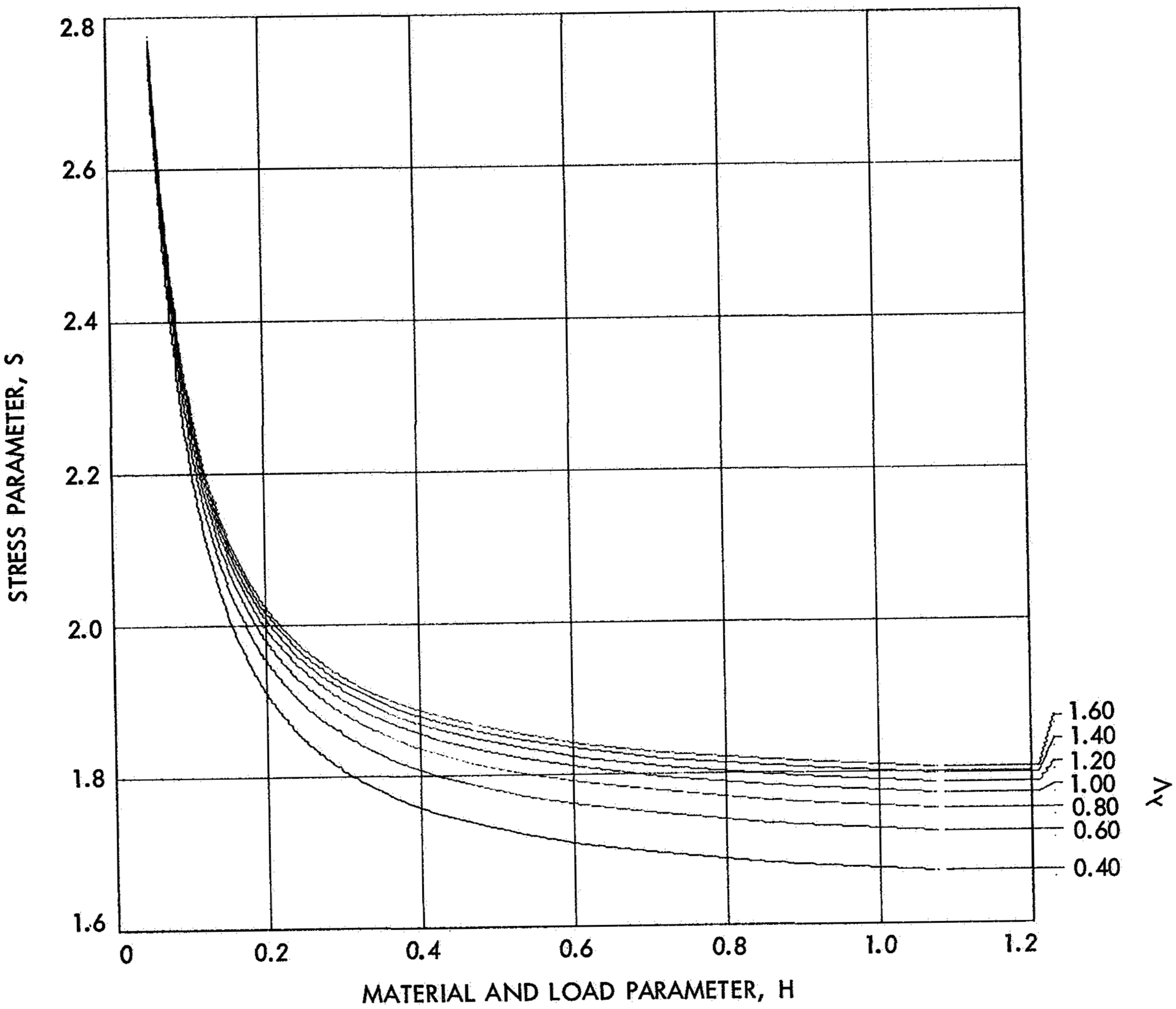
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 0.40$

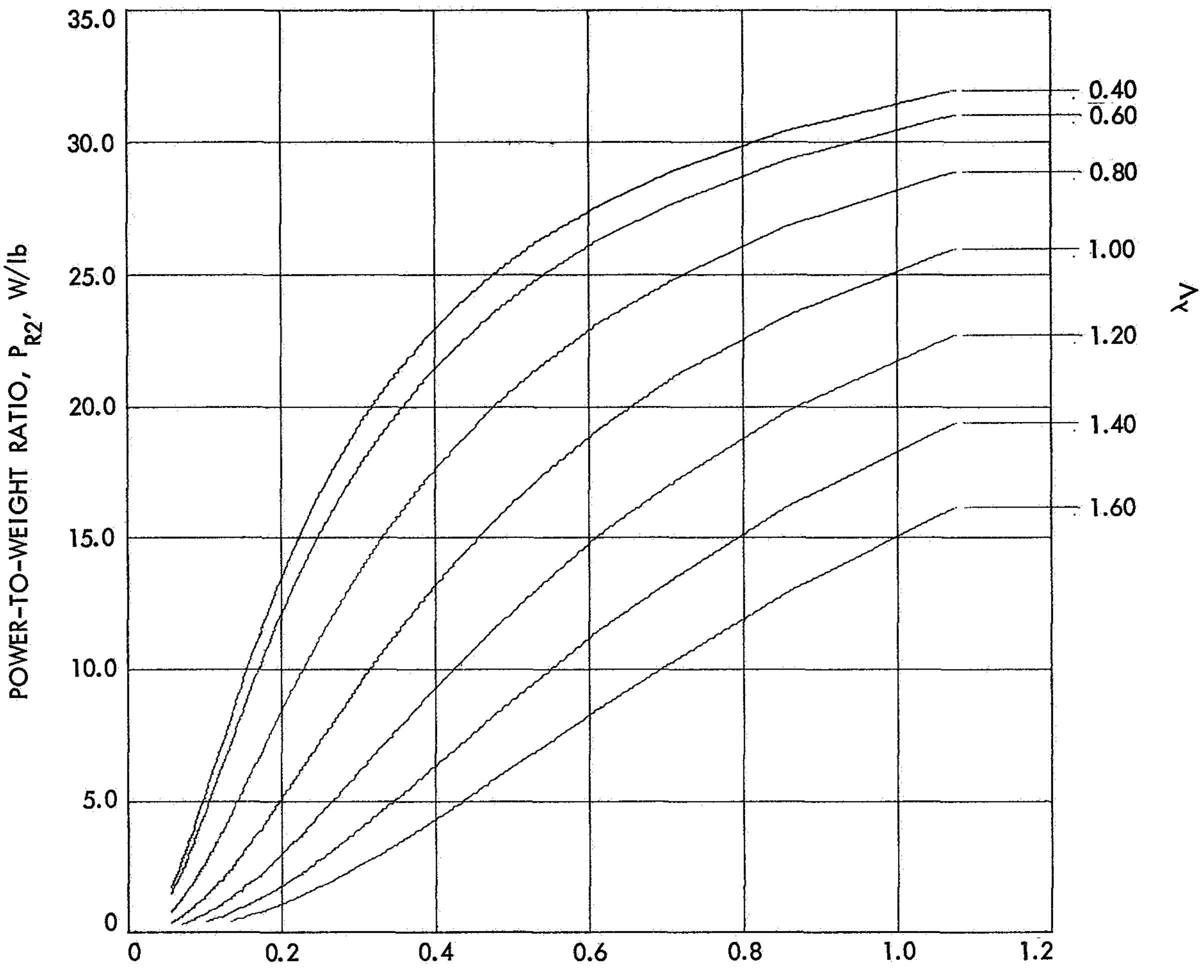
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 0.40$

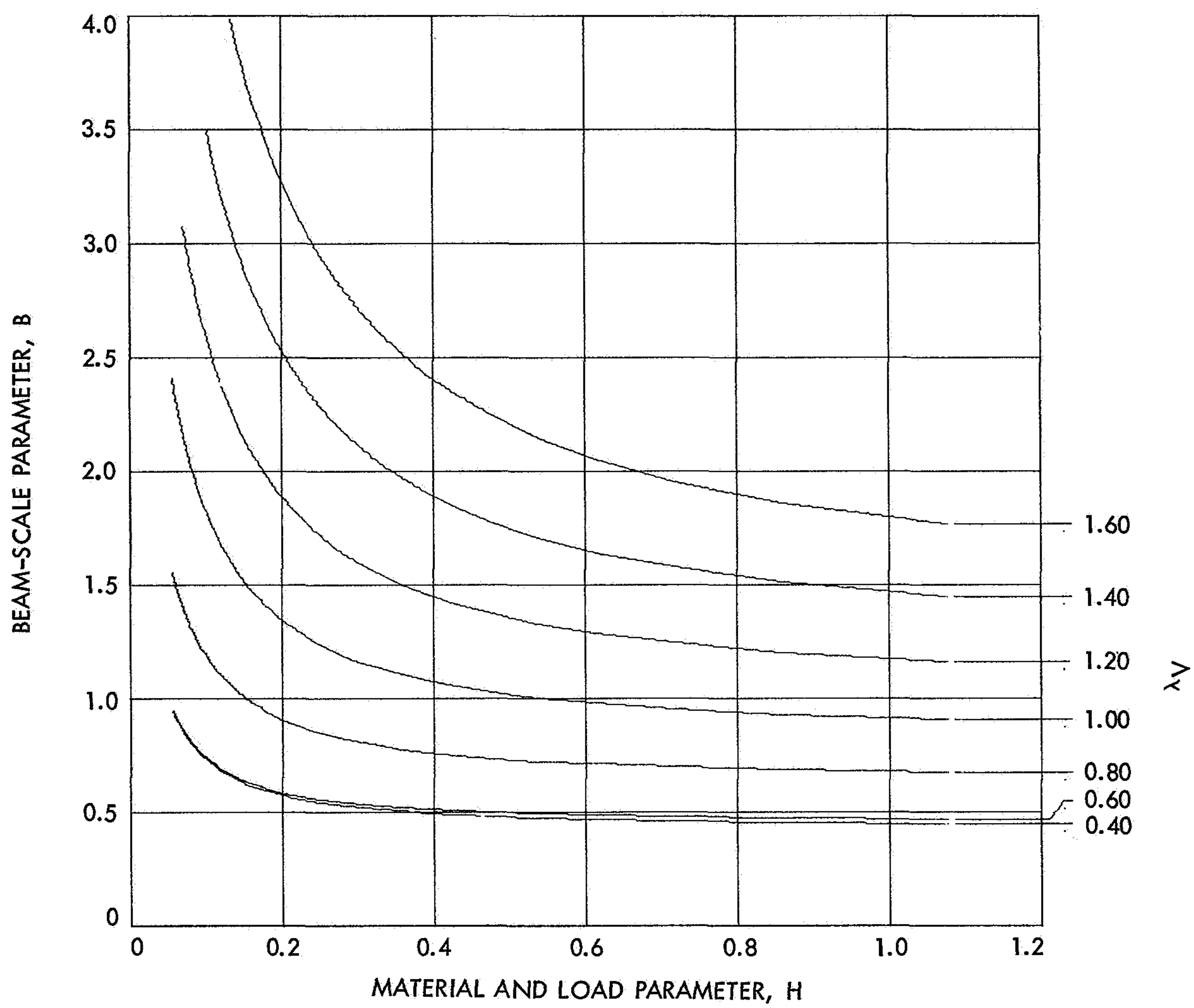
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 0.60$

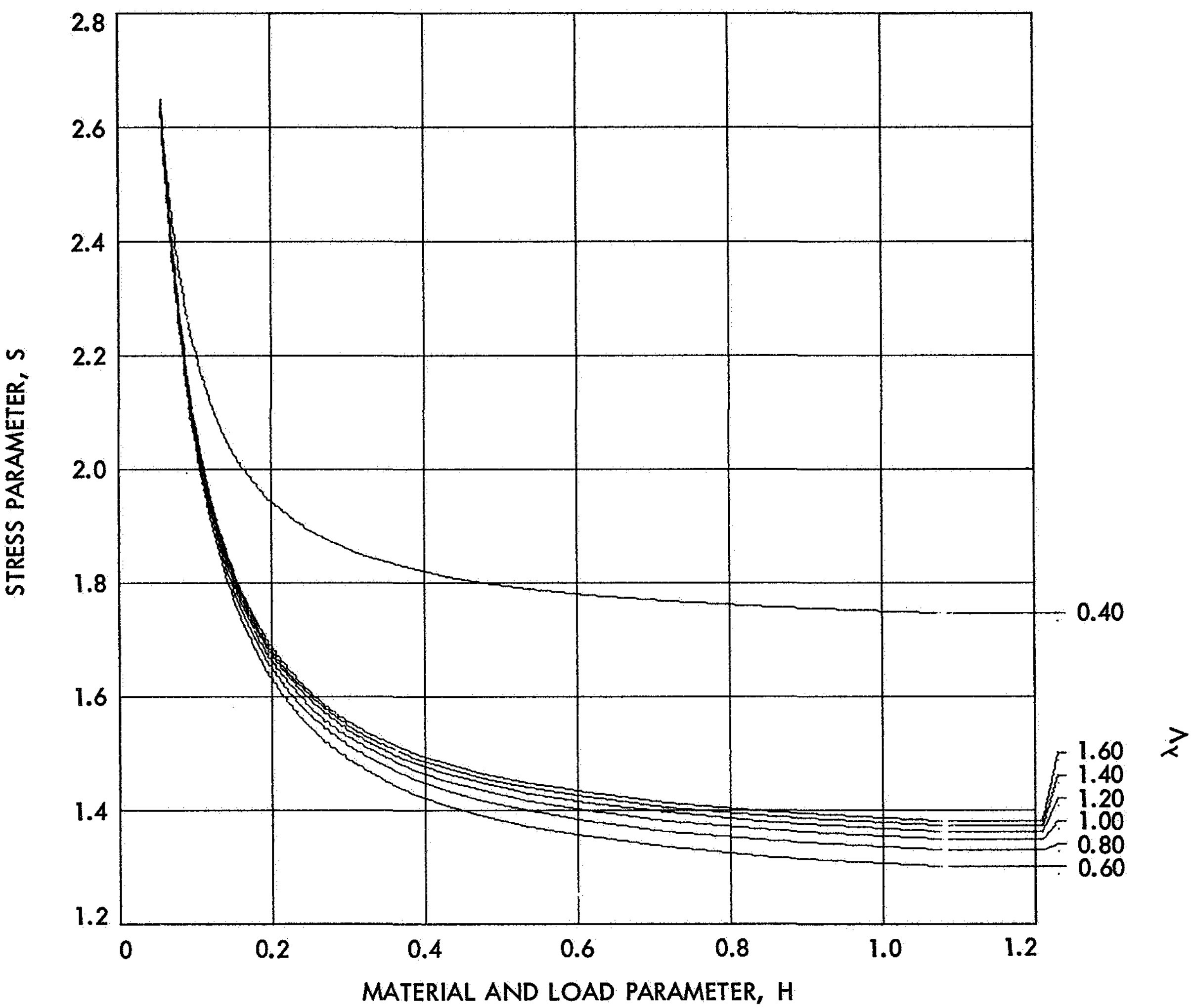
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 0.60$

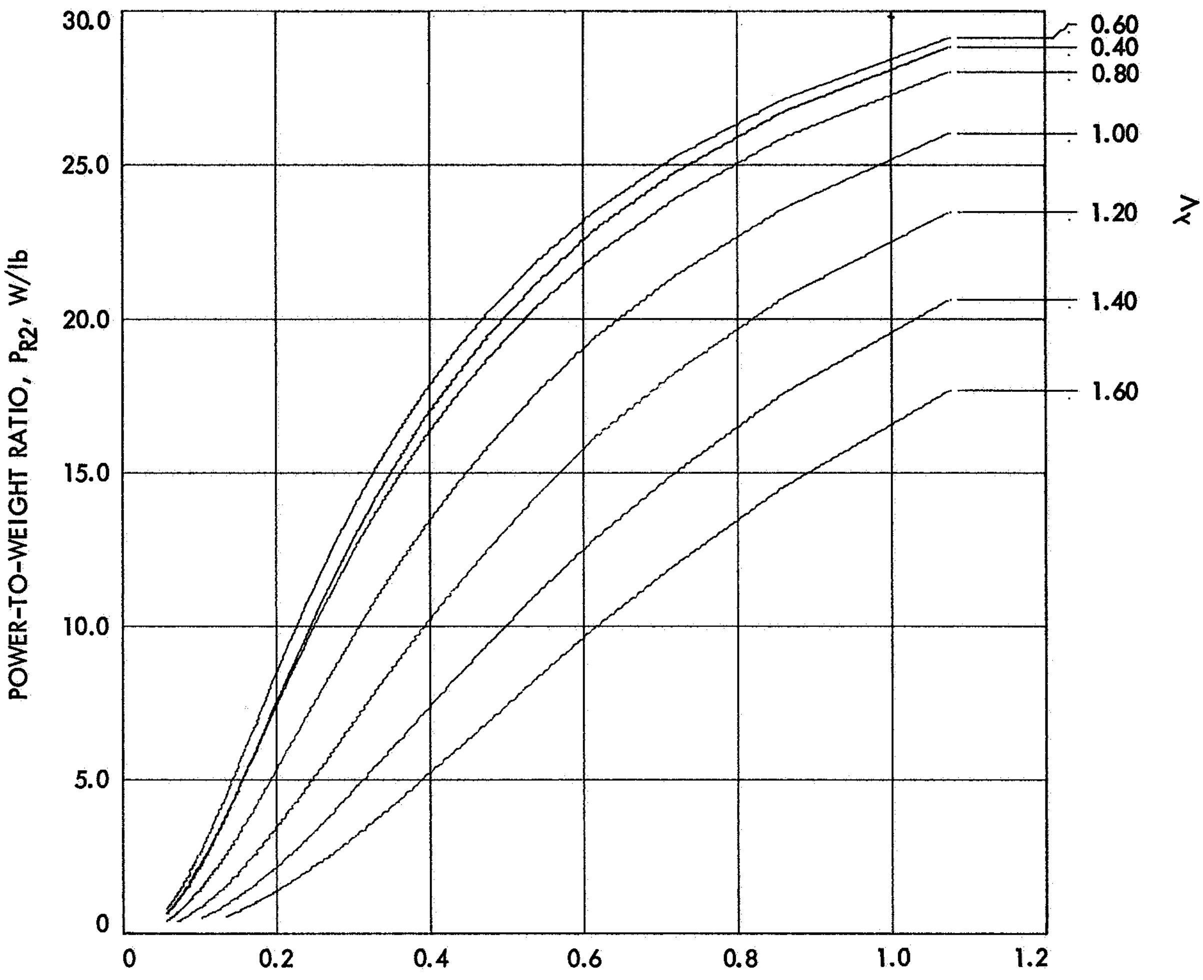
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 0.60$

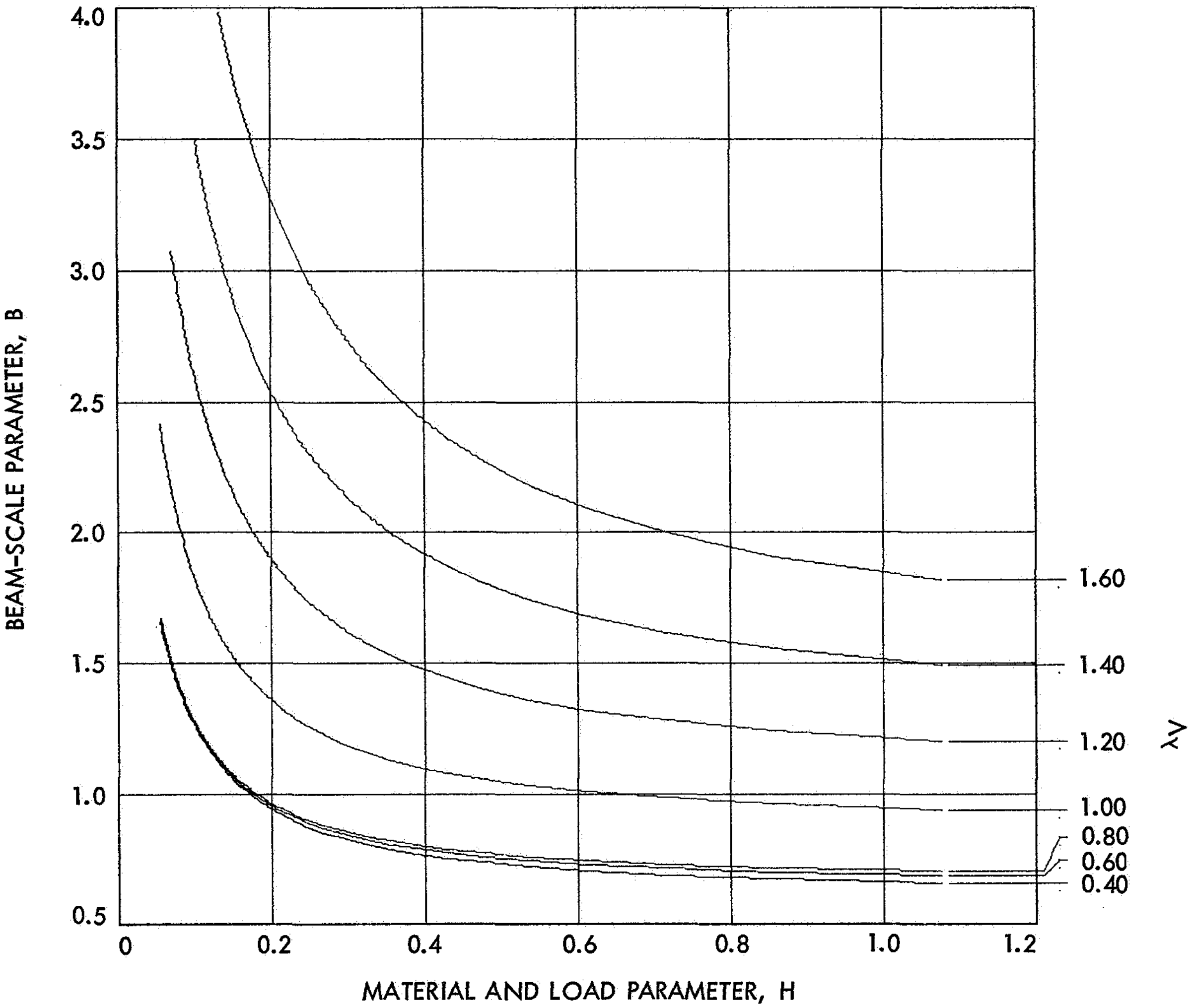
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 0.80$

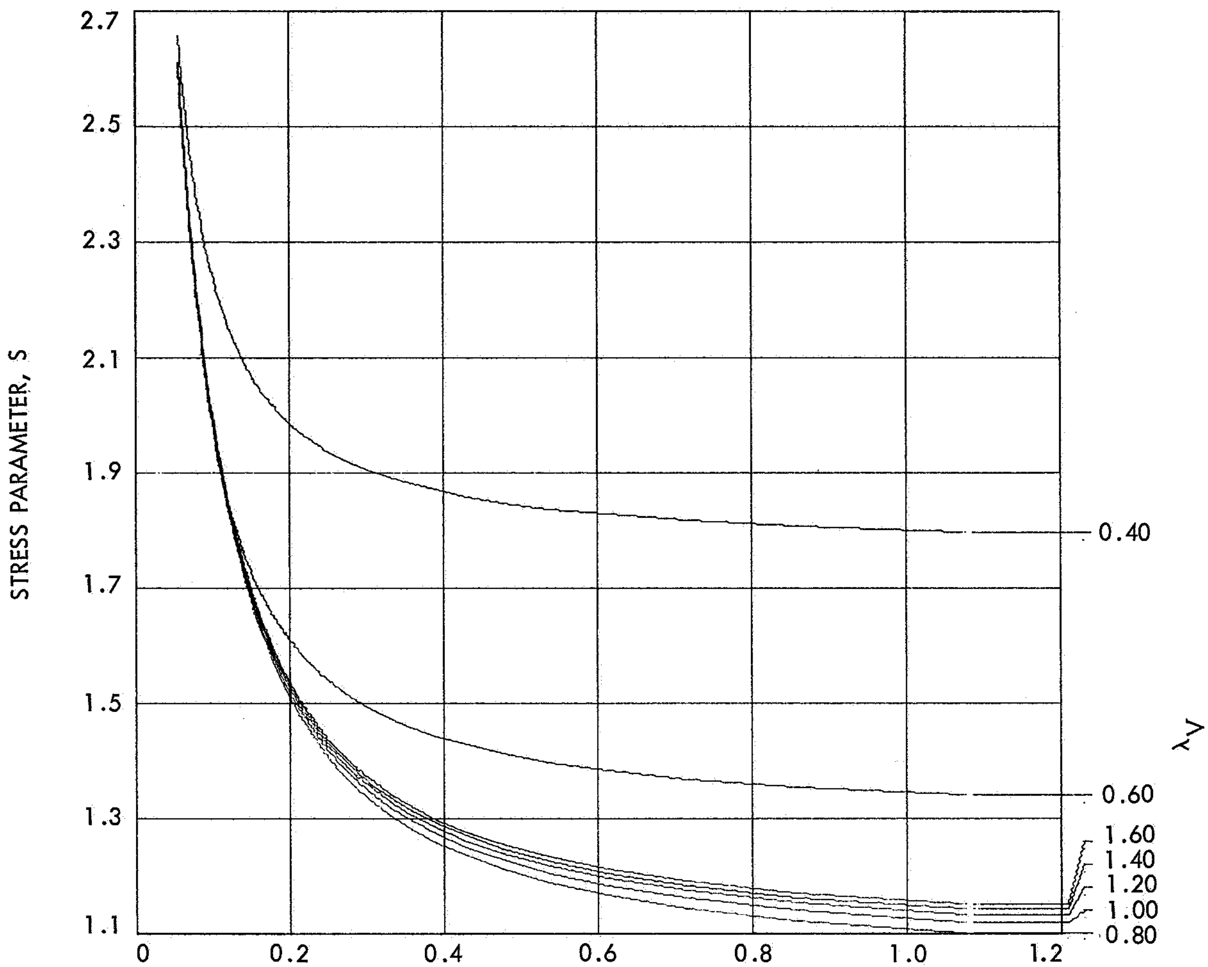
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 0.80$

