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REFRACTION EFFECT IN SATELLITE TRACKING

By

SIMO H. LAURILA

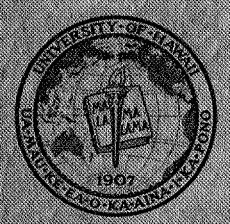
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HAWAII INSTITUTE OF GEOPHYSICS
UNIVERSITY OF HAWAII



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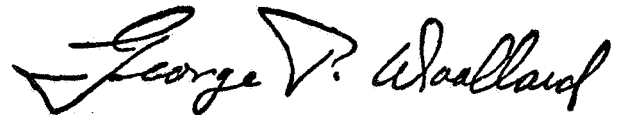
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A handwritten signature in cursive script, reading "George V. Woodland". The signature is written in black ink and is positioned below the "Approved by Director" text.

Date: 6 March 1969

ABSTRACT

The main environmental problem in tracking a satellite through the atmosphere is in finding the most probable value of the mean refractive index. In this report, the mean refractive index is computed as a four-part model. The troposphere is treated as one altitude range from sea level to 9 km, and the stratosphere is divided into three altitude ranges, 9 to 18, 18 to 27, and 27 to 36 kilometers. At 36 km, the N-value is approximately equal to two and reduces rapidly to zero. By the use of the Essen [5] formula in radio wave applications and the modified Kohlraush [7] formula in light-wave applications, point-to-point values of the refractive index are computed through these altitude ranges. The polynomial expansion of second order from the basic exponential function is selected as the model, and the curve-fitting adjustments of the computed values are established separately to each altitude range to obtain coefficients A, B, and C.

A model based on the U. S. Standard Atmosphere, 1962, [10] is used as the reference to which four sets of actual soundings made in Lihue, Hawaii and Fairbanks, Alaska on February 3 and July, 1966, are compared. The results show that the parabolic adjustment has a very high reliability. In the use of standard atmosphere, the standard error of the mean refractive index through the total altitude range of 0 to 36 km, and at the 70° zenith distance, equals only ± 7 mm of distance when radio waves are utilized, and ± 3 mm of distance when light waves are utilized.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	iii
TABLE OF CONTENTS	v
1. Geometry	1
1.1 Curvature of the Ray Path	1
1.2 Model of the Refractive Index as a Function of the Atmosphere	3
2. Point-to-Point Determination of the Refractive Index for the Radio Waves	7
2.1 Sensitivity of the Atmospheric Parameters	9
3. Standard Atmosphere	10
3.1 Strength of the Parabolic Assumption	12
4. Model of the Refractive Index of Radio Waves from Actual Meteorological Soundings	17
5. Point-to-Point Determination of the Refractive Index for Light Waves	26
5.1 Sensitivity of the Atmospheric Parameters	29
6. Model of the Refractive Index of Light Waves from Standard Atmosphere	31
7. Model of the Refractive Index of Light Waves from Actual Meteorological Soundings	34
8. Conclusion and Recommendations	40
LIST OF REFERENCES	44

1. Geometry

In this report only the refraction effect affecting the arc-chord correction and the velocity of radio and light waves is discussed, e.g., the problem of distance measurements. In the tracking of satellites various zenith distances should be anticipated. Because of the short distances involved, the various altitude ranges inside the atmosphere are considered as parallels to the flat earth, and the tracking distance S as a function of the altitude range h can be written:

$$S = \frac{h}{\cos \psi} \quad (1)$$

where ψ is the zenith angle.

Corresponding to the zenith angles, $\psi = 0^\circ, 30^\circ, 50^\circ, 60^\circ,$ and $70^\circ,$ the tracking distances, equivalent to the 9-km altitude range, are $S = 9.0, 10.4, 14.0, 18.0,$ and 26.3 kilometers.

1.1 Curvature of the Ray Path

To investigate the significance of the geometric correction to be applied in order to reduce the arc-distance of the ray path to the chord-distance, and taking into account the fact that at different zenith angles the electromagnetic energy propagates through proportionally slightly different atmospheric layers, and thus affects the velocity, the following formula is used [5] [6]:

$$Q = - \left[B + C(h + k) - \frac{CS^2}{6} \left\{ \frac{1}{R} + B + C(h + k) \right\} \right] \cos \alpha \quad (2)$$

where Q is the curvature of the ray path, R is the curvature radius of the earth in kilometers, S is the tracking distance at various zenith angles in kilometers, h is the elevation of the altitude range in

kilometers, k is the elevation of the tracking station in kilometers, α is an angle between the tracking sighting and the horizontal ($\alpha = 90^\circ - \psi$), and B and C are meteorological constants obtained from the NACA moist atmosphere [5]:

$$B = -0.00003670 \tag{3}$$

$$C = 0.000001350$$

The radius of curvature, $R' = 1/Q$, of the ray path was computed for the zenith angles $\psi = 0^\circ, 30^\circ, 50^\circ, 60^\circ$, and 70° and for the altitude range of 0 to 9 km by the assumption that $k = 0$ and $h = 9$ kilometers. The results in kilometers are given in Table I as follows:

TABLE I

$\psi = 0^\circ$	30°	50°	60°	70°
$R' = \infty$	81,453	53,160	47,016	43,311

The correction of the arc-distance to chord-distance is obtained by the following well-known formula:

$$\Delta S = \frac{S^3}{24 R'^2} \tag{4}$$

where S is the arc distance of the ray path, and R' is the curvature radius of the ray path.

By selecting the most favorable zenith distance, $\psi = 70^\circ$, for comparison, and by applying $S = 26.3$ km and $R' = 43,311$ km into Eq. (4),

the correction will be $\Delta S = 0.4$ mm, which in all practical and even theoretical applications can be neglected.

1.2 Model of the Refractive Index as a Function of the Atmosphere

Models of the refractive index can be established on the basis of a standard atmosphere or an actual atmosphere based on meteorological soundings. There are several approaches in forming the model from standard atmosphere: four of them are discussed here.

Henriksen [2] (Army Map Service) expresses the N-value, which is equivalent to the sixth decimal of the refractive index [$N = (n - 1) \cdot 10^6$], from zero altitude to the upper limit of the atmosphere by the exponential curve:

$$N = 325 \cdot e^{-.1312 \cdot h \cdot 10^{-3}} \quad (5)$$

where the sea-level value is assumed to be $n_0 = 1.000325$, and h is the height above sea level in meters.

Bean [1] [2] (The National Bureau of Standards) expresses the refractive index through atmosphere as a three-part model. He assumes linear decrease in N for the first kilometer above the station. From $(h_s + 1)$ to 9 km, he uses the exponential function:

$$N = N_1 \cdot e^{-c(h - h_s - 1)} \quad (6)$$

where

$$c = \frac{1}{8 - h_s} \cdot \ln \frac{N_1}{105}$$

and h_s is the height of the station in kilometers. Above 9 km, the

following exponential formula is utilized:

$$N = 105 \cdot e^{-.1424 (h - 9)} \quad (7)$$

Jacobsen [5] (U. S. Air Force) developed a formula which took into consideration also the curvature of the ray path, Q, the curvature of the earth, 1/R, and the slope distance, S, of the tracking at large zenith angles, as follows:

$$n = 1 + A + \frac{B(h + k)}{2} + \frac{C}{3} [(h + k)^2 - hk] - \frac{S^2}{12} \left[\frac{1}{R} - Q \right] \quad (8)$$
$$\left[B + C(h + k) - \frac{CS^2}{10} \left(\frac{1}{R} - Q \right) \right]$$

where A = 0.000321. B and C are the same as in (3).

