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SOME PECULIARITIES OF PLASMA MOTIONS IN THE EARTH'S MAGNETOSPHERE

by

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SUMMARY

The motion of matter inside the Earth's magnetosphere is investigated in this paper. It is shown that at flow with free boundary inside the magnetosphere, there sets in an eddy current, whereupon the matter velocity changes sign as it passes through the frontal point. As an example, the structure of the vortex flow is investigated in the incompressible fluid with forbidden zone having the shape of a cylinder or a sphere. Investigated also is the quasi-hydrodynamic flow in the vicinity of the frontal point.

* * *

As is shown in [1], in the quasi-hydrodynamic approximation at supersonic flow past the dipole, a forbidden zone is formed, which may be identified with the Earth's magnetosphere. The flow structure outside the forbidden zone in the equatorial plane of the magnetic field is investigated by the method of consecutive approximations; however, inside the forbidden zone this method is impracticable [1]. As is to be expected, an eddy current forms inside the forbidden zone, which in the vicinity of frontal point is topologically similar to the flow outside the forbidden zone, and roughly speaking, appears to be the specular reflection relative to a smaller portion of the boundary near the front point. If the instabilities of such a motion are suppressed by the nonlinear interaction and the boundary of the forbidden zone is present, the vortex flow, which may be interpreted as a fountain effect in the forbidden zone, induces a convective motion in the tail of the Earth's magnetosphere. The hypothesis on convection in the magnetosphere tail [2] is connected with an unknown mechanism of "viscous-like interaction" with solar wind; however, it is quite clear, that the acceptance of the existence of such a mechanism is superfluous.

In the present work, some peculiarities are investigated of such a flow within the Earth's magnetosphere in a series of models; examined in particular is the structure of the vortex flow of an incompressible fluid, excited within the forbidden zone in the shape of a cylinder or a sphere, when flown at infinity by a uniform stream. Studied here also is the so called frontal point instability, which leads to generation of acoustic waves.

CONTINUITY OF VELOCITY AT THE BOUNDARY OF FORBIDDEN ZONE. We shall define as the frontal point an isolated point at which the velocity vector of the fluid vanishes. We shall define the boundary of forbidden zone, or for brevity, as the boundary, the geometrical position of current lines, to which the frontal point pertains.

We shall define the regions of the fluid lying on various sides of the boundary, as the inner and the outer, and the physical quantities pertaining to these regions will be respectively denoted by the indices 1 and 2. Let us note that generally speaking, for the inner and outer regions, the frontal points could be different though actually lying in the same surface, which is the boundary. The equation of motions inside the inner and outer regions may be presented in the form

$$v_1 \frac{\partial v_1}{\partial \lambda} + \frac{1}{\rho_1}\frac{\partial p_1}{\partial \lambda} = 0, \quad v_2 \frac{\partial v_2}{\partial \lambda} + \frac{1}{\rho_2}\frac{\partial p_2}{\partial \lambda} = 0, \tag{1}$$

where λ is the coordinate along the current line (in parametrical representation the current line is $x = x(\lambda)$, $y = y(\lambda)$, $z = z(\lambda)$); v, ρ, p are respectively the absolute value of the velocity, the density and the pressure of the fluid. Let us call trivial the case when the velocities v_1 and v_2 along the current line do not depend on λ.

Let us demonstrate the following theorem: with the exception of the trivial case, the velocities on both sides of the boundary of the forbidden zone are identical only when the densities are the same.

Let the velocities v_1 and v_2 be identical along the boundary. Inasmuch as in the state of equilibrium the pressures p_1 and p_2 are identical along the boundary, it follows from the expressions (1) that with the exception of the trivial case, $\rho_1 = \rho_2$.

