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PLANETARY ATMOSPHERE ALBEDO

by

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SUMMARY

The plane and spherical albedo of planetary atmosphere are computed assuming that the atmosphere's optical thickness is infinitely great. Applying the obtained formula for $A(\zeta)$– spherical albedo to planet Venus, which is 0.76 in the visible part of the spectrum, and using the value of x = 2.1, recently obtained by the present author, it is found that $1 - \lambda = 0.0045$.

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The object of the present note is the obtaining of formulas for planet's plane and spherical albedo, when it is surrounded by an atmosphere of infinitely great optical thickness. The scattering indicatrix $x(\gamma)$ is considered arbitrary and the quantity λ, representing the ratio of the scattering coefficient to the sum of scattering coefficients and true absorption, is assumed to be close to the unity (i. e., $1 - \lambda \ll 1$).

Let the cosine of the incident angle of solar rays be ζ at the given spot, while the atmosphere's upper boundary illumination, induced by them is $\pi S\zeta$. The averaged by azimuth radiation intensity, diffusively reflected by the atmosphere at an angle arc $\cos\eta$ to the normal, shall be represented in the form

$$I(\eta, \zeta) = S\rho(\eta, \zeta)\zeta. \qquad (1)$$

Then the atmosphere plane albedo will be determined by the formula

$$A(\zeta) = 2\int_0^1 \rho(\eta, \zeta)\,\eta\,d\eta, \qquad (2)$$

and the spherical albedo of the planet will be

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$$A_* = 2\int_0^1 A(\zeta)\zeta\, d\zeta. \tag{3}$$

The quantities $A(\zeta)$ and A may be expanded in series by powers $\sqrt{1-\lambda}$. Earlier [1], the zero and the first terms of this expansion have been obtained. We shall now apply the same method to find the term of the order $1-\lambda$.

We shall start from the following two relations, linking among themselves the reflection factor of the atmosphere $\rho(\eta, \zeta)$, its transmission factor $u(\eta)$ and the relative radiation intensity $i(\eta)$ in the deep layers of the medium:

$$i(-\zeta) = 2\int_0^1 \rho(\eta, \zeta)\, i(\eta)\, \eta\, d\eta, \tag{4}$$

$$Mu(\zeta) = i(\zeta) - 2\int_0^1 \rho(\eta, \zeta)\, i(-\eta)\, \eta\, d\eta. \tag{5}$$

Here

$$M = 2\int_{-1}^{1} i^2(\eta)^1 d\eta, \tag{6}$$

while functions $u(\eta)$ and $i(\eta)$ are normalized according to conditions

$$2\int_0^1 u(\eta)\, i(\eta)\, \eta\, d\eta = 1, \tag{7}$$

$$\frac{\lambda}{2}\int_{-1}^{1} i(\eta)\, d\eta = 1. \tag{8}$$

To determine the function $i(\eta)$, we shall make use of the integral equation obtained by V. A. Ambartsumyan [2]. From that equation we find

$$i(\eta) = 1 + 3\eta\sqrt{\frac{1-\lambda}{3-x_1}} + \frac{15\eta^2 - x_2}{5 - x_2}(1-\lambda) + \ldots, \tag{9}$$

where x_1 and x_2 are the first and second coefficients of expansion of the scattering indicatrix $x(\gamma)$ by Legendre polynomials.

In accordance with one of the results of work [1], function $u(\eta)$ may be represented in the form

$$u(\eta) = u_0(\eta)\left(1 - C\sqrt{1-\lambda}\right) + \ldots, \tag{10}$$

where $u_0(\eta)$ is the atmosphere's transmission factor in case of pure scattering and C is a constant. Substituting (9) and (10) into (7), we shall obtain

$$C = \frac{6}{\sqrt{3-x_1}}\int_0^1 u_0(\eta)\,\eta^2\,d\eta. \tag{11}$$

Let us postulate

$$A(\zeta) = 1 + A_1(\zeta)\sqrt{1-\lambda} + A_2(\zeta)(1-\lambda) + \ldots, \tag{12}$$

$$\rho(\eta,\zeta) = \rho_0(\eta,\zeta) + \rho_1(\eta,\zeta)\sqrt{1-\lambda} + \rho_2(\eta,\zeta)(1-\lambda) + \ldots, \tag{13}$$

where $\rho_0(\eta, \zeta)$ is atmosphere's reflection factor at $\lambda = 1$. The quantity $\rho_0(\eta, \zeta)$ satisfies the condition

$$2\int_0^1 \rho_0(\eta,\zeta)\,\eta\,d\eta = 1. \tag{14}$$

Moreover, we have

$$2\int_0^1 \rho_1(\eta,\zeta)\,\eta\,d\eta = A_1(\zeta), \quad 2\int_0^1 \rho_2(\eta,\zeta)\,\eta\,d\eta = A_2(\zeta). \tag{15}$$

