## NASA CR-61259

## THUNDERSTORM PERSISTENCE AT CAPE KENNEDY, FLORIDA

Prepared under Government Order No, H-76789 by Russell F. Lee, James W. Ownbey, and Frank T. Quinlan NATIONAL WEATHER RECORDS CENTER


For
NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama

THUNDERSTORM PERSISTENCE AT CAPE KENNEDY, FLORIDA

By<br>Russell F. Lee, James W. Ownbey and

Frank T. Quinlan

Prepared under Government Order 76789 by
NATIONAL WEATHER RECORDS CENTER
Asheville, North Carolina

Contract Monitors: Orvel E. Smith and S. C. Brown Aerospace Environment Division Aero-Astrodynamics Laboratory

Distribution of this report is provided in the interest of information exchange. Responsibility for the contents resides in the author or organization that prepared it.
TABLE OF CONTENTS4
Page
I. INTRODUCTION ..... 2
II. STATEMENT OF THF PROBLEM ..... 2
III. SOURCE OF DATA ..... 3
IV. PROCEDURES ..... 3
V. RESULTS ..... 8
VI. CONCLUSIONS ..... 9
APPENDIX A ..... 23
APPENDIX B ..... 63

SUMMARY

The probabilities and conditional probabilities of sequences of days with and without thunderstorms at Cape Kennedy, Florida, are determined for given months and seasons, and for specified periods of the day. These periods are as follows:
(1) Each hour of the day beginning with 0000-0059 EST (for thunderstorm probabilities only);
(2) The 4-hour periods 0000-0359, 0400-0759, 0800-1159, 1200-1559, 1600-1959 and 2000-2359 EST;
(3) The 8 -hour periods 0000-0759, 0400-1159, 0800-1559, 1200-1959 and 1600-2359 EST; and
(4) The 24-hour period 0000-2359 EST.

Data used in this study are for the period January 1957 through Decenber 1962.

Conditional probabilities of sequences of thunderstorms are computed for all times except the one-hour periods. The highest wind speeds associated with thunderstorms are examined for the 24 -hour period for each month and season of the year and for the period 1200 through 1959 EST for June, July, and August and for the summer season only (June, July, and August combined).

The distributions of sequences of days with and without thunderstorms during the hours 1200 through 1959 EST for June, July, August and the summer season are compared with the persistence series suggested by Eggenberger and Polya, and the Markov chain model of the zero, first and second orders. The comparisons are tested by using the Chi-square and the Kolmogorov-Smirnov statistical tests.

## I. INTRODUCTION

This report represents part of a continuing evaluation of meteorological data affecting aerospace operations at Cape Kennedy, Flcrida. Relative frequencies of thunderstorm occurrences at Cape Kennedy are available in the Summary of Monthly Aerological Records (SMAR) format for each month, which was prepared for the National Aeronautics and Space Administration by the National Weather Records Center in Asheville, North Carolina.

Because the lightning and high surface winds associated with thunderstorms hinder the launch operations at Cape Kennedy, information is presented on probabilities and conditional probabilities of thunderstorms and on the distribution of the highest wind speeds during thunderstorms.

Dr. Harold L. Crutcher's consultation and guidance in the preparation of this report are gratefuliy appreciated.

## II. STATEMENT OF THE PROBLEM

The probabilities and conditional probabilities of sequences of thunderstorms at Cape Kennedy, Florida, are needed for given months and seasons, and for specified periods of the day. These periods are as follows:
(a) Each hour of the day beginning with 0000-0059 EST (for thunderstorm probabilities only);
(b) The 4 -hour periods 0000-0359, 0400-0759, 0800-1159, 1200-1559, 1600-1959 and 2000-2359 EST;
(c) The 8 -hour periods 0000-0759, 0400-1159, 0800-1559, 1200-1959 and 1600-2359 EST; and
(d) The 24-hour period 0000-2359 EST.

Conditional probabilities of sequences of thunderstorms are to be computed for all times except the one-hour periods. The highest wind speeds associated with thunderstorms are to be examined for the 24 -hour period and for the period 1200 through 1959 EST for the summer season only. To answer specific questions concerning sequences of thunderstorm occurrences, the distribution of sequences of days without thunderstorms also must be determined. The condilional probabilities for
sequences of days with and without thunderstorms for only the summer season will be examined because most of the thunderstorms at Cape Kennedy occur during the summer months.

## III. SOURCE OF DATA

All of the data for this study are from the Surface Weather Observation Record (Forms WBAN $10-\mathrm{A}$ and B ) for Cape Kennedy, Florida, for the period January 1957 through December 1962. A thunderstorm is reported on Form WBAN 10 whenever thunder is heard at the station within the 15 minutes $b \geqslant f o r e ~ t h e ~ o b s e r v a t i o n ~[1] . ~$

## IV. PROCEDURES

## A. Data Organization

The occurrences of thunderstorms are first listed by year, month, day and hour. The frequencies of days with thunderstorms during each hour are tallied by month and by season. Relative frequercies, computed by dividing these counts by the total number of days, are shown in tables Al. 1 and Al. 2 in Appendix A.

The frequency of days with thunderstorms during the specified $4-$, 8 -, and 24 -hour periods also aro tallied by month for June, July ard August, and for the summer season (June through August combined). In addition, sequences of $2,3, \ldots, 12$ days are tallied with thunderstorms and without thunderstorms during the specified time periods. Each sequence is tallied ir the month in which the first day of that sequence fell. For example, if thunderstorms were observed on July 30 and 31 , and August 1 and 2 , two sequences of two (30-31 and 31-1) would be tallied for July and one for August (August 1 and 2).

To further illustrate how the thunderstorm sequences are tallied, consider the following sequences of days with thunderstorms (indicated by $T$ ) and with no thunderstorms (indicated by $N$ ) from Cape Kennedy, Florida, WBAN 10 records for the hours 1200-1559 EST during July 1957.

| July 1957 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |  | 2 | 13 | 1 |  | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Thunderstorm | N | T | T | T | N | N | N | N | N | T |  |  | T | T |  |  | T |
| July 1957 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |  |
| Thunderstorm Occurrence | N | T | N | T | N | N | N | N | T | T | N | N | T | T | T | T |  |


| August 1957 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Thunderstorm <br> Uccurrence | T | T | T | T | N |

The frequencies of sequences of various lengths are tallied as follows:

| Length of |
| :--- |
| Sequence of |
| Thunderstorm |
| Days |


| 1 |
| :--- | | No. off |
| :--- |
| Sequences |
| Tallied for |
| July 1957 |

2

Notice that, while the initial day of each sequence must be in the month being tallied, successive days within that sequence need not be. The 2 -day sequence of July 31 through August 1 is tallied in July, but the sequense August 1 through 2 is counted in August. From July 28 to August 4 an 8 -day sequence is tallied in July.

Notice, also, that one run may contain a large number of sequences. A run of 4 days for example, contains one sequence of four, 2 sequences of three, 3 sequences of two, and 4 sequences of one.

