General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

Quarterly Progress Report

FAILURE CRITERIA

FOR VISCOELASTIC MATERIALS

W. G. Knauss

GALCIT SM 69-3   MARCH 1969

Firestone Flight Sciences Laboratory

Graduate Aeronautical Laboratories

California Institute of Technology

Pasadena, California

The work carried out under the Grant NsG-172-60, GALCIT 120, has dealt with

a) extending results obtained for crack propagation in special geometries and under special load conditions to the most general situation

b) solidifying earlier work

c) initiate work of crack propagation on solid propellant

STEADY STATE CRACK PROPAGATION

The work on crack propagation in a viscoelastic strip has been written up for publication. The parallel work on the meaning of surface energy, as presented at the National Meeting of the Society of Rheology has also been written up for publication and copies will be forwarded soon.

There is a continual confusion as to the meaning of surface energy in fracture. The problem is one primarily of semantics, but is seldom recognized as being so simple.

One may, on the one hand, define as "surface energy" that work necessary to create a unit of new surface, regardless of the processes involved in surface creation. From a continuum mechanical viewpoint this definition is entirely advisable and sufficient, though not necessarily always useful.

On the other hand, one may consider that in creating new surface one may have to do work against primary interatomic forces (valence bonds) and against secondary forces such as van der Waals forces in re-orienting mocules or atoms near a new surface. The

amounts of work done against different types of forces in such a molecular process vary in size. Classically speaking the surface tension of a liquid is only the work done against secondary inter-molecular forces. Griffith assumed that this same surface tension acts in the solid state. Since glass is a super cooled liquid, that assumption seems reasonable.

When the fracture of cross-linked polymers is concerned, one must consider, from a molecular viewpoint, the work spent in the breaking of primary bonds and not only the work against the molecular interaction forces. The latter, incidentally, cause the rate sensitivity of the material; in the equilibrium state of slow crack growth they are smaller by orders of magnitude than the valence bond forces. Thus the intrinsic fracture energy for creating new surface is dependent on the bond strength. Surface tension in the classical molecular sense may not be important.

This analysis of the fracture energy is, in principle, simple. Nevertheless, we elaborate on it here to point out that the lack of understanding of so simple a problem can jeopardize the understanding of fracture problems. Because fracture is a problem involving the "continuum" and the molecular structure, the investigators in physical chemistry often stumble over the "surface tension" concept in fracture and accept or reject a theory on such a superficial distinction. The continuum mechanist, on the other hand, is not, in principle, disturbed by this problem and tends to leave the molecule-oriented physicist alone in his dilemma by being needlessly vague or magnanimous about the meaning of the fracture energy. By addressing ourselves to this problem

we hope to improve the understanding of fracture from both the microscopic and macroscopic point of view.

As a practical point of interest it should be noted that the fracture energy determined for Solithane 113 agrees closely with the theoretical value determined by Lake and Thomas.*

THE GENERAL PROBLEM OF CRACK PROPAGATION

Following the method outlined in an earlier report** and as documented in the forthcoming paper on crack propagation in a strip we have determined the law governing crack propagation in arbitrary geometries and under arbitrary load histories. The result is as follows: Let Γ be the fracture energy and $K = K(c,t) = K^*(c)\, \sigma_o(t)$ be the stress intensity factor which depends on the geometry of the structure (indicated here symbolically only by the crack length as the argument) and explicitly on the load history $\sigma_o(t)$. Then one has for the general case

$$\frac{\Gamma}{\pi D\,(\infty)} = \sum_{n,\,m=0} \frac{1}{n!\,m!\,(n+m+2)} \left(\frac{\alpha}{\dot{c}}\right)^{m+n} K^{(m)} \left[(n+1)K^{(n)} - \left(\frac{\alpha}{\dot{c}}\right) K^{(n+1)}\right] B_{nm}\left(\frac{\alpha}{\dot{c}}\right)$$

(1)

where $K^{(n)} \quad = \quad \partial^n K/\partial t^n$

$\dot{c} \qquad = \quad$ crack velocity

$\alpha \qquad = \quad$ length parameter (small)

$$B_{nm}(t) \quad = \quad \frac{(m+1)(m+n+2)}{t^{m+n+2}} \int_o^t \xi^n \int_o^\xi (\xi - \eta)^m \; \frac{D(\eta)}{D(\infty)} \; d\eta \; d\xi$$

* Lake, G. J.; Thomas, A. E., "The Strength of Highly Elastic Materials", Proceedings of the Royal Society of London, Series, A., Vol. 300, September 1967, pp. 108-119.

** Knauss, W. G. Delayed Failure--The Griffith Problem for Linearly Viscoelastic Materials, GALCIT SM 68-15, Calfiornia Institute of Technology, Pasadena, California, September 1968.

It should be noted that all of the functions $B_{nm}(t)$ tend to unity as the argument t tends to infinity and

$$\lim_{t \to o} B_{nm}(t) = \frac{D(o)}{D(\infty)}$$

$$\lim_{t \to \infty} B_{nm}(t) = \max \left| B_{nm}(t) \right| = 1$$

Because B_{nm} are bounded from above by unity it is easy to recognize that the double series converges always as long as $\dot{c} \neq 0$. The case $\dot{c} = 0$ exists only at the time limit of small applied loads, which limit is given by a Griffith criterion involving only the long time properties of the material.