On the contrary, let the densities ρ_1 and ρ_2 be identical. Then, as follows from expressions (1), in the state of equilibrium along a certain current line, the following relations are fulfilled

$$v_1 \frac{\partial v_1}{\partial \lambda} = v_2 \frac{\partial v_2}{\partial \lambda}, \tag{2}$$

$$v_1^2 - v_2^2 = C, \tag{3}$$

where C is a constant. Let us note that, according to (3), if the velocities v_1 and v_2 are identical at any point of a certain current line, they are identical everywhere over the same current line. Let us examine the frontal point of the inner region. According to definition the velocity v_1 vanishes at this point. Then, as follows from relation (2), at the frontal point of the inner region, we have either $v_2 = 0$, or $\partial v_2/\partial\lambda = 0$. If $v_2 = 0$, the frontal points of the inner and outer regions coincide, $C = 0$, and the theorem is demonstrated.

Let us examine the second case. As follows from (3), $C \leqslant 0$. Now let us examine the frontal point of the outer region. At this point $v_2 = 0$, and analogous discussions lead to the conclusion that $C \geqslant 0$. Hence $C = 0$, and the theorem is demonstrated.

SIMPLIFICATION OF TWO-DIMENSIONAL QUASI-HYDRODYNAMICS' EQUATIONS. The quasi-hydrodynamics equations may be represented in the form [1]

$$(\zeta\nabla)\zeta = -\frac{1}{M^2}\nabla\varphi, \qquad \operatorname{div}\varphi\zeta = 0,$$

where ζ is the velocity, related to the velocity v_0 at infinity; $\overline{\phi}$ is the density, the magnetic field or the square root of the pressure, related to the corresponding values ρ_0, B_0 or $\sqrt{p_0}$ at infinity; $M = v_0/\sqrt{2p_0/\rho_0 + B_0^2/(4\pi\rho_0)}$ is the Mach number (the magnetic lines of force are directed along z, $\partial/\partial z = 0$). Eqs. (1) describe a two-dimensional flow of compressible gas with adiabatic exponent 2; and to them we shall apply the usual mathematical pparatus of gas dynamics, in particular, the abovemention theorem. The quantity $\zeta^2/2 + \phi/M^2$

is a function of the current line, but in the absence of vortex this value constitutes an invariant (Bernulli equation).

A single unknown quantity can be eliminated from the system of Eqs.(1). Let us introduce the function of current Q determined by the relation

$$\varphi\zeta = \nabla Q \times \mathbf{k},$$

where $\vec{k}$ is the unitary ort along $\underline{z}$. Then the irrotational plane quasi-hydrodynamic flow is described by the system of equations.

$$(M^2+2)\varphi^2 = 2\varphi^3 + M^2(\operatorname{grad} Q)^2, \quad \varphi\Delta Q - (\nabla\varphi\nabla Q) = 0,$$

and the vortex flow by the system of equations

$$\frac{\operatorname{grad} Q}{\varphi}\operatorname{div}\left(\frac{\operatorname{grad} Q}{\varphi}\right) = \operatorname{grad}\left[\frac{1}{2}\left(\frac{\operatorname{grad} Q}{\varphi}\right)^2 + \frac{\varphi}{M^2}\right]. \tag{4}$$

Eqs.(4) describe approximately the supersonic flow of solar wind past the geomagnetic dipole in the geomagnetic equatorial plane [1]. We have demonstrated that in the state of equilibrium there exists within the forbidden zone (the magnetosphere) a certain current. If in the outer region of the examined magnetospheric model the flow of solar plasma is irrotational [1], inside it the corresponding flow is vortical, inasmuch as the integral, taken from the velocity over a certain current line is not zero (Fig.1, where arrows indicate the direction of the flow of matter).

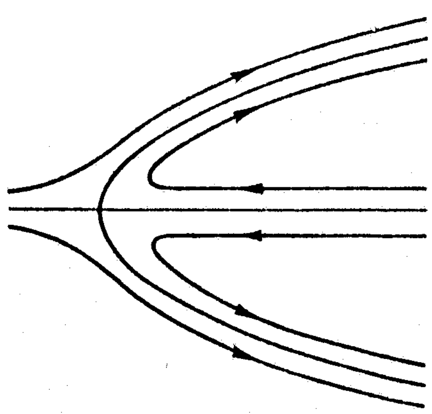

Fig.1.