The conditional probabilities are computed from the frequencies of sequences by the equation $p(k \mid i)=F(k+i) / F(i)$, where $p(k \mid i)$ is the conditional probability of having $k$ additional consecutive days of an occurrence of an event (either a thunderstorm or lack of a thunderstorm), given that $i$ consecutive days of this event have just occurred with no
information about events prior to this; $F(i)$ is the frequency of occurrence of a sequence of length $i$; and $F(k+i)$ is the frequency of occurrence of a sequence of length $k+i$.

When $i=0$, then $F(0)$ is the frequency of days for which the presence or absence of the event is not specified, hence $F(0)=N$, which is the total number of days in the sample. The probability of $k$, given no previous information, is given by $p(k \mid 0)$. Thus, $f(k \mid 0)$ may be read as, simply, $p(k)$.

The conditional probabilities of sequences of thunderstorms occurring within the specified 4 -hour periods are shown in tables A2.1 through A2.5 in Appendix A by month and season, and within the 8 -hour and 24 -hour periods in tables A3.l through A3.5 by month and season.

The conditional probabilities of thunderstorms not occurrin ${ }_{5}$ within the specified 4 -hour periods are shown in tables A4.1 through A4.12 in Appendix A by month and season, and not occurring within the 8 -hour and 24 -hour periods in tables A5.1 through A5. 12 by month and season.

The use of these tables is described by two examples. Suppose the probability is required of the occurrence of two consecutive August days with thunderstorms between 1200 and 1559 EST at Cape Kennedy, Florida, given that thunderstorms have occurred during these hours on each or the previous three consecutive days. The answer is in table A2.3 in Appendix A, which is the table corresponding to the hours 1200 through 1559 EST. The intersection of the row $i=3$ (three consecutive days of thunderstorms have already occurred) with column $k=2$ (to find the probability of two additional days of thunderstorms) in the tabulation for August yields the probability of .400 .

If the probability is required of three consecutive days during the summer with no thunderstorms between 1200 and 1959 EST, locate the table for these hours, which is table A5.8 on page 32 of Appendix A.

As there is no information on the occurrence or non-occurrence of thunderstorms prior to the time in question, the desired probability is found at the intersection of row $i=0$ and column $k=3$ in the tabulation for the summer season. The probability of three consecutive summer days with no thunderstorms is found to be . 317.

The frequency distribution of maximum wind speeds associated with thunderstorms is presented in tables A6.1 and A6. 2 in Appendix A. This distribution is determined by examining the Forms WBAN $10-A$ and $B$ for each day during which a thunderstorm occurred. The peak thunderstorm gust recorded for the day is used. If this figure is not available, the highest reported wind during and within 30 minutes either side of the thunderstorms of the day is substituted.

## B. Development of Theoretical Models

The distribution of sequences of days with and without thunderstorms between the hours 1200 and 1959 EST during the months of June, July, and August and the summer seas on (June through Augus combined) is compared with the persistence series and the Markov chain model of the zero, first and second orders. This season and time of day are selected because most of the thunderstorms at Cape Kennedy occur between these afternoon hours and during the summer months. Other time periods have insufficient numbers of thunderstorms to provide good comparisons with the theoretical models.

The observed frequencies of sequences of thunderstorm and of nonthunderstorm occurrences are compared with the theoretical distributions predicted by the Markov chain model and by the persistence series of Eggenberger and Polya [3]. According to Brooks and Carruthers [2], following Eggenberger and Polya [3], the persistence series is defined as:

$$
\begin{aligned}
& \frac{1}{\left(1+d^{\prime}\right)^{m / d^{\prime}}}, \frac{m}{1!\left(1+d^{\prime}\right)^{m / d^{\prime}+1}}, \quad \frac{m\left(m+d^{\prime}\right)}{2!\left(1+d^{\prime}\right)^{m / d^{\prime}+2}}, \cdots, \\
& \quad \frac{m\left(m+d^{\prime}\right)\left(m+2 d^{\prime}\right) \ldots\left(m+[i-1] d^{\prime}\right)}{i!\left(i+d^{\prime}\right)^{m^{\prime} d^{\prime}+i}}
\end{aligned}
$$

where $m$ is the mean of the observed series and $d^{\prime}=(1, \ldots-1$. The variance is $\sigma^{2}$ and $i=\cdot 1$, where $p$ is the length of $t: \cdot$ longest sequence to be determ1tiva.

The zero order Mark $v$ model [5] is constructed $r$...suming that the probability of an event occurring on a given daj i: independent of any previous occurrences. It the probability of o :rajerstorm occurring on a certain day is .400 , then the theoret: il : obability of thunderstorns occurring on two suceessive diys $\quad . \quad 100=.160$. Likewise, the nrobability of a three-day sequene wi thunderstorms is $(.400)^{i j}=.064$. Thus, assuming a zers order Maybov model, the probability of a sequence of $n$ days of events ( $E$ ) counting from any given day is $P_{n}=[p(E)]^{n}$. For the purposes of this paper, an event will be either the occurrence or the non-occurrence of a thanderstorm.

The assumption of a first order Markov process is the assumption that the probability of an event occurring depends upon the occurrence or non-sccurrence of the event on the previous day, but is independent
of any other prior occurrences. Then the probability of an event occurring two days in succession is $p_{2}=p(E) \times p(E \mid E)$ where $p(E \mid E)$ is the robability of the event $E$, given that such an event has occurred on 'he previous day. Thus, the probability of sequence of $n$ events is

$$
p_{n}=p(E) \times[p(E \mid E)]^{n-1}
$$

In a similar manner, the second order Markov $, \ldots, i$ assumes that the probability of an event i.s dependent only upon t ts occurrence or nenoccurrence during the previous two days. The equation for the second order Markov seiies is a logical extension of the Eirst order:

$$
p_{n}=p(E) \times p(E \mid E) \times[p(E \mid E E)]^{n-2} \quad \text { for } n=2 \text {, }
$$

where $P(E \mid E E)$ is the probability of an event occurring, given that it has already occurred on each of the two preceding days.

If a specific distribution of sequences of thunderstorm and nonthunderstorm afternoons is assumed to be Markovian, then the order of the Markov series best representing the data can be determined by use of the asymptotic Chi-square statistic, which is descrired in Appendix B [5]. This test was applied to the sequences of the occurrences and nonoccurrences of thunderstorms during the summer within the period 1200 through 1959 EST. The resulting $X^{2}$ values are shown in table 1.1.

The Markov processes, zero through second order, and the persistence series are compared with the observed relative frequency distributions of the occurrences of thunderstorms within the same time periods used above.

The Kolmogorov-Smirnov test [4] is used to test the goodness of fit. This test uses the maximum difference between the observed and theoretical cumulative distributions of days with or without thunderstoms as the argument with which to determine whether the distributions are significantly different. As the longer sequences anclude the shorter sequences within them, frequencies of sequences of days with and without thunderstorms shown in tables 2.1 and 2.2 are already in the form of cumulative distributions beginning with the longer sequences. It is only necessary to divide each frequency by the freq'ency of the shortest sequence, namely, the frequency of the sequences of one day with or without thunderstorms. Then take the maximum difference between the observed and theoretical distributions and compare it with the value for $\alpha=.20$. If the argument is greater than the limiting argument
for $a=.20$, then the null hypothes is is not accepted that the distribution of sequences of thunderstorms is the same as the theoretical distribution.

