NASA CR-54634
HTL TR NO. 88

# MEASUREMENT OF PRANDTL NUMBER AND THERMAL CONDUCTIVITY

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prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

NASA CONTRACT NO. NAS-3-7910

HEAT TRANSFER LABORATORY

MECHANICAL ENGINEERING DEPARTMENT

UNIVERSITY OF MINNESOTA

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# SUMMARY REPORT

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December 15, 1968

CONTRACT NAS 3-7910
TECHNICAL MANAGEMENT
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# MEASUREMENT OF PRANDTL NUMBER AND THERMAL CONDUCTIVITY

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## SUMMARY

Prandtl number measurements are performed using air, argon, and the combustion products of hydrocarbon fuel $(CH_2)_n$ burned in air at various fuel-air ratios. These ratios include mass of fuel to mass of air equal to 0.0684 (stoichiometric), 0.0300, and 0.0150. The combustion products mixtures are made by combining the proper proportions of air, nitrogen, carbon dioxide, and water vapour. The pressure for these measurements is one atmosphere while the temperature is over a range of 811 to $1367^{o}$K (1000 to $2000^{o}$F). The Prandtl numbers are obtained by measuring the adiabatic recovery factor of the gas flowing over a simulated flat plate, the square of the recovery factor giving the Prandtl number.

Thermal conductivities are calculated for the same gases at the same temperatures and pressure using these measured Prandtl numbers and predicted values of specific heats and viscosities. The viscosities were calculated using kinetic theory with the assumption that the intermolecular potential is of the form of the Lennard - Jones 6-12. The force constants needed were determined from experimental viscosity data as reported in the literature. The specific heats were calculated using data taken from the literature along with the simple combination formula used for mixtures. Results are given on graphs and in tables using both the c.g.s. units and engineering units.

# MEASUREMENT OF PRANDTL NUMBER AND THERMAL CONDUCTIVITY

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## ABSTRACT

Measurements of Prandtl numbers of air, argon, and combustion products of hydrocarbon fuel burned in air have been conducted over a temperature range of 800-1350$^{\circ}$K. Using these measured Prandtl numbers, along with predicted values of specific heats and viscosities, the thermal conductivities are calculated for the same gases and over the same temperature range.

It is found that both the Prandtl number and the thermal conductivity of the combustion products mixtures are only very slightly dependent on the amount of air used in the combustion process in the range of this experiment. A fairly complete error analysis shows that the results found should be accurate to within 2.5%.

# INTRODUCTION

Accurate determinations of the Prandtl number of combustion products are of interest in engineering and science. This is largely due to the demand for accurate knowledge of transport properties of these gases by engineers interested in jet propulsion, for the design of such devices characteristically makes heavy claims for accurate data over an extended temperature range. For example, transport property information is desired for hydrocarbon combustion products at temperatures of $1200^{o}K$ or higher, since such temperatures are commonly encountered in modern supersonic jet engines.

The need for such information has prompted recent work in this area; unfortunately, the work is either for temperatures below $700^{o}K$ (1)* or based entirely on theoretical calculations (2). The properties of primary interest are the density, specific heat, viscosity, and thermal conductivity for gas mixtures. There is little problem with the density or specific heat of a mixture, since these values for nonreactive mixtures can be determined easily from the component properties. The thermal conductivity and viscosity, however, are more difficult. Computation of either of these properties by simple molar averages of the components will lead to values that are unsatisfactory when compared with experimental measurements. Fortunately, the viscosity of a mixture can be calculated by more sophisticated techniques (3). One such technique will be elaborated in the section dealing with viscosity predictions. However, when such techniques are applied to compute the thermal conductivity,

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*Numbers in parentheses designate references at the end of paper.

one finds discrepancies of as much as 15%, depending on which molecular model is used. Thus, not only is experimental determination of thermal conductivity valuable per se, but such information is helpful to the scientist in studying molecular models. This report deals with the experimental determination of a ratio of transport properties, the Prandtl number, and is limited to hydrocarbon combustion products, air, and argon.

Not only the Prandtl number of the gas mixture, but the thermal conductivity is also needed in the design of jet engines, especially in considering various schemes for cooling the turbine blades and other parts where high temperature effects limit the operating capabilities. The thermal conductivity of various mixtures can be determined from the Prandtl number and from accurate data for the viscosity and specific heat.

The advantage of finding the thermal conductivity by this method is apparent when the accuracy with which this property is known is examined. Data available on thermal conductivity are generally accurate to something less than 10 percent, however, specific heats are known within one tenth of one percent and viscosity within one percent. Therefore, when the Prandtl number is found to within two and one half per cent, the thermal conductivity is known to approximately the same accuracy. Finding the thermal conductivity by a direct method is usually less accurate due to the particular errors associated with such experiments. The usual method is to measure the heat flow through a small sample of the gas and calculate the thermal conductivity according to Fourier's equation. End losses are a serious problem because of the low thermal conductivity of a gas, such losses comprising a significant part of the total heat transfer. Furthermore, heat transfer by convection and radiation must either be eliminated or accurately accounted for, since these can cause a large error at high temperatures.

The method that is used in this report for finding the Prandtl number, and indirectly the thermal conductivity, by measuring the recovery factor of the gas was initially developed by Eckert and Irvine and reported in references (4) and (5).

## SYMBOLS

| | |
|---|---|
| a | radius of cylinder, (cm), (ft) |
| $c_p$ | specific heat at constant pressure, (cal/gm-$^{o}$K), (Btu/$lb_m$ - $^{o}$R) |
| $c_v$ | specific heat at constant volume, (cal/gm - $^{o}$K), (Btu/$lb_m$ - $^{o}$R) |
| $\bar{c}_p$ | molar specific heat at constant pressure, (cal/gm - mole $^{o}$K), (Btu/$lb_m$ - mole $^{o}$R) |
| $\bar{c}_v$ | molar specific heat at constant volume, (cal/gm - mole $^{o}$K), (Btu/$lb_m$ mole $^{o}$R) |
| E | thermocouple output, (emf) |
| g | function of temperature in error equation (15) |
| h | heat transfer coefficient, (cal/sec-cm$^{2}$-$^{o}$K), (Btu/hr-ft$^{2}$-$^{o}$R) |
| k | thermal conductivity, (cal/cm-sec-$^{o}$K), (Btu/hr-ft-$^{o}$R) |

| | |
|---|---|
| $M_f$ | mass of fuel (gm), (lbm) |
| $M_g$ | mass of gas (gm), ($lb_m$) |
| n | number of moles of gas |
| P | pressure, (cm of Hg), (dynes/$cm^2$), ($lb_f/in^2$) |
| $P_d$ | dimensionless pressure used in equation (17) |
| $P_s$ | static pressure of gas, (dynes/$cm^2$), ($lb_f/in^2$) |
| $P_t$ | total pressure of gas, (dynes/$cm^2$), ($lb_f/in^2$) |
| Pr | Prandtl number |
| q | heat flow per unit area, (cal/sec-$cm^2$), (Btu/sec-$ft^2$) |
| r | recovery factor |
| $r_{cy}$ | recovery factor of cylinder |
| $r_{fp}$ | recovery factor of flat plate |
| R | gas constant, (ergs/mole-$^oK$), (Btu/mole-$^oR$) |
| s | length measurement, (cm), (ft) |
| T | temperature, ($^oK$), ($^oR$) |
| $T_r$ | recovery temperature, ($^oK$), ($^oR$) |
| $T_s$ | static temperature, ($^oK$), ($^oR$) |
| $T_t$ | total temperature of gas flow, ($^oK$), ($^oR$) |
| $T_{tc}$ | thermocouple temperature, ($^oK$), ($^oR$) |
| $T_w$ | wall temperature, ($^oK$), ($^oR$) |
| T* | reference temperature associated with measured Prandtl number, ($^oK$), ($^oR$) |
| U | stream velocity, (cm/sec), (ft/sec) |
| V | volume, ($cm^3$), ($ft^3$) |
| W | molecular weight, (gm/mole), ($lb_m$/mole) |
| x | mole fraction |
| Z | compressibility factor |

| | |
|---|---|
| $\gamma$ | ratio of specific heats |
| $\varepsilon$ | force constant, depth of potential well, (ergs) |
| $\mu$ | dynamic viscosity, (gm/cm-sec), ($lb_m$/ft-sec) |
| $\nu$ | kinematic viscosity, ($cm^2$/sec), ($ft^2$/sec) |
| $\xi$ | functions in error equation (15) |
| $\sigma$ | force constant, collision diameter, (Angstroms) |

Subscripts

| | |
|---|---|
| $CO_2$ | carbon dioxide |
| $H_2O$ | water |
| $N_2$ | nitrogen |
| m | mixture |

## GENERAL THEORY

The property which relates the Prandtl number to quantities which can be measured directly is the recovery factor. The recovery factor is defined by

$$r = \frac{T_r - T_s}{T_t - T_s} \tag{1}$$

where $T_t$ is the total temperature of the gas, $T_s$ is the static temperature of the gas, and $T_r$ is the recovery temperature which an adiabatic surface would assume if placed in the gas stream. $T_r$ is greater than $T_s$ due to aerodynamic heating. E. Pohlhausen (6) solved the heat transfer equation for a laminar, steady boundary layer on a flat plate and found that for Prandtl numbers between 0.6 and 1.0 the recovery factor can be calculated within 0.3% by

$$r = (Pr)^{1/2} \tag{2}$$

Eckert ( 7 ) showed that a reference temperature, T*, given by

$$T^* = T_s + 0.72(T_r - T_s) \tag{3}$$

should be used in applying equation (2) to account for the variation of properties in the boundary layer. This is the temperature at which the Prandtl number is measured and reported.

Combining equations (1) and (2) gives

$$(Pr)^{1/2} = \frac{T_r - T_s}{T_t - T_s} \tag{4}$$

Thus three temperature measurements, $T_r$, $T_s$, and $T_t$ are needed to calculate the Prandtl number. $T_s$ is found indirectly. In the experiment, the gas is expanded through a nozzle with an upstream pressure $P_t$ and an upstream static temperature which is taken equal to $T_t$ because the velocity is very low (5-7 m/sec). The flow through the nozzle is essentially isentropic. The downstream static temperature, $T_s$, is therefore related to the downstream static pressure, $P_s$, by

$$\frac{T_s}{T_t} = \left(\frac{P_s}{P_t}\right)^{\frac{\gamma-1}{\gamma}} \tag{5}$$

where $\gamma$ is the heat capacity ratio of the gas. This assumes the gas is behaving as a perfect gas. Such an assumption is valid for low pressure and high temperature, and since, in this work $P_s$ is one atmosphere, $P_t$ is less than two atmospheres, and the temperatures are greater than 800°K, equation (5) is readily used. This equation also assumes that $\gamma$ is constant for the gas. Although the gas experiences a temperature change of up to about 20°K during an expansion through the nozzle, the change in the specific heats is slight and the ratio of the specific heats $\gamma$ is considered a constant. Equation (4) can be rearranged to give:

$$(Pr)^{1/2} = 1 - \frac{T_t - T_r}{T_t(1 - \frac{T_s}{T_t})} \qquad (6)$$

Eliminating $T_s/T_t$ by equation (5) gives the working equation for the experiment

$$(Pr)^{1/2} = 1 - \frac{T_t - T_r}{T_t\,(1 - (\frac{P_s}{P_t})^{\frac{\gamma-1}{\gamma}})} \qquad (7)$$

The basic measurements thus consist of a temperature difference $(T_t - T_r)$, a temperature $T_t$, and a pressure ratio $P_s/P_t$ for a gas (pure or a mixture) with known $\gamma$. The specific heat ratio, $\gamma$, is found according to

$$\gamma = \frac{\Sigma\, x_i\, \bar{c}_{pi}}{\Sigma\, x_i\, \bar{c}_{vi}} \qquad (8)$$

This equation assumes the use of a perfect gas, an assumption valid in the present work.