To investigate the effect of the slope propagation of electromagnetic energy in the refractive index at various zenith angles through the basic altitude range from 0 to 9 km, the following values were applied to Eq. (8):

A = 0.000321	R = 6371 kilometers	
B = -0.00003670	h = 9.0 kilometers	(9)
C = 0.000001350	k = 0	

R'-values were taken from Table I, and the results are given in Table II as follows:

TABLE II

$\psi = 0^\circ$	30°	50°	60°	70°
S = 9.0 km	10.4 km	14.0 km	18.0 km	26.3 km
n = 1.00019233	1.00019233	1.00019236	1.00019239	1.00019249

The largest difference in n between the zenith angles of 0° and 70° is only $\Delta n = 1.6 \cdot 10^{-7}$, which at the distance of S = 26.3 km is equivalent to 4.2 millimeters. This suggests strongly that at any practical application the effect of the sloping distance in the refractive index can be neglected and that the mean refractive index can be computed on the basis of the vertical propagation of atmospheric parameters.

In the Jacobsen formula, Eq. (8), several approximations were anticipated. To investigate the validity of these approximations, a vertical propagation of the electromagnetic waves can be assumed from sea level. Under this condition $k = 0$, $S = h$, and $Q = 0$, and Eq. (8) can be rewritten as follows:

$$n = 1 + A + \frac{B}{2} h + \frac{C}{3} h^2 + dn \tag{10}$$

where

$$dn = - \left[\frac{h^2 B}{12R} + \frac{h^3 C}{12R} - \frac{h^4 C}{120R^2} \right] \tag{11}$$

By the application of the values (9) into Eq. (11), dn yields the following magnitude, $dn = 0.00000003$, or $3 \cdot 10^{-8}$ of the distance. At the most unfavorable slope distance of 26.3 km, $dn = 0.8$ mm, which is completely negligible, and Eq. (10) minus dn thus represents the mean

refractive index between sea level and an altitude of h .

Because of the development of a polynom of second order only, the Jacobsen formula was originally used for measuring distances to aircrafts not exceeding the altitude of about 10 kilometers.

The author presents the refractive index through the atmosphere as a four-part model. The second order parabolic development of the basic exponential function is applied as one altitude range through the troposphere, and as three separate altitude ranges through the stratosphere.

The second order parabola:

$$n = 1 + A + Bh + Ch^2 \quad (12)$$

is valid only for the point-to-point determination of the refractive index when the coefficients A , B , and C and the altitude h is known. In the utilization of refractive index through the atmosphere for satellite tracking, the best-fitting mean refractive index obtained through various altitude ranges is more significant than the point-to-point values, and therefore, instead of Eq. (12), the mean value should be used as obtained from the following expression:

$$\bar{n} = \frac{1}{h} \int_0^h n \, dh \quad (13)$$

or

$$\bar{n} = A + \frac{h}{2} B + \frac{h^2}{3} C \quad (14)$$

which is practically the same as Eq. (10).

Also, the most realistic criterion in investigating the reliability, or validity, of such mean refractive index is to compute the standard error of function (14). According to the general error propagation it will be:

$$m_n = \pm \mu \sqrt{[FF]} \quad (15)$$

where μ is the standard error of the weight unit, and $[FF]$ is the weight number of function (14) obtained as follows:

$$[FF] = f'_A{}^2 [\alpha\alpha] + 2f'_A f'_B [\alpha\beta] + 2f'_A f'_C [\alpha\gamma] + f'_B{}^2 [\beta\beta] \\ + 2f'_B f'_C [\beta\gamma] + f'_C{}^2 [\gamma\gamma] \quad (16)$$

$[\alpha\alpha]$, $[\beta\beta]$, and $[\gamma\gamma]$ are the weight numbers and $[\alpha\beta]$, $[\alpha\gamma]$, and $[\beta\gamma]$ are the correlation numbers of unknowns A, B, and C. Partial derivatives for the unknowns are denoted by f'_A , f'_B , and f'_C , correspondingly. When applied to Eq. (14), the following expression for $[FF]$ can be written:

$$[FF] = [\alpha\alpha] + h[\alpha\beta] + \frac{2}{3} h^2 [\alpha\gamma] + \frac{h^2}{4} [\beta\beta] + \frac{h^3}{3} [\beta\gamma] + \frac{h^4}{9} [\gamma\gamma] \quad (17)$$

2. Point-to-Point Determination of the Refractive Index for the Radio Waves.

To determine the coefficients of any model by the least-squares curve-fitting method, observed values of the refractive index must first be computed. The three formulas which follow are commonly used in the determination of the refractive index as a function of the atmospheric parameters--temperature, pressure, and humidity:

Essen Formula:

$$N = (n - 1) \cdot 10^6 = \frac{77.62}{T} P - \left(\frac{12.92}{T} - \frac{37.19 \cdot 10^4}{T^2} \right) e \quad (18)$$

Essen-Froome Formula:

$$N = (n - 1) \cdot 10^6 = \frac{103.49}{T} (P - e) + \frac{86.26}{T} \left(1 + \frac{5748}{T}\right) e \quad (19)$$

Smith-Weintraub Formula:

$$N = (n - 1) \cdot 10^6 = \frac{77.6}{T} (P + 4.81 \cdot 10^3 \cdot \frac{e}{T}) \quad (20)$$

In the Essen and Smith-Weintraub formulas, T is the temperature in Kelvin units, P is the total pressure in millibars, and e is the partial pressure of water vapor in millibars. At the 1960 meeting of the International Union of Geodesy and Geophysics [4] it was recommended that the Essen-Froome formula be used to achieve uniformity in calculations of electronic distance measurements. It should be noted, however, that the Essen-Froome formula is actually the Essen formula obtained by rearranging terms and stating pressures in millimeters of mercury instead of millibars.

Essen and Smith-Weintraub formulas were compared by applying to both formulas T and P values from the U. S. Standard Atmosphere 1962 [10] and e value from Eq. (30) as follows:

TABLE III

Altitude, km	Essen N-value	Smith-Weintraub N-value	Difference, ΔN
0	321.0	321.6	-0.6
1	285.6	286.1	-0.5
2	253.0	253.3	-0.3
3	223.0	223.2	-0.2
4	195.7	195.9	-0.2
5	171.1	171.2	-0.1

Consequently, the use of either formula is justified. In this report, the Essen formula has been utilized because, in actual soundings, the total pressure is given in millibars and the partial pressure of water vapor is given either as the dew point temperature or as the relative humidity. In both cases it is easy to convert them to millibars by using the Smithsonian Meteorological Tables.

2.1 Sensitivity of the Atmospheric Parameters

To evaluate the error in n caused by the uncertainty in observing the parameters T , P , and e , the Essen formula is differentiated as follows:

If

$$N = (n - 1) \cdot 10^6 = \frac{K}{T} P - \left(\frac{L}{T} - \frac{M \cdot 10^4}{T^2} \right) e \quad (21)$$

where $K = 77.62$, $L = 12.92$, and $M = 37.19$, it is obtained:

$$dN_T = \left[-\frac{KP}{T^2} + \left(\frac{L}{T^2} - \frac{2 \cdot M \cdot 10^4}{T^3} \right) e \right] dT$$
$$dN_P = \frac{K}{T} dP \quad (22)$$

$$dN_e = \left(-\frac{L}{T} + \frac{M \cdot 10^4}{T^2} \right) de$$

If the criterion for the sensitivity of observing the parameters is set forth to be 1 ppm, and K , P , and e are taken from the U. S. Standard Atmosphere and from Eq. (30) to correspond to the altitude of 4.5 km, the following values are applied to Eq. (22):

$$\begin{aligned} T &= 258^{\circ}.9 \\ P &= 577 \text{ mb} \\ e &= 1.84 \text{ mb} \end{aligned} \tag{23}$$

The standard error of each individual parameter to cause an error of 1 ppm in n thus will be:

$$\begin{aligned} m_T &= \pm 1^{\circ}.3 \\ m_P &= \pm 3.3 \text{ mb} \\ m_e &= \pm 0.2 \text{ mb} \end{aligned} \tag{24}$$

From (24) it is clearly seen that, in tracking satellites by the use of radio waves, the determination of the humidity is the most critical phase in the operation.