REFERENCE BOEING LASA

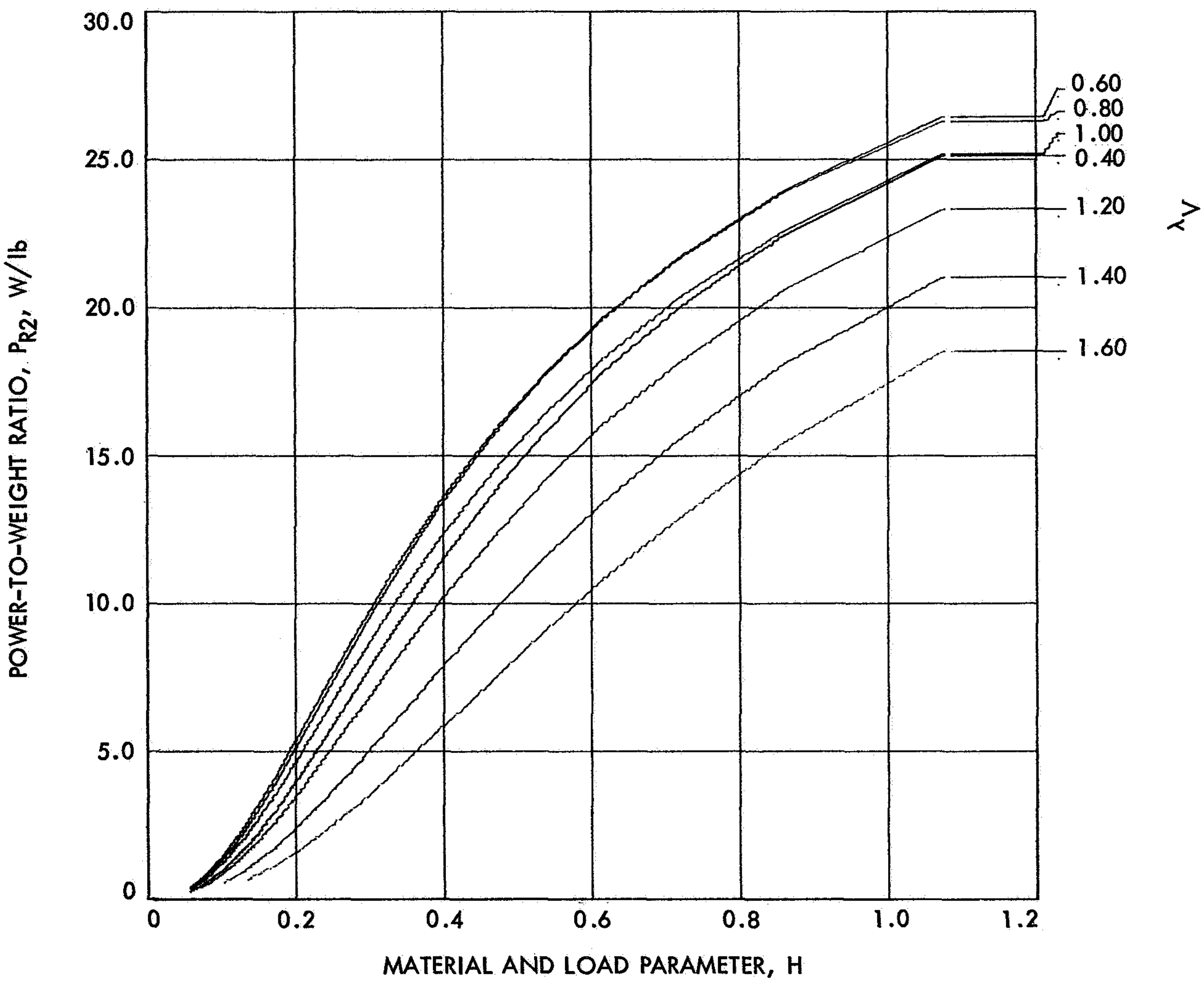


MATERIAL AND LOAD PARAMETER, H

DEFLECTION-LIMITED

$\lambda_H = 0.80$

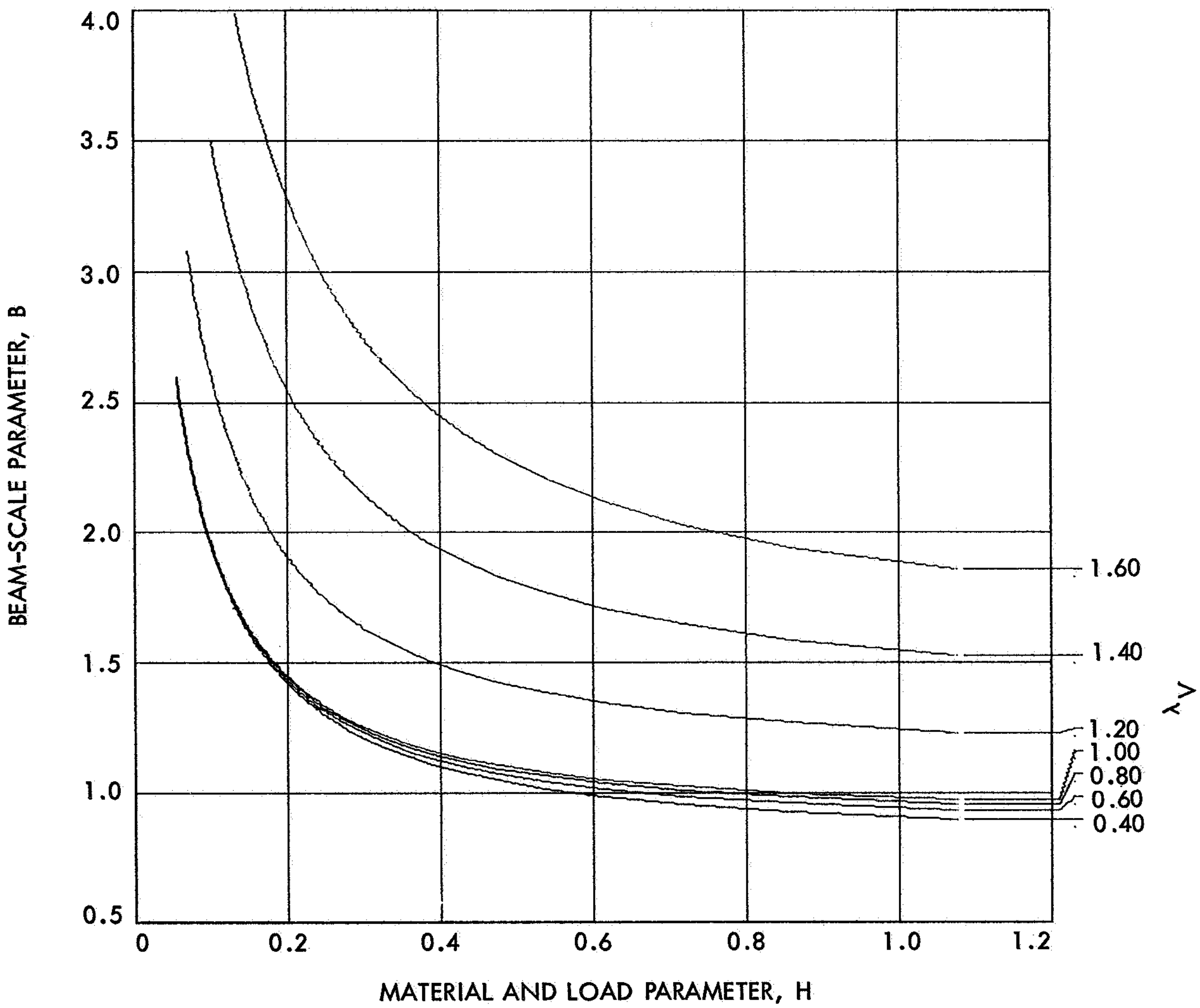
REFERENCE BOEING LASA



DEFLECTION-LIMITED

$\lambda_H = 1.00$

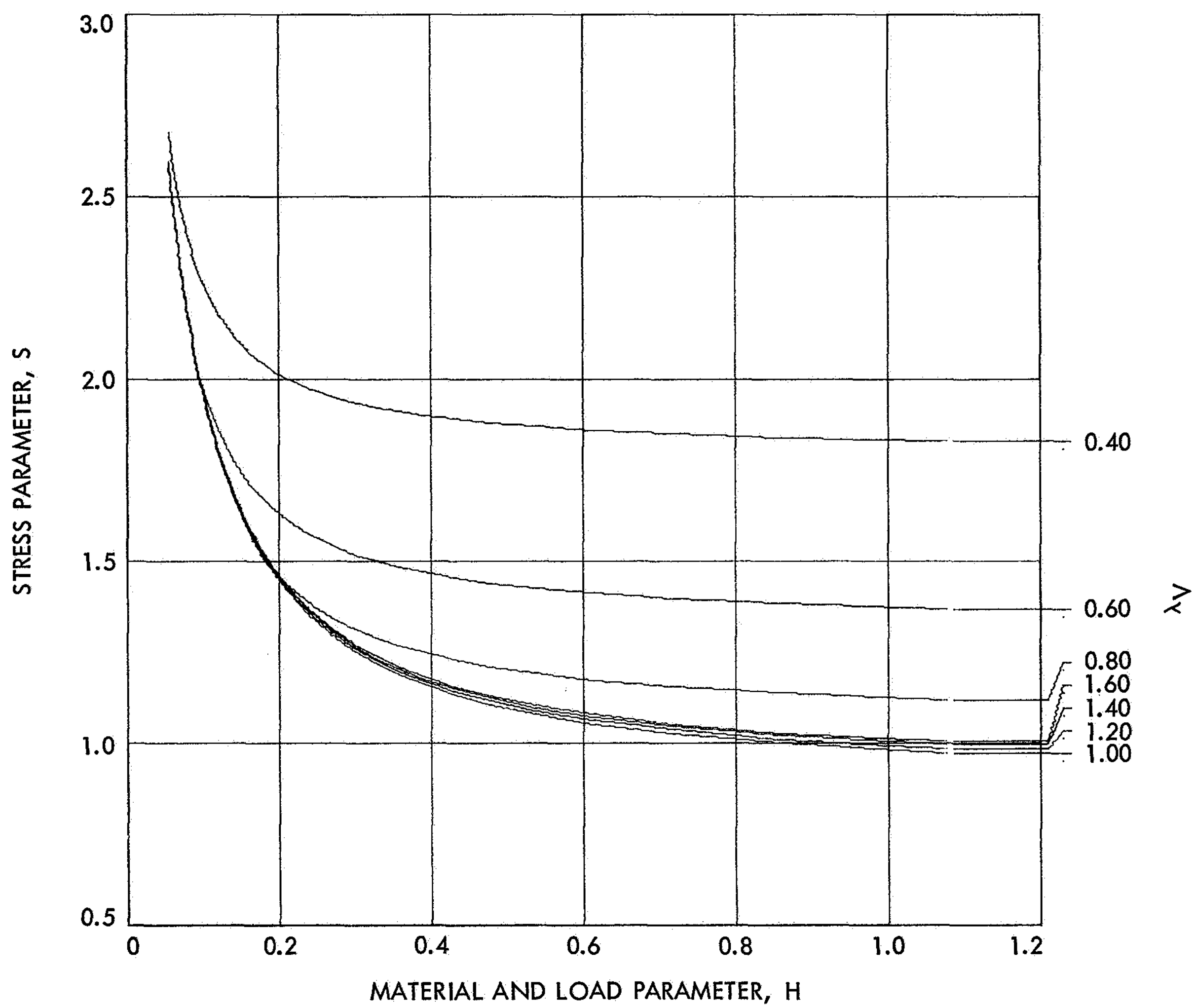
REFERENCE BOEING LASA



DEFLECTION-LIMITED

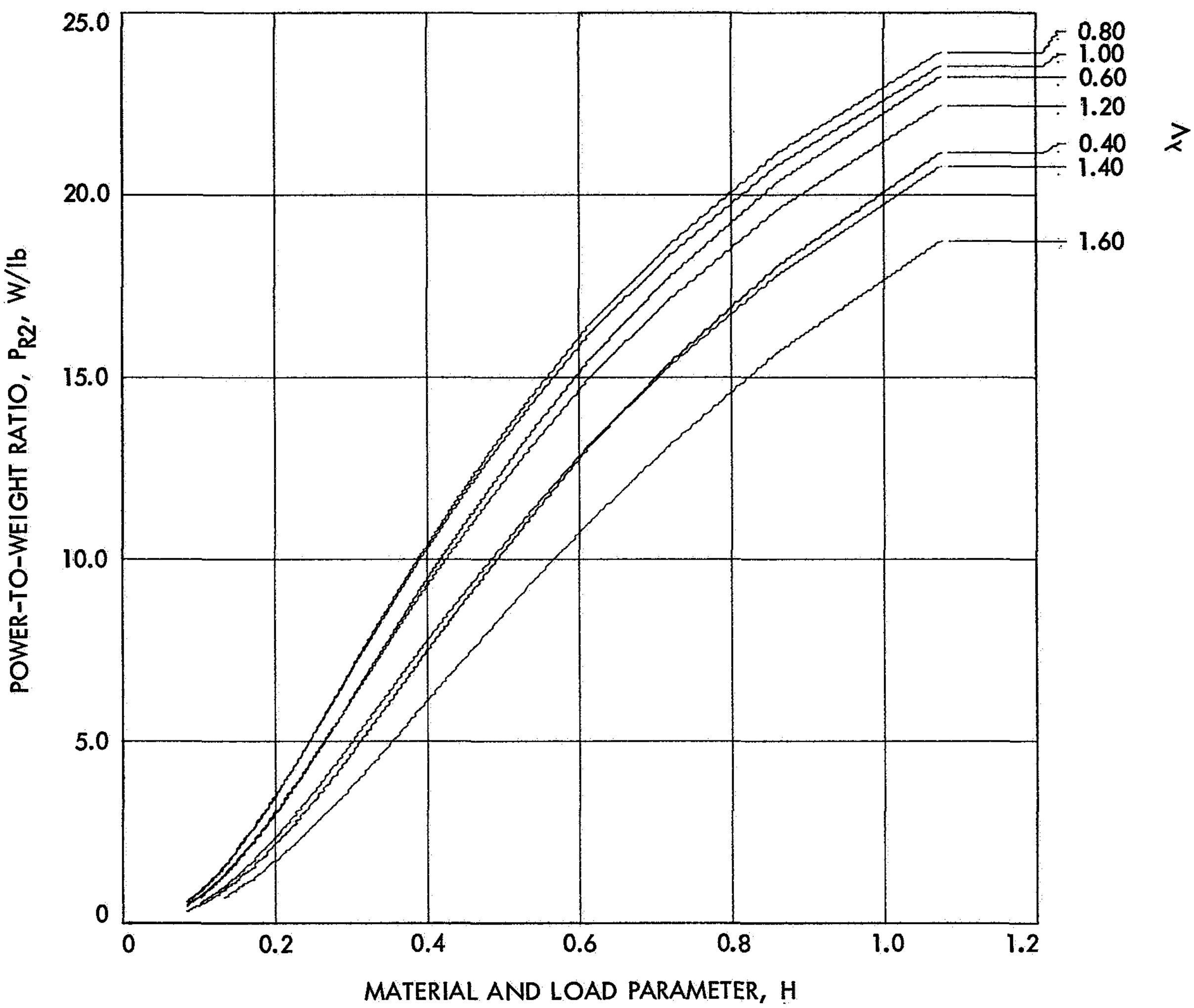
$\lambda_H = 1.00$

REFERENCE BOEING LASA



DEFLECTION - LIMITED

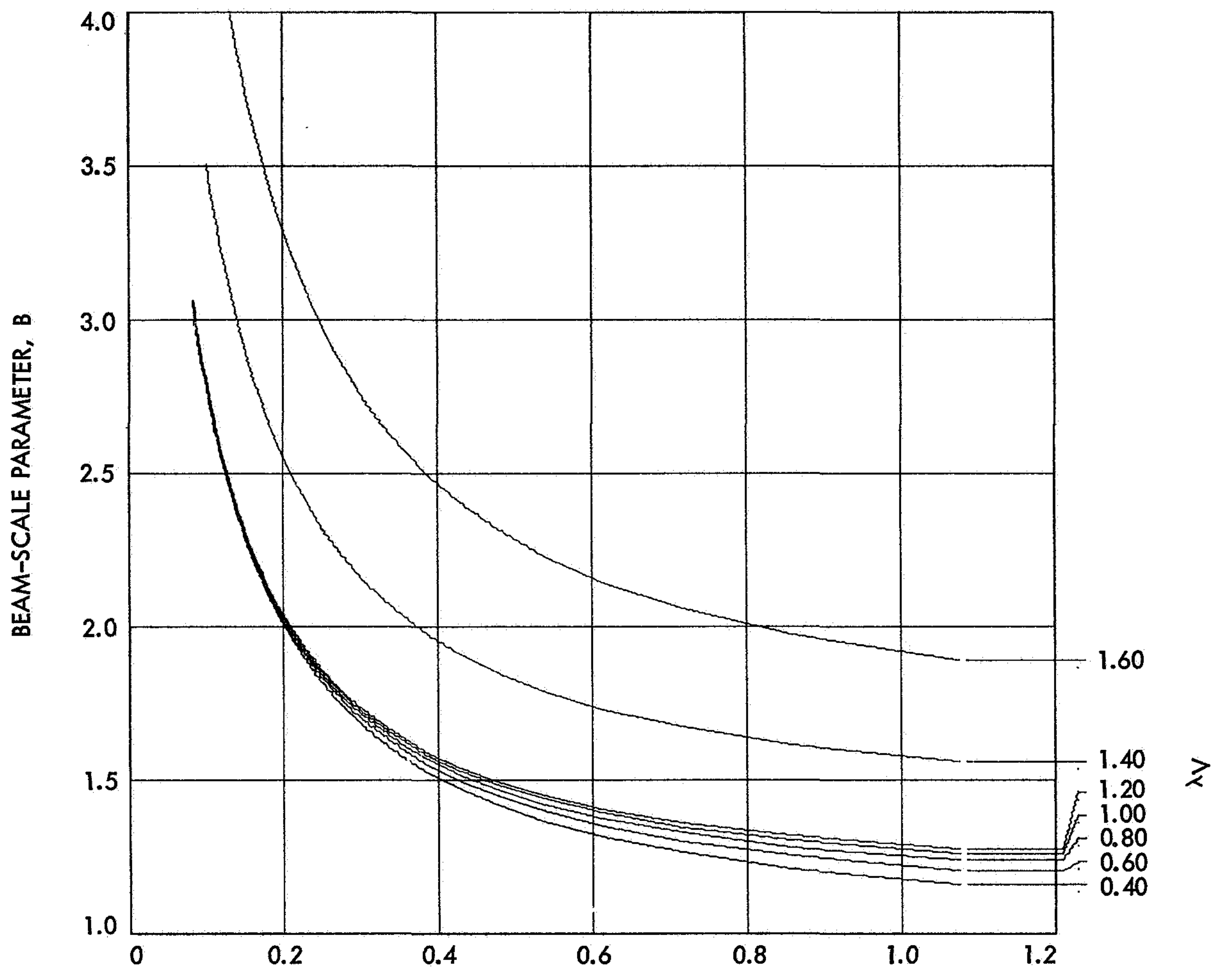
$\lambda_H = 1.00$
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 1.20$