# EXPERIMENTAL PROCEDURE

An experimental program measured the Prandtl number of air, argon, and the combustion products of hydrocarbon fuel burned in air. The experimental procedure was based on the fact that a unique relationship exists between the Prandtl number and the flat plate recovery factor for laminar, high-velocity, boundary-layer flow as described in reference (4).

To simulate the conditions of a flat plate as required in equation (7), a cylinder is used whose axis is parallel to the direction of flow. This avoids the edge effects that are inherent in the use of a true flat plate, and simulates an infinitely wide flat plate. This cylinder, which is actually a wire of radius 0.0635 cm., is positioned so that it lies axially along the center line of a small 1.016 cm diameter converging nozzle. The nozzle serves to accelerate the fluid to high speed, providing the stable laminar boundary layer required for equation (7) to be valid. The recovery temperature of the cylinder needed in equation (7) is obtained by making the cylinder a temperature sensing device. This occurs by making the cylinder to form a butt-welded thermocouple. One junction is placed at the nozzle throat to read $T_r$ and the other junction is located upstream of the nozzle throat in the low velocity region (about 7 m/sec) where the wire junction assumes the total temperature of the gas. Thus the temperature difference, $T_t - T_r$, is obtainable directly from the thermocouple.

Because this thermocouple is exposed to high temperatures for extended periods of time and required to measure a small temperature difference, (less than $30^{\circ}K$), it is made from platinum/platinum - 10% rhodium. These materials provide a very stable thermocouple pair as reported in reference (8). The thermocouple required to measure the absolute total gas temperature, $T_r$ does not need to be exotic since the possibility of a calibration change (so often found in differential thermocouples) is much less in an absolute temperature thermocouple. It was made of chromel/alumel, a material pair that according to reference (8) is considered suitable.

Figure (1) is a schematic diagram of the apparatus. The gas enters a 1.3 meter long heat exchanger filled with 3 mm diameter aluminum oxide spheres. The spheres are tightly packed in a type 310 stainless steel pipe, 10.16 cm I.D., 11.43 cm O.D.

After the gas flows through the heat exchanger, where it assumes the temperature of the aluminum oxide spheres, it passes through a nozzle and accelerates to approximately Mach 0.9. Before passing through the nozzle, however, the total temperature and total pressure are recorded. A double radiation shield protects the total temperature thermocouple and the upstream junction of the differential thermocouple from radiative heat exchange with the surrounding walls. The nozzle, a standard German measuring nozzle reported in (9), has an exit diameter of 1.016 cm.

To permit adjustment and easy removal, the differential thermocouple is strung the entire length of the apparatus. Twelve centimeters downstream of the nozzle it passes through an electrically insulated support which prevents vibration in the wire. A braided cord connected to a 250 gm weight passes over a pulley and attaches to the thermocouple wire. This weight keeps the wire taut and, at the same time, allows for thermal expansion. The downstream shield is moved against the nozzle face during a run in order to minimize radiation loss from the recovery junction.

The pressure ratio in Equation (7) is found in the following manner: The static pressure of the high velocity jet assumes ambient atmospheric pressure and is measured by a precision Fortrin barometer near the apparatus. A 0.3 cm hole is drilled through the wall of the central pipe upstream of the nozzle and a stainless steel tube is inserted (press fit) into the hole to serve as an upstream static pressure tap. This upstream static pressure is exactly the downstream total pressure since the process of airflow between these two regions is isentropic and the upstream velocity is negligibly small. The upstream pressure is measured by a Wallace and Tiernan manometer, accurate to 0.025 cm of mercury.

The thermocouple emf of the temperature measurements needed in Equation (7) are recorded by a Hewlett-Packard Dymec integrating voltmeter. This voltmeter is calibrated with the Wenner Bridge operated by the Heat Transfer Calibration Laboratory, University of Minnesota. This bridge, which has a certified accuracy of .001 mv by the National Bureau of Standards, has never indicated more than a .05% error in the voltmeter. The voltmeter is fitted with a multi-channel scanner which permits reading and recording of six temperatures, $T_t$, $(T_t - T_r)$, and four on the radiation shield downstream of the nozzle, every 1.5 seconds.

The operation of the apparatus proceeds as follows: Heaters first bring the heat exchanger to the temperature level at which the test is to be made. The current to the heaters is then turned off and the test gas is permitted to flow through the apparatus at a total pressure of up to 125 cm of mercury. The operator maintains this pressure for approximately 45 seconds by a needle valve located in the gas supply line. Meanwhile, the voltmeter continuously records the emf's of the two thermocouples listed above as well as the emf's of the four reference thermocouples in the radiation shield downstream of the nozzle. During this time interval, the conditions demanded by equation (7) are satisfied; that is a subsonic, laminar, isentropic flow passes over a flat plate (in this case, a wire) which experiences a zero heat flux at its surface.

## MIXING APPARATUS AND MIXING PROCEDURE

The chemical formula which most closely represents the hydrocarbon used as a fuel is $C_nH_{2n}$. The combustion of this fuel with an excessive amount of air can be written

$$C_nH_{2n} + \text{air} \rightarrow H_2O + CO_2 + N_2 + \text{air} \qquad (9)$$

The amount of air on the right depends upon the molar ratio of $C_nH_{2n}$ to air on the left.

The purpose of this investigation is to find the Prandtl numbers of mixtures occuring from the combustion of $C_nH_{2n}$ and air in varying proportions, i.e., of the reactants on the left side of equation (9). The actual combustion of $C_nH_{2n}$ with air to provide the gas mixtures presents some difficulties for the type of measurements anticipated. Thus it was decided to synthesize the combustion products by mixing the pure components $CO_2$, $N_2$, $H_2O$, and air in the necessary proportions.

Air is assumed to be composed entirely of $O_2$ and $N_2$ with the molar ratio of $N_2$ to $O_2$ being 3.73. If the combustion of $C_nH_{2n}$ with air is complete, there is no air left and the balanced equation is

$$2C_nH_{2n} + 3nO_2 + 3.73(3n)N_2 = 2nCO_2 + 2nH_2O + 3.73(3n)N_2 \qquad (10)$$

The molar proportion of the constituents $CO_2$, $H_2O$, and $N_2$ respectively are 1 to 1 to 5.59.

The apparatus designed to synthesize the mixtures consists of a mixing tank (made of type 302 stainless steel, 150 cm long, 20.32 cm I.D., 21.84 cm O.D.) which is fitted with a chromel-alumel thermocouple to measure the gas temperature. The tank is wrapped with electric heating wire and insulation to provide a temperature capability which would ensure complete vaporization of the $H_2O$.

From the supply bottles, high pressure brass pipe permits addition of the nitrogen, carbon dioxide, and air first into the mixing tank and then into the heating tank. At any time during the mixing procedure, the pressure of the mixture can be read from a Hiese pressure gauge calibrated from 0 to 1200 $lb_f/in^2$ in steps of 1 $lb_f/in^2$. A stainless steel tube runs from a 250 ml burette directly into the heating tank to enable the addition of liquid $H_2O$.

The following is the procedure used to synthesize a mixture:

1) The mixing tank is evacuated to a pressure less than 50 microns.
2) $n_{H_2O}$ moles of liquid water are added at room temperature.

3) An equal number of moles of $CO_2$ is added by admitting the gas while the sum of the partial pressures of $CO_2$ and $H_2O$ is read on the pressure gauge. The partial pressure of carbon dioxide is obtained from

$$P_{CO_2} = \frac{n_{CO_2} \; R \; T_{CO_2} \; Z_{CO_2}}{V} \qquad (11)$$

4) After four or five hours have lapsed to give the mixture time to reach equilibrium, $n_{N_2} = 5.59 \; n_{H_2O}$ moles of nitrogen are added by admitting gas until a pressure $P_{CO_2} + P_{H_2O} + P_{N_2}$ is read on the gauge, where

$$P_{N_2} = \frac{n_{N_2} \; R \; T_{N_2} Z_{N_2}}{V} \qquad (12)$$

5) Before admitting the mixture into the heat exchanger pipe containing the $alO_3$ spheres the mixture is heated in the mixture tank to a temperature high enough to ensure that the water is completely evaporated. This temperature was chosen to be greater than the dew point temperature at the partial pressure of $H_2O$. The partial pressure can be inferred from the equation $P_{H_2O} = x_{H_2O}P_t$. $P_t$ the total pressure in the tank after heating, was measured and the mole fraction of $H_2O$, $x_{H_2O}$, was known.

## PREDICTION OF SPECIFIC HEAT AND VISCOSITY

To obtain the thermal conductivity of the combustion products mixture from the experimental Prandtl number using

$$Pr = \frac{\mu c_p}{k} \tag{13}$$

values for the viscosity and specific heat are needed. As data for these properties are not readily available, a method of predicting them by some analytical or empirical means is needed. This creates no problem for the specific heat as the simple combination formula

$$c_{p_m} = (1/W_m)\Sigma_i x_i \bar{c}_{p_i} \tag{14}$$

for specific heats of mixtures gives very good results. This combined with the fact that the specific heats of the individual gases are known to great accuracy allows us to predict the specific heat of any given mixture. The values of the specific heats for the individual gases used in this work were taken from references (10) and (11).

The prediction for the viscosity of a gas mixture is not nearly as simple or direct as that of the specific heat. Several methods of predicting mixture viscosities are available (12)-(19) and all are much more complicated than the specific heat equation. By comparing these different methods with the available experimental data of mixture viscosities Pulkrabek (3) found that the method developed by Chapman and Cowling (17) and put forth by Hirschfelder, Curtiss and Bird (18),using exclusively the methods of kinetic theory, was by far the best of all those available.

The advantage of using this method, and, indeed, the advantage of using kinetic theory in general, is that the only dependent quantities are the force constants, $\sigma$ and $\epsilon$, and the mole fractions. The mole fractions are listed in Table (1) for fuel to air ratios (mass fuel/mass air) of .0684, .0300, and .0150. These fuel-to-air ratios represent the three mixtures investigated in this report. The fuel-to-air ratio of .0684 represents a mixture that has undergone complete combustion of the hydrocarbons with no excess air present (i.e., stoichiometric combustion) while .0300 and .0150 are leaner mixtures that contain unburned air after combustion.

For this method the type of interaction between the various gas molecules must be approximated. The form selected for the interaction potential between the molecules is the Lennard-Jones 6-12 potential. Using this potential and the force constants for the various gases found in references (18), (19), and (20), the viscosities of the gas mixtures were calculated, Figure (7) and that of argon, Figure (8). Although this method is developed for non-polar gases it is used here to include water vapour which has a polar molecule. This is done using "effective" force constants found in references (19) and (20). This should create no problem as the mole fraction of water in any of the mixture ratios is never very high. The force constants used in this work are tabulated in Table (2).