Special Cases. We consider now a few special cases of this general result. Consider, as in earlier work, crack propagation in an infinite strip such that the only variation of the stress intensity factor arises from the variation of the applied strain, i. e.,

$$K(t) = \overline{K} \sigma_o(t) \quad ; \quad \overline{K} = \text{const.}$$

a) constant stress $\sigma_o(t) = \sigma_o = \text{const.}$

$$\frac{2\Gamma}{\pi \overline{K}^2 \sigma_o^2 D(\infty)} = B_{oo}\left(\frac{\alpha}{\dot{c}}\right) \tag{2}$$

This is the same result as obtained earlier in the work now being readied for publication.

b) constant stress rate

$$\sigma_o(t) = \dot{\sigma}_o \cdot t$$

$$\frac{2\Gamma}{\pi \overline{K}^2 \dot{\sigma}_o^2 D(\infty)} = t\left(t - \frac{\alpha}{c}\right) B_{0\,0}\left(\frac{\alpha}{c}\right) + \frac{2}{3}\frac{\alpha}{c}\left(t - \frac{\alpha}{c}\right)B_{0\,1}\left(\frac{\alpha}{c}\right)$$

$$+ \frac{4}{3} t \frac{\alpha}{c} B_{1\,0}\left(\frac{\alpha}{c}\right) + \left(\frac{\alpha}{c}\right)^2 B_{1\,1}\left(\frac{\alpha}{c}\right) \tag{3}$$

c) sinusoidal stress history (fatigue)

$$K = \overline{K}\sigma_o(t) \quad \sigma_o(t) = \sigma_o(A + \sin\omega t) = \mathbf{Re}\left\{\sigma_o(A - ie^{i\omega t})\right\}$$

$$\frac{\Gamma}{\pi \overline{K}^2 \sigma_o^2 D(\infty)} = \mathbf{Re}\left\{\frac{1}{2}A^2 B_{0\,0}\right.$$

$$- A\sum_{m=0}^{1} i^{m+1} a^m e^{i\omega t} C_{om} B_{om}$$

$$+ A\sum_{n=0}^{\infty} i^{n+1} a^n e^{i\omega t} (n+1-ia) C_{no} B_{no}$$

$$+ \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} i^{n+m+2} a^{n+m} e^{2i\omega t} (n+1-ia) C_{nm} B_{nm}\right\} \tag{4}$$

where $\quad a \equiv \frac{\alpha}{c}\omega$

$$C_{nm} = \frac{1}{n!}\,\frac{1}{m!}\,\frac{1}{n+m+2}$$

-5-

CRACK PROPAGATION IN SOLID PROPELLANTS

Inasmuch as the prediction of crack propagation and failure in an unfilled elastomer seems to lead to reasonable agreement with experiment it is desirable to attempt application of the theory to filled polymers, solid propellants in particular. To that end we have convened with Messrs. G. Lewis and E. Duran of the Jet Propulsion Laboratory for providing appropriate sheet material. This work is now in progress.

CRACK PROPAGATION AROUND INCLUSIONS

In order to understand the propagation of failure in particulate filled elastomers like solid propellants, one needs to understand the propagation of a crack through or around a particle. As a model for failure propagation in the vicinity of a particle we are investigating experimentally the growth of a crack in the immediate vicinity of a circular inclusion in a sheet. It has been found so far that a crack running on the axis of symmetry for the geometry will deviate from that symmetry line when it comes close to the circular inclusion and approaches the rim of the inclusion tangentially. This finding is important for knowing what kind of stress analysis has to be performed prior to calculation of crack propagation rate around inclusions.

Such rate calculations will allow one to estimate the effect of filler particles on retarding crack propagation. It would appear, therefore, that one could estimate from the volume fraction of filler and the filler-elastomer bond properties a lengthening of the failure times of filled elastomers.

CRACK GROWTH IN A SHEAR FIELD

It is often convenient to consider crack propagation in anti-plane shear or tearing (Fig. 1). The mathematical analysis is simpler than for in-plane loading (Fig. 2) and therefore attractive from this viewpoint. However, when it comes to evaluating fracture criteria from such a simpler analysis, one must assume that crack propagation occurs in the plane of the crack. This, more often than not is accepted as true and no conclusive evidence exists in the literature to contradict this assumption.

We have prepared molds to cast specimens of the type shown in Fig. 1 and are about to show experimentally that the crack will not propagate in its own plane but in a much more complicated fashion. This has been done once before in our laboratory, except that the test sample was too small and the results therefore not sufficiently clear for argumentative presentation.

FRACTURE WORK

Work on all the aspects described above is in progress. A main effort will be expended in evaluating the crack propagation laws cited above, equations (1-4). In doing so we shall explore specifically how many terms have to be retained in the infinite double series with an aim to make the result as simple as possible for maximum ease of practical application.