3. Standard Atmosphere

To evaluate the magnitude of errors made in obtaining the meteorological parameters from a standard atmosphere instead of a local atmosphere based on actual soundings, the U. S. Standard Atmosphere, 1962, [10] has been selected as the basis for this analysis. This atmosphere is fundamentally defined in terms of an ideal air obeying the perfect gas law. It is based upon accepted standard values of air density, temperature, and pressure at sea level as follows:

$$\begin{aligned} \text{Density} \quad p_o &= 1.2250 \text{ kg/m}^3 \\ \text{Temperature } T_o &= 288^{\circ}.15 \text{ K} \\ \text{Pressure } P_o &= 1013.25 \text{ millibars} \end{aligned} \tag{25}$$

The density for tabulated altitudes is obtained from the following expression:

$$P = \frac{M \cdot P}{R \cdot T} \quad (26)$$

Where $M = 28.9644$ is the molecular weight of air and is considered as a constant up to an altitude of 90 km, P is the pressure, R is the universal gas constant, and T is the absolute temperature.

The temperature variation is defined as a series of connected segments, linear in latitude. The general formula for each linear segment is:

$$T = T_n + L(H + H_n) \quad (27)$$

where L is the gradient of the temperature with altitude dT/dH .

Between $H_n = 0$ and $H = 11$ km, $L = -6^\circ.5$ K/1 km; between $H_n = 11$ km and $H = 20$ km, $L = 0^\circ.0$ K/1 km; between $H_n = 20$ km and $H = 32$ km, $L = +1^\circ.0$ K/1 km; and between $H_n = 32$ km and $H = 47$ km, $L = +2^\circ.8$ K/1 km.

Within an atmospheric layer throughout which T is a linear function of H , the pressure is computed by the following formula when $L \neq 0$:

$$\text{Log}_e P = \text{Log}_e P_n - \frac{g_o M}{L \cdot R} \text{Log}_e \frac{L(H - H_n) + T}{T} \quad (28)$$

where $g_o = 9.80665$ m/sec².

If $L = 0$, the following expression for P is used:

$$\text{Log}_e P = \text{Log}_e P_n - \frac{g_o M}{R \cdot T} (H - H_n) \quad (29)$$

Partial pressure of water vapor is not readily obtainable from the U. S. Standard Atmosphere. For its determination, the authors established

the following procedure [8]: Coefficients A, B, and C from (9) were applied to Eq. (12) which then was solved simultaneously with the Essen formula, Eq. (21) for e:

$$e = \frac{T^2 \left(\frac{K}{T} P \cdot 10^{-6} - A - Bh - Ch^2 \right)}{(TL - M \cdot 10^4) \cdot 10^{-6}} \quad (30)$$

All altitude values, H, in Eqs. (27) - (29) are geopotential altitudes, while h in Eq. (30) is geometric altitude.

3.1 Strength of the Parabolic Assumption

To investigate the strength of the parabolic assumption, 74 values of T and P were taken from the U. S. Standard Atmosphere, together with the e-values from Eq. (30) at intervals of 0.5 km from sea level to an altitude of 36.5 kilometers. By the use of the Essen formula, the N-values were computed, Table IV.

TABLE IV

No.	Geom. Alt., km	T, °K	P, mb	e, mb	N
1	0	288.2	1013.0	10.87	321.0
2	0.5	284.9	955.0	9.43	302.9
3	1.0	281.7	899.0	8.18	285.7
4	1.5	278.4	846.0	6.96	268.9
5	2.0	275.2	795.0	5.91	252.9
6	2.5	271.9	747.0	4.90	237.7
7	3.0	268.7	701.0	4.02	223.0
8	3.5	265.4	658.0	3.17	209.0
9	4.0	262.2	617.0	2.44	195.7
10	4.5	258.9	577.0	1.84	183.1
11	5.0	255.7	540.0	1.28	171.1

TABLE IV (continued)

12	5.5	252.4	505.0	0.79	159.9
13	6.0	249.2	472.0	0.39	149.3
14	6.5	245.9	441.0	0.03	139.4
15	7.0	242.7	411.0	0	131.4
16	7.5	239.5	383.0	0	124.1
17	8.0	236.2	357.0	0	117.3
18	8.5	232.9	332.0	0	110.6
19	9.0	229.7	308.0	0	104.1
20	9.5	226.5	286.0	0	98.0
21	10.0	223.3	265.0	0	92.1
22	10.5	220.0	245.0	0	86.5
23	11.0	216.7	227.0	0	81.3
24	11.5	216.7	210.0	0	75.2
25	12.0	216.7	194.0	0	69.5
26	12.5	216.7	179.0	0	64.1
27	13.0	216.7	166.0	0	59.5
28	13.5	216.7	153.0	0	54.8
29	14.0	216.7	142.0	0	50.9
30	14.5	216.7	131.0	0	46.9
31	15.0	216.7	121.0	0	43.3
32	15.5	216.7	112.0	0	40.1
33	16.0	216.7	104.0	0	37.3
34	16.5	216.7	96.0	0	34.3
35	17.0	216.7	88.0	0	31.5
36	17.5	216.7	82.0	0	29.4
37	18.0	216.7	76.0	0	27.2
38	18.5	216.7	70.0	0	25.1
39	19.0	216.7	65.0	0	23.3
40	19.5	216.7	60.0	0	21.5
41	20.0	216.7	55.0	0	19.7
42	20.5	217.1	51.0	0	18.2
43	21.0	217.6	47.0	0	16.8
44	21.5	218.1	44.0	0	15.7

TABLE IV (continued)

45	22.0	218.6	40.0	0	14.2
46	22.5	219.1	37.0	0	13.1
47	23.0	219.7	35.0	0	12.4
48	23.5	220.1	32.0	0	11.3
49	24.0	220.6	30.0	0	10.6
50	24.5	221.1	28.0	0	9.8
51	25.0	221.6	25.0	0	8.8
52	25.5	222.0	23.0	0	8.0
53	26.0	222.5	22.0	0	7.7
54	26.5	223.0	20.0	0	7.0
55	27.0	223.5	19.0	0	6.6
56	27.5	224.0	17.0	0	5.9
57	28.0	224.5	16.0	0	5.5
58	28.5	225.0	15.0	0	5.2
59	29.0	225.5	14.0	0	4.8
60	29.5	226.0	13.0	0	4.4
61	30.0	226.5	12.0	0	4.1
62	30.5	227.0	11.1	0	3.8
63	31.0	227.5	10.3	0	3.5
64	31.5	228.0	9.6	0	3.3
65	32.0	228.5	8.9	0	3.0
66	32.5	229.6	8.2	0	2.8
67	33.0	231.0	7.7	0	2.6
68	33.5	232.3	7.3	0	2.4
69	34.0	233.7	6.6	0	2.2
70	34.5	234.6	6.2	0	2.0
71	35.0	236.5	5.7	0	1.9
72	35.5	237.9	5.4	0	1.8
73	36.0	239.3	5.0	0	1.6
74	36.5	240.6	4.6	0	1.5

The computed N-values were adjusted separately for the four altitude ranges, 0 to 9, 9 to 18, 18 to 27, and 27 to 36 kilometers. Curve-fitting adjustment of Eq. (12) yielded the following constants, A, B, and C, given in Table V.