Substituting expressions (9), (10), (12) and (13) into relations (4) and (5), we obtain

$$A_1(\zeta) = -\frac{4}{\sqrt{3-x_1}}\,u_0(\zeta), \tag{16}$$

$$A_2(\zeta) = \frac{15}{5-x_2}\,v_0(\zeta) + \frac{4C}{\sqrt{3-x_1}}\,u_0(\zeta), \tag{17}$$

where

$$u_0(\zeta) = \frac{3}{4}\left[\zeta + 2\int_0^1 \rho_0(\eta,\zeta)\,\eta^2\,d\eta\right], \tag{18}$$

$$v_0(\zeta) = \zeta^2 - 2\int_0^1 \rho_0(\eta,\zeta)\,\eta^3\,d\eta. \tag{19}$$

Introducing (16) and (17) into (12, and denoting

$$D = 24\int_0^1 u_0(\eta)\,\eta^2\,d\eta, \tag{20}$$

we find

$$A(\zeta) = 1 - 4\sqrt{\frac{1-\lambda}{3-x_1}}\,u_0(\zeta) + \left[\frac{15}{5-x_2}\,v_0(\zeta) + \frac{D}{3-x_1}\,u_0(\zeta)\right](1-\lambda). \tag{21}$$

This asymptotic formula is precisely the one that determines the atmosphere albedo for the angle of incidence of solar rays arc cos ζ.

The functions $u_0(\zeta)$ and $v_0(\zeta)$, entering into (21), satisfy the conditions

$$2\int_0^1 u_0(\zeta)\,\zeta\,d\zeta = 1, \quad \int_0^1 v_0(\zeta)\,\zeta\,d\zeta = 0. \tag{22}$$

Substituting (21) into (3), and taking advantage of (22), we obtain the following asymptotic formula for the spherical albedo of the planet

$$A_* = 1 - 4\sqrt{\frac{1-\lambda}{3-x_1}} + D\,\frac{1-\lambda}{3-x_1}. \tag{23}$$

We see that when taking into account only the first two terms of the expansion by powers $1 - \lambda$, the spherical albedo depends not on the entire scattering indicatrix, but only on parameter x_1. This property is also approximately preserved when taking into account the term of the order $1 - \lambda$, since the magnitude of D is feebly dependent on the scattering indicatrix (this quantity's magnitude may be also taken at isotropic scattering as $D = 8.5$).

The smaller $1 - \lambda$, the more precise the formulas (21) and (23). The presently obtained terms of the order $1 - \lambda$ often substantially increase the precision of determination of the quantities $A(\zeta)$ and A_*.

As an example, we compiled in Table 1 the values of $A(\zeta)$ for a scattering indicatrix $x(\gamma) = 1 + \cos\gamma + P_2(\gamma)$ at $\lambda = 0.99$. The approximate values are obtained by formula (21) without the term $1 - \lambda$ at the outset, and then with it. The precise values are computed by formulas borrowed from the work [3].

Let us apply formula (23) to the atmosphere of Venus, considering its optical thickness as being infinitely great. It is well known from observations (see [4]) that the spherical albedo of Venus is 0.76 in the visual part of the spectrum. For parameter x_1 , a value of 2.1 was recently obtained from analysis of polarimetric observations [5]. Substituting these values of A_* and x_1 into formula (23), we find $1 - \lambda = 0.0045$.

TABLE 1

VALUES OF ALBEDO A(ζ)

ζ	APPROXIMATE		PRECISE	ζ	APPROXIMATE		PRECISE
0	0.886	0.889	0.889	0.6	0.731	0.767	0.764
0.1	0.854	0.860	0.860	0.7	0.709	0.752	0.748
0.2	0.827	0.838	0.838	0.8	0.686	0.738	0.732
0.3	0.803	0.819	0.818	0.9	0.663	0.725	0.717
0.4	0.779	0.800	0.799	1.0	0.641	0.712	0.702
0.5	0.775	0.783	0.781				

*** T H E   E N D ***

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