REFERENCE BOEING LASA

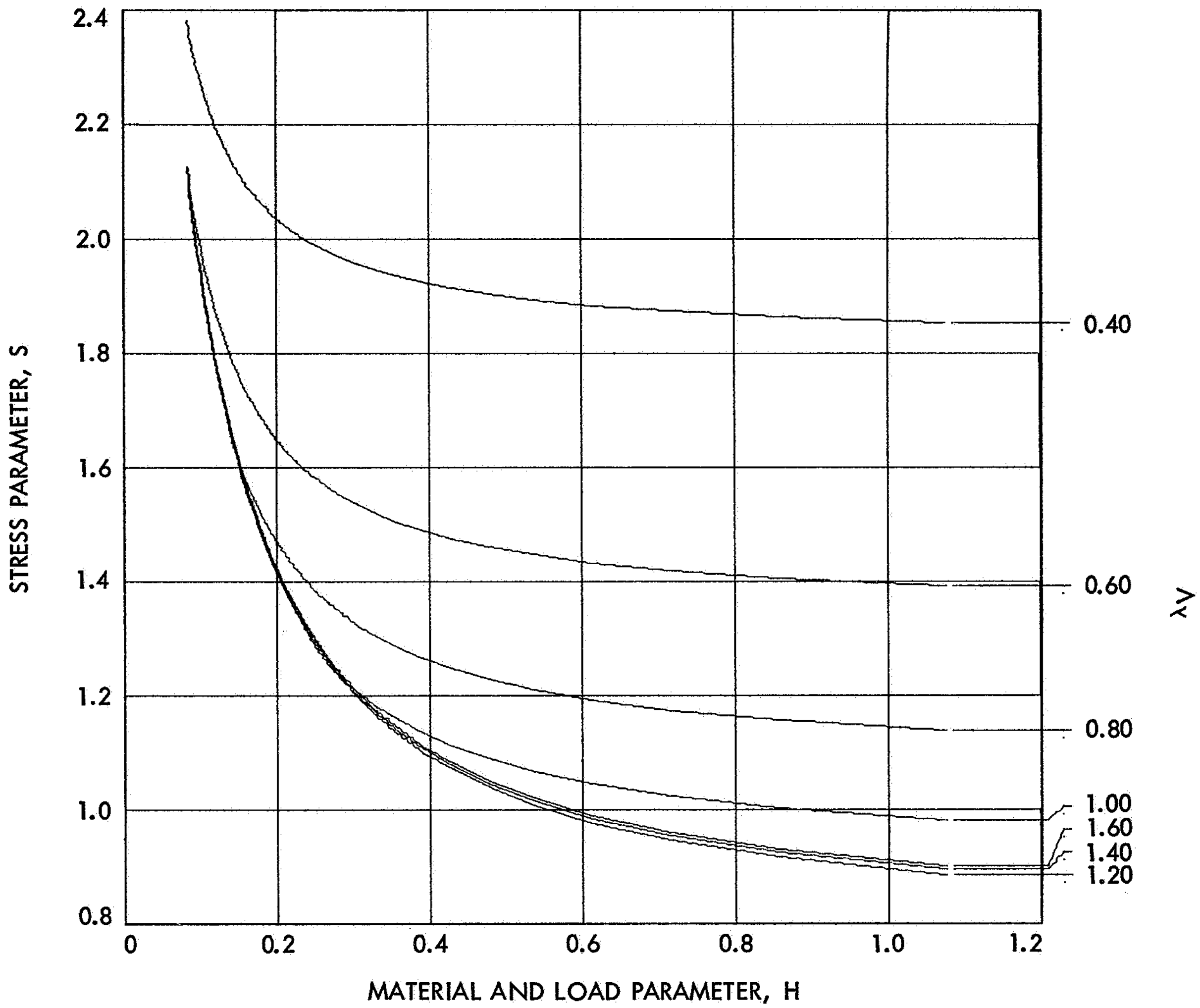


MATERIAL AND LOAD PARAMETER, H

DEFLECTION - LIMITED

$\lambda_H = 1.20$

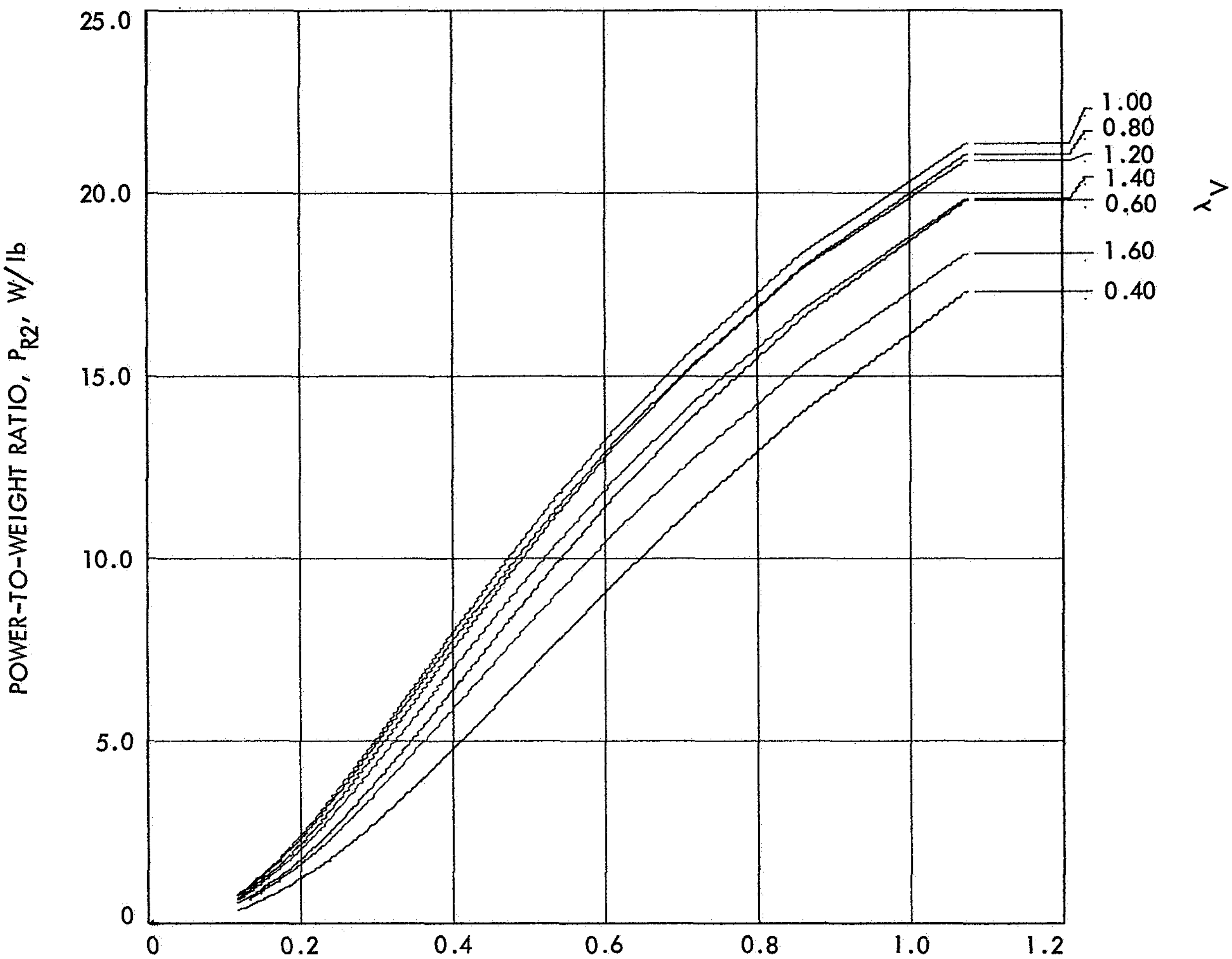
REFERENCE BOEING LASA



DEFLECTION - LIMITED

$\lambda_H = 1.20$

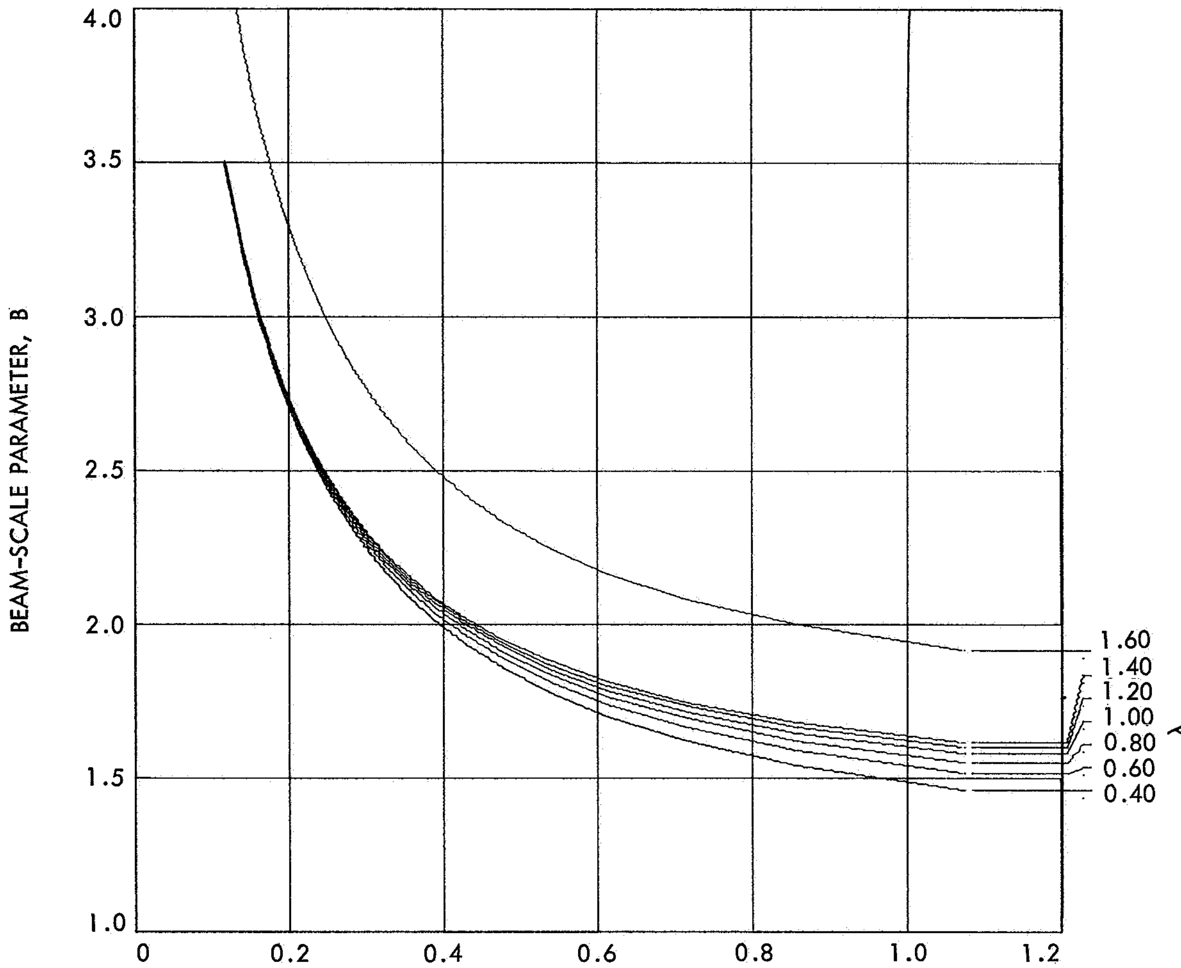
REFERENCE BOEING LASA



DEFLECTION-LIMITED

$\lambda_H = 1.40$

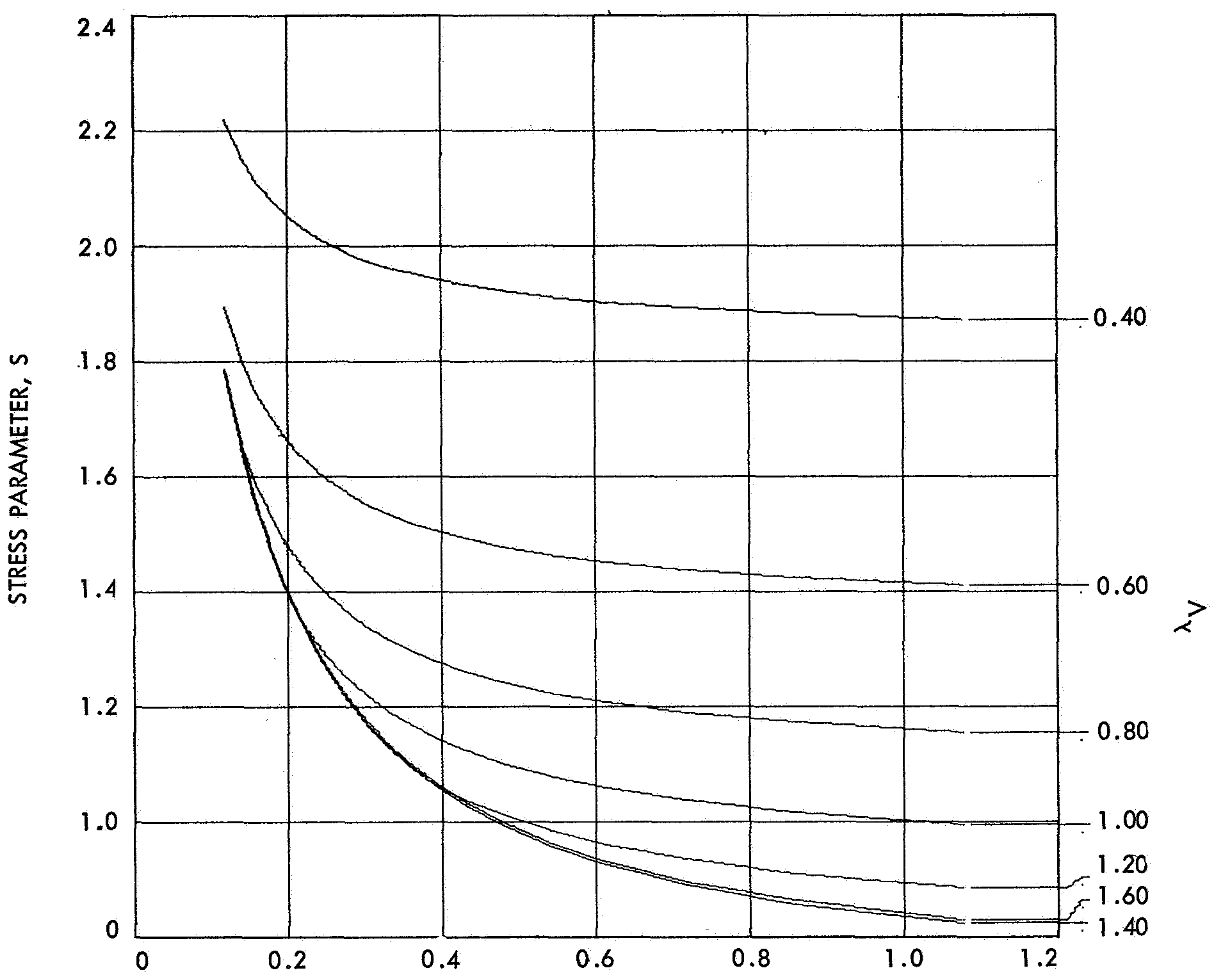
REFERENCE BOEING LASA



DEFLECTION-LIMITED

$\lambda_H = 1.40$

REFERENCE BOEING LASA

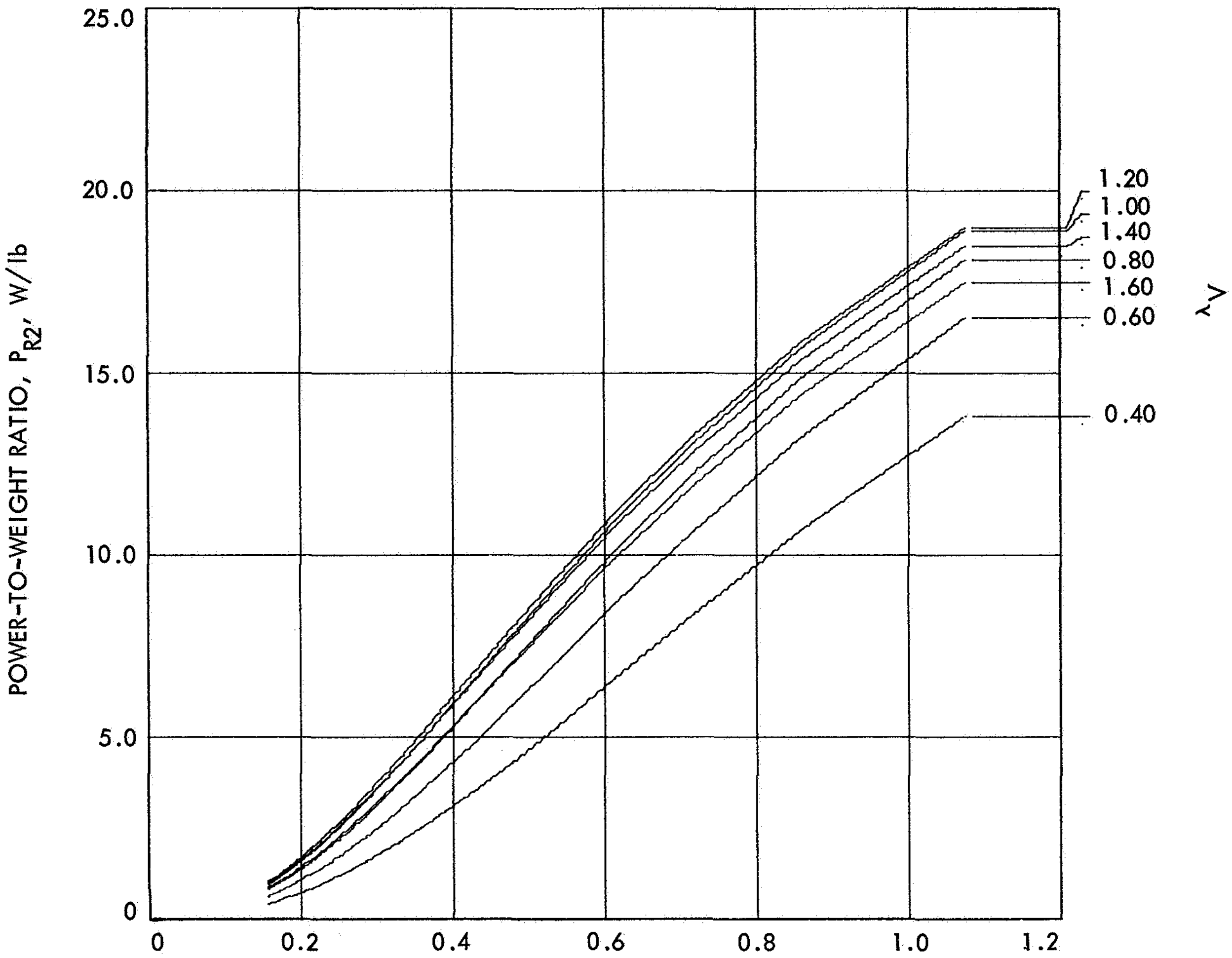


MATERIAL AND LOAD PARAMETER, H

DEFLECTION-LIMITED

$\lambda_H = 1.40$

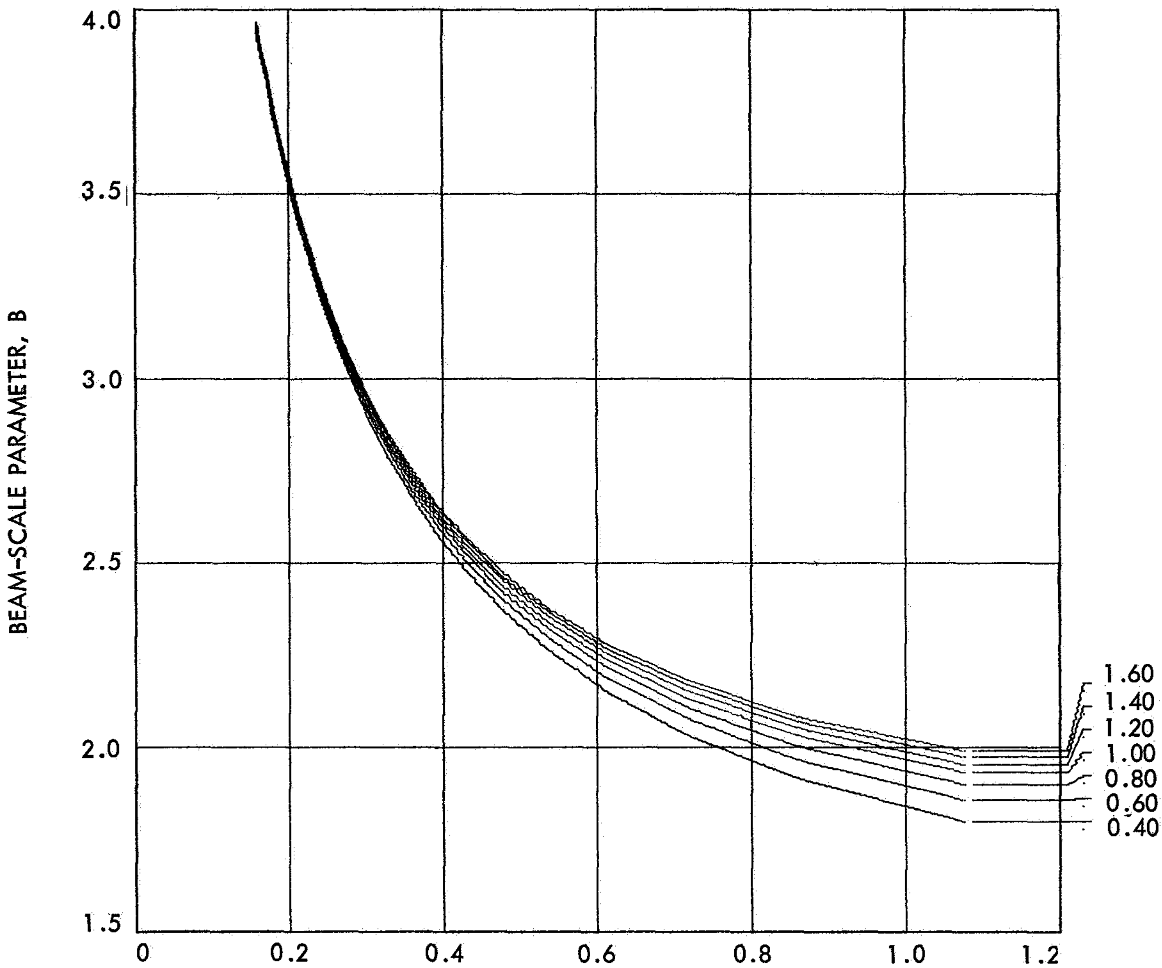
REFERENCE BOEING LASA



DEFLECTION-LIMITED

$\lambda_H = 1.60$

REFERENCE BOEING LASA

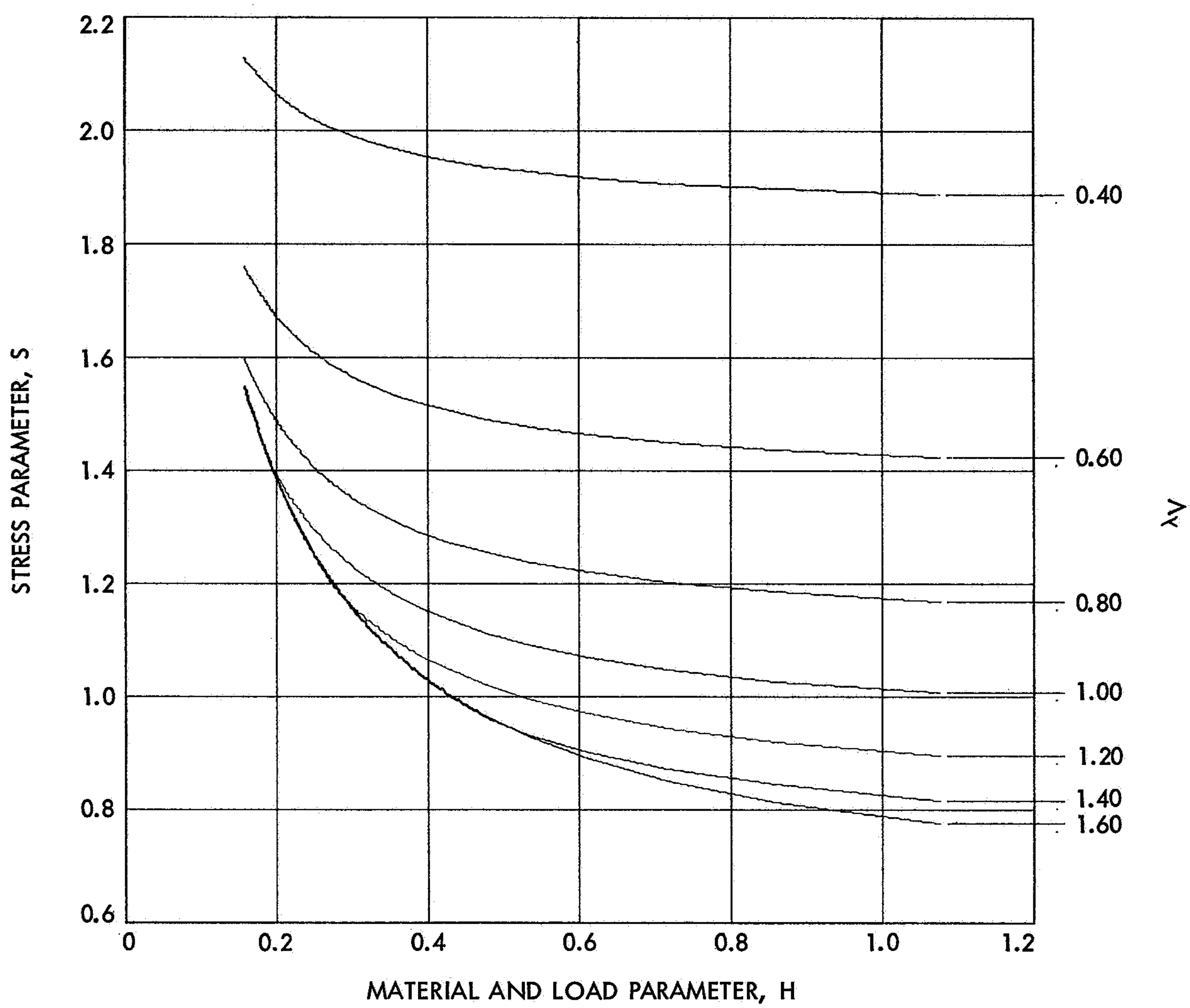


MATERIAL AND LOAD PARAMETER, H

DEFLECTION-LIMITED

$\lambda_H = 1.60$

REFERENCE BOEING LASA

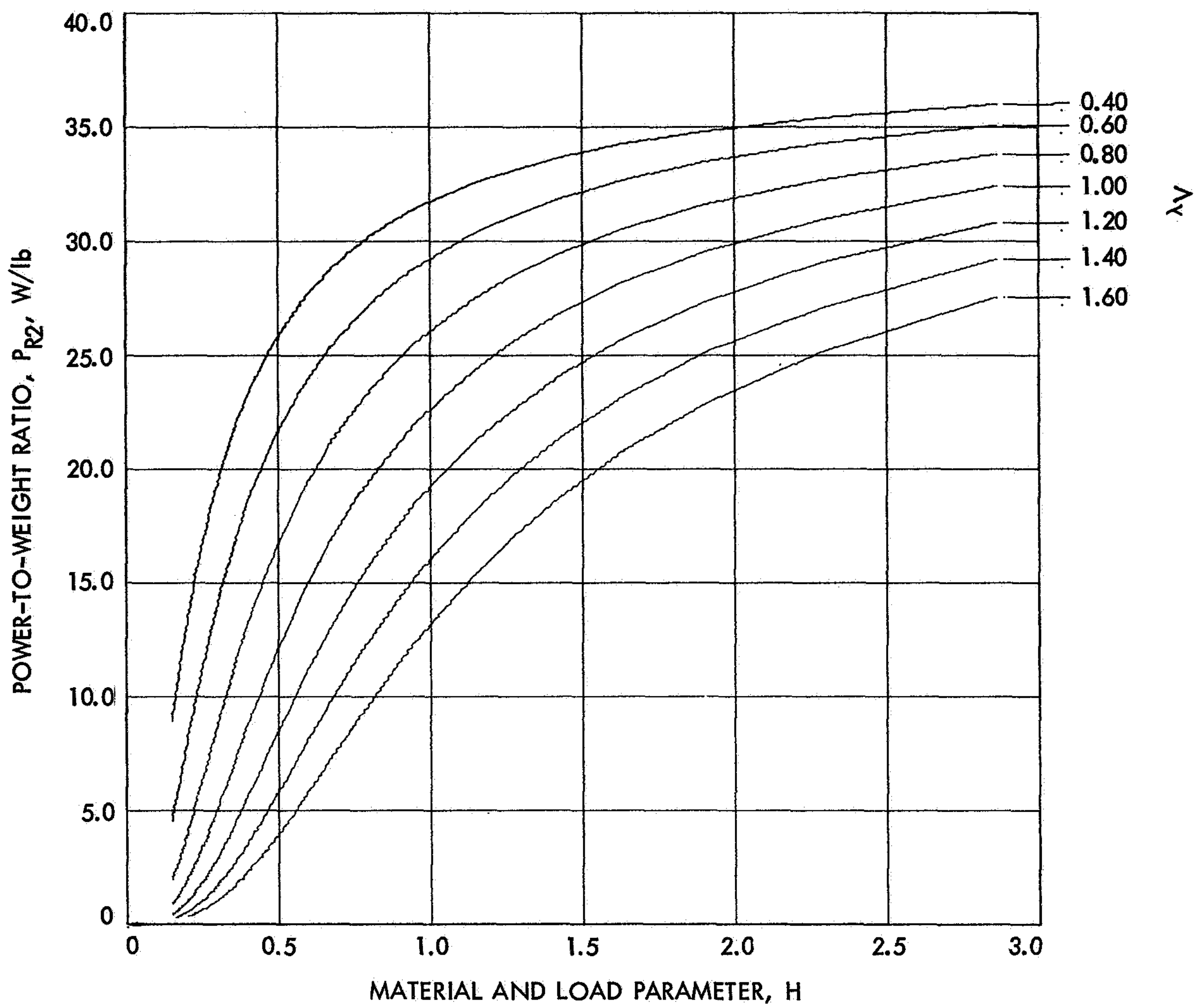


DEFLECTION-LIMITED

$\lambda_H = 1.60$
REFERENCE BOEING LASA

II. Stress-Limited Design—Condition B

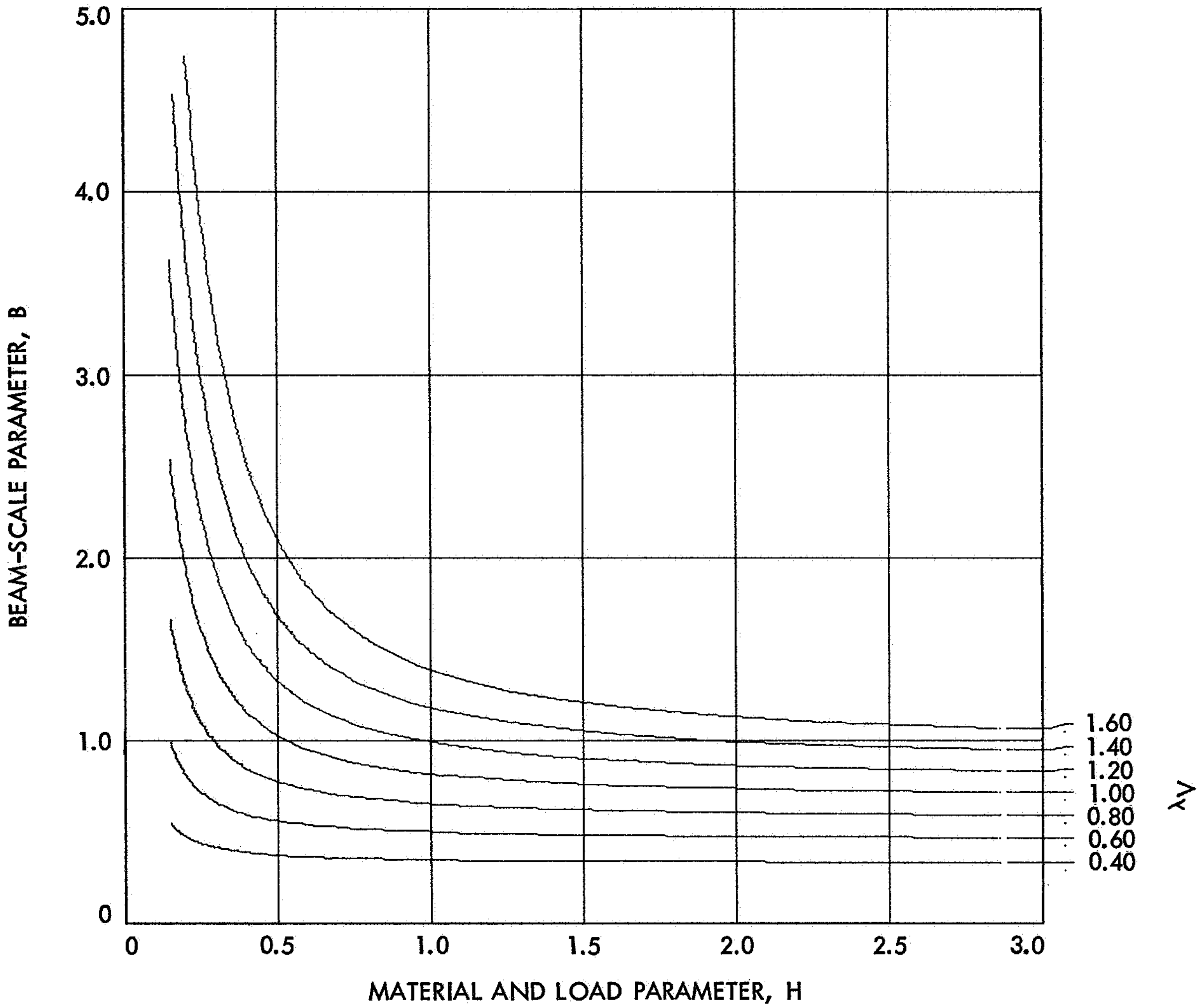
Each set of constant values of λ_H contains four graphs plotted for the parameters P_{R2}, B, F, and D, defined in Section V-B; these are plotted vs the parameter H for constant values of λ_V.



STRESS - LIMITED

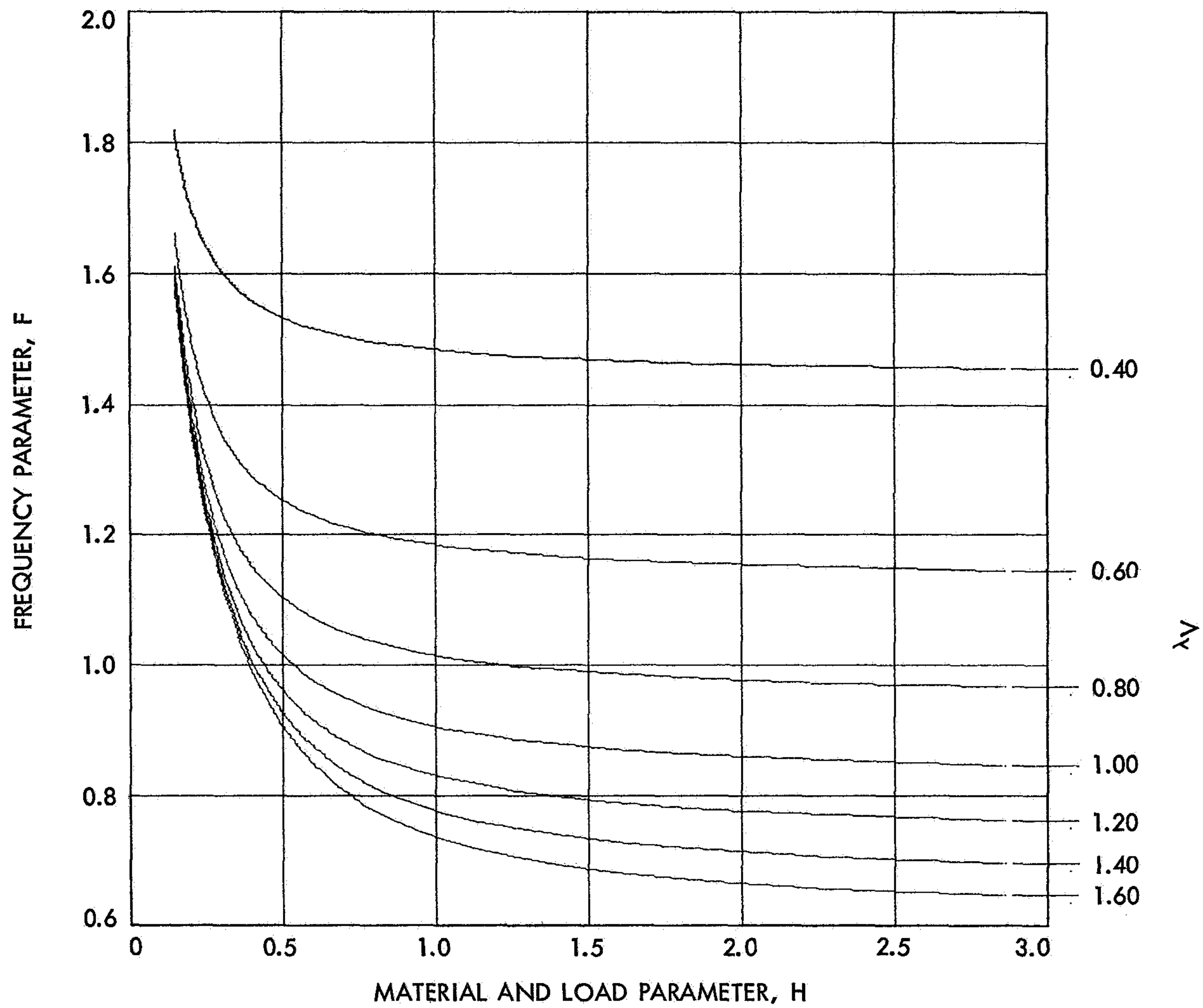
$\lambda_H = 0.40$

REFERENCE BOEING LASA



STRESS - LIMITED

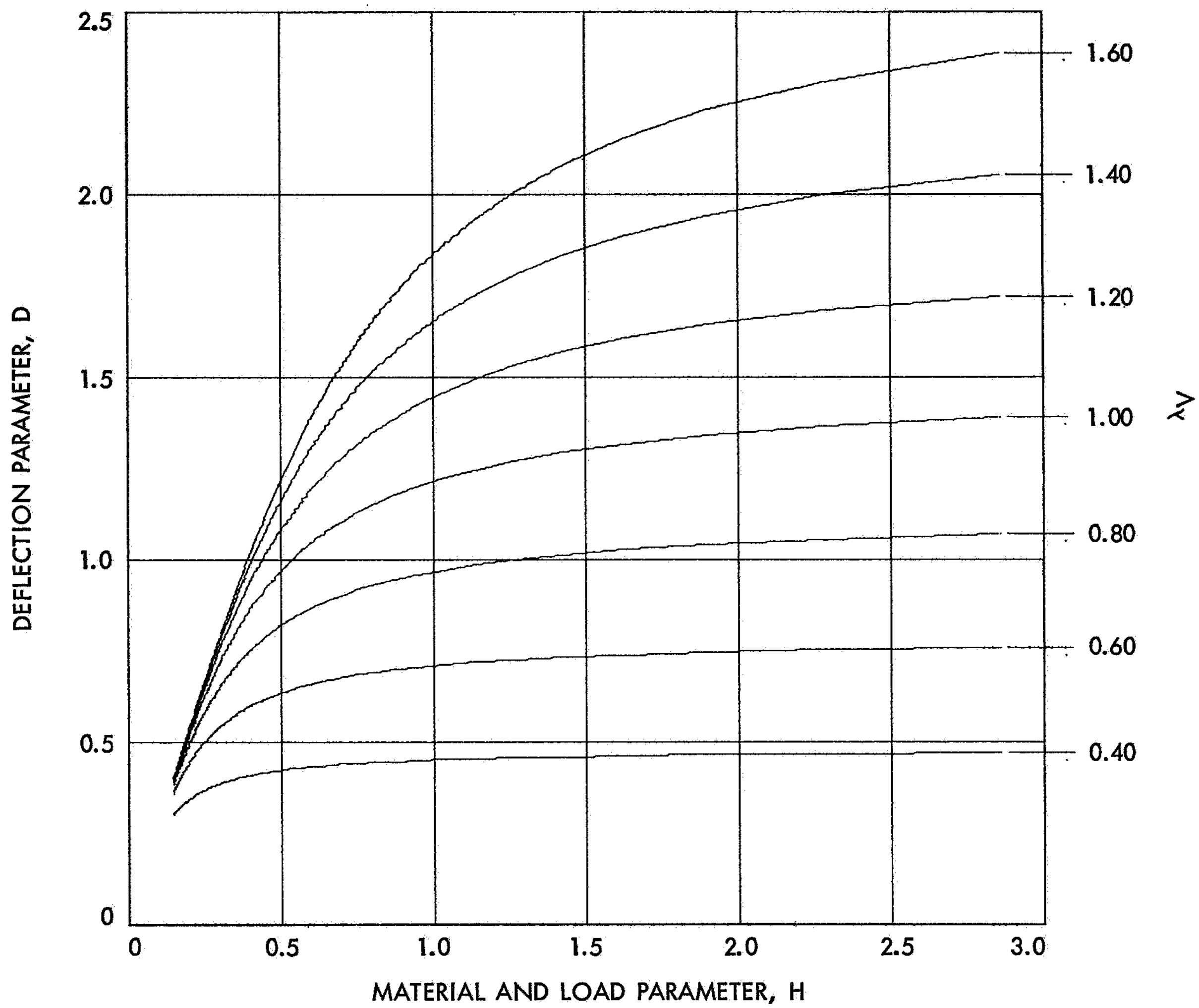
$\lambda_H = 0.40$
REFERENCE BOEING LASA



STRESS - LIMITED

$\lambda_H = 0.40$

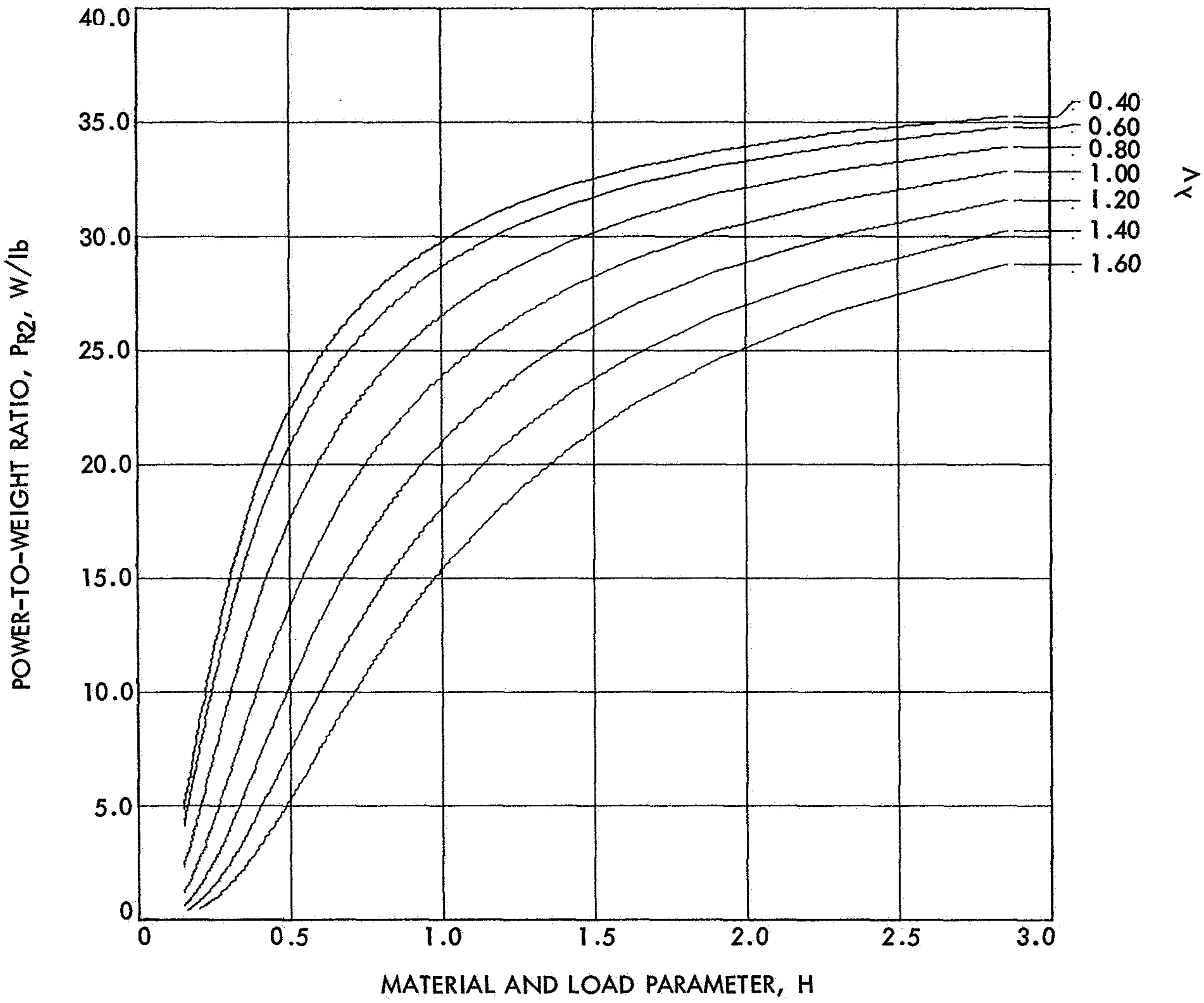
REFERENCE BOEING LASA



STRESS - LIMITED

$\lambda_H = 0.40$

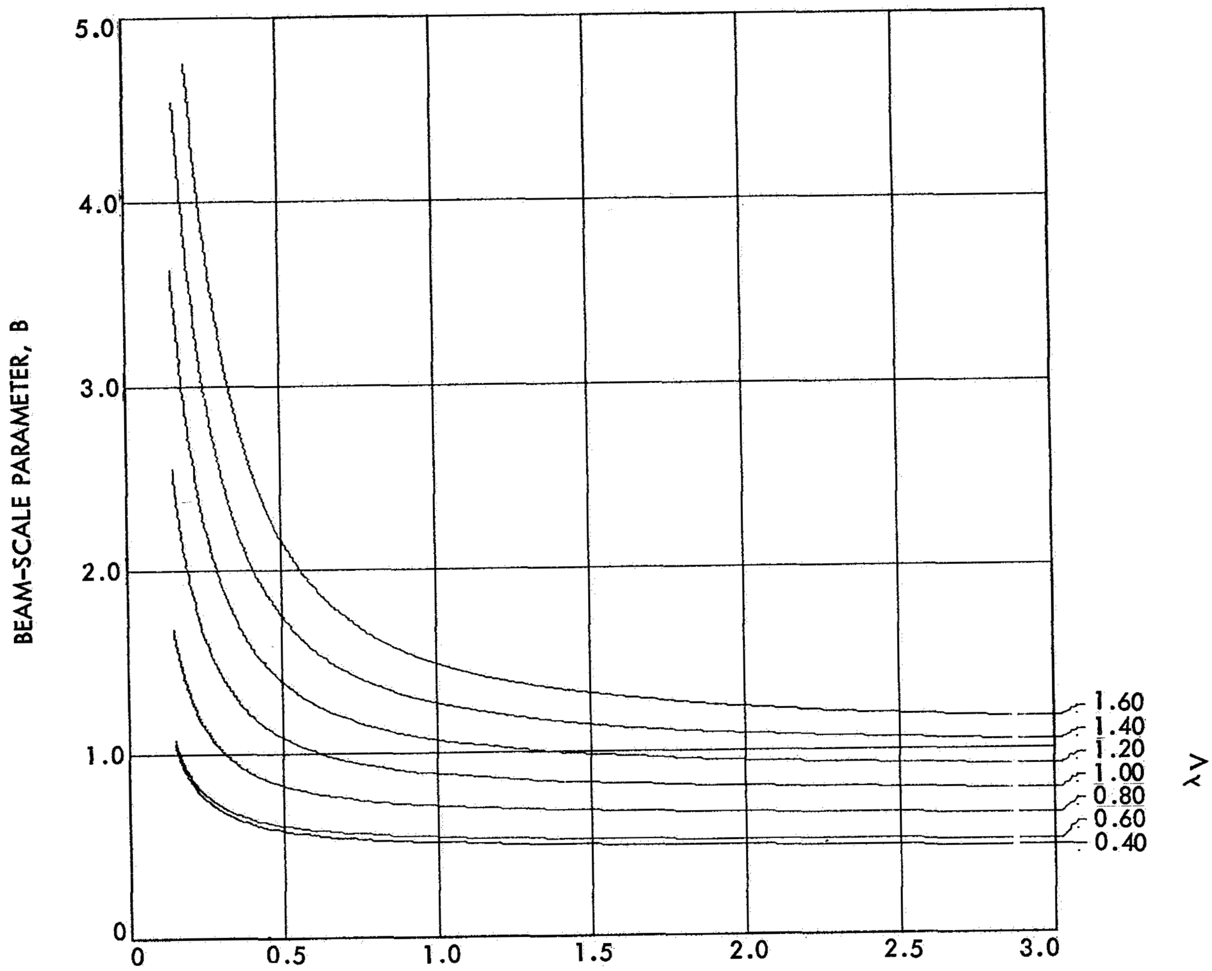
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 0.60$