Let us designate such motion of matter as a fountain effect in the forbidden zone. Thus, at least within the framework of quasi-hydrodynamic model, the convective motion in the Earth's magnetosphere is the consequence of gas-dynamic flow with free boundary and it is not linked with the presence of "viscous-like friction" mechanism, as was assumed in [2]; the tangential plasma velocities on both sides of the magnetosphere boundary are identical and the front points coincide.

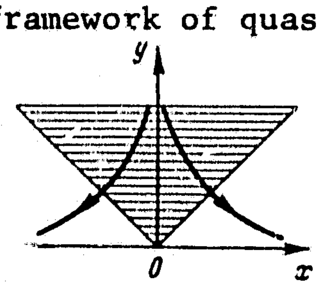

Fig.2.

The investigati n of vortex flow of matter inside the Earth's magnetosphere, even within the framework of two-dimensional quasi-hydrodynamics constitutes a complex problem of gas dynamics, whereupon it is unknown, whether there exists a single solution. The vortex flow in an incompressible fluid, excited by a uniform flow at infinity, within the forbidden zone having the shape of a cylinder or a sphere, is considered in the Appendix. As a consequence of simplicity of geometry and boundary conditions, obtained from the solution of the outer problem, it is possible to construct an analytical solution of the stated problem in both cases. In the case of two-dimension geometry (cylinder) there exists a countable set of solutions, while for the sphere only a single one is obtained. Both examined cases are precise solutions of nonlinear equ tions in partial derivatives of third order with assigned distribution of velocity and pressure at the boundary of the forbidden zone and may serve as illustrations for the recently worked out theory on the nonuniqueness of solution of problems of plane hydrodynamic vortex flows [3].

Thus, the equations of hydrodynamics for an ideal fluid admit solutions with a forbidden zone in the shape of a circle or a sphere, whereupon there exists inside the forbidden zone an eddy with closed current lines without singular points. However, as is shown below, such formations are unstable. In the vicinity of the frontal point, exponentially accruing acoustic waves are excited.

INSTABILITY OF THE FRONTAL POINT. Let us investigate the instability of the frontal point in standard hydrodynamics. In the assumption of process' adiabaticity the basic equations have the form

$$\rho\left[\frac{\partial \mathrm{w}}{\partial t}+(\mathrm{w}\nabla)\mathrm{w}\right]=-\nabla p, \quad \frac{\partial \rho}{\partial t}+\operatorname{div}\rho\mathrm{w}=0, \quad p\rho^{-\gamma}=\mathrm{const.} \tag{5}$$

Here ρ, p, $\vec{w}$, are the density, pressure and velocity of matter; γ is the adiabatic exponent. For simplicity, let us examine the two-dimensional perturbations in the vicinity of the frontal point of a two-dimensional flow. In the state of equilibrium in the neighborhood of the frontal point $u_0 = bx$, $v_0 = -by$, $p_0 = P_0 - \frac{\rho_0}{2} b^2(x^2+y^2)$, where the unperturbed quantities are marked by zero index;

$\underline{b}$ is a constant, ρ_0 and P_0 are the density and pressure at the frontal point; $\underline{u}$ and $\underline{v}$ are the projections of the velocity $\vec{w}$ on the axes $\underline{x}$ and $\underline{y}$ ($\partial/\partial z = 0$). Denoting the corresponding small perturbations without subscripts and linearizing Eqs.(5) we obtain

$$\rho_0\left[\frac{\partial \mathrm{w}}{\partial t}+(\mathrm{w}\nabla)\mathrm{w}_0+(\mathrm{w}_0\nabla)\mathrm{w}\right]=-\gamma\frac{p_0}{\rho_0}\nabla\rho-(\gamma-1)\frac{\rho}{\rho_0}\nabla p_0,$$
$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho_0\mathrm{w}+\rho\mathrm{w}_0)=0. \qquad (6)$$