## RESULTS

The experimental results of the Prandtl number measurements for the various gases and gas mixtures are shown in Figures (2) to (6), and Tables (3) to (7). Figure (2) shows that the Prandtl number of air decreases slowly as the temperature increases, reaches a minimum, and then rises again at the higher temperatures. The same occurance is true for the Prandtl number of the combustion product mixture as shown in Figures (3), (4) and (5). Again a decrease with temperature to a minimum and then an upward swing at the higher temperatures. In Figures (2) through (5) additional lines are added one and one-half percent above and below the line fitted to the data points. It can be seen that most of the experimental points fall within these additional lines. It can also be seen that the scatter in the data points becomes greater at the higher temperature levels. This can be explained by the great increase in radiation problems at the thermocouple junctions at these higher temperatures. It is interesting to note that the Prandtl number data for the combustion products mixtures for all fuel to air ratios are approximately the same at corresponding temperatures. It was found that if the data for all three mixture ratios were plotted on one graph, one line could be drawn to represent the Prandtl number of any of the fuel to air ratio mixtures. In other words, the Prandtl number of the combustion products has a very small dependency on the amount of air used to dilute the mixture.

Figure (6) shows the data points of the argon experiments. As can be seen, all points fall very close to the expected Prandtl number value of two-thirds, predicted by kinetic theory. It can also be seen that argon is a "well-behaved" gas with most of the data points falling within one-half percent of the expected value. This is very encouraging, since the primary reason argon was included in the experiment was to periodically check the equipment and the method of interpreting the results. The good results of the argon data gives added acceptability to the data of the other gases.

Tables (3) through (7) gives the results of the Prandtl number measurements as well as the predicted values of the specific heats and viscosities at the same temperatures. The method of these property predictions is described in an earlier section of the paper. Also included in these tables are the thermal conductivity values obtained by equation (13) using these Prandtl numbers, specific heats, and viscosities.

Figure (9) shows the thermal conductivity values obtained from the measured Prandtl numbers and the predicted specific heats and viscosities. It can be seen that the thermal conductivity of the combustion products, like the Prandtl number, is only slightly dependent on the amount of air in the mixture.

This experiment was conducted over an operating temperature range of 811-1367$^{\circ}$K (1000-2000$^{\circ}$F). This covered the complete range for which the equipment was designed and was capable of obtaining data. The lower end of the range was governed by the emf output of the differential thermocouple. At temperatures below 800$^{\circ}$K the emf output of the thermocouple was too small to give accurate results. The upper temperature level was governed by the melting point of the materials used in the construction of the system. To have an operating temperature of 1400$^{\circ}$K meant that there were local spots in the heat exchanger that had temperatures appreciably above this. It was not uncommon to have heaters, thermocouples, etc. burn out periodically.

Due to the method of finding the reference temperature associated with each Prandtl number datum point as given by equation (3), the range of reported data given here is 802-1336$^\circ$K (984-1945$^\circ$F). That is, the reference temperature associated with each datum point is slightly less than the operating temperature of the apparatus.

## ANALYSIS OF EXPERIMENTAL ERROR

### I. Flat Plate Assumption

In addition to the errors associated with the measurement of the quantities in equation (7), the validity of the equation for a cylindrical geometry must be examined. That is, equation (7) assumes the use of a flat plate to measure the Prandtl number; in the present work, however, a circular cylinder is substituted for a flat plate. It is expected, then, that some error would result since the recovery factor for a flat plate is not the same as for a cylinder.

In reference (21) Sparrow and Fleming make a study of the difference between the recovery factor for a flat plate and the recovery factor for a circular cylinder. They conclude that

$$\frac{r_{cy} - r_{fp}}{r_{fp}} = \xi \left(\frac{g_1(0)}{g_o(0)}\right) + \xi^2 \left(\frac{g_2(0)}{g_o(0)}\right) + \ldots \qquad (15)$$

where $r_{cy}$ = recovery factor for cylinder

$f_{fp}$ = recovery factor for flat plate

$g_o(0)$ = temperature distribution of the flat plate boundary layer evaluated at the surface

$g_1(0), g_2(0)$ = perturbations on the flat plate distributions for the circular cylinder solution

$$\xi = \frac{4}{a} \left(\frac{\nu s}{U_\infty}\right)^{1/2}$$

$a$ = radius of cylinder, 0.0635 cm in this case

$U_\infty$ = stream velocity

$\nu$ = viscosity

$s$ = distance from start of boundary layer

The greatest error occurs for the greatest $\xi$. Since the ratio $\nu/U_\infty$ appears in $\xi$ and since $\nu$ and $U_\infty$ vary with temperature and gas mixture, $\xi$ is computed for the run with the greatest $\nu/U_\infty$. This would be at the highest temperature using argon. Not only is the viscosity of argon higher than that of air but the free stream velocity is lower for the same pressure drop across the nozzle. For the argon run with the highest temperature we have: $\nu = 1.90$ cm$^2$/sec, $U_\infty = 433$ m/sec.

The values of $g_0(0)$, $g_1(0)$, and $g_2(0)$ are tabulated in reference (21). The calculation of distance s presents a minor problem since the boundary layer forms at a point inside the nozzle where a pressure gradient exists. Irvine (4) using a nozzle with exactly the same profile, finds that the boundary layer produced by the accelerating flow begins at a point .36L from the nozzle exit, where L is the length of the nozzle. This point is a distance 1.37 cm from the thermocouple junction which measures the recovery temperature.

Using the above data, equation (15) becomes

$$\frac{r_{cy} - r_{fp}}{r_{fp}} = -.0128$$

that is, a 1.28% maximum error is incurred by using a cylinder to measure the recovery factor instead of a flat plate.

## II. Thermocouple Calibration

As would be expected, the pressure measurements are not as susceptable to errors as are the temperature measurements. For this reason, a careful description of the calibration and errors involved with measuring $T_t$ and $T_t - T_r$ is given.

The calibration of the thermocouples was done at the University of Minnesota Heat Transfer Calibration Laboratory using the following procedure: One thermocouple junction is placed in a vacuum container filled with distilled water and crushed ice; the other is placed in an oil bath which is modeled after a similar device in use at the National Bureau of Standards. This oil bath produces an isothermal zone which encompasses the thermocouple and a platinum resistance thermometer. The resistance thermometer, together with the Mueller bridge which measures the resistance, have an accuracy of $.001^{\circ}F$ as certified by the National Bureau of Standards.

The above calibration procedure was carried out from $273^{\circ}K$ to $1250^{\circ}K$ for both the chromel/alumel pair and the platinum/90% platinum-10% rhodium pair. For the latter, we are especially interested in the temperature versus the rate of change of voltage with temperature, rather than in the temperature vs emf relation.

A least square procedure is used to fit an equation of the form

$$E = aT + bT^2 + cT^3 + dT^4$$

to the calibration data by the use of a computerized program. For the differential temperature measurement $T_t - T_r$ this is differentiated to give

$$\frac{dE}{dT} = a + 2bT + 3cT^2 + 4dT^3 \tag{16}$$

The equipment of the calibration laboratory was not capable of extending above 1250$^{\circ}$K, so standard tables obtained from Leeds and Northrup were used above this point (22).

The important question is the accuracy of the calibration. Irvine (4), using the same calibration equipment and the same method of curve fitting, concludes that a reasonable uncertainty to be associated with the calibration $\frac{dL}{dT}$ is .25% and .1% for that of $T_t$. These errors will be used in the present work.

## III. Conduction and Radiation ~~Erros~~ Errors

As with any thermocouple measurement, there is always radiative and conductive heat exchange which produce errors in the temperature reading. For the Prandtl number apparatus, these errors are summarized below:

1) An error in the total temperature sensed by the upstream differential thermocouple due to a) conduction to its upstream support, and b) radiation to its surroundings.

2) An error in the recovery temperature sensed by the downstream differential thermocouple due to a) conduction to the downstream support, and b) radiation to its surroundings.

3) An error in $T_t$ - $T_r$ caused by conduction between the junctions.

4) An error in the total temperature thermocouple caused by a) conduction along the thermocouple, and b) radiation to its surroundings.

The following discussion is an analysis of the above errors, i.e., which are important and how they effect the accuracy of the Prandtl number.

## IV. Radiation Error Estimates

The apparatus is equipped with guard heaters surrounding the nozzle and central tube in order to produce a uniform temperature in the upstream region. These heaters are carefully adjusted to minimize temperature gradients that cause radiative heat exchange. Even with these precautions, however, a temperature difference is usually found between $T_t$ of the gas and the surrounding walls.

Before each Prandtl number measurement, thermocouples in the inside wall are read to see how the temperatures differ from $T_t$. In making this check, the wall temperature never differs from $T_t$ by more than $6^oK$. With $T_w - T_t = 6^oK$, a heat balance on the upstream thermocouple junction gives a radiation error $T_{tc} - T_t$ less than $0.057^oK$. Using this error for a run at the upper end of the temperature range, this would represent a maximum error of 0.27 percent.

The calculation of a radiation error on the total temperature thermocouple is the same as that given above for the differential thermocouple. The total temperature thermocouple is also situated within the two shields. However, due to the difference in shape of the total temperature thermocouple, the convective heat transfer coefficient is different and the resulting error is much less than the total temperature junction on the differential thermocouple. Again with $T_w - T_t = 6^oK$, the error in the total temperature, $T_{tc} - T_t$ is $0.01^oK$. This is a negligible error since $T_t$ is of the order $1100^oK$.

The third and last radiation error is that associated with the recovery temperature in the quantity $T_t$ - $T_r$ sensed by the differential thermocouple. The recovery junction is also surrounded by a radiation shield, as seen in Figure (1). The temperature of the shield varies along its length. Furthermore, its temperature decreases with time due to cooler air entering from the outside environment. To measure this change in temperature, four thermocouples are located in the shield region.

The aim, of course, is to achieve a net radiation heat transfer to the recovery junction equal to zero. It is found that at any instant during the run, some regions have a temperature greater than $T_r$ while others have a temperature less than $T_r$. This means that the recovery junction is gaining energy from some regions and losing energy to others. It is possible, then, to find the time when the net energy transfer is zero.

The shield area, representing the solid angle seen by the downstream or recovery thermocouple junction, is divided into several sections. By using the appropriate emissivities and angle-factors for the various sections, along with the temperatures as read by the four shield thermocouples, an energy balance is made on the recovery thermocouple junction. At the time when the incoming radiation is balanced by the outgoing radiation the net energy exchange is zero. For this condition, the quantity $T_t$ - $T_r$ is used to compute the Prandt number.

By using the above procedure, the radiation heat exchange to the recovery junction is zero. The radiation associated with $T_r$ is therefore negligible.

## V. Conduction Error Estimates

Conduction also affords a possibility of temperature measurement error. If thermocouples are used to measure the temperature of a gas, and if a temperature difference exists between the thermocouple junction and, say, a wall where the lead wires are attached, then heat is conducted from (or to) the junction. This heat transfer, then, decreases (or increases) the temperature sensed by the thermocouple junction, causing a temperature measurement error.

Whenever possible, however, this problem can be eliminated by the following procedure. From the junction, the lead wires are run in a direction parallel to the gas flow. If it is necessary for the wires to be strung across the flow direction, the thermocouple wires should be bent such that a length immediately adjacent to the junction runs parallel to the flow. In this way, heat is conducted primarily from the bend location rather than the thermocouple junction.

The above procedure is used to eliminate a conduction error in the total temperature thermocouple, and since the differential thermocouple is strung parallel to the gas flow, conduction between the upstream junction is also negligible. Moreover, the same claim is made for the downstream junction since the high velocity jet, which bathes the recovery portion of the thermocouple, extends downstream more than three inches from the junction. Thus there is a relatively large region, at constant temperature equal to the recovery temperature, in which no conduction occurs.