TABLE V

Altitude Range, km	A	B	C
0 - 9	0.0003212	-0.000037141	0.0000014370
9 - 18	0.0001062	-0.000013897	0.0000005693
18 - 27	0.0000267	-0.000003786	0.0000001766
27 - 36	0.0000064	-0.000000870	0.0000000380

The mean refractive indexes \bar{n} and their standard errors in N-units for each altitude range are given in Table VI. In Table VII, residuals after the adjustment are given at intervals of 1 km from 0 to 15 km, and at intervals of 5 km from 15 to 35 kilometers. E denotes the Essen point-to-point values, L denotes the values read from the adjusted parabolas, and ΔN denotes their differences.

TABLE VI

Altitude Range, km	\bar{n}	$m_{\bar{N}}$
0 - 9	1.0001929	± 0.18
9 - 18	1.0000591	± 0.09
18 - 27	1.0000144	± 0.05
27 - 36	1.0000036	± 0.02

TABLE VII

Altitude, km	E, N- Value	L, N- Value	L, ΔN- Value	Altitude, km	E, N- Value	L, N- Value	L, ΔN- Value
0	321.0	321.2	-0.2	11	81.3	80.7	+0.6
1	285.7	285.5	+0.2	12	69.5	69.6	-0.1
2	252.9	252.6	+0.3	13	59.5	59.7	-0.2
3	223.0	222.7	+0.3	14	50.9	50.9	0
4	195.7	195.6	+0.1	15	43.3	43.3	0
5	171.1	171.4	-0.3	20	19.7	19.8	-0.1
6	149.3	150.2	-0.9	25	8.8	8.9	-0.1
7	131.4	131.6	-0.2	30	4.1	4.2	-0.1
8	117.3	116.1	+1.2	35	1.9	1.8	+0.1
9	104.1	103.3	+0.8				
10	92.1	92.9	-0.8				

The standard errors from Table VI were converted in meters of distance corresponding to the zenith angles of $\psi = 70^\circ, 60^\circ, 50^\circ, 30^\circ,$ and 0° . Since each altitude range was adjusted independently, the total cumulative standard error for the entire altitude range from sea level to 36 km was obtained by the normal, quadratic error cumulation. The results are given in Table VIII.

TABLE VIII

Altitude Range, km	Zenith Angle				
	70°	60°	50°	30°	0°
0 - 9	±0.005	+0.003	±0.002	±0.002	±0.002
9 - 18	0.003	0.002	0.001	0.001	0.001
18 - 27	0.002	0.001	0.001	0.000	0.000
27 - 36	0.001	0.000	0.000	0.000	0.000
0 - 36	0.007	0.004	0.002	0.002	0.002

The largest standard error at the poorest zenith angle, $\psi = 70^\circ$, and through the most vulnerable altitude range, 0 to 9 km, where moisture is present, is only ± 5 millimeters. This, together with the small residuals in Table VII prove the parabolic curve-fitting adjustment to be very reliable.

4. Model of the Refractive Index of Radio Waves from Actual Meteorological Soundings

The effect of the use of actual meteorological soundings in the vicinity of the tracking station was investigated by selecting four widely separated climatological samples as reference. The sounding data were obtained from the U. S. Weather Bureau stations at Lihue, February 3, 1966; Lihue, July 2, 1966; Fairbanks, February 3, 1966; Fairbanks, July 2, 1966. All soundings were made at 00.00 Greenwich time.

These samples represented hot and humid, cool and dry, and cold and dry climatological conditions, with the rather anomalous temperature distribution as shown in Table IX, given in centigrade:

TABLE IX

Altitude, km	Lihue July	Lihue February	Fairbanks July	Fairbanks February	U. S. Standard
0.5	21	17	23	- 8	12
5.0	0	- 5	-15	-30	-17
10.0	-38	-35	-43	-54	-50
15.0	-69	-71	-45	-46	-57
17.0	-69	-75	-45	-47	-57

All sounding altitudes were given as geopotential heights, which were converted into geometric heights by the use of the Smithsonian

Meteorological Tables, based on the following formula:

$$H = \frac{M \cdot Z}{\left(\frac{g_{\phi} \cdot M}{9.8}\right) - Z} \quad (31)$$

where Z is the geopotential height, g_{ϕ} is the actual acceleration of gravity at latitude ϕ , and M is the meridian curvature radius at that latitude.

After this conversion, actual soundings were applied to the Essen formula, Eq. (18) and are presented in tables X - XIII. Curve-fitting adjustment of Essen values from these tables into the parabola (12) yielded the following constants, A, B, and C given in tables XIV - XVII. The mean refractive indexes through the four altitude ranges and their standard errors in N-units are presented in tables XVIII - XXI. These standard errors were converted to correspond to standard errors in distance, in meters, for the zenith angles $\psi = 0^{\circ}, 30^{\circ}, 50^{\circ}, 60^{\circ},$ and 70° and for the altitude ranges of 0 to 9, 9 to 18, 18 to 27, and 27 to 36 km. Also, the constant shift, which occurs in the tracking of satellites when the mean refractive index is taken from the U. S. Standard Atmosphere, instead of from the local atmosphere based on soundings, was computed. It was obtained as the difference of the mean refractive indexes from tables XVIII - XXI and those from Table VI, in meters of distance. The results are given in tables XXII - XXV.

TABLE X

Lihue, Hawaii, February 3, 1966

No.	Geom. Alt., km	T, °K	P, mb	e, mb	N
1	0.036	294.7	1009	15.83	332.9
2	0.114	293.9	1000	12.36	316.8
3	0.552	289.7	950	9.50	296.2
4	1.012	285.3	900	9.30	286.9

TABLE X (continued)

5	1.490	283.6	850	4.47	253.1
6	1.994	282.9	800	2.80	232.4
7	2.505	281.2	750	2.17	217.1
8	3.099	279.0	700	1.87	203.6
9	3.704	276.0	650	1.51	190.1
10	4.348	272.7	600	1.20	176.7
11	5.031	268.0	550	0.88	163.8
12	5.785	262.0	500	0.58	151.2
13	6.575	257.0	450	0.40	138.1
14	7.472	251.6	400	0.22	124.7
15	8.451	247.9	350	0.17	110.6
16	9.564	242.3	300	0.10	96.7
17	10.839	234.0	250	0.05	83.3
18	12.335	222.5	200	0	69.8
19	13.202	216.4	175	0	62.8
20	14.167	209.7	150	0	55.5
21	15.273	201.9	125	0	48.1
22	16.580	197.9	100	0	39.2
23	17.884	197.8	80	0	31.3
24	18.664	198.5	70	0	27.4
25	19.582	206.0	60	0	22.6
26	20.695	208.0	50	0	18.7
27	22.083	212.0	40	0	14.6
28	23.915	217.1	30	0	10.7
29	25.130	220.0	25	0	8.8
30	26.450	221.7	20	0	7.0
31	28.442	222.6	15	0	5.2
32	31.137	228.1	10	0	3.4
33	33.540	230.0	7	0	2.3

TABLE XI

Lihue, Hawaii, July 2, 1966

No.	Geom. Alt., km	T, °K	P, mb	e, mb	N
1	0.036	300.0	1013	24.42	362.0
2	0.150	296.7	1000	18.17	337.6
3	0.591	292.9	950	15.79	319.5
4	1.062	289.3	900	12.78	297.7
5	1.547	286.6	850	10.43	276.9
6	2.055	284.7	800	7.42	251.8
7	2.594	281.8	750	6.68	237.6
8	3.170	284.0	700	1.97	200.3
9	3.777	281.9	650	1.82	187.4
10	4.446	277.2	600	1.31	174.3
11	5.131	272.0	550	0.91	161.5
12	5.903	266.3	500	0.63	149.0
13	6.717	260.0	450	0.38	136.4
14	7.612	254.7	400	0.25	123.3
15	8.597	247.2	350	0.14	110.7
16	9.699	238.7	300	0.07	98.0
17	10.960	231.1	250	0	84.0
18	12.447	220.0	200	0	70.6
19	13.300	212.3	175	0	64.0
20	14.253	206.4	150	0	56.4
21	15.344	203.0	125	0	47.8
22	16.689	203.6	100	0	38.1
23	18.040	207.2	80	0	30.0
24	18.855	208.4	70	0	26.1
25	19.805	209.9	60	0	22.2
26	20.940	211.7	50	0	18.3
27	22.347	215.5	40	0	14.4
28	24.202	220.8	30	0	10.5
29	25.406	225.9	25	0	8.6