## V. RESULTS

As shown in table Al. 2, 46.6 percent of the summer days contain thunderstorms, while less than 20 percent of the days in any of the other seasons contain thunderstorms. The probability of a thunderstorm occurring is greatest during the period 1200 through 1959 EST, that probability being .420 for the summer season. The probability of a thunderstorm occurring during an adjecent period, 0400 through 1159 EST, is only . 109. A comparison of tables A3.3 and A3.5 reveals that the distributions of sequences of thunderstorms are essentially the same for the period 1200 through 1959 EST as for the entire day ( 0000 through 2359 EST). In fact, during the summer season, more than 90 percent of the days during which thunderstorms are reported contain thunderstorms during the period 1200 through 1959 EST.

If the observed distribution of sequences of days with and without thunderstorms is Markovian, then the order of the Markov distribution whicn fits the observed distribution is determined by using the Chisquare test. The results of such a test are shown in table 1.1. The zero order Markov model is not accepted at the $a_{0}=.01$ level for any of the summer months nor for the summer season. The first and second order Markov models fit the distributions of sequences of days with and sequences of days without thunderstorms. Since the second order Markov model requires more input data, the first order model is considered adequate.

The results of the Kolmogorov-Smirnov test comparing the distribution of the sequences of thunderstorms with the distribution predicted by the persistence series of Eggenberger and Polya and the zero, first, and second order Markov chain models (table 3.1) indicate that all the distributions except the zero order Markov are acceptable approximations to the observed distribution. The persistence series, while roughly equivalent to the first order Markov in accuracy, is much more difficult to compute. It is for this reason that the persistence series is omitted from the study of sequences of days without thunderstorms. The actual distributions and the first order Markov expected distributions of the sequences of days with thunderstorms during the hours 1200 thrcugh 1959 EST in June, July, and August and the summer season are shown graphically in figures 1 through 4.

Table 3.2 shows the results of the Kolmogorov-Smirnov test applied to the zero, first, and second order estimates of the distribution of sequences of days without thunderstorms between 1200 and 1959 EST. In this case, the first order Markov model is not accepted at the $\alpha=.20$ level for Junc and the summer season, but is accepted for July and August.

The first order Markov model is found to fit the distributions of sequences of days without afternoon thunderstorms during July and August; however, the second order Markov model is required to fit the distributions of sequences during June and the summer season.

Tables A6.1 and A6.2 in Appendix A show the distribution of thunderstorm winds. Notice that the median wind speeds for the summer season and for the year as a whole are between 15 and $1 y$ knots. No comparison between the thunderstorm wind distribution for days with thunderstorms and the distributions for days without thunderstorms is made.

Partial results of this study were presented [6] at the symposium held at the Marshall Space Flight Center in December 1966.

## VI. CONCLUSIONS

The first order Markov model may be used to approximate the dis tribution of sequences of summer afternoons with thunderstorms. The second order Markov model may be used to approximate the distribution of sequences of summer afternoons without thunderstorms.

Table 1.1
Results of the Chi-square test to determine which order of the Markov model applies assuming the distribution is Markovian. The data being tested are the distributions of sequences of days with and without thunderstorms occurring at Cape Kennedy, Florida, between 1200 and 1959 EST during June, July, August and the summer season, for the years 1957 through 1962.

|  | Zero <br> Order | First <br> Order | Second <br> Order |
| :---: | :---: | :---: | :---: |
| June | $23.68453^{* *}$ | 8.36120 | 3.11543 |
| July | $39.56622^{* *}$ | 5.41721 | 4.57970 |
| August | $21.21101^{* *}$ | 4.59896 | 4.30371 |
| Summer | $76.39713^{* *}$ | 9.82338 | 6.18184 |
| df | 7 | 6 | 4 |
|  |  |  |  |

[^0]Table 2.1
The observed and theoretical relative frequencies of sequences of days
with thunderstorms between 1200 and 1959 EST at Cape Kennedy, Florida,
for the years 1957 through 1962.

| Length of Secuences in Days |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| June |  |  |  |  |  |  |  |  |  |  |  |
| . 406 | . 228 | . 133 | . 073 | . 033 | . 017 | . 006 |  |  |  |  |  |
| . 407 | . 229 | . 125 | . 067 | . 035 | . 018 | . 010 |  |  |  |  |  |
| . 406 | . 164 | . 067 | . 027 | . 011 | . 004 | . 002 |  |  |  |  |  |
| . 406 | . 228 | . 128 | . 072 | . 040 | . 023 | . 013 |  |  |  |  |  |
| . 406 | . 228 | . 133 | . 078 | . 046 | . 027 | . 016 |  |  |  |  |  |
| July |  |  |  |  |  |  |  |  |  |  |  |
| . 441 | . 312 | . 226 | . 161 | . 113 | . 081 | . 054 | . 038 | . 022 | . 016 | . 011 | . 005 |
| . 431 | . 303 | . 210 | . 145 | . 100 | . 068 | . 046 | . 030 | . 020 | . 013 | . 008 | . 004 |
| . 441 | . 194 | . 086 | . 038 | . 017 | . 007 | . 003 | . 001 | . 001 | . 000 | . 000 | . 000 |
| . 441 | . 312 | . 221 | . 156 | . 110 | . 078 | . 055 | . 039 | . 028 | . 020 | . 014 | . 011 |
| . 441 | . 312 | . 226 | . 164 | . 118 | .c86 | . 062 | . 045 | . 033 | . 024 | . 017 | . 012 |
| August |  |  |  |  |  |  |  |  |  |  |  |
| . 414 | .237 | . 140 | . 091 | . 059 | . 038 | . c 22 | . 016 | . Oll | . 005 |  |  |
| . 371 | . 214 | . 105 | . 043 | . 008 | . 003 |  |  |  |  |  |  |
| . 414 | . 171 | . 021 | .0̌23 | . 012 | . 005 | . 002 | . 001 | . 000 | . 000 |  |  |
| . 414 | . 237 | . 135 | . 077 | . 044 | . 025 | . 014 | . 008 | . 005 | . 003 |  |  |
| . 414 | .237 | . 140 | . 083 | . 049 | . 029 | . 017 | . 010 | . 006 | . 004 |  |  |
| Summer |  |  |  |  |  |  |  |  |  |  |  |
| . 420 | . 259 | . 167 | . 109 | . 069 | . 045 | . 027 | . 018 | . 011 | . 007 | . 004 | . 002 |
| . 416 | . 255 | . 16.1 | . 102 | . 065 | . 041 | . 037 | . 017 | . 011 | . 007 | . 004 | . 003 |
| . 420 | . 177 | . 074 | . 031 | . 013 | . 006 | . 002 | . 001 | . 000 | . 000 | . 000 | . 000 |
| . 420 | . 259 | . 160 | . 098 | . 061 | . 037 | . 023 | . 014 | . 009 | . 005 | . 003 | . 002 |
| . 420 | . 259 | .167 | . 107 | . 069 | . 044 | . 029 | . 018 | . 012 | . 008 | . 005 | . 003 |