REFERENCE BOEING LASA

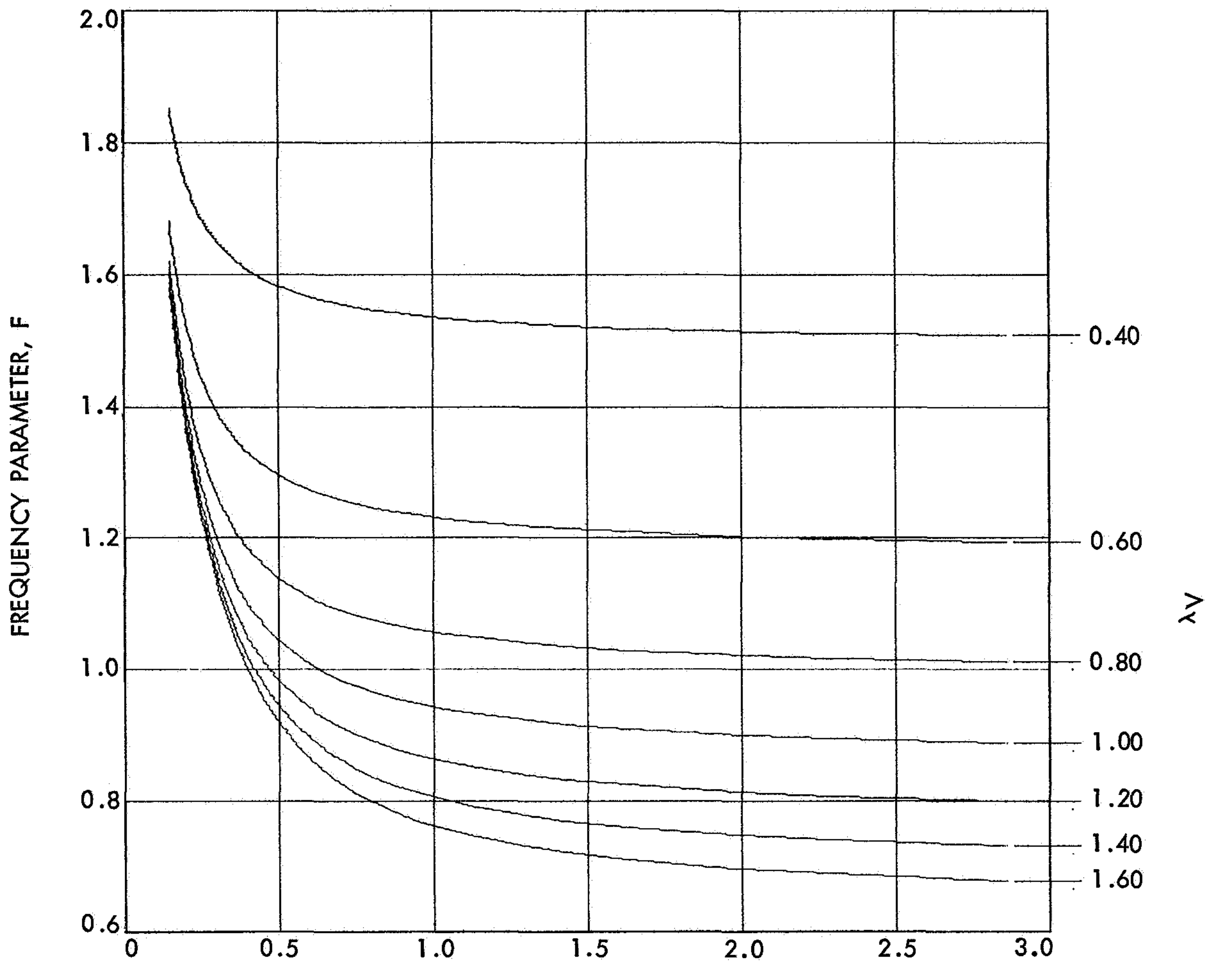


MATERIAL AND LOAD PARAMETER, H

STRESS-LIMITED

$\lambda_H = 0.60$

REFERENCE BOEING LASA

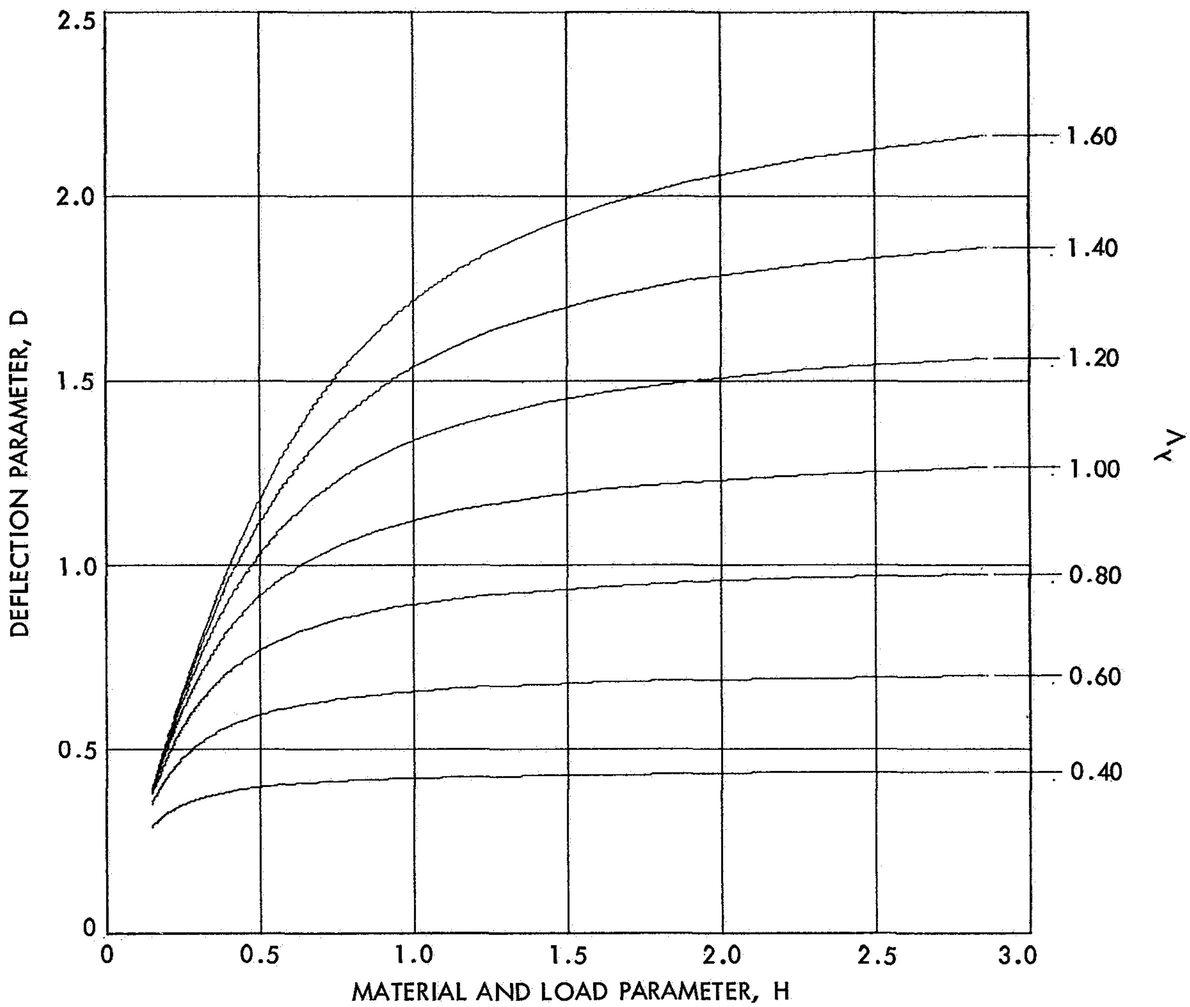


MATERIAL AND LOAD PARAMETER, H

STRESS-LIMITED

$\lambda_H = 0.60$

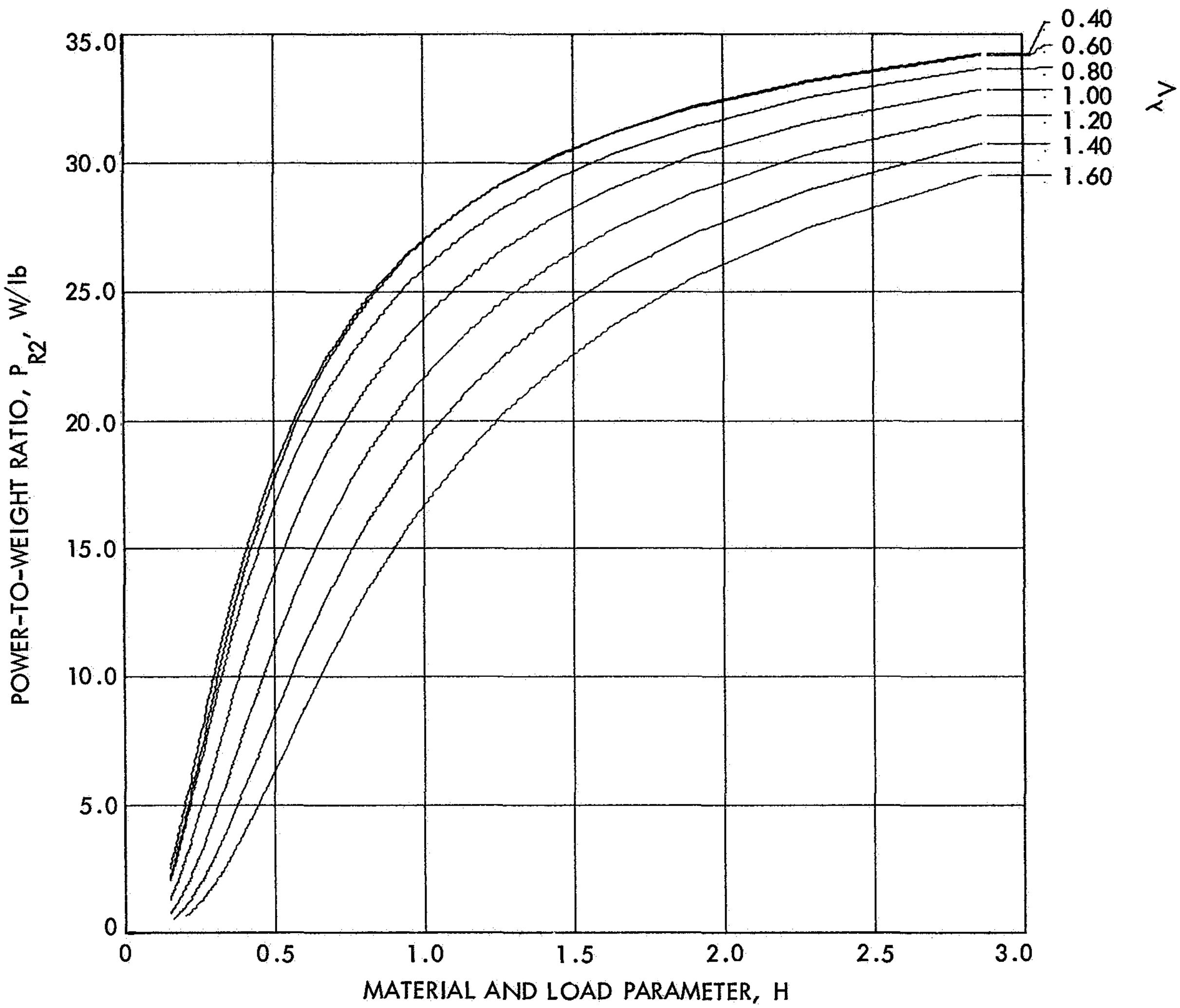
REFERENCE BOEING LASA



STRESS- LIMITED

$\lambda_H = 0.60$

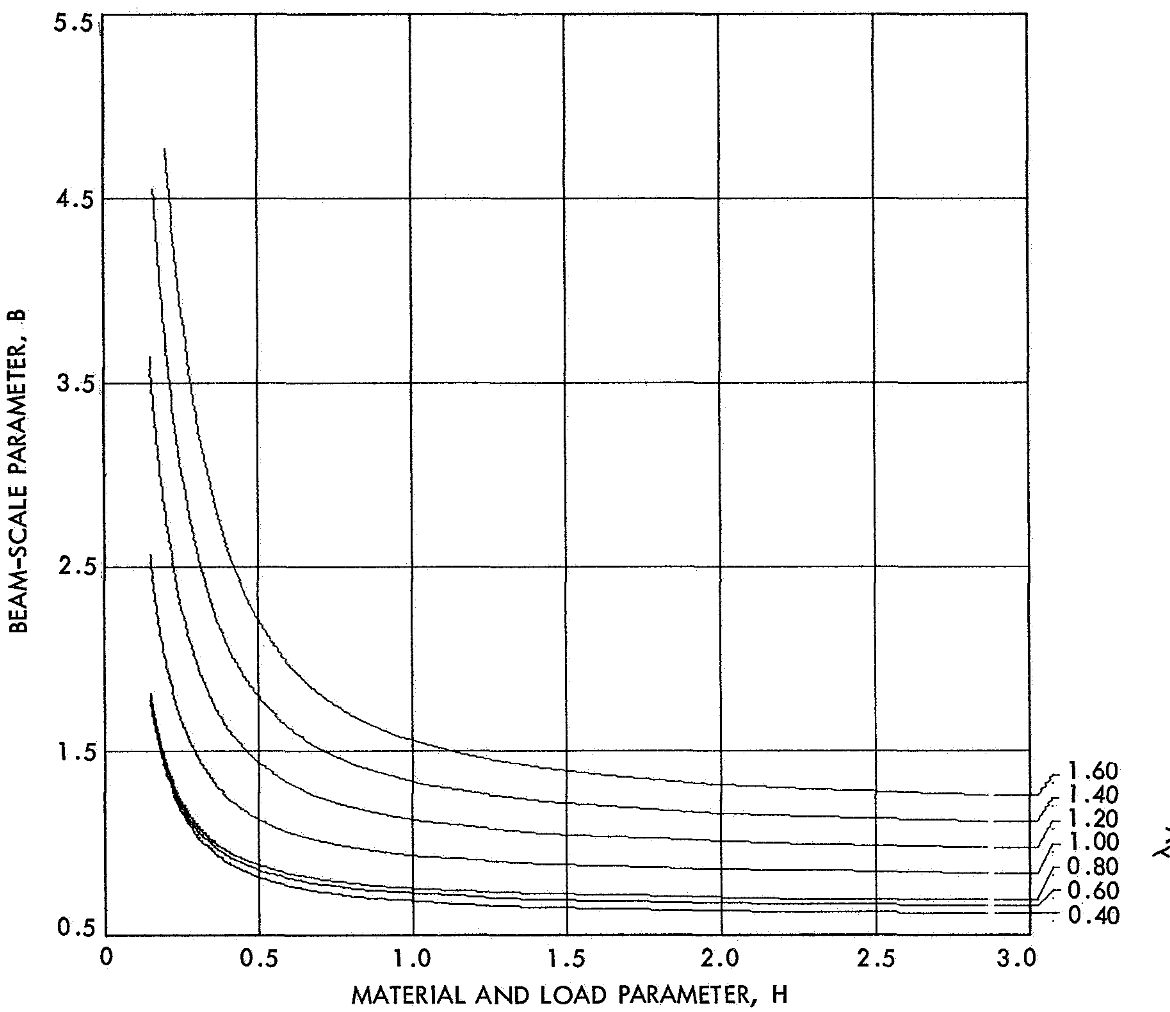
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 0.80$

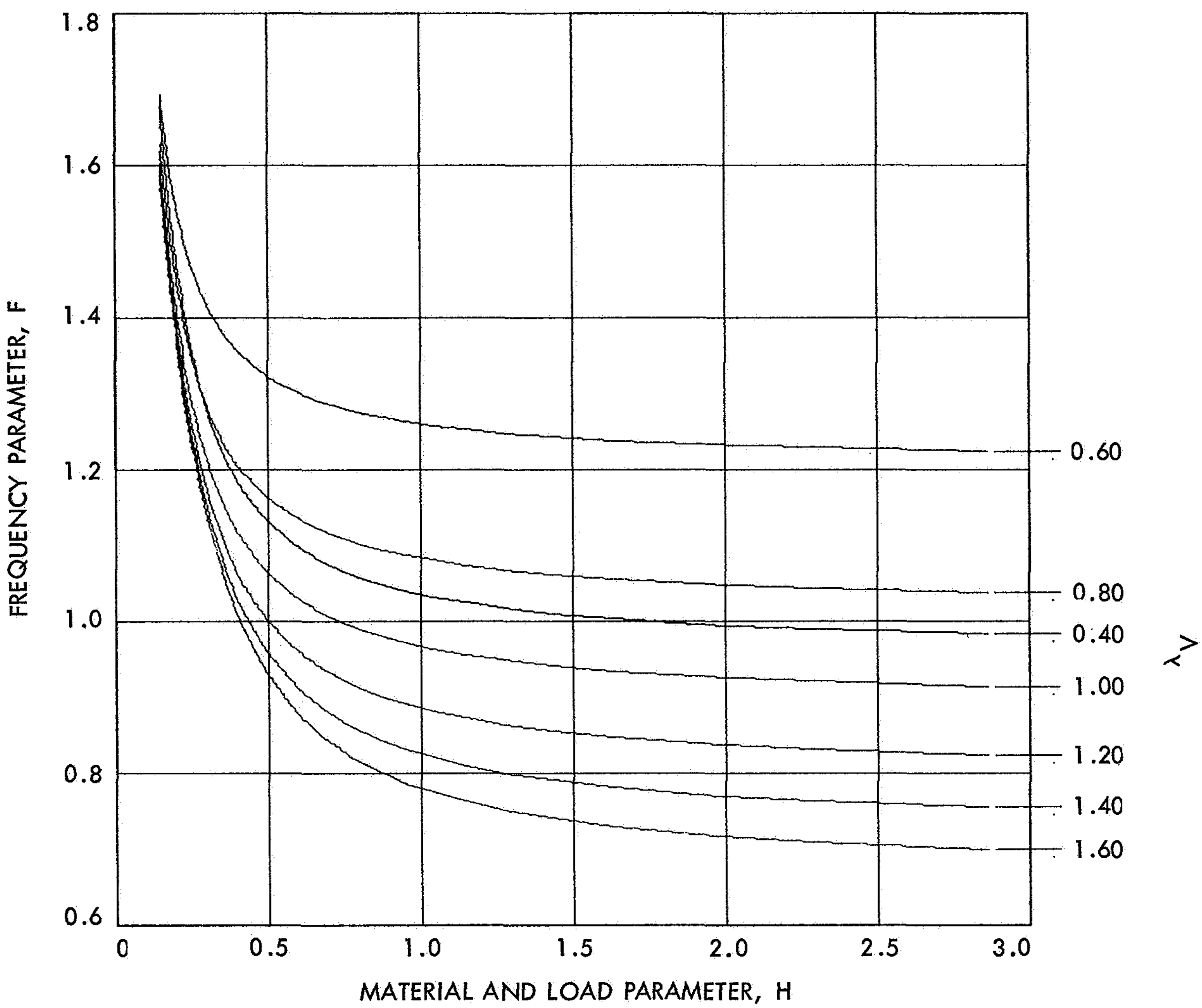
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 0.80$

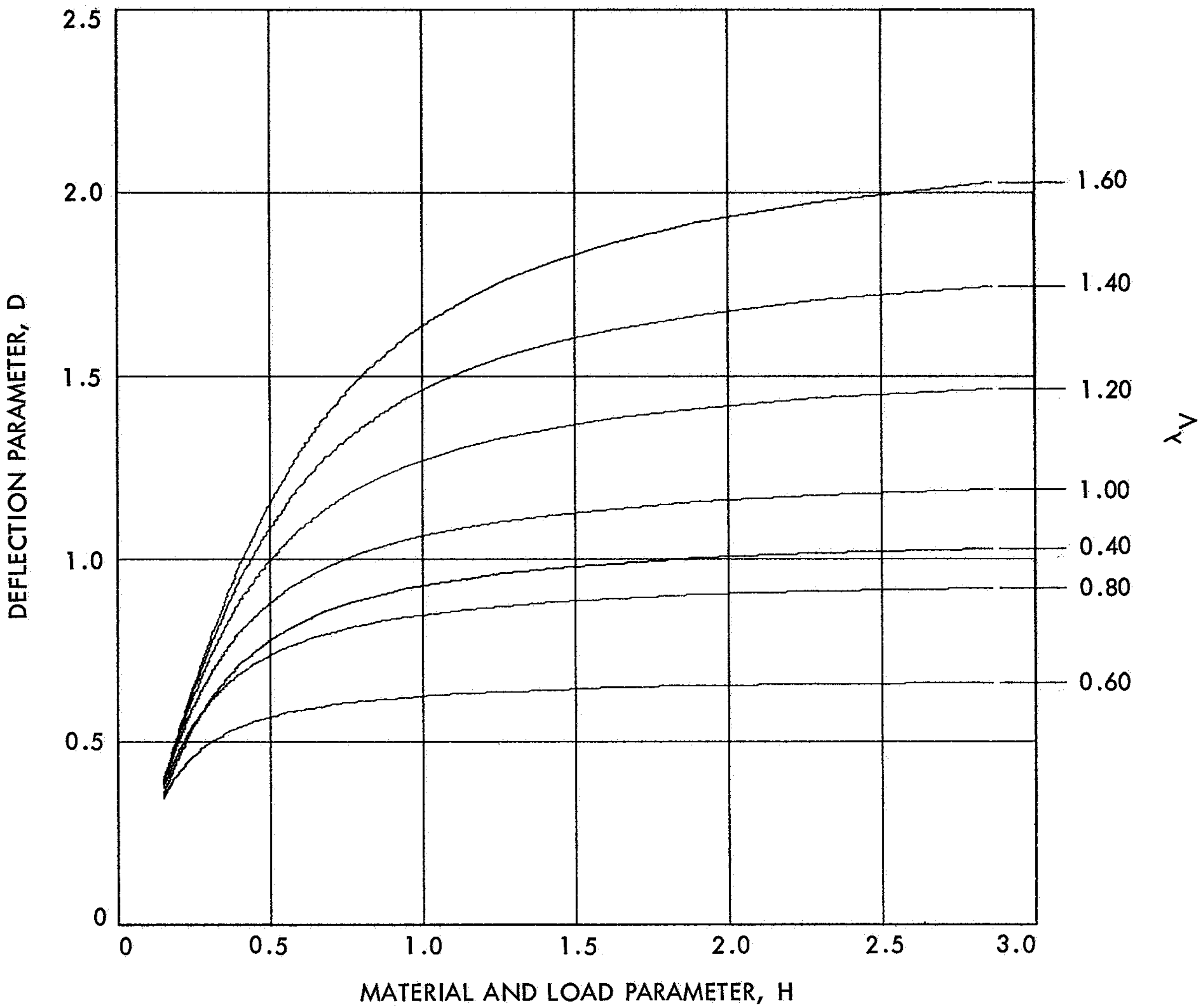
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 0.80$

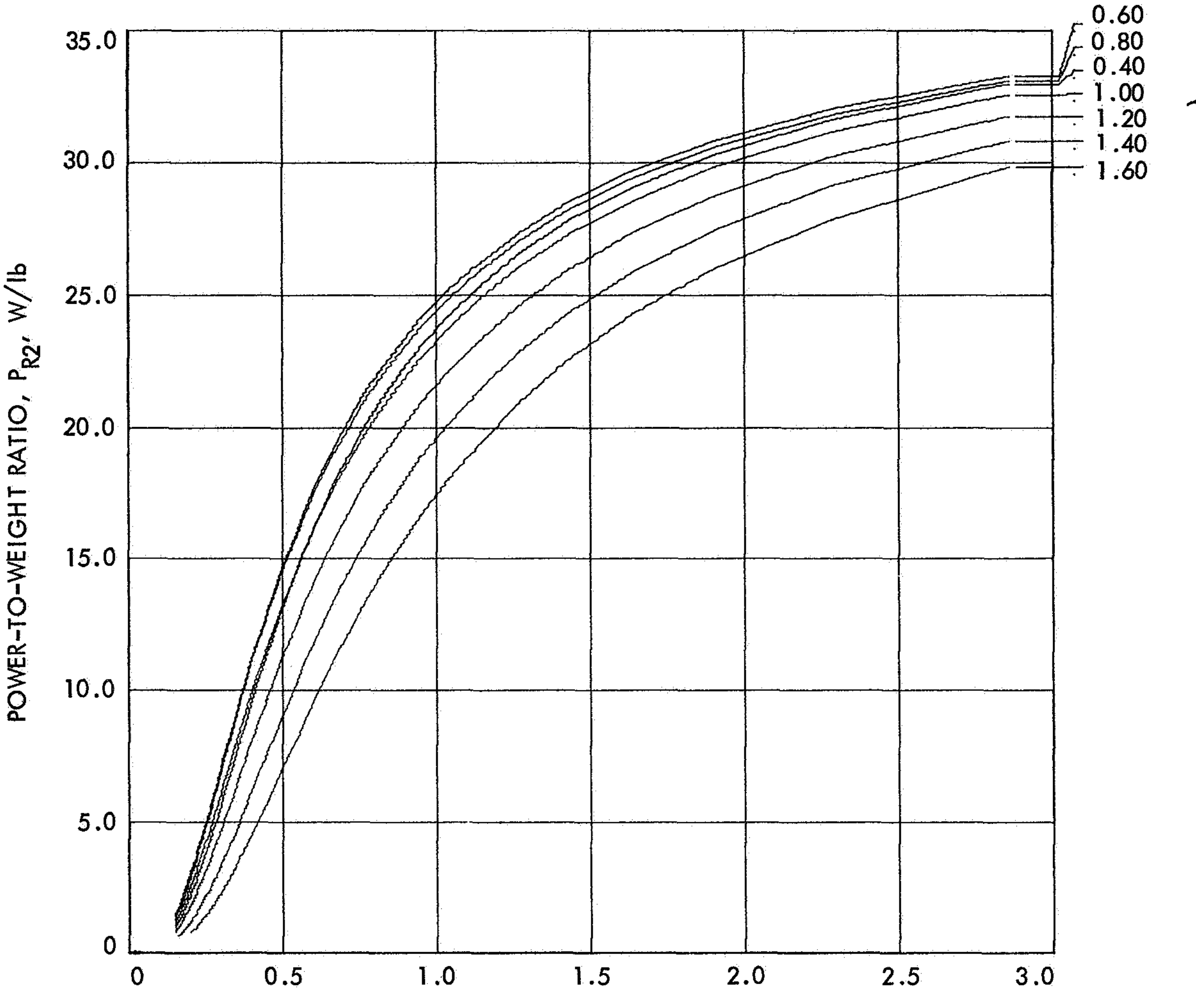
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 0.80$

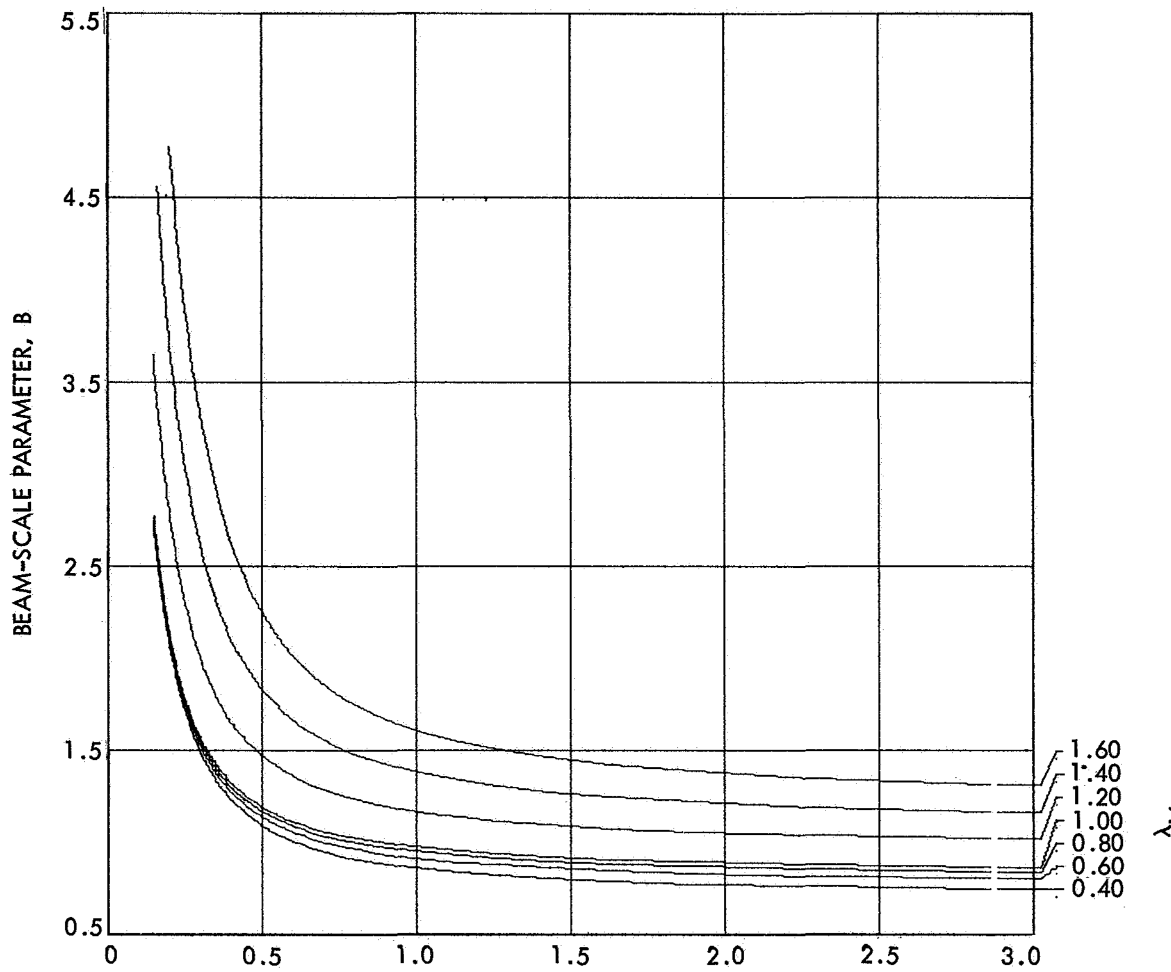
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 1.00$

REFERENCE BOEING LASA

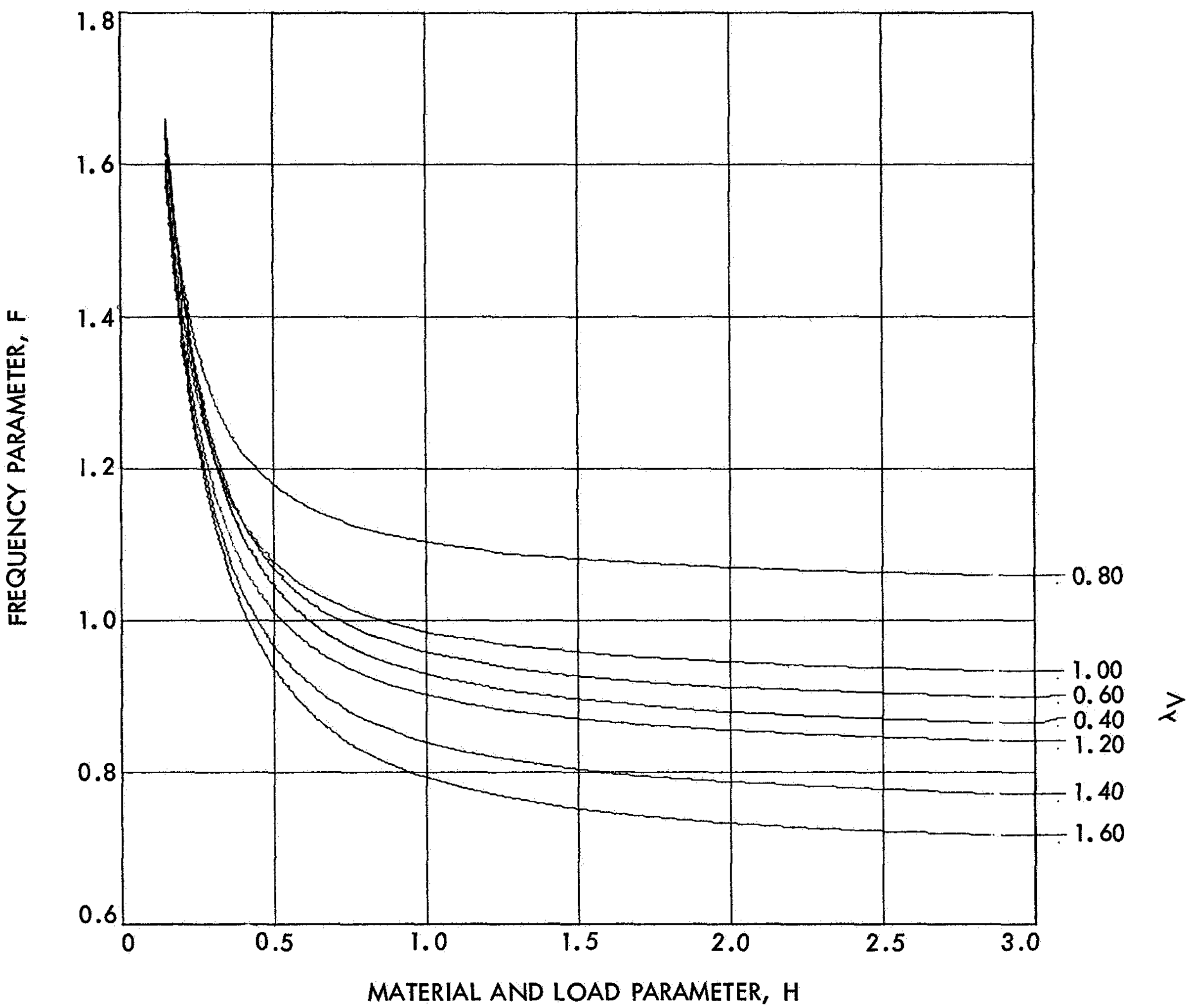


MATERIAL AND LOAD PARAMETER, H

STRESS-LIMITED

$\lambda_H = 1.00$

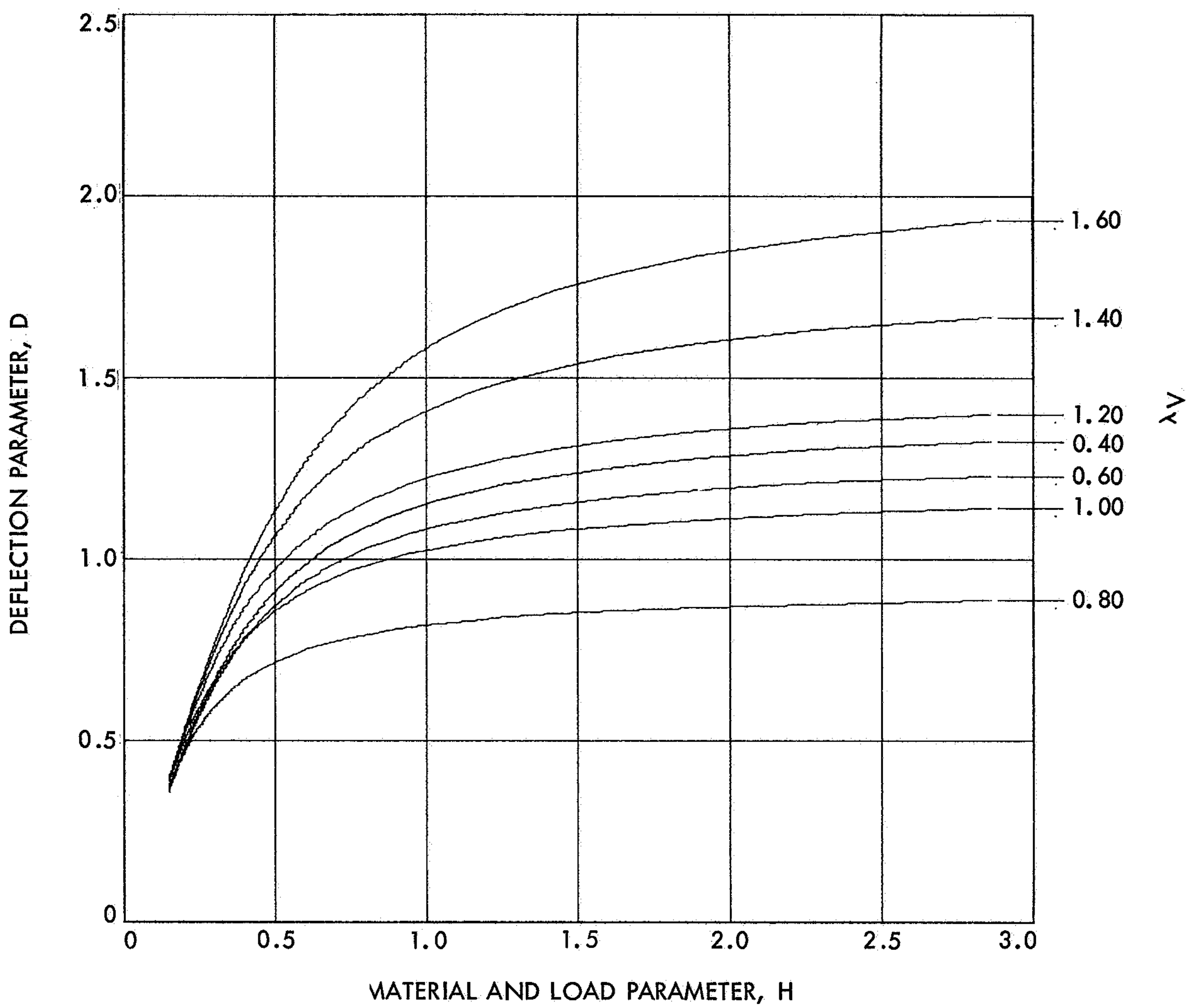
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 1.00$

