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QUARTERLY PROGRESS REPORT
CONVOLUTIONAL CODING TECHNIQUES
FOR DATA PROTECTION

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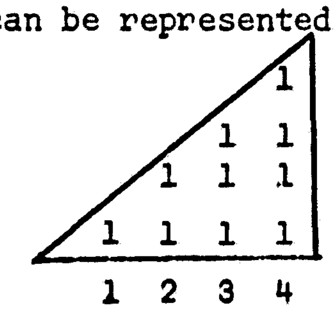
1. Research in Sequential Decoding

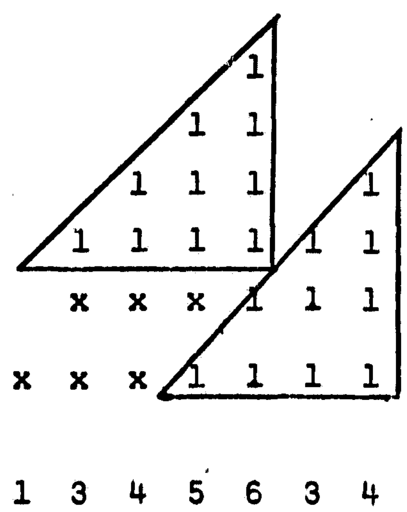
Research has continued into the problem of characterizing the "typical" behavior of a sequential decoder, i.e. that behavior which occurs with probability $1-\varepsilon$ or more where ε is a small number, say 10^{-3}. The objective is to obtain the probability distribution for performance in this typical region in terms of (a) code parameters which can be easily determined, and (b) parameters of the Fano decoding algorithm, namely the threshold increment Δ and the branch metric values. It is then anticipated that an analytical optimization of all these parameters can be carried out--a result that could be of considerable practical value either by showing how the typical performance of existing decoders could be improved or by showing that no additional work is needed if existing decoders prove to be already near optimal.

Some tentative results in this direction have been obtained by J. Geist in work that is continuing. Realizing that most of the typical computation is performed in searches along the "correct path" through the decoding tree (in a sense when the decoder is reassuring itself that it is indeed on the correct path), Geist has concentrated on this phase of the decoding effort and as one interesting example of his results has been able to obtain a geometric construction which might be termed "superposition of identical triangles" for determining the computation done at each branch. An example will illustrate the method:

Example: Consider a rate one-half Fano sequential decoder for a binary symmetric channel with $\Delta = 1$ and branch weights of +1, -4, and -9 depending on whether the two branch digits agree exactly, disagree in one place, or disagree entirely with the received digits. Then an isolated single channel error causes 1 extra computation on the third branch preceding the error, 2 extra on the second, 3 extra on the first, and 4 extra on the branch with the error. This extra computation can be represented by the triangular array



The additional computation due to two isolated errors separated by an error free branch can be determined geometrically as



That is, there will be one extra computation fourth branch preceding the first error, 3 extra on the third preceding, etc.

This geometric rule turns out to be completely general for arbitrary numbers of errors, and is expected to provide needed insight into the typical "bulges" in computation due to several clustered errors in a decoding frame.

2. Research into Convolutional Codes

D. Costello has continued his investigations into the theoretical structure of convolutional codes. An aspect of particular attention has been the so-called "free distance" d_{free} of the code which is the minimum Hamming distance between two arbitrarily long encoded sequences with different first information digits (in contrast to the usual minimum distance d_{min} in which the minimum is taken only over the first constraint length.) Simulations as reported in

> D. J. Costello, Jr., "A Construction Technique for Random Error Correcting Convolutional Codes," Memo. EE-6822, Dept. of Elec. Engr., Univ. of Notre Dame, Nov. 14, 1968. (Preprint of Correspondence Article to appear in IEEE Transactions on Information Theory.)

have established a close correlation between d_{free} and the undetected error probability of sequential decoding, as had been conjectured theoretically. Costello has recently obtained an improved Plotkin-like upper bound on d_{free} applicable to all code rates and code types (systematic or non-systematic) and has obtained an improved Gilbert-type lower bound which suggests that d_{free} can be made roughly twice d_{min} over an interesting range of rates. This work will be part of a later report summarizing all our theoretical investigations of convolutional codes.