1200-1959 EST

Persistence Series Persistence Series First Order Markov Second Order Markov 1200-1959 EST Observed Persistence Series Zero Order Markov First Order Markov Second Order Markov 1200-1959 EST Observed Series Persistence Markov Zero Order Markov Second Order Markov 1200-1959 EST eries Zero Order Markov First Order Markov Second Order Markov
The observed and theoretical relative frequencies of sequences of days
without thunderstorms between 1200 and 1959 EST at Cape Kennedy, Florida,
for the years 1957 through 1962.

| Length of Sequences in Days |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| June |  |  |  |  |  |  |  |  |  |  |  |
| . 594 | . 417 | . 322 | . 261 | . 211 | . 161 | . 133 | . 106 | . 083 | . 067 | . 050 | . 044 |
| . 594 | .353 | . 210 | . 125 | .074 | . 044 | . 026 | .016 | . 009 | . 006 | . 003 | . 002 |
| . 594 | . 417 | . 292 | . 205 | .143 | . 101 | . 070 | . 049 | . 035 | . 024 | . 017 | . 012 ẽ |
| . 594 | .417 | . 322 | .249 | .193 | .149 | .115 | . 089 | . 069 | . 053 | .041 | . 032 |
| July |  |  |  |  |  |  |  |  |  |  |  |
| . 559 | . 419 | . 312 | .237 | .183 | .151 | .129 | . 108 | .097 | . 086 | .075 | . 065 |
| . 559 | . 313 | .175 | . 098 | . 055 | . 031 | . 017 | . 010 | . 005 | .003 | . 002 | . 001 |
| . 559 | . 419 | . 315 | .236 | .177 | .133 | .100 | . 075 | . 056 | . 042 | .031 | . 024 |
| . 559 | . 419 | . 312 | . 232 | .172 | . 128 | . 095 | . 071 | . 053 | . 039 | . 029 | . 022 |
| August |  |  |  |  |  |  |  |  |  |  |  |
| . 586 | .419 | . 317 | .253 | . 188 | . 134 | . 097 | . 065 | . 043 | . 032 | . 022 | . 011 |
| . 586 | .343 | .201 | .118 | . 069 | . 041 | . 024 | . 014 | .008 | . 005 | . 003 | . 002 |
| . 586 | . 419 | . 300 | . 215 | . 154 | . 110 | . 079 | . 056 | . 040 | . 029 | . 021 | .015 |
| .586 | . 419 | .317 | . 240 | . 181 | .137 | .104 | . 079 | . 059 | .045 | .034 | .026 |
| Summer |  |  |  |  |  |  |  |  |  |  |  |
| . 580 | . 418 | .317 | .250 | .194 | . 149 | . 120 | . 092 | .074 | . 062 | . 049 | . 040 |
| . 580 | .336 | . 195 | .113 | . 065 | . 038 | . 022 | . 013 | . 007 | . 004 | . 002 | . 001 |
| . 580 | . 418 | . 302 | . 218 | .157 | . 114 | . 082 | .05.7 | . 043 | .031 | . 022 | . 016 |
| . 580 | .418 | . 317 | .240 | . 182 | .138 | .104 | . 079 | . 060 | . 045 | . 034 | . 026 |

1200-1959 EST
Order Markov
First Order Markov Second Order Markov 1200-1959 EST

Zero Order Markov First Order Markov Second Order Markov

1200-1959 EST
Zero Order Markov First Order Markov Second Order Markov

1200-1959 EST
Zero Order Markov First Order Markov Second Order Markov

Table 3.1
The application of the "Kolmogorov-Smirnov" test to the maximum differences between the observed and theoretical cumulative distributions of sequences of days with thunderstorms occurring at Cape Kennedy, Florida, between 1200 and 1959 EST for June, July, August and the summer season (1957-1962).

|  | June | July | August | Summer |
| :--- | :---: | :---: | :---: | :---: |
| Persistence Series | .022 | .036 | .016 | .016 |
| Zero Order Markov | $.164^{*}$ | $.318^{*}$ | $.166 *$ | $.225^{*}$ |
| First Order Markov | .018 | .014 | .036 | .030 |
| Second Order Markov | .030 | .025 | .025 | .007 |
| N-Count | 73 | 82 | 77 | 232 |
| $\alpha=0.20$ | .125 | .118 | .122 | .070 |

*Indicates the null hypothesis is not accepted.

Null hypothesis: The distribution of sequences of days with thunderstorms between 1200 and 1959 EST at Cape Kennedy, Florida, occurs according to the indicated theoretical distributions.

Results: For all periods considered, the zero order Markov model is not accepted. The first and second order Markov models and the persistence series are accepted.

Table 3.2

The application of the "Kolmogorov-Smirnov" test to the maximum differences between the observed and theoretical cumulative distributions of sequences of days without thunderstorms occurring at Cape Kennedy, Florida, between 1200 and 1959 EST for June, July, August and the summer season (1957-1962).

|  | June | July | August | Summer |
| :--- | :---: | :---: | :---: | :---: |
| Zero Order Markov | $.230^{*}$ | $.24^{*}$ | $.230^{*}$ | $.237^{*}$ |
| First Order Markov | $.24^{*}$ | .079 | .065 | $.065^{*}$ |
| Second Order Markov | .031 | .084 | .028 | .028 |
| N-Count | 107 | 104 | 109 | 320 |
| $\alpha=.20$ | .103 | .105 | .102 | .060 |
|  |  |  |  |  |

*Indicates the null hypothesis is not accepted.

Null hypothesis: The distribution of sequences of days without thunderstorms between 1200 and 1959 EST at Cape Kennedy, Florida, occurs according to the indicated theoretical distributions.

Results: For June and the summer season, the zero and first order Markov models are not accepted; the second order Markov model is accepted. For July and August, the zero order Markov model is not accepted; the first and second order Markov models are accepted.


Figure 1. Comparison of the observed and the first order Markov predicted relative frequencies of sequences of days in June with thunderstorms occurring between 1200 and 1959 EST.


Figure 2. Comparison of the observed and the first order Markov predicted relative frequencies of sequences of days in July with thunderstorms occurring between 1200 and 1959 EST.


Figure 3. Comparison of the observed and the first order Markov predicted relative frequencies of sequences of days in August with thunderstorms occurring between 1200 and 1959 EST.


Figure 4. Comparison of the ouserved and the first order Markov predicted relative frequencies of sequences of days during the sumer seascin with ti.uncierstorms occurring between 1200 and 1959 EST.


Pigure 5. Comparison of the observed and the first and second order Markov predicted relative frequencies of sequences of day in June without thunderstorms occurring between 1200 and 1959 EST.


Figure 6. Comparison of the observed and the first and second order Markov predicted relative frequencies of sequences of days in July without thunderstorms occurring between 1200 and 1959 EST.


Figure 7. Comparison of the observed and the first and second order Markov predicted relative frequencies of sequences of days in August without thunderstorms occurring between 1200 and 1959 EST.