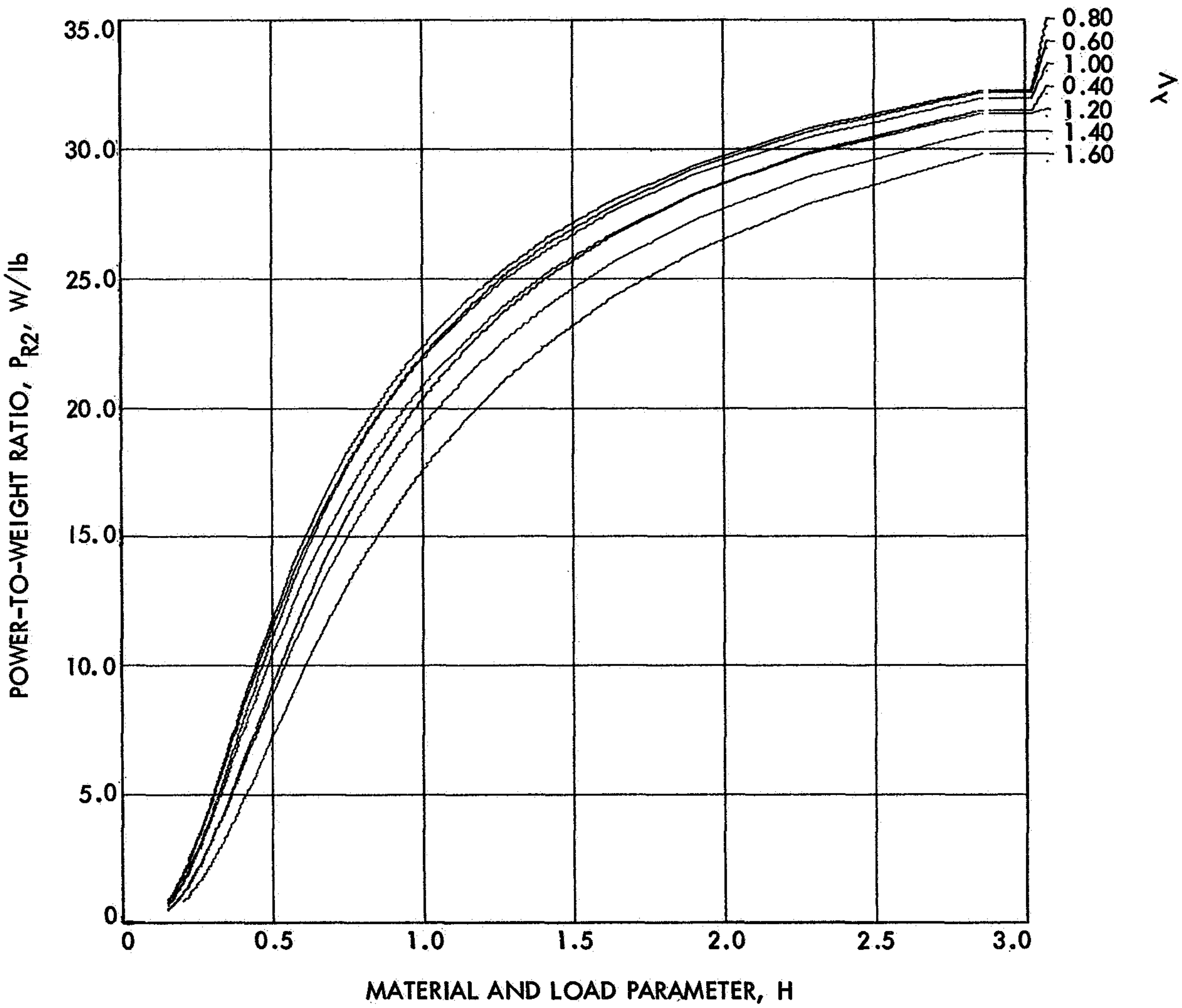
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 1.00$

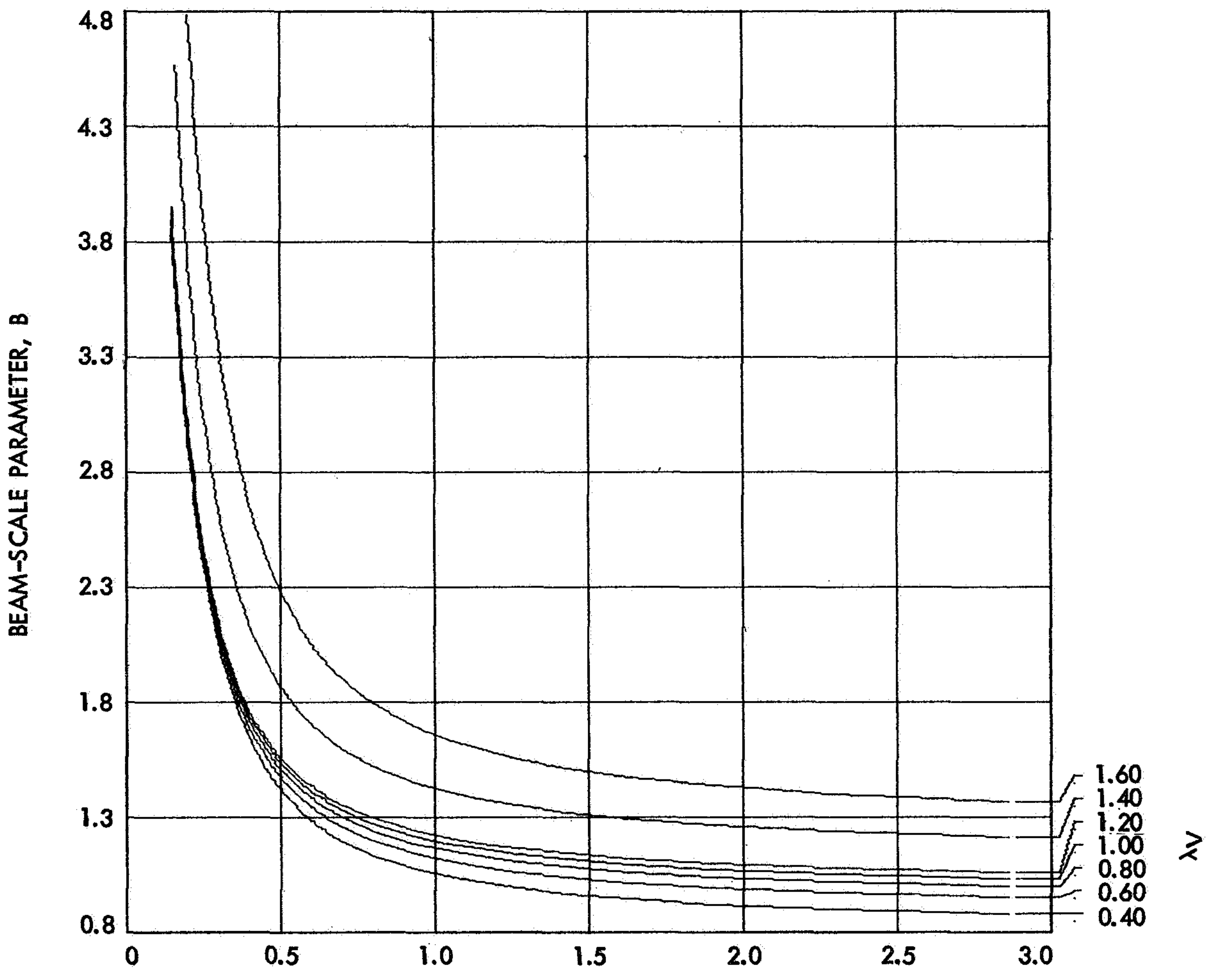
REFERENCE BOEING LASA



STRESS-LIMITED

$\lambda_H = 1.20$

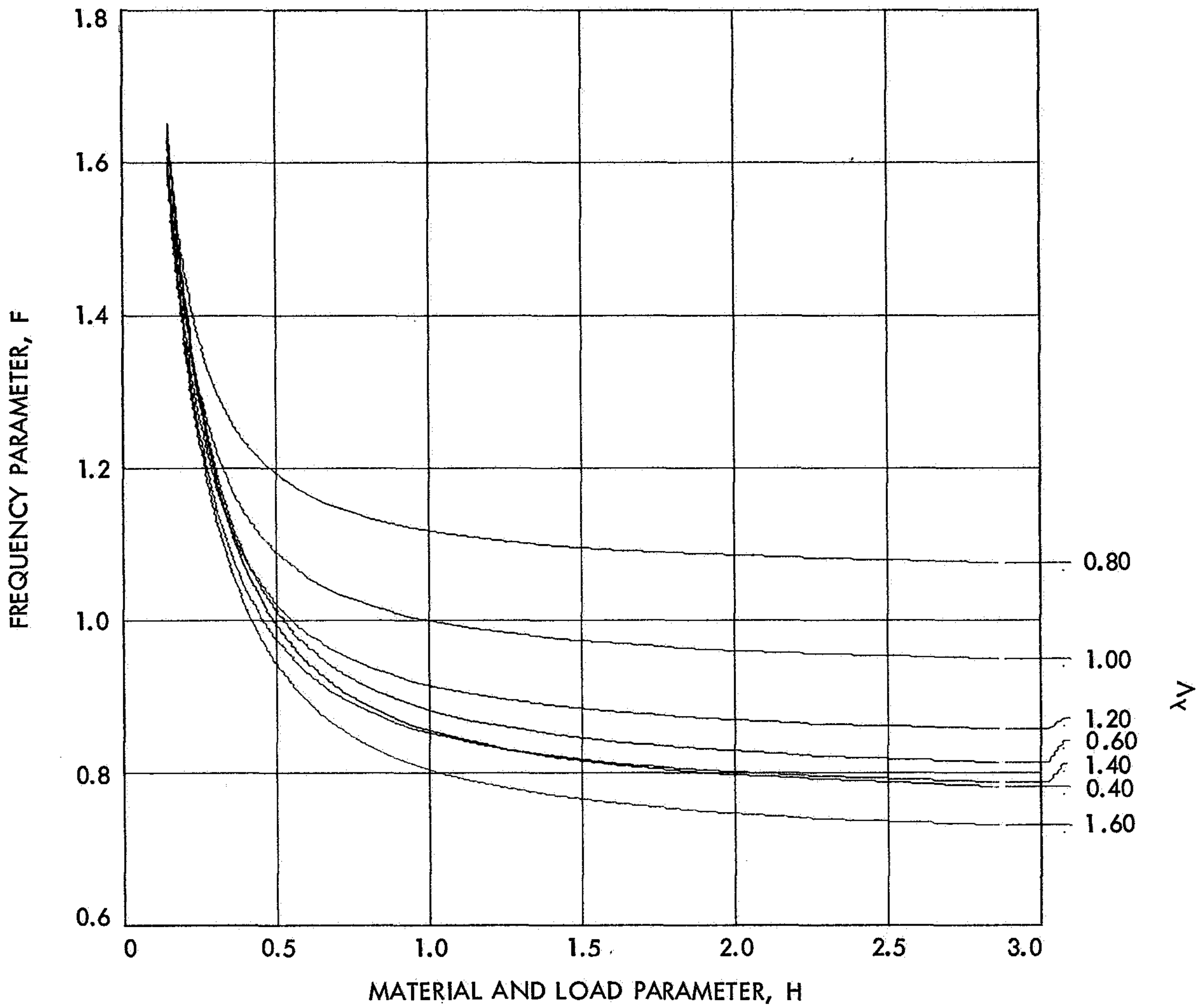
REFERENCE BOEING LASA



MATERIAL AND LOAD PARAMETER, H

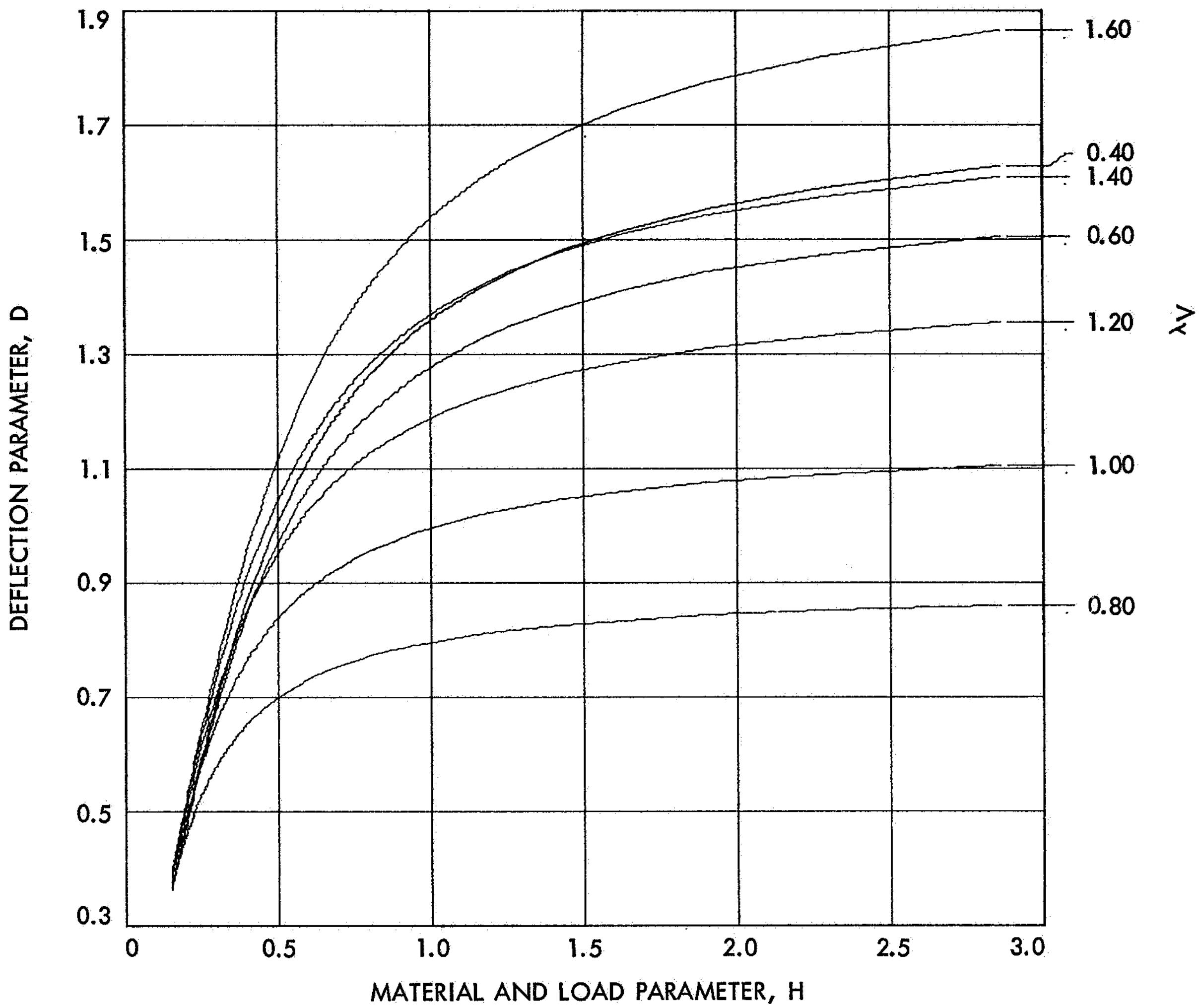
STRESS - LIMITED

$\lambda_H = 1.20$
REFERENCE BOEING LASA



STRESS - LIMITED

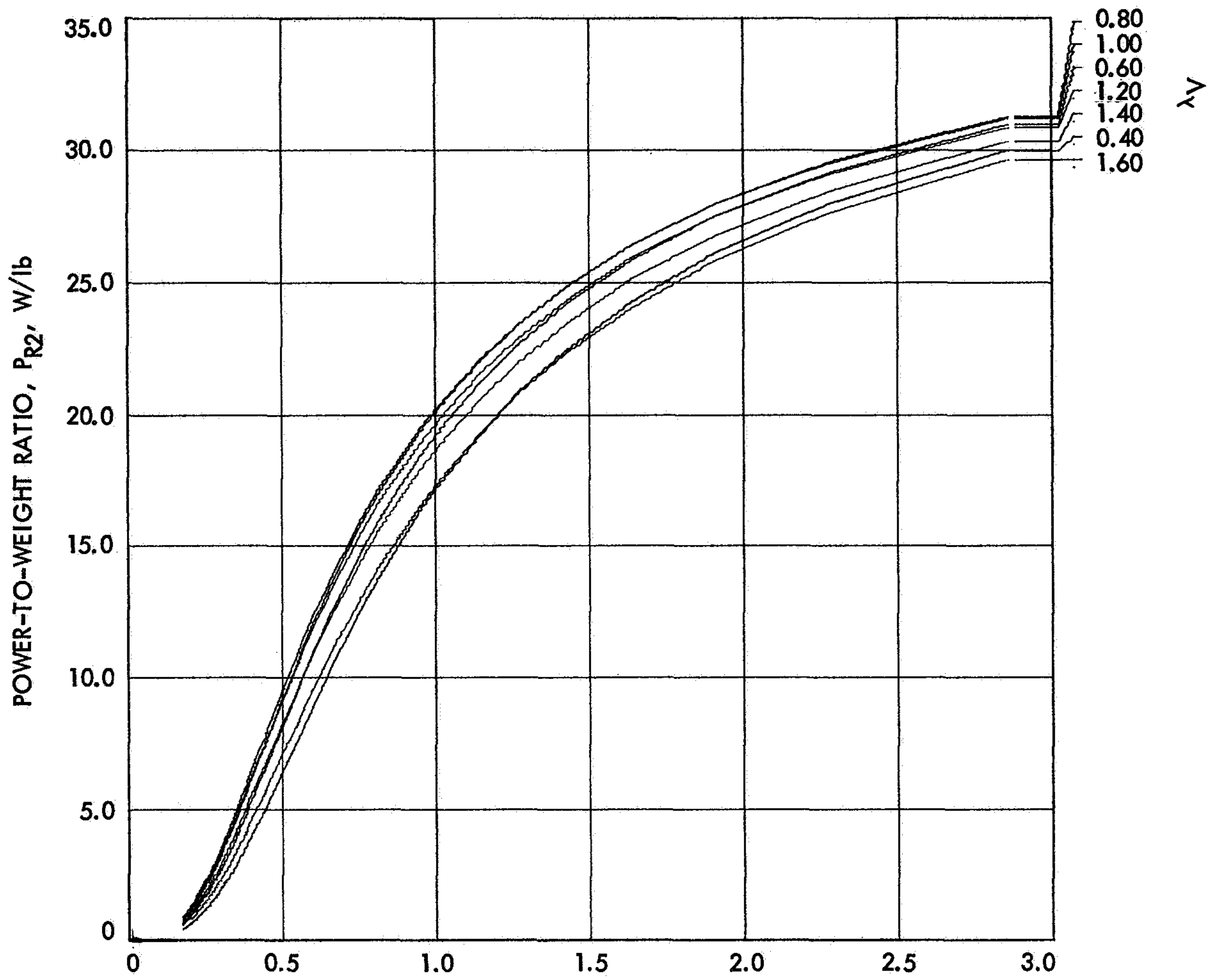
$\lambda_H = 1.20$
REFERENCE BOEING LASA



