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Transient Electromagnetic Interaction of the Moon with the Solar Wind

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ABSTRACT

The response of a homogeneous conducting moon to variations and irregularities in the interplanetary magnetic field is determined. The significant lunar response is forced by fluctuations in the motional electric field. In the limit of a highly conducting plasma surrounding the moon and for length scales of the interplanetary magnetic field irregularities much larger than the moon's radius, the magnetic field fluctuations in the lunar interior are the sum of variations in the driving interplanetary magnetic field plus field fluctuations driven by the motional electric field. The magnitude of the motionally induced time-varying magnetic field inside the moon is proportional to the product of the magnitude of the interplanetary magnetic field fluctuations and the lunar magnetic Reynolds number. This induced field is toroidal about an axis in the direction of the forcing motional electric field. The motionally induced magnetic field in the plasma is also toroidal and is confined to a thin layer near the lunar surface. The electric field is everywhere equal to the forcing motional electric field.

INTRODUCTION

The steady state interaction of the moon and the solar wind plasma has been described by SONETT and COLBURN [1967, 1968] in terms of a unipolar generator. HOLLWEG [1968] has applied this induction generator model to the steady interaction of a two-layered moon with the solar wind. The motional electric field measured by an observer moving with the moon generates currents in the lunar interior which, in the steady state, close in the highly conducting interplanetary plasma. In this paper we present a theory for the time-dependent interaction of the moon and the solar wind. Time variations in the driving or incident interplanetary magnetic field appear as time-dependent magnetic field fluctuations in the lunar coordinate system. In addition, the relative motion of the moon with respect to spatial irregularities in the incident interplanetary magnetic field produces time-dependent magnetic field fluctuations in the reference frame moving with the moon. In fact, the power density spectrum of the interplanetary magnetic field [COLEMAN, 1968] shows that a significant fraction of the power is at the frequencies associated with field reversals caused by the corotation of the field's sector structure with the sun. Also, the time scale of the sector structure field reversals is in the range of possible Cowling diffusion times for the moon.

In addition to forcing a direct time-varying response in the lunar interior, variations and irregularities in the interplanetary magnetic field generate a time-dependent motional electric field in the reference frame moving with the moon which, as in the steady state unipolar generator model, forces an important time-dependent lunar response. BLANK and SILL [1969] in their ad hoc model of the transient interaction problem, have failed to consider this significant phenomenon. We solve for the fluctuating electromagnetic fields in the moon and in the plasma which are driven by the superposition of both a time-varying magnetic field and a time-varying motional electric field. The magnetic field fluctuations in the lunar interior are the sum of the fluctuations in the incident field and those induced by the incident motional electric field. The motional electric field also induces magnetic field fluctuations in the solar wind which are confined to a thin layer surrounding the moon. For frequencies of the order of 10^{-5} sec^{-1} and a plasma conductivity of 10^{4} mhos/m the length scale of this layer is approximately 4 km. The theory presented in this paper is necessary for the correct interpretation of data from lunar surface magnetometer experiments which can provide information on the electrical conductivity and consequently the temperature of the lunar interior.

THEORETICAL MODEL AND SOLUTION

Consider the case of a homogeneous moon with electrical conductivity σ_1 and magnetic permeability μ immersed in a plasma of conductivity σ_2 and permeability μ. In a coordinate system moving with the moon let the driving electromagnetic fields be of the form $\underline{a}_y H_0 \exp[i(2\pi z/\lambda - \omega t)]$ and $\underline{a}_x \mu v H_0 \exp[i(2\pi z/\lambda - \omega t)]$, where H_0 is the amplitude of the magnetic field oscillation, $\mu v H_0 = E_m$ is the amplitude of the motional electric field fluctuation associated with the relative velocity v of the moon and the spatially periodic magnetic field of length scale λ, the frequency $\frac{\omega}{2\pi} = \frac{v}{\lambda}$, and $\underline{a}_x$ and $\underline{a}_y$ are unit vectors in the x and y-directions, respectively. This form for the forcing electromagnetic field models the relative motion of the moon and the rotating sector structure of the interplanetary magnetic field.

In the lunar interior the forcing fields produce the response $\underline{E}_t \exp(-i\omega t)$, $\underline{H}_t \exp(-i\omega t)$ and in the plasma, the electromagnetic field is the sum of the forcing field and the reflected field $\underline{E}_r \exp(-i\omega t)$, $\underline{H}_r \exp(-i\omega t)$. Since the driving fields are in mutually perpendicular directions, the transmitted and reflected fields possess no simple symmetry properties.