Figure 8. Comparison of the observed and the first, and second order Markov predictea relative frequencies of sequences of days during the summer without thundrrstorns occurring between 1200 and 1959 EST.

APPENDIX A

## TABLE Al. 1

Empirical Probability of Thunderstorm
Occurrence by Hour by Month
Cape Kennedy, Florida (1957-1962)


Each hour represents the 60 minutes beginning at the indicated hour; e.g. " 00 " represents the period 0000-0059 EST.

TABLE Al. 2
Empirical Probability of Thunderstorm
Occurrence by Hour by Season
Cape Kennedy, Florida (1957-1962)

|  | Spring | Summer | Fall | Winter | Ann. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | . 011 | . 022 | . 007 | . 004 | . 011 |
| 01 | . 009 | . 018 | . 013 | . 002 | . 011 |
| 02 | . 005 | . 020 | . 009 | * | . 009 |
| 03 | . 004 | . 013 | . 009 | * | . 006 |
| 04 | . 005 | . 007 | . 007 | . 002 | . 005 |
| 05 | . 011 | . 016 | . 011 | * | . 010 |
| 06 | . 005 | . 018 | . 007 | . 002 | . 008 |
| 07 | * | . 009 | . 011 | . 002 | . 006 |
| 08 | . 004 | . 004 | . 015 | . 004 | . 006 |
| 09 | . 011 | . 002 | . 020 | . 004 | . 009 |
| 10 | . 013 | . 038 | . 015 | . 004 | . 018 |
| 11 | . 016 | . 087 | . 018 | . 002 | . 031 |
| 12 | . 018 | . 138 | . 033 | . 004 | . 048 |
| 13 | . 033 | . 205 | . 038 | . 002 | . 070 |
| 14 | . 042 | . 228 | . 040 | * | . 078 |
| 15 | . 043 | . 232 | . 053 | . 004 | . 083 |
| 16 | . 051 | . 228 | . 049 | . 006 | . 083 |
| 17 | . 060 | . 197 | . 051 | . 004 | . 078 |
| 18 | . 062 | . 154 | . 053 | * | . 067 |
| 19 | . 042 | . 118 | . 046 | . 004 | . 052 |
| 20 | . 034 | . 076 | . 027 | . 006 | . 036 |
| 21 | . 027 | . 056 | . 015 | . 006 | . 026 |
| 22 | . 031 | . 031 | . 013 | . 006 | . 020 |
| 23 | . 022 | . 029 | . 009 | . 004 | . 016 |
| Avg. | . 023 | . 081 | . 024 | . 003 | . 033 |
| 2359 | . 163 | . 466 | . 161 | . 030 | . 205 |

TABLE A2.1
EMPIRICAL ODNDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms during the indicated hours, given that $i$ consecutive days with thunderstorms have just occurred


TABLE A2. 2
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms during the indicated hours, given that $i$ consecutive days with thunderstorms have just occurred


TABLE A2. 3
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms during the indicated hours, given that i consecutive days
with thunderstorms have just occurred

table A2. 4

## EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms during the indicated hours, given that i consecutive days with thunderstorms have just occurred


TABLE A2.5
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days with thunderstorms during the indicated hours, given that i consecutive days
with thunderstorms have just occurred


TABLE A3. 1
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms during the indicated hours, given that i consecutive days
with thunderstorms have just occurred


TABLE A3. 2
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms during the indicated hours, given that $i$ consecutive days
with thunderstorms have just occurred


TABLE A3. 3
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms
during the indicated hours, given that $i$ consecutive days
with thunderstorms have just occurred

| $\begin{aligned} & 1200- \\ & 1959 \\ & \hline \end{aligned}$ | k |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| June $\begin{array}{rr}\text { i } & =0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6\end{array}$ | . 406 | . 228 | . 133 | . 072 | . 033 | . 017 | . 006 |  |  |  |  |  |
|  | . 562 | . 329 | . 178 | . 082 | . 041 | . 014 |  |  |  |  |  |  |
|  | . 585 | . 317 | . 146 | . 073 | . 024 |  |  |  |  |  |  |  |
|  | . 542 | . 250 | . 125 | . 042 |  |  |  |  |  |  |  |  |
|  | . 462 | . 231 | . 077 |  |  |  |  |  |  |  |  |  |
|  | . 500 | . 167 |  |  |  |  |  |  |  |  |  |  |
|  | . 333 |  |  |  |  |  |  |  |  |  |  |  |
| July $\begin{array}{lr}\text { i } & \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \\ & 7 \\ & 8 \\ & 9 \\ & 10 \\ & 11\end{array}$ | . 441 | . 312 | . 226 | . 161 | . 113 | . 081 | . 054 | . 038 | . 022 | . 016 | . 011 | . 005 |
|  | . 707 | . 512 | . 366 | . 256 | . 183 | . 122 | . 085 | . 049 | . 037 | . 024 | . 012 |  |
|  | . 724 | . 517 | . 362 | . 259 | . 172 | . 121 | . 069 | . 052 | . 034 | . 017 |  |  |
|  | . 714 | . 500 | . 357 | . 238 | . 167 | . 095 | . 071 | . 048 | . 024 |  |  |  |
|  | . 700 | . 500 | . 333 | . 233 | . 133 | . 100 | . 067 | . 033 |  |  |  |  |
|  | . 714 | . 476 | . 333 | . 190 | . 143 | . 095 | . 048 |  |  |  |  |  |
|  | . 667 | . 467 | . 267 | . 200 | . 133 | . 067 |  |  |  |  |  |  |
|  | . 700 | . 400 | . 300 | . 200 | . 100 |  |  |  |  |  |  |  |
|  | . 571 | . 429 | . 286 | . 143 |  |  |  |  |  |  |  |  |
|  | . 750 | . 500 | . 250 |  |  |  |  |  |  |  |  |  |
|  | . 667 | . 333 |  |  |  |  |  |  |  |  |  |  |
|  | . 500 |  |  |  |  |  |  |  |  |  |  |  |
| August i=0 | . 414 | . 237 | . 140 | . 091 | . 059 | . 038 | . 022 | . 016 | . 011 | . 005 |  |  |
| 1 | . 571 | . 338 | . 221 | . 143 | . 091 | . 052 | . 039 | . 026 | . 013 |  |  |  |
| 2 | . 591 | . 386 | . 250 | . 159 | . 091 | . 068 | . 045 | . 023 |  |  |  |  |
| 3 | . 654 | . 423 | . 269 | . 154 | . 115 | . 077 | . 038 |  |  |  |  |  |
| 4 | . 647 | . 412 | . 235 | . 176 | . 118 | . 059 |  |  |  |  |  |  |
| 5 | . 636 | . 364 | . 273 | . 182 | . 091 |  |  |  |  |  |  |  |
| 6 | . 571 | . 429 | . 286 | . 143 |  |  |  |  |  |  |  |  |
|  | . 750 | . 500 | . 250 |  |  |  |  |  |  |  |  |  |
| 8 | . 667 | . 333 |  |  |  |  |  |  |  |  |  |  |
| 9 | . 500 |  |  |  |  |  |  |  |  |  |  |  |
| Summer i=0 | . 420 | . 259 | . 167 | . 109 | . 069 | . 045 | . 027 | . 018 | . 011 | . 007 | . 004 | . 002 |
| 1 | . 616 | . 397 | . 259 | . 164 | . 108 | . 065 | . 043 | . 026 | . 017 | . 009 | . 004 |  |
| 2 | . 643 | . 420 | . 266 | . 175 | . 105 | . 070 | . 042 | . 028 | . 014 | . 007 |  |  |
| 3 | . 652 | . 413 | . 272 | . 163 | . 109 | . 065 | . 043 | . 022 | . 011 |  |  |  |
| 4 | . 633 | . 417 | . 250 | . 167 | . 100 | . 067 | . 033 | . 017 |  |  |  |  |
| 5 | . 658 | . 395 | . 263 | . 158 | . 105 | . 053 | . 026 |  |  |  |  |  |
| 6 | . 600 | . 400 | . 240 | . 160 | . 085 | . 040 |  |  |  |  |  |  |
|  | . 667 | . 400 | . 267 | . 133 | . 067 |  |  |  |  |  |  |  |
| 8 | . 600 | . 400 | . 200 | . 100 |  |  |  |  |  |  |  |  |
| 9 | . 667 | . 333 | . 167 |  |  |  |  |  |  |  |  |  |
| 10 | . 500 | . 250 |  |  |  |  |  |  |  |  |  |  |
|  | . 500 |  |  |  |  |  |  |  |  |  |  |  |