If we are to assume the time dependence for the perturbations in the form $\exp(i\omega t)$, where ω is a complex quantity and substitute the unperturbed values of velocity and pressure for a plane flow in the neighborhood of the frontal point, Eqs.(6) will take form

$$i\omega\rho_0\mathrm{w}+b\rho_0(\mathrm{u}-\mathrm{v})+b\rho_0\left(x\frac{\partial \mathrm{w}}{\partial x}-y\frac{\partial \mathrm{w}}{\partial y}\right)=$$
$$=-\gamma\left[\frac{P_0}{\rho_0}-\frac{1}{2}b^2(x^2+y^2)\right]\nabla\rho+(\gamma-1)b^2\mathrm{r}\rho, \qquad (7)$$
$$i\omega\rho+\rho_0\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+b\left(x\frac{\partial\rho}{\partial x}-y\frac{\partial\rho}{\partial y}\right)=0,$$

where $\vec{r}$ is the radius-vector in the plane z = const

Let us examine the perturbations of small wavelength. Using the method of geometrical optics, let us take the dependence of perturbations on coordinates in the form $\exp(-\int \mathrm{k}d\mathrm{r})$, where $\vec{k}$ is the wave vector, and the integral is taken over the optical path. Then, taking this dependence into account and equating to zero the determinant of the homogenous system of Eqs.(7) at the frontal point $x = y = 0$, we obtain the dispersion equation

$$(\omega^2+b^2)\omega-k^2c_0^2\omega-ikc_0^2(k_x^2-k_y^2)=0, \qquad (8)$$

where $c_0^2 = \gamma P_0/\rho_0$ is the speed of sound at the frontal point whence, in the assumption of smallness of increment accretion $\delta = -\mathrm{Im}\,\omega$ by comparison with $\mathrm{Re}\,\omega$, we obtain

$$\omega\approx c_0k, \quad \delta\approx\frac{b}{2}\frac{k_{\ }^{2}-k_{\ }^{2}}{k^2}, \qquad (9)$$

whereupon, according to assumption $b \ll \omega$. Thus, for $k_y > k_x$ there is instability at frontal point, connected with the generation of acoustic waves.

Actually the frontal point may be considered as the source of sound. The acoustic wave propagating from the frontal point in the direction of positive and negative x, can only accelerate the fluid, while the acoustic wave propagating from the frontal point in the direction of positive y, can only decelarate it (Fig.2; the region of exponentially accruing waves is crosshatched, while arrows denote the direction of fluid motion). As the energy is preserved, in the first case the wave must damp, in the second case it must accrue, which is found to be in accordance with (9).

It must be noted, that in the examined problem on the excitation of vortex flow in the cylinder, each consecutive root of Bessel's function adds more frontal points in the forbidden zone (Appendix). On the other hand, as is shown above, there is always instability at the frontal point. Therefore with the appearance of such ambiguous solutions instability always sets in, which agrees well with the results obtained by the method of linearization of hydrodynamic equations in the work [3].

Let us examine the instability at the frontal point of a two-dimensional quasi-hydrodynamic flow (the magnetic lines of force appear as straight lines and are directed along z, $\partial/\partial z = 0$).

$$\rho\left[\frac{\partial \mathbf{w}}{\partial t}+(\mathbf{w}\nabla)\mathbf{w}\right] = -\nabla\left(p+\frac{B^2}{8\pi}\right),$$

$$\frac{p}{\rho B} = \mathrm{const}, \quad \frac{\partial \rho}{\partial t}+\operatorname{div}\rho\mathbf{w} = 0, \quad \frac{\partial}{\partial t}\frac{B}{\rho}+(\mathbf{w}\nabla)\frac{B}{\rho} = 0. \tag{10}$$

If we assume the time dependence in the form $\exp(i\omega t)$, and denote, as previously, the perturbed and unperturbed quantities respectively with letters without subscripts and with a zero index and if we linearize Eqs.(10), we shall obtain the following

$$p = p_0\left(\frac{\rho}{\rho_0}+\frac{B}{B_0}\right), \qquad i\omega\rho+\operatorname{div}(\rho\mathbf{w}_0+\rho_0\mathbf{w}) = 0,$$

$$\frac{i\omega}{\rho_0}\left(B-\frac{B_0}{\rho_0}\rho\right)+\frac{1}{\rho_0}(\mathbf{w}_0\nabla)\left(B-\frac{B_0}{\rho_0}\rho\right)+\left(B-\frac{B_0}{\rho_0}\rho\right)(\mathbf{w}_0\nabla)\frac{1}{\rho} = 0, \tag{11}$$

$$\rho_0[i\omega\mathbf{w}+(\mathbf{w}_0\nabla)\mathbf{w}+(\mathbf{w}\nabla)\mathbf{w}_0]+\rho(\mathbf{w}_0\nabla)\mathbf{w}_0 = -\nabla\left(p+\frac{B_0 B}{4\pi}\right).$$