The reflected magnetic field satisfies the vector Helmholtz equation

$$\nabla^2 \underline{H}_r + k_2^2 \underline{H}_r = 0 \quad , \qquad (1)$$

where ∇^2 is the Laplacian operator,

$$k_2^2 = \mu\epsilon\omega^2 + i\sigma_2\mu\omega \quad , \qquad (2)$$

and ϵ is the permittivity of the plasma. Similarly, the magnetic field inside the moon is a solution of the vector Helmholtz equation $\nabla^2 \underline{H}_t + k_1^2 \underline{H}_t = 0$, where $k_1^2 = \mu\epsilon\omega^2 + i\sigma_1\mu\omega$. The reflected electric field and the electric field in the lunar interior are determined from the appropriate forms of Ampere's law. The boundary conditions are the continuity of the tangential components of the electric and magnetic fields at the moon's surface r=a((r,θ,φ) are spherical polar coordinates).

The solution of this problem is readily obtained with the aid of STRATTON [1941]. The spherical harmonic expansions of the forcing fields are

$$\underline{H} = - H_0 e^{-i\omega t} \sum_{n=1}^{\infty} \beta_n (\underline{M}^-_{n,i} + i\, \underline{N}^+_{n,i}) \quad , \qquad (3)$$

$$\underline{E} = E_m e^{-i\omega t} \sum_{n=1}^{\infty} \beta_n (\underline{M}^+_{n,i} - i\, \underline{N}^-_{n,i}) \quad , \qquad (4)$$

where

$$\beta_n = \frac{i^n (2n+1)}{n(n+1)} \quad , \qquad (5)$$

$$\underline{M}^{\pm}_{n,i} = j_n\left(\frac{2\pi r}{\lambda}\right)\left\{\pm \frac{P_n^1(\cos\theta)}{\sin\theta} \begin{matrix}\cos\\ \sin\end{matrix}\,\varphi\,\hat{\underline{\theta}} - \frac{dP_n^1(\cos\theta)}{d\theta} \begin{matrix}\sin\\ \cos\end{matrix}\,\varphi\,\hat{\underline{\varphi}}\right\} \quad , \qquad (6)$$

$$\underline{N}^{+}_{n,i} = \left(\frac{\lambda}{2\pi r}\right) n(n+1) j_n\left(\frac{2\pi r}{\lambda}\right) P_n^1(\cos\theta) \begin{matrix}\sin\\ \cos\end{matrix}\,\varphi\,\hat{\underline{r}} +$$

$$\left(\frac{\lambda}{2\pi r}\right) \frac{d}{dr}\left\{r j_n\left(\frac{2\pi r}{\lambda}\right)\right\}\left\{\frac{d}{d\theta} P_n^1(\cos\theta) \begin{matrix}\sin\\ \cos\end{matrix}\,\varphi\,\hat{\underline{\theta}} + \frac{P_n^1(\cos\theta)}{\sin\theta} \begin{matrix}\cos\\ \sin\end{matrix}\,\varphi\,\hat{\underline{\varphi}}\right\} \quad . \qquad (7)$$

The spherical Bessel functions j_n (and later $h_n^{(1)}$) and the associated Legendre functions $P_n^1(\cos\theta)$ are defined as in STRATTON [1941]. The vectors $\hat{\underline{r}}$, $\hat{\underline{\theta}}$, $\hat{\underline{\varphi}}$ are the orthogonal unit vectors of the spherical coordinate system.

The electric and magnetic fields inside the moon are

$$\underline{H}_t = - vH_0 \frac{k_1}{\omega} \sum_{n=1}^{\infty} \beta_n \left(\delta_n^t \underline{M}^-_{n,t} + i\, \gamma_n^t \underline{N}^+_{n,t}\right) \quad , \qquad (8)$$

$$\underline{E}_t = E_m \sum_{n=1}^{\infty} \beta_n \left(\gamma_n^t \underline{M}^+_{n,t} - i\, \delta_n^t \underline{N}^-_{n,t}\right) \quad , \qquad (9)$$

where $\underline{M}^{\pm}_{n,t}$ and $\underline{N}^{\pm}_{n,t}$ are given by replacing $\frac{2\pi}{\lambda}$ with k_1 in equations (6) and (7), respectively. The reflected fields are

$$\underline{H}_r = - vH_0 \frac{k_2}{\omega} \sum_{n=1}^{\infty} \beta_n\left(\delta_n^r \underline{M}^-_{n,r} + i\, \gamma_n^r \underline{N}^+_{n,r}\right) \quad , \qquad (10)$$