TABLE XI (continued)

30	26.900	228.0	20	0	6.8
31	28.850	230.8	15	0	5.0
32	31.642	236.8	10	0	3.3
33	34.165	239.3	7	0	2.3
34	36.565	241.8	5	0	1.6

TABLE XII

Fairbanks, Alaska, February 3, 1966

No.	Geom. Alt., km	T, °K	P, mb	e, mb	N
1	0.146	254.1	1000	1.14	312.0
2	0.539	266.6	950	2.65	290.3
3	0.960	267.9	900	2.17	271.9
4	1.406	265.6	850	1.65	257.0
5	1.877	263.3	800	1.17	242.1
6	2.376	259.9	750	0.85	228.6
7	2.893	255.3	700	0.73	216.9
8	3.446	250.8	650	0.84	206.1
9	4.026	247.6	600	0.30	189.9
10	4.658	244.5	550	0.21	175.9
11	5.331	240.4	500	0.13	162.3
12	6.056	235.2	450	0.08	149.0
13	6.871	230.0	400	0	135.0
14	7.759	224.6	350	0	121.0
15	8.758	218.4	300	0	106.6
16	9.917	217.6	250	0	89.2
17	11.365	223.1	200	0	69.6
18	12.240	224.8	175	0	60.4
19	13.261	227.7	150	0	51.1

TABLE XII (continued)

20	13.261	227.7	150	0	51.1
21	15.964	226.2	100	0	34.3
22	17.442	225.7	80	0	27.5
23	18.324	225.4	70	0	24.1
24	19.346	225.7	60	0	20.6
25	20.556	225.7	50	0	17.2
26	22.030	226.1	40	0	13.7
27	23.955	227.1	30	0	10.3
28	25.173	227.6	25	0	8.5
29	26.672	228.1	20	0	6.8
30	28.612	228.8	15	0	5.1
31	31.362	231.3	10	0	3.4

TABLE XIII

Fairbanks, Alaska, July 2, 1966

No.	Geom. Alt., km	T, °K	P, mb	e, mb	N
1	0.499	296.5	950	5.21	270.5
2	0.961	291.6	900	4.07	257.2
3	1.445	286.4	850	3.07	244.1
4	1.947	281.3	800	2.73	233.5
5	2.464	276.1	750	2.29	221.9
6	3.027	271.7	700	1.17	205.8
7	3.615	269.0	650	0.73	191.3
8	4.242	264.1	600	0.50	178.9
9	4.900	258.3	550	0.33	167.1
10	5.626	253.8	500	0.24	154.3
11	6.390	251.6	450	0.21	140.0
12	7.263	246.2	400	0.13	126.9

TABLE XIII (continued)

13	8.221	241.2	350	0.09	113.2
14	9.295	233.9	300	0.05	99.9
15	10.520	225.0	250	0	86.2
16	11.973	218.8	200	0	71.0
17	12.837	222.9	175	0	60.9
18	13.855	226.0	150	0	51.5
19	15.070	227.5	125	0	42.6
20	16.560	227.5	100	0	34.1
21	18.055	228.1	80	0	27.2
22	18.950	228.4	70	0	23.8
23	19.986	228.1	60	0	20.4
24	21.203	227.0	50	0	17.1
25	22.697	228.9	40	0	13.6
26	24.650	231.2	30	0	10.1
27	25.895	232.1	25	0	8.4
28	27.411	232.3	20	0	6.7
29	29.400	234.4	15	0	5.0
30	32.215	237.4	10	0	3.3
31	34.720	240.6	7	0	2.3

TABLE XIV

Lihue, February 3, 1966

Altitude Range, km	A	B	C
0 - 9	0.0003210	-0.00004288	0.000002127
9 - 18	0.0001008	-0.00001007	0.000000256
18 - 27	0.0000291	-0.00000447	0.000000229
27 - 36	0.0000065	-0.00000095	0.000000050

TABLE XV

Lihue, July 2, 1966

Altitude Range, km	A	B	C
0 - 9	0.0003498	-0.00005126	0.000002711
9 - 18	0.0001035	-0.00001033	0.000000247
18 - 27	0.0000290	-0.00000416	0.000000188
27 - 36	0.0000064	-0.00000080	0.000000032

TABLE XVI

Fairbanks, February 3, 1966

Altitude Range, km	A	B	C
0 - 9	0.0003070	-0.00003426	0.000001285
9 - 18	0.0000962	-0.00001270	0.000000538
18 - 27	0.0000250	-0.00000349	0.000000173
27 - 36	0.0000065	-0.00000098	0.000000056

TABLE XVII

Fairbanks, July 2, 1966

Altitude Range, km	A	B	C
0 - 9	0.0002844	-0.00002865	0.000000950
9 - 18	0.0001063	-0.00001392	0.000000569
18 - 27	0.0000267	-0.00000349	0.000000148
27 - 36	0.0000071	-0.00000094	0.000000040

TABLE XVIII

Lihue, February 3, 1966

Altitude Range, km	\bar{n}	$m_{\bar{N}}$
0 - 9	1.0001855	± 2.10
9 - 18	1.0000624	± 0.12
18 - 27	1.0000152	± 0.04
27 - 36	1.0000035	± 0.01

TABLE XIX

Lihue, July 2, 1966

Altitude Range, km	\bar{n}	$m_{\bar{N}}$
0 - 9	1.0001923	± 2.10
9 - 18	1.0000637	± 0.21
18 - 27	1.0000154	± 0.06
27 - 36	1.0000036	± 0.03

TABLE XX

Fairbanks, February 3, 1966

Altitude Range, km	\bar{n}	$m_{\bar{N}}$
0 - 9	1.0001811	± 0.31
9 - 18	1.0000590	± 0.27
18 - 27	1.0000150	± 0.00
27 - 36	1.0000042	± 0.03

TABLE XXI

Fairbanks, July 2, 1966

Altitude Range, km	\bar{n}	$m\bar{N}$
0 - 9	1.0001811	± 0.31
9 - 18	1.0000590	± 0.27
18 - 27	1.0000150	± 0.00
27 - 36	1.0000042	± 0.03

5. Point-to-Point Determination of the Refractive Index for Light Waves

For light waves, the group refractive index is first computed by use of the Barrel and Sears [7] formula as follows:

$$(n_g - 1) \cdot 10^7 = A + \frac{3B}{\lambda^2} + \frac{5C}{\lambda^4} \quad (32)$$

where

$$\begin{aligned} A &= 2876.04 \\ B &= 16.288 \\ C &= 0.131 \end{aligned} \quad (33)$$

In the utilization of a ruby laser, the wavelength $\lambda = 6943 \text{ \AA}$ is applied to Eq. (32) and yields the value of the group refractive index as:

$$(n_g - 1) \cdot 10^6 = 297.99 \quad (34)$$

TABLE XXII

Lihue, July 2 Total Altitude Range 0.036 to 36.565 km 35 Soundings

	0 to 9 km	9 to 18 km	18 to 27 km	27 to 36 km	0 to 36 km				
Zenith Angle	m	m	m	m	m				
St. Error, Differ.,	m	m	m	m	m				
St. Error, Differ.,	m	m	m	m	m				
St. Error, Differ.,	m	m	m	m	m				
St. Error, Differ.,	m	m	m	m	m				
0°	±0.019	-0.005	±0.002	+0.042	±0.000	+0.009	±0.000	±0.019	+0.046
30°	0.022	-0.005	0.002	+0.048	0.001	+0.010	0.000	0.000	+0.053
50°	0.029	-0.009	0.003	+0.064	0.001	+0.014	0.000	0.000	+0.069
60°	0.036	-0.014	0.004	+0.082	0.001	+0.018	0.001	0.000	+0.086
70°	0.055	-0.016	0.006	+0.121	0.002	+0.026	0.001	0.000	+0.131