Assuming for perturbations the abovementioned dependence on coordinates, the system of Eqs.(11) may be rewritten for the frontal point in the form

$$\rho_0[i\omega w + b(u - v)] = ikc_0^2\rho, \qquad \omega\rho = \rho_0(kw),$$

where $\vec{u}$ and $\vec{v}$ are the projections of vector $\vec{w}$ on the axes of coordinates x and y; $b = \partial u/\partial x = -\partial v/\partial y$ (the partial derivatives are taken at the frontal point),

$$c_0^2 = 2\frac{p_0}{\rho_0} + \frac{B_0^2}{4\pi\rho_0}, \tag{12}$$

ρ_0, p_0 and B_0 are respectively the unperturbed density, the pressure and the magnetic field at the frontal point. In this case the dispersion equation has the same form (8), with the only difference that velocity c_0 is given by formula (12). Consequently, the frequency ω and the increment of instability accretion δ are given by the expressions (9). Here the instability at the frontal point is linked with the perturbation of magneto-acoustic wave, propagating at right angle in the direction of magnetic lines of force.

In this way, the supersonic flow of solar plasma near the geomagnetic dipole has certain peculiarities. First of all, a shock-wave and a forbidden zone or the magnetosphere are formed, which could be simultaneously obtained within the framework of quasi-hydrodynamics [1]. Secondly, inside the forbidden zone a vortex flow is excited. The investigation of the structure of such a flow constitutes a complex problem of gas dynamics. In the close neighborhood of the frontal point the solution may be obtained with the aid of power series. In fact, in magnetosphere the fountain state differs substantially from the vortex flow of an incompressible fluid in the examined models of forbidden zone (circular cylinder or sphere). In the vicinity of the frontal point, instability always exists and the acoustic waves are perturbed; as is shown by the experiment on cosmic objects, the instability of the frontal point of the magnetopause leads to the formation of a transitional turbulent zone.

I express my gratitude to M.L. Levin for the discussion of the results of this work.

APPENDIX

VORTEX FLOW OF AN INCOMPRESSIBLE FLUID IN A CYLINDER AND A SPHERE

Assume that a forbidden zone has arisen in a uniform flow of an incompressible fluid, with a shape of a circular cylinder. Let us examine the flow outside the cylinder with a uniform stream, directed at infinity along the axis x, and the vortex flow inside the cylinder in the absence of singular points. At boundary circumference the tangential velocity and pressure are continuous. Let us introduce a cylindrical system of coordinates with origin at the center of the circle; from the solution of an extraneous problem, it is known that on the circumference $r = a$ (a is the cylinder radius) $v_\phi \sim \sin \phi$.

If we introduce the function of current Q with the aid of the relation $\vec{v} = \text{rot}\,(Q\vec{k}) = \nabla Q \times \vec{k}$, equation $\text{rot}\,(\vec{v} \times \text{rot}\,\vec{v}) = 0$ will take the form

$$\nabla Q \times \nabla \Delta Q = 0. \tag{I}$$

Let us seek the solution of the intrinsic problem in the form

$$Q = f(r) \sin \varphi. \tag{II}$$

Substituting relation (II) in (I) we obtain the equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right) - \frac{f}{r^2} = -Cf, \tag{III}$$

of which the solution satisfying the condition of absence of singular points in the investigated region is given by the Bessel function $f(r) = AJ_1(r\sqrt{C})$.