Conduction can occur, however, between the upstream junction and the downstream junction since the junctions differ in temperature by approximately 25°K. This conduction would decrease the temperature difference $T_t - T_r$ used in calculating the Prandtl number. The following procedure is used in estimating this temperature decrease.

The situation is taken as a fin problem where the change in temperature of the fin base is calculated, given a fin length subjected to a specified convective heat transfer. In the present problem, the fin is the differential thermocouple. The length, however, is not the full distance between the $T_t$ junction and the $T_r$ junction since upstream of the nozzle the velocity is low and, hence, the convective heat transfer is negligible. The gas has an appreciable velocity only when it enters the nozzle. Therefore, it is assumed that the hypothetical fin extends from the nozzle entrance to the recovery junction. This distance, L, is 3.015 cm.

Irvine (4), with the same situation, solved this fin problem and found that the percent error in measuring $T_t - T_r$ is

$$\%\ \text{error} = \frac{1}{\cosh((hC/kA)^{1/2}L)} \times 100$$

where

h = heat transfer coefficient

C = circumference of fin

k = thermal conductivity of wire

A = area of fin

L = length

With this equation, using the experimental run that gives maximum error, we get % error = 0.31.

## VI. Pressure Errors

In general, pressure measurements are not as vulnerable to errors as temperature measurements. For instance, since $P_s$ is atmospheric pressure, the error associated with $P_s$ is the reading error of the barometer. This is a negligible amount, .02%.

There is, however, a possible error in $P_t$. As mentioned previously, the total pressure is maintained at approximately 125 cm of mercury during the run. In practice, however, it is not possible to maintain the pressure at exactly this amount. The pressure tends to fluctuate about a mean value despite the efforts of the operator. This fluctuation, however, never exceeds 0.25 cm of mercury. This represents a .20% error in $P_t$.

The quantity $1 - (P_s/P_t)^{\frac{\gamma-1}{\gamma}}$ that appears in equation (7) to compute the Prandtl number can be used to define $P_d$ as

$$P_d = 1 - (P_s/P_t)^{\frac{\gamma-1}{\gamma}} \qquad (17)$$

As mentioned above, the error in $P_s$ is negligible; thus, the error in $P_s/P_t$ is the error in $P_t$ i.e., .20%. Since $P_s/P_t$ is typically .585, this corresponds to an error of .0012 in the ratio $P_s/P_t$. To see how this effects $P_d$, take the differential of equation (17) and divide by $P_d$

$$\frac{\Delta P_d}{P_d} = \frac{1-\gamma}{\gamma}\left(\frac{P_s}{P_t}\right)^{-\frac{1}{\gamma}} \frac{\Delta\left(\frac{P_s}{P_t}\right)}{P_d} \qquad (18)$$

$\frac{\Delta P_d}{P_d}$ is the greatest when $\gamma$ is least and, since $\gamma$ decreases as temperature increases, equation (18) is evaluated at the highest temperature level. When this is done, an error is found amounting to

$$\frac{\Delta P_d}{P_d} = .0035 \text{ or } .35\%$$

## VII. Summary of Errors

The errors discussed in the preceding pages are summarized at the end of this section. To see how these errors affect the accuracy of the Prandtl number measurement, the following analysis is made.

First, by taking the differential of equation (2) and dividing by equation (2) one gets $\frac{\Delta Pr}{Pr} = 2\frac{\Delta r}{r}$. That is, the percentage error in the Prandtl number is twice the percentage error in the recovery factor.

Second, rewrite Equation (7) in the form

$$r = 1 - \frac{\delta T}{T_t P_d} \tag{19}$$

where

$$\delta T = T_t - T_r$$

$$P_d = 1 - (P_s/P_t)^{\frac{\gamma-1}{\gamma}}$$

$$T_t = T_t$$

The total differentiation of equation (19) is

$$\Delta r = \frac{\partial r}{\partial(\delta T)}\Delta(\delta T) + \frac{\partial r}{\partial T_t}\Delta T_t + \frac{\partial r}{\partial P_d}\Delta P_d \tag{20}$$

and by performing the differentiation and dividing by equation (19)

$$\frac{\Delta r}{r} = \frac{\Delta(\delta T)}{T_t P_d - \delta T} + \frac{\delta T}{T_t P_d - \delta T}\,\frac{\Delta T_t}{T_t} + \frac{\delta T}{T_t P_d - \delta T}\,\frac{\Delta P_d}{P_d} \qquad (21)$$

Using the experimental run that gives the maximum systematic error and using equation (21) gives,

$$\frac{\Delta r}{r} = \pm 0.0020 \text{ or } \pm 0.20\%$$

The random errors for $T_t$ and $\delta T$ are fixed quantities. Therefore, as $T_t$ and $\delta T$ increase, the effect that these errors produce would decrease. For example, an uncertainty of .01 mv would produce a relatively smaller error at $T_t$ that gives a 40 mv reading than at one that gives a 20 mv reading. The greatest random error occurs, then, for the smallest value of $T_t$ and $\delta T$. Thus, using data from the experimental runs at the lower temperatures and inserting this into equation (21) we get

$$\frac{\Delta r}{r} = 0.0017 \text{ or } 0.17\%$$

The random error in the Prandtl number is twice this or 0.34%.

Besides the systematic errors discussed above, there are two other errors in the recovery factor that were mentioned earlier. They are the Pohlausen approximation, equation (2), and the flat plate assumption. The use of a cylinder instead of a flat plate contributes a positive error while the Pohlausen approximation has an opposite effect. For the recovery factor, then, the errors are

| | | |
|---|---|---|
| 1. | Pohlausen approximation | -.30% |
| 2. | Flat plate assumption | +1.28% |
| 3. | Systematic errors in apparatus | ±0.20% |
| | Total | +1.18% |

The overall systematic error in the Prandtl number is twice this total, or 2.36%.

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## TABLE 1
## MOLE FRACTIONS

| $M_f/M_a$ | Gas | Mole Fraction |
|---|---|---|
| .0684 | $CO_2$ | .1317 |
| | $H_2O$ | .1317 |
| | $N_2$ | .7366 |
| .0300 | $CO_2$ | .1052 |
| | $H_2O$ | .1052 |
| | $N_2$ | .5879 |
| | Air | .2017 |
| .0150 | $CO_2$ | .0773 |
| | $H_2O$ | .0773 |
| | $N_2$ | .4324 |
| | Air | .4130 |

## TABLE 2 FORCE CONSTANTS USED IN THE DETERMINATION OF VISCOSITY VALUES

| Gas | $\sigma$ (Angstroms) | $\varepsilon/k$ (°K) |
|---|---|---|
| Air | 3.689 | 84.0 |
| Argon | 3.465 | 116.0 |
| Water Vapour | 2.685 | 357.0 |
| Nitrogen | 3.749 | 79.8 |
| Carbon Dioxide | 3.897 | 213.0 |

## TABLE 3 THERMAL PROPERTIES OF AIR

| Temperature | | Prandtl Number | Viscosity | | Specific Heat | Thermal Conductivity | |
|---|---|---|---|---|---|---|---|
| ($^{o}$K) | ($^{o}$R) | | $(\frac{gm}{cm\text{-}sec})x10^4$ | $(\frac{lbm}{ft\text{-}sec})x10^5$ | $(\frac{cal}{gm\ ^{o}K})$ | $(\frac{cal}{cm\text{-}sec\text{-}^{o}K})x10^4$ | $(\frac{Btu}{hr\text{-}ft\text{-}^{o}R})x10^2$ |
| 817 | 1470 | 0.6834 | 3.645 | 2.449 | 0.2635 | 1.405 | 3.398 |
| 823 | 1482 | 0.6862 | 3.664 | 2.462 | 0.2639 | 1.409 | 3.407 |
| 873 | 1571 | 0.6803 | 3.807 | 2.558 | 0.2666 | 1.492 | 3.607 |
| 891 | 1604 | 0.6813 | 3.859 | 2.593 | 0.2675 | 1.515 | 3.663 |
| 899 | 1618 | 0.6844 | 3.881 | 2.608 | 0.2679 | 1.519 | 3.673 |
| 911 | 1639 | 0.6856 | 3.914 | 2.630 | 0.2685 | 1.533 | 3.706 |
| 943 | 1698 | 0.6809 | 4.005 | 2.691 | 0.2702 | 1.589 | 3.842 |
| 964 | 1736 | 0.6717 | 4.063 | 2.730 | 0.2713 | 1.641 | 3.967 |
| 975 | 1755 | 0.6739 | 4.092 | 2.750 | 0.2717 | 1.650 | 3.988 |
| 979 | 1762 | 0.6778 | 4.103 | 2.757 | 0.2719 | 1.646 | 3.979 |
| 1006 | 1810 | 0.6698 | 4.175 | 2.805 | 0.2733 | 1.704 | 4.118 |
| 1034 | 1862 | 0.6749 | 4.253 | 2.858 | 0.2746 | 1.730 | 4.183 |
| 1078 | 1941 | 0.6961 | 4.369 | 2.936 | 0.2765 | 1.735 | 4 195 |
| 1082 | 1947 | 0.7008 | 4.378 | 2.942 | 0.2767 | 1.729 | .179 |
| 1098 | 1977 | 0.6847 | 4.422 | 2.971 | 0.2775 | 1.792 | 4.333 |
| 1118 | 2012 | 0.6925 | 4.472 | 3.005 | 0.2784 | 1.798 | 4.346 |
| 1132 | 2037 | 0.6826 | 4.508 | 3.029 | 0.2789 | 1.842 | 4.453 |
| 1132 | 2037 | 0.6953 | 4.508 | 3.029 | 0.2789 | 1.808 | 4.371 |
| 1171 | 2108 | 0.6856 | 4.610 | 3.098 | 0.2807 | 1.887 | 4.563 |
| 1176 | 2117 | 0.6892 | 4.622 | 3.106 | 0.2809 | 1.884 | 4.554 |
| 1228 | 2211 | 0.6985 | 4.755 | 3.195 | 0.2831 | 1.927 | 4.659 |
| 1286 | 2314 | 0.7009 | 4.897 | 3.291 | 0.2855 | 1.995 | 4.822 |
| 1307 | 2352 | 0.6856 | 4.949 | 3.326 | 0.2864 | 2.067 | 4.998 |
| 1332 | 2398 | 0.6942 | 5.011 | 3.367 | 0.2875 | 2.075 | 5.017 |