TABLE XXIII

Lihue, February 3 Total Altitude Range 0.036 to 33.540 km 34 Soundings

	0 to 9 km	9 to 18 km	18 to 27 km	27 to 36 km	0 to 36 km				
Zenith Angle	m	m	m	m	m				
St. Error, Differ.,	m	m	m	m	m				
St. Error, Differ.,	m	m	m	m	m				
St. Error, Differ.,	m	m	m	m	m				
St. Error, Differ.,	m	m	m	m	m				
0°	±0.019	-0.067	±0.001	+0.030	±0.000	+0.007	±0.001	±0.019	-0.031
30°	0.022	-0.077	0.001	+0.034	0.000	+0.008	0.000	0.022	-0.036
50°	0.029	-0.104	0.002	+0.046	0.001	+0.011	0.000	0.029	-0.049
60°	0.038	-0.133	0.002	+0.059	0.001	+0.014	0.000	0.038	-0.062
70°	0.055	-0.195	0.003	+0.089	0.002	+0.021	0.000	0.055	-0.088

TABLE XXIV

Fairbanks, July 2 Total Altitude Range 0.499 to 34.720 km 31 Soundings

Zenith Angle	0 to 9 km		9 to 18 km		18 to 27 km		27 to 36 km		0 to 36 km	
	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m
0°	±0.003	-0.106	±0.003	-0.001	±0.000	+0.005	±0.000	+0.005	±0.005	-0.097
30°	0.003	-0.123	0.003	-0.001	0.000	+0.007	0.000	+0.007	0.004	-0.110
50°	0.004	-0.165	0.004	-0.002	0.000	+0.011	0.000	+0.011	0.006	-0.145
60°	0.006	-0.212	0.005	-0.002	0.000	+0.014	0.001	+0.014	0.008	-0.186
70°	0.008	-0.310	0.008	-0.002	0.000	+0.016	0.001	+0.016	0.011	-0.280

TABLE XXV

Fairbanks, February 3 Total Altitude Range 0.146 to 31.362 km 31 Soundings

Zenith Angle	0 to 9 km		9 to 18 km		18 to 27 km		27 to 36 km		0 to 36 km	
	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m
0°	±0.008	-0.055	±0.003	-0.052	±0.000	-0.004	±0.000	±0.000	±0.009	-0.111
30°	0.010	-0.063	0.003	-0.060	0.000	-0.004	0.001	0.000	0.011	-0.127
50°	0.013	-0.085	0.004	-0.081	0.001	-0.006	0.001	0.000	0.014	-0.172
60°	0.016	-0.110	0.005	-0.104	0.001	-0.007	0.002	0.000	0.017	-0.221
70°	0.024	-0.160	0.008	-0.152	0.001	-0.011	0.002	0.000	0.026	-0.323

To compute the ambient refractive index for light waves, the Kohlrausch [7] formula is usually used, as follows:

$$N = (n - 1) \cdot 10^6 = \left[\frac{n_g - 1}{1 + \alpha t} - \frac{P}{760} - \frac{0.000000055 \cdot e}{1 + \alpha t} \right] 10^6 \quad (35)$$

where P is the total pressure in millimeters of mercury, e is the partial pressure of water vapor in millimeters of mercury, α is the heat expansion coefficient of air and equals 0.00367, and t is the temperature in centigrade.

Since in actual soundings the total pressure is given in millibars, the author has modified Eq. (35) to be more suitable for computer programming in the following way [9]:

$$N = (n - 1) \cdot 10^6 = \frac{298.0 \cdot P - 41.8 \cdot e}{3.709 \cdot T} \quad (36)$$

where P and e are given in millibars, and T is the temperature in Kelvin units. In the derivation of Eq. (36) it was anticipated that $T_0 = 273.15$, which yields the heat expansion coefficient $\alpha = 0.00366$.

5.1 Sensitivity of the Atmospheric Parameters

To investigate the sensitivity of atmospheric parameters when the light wave propagation is concerned, Eq. (36) must be differentiated as follows:

$$\begin{aligned} dN_T &= \frac{1}{T^2} \left(-\frac{298.0}{3.709} P + \frac{41.8}{3.709} e \right) dT \\ dN_P &= \frac{298.0}{3.709 \cdot T} \cdot dP \\ dN_e &= -\frac{41.8}{3.709 \cdot T} \cdot de \end{aligned} \quad (37)$$

By the application of the T, P, and E values from (23) into (37) to correspond to an altitude of 4.5 km, the following relationships are obtained:

$$\begin{aligned}dN_T &= 0.691 \cdot dt \\dN_P &= 0.310 \cdot dP \\dN_e &= -0.044 \cdot d_e\end{aligned}\tag{38}$$

Again, if 1 ppm is set as reference criterion, the following standard errors of each parameter are valid:

$$\begin{aligned}m_T &= \pm 1.4 \\m_P &= \pm 3.2 \text{ mb} \\m_e &= \pm 22.9 \text{ mb}\end{aligned}\tag{39}$$

On comparing values (39) to those of (24), it becomes very evident that the temperature and the pressure in both cases (propagation of radio and light waves) are of the same magnitude of sensitivity. While the humidity in the radio-wave propagation was the major factor in accuracy, in the light-wave propagation it is practically negligible. Even in hot and humid climate the partial pressure of water vapor at sea level rarely exceeds 23 mb, which means that Eq. (36) can be rewritten for the cool and dry climate as follows:

$$N = (n - 1) \cdot 10^6 = 80.35 \frac{P}{T}\tag{40}$$

6. Model of the Refractive Index of Light Waves from Standard Atmosphere

Similarly, as was done in the case of radio-wave propagation, the U. S. Standard Atmosphere was selected as the basis for the analysis. In Table XXVI, N-values obtained from Eq. (36) by the use of the T, P, and e-values from Table IV, are presented together with the altitudes.

The computed N-values were adjusted separately for the four altitude ranges, 0 to 9, 9 to 18, 18 to 27, and 27 to 36 kilometers. Curve-fitting adjustment of Eq. (12) yielded the following constants A, B, and C, given in Table XXVII.

The mean refractive indexes \bar{n} and their standard errors in N-units for each altitude range are given in Table XXVIII. In Table XXIX, residuals after the adjustment are given at intervals of 1 km from 0 to 15 km, and at intervals of 5 km from 15 to 35 kilometers. L denotes the point-to-point value obtained from Eq. (36), P denotes the value read from the adjusted parabola, and ΔP denotes their difference.

The standard errors from Table XXVIII were converted in meters of distance corresponding to the zenith angles of $\psi = 70^\circ, 60^\circ, 50^\circ, 30^\circ,$ and 0° . These, together with the cumulative standard errors from sea level to 36 kilometers are given in Table XXX.

The largest standard error at the poorest zenith angle, $\psi = 70^\circ$, and through the altitude range, 0 to 9 km, is only ± 1 millimeter. This accuracy is five times better than that obtained by the use of radio waves through the same altitude range. Through the altitude range of 9 to 18 km, standard errors in both cases are ± 3 millimeters. This comparison justifies the use of the modified formula (40), even in the lower atmosphere where moisture is present. Tables XXVI - XXX follow.