TABLE A3. 4
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms during the indicated hours, given that i consecutive days
with thunderstorms have just occurred

| $\begin{array}{ll} 1600- \\ 2359 & \text { EST } \end{array}$ | k |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| June $\quad i=$ | . 283 | . 133 | . 067 | . 022 | . 005 |  |  |  |  |  |  |  |
|  | . 471 | . 235 | . 078 | . 020 |  |  |  |  |  |  |  |  |
|  | . 500 | . 167 | . 042 |  |  |  |  |  |  |  |  |  |
|  | . 333 | . 083 |  |  |  |  |  |  |  |  |  |  |
|  | . 250 |  |  |  |  |  |  |  |  |  |  |  |
| July $\begin{array}{rr}\text { i } & =0 \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6\end{array}$ | . 376 | . 167 | . 124 | . 065 | . 038 | . 027 | . 011 |  |  |  |  |  |
|  | . 443 | . 329 | . 171 | . 100 | . 071 | . 029 |  |  |  |  |  |  |
|  | . 742 | . 387 | . 226 | . 161 | . 065 |  |  |  |  |  |  |  |
|  | . 522 | . 304 | . 217 | . 087 |  |  |  |  |  |  |  |  |
|  | . 583 | . 417 | . 167 |  |  |  |  |  |  |  |  |  |
|  | . 714 | . 286 |  |  |  |  |  |  |  |  |  |  |
|  | . 400 |  |  |  |  |  |  |  |  |  |  |  |
| August i=0 | . 280 | . 102 | . 054 | . 027 | . 016 | د.00. |  |  |  |  |  |  |
| 1 | . 365 | . 192 | . 096 | . 058 | . 019 |  |  |  |  |  |  |  |
| 2 | . 526 | . 263 | . 158 | .05, |  |  |  |  |  |  |  |  |
| 3 | . 500 | . 300 | . 100 |  |  |  |  |  |  |  |  |  |
| 4 | . 600 | . 200 |  |  |  |  |  |  |  |  |  |  |
| 5 | . 333 |  |  |  |  |  |  |  |  |  |  |  |
| Summer $\begin{array}{r}\text { i }\end{array}$ 0 ${ }^{1} \mathrm{l}$ | . 313 | . 134 | . 082 | . 038 | . 020 | . 011 | . 004 |  |  |  |  |  |
|  | . 428 | . 260 | . 121 | . 064 | . 025 | . 012 |  |  |  |  |  |  |
|  | . 608 | . 284 | . 149 | . 081 | . 027 |  |  |  |  |  |  |  |
|  | . 467 | . 244 | . 133 | . 044 |  |  |  |  |  |  |  |  |
|  | . 524 | . 286 | . 095 |  |  |  |  |  |  |  |  |  |
|  | . 545 | . 182 |  |  |  |  |  |  |  |  |  |  |
|  | . 333 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

TABLE A3. 5

## EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days with thunderstorms during the indicated hours, given that i consecutive days
with thunderstorms have just occurred

| $\begin{aligned} & 0000- \\ & 2359 \end{aligned}$ | EST | k |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| June | $\mathrm{i}=0$ | . 444 | . 278 | . 172 | . 100 | . 050 | . 022 | . 006 |  |  |  |  |  |
|  | 1 | . 625 | . 388 | . 225 | . 113 | . 050 | - 013 |  |  |  |  |  |  |
|  | 2 | .62C | 360 | . 180 | . 080 | . 020 |  |  |  |  |  |  |  |
|  | 3 | . $58{ }^{\text {i }}$ | . 290 | . 129 | . 032 |  |  |  |  |  |  |  |  |
|  | 4 | . 500 | . 222 | . 056 |  |  |  |  |  |  |  |  |  |
|  | 5 | . 444 | . 111 |  |  |  |  |  |  |  |  |  |  |
|  | 6 | . 250 |  |  |  |  |  |  |  |  |  |  |  |
| July | $\mathrm{i}=0$ | . 452 | . 317 | . 231 | . 161 | . 113 | . 081 | . 054 | . 038 | . 022 | . 016 | . 011 | . 005 |
|  | 1 | . 702 | . 512 | . 357 | . 250 | . 179 | . 119 | . 083 | . 048 | . 036 | . 024 | . 012 |  |
|  | 2 | . 729 | . 508 | . 356 | . 254 | . 169 | . 119 | . 068 | . 051 | . 034 | . 017 |  |  |
|  | 3 | . 698 | . 488 | . 349 | . 233 | . 163 | . 093 | . 070 | . 047 | . 023 |  |  |  |
|  | 4 | . 700 | . 500 | . 333 | . 233 | . 133 | . 100 | . 067 | . 033 |  |  |  |  |
|  | 5 | . 714 | . 476 | . 333 | . 190 | . 143 | . 095 | . 048 |  |  |  |  |  |
|  | 6 | . 667 | . 467 | . 267 | . 200 | . 133 | . 067 |  |  |  |  |  |  |
|  | 7 | . 700 | . 400 | . 300 | . 200 | . 100 |  |  |  |  |  |  |  |
|  | 8 | . 571 | . 429 | . 286 | . 143 |  |  |  |  |  |  |  |  |
|  | 9 | . 750 | . 500 | . 250 |  |  |  |  |  |  |  |  |  |
|  | 10 | . 667 | . 333 |  |  |  |  |  |  |  |  |  |  |
|  | 11 | . 500 |  |  |  |  |  |  |  |  |  |  |  |
| August | $i=0$ | . 500 | . 312 | . 199 | . 134 | . 086 | . 054 | . 032 | . 016 | . 011 | . 005 |  |  |
|  | 1 | . 624 | . 398 | . 269 | . 172 | . 108 | . 065 | . 032 | . 022 | . 011 |  |  |  |
|  | 2 | . 638 | . 431 | . 276 | . 172 | . 103 | . 052 | . 034 | . 017 |  |  |  |  |
|  | 3 | . 676 | . 432 | . 270 | . 162 | . 081 | . 054 | . 027 |  |  |  |  |  |
|  | 4 | . 640 | . 400 | . 240 | . 120 | . 080 | . 040 |  |  |  |  |  |  |
|  | 5 | . 625 | . 375 | . 188 | . 125 | . 063 |  |  |  |  |  |  |  |
|  | 6 | . 600 | . 300 | . 200 | . 100 |  |  |  |  |  |  |  |  |
|  | 7 | . 500 | . 333 | . 167 |  |  |  |  |  |  |  |  |  |
|  | 8 | . 667 | . 333 |  |  |  |  |  |  |  |  |  |  |
|  | 9 | . 500 |  |  |  |  |  |  |  |  |  |  |  |
| Summer | $i=0$ | . 466 | . 303 | . 201 | . 132 | . 083 | . 053 | . 031 | . 018 | . 011 | . 007 | . 004 | . 002 |
|  | 1 | . 650 | . 432 | . 284 | . 179 | . 113 | . 066 | . 039 | . 023 | . 016 | . 008 | . 004 |  |
|  | 2 | . 665 | . 437 | . 275 | . 174 | . 102 | . 060 | . 036 | . 024 | . 012 | . 006 |  |  |
|  | 3 | . 658 | . 414 | . 261 | . 153 | . 090 | . 054 | . 036 | . 018 | . 009 |  |  |  |
|  | 4 | . 630 | . 397 | . 233 | . 137 | . 082 | . 055 | . 027 | . 014 |  |  |  |  |
|  | 5 | . 630 | . 370 | . 217 | . 130 | . 087 | . 043 | . 022 |  |  |  |  |  |
|  | 6 | . 586 | . 345 | . 207 | . 138 | . 069 | . 034 |  |  |  |  |  |  |
|  | 7 | . 588 | . 353 | . 235 | . 118 | . 059 |  |  |  |  |  |  |  |
|  | 8 | . 600 | . 400 | . 200 | . 100 |  |  |  |  |  |  |  |  |
|  | 9 | . 667 | . 333 | . 167 |  |  |  |  |  |  |  |  |  |
|  | 10 | . 500 | . 250 |  |  |  |  |  |  |  |  |  |  |
|  | 11 | . 500 |  |  |  |  |  |  |  |  |  |  |  |