TABLE 4 THERMAL PROPERTIES OF COMBUSTION PRODUCTS
WITH (MASS FUEL/MASS AIR) = 0.0684

| Temperature | | Prandtl Number | Viscosity | | Specific Heat | Thermal Conductivity | |
|---|---|---|---|---|---|---|---|
| ($^oK$) | ($^oR$) | | $(\frac{gm}{cm\text{-}sec})x10^4$ | $(\frac{lbm}{ft\text{-}sec})x10^5$ | $(\frac{cal}{gm\ ^oK})$ | $(\frac{cal}{cm\text{-}sec\text{-}^oK})x10^4$ | $(\frac{Btu}{hr\text{-}ft\text{-}^oR})x10^2$ |
| 810 | 1458 | 0.7256 | 3.532 | 2.373 | 0.2913 | 1.418 | 3.428 |
| 867 | 1561 | 0.7137 | 3.698 | 2.485 | 0.2954 | 1.531 | 3.701 |
| 898 | 1616 | 0.7120 | 3.784 | 2.543 | 0.2976 | 1.582 | 3.824 |
| 998 | 1797 | 0.7086 | 4.061 | 2.729 | 0.3041 | 1.743 | 4.214 |
| 1044 | 1879 | 0.7125 | 4.183 | 2.811 | 0.3069 | 1.802 | 4.356 |
| 1113 | 2004 | 0.7200 | 4.365 | 2.933 | 0.3108 | 1.884 | 4.555 |
| 1174 | 2113 | 0.7055 | 4.520 | 3.027 | 0.3141 | 2.027 | 4.900 |
| 1187 | 2136 | 0.6935 | 4.552 | 3.059 | 0.3148 | 2.052 | 4.961 |
| 1243 | 2237 | 0.7156 | 4.693 | 3.154 | 0.3175 | 2.082 | 5.033 |
| 1287 | 2317 | 0.7017 | 4.802 | 3.227 | 0.3196 | 2.187 | 5.287 |
| 1310 | 2358 | 0.7124 | 4.857 | 3.264 | 0.3206 | 2.186 | 5.285 |
| 1336 | 2404 | 0.7186 | 4.919 | 3.305 | 0.3217 | 2.202 | 5.323 |

## TABLE 5 THERMAL PROPERTIES OF COMBUSTION PRODUCTS WITH (MASS FUEL/MASS AIR) = 0.0300

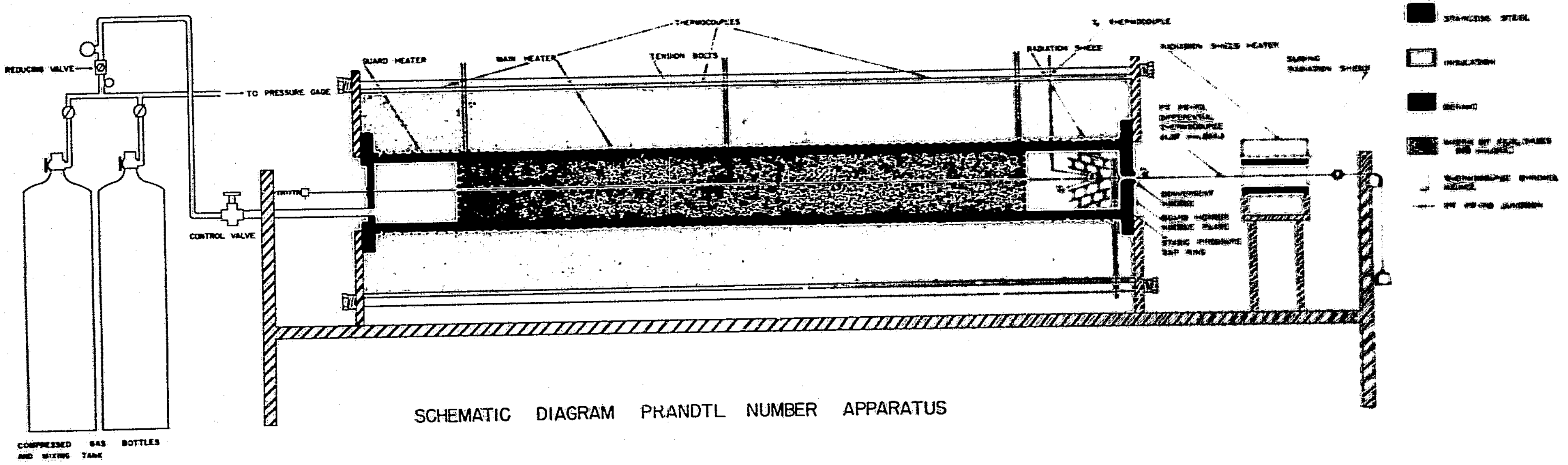
| Temperature | | Prandtl Number | Viscosity | | Specific Heat | Thermal Conductivity | |
|---|---|---|---|---|---|---|---|
| ($^o$K) | ($^o$R) | | $(\frac{gm}{cm\text{-}sec})\times10^4$ | $(\frac{lbm}{ft\text{-}sec})\times10^5$ | $(\frac{cal}{gm\ ^oK})$ | $(\frac{cal}{cm\text{-}sec\text{-}^oK})\times10^4$ | $(\frac{Btu}{hr\text{-}ft\text{-}^oR})\times10^2$ |
| 802 | 1443 | 0.7250 | 3.532 | 2.373 | 0.2840 | 1.384 | 3.346 |
| 858 | 1545 | 0.7177 | 3.697 | 2.484 | 0.2878 | 1.483 | 3.585 |
| 924 | 1664 | 0.7162 | 3.884 | 2.610 | 0.2932 | 1.590 | 3.844 |
| 987 | 1776 | 0.7040 | 4.056 | 2.726 | 0.2971 | 1.712 | 4.139 |
| 1052 | 1893 | 0.7164 | 4.230 | 2.842 | 0.3009 | 1.777 | 4.926 |
| 1116 | 2009 | 0.7138 | 4.399 | 2.956 | 0.3044 | 1.876 | 4.535 |
| 1168 | 2103 | 0.6950 | 4.533 | 3.046 | 0.3071 | 2.003 | 4.842 |
| 1173 | 2112 | 0.7204 | 4.545 | 3.054 | 0.3073 | 1.939 | 4.688 |
| 1243 | 2237 | 0.7071 | 4.720 | 3.172 | 0.3107 | 2.074 | 5.014 |
| 1287 | 2317 | 0.7163 | 4.830 | 3.246 | 0.3127 | 2.109 | 5.099 |
| 1307 | 2353 | 0.6970 | 4.879 | 3.279 | 0.3136 | 2.195 | 5.306 |
| 1334 | 2402 | 0.7083 | 4.945 | 3.323 | 0.3147 | 2.197 | 5.311 |

TABLE 6 THERMAL PROPERTIES OF COMBUSTION PRODUCTS WITH (MASS FUEL/MASS AIR) = 0.0150

| Temperature | | Prandtl Number | Viscosity | | Specific Heat | Thermal Conductivity | |
|---|---|---|---|---|---|---|---|
| ($^oK$) | ($^oR$) | | $(\frac{gm}{cm\text{-}sec})x10^4$ | $(\frac{lbm}{ft\text{-}sec})x10^5$ | $(\frac{cal}{gm\ ^oK})$ = | $(\frac{cal}{cm\text{-}sec\text{-}^oK})x10^4$ | $(\frac{Btu}{hr\text{-}ft\text{-}^oR})x10^2$ |
| 806 | 1451 | 0.7264 | 3.568 | 2.398 | 0.2794 | 1.372 | 3.317 |
| 848 | 1527 | 0.7203 | 3.691 | 2.480 | 0.2821 | 1.446 | 3.496 |
| 918 | 1652 | 0.7125 | 3.889 | 2.613 | 0.2865 | 1.564 | 3.781 |
| 1000 | 1800 | 0.7120 | 4.115 | 2.765 | 0.2913 | 1.684 | 4.071 |
| 1037 | 1867 | 0.7160 | 4.215 | 2.832 | 0.2933 | 1.727 | 4.175 |
| 1111 | 2000 | 0.7115 | 4.410 | 2.963 | 0.2972 | 1.842 | 4.453 |
| 1173 | 2111 | 0.7213 | 4.568 | 3.070 | 0.3002 | 1.901 | 4.596 |
| 1229 | 2213 | 0.7199 | 4.711 | 3.166 | 0.3029 | 1.982 | 4.791 |
| 1282 | 2307 | 0.6982 | 4.841 | 3.253 | 0.3053 | 2.117 | 5.118 |
| 1308 | 2354 | 0.6929 | 4.905 | 3.296 | 0.3064 | 2.169 | 5.244 |
| 1335 | 2403 | 0.7102 | 4.972 | 3.341 | 0.3076 | 2.153 | 5.205 |

## TABLE 7 THERMAL PROPERTIES OF ARGON

| Temperature | | Prandtl Number | Viscosity | | Specific Heat | Thermal Conductivity | |
|---|---|---|---|---|---|---|---|
| ($^oK$) | ($^oR$) | | $(\frac{gm}{cm\text{-}sec})x10^4$ | $(\frac{lbm}{ft\text{-}sec})x10^5$ | $(\frac{cal}{gm\ ^oK})$ | $(\frac{cal}{cm\text{-}sec\text{-}^oK})x10^5$ | $(\frac{Btu}{hr\text{-}ft\text{-}^oR})x10^2$ |
| 803 | 1446 | 0.6686 | 4.555 | 3.061 | 0.1244 | 8.475 | 2.049 |
| 831 | 1496 | 0.6698 | 4.660 | 3.131 | 0.1244 | 8.655 | 2.092 |
| 887 | 1596 | 0.6671 | 4.865 | 3.269 | 0.1244 | 9.072 | 2.193 |
| 928 | 1670 | 0.6688 | 5.013 | 3.369 | 0.1244 | 9.324 | 2.254 |
| 932 | 1678 | 0.6657 | 5.029 | 3.379 | 0.1244 | 9.398 | 2.272 |
| 942 | 1695 | 0.6629 | 5.062 | 3.401 | 0.1244 | 9.499 | 2.296 |
| 982 | 1767 | 0.6732 | 5.203 | 3.496 | 0.1244 | 9.615 | 2.324 |
| 1008 | 1815 | 0.6693 | 5.296 | 3.559 | 0.1244 | 9.843 | 2.380 |
| 1016 | 1829 | 0.6619 | 5.323 | 3.577 | 0.1244 | 10.004 | 2.419 |
| 1019 | 1834 | 0.6639 | 5.333 | 3.584 | 0.1244 | 9.993 | 2.416 |
| 1085 | 1953 | 0.6657 | 5.558 | 3.735 | 0.1244 | 10.386 | 2.511 |
| 1114 | 2006 | 0.6650 | 5.656 | 3.801 | 0.1244 | 10.581 | 2.558 |
| 1116 | 2008 | 0.6680 | 5.660 | 3.803 | 0.1244 | 10.540 | 2.548 |
| 1123 | 2022 | 0.6694 | 5.685 | 3.820 | 0.1244 | 10.565 | 2.554 |
| 1128 | 2031 | 0.6717 | 5.702 | 3.832 | 0.1244 | 10.560 | 2.553 |
| 1131 | 2035 | 0.6668 | 5.710 | 3.837 | 0.1244 | 10.653 | 2.575 |
| 1141 | 2054 | 0.6658 | 5.744 | 3.860 | 0.1244 | 10.732 | 2.595 |
| 1172 | 2109 | 0.6656 | 5.845 | 3.928 | 0.1244 | 10.924 | 2.641 |
| 1239 | 2231 | 0.6693 | 6.063 | 4.074 | 0.1244 | 11.269 | 2.724 |
| 1258 | 2265 | 0.6687 | 6.124 | 4.115 | 0.1244 | 11.393 | 2.754 |
| 1277 | 2299 | 0.6652 | 6.183 | 4.155 | 0.1244 | 11.563 | 2.795 |