TABLE XXVI

No.	Geom. Alt., km	N	No.	Geom. Alt., km	N
1	0	282.0	38	18.5	26.0
2	0.5	268.9	39	19.0	24.1
3	1.0	256.1	40	19.5	22.2

TABLE XXVI (continued)

4	1.5	243.9	41	20.0	20.4
5	2.0	231.9	42	20.5	18.9
6	2.5	220.5	43	21.0	17.4
7	3.0	209.4	44	21.5	16.2
8	3.5	199.1	45	22.0	14.7
9	4.0	189.0	46	22.5	13.6
10	4.5	179.0	47	23.0	12.8
11	5.0	169.6	48	23.5	11.7
12	5.5	160.7	49	24.0	10.9
13	6.0	152.2	50	24.5	10.2
14	6.5	144.1	51	25.0	9.1
15	7.0	136.1	52	25.5	8.3
16	7.5	128.5	53	26.0	7.9
17	8.0	121.4	54	26.5	7.2
18	8.5	114.5	55	27.0	6.8
19	9.0	107.7	56	27.5	6.1
20	9.5	101.5	57	28.0	5.7
21	10.0	95.3	58	28.5	5.4
22	10.5	89.5	59	29.0	5.0
23	11.0	84.1	60	29.5	4.6
24	11.5	77.9	61	30.0	4.3
25	12.0	71.9	62	30.5	3.9
26	12.5	66.4	63	31.0	3.6
27	13.0	61.5	64	31.5	3.4
28	13.5	56.7	65	32.0	3.1
29	14.0	52.6	66	32.5	2.9
30	14.5	48.6	67	33.0	2.7
31	15.0	44.9	68	33.5	2.5
32	15.5	41.5	69	34.0	2.3
33	16.0	38.6	70	34.5	2.1
34	16.5	35.6	71	35.0	1.9
35	17.0	32.6	72	35.5	1.8
36	17.5	30.4	73	36.0	1.7
37	18.0	28.2	74	36.5	1.5

TABLE XXVII

Altitude Range, km	A	B	C
0 - 9	0.0002818	-0.000026408	0.0000007964
9 - 18	0.0001085	-0.000013852	0.0000005465
18 - 27	0.0000279	-0.000003998	0.0000001877
27 - 36	0.0000066	-0.000000907	0.0000000400

TABLE XXVIII

Altitude Range, km	\bar{n}	$m\bar{N}$
0 - 9	1.0001845	± 0.04
9 - 18	1.0000610	± 0.11
18 - 27	1.0000150	± 0.05
27 - 36	1.0000036	± 0.02

TABLE XXIX

Altitude, km	L, N- Value	P, N- Value	ΔP , N- Value	Altitude, km	L, N- Value	P, N- Value	ΔP , N- Value
0	282.0	281.8	+0.2	10	95.3	95.1	+0.2
1	256.1	256.2	-0.1	11	84.1	83.0	+1.1
2	231.9	232.2	-0.3	12	71.9	71.8	+0.1
3	209.4	209.8	-0.4	13	61.5	61.9	-0.4
4	189.0	188.9	+0.1	14	52.6	52.9	-0.3
5	169.6	169.6	± 0	15	44.9	45.1	-0.2

TABLE XXIX (continued)

6	152.2	152.0	+0.2	20	20.4	20.7	-0.3
7	136.1	135.9	+0.2	25	9.1	9.1	±0
8	121.4	121.4	±0	30	4.3	4.3	±0
9	107.7	108.5	-0.8	35	1.9	1.9	±0

TABLE XXX

Altitude Range, km	Zenith Angle				
	70°	60°	50°	30°	0°
0 - 9	±0.001	±0.001	±0.001	±0.000	±0.000
9 - 18	0.003	0.002	0.002	0.001	0.001
18 - 27	0.001	0.001	0.000	0.000	0.000
27 - 36	0.000	0.000	0.000	0.000	0.000
0 - 36	0.003	0.002	0.002	0.001	0.001

7. Model of the Refractive Index of Light Waves from Actual Meteorological Soundings

Similarly with the case of the use of radio waves, actual meteorological soundings made in Lihue, Hawaii and Fairbanks, Alaska, February 3, 1966 and July 2, 1966, were used in the analysis of the light waves.

Actual soundings with the T, P, and e values from tables X - XIII were applied to Eq. (36) to obtain the N-values and are presented in tables XXXI - XXXIV. Curve-fitting adjustment of the N-values with the parabola (12) yielded the following constants A, B, and C given in tables XXXV - XXXVIII. The mean refractive indexes through the four altitude ranges and their standard errors in N-units are presented in tables XXXIX - XLII. These standard errors were converted to correspond to standard errors in distances, in meters, for the previously used zenith angles

and altitude ranges. Also, the constant shift between the use of the U. S. Standard Atmosphere and the local, actual atmosphere was computed. The results are given in tables XLIII - XLVI. Tables XXXI - XLVI follow.

TABLE XXXI

Lihue, Hawaii, February 3, 1966

No.	Geom. alt., km	N	No.	Geom. alt., km	N
1	0.036	274.5	18	12.335	72.2
2	0.114	272.9	19	13.202	64.9
3	0.552	263.1	20	14.167	57.5
4	1.012	253.1	21	15.273	49.7
5	1.490	240.6	22	16.580	40.6
6	1.994	227.1	23	17.884	32.5
7	2.505	214.2	24	18.664	28.3
8	3.099	201.5	25	19.582	23.4
10	4.348	176.7	27	22.083	15.2
11	5.031	164.9	28	23.915	11.1
12	5.785	153.3	29	25.130	9.1
13	6.575	140.7	30	26.450	7.2
14	7.472	127.7	31	28.442	5.4
15	8.451	113.4	32	31.137	3.5
16	9.564	99.5	33	33.540	2.4
17	10.839	85.8			

TABLE XXXII

Lihue, Hawaii, July 2, 1966

No.	Geom. Alt., km	N	No.	Geom. Alt., km	N
1	0.036	270.4	18	12.447	73.0
2	0.150	270.1	19	13.300	66.2

TABLE XXXII (continued)

3	0.591	259.9	20	14.253	53.4
4	1.062	249.5	21	15.344	49.5
5	1.547	237.9	22	16.689	39.5
6	2.055	225.5	23	18.040	31.0
7	2.594	213.6	24	18.855	26.9
8	3.170	197.9	25	19.805	22.9
9	3.777	185.2	26	20.940	18.9
10	4.446	173.9	27	22.347	14.9
11	5.131	162.4	28	24.202	10.9
12	5.903	150.8	29	25.406	8.9
13	6.717	139.0	30	26.900	7.0
14	7.612	126.2	31	28.850	5.2
15	8.597	113.8	32	31.642	3.4
16	9.699	101.0	33	34.165	2.4
17	10.960	86.9	34	36.565	1.7

TABLE XXXIII

Fairbanks, Alaska, February 3, 1966

No.	Geom. Alt., km	N	No.	Geom. Alt., km	N
1	0.146	316.1	18	12.240	62.5
2	0.539	286.2	19	13.261	52.9
3	0.960	269.8	20	14.480	44.3
4	1.406	257.1	21	15.964	35.5
5	1.877	244.1	22	17.442	28.5
6	2.376	231.8	23	18.324	24.9
7	2.893	220.3	24	19.346	21.4
8	3.446	208.2	25	20.556	17.8
9	4.026	194.7	26	22.030	14.2
10	4.658	180.7	27	23.955	10.6

TABLE XXXIII

11	5.331	167.1	28	25.173	8.5
12	6.056	153.7	29	26.672	6.8
13	6.871	139.7	30	28.612	5.1
14	7.759	125.2	31	31.362	3.3
15	8.758	110.4			
16	9.917	92.3			
17	11.365	72.0			