TABLE A4.1

## EMPIRICAL OONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4.2

## EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4. 3

## EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days without thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4.4
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days withou thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4. 5
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4. 6

EMPIRICAL ONNDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days without thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4.7

EMPIRICAL CONDITIONAL PROBAPTLITILS

Probability of $\mathbf{k}$ additional consecutive days without thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4. 8
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4. 9
EmPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days withou ${ }^{+}$thunderstorms have just occurred.


TABLE A4. 10

## EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive diys without thunderstorms have just occurred.


TABLE A4. 11
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A4. 12

## EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A5.1
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicatea hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A5. 2
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without thunderstorms during the indicated hours, given that

1 consecutive days without thunderstorms have just occurred


TABLE A5. 3
EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A5. 4

EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecuive aays whrnout thunderstorms have just occurrea


TABLE A5. 5

## EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutıve days without
thuncerstorms during the indicated hours, given that
i consecutive days withuut tuunuerstorms have just occurred

tarlr A5. 6
ERPIRICAL CONDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days without thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred


TABLE A5. 7

EMPIRICAL OONDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred


TABLE A5. 8

## EMPIRICAL CONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occuried.


## EmpIRICAL CONDITIONAL PROBABILITIES

Probability of $\mathbf{k}$ additional consecutive days without thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


## Empirical Conditional probabilities

Probability of $\mathbf{k}$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


TABLE A5. 11

EMPIP $\because$ ONDITIONAL PROBABILITIES

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thuriderstorms have just occurred

| $\begin{aligned} & 0000- \\ & 2359 \end{aligned}$ | EST | k |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| June | $i=0$ | . 556 | . 394 | . 294 | . 233 | . 178 | . 133 | . 106 | . 083 | . 067 | . 056 | . 044 | . 039 |
|  | 1 | . 710 | . 530 | . 420 | . 320 | . 240 | . 190 | . 150 | . 120 | . 100 | . 080 | . 070 | . 060 |
|  | 2 | . 746 | . 592 | . 451 | . 338 | . 268 | . 211 | . 169 | . 141 | . 113 | . 099 | . 085 | . 070 |
|  | 3 | . 792 | . 604 | . 453 | . 358 | . 283 | . 226 | . 189 | . 15 ? | . 132 | . 113 | . 094 | . 075 |
|  | 4 | . 762 | . 571 | . 452 | . 357 | . 286 | . 238 | . 190 | . 167 | . 143 | . 119 | . 095 |  |
|  | 5 | . 750 | . 594 | . 469 | . 375 | . 313 | . 250 | . 219 | . 188 | . 156 | . 125 |  |  |
|  | 6 | . 792 | . 625 | . 500 | . 417 | . 333 | . 292 | . 250 | . 208 | . 167 |  |  |  |
|  | 7 | . 789 | . 632 | . 526 | . 421 | . 353 | . 316 | . 263 | . 211 |  |  |  |  |
|  | 8 | . 800 | . 667 | . 533 | . 467 | $\therefore$. | . 333 | . 267 |  |  |  |  |  |
|  | 9 | . 833 | . 667 | . 583 | . 500 | 1:3\% | . 333 |  |  |  |  |  |  |
|  | 10 | . 300 | . 700 | . 600 | . 500 | . 400 |  |  |  |  |  |  |  |
|  | 11 | . 875 | . 750 | . 625 | . 500 |  |  |  |  |  |  |  |  |
|  | 12 | . 857 | . 714 | . 571 |  |  |  |  |  |  |  |  |  |
| July | $i=0$ | . 548 | . 398 | . 285 | . 204 | 1.151 | . 118 | . 097 | . 075 | . 065 | . 054 | . 043 | . 032 |
|  | 1 | . 725 | . 520 | . 373 | . 275 | . 216 | . 176 | . 137 | . 118 | . 098 | . 078 | . 059 | . 039 |
|  | 2 | . 716 | . 514 | . 378 | . 297 | . 243 | . 189 | . 162 | . 135 | . 108 | . 081 | . 054 | . 041 |
|  | 3 | . 717 | . 528 | . 415 | . 340 | . 264 | . 226 | . 189 | . 151 | . 113 | . 075 | . 057 | . 038 |
|  | 4 | . 737 | . 579 | . 474 | . 368 | . 316 | . 263 | . 211 | . 158 | . 105 | . 079 | . 053 |  |
|  | 5 | . 786 | . 643 | . 500 | . 429 | . 357 | . 286 | . 214 | . 143 | . 107. | . 071 |  |  |
|  | 6 | . 818 | . 636 | . 545 | . 455 | . 364 | ). 273 | . 182 | . 136 | . 091 |  |  |  |
|  | 7 | . 778 | . 667 | . 556 | . 444 | . 333 | . 222 | . 167 | . 111 |  |  |  |  |
|  | 8 | . 857 | . 714 | . 571 | . 429 | . 286 | . 214 | . 143 |  |  |  |  |  |
|  | 9 | . 833 | . 667 | . 500 | . 333 | . 250 | -167 |  |  |  |  |  |  |
|  | 10 | . 800 | . 600 | . 400 | . 300 | . 200 |  |  |  |  |  |  |  |
|  | 11 | .750 | . 500 | . 375 | . 250 |  |  |  |  |  |  |  |  |
|  | 12 | . 667 | .500 | . 333 |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Probability of $k$ additional consecutive days without
thunderstorms during the indicated hours, given that
i consecutive days without thunderstorms have just occurred.