SCHEMATIC DIAGRAM PRANDTL NUMBER APPARATUS

Figure 1

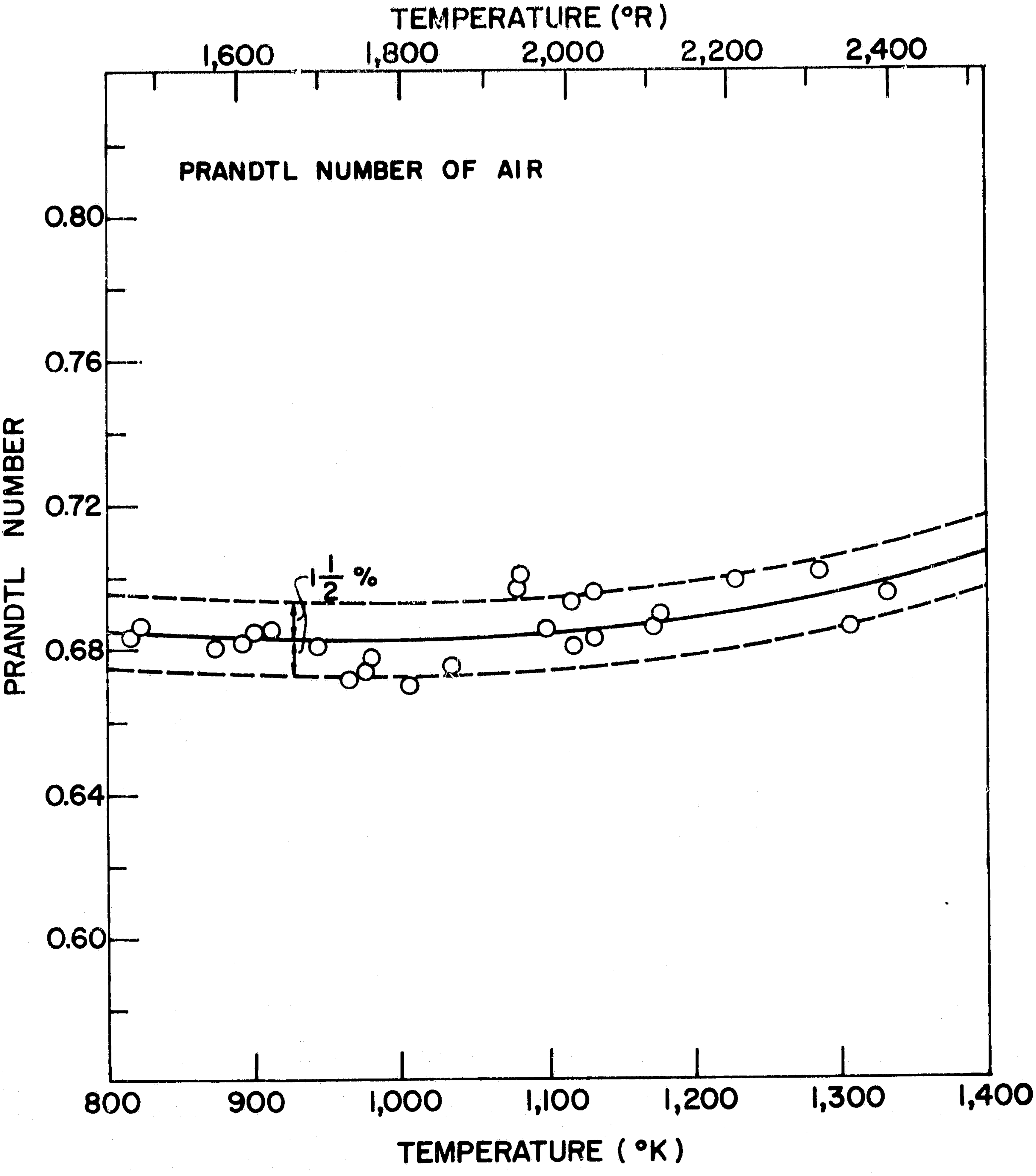

TEMPERATURE (°R)
1,600
1,800
2,000
2,200
2,400
PRANDTL NUMBER OF AIR
0.80
0.76
0.72
0.68
0.64
0.60
PRANDTL NUMBER
$1\frac{1}{2}$ %
800
900
1,000
1,100
1,200
1,300
1,400
TEMPERATURE (°K)

Figure 2

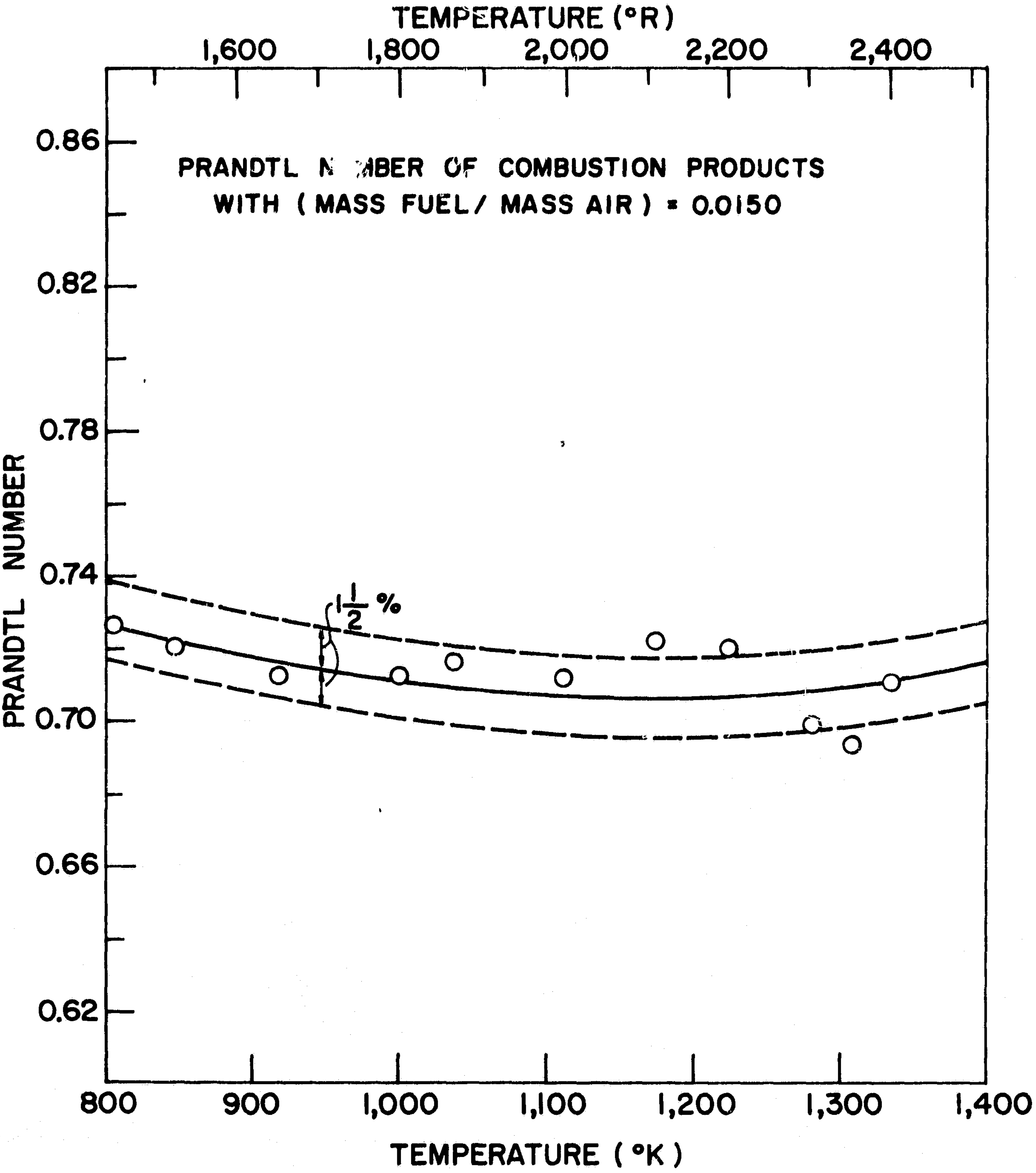

TEMPERATURE (°R)
1,600
1,800
2,000
2,200
2,400
PRANDTL NUMBER OF COMBUSTION PRODUCTS
WITH (MASS FUEL/ MASS AIR) = 0.0150
0.86
0.82
0.78
0.74
0.70
0.66
0.62
PRANDTL NUMBER
$1\frac{1}{2}\%$
800
900
1,000
1,100
1,200
1,300
1,400
TEMPERATURE (°K)

Figure 3

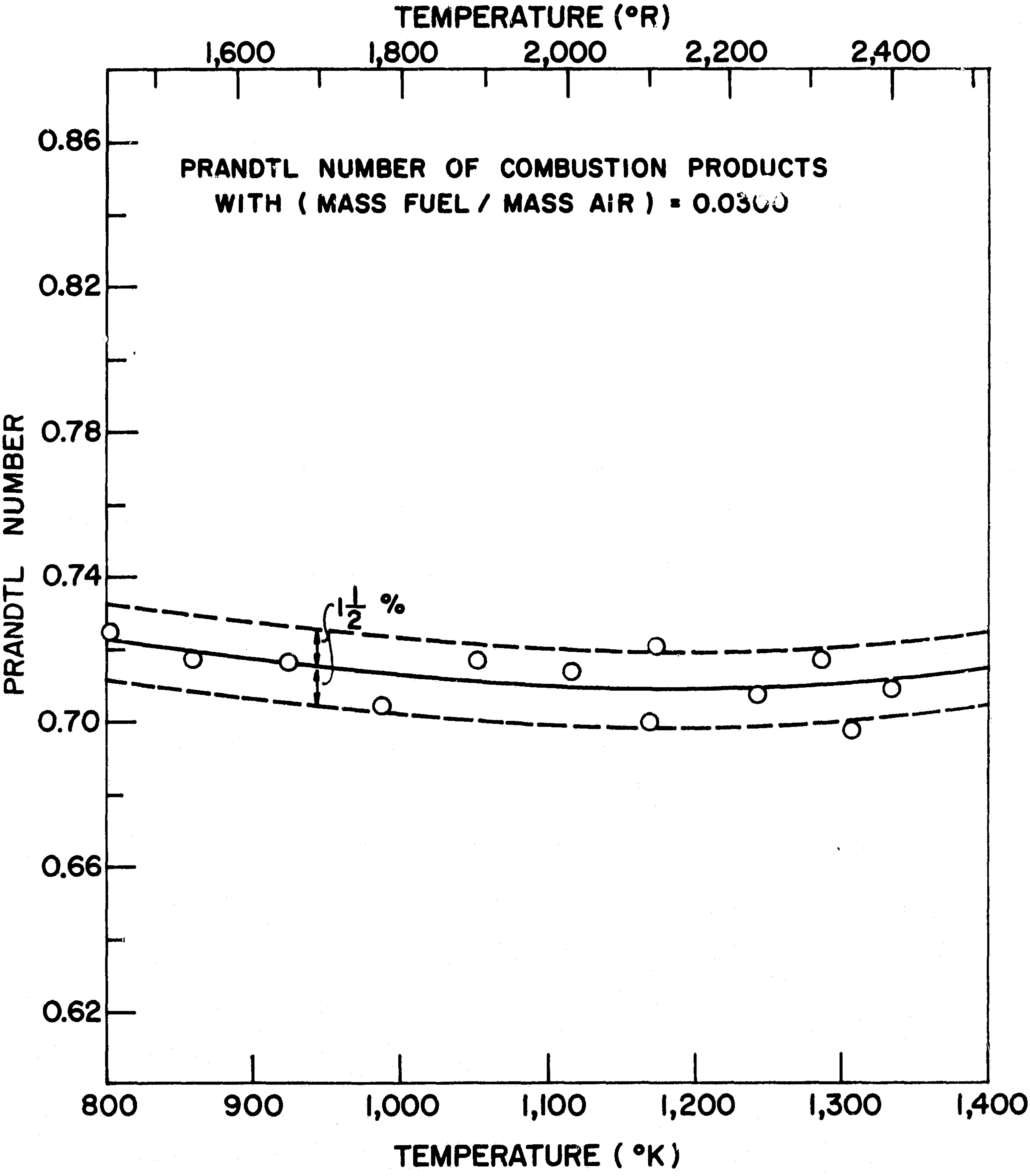

TEMPERATURE (°R)
1,600
1,800
2,000
2,200
2,400
PRANDTL NUMBER OF COMBUSTION PRODUCTS
WITH ( MASS FUEL / MASS AIR ) = 0.0300
$1\frac{1}{2}$ %
PRANDTL NUMBER
0.86
0.82
0.78
0.74
0.70
0.66
0.62
800
900
1,000
1,100
1,200
1,300
1,400
TEMPERATURE ( °K )