STRESS - LIMITED

$\lambda_H = 1.20$

REFERENCE BOEING LASA

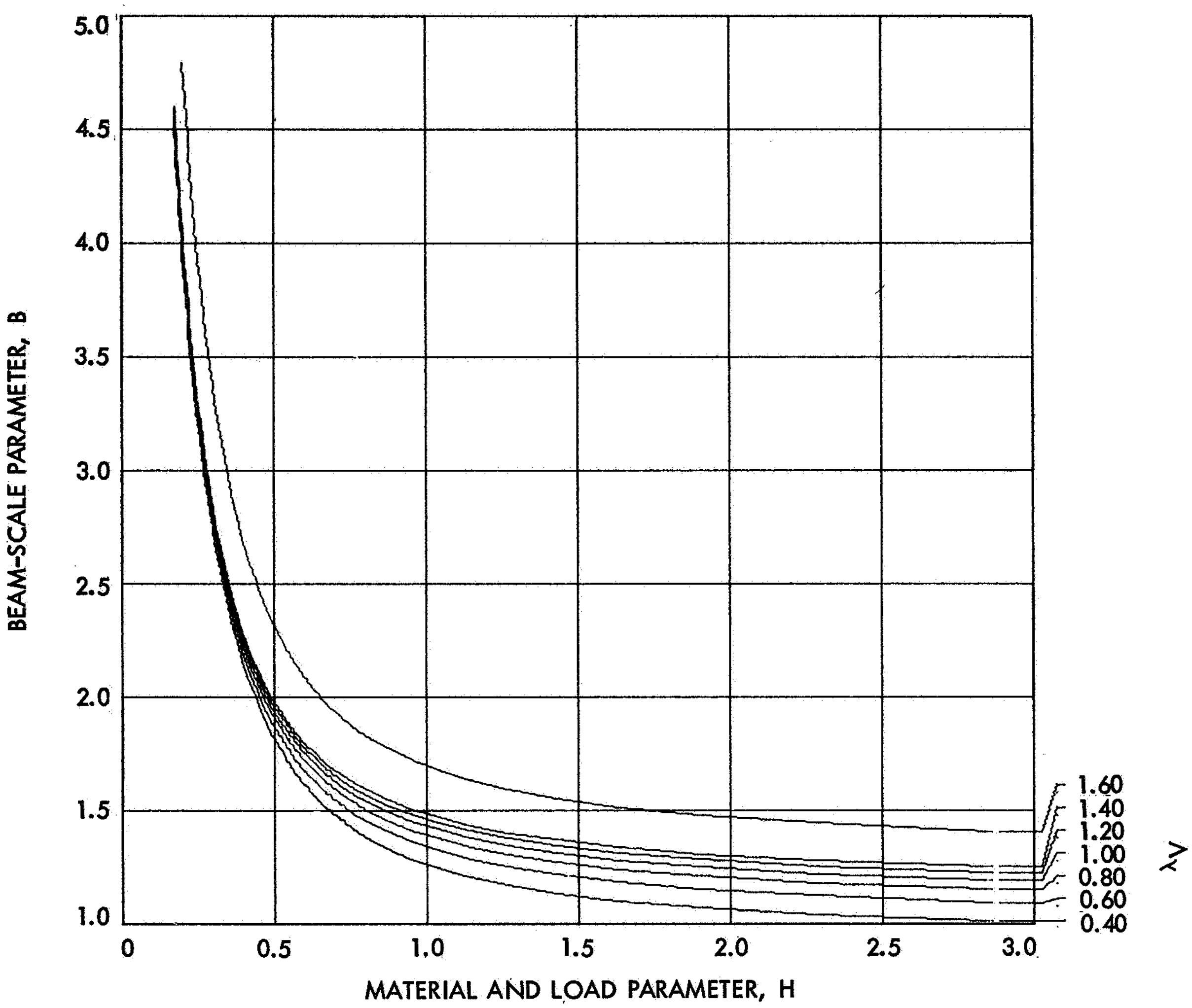


MATERIAL AND LOAD PARAMETER, H

STRESS - LIMITED

$\lambda_H = 1.40$

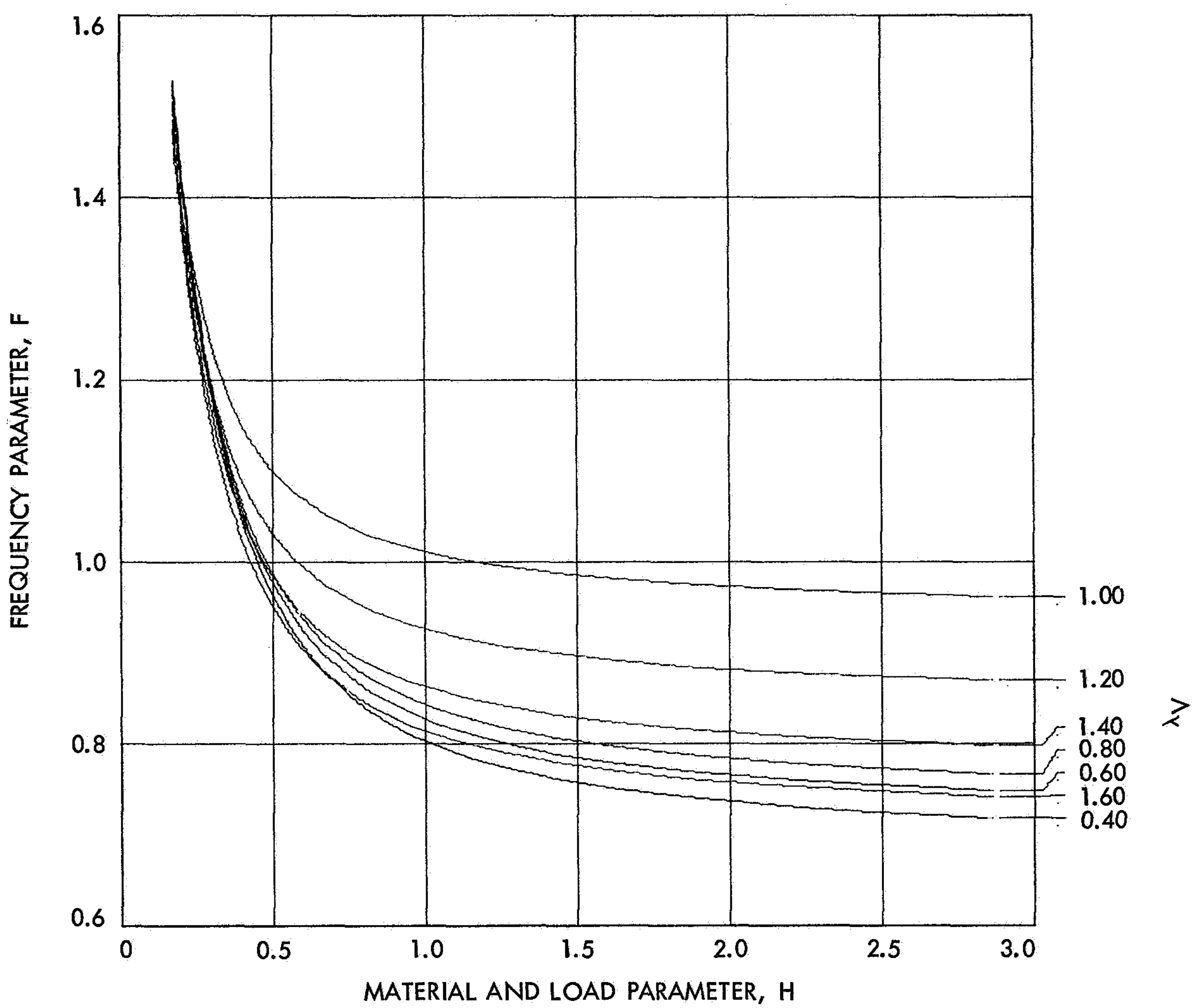
REFERENCE BOEING LASA



STRESS – LIMITED

$\lambda_H = 1.40$

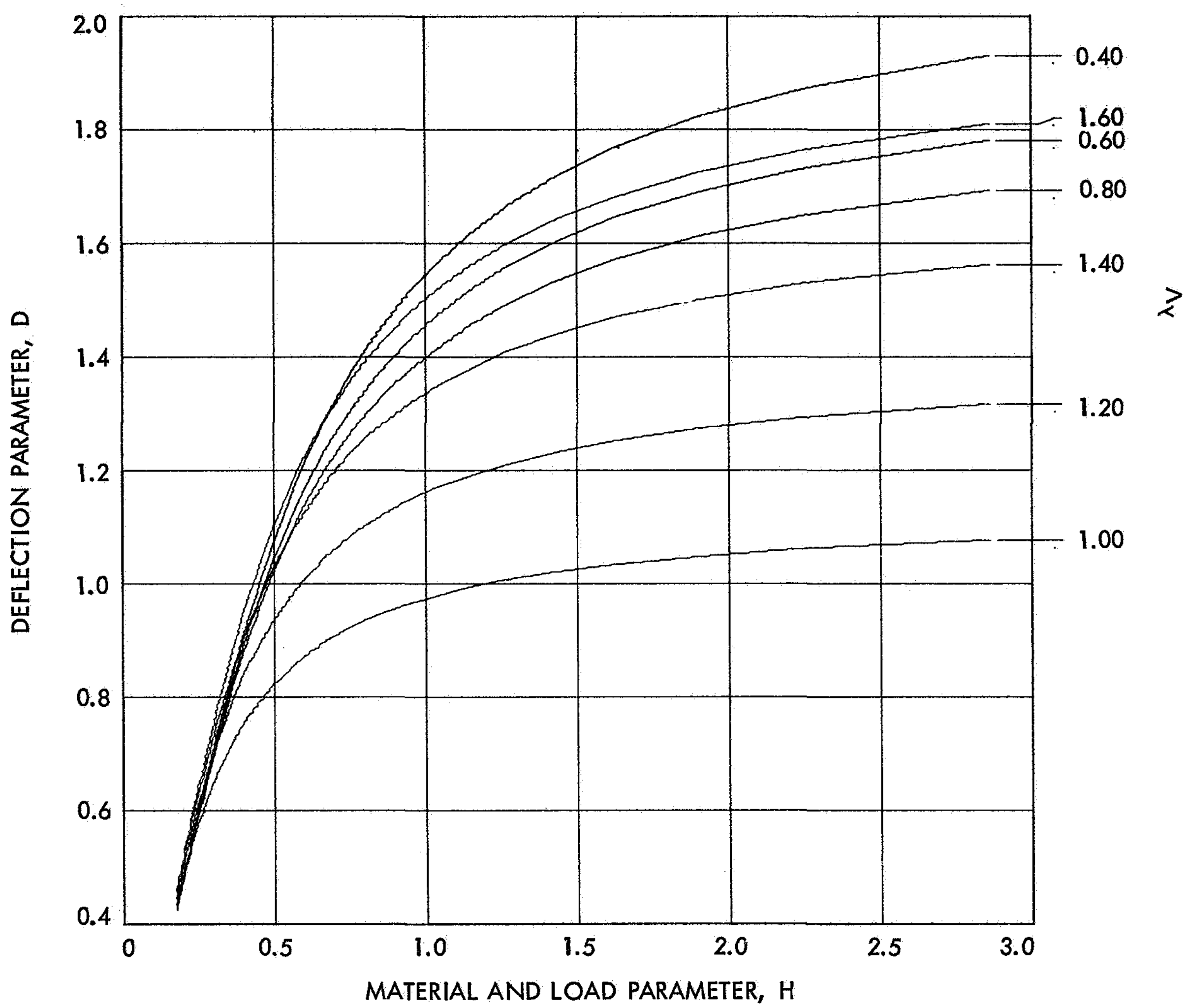
REFERENCE BOEING LASA



STRESS - LIMITED

$\lambda_H = 1.40$

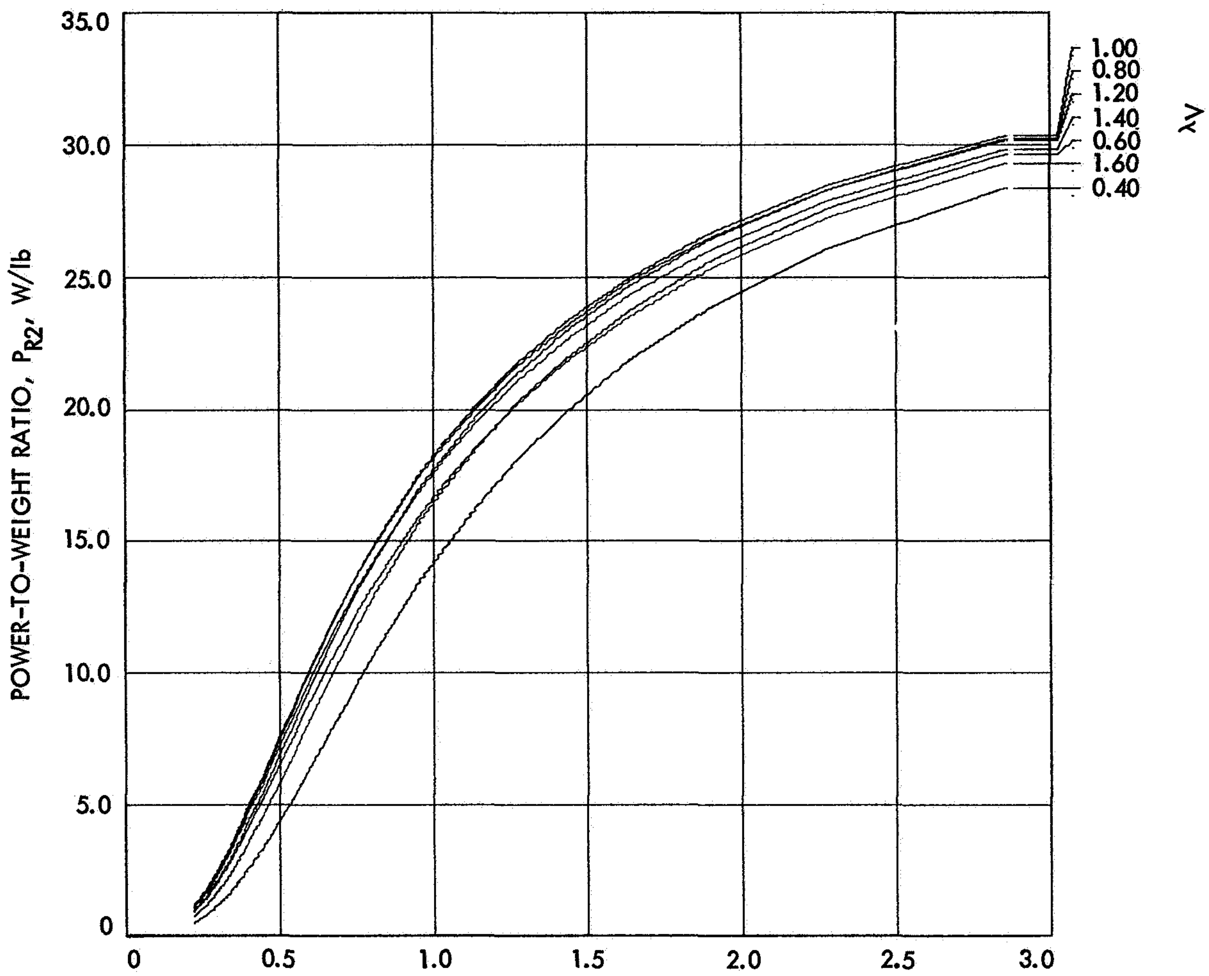
REFERENCE BOEING LASA



STRESS - LIMITED

$\lambda_H = 1.40$

REFERENCE BOEING LASA

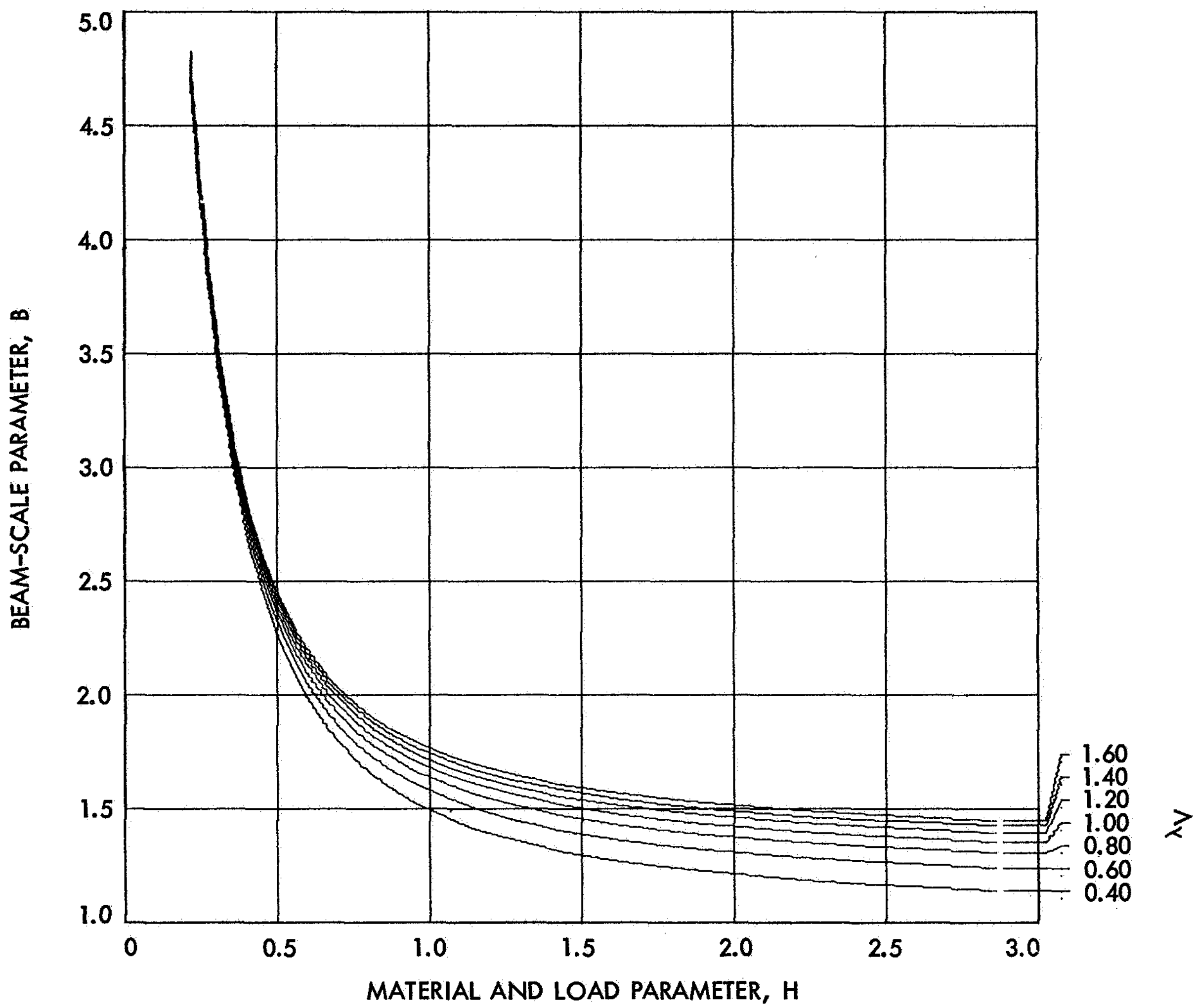


MATERIAL AND LOAD PARAMETER, H

STRESS - LIMITED

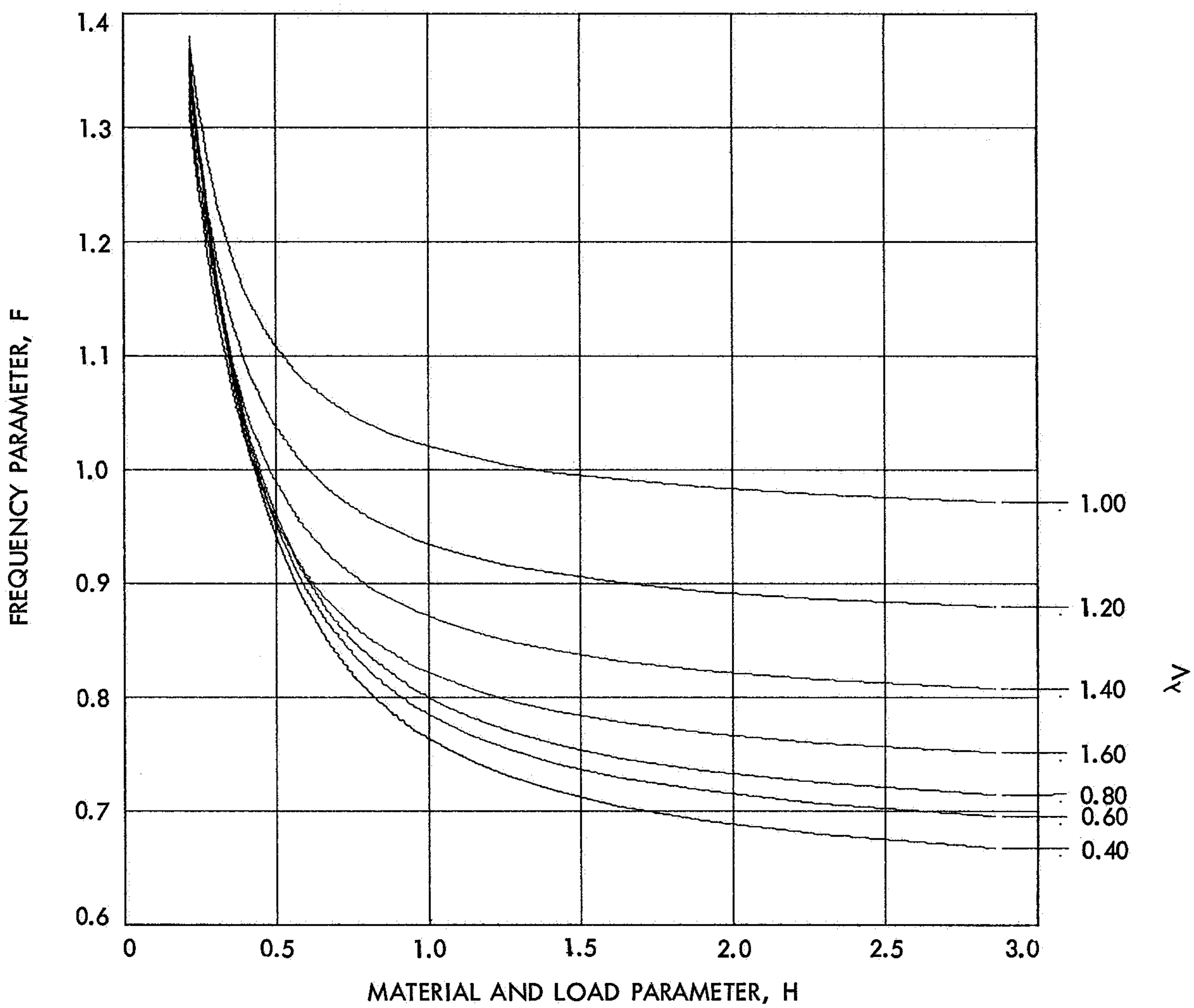
$\lambda_H = 1.60$

REFERENCE BOEING LASA



STRESS - LIMITED

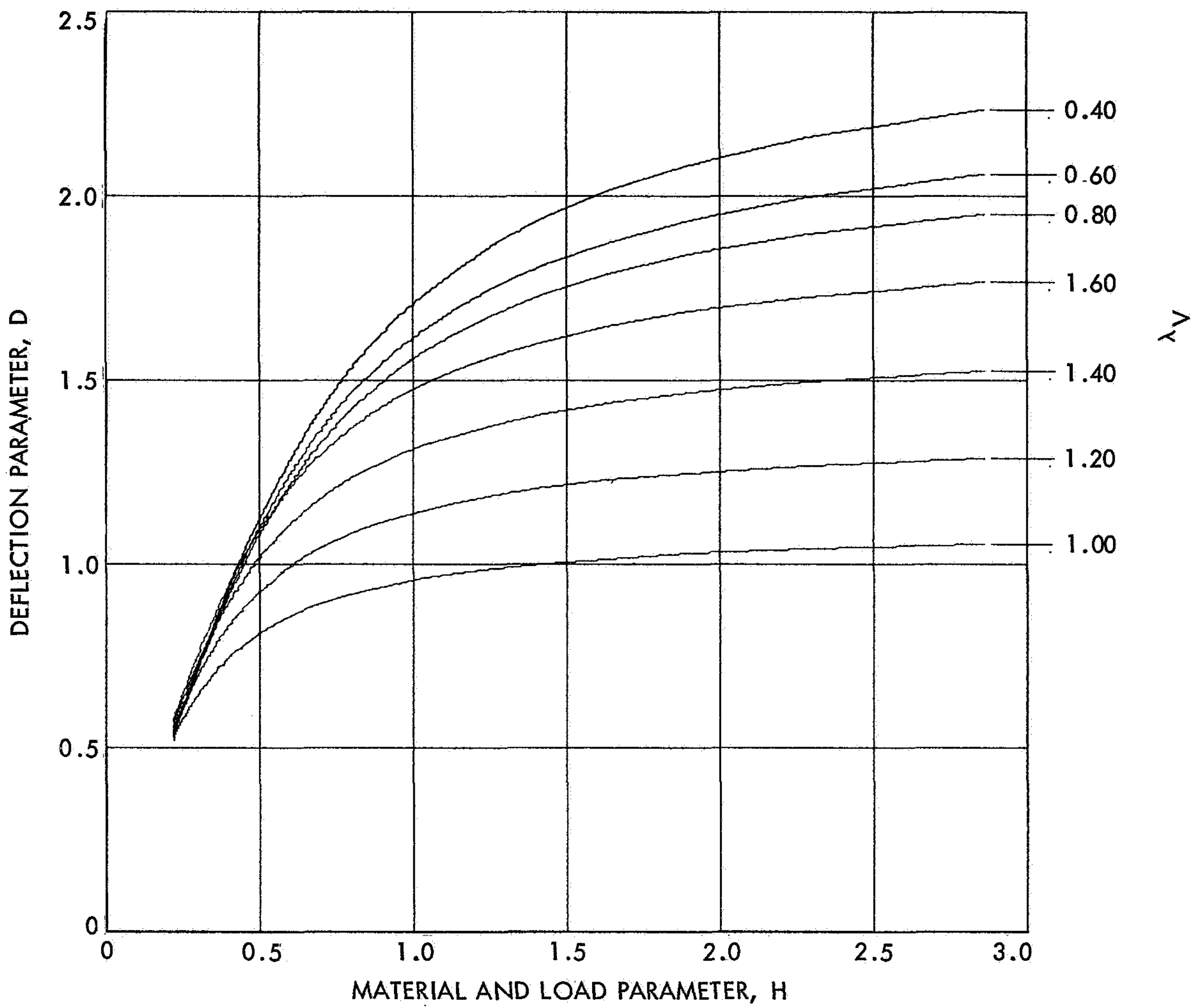
$\lambda_H = 1.60$
REFERENCE BOEING LASA



STRESS - LIMITED

$\lambda_H = 1.60$

REFERENCE BOEING LASA



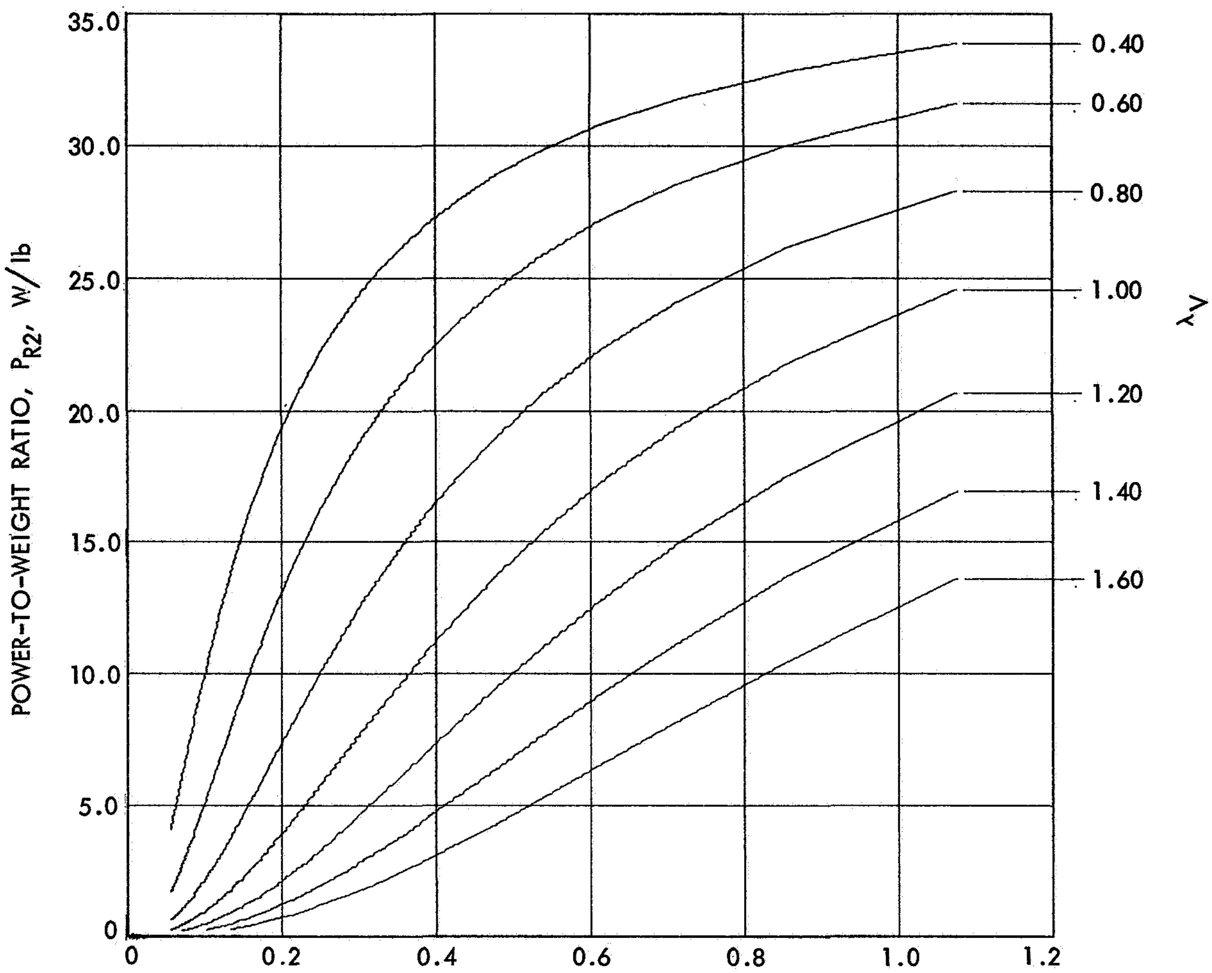
STRESS-LIMITED

$\lambda_H = 1.60$

REFERENCE BOEING LASA

III. Frequency-Limited Design—Condition C

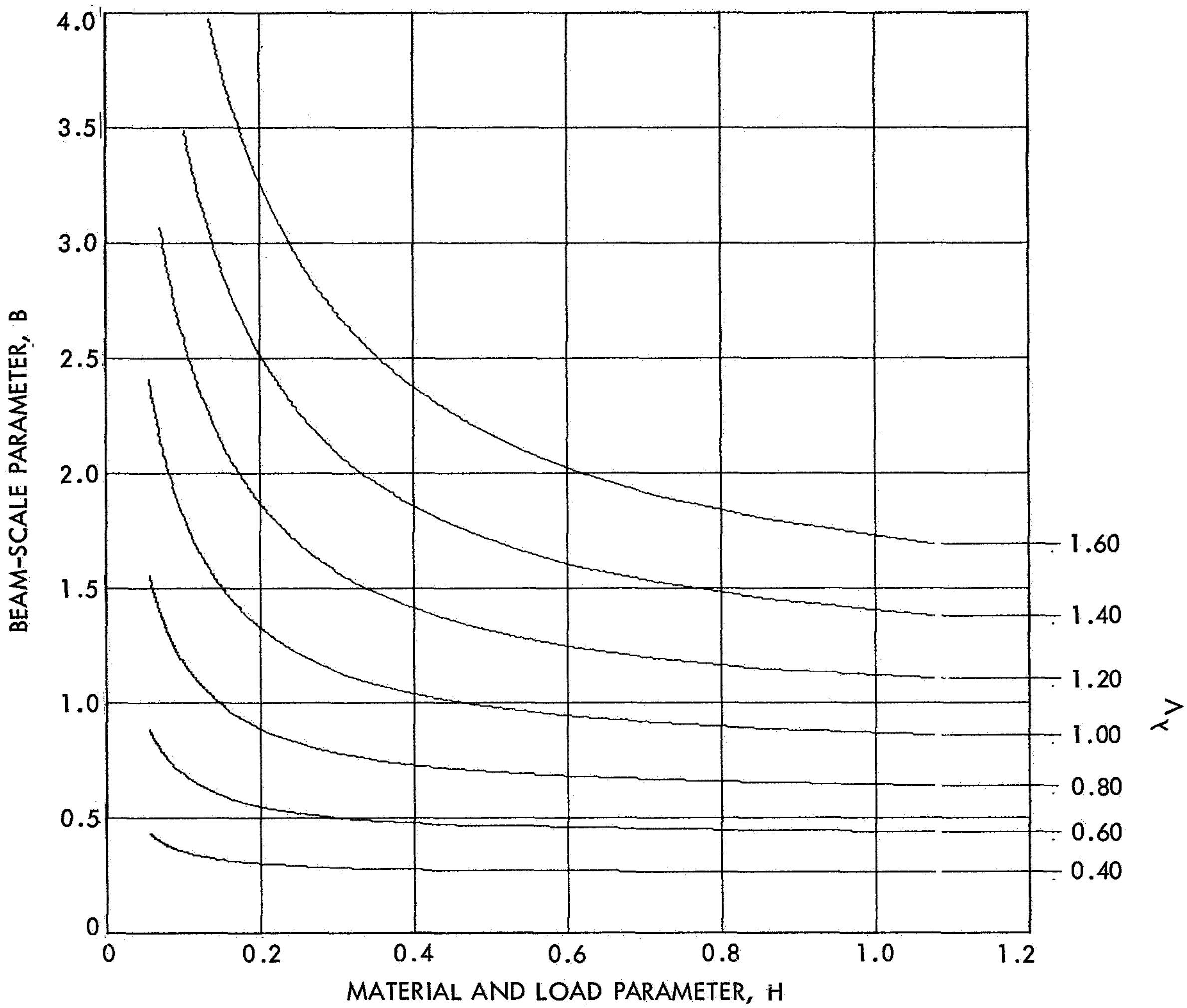
Each set of constant values of λ_H contains three graphs plotted for the parameters of P_{R2}, B, and S, defined in Section V-C; these are plotted vs the parameter H for constant values of λ_H.



FREQUENCY-LIMITED

$\lambda_H = 0.40$

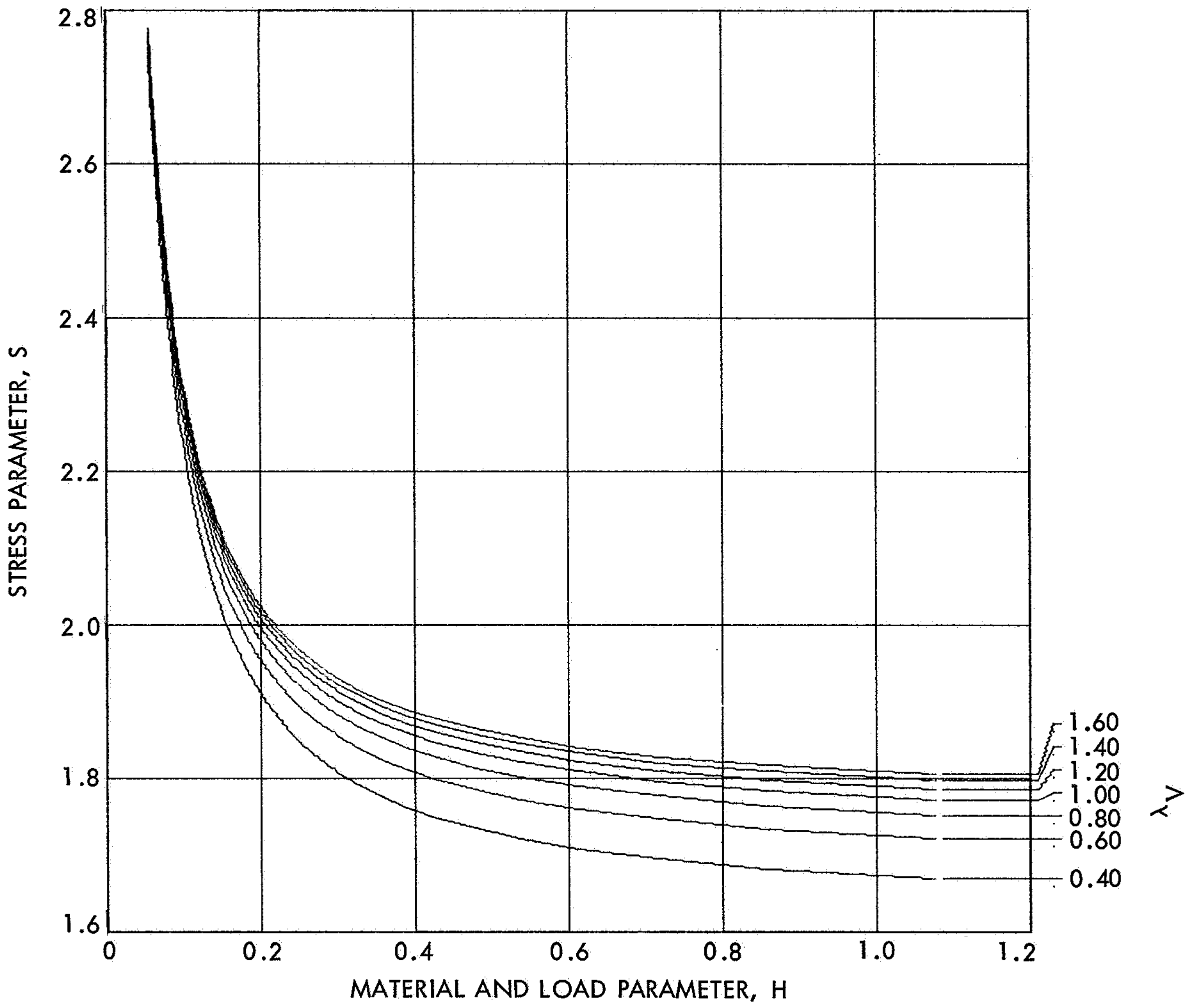
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 0.40$

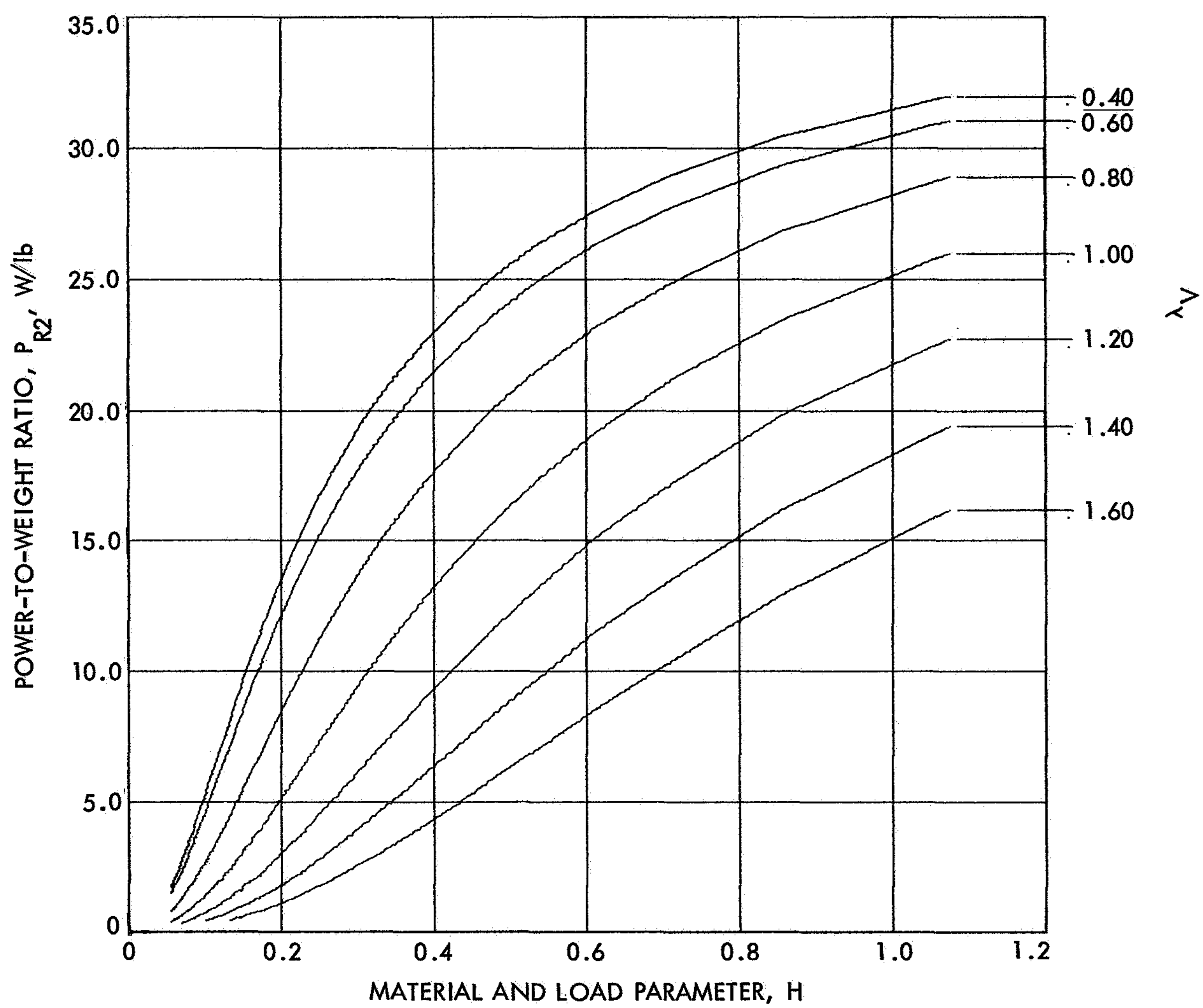
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 0.40$