Inasmuch as at cylinder boundary the frontal point exists, function $f(a) = 0$, whence $a\sqrt{C} = \nu_{1m}$, where ν_{1m} is the m-th root of Bessel function $J_1(x)$. From the solution of the extraneous problem follows the boundary condition on the upper semicircumference $v_\phi = -2v_\infty \sin\phi$ (v_∞ is the flow velocity at infinity); this condition yields $A = 2av_\infty/\nu_{1m}J_1'(\nu_{1m})$.

Thus, the solution of the problem has the form

$$Q = \frac{2av_\infty}{\nu_{1m}J_0(\nu_{1m})} J_1\left(\frac{\nu_{1m}}{a}r\right)\sin\varphi.$$

Let us find the pressure distribution inside the cylinder. It is obrious that

$$\nabla\frac{p}{\rho} = \Delta Q \nabla Q - \frac{1}{2}\nabla(\nabla Q)^2.$$

Inasmuch as according to (III), $\nabla Q = -CQ$, this equation may be integrated, and we finally obtain

$$\frac{p}{\rho} + \frac{2v_\infty^2}{J_0^2(\nu_{1m})}\left\{\sin^2\varphi\left[J_0^2\left(\frac{\nu_{1m}}{a}r\right) + J_1^2\left(\frac{\nu_{1m}}{a}r\right)\right] + \right.$$
$$\left. + \cos^2\varphi \frac{a^2}{r^2\nu_{1m}^2} J_1^2\left(\frac{\nu_{1m}}{a}r\right)\right\} = \frac{p_\infty}{\rho} + \frac{v_\infty^2}{2},$$

where p_∞ is the pressure at infinity.

Suppose that the forbidden zone is a sphere and that the velocity of the incompressible fluid at infinity is directed along the axis z.

Let us introduce a spherical system of coordinates with origin at the center of the sphere and polar axis z, and a function Q determined by the relation $\vec{v} = \text{rot}\,(Q\vec{e}_\varphi)$

Instead of (I) we obtain the following equation:

$$\frac{\partial}{\partial r}\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta Q)\left\{\frac{1}{r}\frac{\partial^2}{\partial r^2}(rQ) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta Q)'\right]\right\}\right\} =$$
$$= \frac{\partial}{\partial\theta}\left\{\frac{1}{r}\frac{\partial}{\partial r}(rQ)\left\{\frac{1}{r}\frac{\partial^2}{\partial r^2}(rQ) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta Q)\right]\right\}\right\}. \quad \text{(IV)}$$

It is easily seen that the function Q is linked with the function of current ψ by the relation $r\sin\theta Q = \psi$. With irrotational streamline flow past the sphere, uniform at infinity, whose velocity v_∞ is directed along z, we have

$$\psi = \frac{1}{2} v_\infty r^2 \sin^2\theta\left[1 - \left(\frac{a}{r}\right)^3\right],$$

where a is the radius of the sphere. Therefore analogously with (II), we shall seek the solution of the intrinsic problem in the form $Q = R(r) \sin\theta$.

Then (IV) is reduced to the ordinary differential equation

$$\frac{1}{r}\frac{d^2}{dr^2}(rR) - \frac{2R}{r^2} = Cr,$$

whose solution satisfying the condition of absence of singular point at the origin of coordinates and the condition $R(a) = 0$, is given by the form

$$R(r) = Ar\left(1 - \frac{r^2}{a^2}\right).$$

The velocity distribution on the surface of the sphere follows from the solution of the extraneous problem and is characterized by the expression $v_\phi = -{}^3\!/_2 v_\infty \sin\theta$.

Equating the tangential velocities inside and outside, we ultimately obtain $A = -{}^3\!/_4 v_\infty$,

$$Q = -\frac{3}{4} v_\infty r\left(1 - \frac{r^2}{a^2}\right)\sin\theta.$$

The distribution of pressure inside the sphere has the form

$$p = p_\infty + \frac{\rho v_\infty^2}{2} - \frac{9}{8}\rho v_\infty^2\left[\left(1 + \frac{r^2}{a^2} - \frac{r^4}{a^4}\right)\sin^2\theta + \left(1 - \frac{r^2}{a^2}\right)^2\cos^2\theta\right],$$

where p_∞ is the pressure in the flow at infinity.

* * * THE END * * *

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