Figure 4

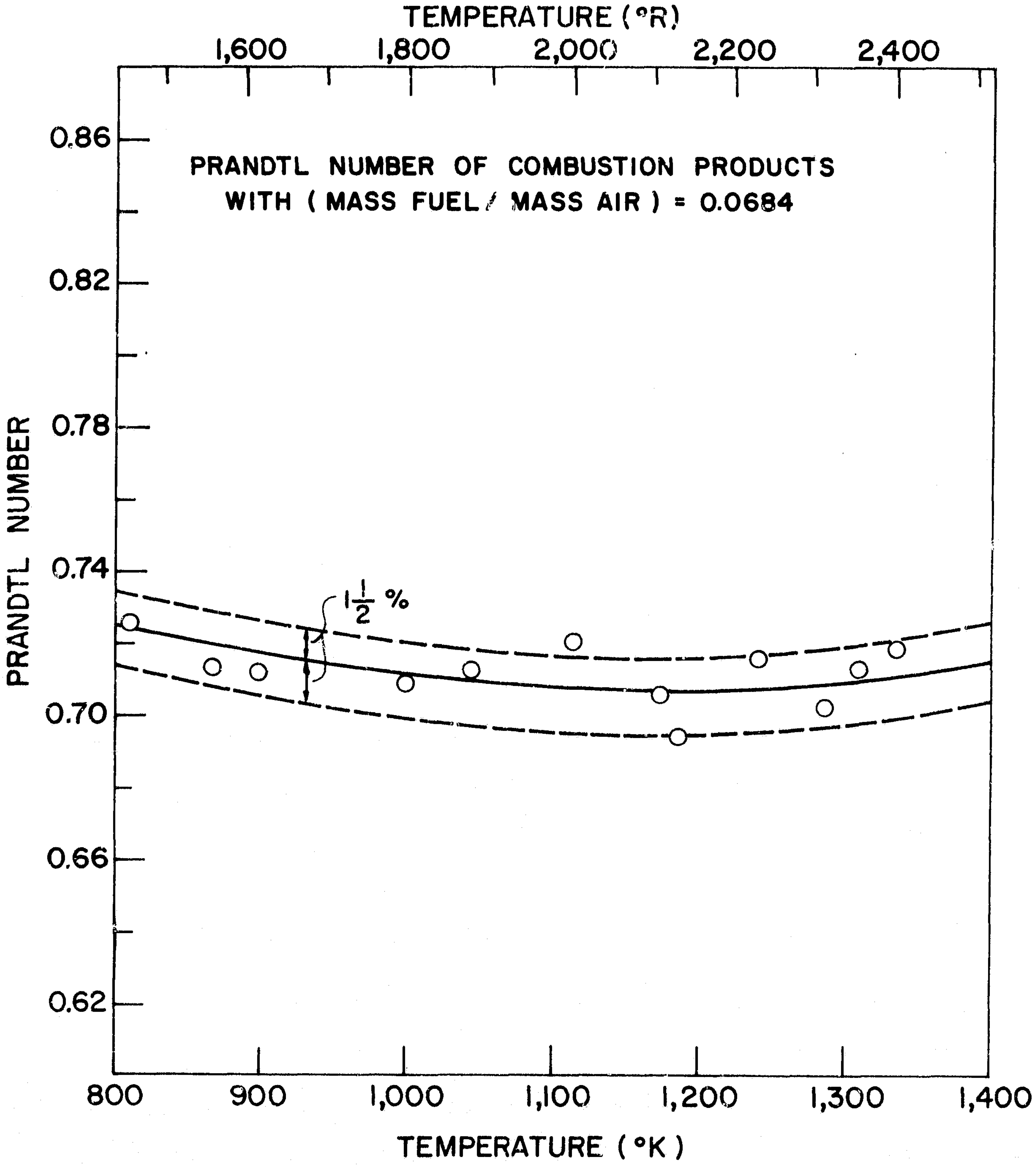

TEMPERATURE (°R)
1,600
1,800
2,000
2,200
2,400
PRANDTL NUMBER OF COMBUSTION PRODUCTS
WITH ( MASS FUEL / MASS AIR ) = 0.0684
0.86
0.82
0.78
0.74
0.70
0.66
0.62
PRANDTL NUMBER
$1\frac{1}{2}$ %
800
900
1,000
1,100
1,200
1,300
1,400
TEMPERATURE ( °K )

Figure 5

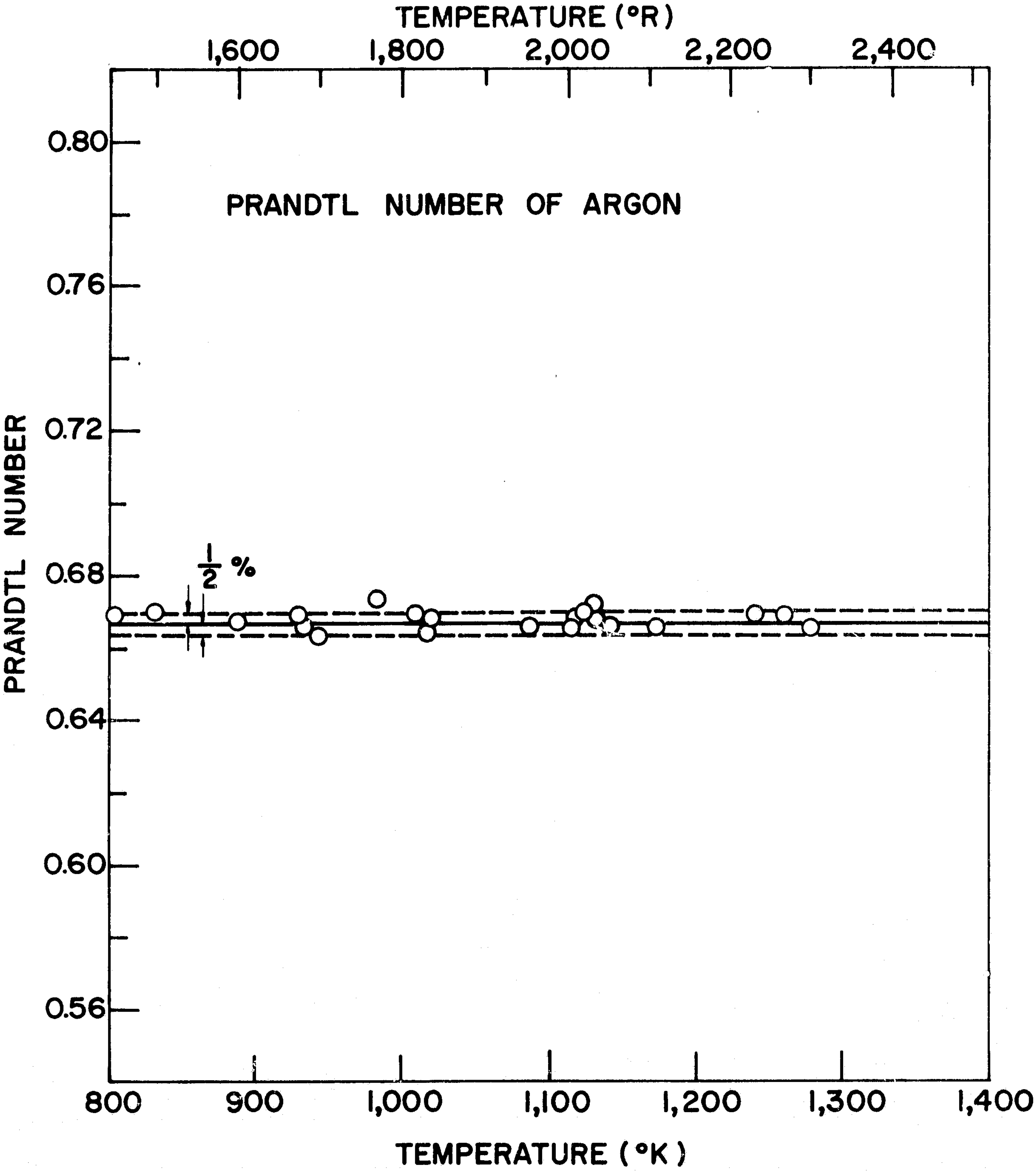

TEMPERATURE (°R)
1,600
1,800
2,000
2,200
2,400
PRANDTL NUMBER OF ARGON
0.80
0.76
0.72
0.68
0.64
0.60
0.56
PRANDTL NUMBER
$\frac{1}{2}$ %
800
900
1,000
1,100
1,200
1,300
1,400
TEMPERATURE (°K)