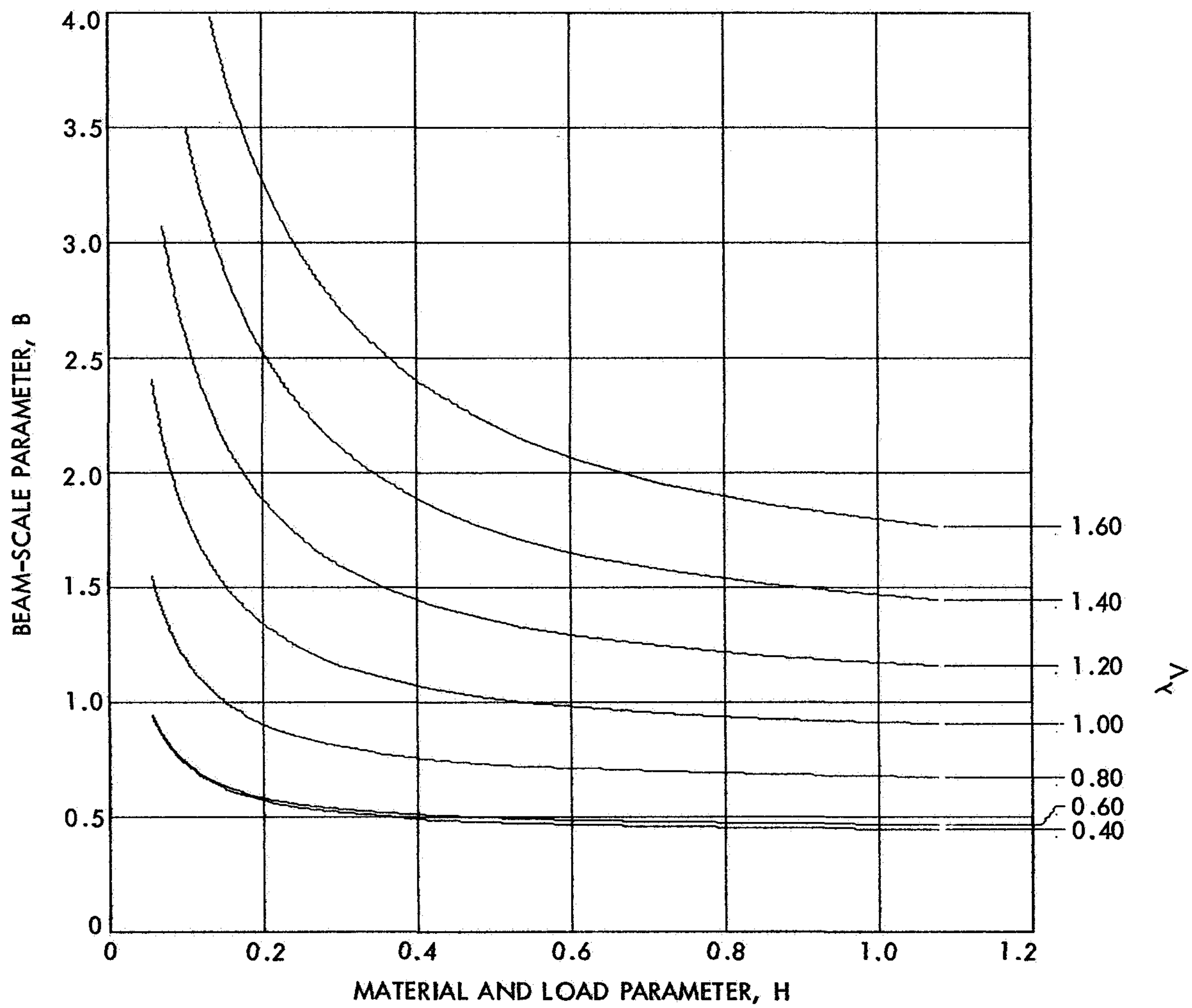
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 0.60$

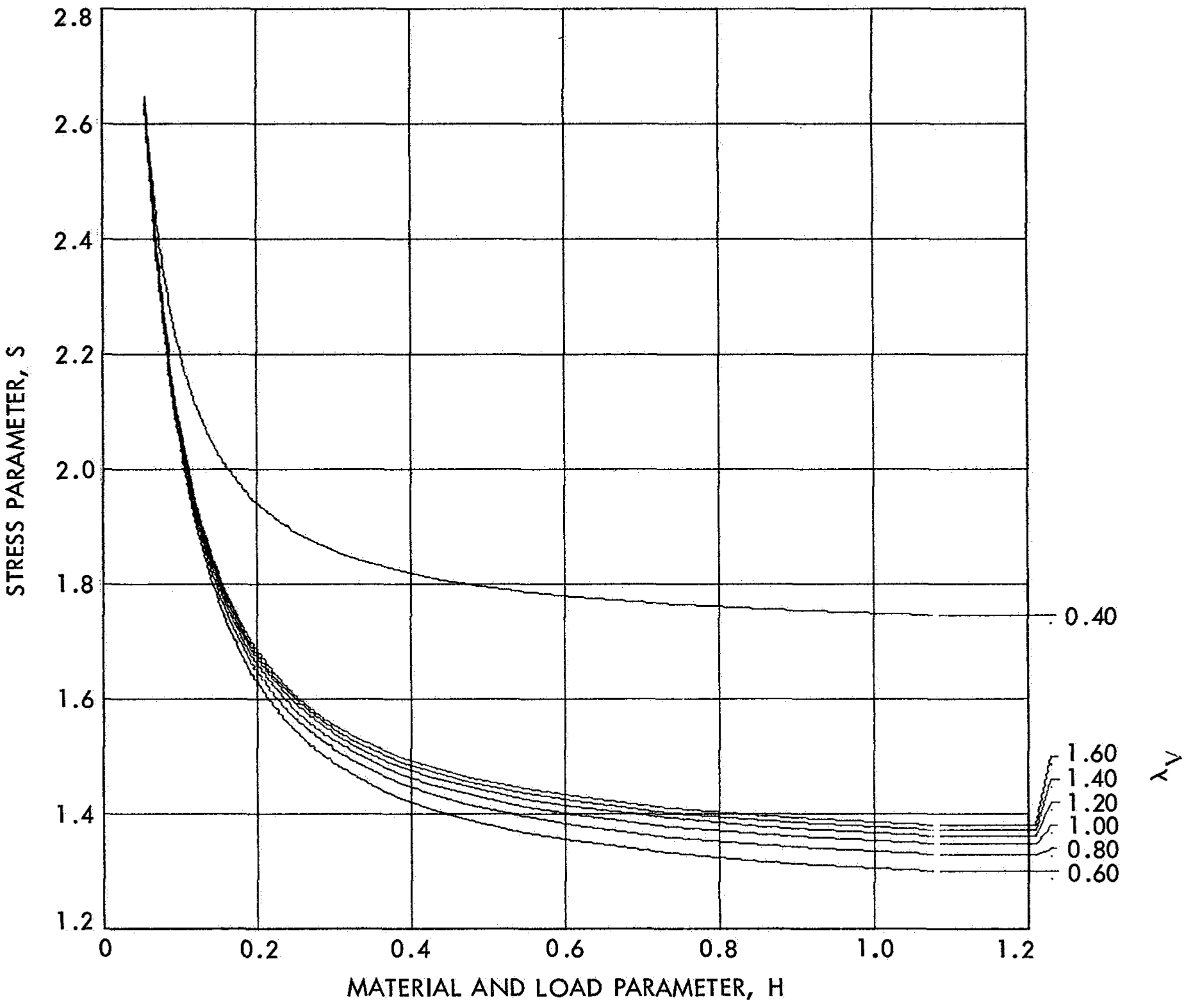
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 0.60$

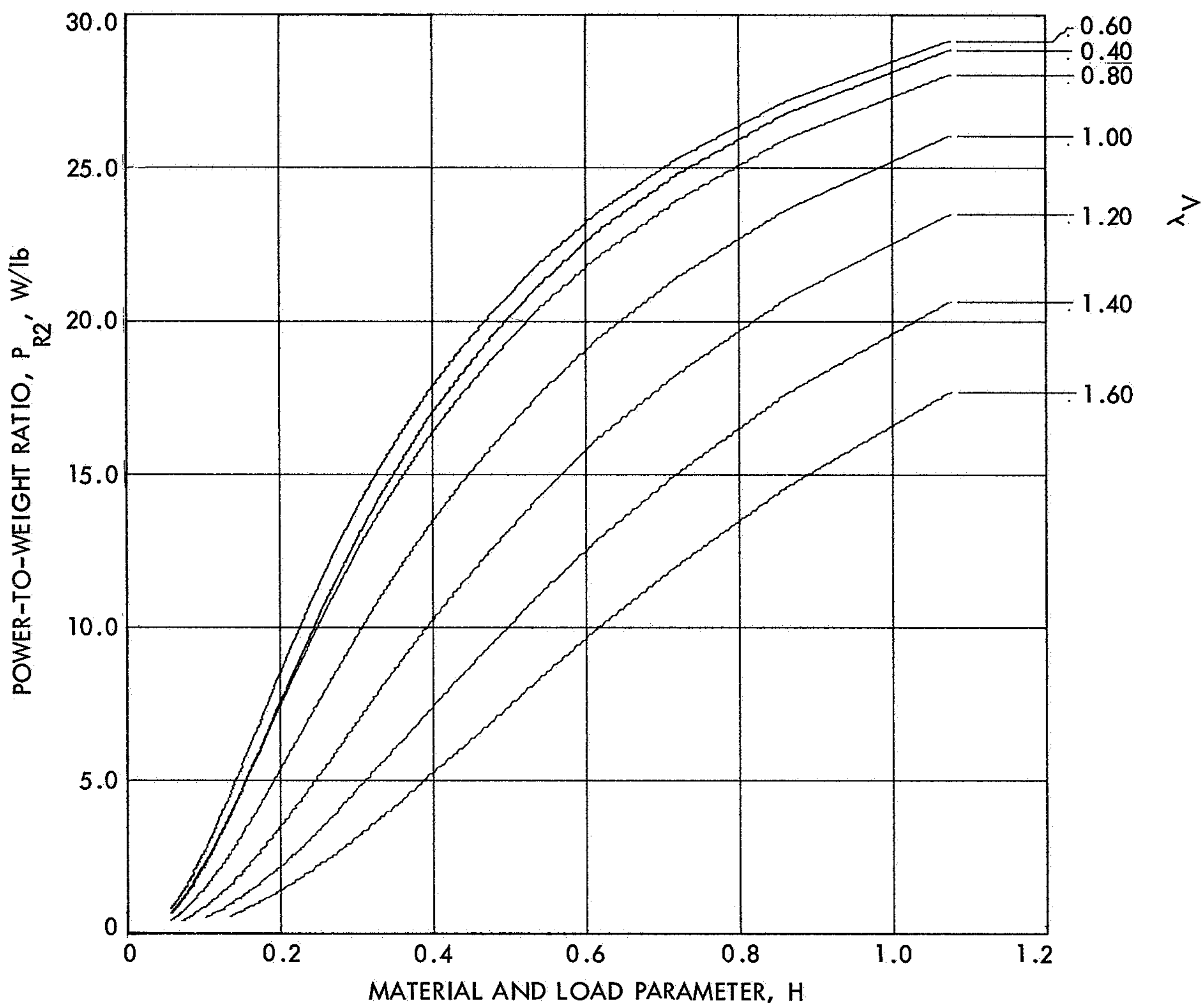
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 0.60$

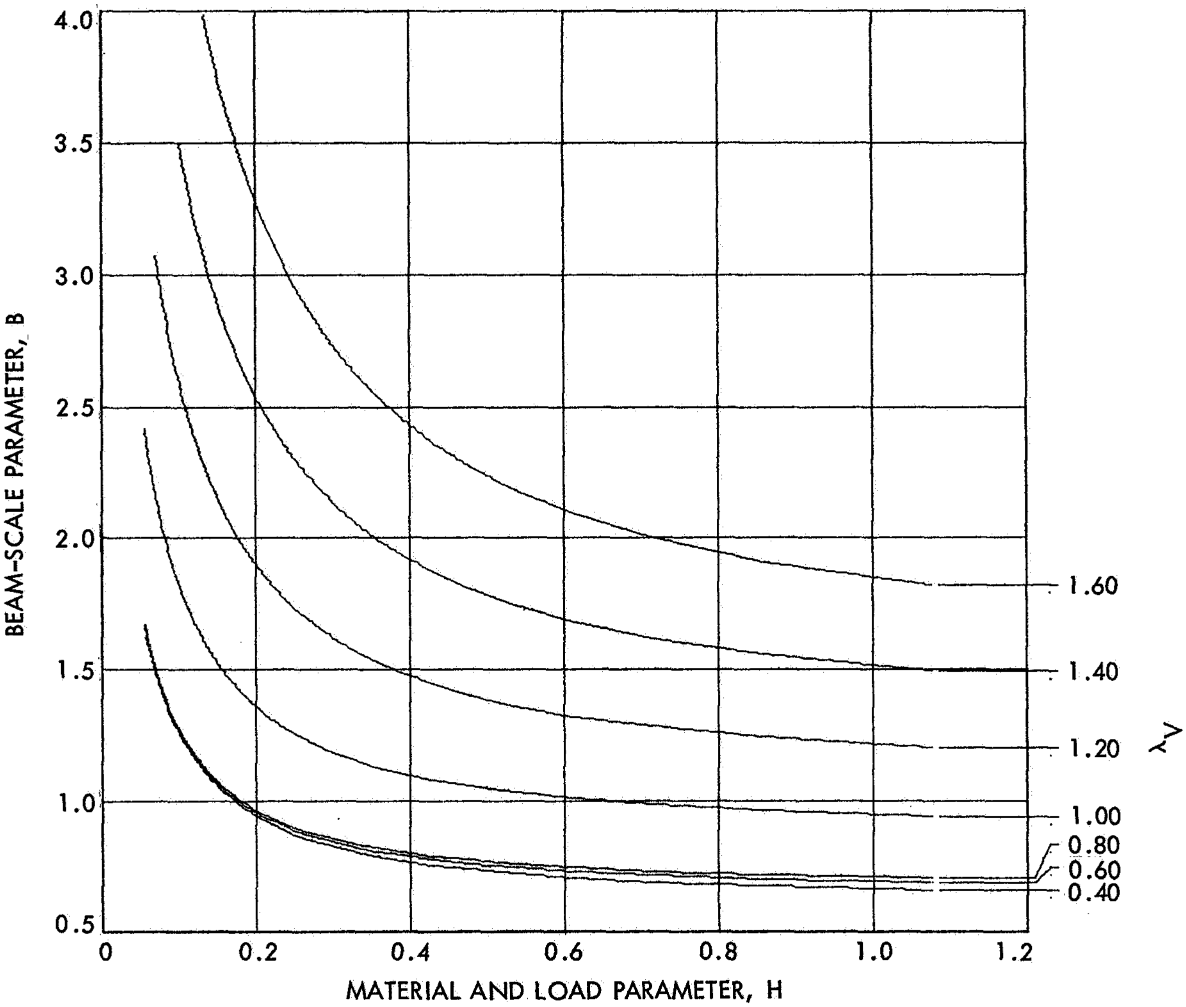
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 0.80$

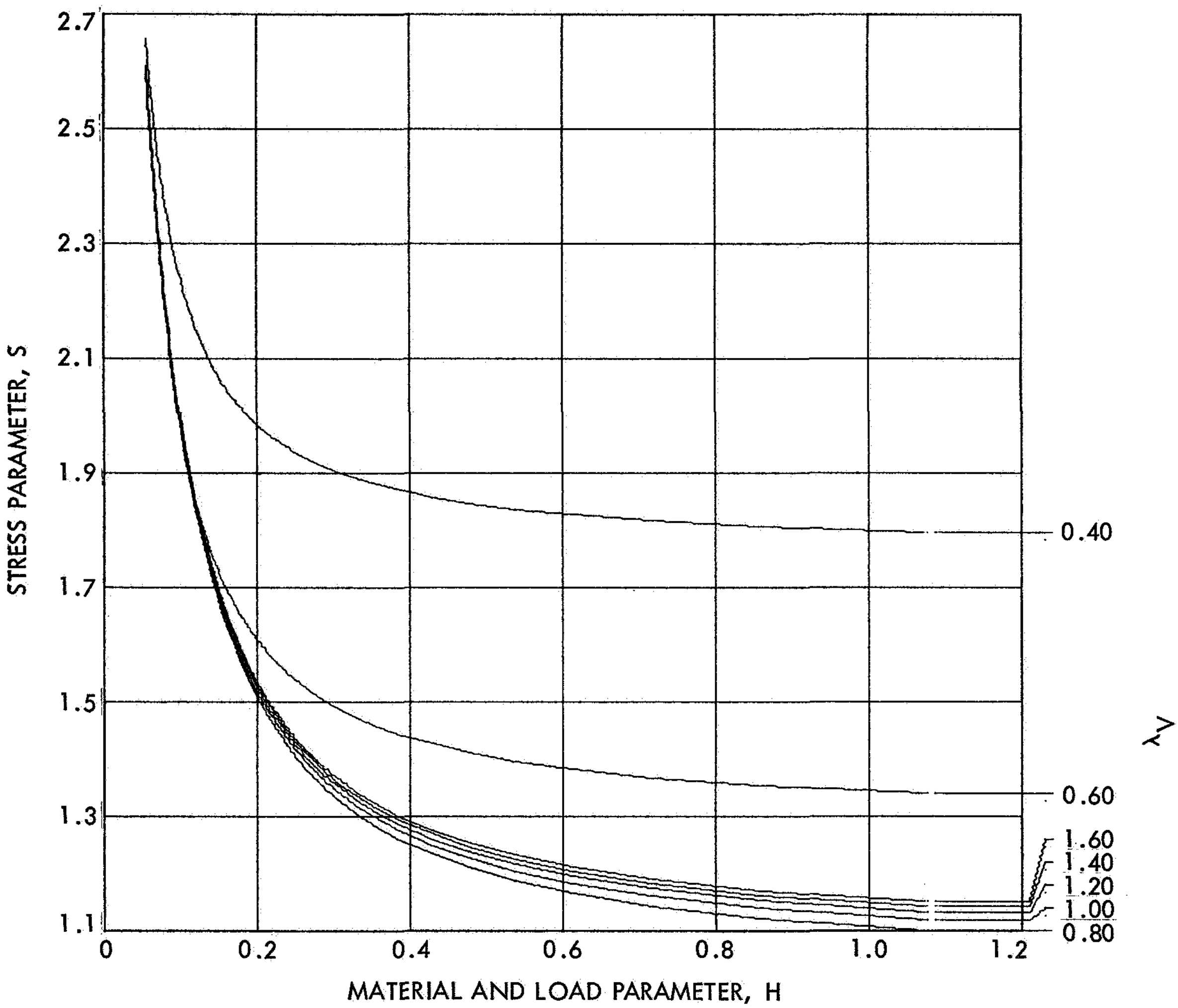
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 0.80$

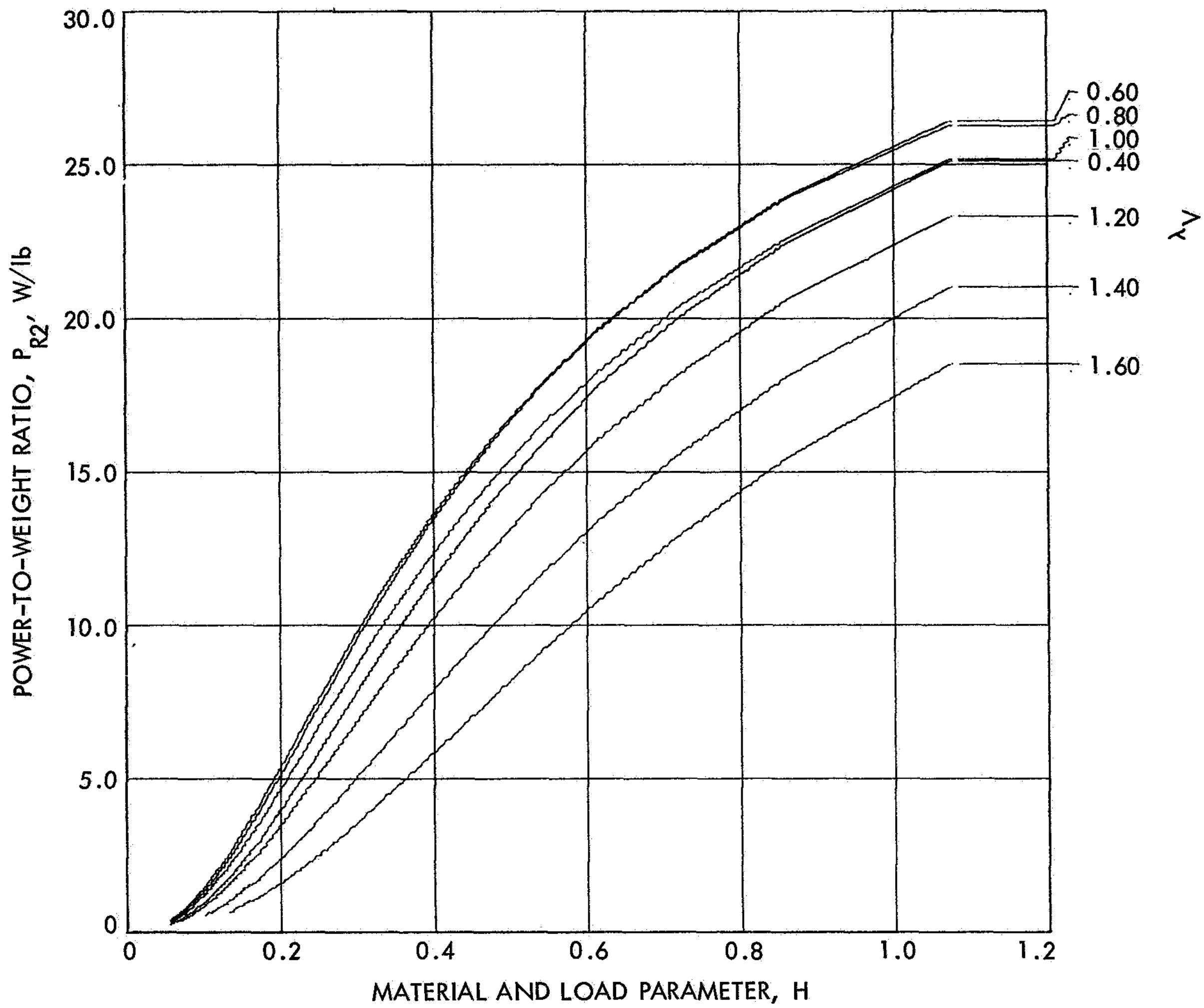
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 0.80$

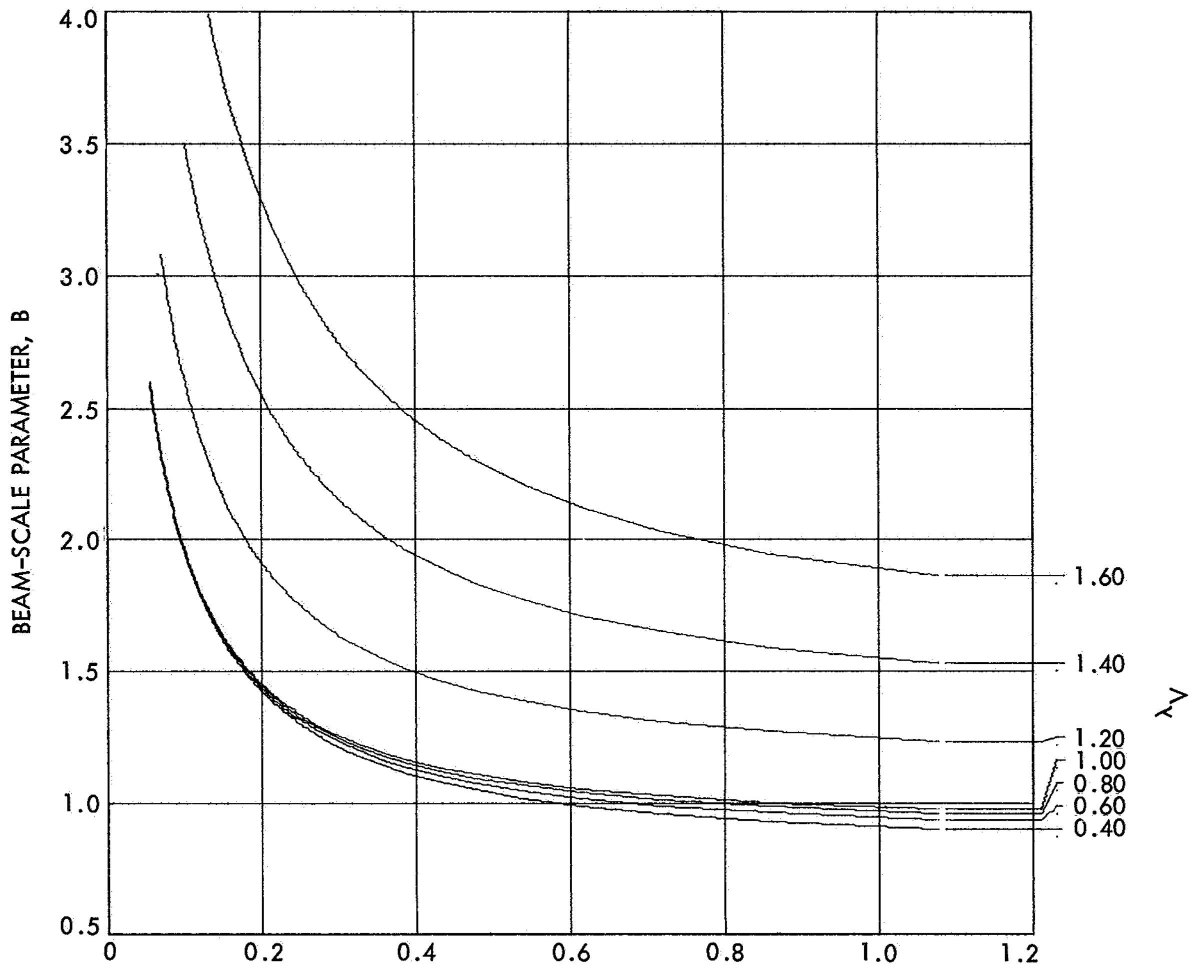
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 1.00$

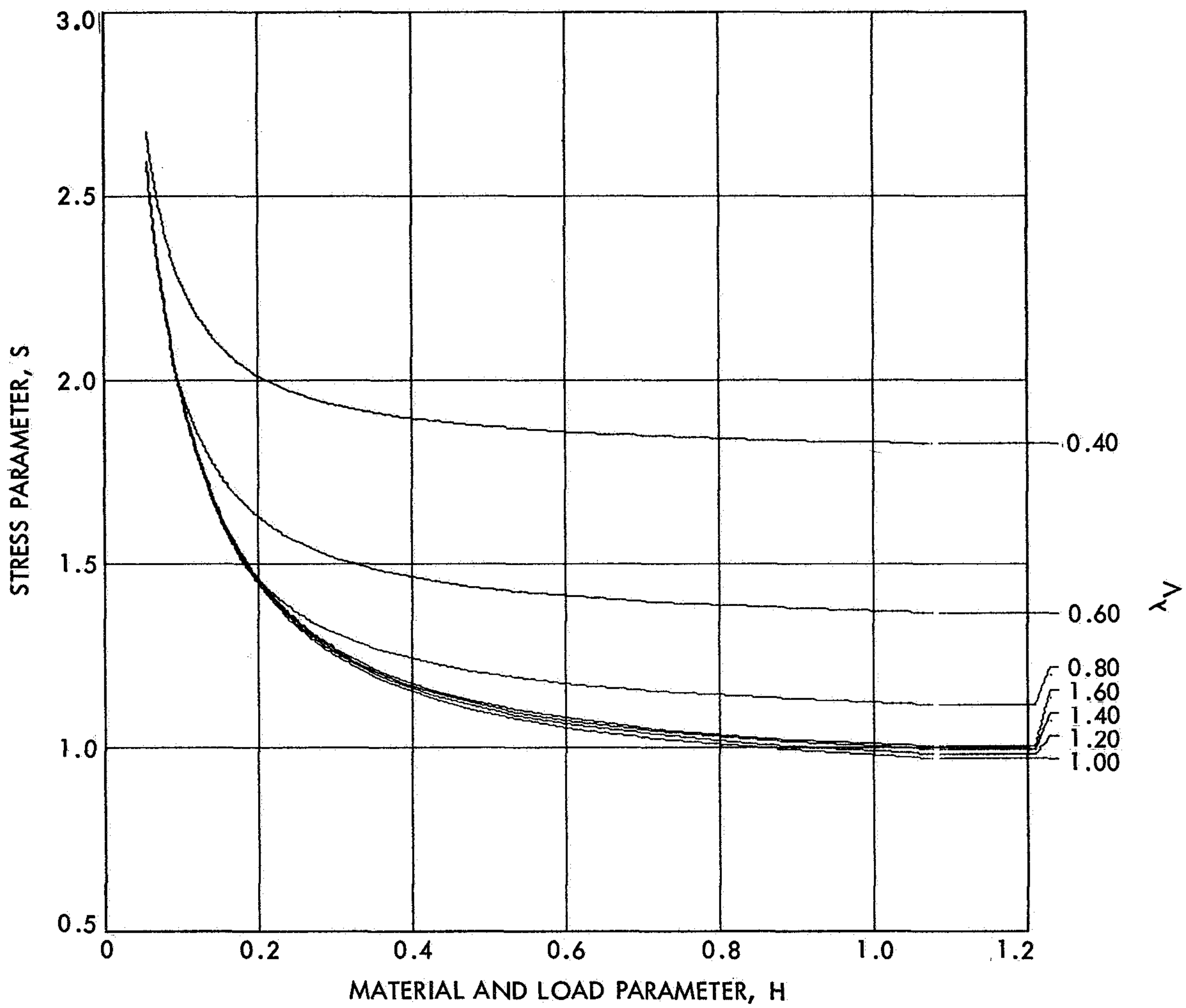
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 1.00$

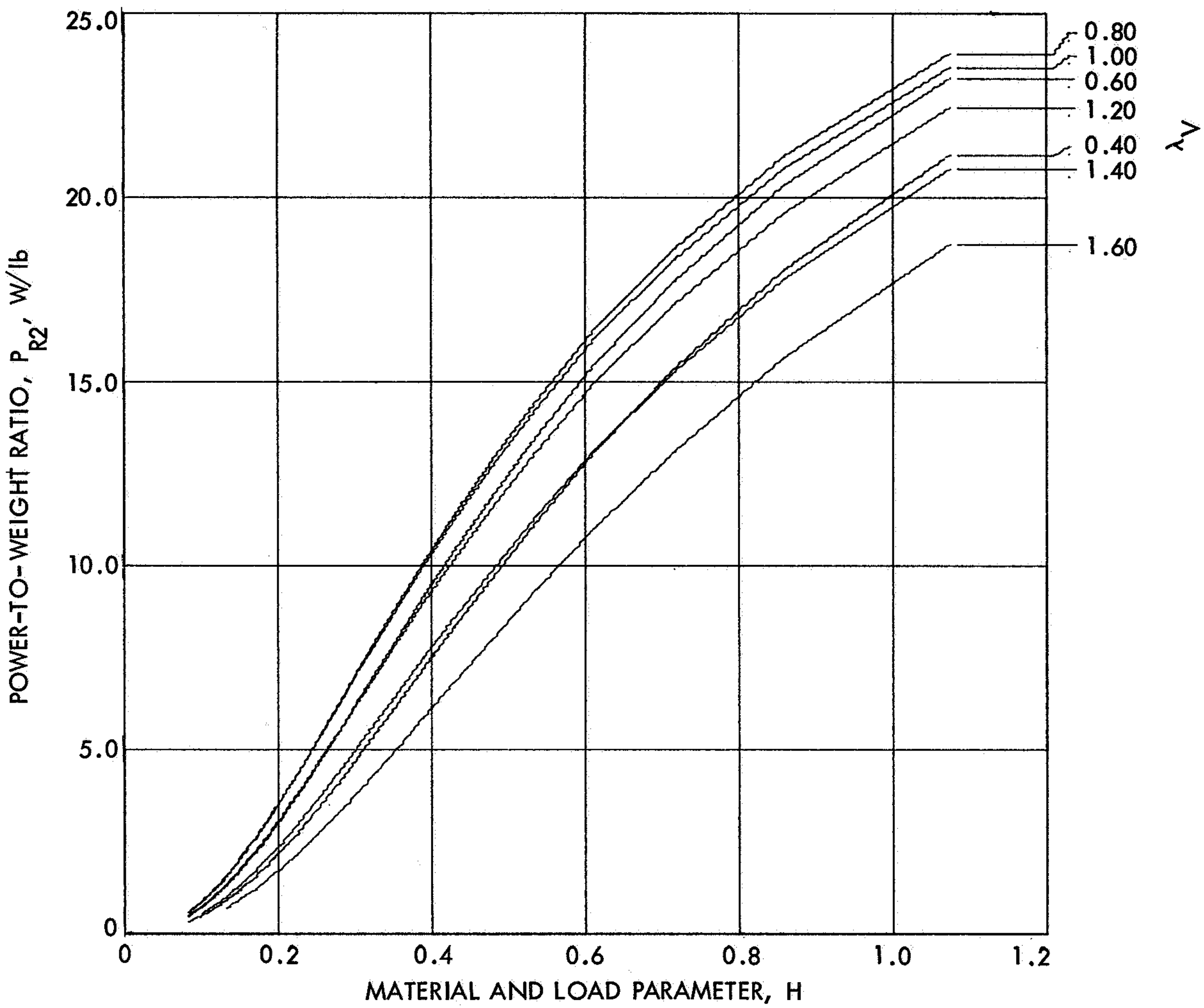
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 1.00$