$$\underline{E}_r = E_m \sum_{n=1}^{\infty} \beta_n \left(\gamma_n^r \, \underline{M}_{n,r}^{+} - i \, \delta_n^r \, \underline{N}_{n,r}^{-} \right) \quad , \qquad (11)$$

where $\underline{M}_{n,r}^{\pm}$ are given by replacing $\frac{2\pi}{\lambda}$ with k_2 and j_n with $h_n^{(1)}$ in equations (6) and (7), respectively. Continuity of the tangential components of the $\underline{E}$ and $\underline{H}$ fields at r = a leads to

$$\gamma_n^t = \left[-j_n\left(\frac{2\pi a}{\lambda}\right) \frac{d}{da} \left\{ a \, h_n^{(1)}(k_2 a) \right\} + h_n^{(1)}(k_2 a) \frac{d}{da} \left\{ a \, j_n\left(\frac{2\pi a}{\lambda}\right) \right\} \right] d_\gamma^{-1} , \qquad (12)$$

$$\gamma_n^r = \left[j_n(k_1 a) \frac{d}{da} \left\{ a \, j_n\left(\frac{2\pi a}{\lambda}\right) \right\} - j_n\left(\frac{2\pi a}{\lambda}\right) \frac{d}{da} \left\{ a \, j_n(k_1 a) \right\} \right] d_\gamma^{-1} \quad , \qquad (13)$$

$$\delta_n^t = \left[\frac{k_2 \lambda}{2\pi} h_n^{(1)}(k_2 a) \frac{d}{da} \left\{ a \, j_n\left(\frac{2\pi a}{\lambda}\right) \right\} - \frac{2\pi}{k_2 \lambda} j_n\left(\frac{2\pi a}{\lambda}\right) \frac{d}{da} \left\{ a h_n^{(1)}(k_2 a) \right\} \right] d_\delta^{-1} \quad , \qquad (14)$$

$$\delta_n^r = \left[\frac{k_1 \lambda}{2\pi} j_n(k_1 a) \frac{d}{da} \left\{ a \, j_n\left(\frac{2\pi a}{\lambda}\right) \right\} - \frac{2\pi}{k_1 \lambda} j_n\left(\frac{2\pi a}{\lambda}\right) \frac{d}{da} \left\{ a j_n(k_1 a) \right\} \right] d_\delta^{-1} \quad , \qquad (15)$$

where

$$d_\gamma = h_n^{(1)}(k_2 a) \frac{d}{da} \left\{ a \, j_n(k_1 a) \right\} - j_n(k_1 a) \frac{d}{da} \left\{ a \, h_n^{(1)}(k_2 a) \right\} \quad , \qquad (16)$$

$$d_\delta = \frac{k_2}{k_1} h_n^{(1)}(k_2 a) \frac{d}{da} \left\{ a \, j_n(k_1 a) \right\} - \frac{k_1}{k_2} j_n(k_1 a) \frac{d}{da} \left\{ a h_n^{(1)}(k_2 a) \right\} . \qquad (17)$$

DISCUSSION OF THE SOLUTION

The formulas of the preceding section can be simplified considerably for the values of the parameters of interest here. Since we are mainly concerned with the transient response of the moon to the rotating sector structure of the interplanetary magnetic field we will take $\omega = 0(10^{-5}\ \sec^{-1})$. The electrical conductivity of the solar wind plasma lies in the range 10^2 to 10^4 mhos/m (Sonett and Colburn 1968) and the absence of a lunar bow shock indicates that a representative lunar conductivity would be smaller than 10^{-5} mhos/m. The actual value of σ_1 depends on the thermal state of the lunar interior. Thus $k_{1,2}^2 \approx i\mu\omega\sigma_{1,2}$ and $|k_2 a|$ is ≈ 60 for $\sigma_2 \approx 10^2$ mhos/m. The parameter $2\pi a/\lambda = \omega a/v \approx 10^{-4}$ for v of $0(10^2$ km/sec). Thus we will be interested in the limits $2\pi a/\lambda \to 0$ and $k_2 a \to \infty$.