Figure 6

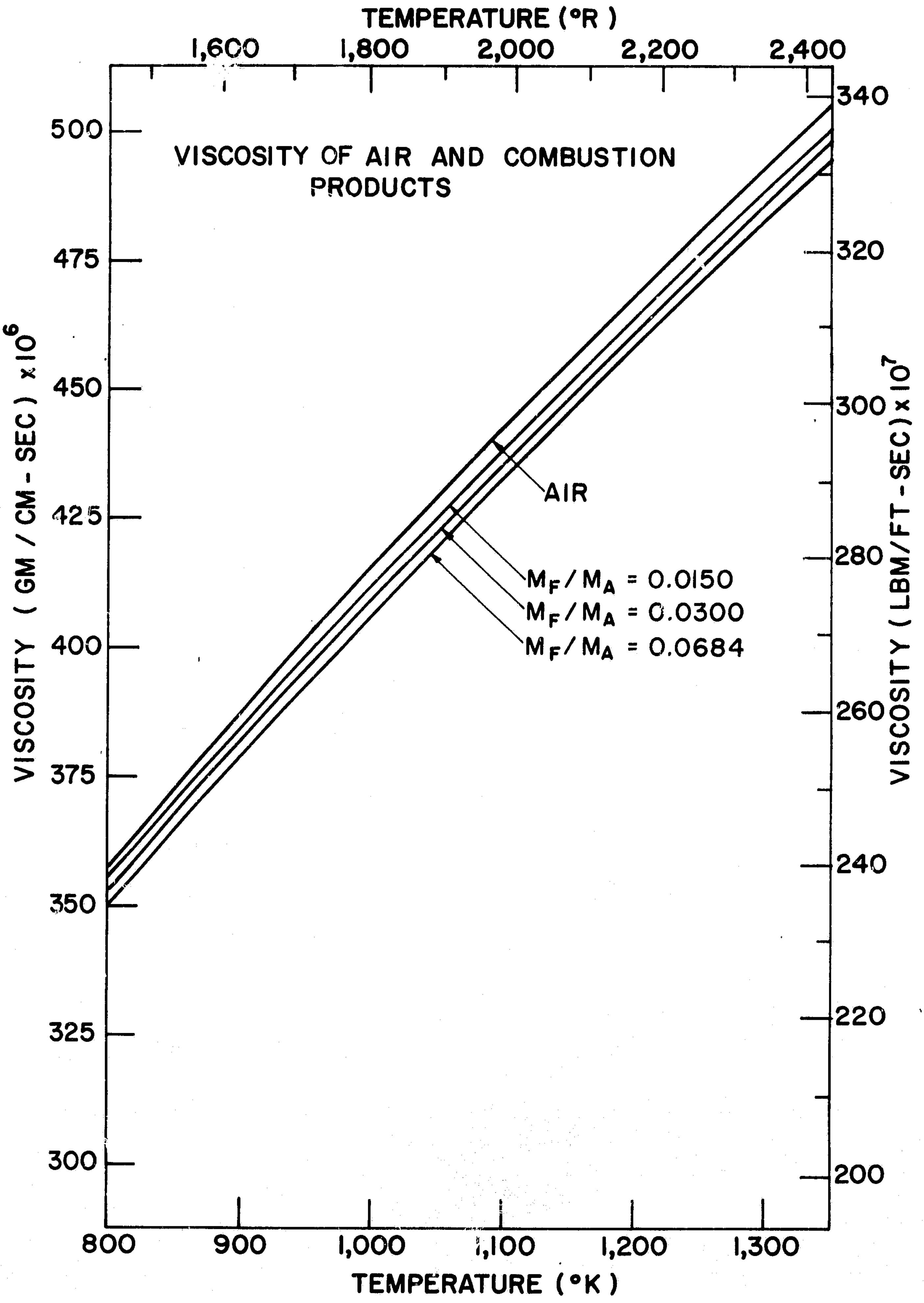

TEMPERATURE (°R)
1,600
1,800
2,000
2,200
2,400
VISCOSITY OF AIR AND COMBUSTION PRODUCTS
AIR
$M_F/M_A$ = 0.0150
$M_F/M_A$ = 0.0300
$M_F/M_A$ = 0.0684
VISCOSITY (GM/CM-SEC) x $10^6$
500
475
450
425
400
375
350
325
300
VISCOSITY (LBM/FT-SEC) x $10^7$
340
320
300
280
260
240
220
200
800
900
1,000
1,100
1,200
1,300
TEMPERATURE (°K)

Figure 7

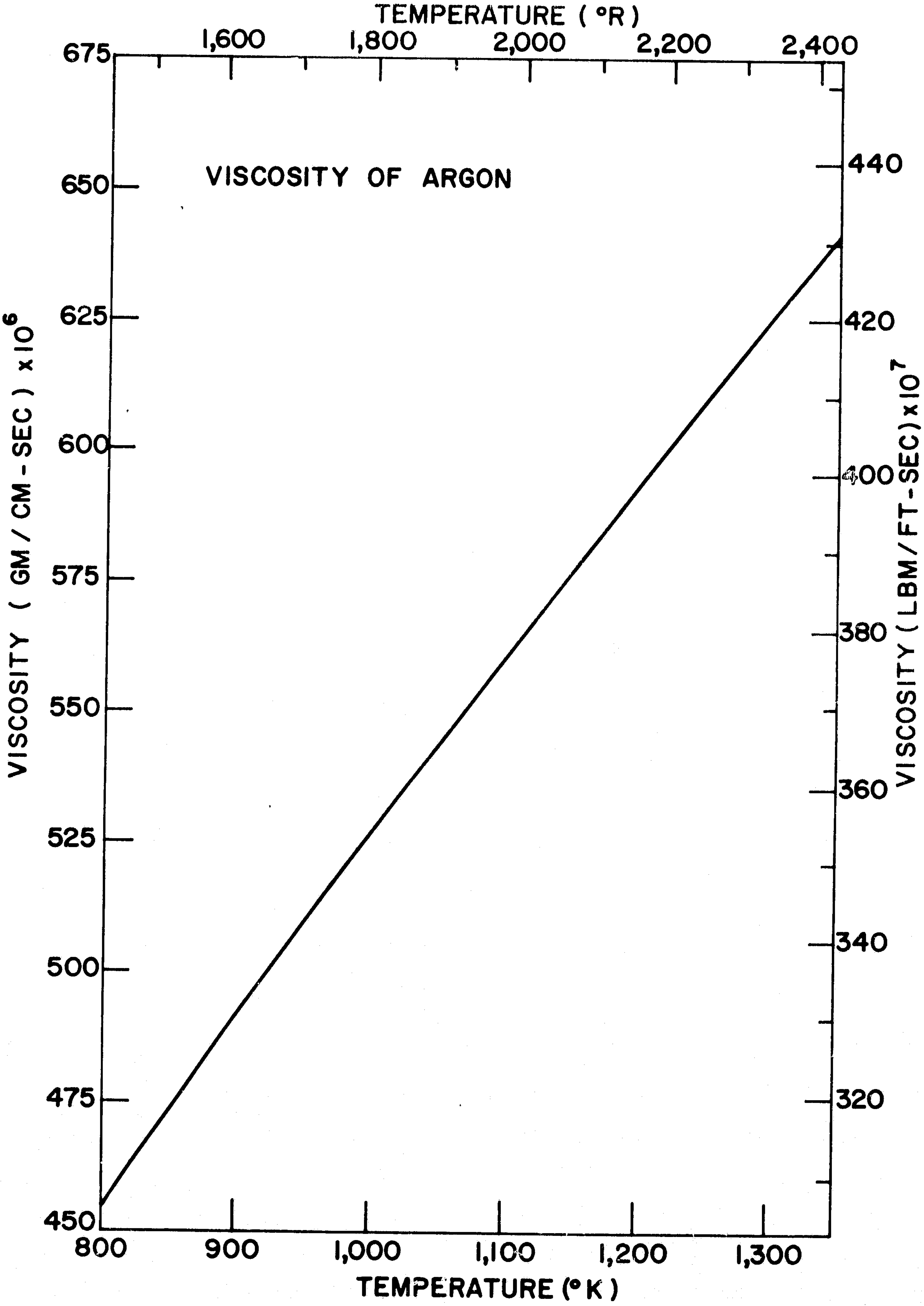

TEMPERATURE ( °R )
1,600
1,800
2,000
2,200
2,400
VISCOSITY OF ARGON
VISCOSITY ( GM / CM - SEC ) x 10^6
675
650
625
600
575
550
525
500
475
450
VISCOSITY ( LBM / FT - SEC ) x 10^7
440
420
400
380
360
340
320
800
900
1,000
1,100
1,200
1,300
TEMPERATURE (° K )

Figure 8

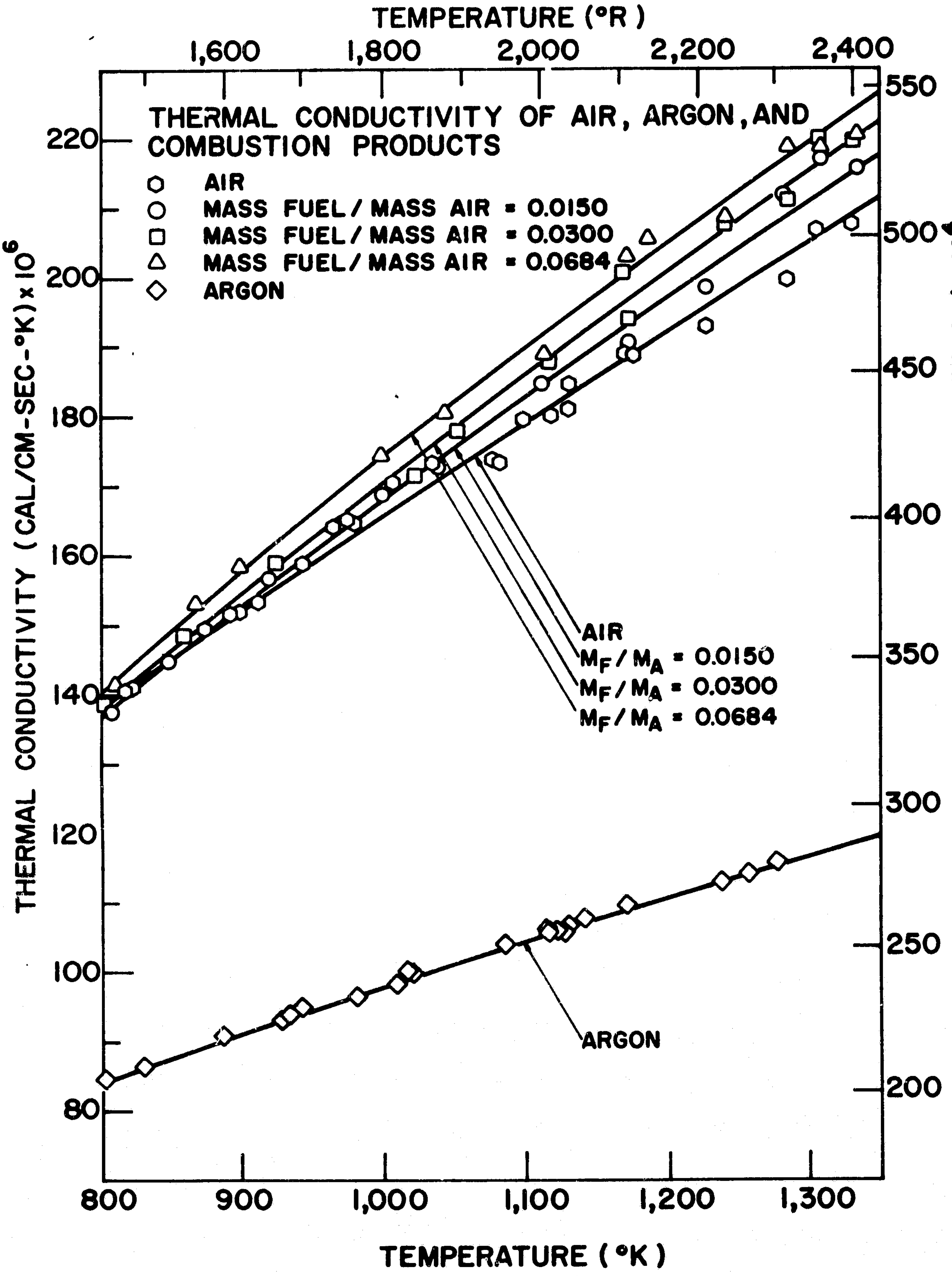

TEMPERATURE (°R)
1,600
1,800
2,000
2,200
2,400
THERMAL CONDUCTIVITY OF AIR, ARGON, AND COMBUSTION PRODUCTS
⬡ AIR
○ MASS FUEL / MASS AIR = 0.0150
□ MASS FUEL / MASS AIR = 0.0300
△ MASS FUEL / MASS AIR = 0.0684
◇ ARGON
AIR
$M_F / M_A$ = 0.0150
$M_F / M_A$ = 0.0300
$M_F / M_A$ = 0.0684
ARGON
THERMAL CONDUCTIVITY (CAL/CM-SEC-°K) x 10$^6$
220
200
180
160
140
120
100
80
THERMAL CONDUCTIVITY (BTU/HR-FT-°R) x 10$^4$
550
500
450
400
350
300
250
200
800
900
1,000
1,100
1,200
1,300
TEMPERATURE (°K)

Figure 9