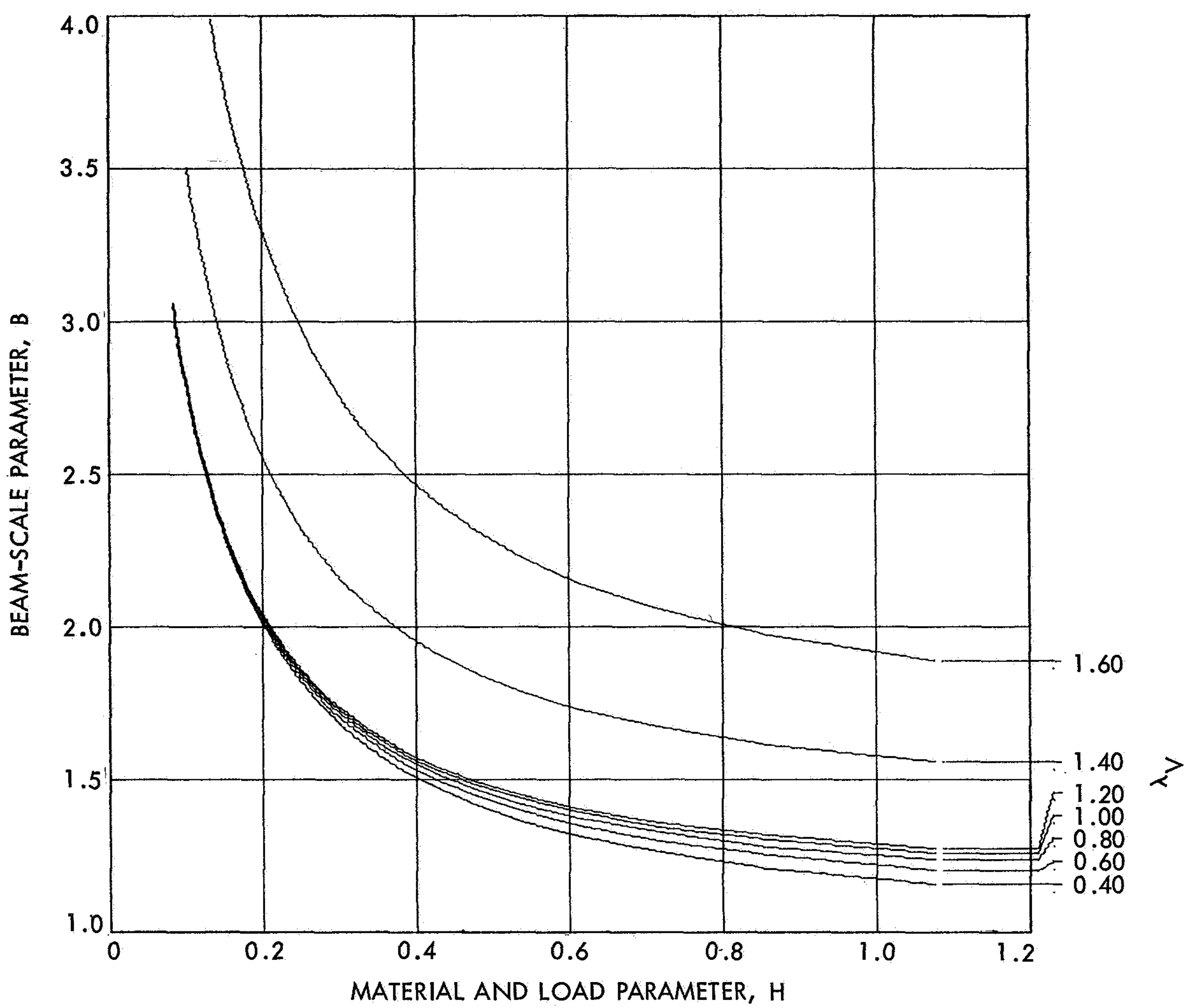
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 1.20$

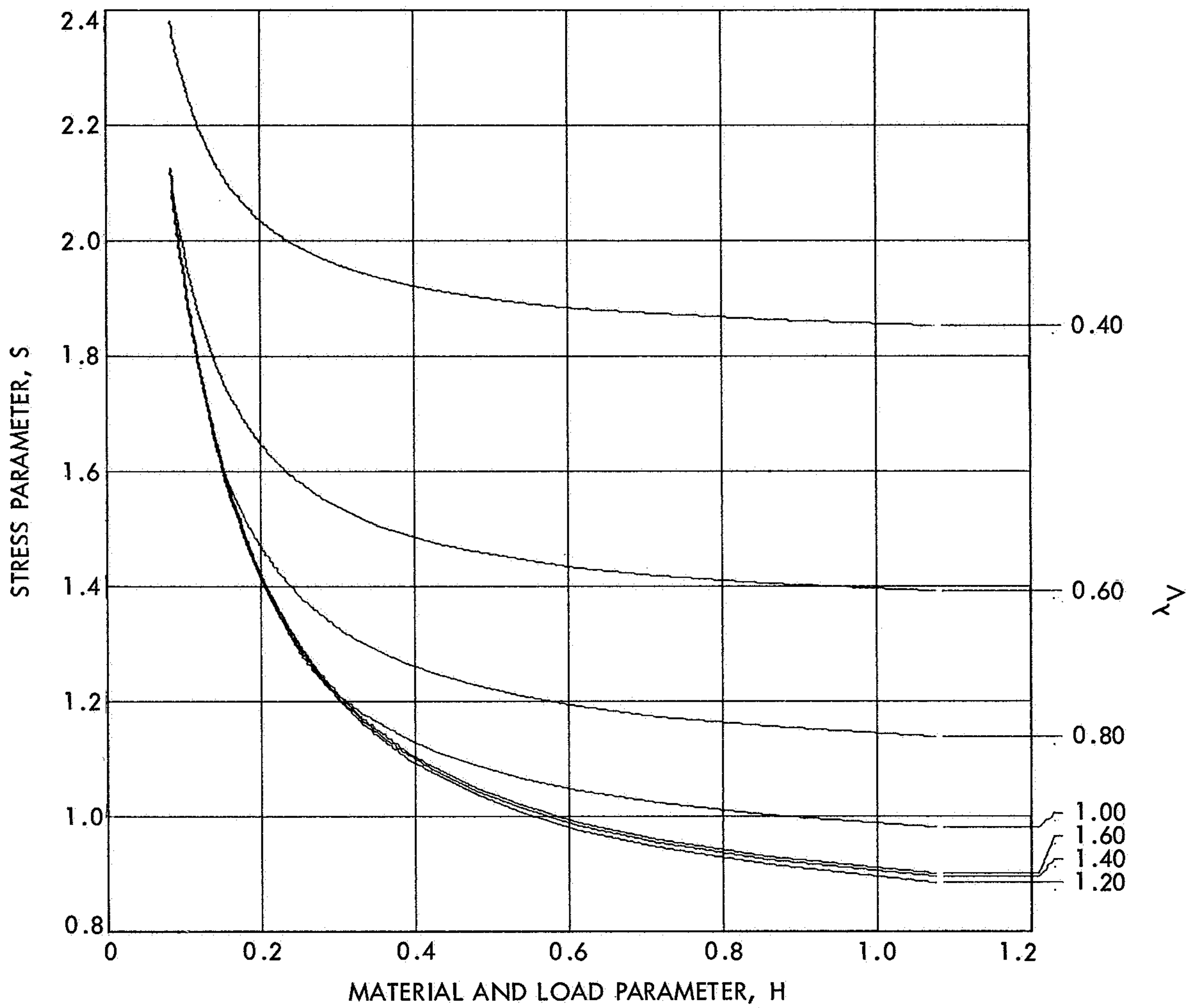
REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 1.20$

REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 1.20$

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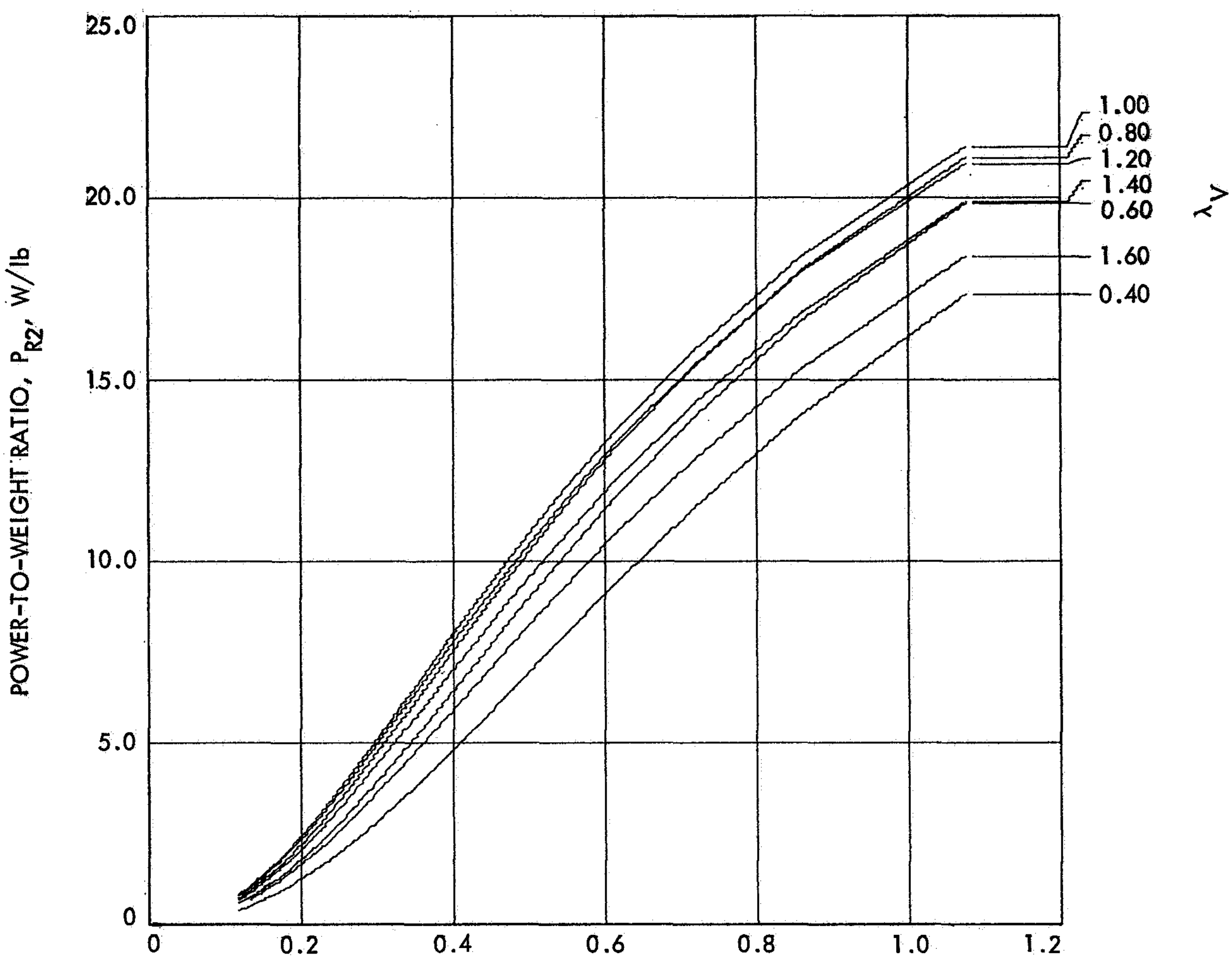
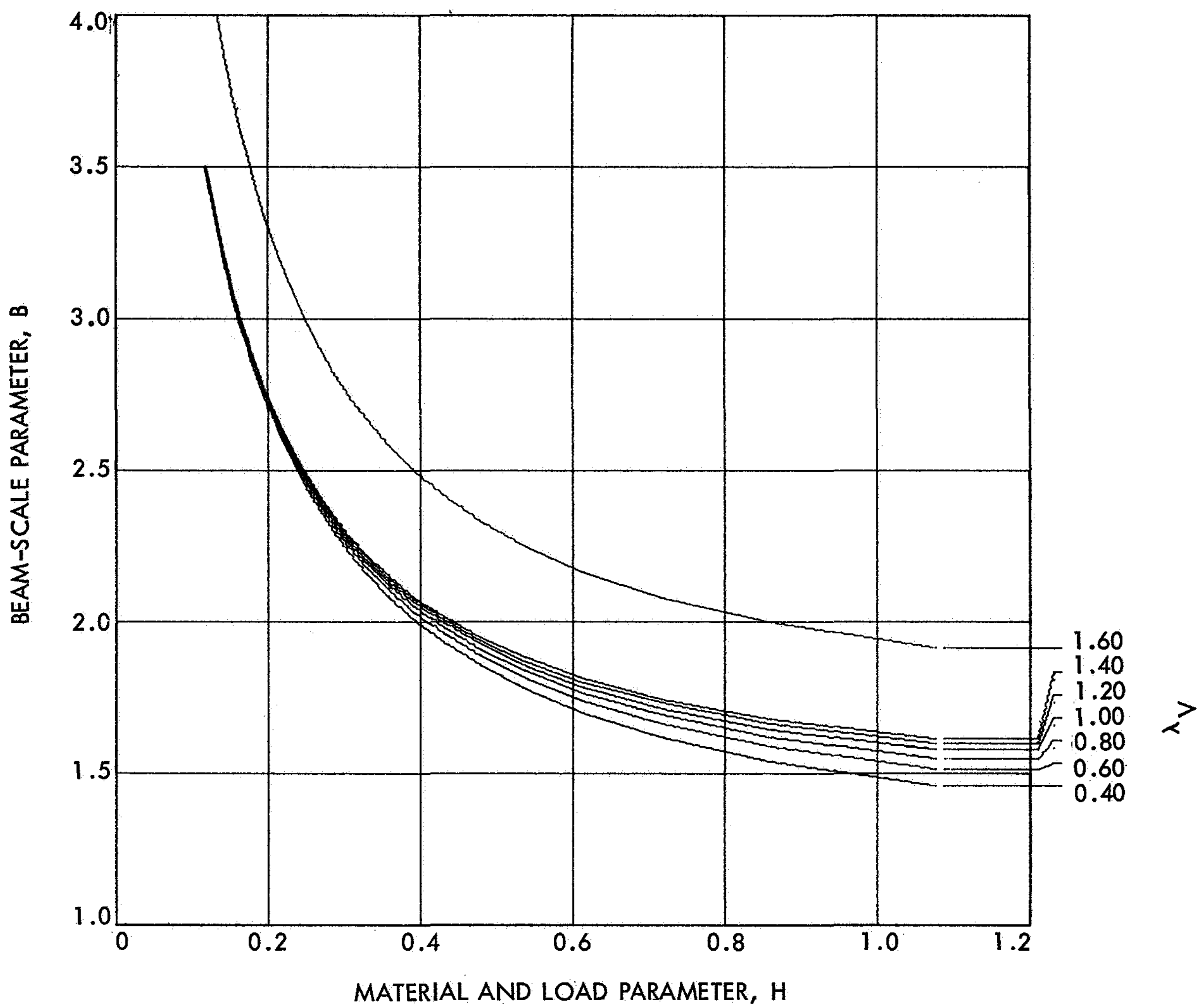


MATERIAL AND LOAD PARAMETER, H

FREQUENCY-LIMITED

$\lambda_H = 1.40$

REFERENCE BOEING LASA



FREQUENCY-LIMITED

$\lambda_H = 1.40$

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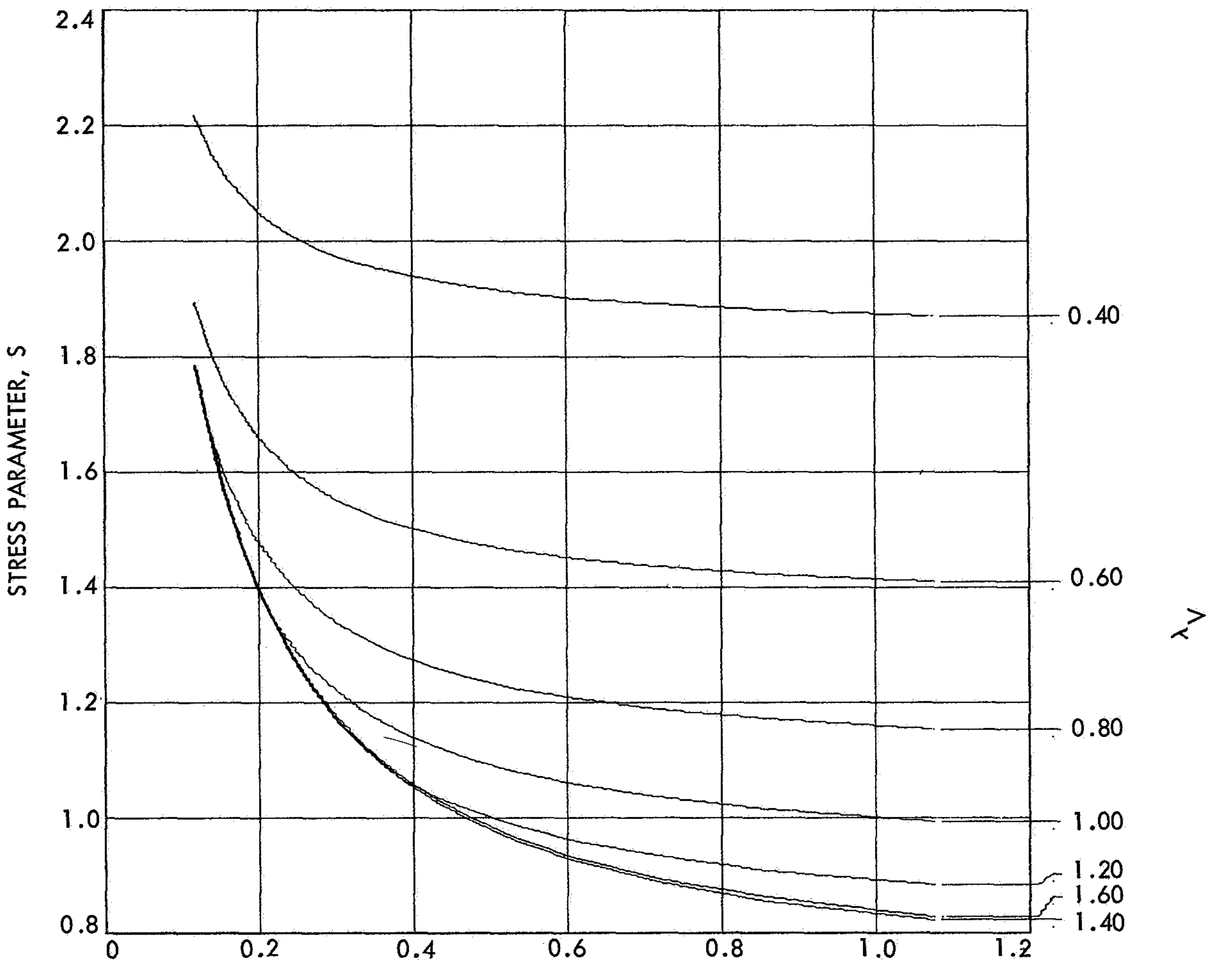
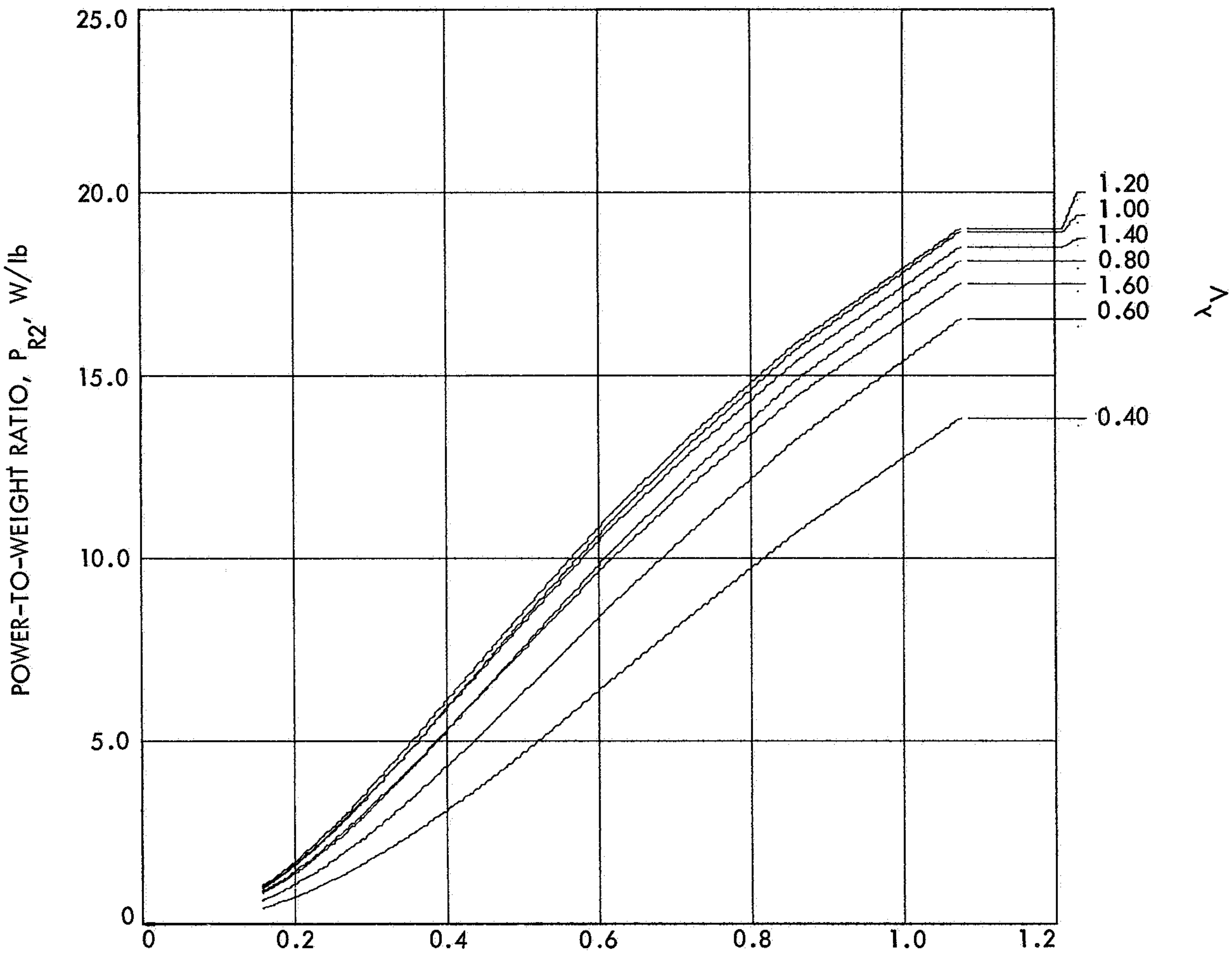


MATERIAL AND LOAD PARAMETER, H

FREQUENCY-LIMITED

$\lambda_H = 1.40$

REFERENCE BOEING LASA

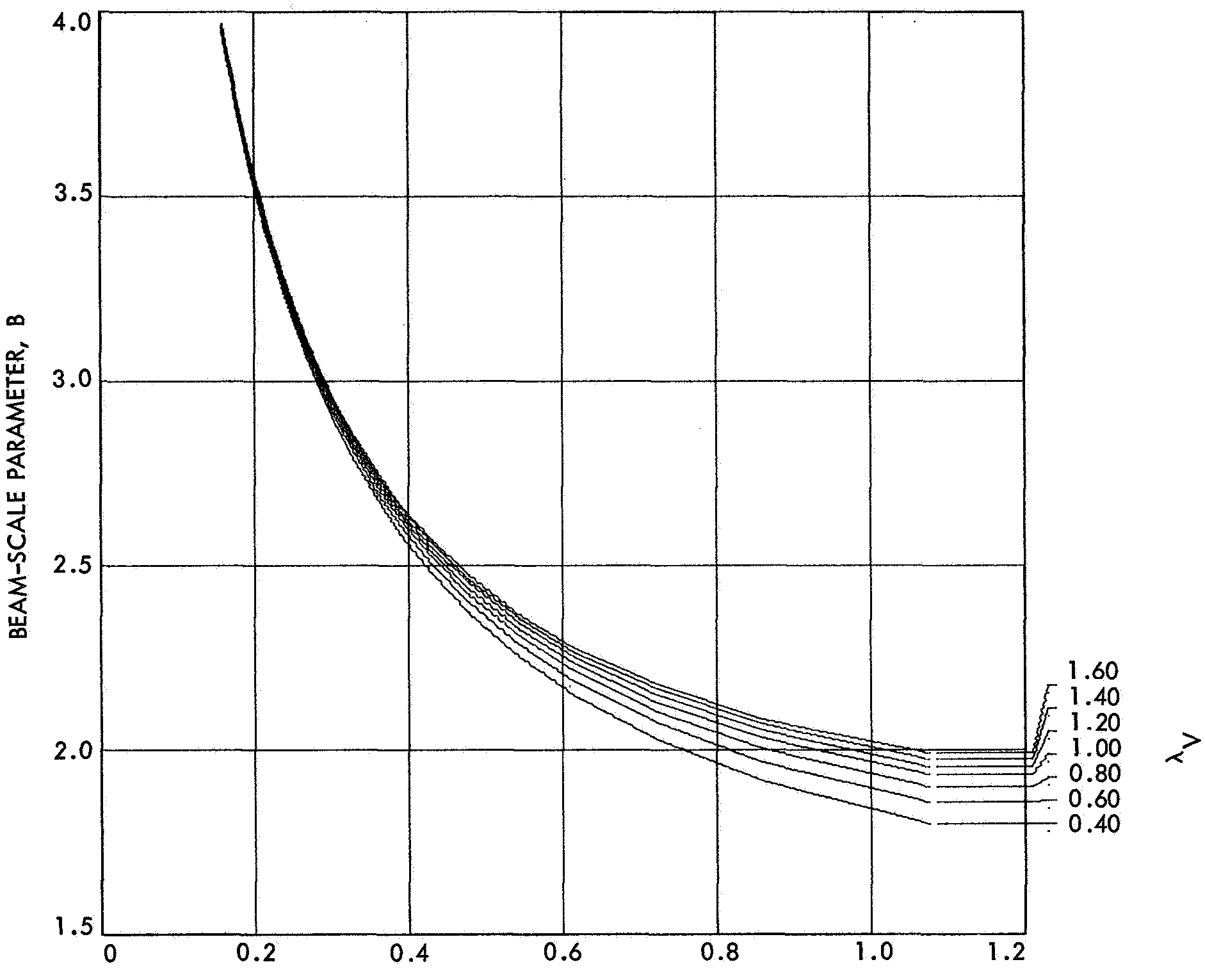


MATERIAL AND LOAD PARAMETER, H

FREQUENCY-LIMITED

$\lambda_H = 1.60$

REFERENCE BOEING LASA

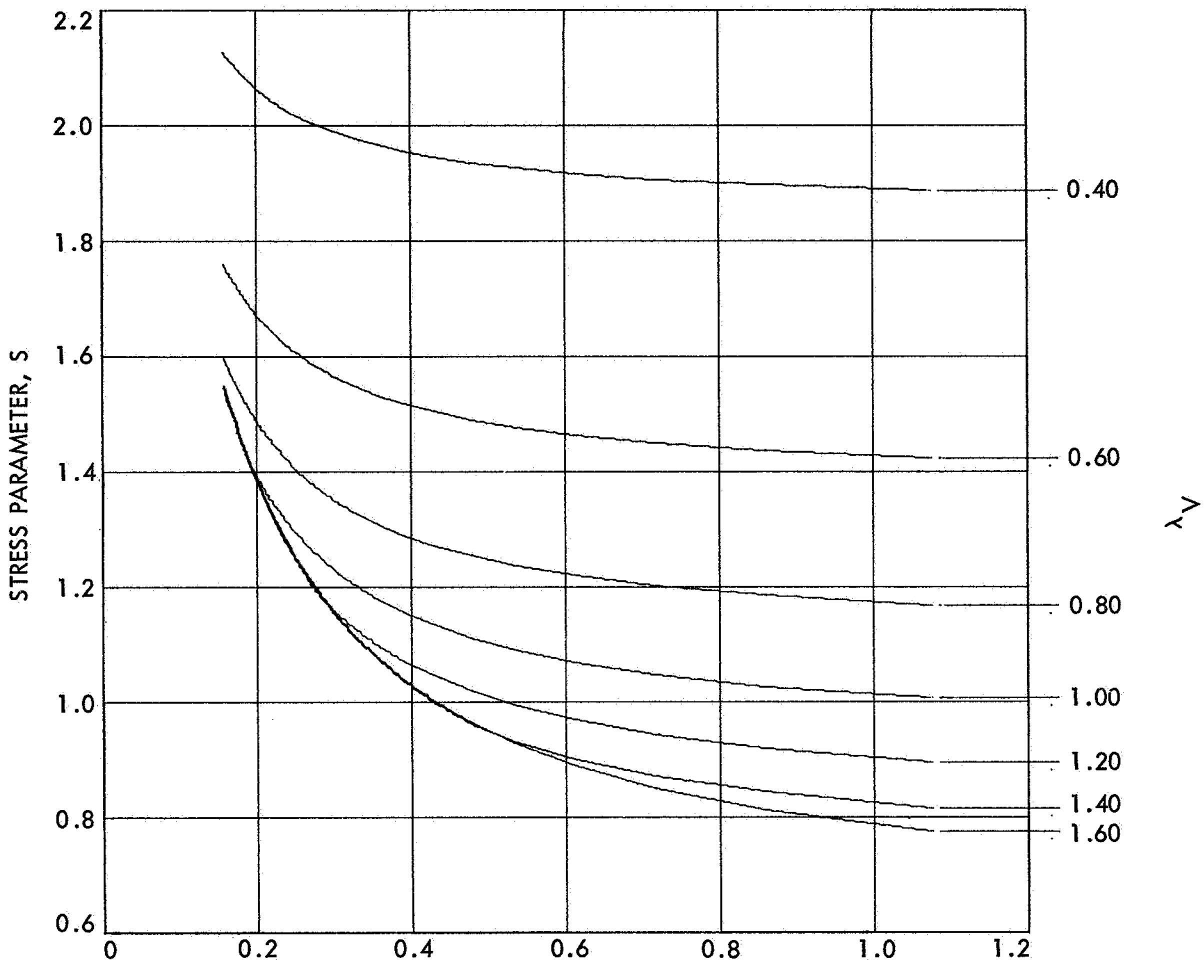


MATERIAL AND LOAD PARAMETER, H

FREQUENCY-LIMITED

$\lambda_H = 1.60$

REFERENCE BOEING LASA



MATERIAL AND LOAD PARAMETER, H

FREQUENCY-LIMITED

$\lambda_H = 1.60$

REFERENCE BOEING LASA