$$
\begin{aligned}
& \begin{array}{l}
0000-\mathrm{kST} \\
2359 \mathrm{k} \\
0-4 \mathrm{kt} \\
5-9 \mathrm{kt} \\
10-14 \mathrm{kt} \\
15-19 \mathrm{kt} \\
20-24 \mathrm{kt} \\
25-29 \mathrm{kt} \\
30-34 \mathrm{kt} \\
35-39 \mathrm{kt} \\
40-44 \mathrm{kt} \\
45-49 \mathrm{kt} \\
\text { Totals }
\end{array}
\end{aligned}
$$

TABLE A6. 2
Frequency Distribution of Maximum Wind Speeds
Observed with Thunderstorms at Cape Kennedy, Florida (1957-1962)



APPENDIX B

## Testing Procedure for Order for Markov Chains

Adotevi-Akue et al [l] have shown that the appropriate forms of the asymptotically Chi-square statistic as derived by Billingsley [2] for testing the order of Markov chains (with two states and 4-day sequences) are:

For zero order,

$$
\chi^{2}=\Sigma a_{1} \ldots a_{4} \frac{\left(f a_{1} \ldots a_{4}-\frac{f a_{1} \ldots a_{3} f a_{4}}{N}\right)^{2}}{\frac{f a_{1} \ldots a_{3} f a_{4}}{\hat{N}}}
$$

with 7 degrees of freedom where the f's are the sequence frequencies and N is the total numijer of transitions.

For first order,

$$
\chi^{2}=\Sigma a_{1} \ldots a_{4} \frac{\left(f a_{1} \ldots a_{4}-\frac{f a_{1} \ldots a_{3} f a_{3} a_{4}}{f a_{3}}\right)^{2}}{\frac{f a_{1} \cdots a_{3} f a_{3} a_{4}}{f a_{3}}}
$$

with 6 degrees of freedom.

For second order,

$$
\chi^{2}=\sum_{a_{1} \ldots a_{4}} \frac{\left(f a_{1} \ldots a_{4}-f a_{1} \ldots a_{3} f a_{2} \ldots a_{4}\right)^{2}}{\frac{f a_{2} a_{3}}{f a_{2} \cdots a_{3} f a_{2} \cdots a_{4}}} \frac{f a_{2} a_{3}}{}
$$

with 4 degrees of freedom.

Degrees of freedom are determined by the relation

$$
d f=\left(s^{t+1}-s^{t}\right)-\left(s^{r+1}-s^{r}\right)
$$

where $S$ is the number of outcome, $t$ is the number of days in the sequence minus one and $r$ is the order of the model.

## Sample Calculations of Chi-square

The follow ing frequencies were extracted from 6 years, 1957-196?, of data at Cape Kennedy, Florida for the summer season. The time period is 1200-1959 EST. The 4-day sequences beginning on each day of the summer period were tabulated yielding a total of 552 frequencies for the 6 -year period.

| Sequence | Frequency | Sequence | Frequency |
| :---: | :---: | :---: | :---: |
| NNNN | 138 | TNNN | 38 |
| NNNT | 37 | TNNT | 20 |
| NNTN | 23 | TNTN | 12 |
| NNTT | 33 | TNTTT | 19 |
| NTNN | 22 | TITNN | 37 |
| NINT | 16 | TTNT | 15 |
| NTHN | 20 | TTMN | 31 |
| NTHT | 32 | THTT | 59 |

The letter " $T$ " denotes thunderstorm occurrence and " N " non-occurrence.

If it is desired to test the above sequences for first order the appropriate statistic would be sumned over the 16 sequences. For example, if we denote $\Delta$ as the centributum of each individual sequence to the total Chi-square, the $\Delta$ value for the first sequence (NNNN) would be:

$$
\Delta=\frac{\left(138-\frac{(175)(235)}{323}\right)^{2}}{\frac{(175)(235)}{323}}=0.895526
$$

where

$$
f a_{1} \ldots a_{4}=138 ; f a_{1} \ldots a_{3}=175 ; f a_{3} a_{4}=235 ; f a_{3}=323
$$

The Cini-square velue is,

$$
\chi^{2}=\sum_{i=1}^{16} \quad \Delta i=9.82338
$$

The null kypothesis tested is that within the assumption that the process is Markovian, the order of dependence is one. Interpretation as to the significance of the above Chi-square value cannot be made until the resilts of the zero and second order tests can be evaluated.

## REFERENCES

1. Manual of Surface Observations (WBAN) Circular N, Seventh Edition, Government Printing Office, Washington, D. C., 1957, 173 pp.
2. Brooks, C. E. P. and Carruthers, N., Handbook of Statistical Methods in Meteorology, M. 0. 538 Air Ministry, Her Majesty's Stationery Office, London, 1953, pp. 315-317.
3. Eggengerger, F. and Polya, G., "Über die Statistik Verketteter Vorgänge," Z. angew. Math. Mech., Berlin, 1923, p. 279.
4. Siegel, Sidney, Nonparametric Statistics for the Behavioral Sciences, McGraw-Hill Book Company, New York, 1956, pp. 47-52.
5. Quinlan, F. T., "An Investigation of a Markov Model to Describe Sequences of Wet and Dry Days," (unpublished), Rutgers, The State University, New Brunswick, New Jersey, 1966, 63 pp.
6. Crutcher, H. L., Quinlan, F. T., Lee, R. F., and Ownbey, J. W., "Thunderstorm Persistence at Cape Kennedy, Florida," a paper given at the symposium on "Recent Contributions in Dynamical and Statistical Meteorology to Aerospace Vehicle Problems" held at the NASA-George C. Marshall Space Flight Center, Huntsville, Alabama, on December 15, 1966.
7. Adotevi-Akue, G. M., et al., "F aluation of Alternative Markovian Models for Precipitation Occurrence in Oregon," Oregon State University, Progress Report to the Technical Committee, Regional Research Project W-48, May 1965, 29 pp.
8. Billingsley, P., "Statistical Methods in Markov Chains," Annals of Mathematical Statistics, Vol. 32, No. 1, March 1961, pp. 12-40.

[^0]:    **Indicates theoretical and orserved distributions are significantly different at the 0.01 probability level.