The parameter $k_1^2 a^2$ is equal to the product of $i\ 2\pi a/\lambda$ and the magnetic Reynolds number $\sigma_1 \mu v a$. The magnetic Reynolds number for the moon is $\lesssim 1$ so that in the limit $2\pi a/\lambda \to 0$ we will assume $\sigma_1 \mu v a$ is $0(1)$ and $k_1 a$ is $0(\ (2\pi a/\lambda)^{\frac{1}{2}})$. This insures that the lunar transient response to the forcing motional field fluctuations is consistent with the absence of a lunar bow shock. From equations (12) - (17) we find that in the limit $\left(\frac{2\pi a}{\lambda}\right) \to 0$,

$$\gamma_n^t,\ \gamma_n^r,\ \delta_n^t,\ \delta_n^r = 0\left(\frac{2\pi a}{\lambda}\right)^{\frac{n}{2},n,\frac{n}{2}-\frac{1}{2},n} \qquad (18)$$

We also note that as $\left(\frac{2\pi a}{\lambda}\right) \to 0$

$$\underline{M}^{\pm}_{n,i} \ , \ \underline{M}^{\pm}_{n,t} \ , \ \underline{M}^{\pm}_{n,r} = 0\left(\frac{2\pi a}{\lambda}\right)^{n,\frac{n}{2},0} \quad , \qquad (19)$$

$$\underline{N}^{\pm}_{n,i} \ , \ \underline{N}^{\pm}_{n,t} \ , \ \underline{N}^{\pm}_{n,r} = 0\left(\frac{2\pi a}{\lambda}\right)^{n-1,\frac{n}{2}-\frac{1}{2},\ 0} \quad . \qquad (20)$$

The dimensionless transmitted magnetic field $\underline{H}_t/H_0$ is thus the sum of terms of order $(2\pi a/\lambda)^{n-1}$. The terms involving δ_n^t and γ_n^t are of the same order. The dimensionless lunar electric field $\underline{E}_t/E_m$ is the sum of terms of order $(2\pi a/\lambda)^n$ (associated with γ_n^t) and terms of order $(2\pi a/\lambda)^{n-1}$ (associated with δ_n^t). Similarly, the nth term in the summation for $\underline{H}_r/H_0$ is of order $(2\pi a/\lambda)^{n-1}$ and the nth term in the expression for $\underline{E}_r/E_m$ is of order $(2\pi a/\lambda)^n$ (δ_n^r and γ_n^r terms are of the same order). The following are formulas for $\underline{H}_t/H_0$, $\underline{H}_r/H_0$, $\underline{E}_t/E_m$ and $\underline{E}_r/E_m$ correct to zeroth order in the small parameter $2\pi a/\lambda$ (the simplification $k_2 a \to \infty$ has also been employed):

$$\underline{H}_t/H_0 = \underline{a}_y - \frac{1}{2}(\sigma_1 \mu v a)\left(\frac{r}{a}\right)(\sin\varphi\underline{\hat{\theta}} + \cos\varphi\cos\theta\ \underline{\hat{\varphi}}) \quad , \qquad (21)$$

$$\underline{E}_t/E_m = \underline{a}_x \quad , \qquad (22)$$

$$\underline{E}_r/E_m = \underline{0} \quad , \qquad (23)$$

$$\underline{H}_r/H_0 = -\frac{1}{2}(\sigma_1 \mu v a)\ \frac{a}{r}\ e^{ik_2(r-a)}(\sin\varphi\underline{\hat{\theta}} + \cos\varphi\cos\theta\ \underline{\hat{\varphi}}) \ . \qquad (24)$$

To zeroth order, the electric field both in the solar wind and in the lunar interior is the incident motional electric field. To this same order, the magnetic field in the lunar interior is the sum of the incident interplanetary magnetic field and a field induced by the motional electric field fluctuation. The magnitude of this motionally induced magnetic field fluctuation is proportional to H_0 times the lunar magnetic Reynolds number and is toroidal with respect to the x-axis, the direction of the incident motional electric field. It is the time-dependent analog of the steady state unipolar generator solution for the induced magnetic field inside the moon [SONETT and COLBURN, 1968]. The disturbance magnetic field in the solar wind is also toroidal about the x-axis and is confined to a thin layer with length scale $(\sigma_2 \mu \omega)^{-\frac{1}{2}}$ adjacent to the lunar surface.

From the theory presented here [see equation (21)], data from lunar surface magnetometer experiments can be used to evaluate σ_1 and consequently obtain knowledge of the thermal state of the lunar interior. The normal component of the magnetic field fluctuation at the lunar surface will equal the normal component of the incident interplanetary magnetic field. After subtracting the tangential components of the forcing interplanetary field from the lunar surface field, the net fluctuations will give a direct measure of the lunar magnetic Reynolds number. In a future publication we will consider the application of this theory to a two-layered lunar conductivity model.

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