TABLE XXXIV

Fairbanks, Alaska, July 2, 1966

No.	Geom. Alt., km	N	No.	Geom. Alt., km	N
1	0.499	257.2	17	12.837	63.1
2	0.961	247.8	18	13.855	53.3
3	1.445	238.3	19	15.070	44.1
4	1.947	228.4	20	16.560	35.3
5	2.464	218.2	21	18.055	28.2
6	3.027	206.9	22	18.950	24.6
7	3.615	194.1	23	19.986	21.1
8	4.242	182.5	24	21.203	17.7
9	4.900	171.1	25	22.697	14.0
10	5.626	158.3	26	24.650	10.4
11	6.390	143.7	27	25.895	8.6
12	7.263	130.5	28	27.411	6.9
13	8.221	116.6	29	29.400	5.1
14	9.295	103.0	30	32.215	3.4
15	10.520	89.3	31	34.720	2.3
16	11.973	73.4			

TABLE XXXV

Lihue, February 3, 1966

Alt. Range, km	A	B	C
0 - 9	0.0002767	-0.00002666	0.000000883
9 - 18	0.0001049	-0.00001070	0.000000288
18 - 27	0.0000300	-0.00000456	0.000000230
27 - 36	0.0000067	-0.00000095	0.000000046

TABLE XXXVI

Lihue, July 2, 1966

Alt. Range, km	A	B	C
0 - 9	0.0002744	-0.00002632	0.000000893
9 - 18	0.0001080	-0.00001096	0.000000270
18 - 27	0.0000304	-0.00000444	0.000000206
27 - 36	0.0000066	-0.00000085	0.000000035

TABLE XXXVII

Fairbanks, February 3, 1966

Alt. Range, km	A	B	C
0 - 9	0.0003071	-0.00003378	0.000001335
9 - 18	0.0001050	-0.00001540	0.000000760
18 - 27	0.0000261	-0.00000370	0.000000185
27 - 36	0.0000067	-0.00000101	0.000000063

TABLE XXXVIII
Fairbanks, July 2, 1966

Alt. Range, km	A	B	C
0 - 9	0.0002689	-0.00002199	0.000000408
9 - 18	0.0001079	-0.00001365	0.000000532
18 - 27	0.0000279	-0.00000372	0.000000162
27 - 36	0.0000073	-0.00000098	0.000000044

TABLE XXXIX
Lihue, February 3, 1966

Alt. Range, km	\bar{n}	$m\bar{N}$
0 - 9	1.0001806	± 0.37
9 - 18	1.0000645	± 0.15
18 - 27	1.0000157	± 0.03
27 - 36	1.0000037	± 0.01

TABLE XL
Lihue, July 2, 1966

Alt. Range, km	\bar{n}	$m\bar{N}$
0 - 9	1.0001801	± 0.49
9 - 18	1.0000660	± 0.20
18 - 27	1.0000160	± 0.08
27 - 36	1.0000037	± 0.04

TABLE XLI
Fairbanks, February 3, 1966

Alt. Range, km	\bar{n}	$m_{\bar{N}}$
0 - 9	1.0001911	±1.52
9 - 18	1.0000562	±0.39
18 - 27	1.0000144	±0.03
27 - 36	1.0000038	±0.14

TABLE XLII
Fairbanks, July 2, 1966

Alt. Range, km	\bar{n}	$m_{\bar{N}}$
0 - 9	1.0001810	±0.25
9 - 18	1.0000608	±0.35
18 - 27	1.0000155	±0.03
27 - 36	1.0000041	±0.05

8. Conclusion and Recommendations

Due to the limited number of samples analyzed, no final conclusion can be drawn. However, a very clear correlation exists between standard errors obtained in a hot and humid climate (Hawaii) and in a cold and dry climate (Alaska) when radio-wave propagation is utilized. The total standard errors from sea level to 36 kilometers were ±6 cm and ±6 cm, based on soundings made in Hawaii during the months of July and February, and ±1 cm and ±3 cm, based on soundings made in Alaska during the months of February and July.

TABLE XLIII

Lihue, July 2 35 Soundings
 Total Altitude Range 0.036 to 36.565 km

Zenith Angle	0 to 9 km		9 to 18 km		18 to 27 km		27 to 36 km		0 to 36 km	
	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m
0°	±0.004	-0.040	±0.002	+0.045	±0.001	+0.009	±0.000	+0.001	±0.005	+0.015
30°	0.005	-0.046	0.002	+0.052	0.001	+0.010	0.000	+0.001	0.005	+0.017
50°	0.007	-0.062	0.003	+0.070	0.001	+0.014	0.001	+0.001	0.008	+0.023
60°	0.009	-0.079	0.004	+0.090	0.001	+0.018	0.001	+0.002	0.010	+0.031
70°	0.013	-0.116	0.005	+0.132	0.002	+0.026	0.001	+0.003	0.014	+0.045

TABLE XLIV

Lihue, February 3 34 Soundings
 Total Altitude Range 0.036 to 33.540 km

Zenith Angle	0 to 9 km		9 to 18 km		18 to 27 km		27 to 36 km		0 to 36 km	
	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m
0°	±0.003	-0.035	±0.001	+0.031	±0.000	+0.006	±0.000	+0.001	±0.003	+0.003
30°	0.004	-0.041	0.002	+0.036	0.000	+0.007	0.000	+0.001	0.004	+0.003
50°	0.005	-0.055	0.002	+0.049	0.000	+0.010	0.000	+0.001	0.005	+0.005
60°	0.007	-0.070	0.003	+0.063	0.001	+0.013	0.000	+0.002	0.008	+0.008
70°	0.010	-0.103	0.004	+0.092	0.001	+0.018	0.000	+0.003	0.011	+0.010

TABLE XLV

Fairbanks, July 2 Total Altitude Range 0.499 to 34.720 km 31 Soundings

	0 to 9 km		9 to 18 km		18 to 27 km		27 to 36 km		0 to 36 km	
Zenith Angle	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m
0°	±0.002	-0.032	±0.003	-0.002	±0.000	+0.005	±0.000	+0.005	±0.004	-0.024
30°	0.003	-0.036	0.004	-0.002	0.000	+0.005	0.001	+0.005	0.005	-0.028
50°	0.004	-0.049	0.005	-0.003	0.000	+0.007	0.001	+0.007	0.007	-0.038
60°	0.005	-0.063	0.006	-0.004	0.001	+0.009	0.001	+0.009	0.008	-0.049
70°	0.007	-0.092	0.009	-0.005	0.001	+0.013	0.001	+0.013	0.012	-0.071

TABLE XLVI

Fairbanks, February 3 Total Altitude Range 0.146 to 31.362 km 31 Soundings

	0 to 9 km		9 to 18 km		18 to 27 km		27 to 36 km		0 to 36 km	
Zenith Angle	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m	St. Error, m	Differ., m
0°	±0.014	+0.059	±0.004	-0.043	±0.000	-0.006	±0.001	+0.002	±0.015	+0.012
30°	0.015	+0.069	0.004	-0.050	0.000	-0.006	0.002	+0.002	0.016	+0.015
50°	0.021	+0.092	0.005	-0.067	0.000	-0.008	0.002	+0.003	0.022	+0.020
60°	0.027	+0.119	0.007	-0.086	0.001	-0.011	0.003	+0.004	0.028	+0.026
70	0.040	+0.173	0.010	-0.124	0.001	-0.016	0.004	+0.005	0.042	+0.038

The largest difference between distances based on standard-atmosphere data and local-atmosphere data (Fairbanks, February 3, 1966) was -32 cm, which shows that to achieve the ultimate accuracy in utilizing the radio-wave propagation, soundings should be used that have been made in the vicinity or proximity of the tracking station.

In the case of laser and, consequently, light-wave propagation, the influence of humidity is practically negligible. Therefore, standard errors obtained in measurements based on either humid- or dry-climate soundings are of the same order. Due to the absence of the humidity effect, the accuracy of laser tracking is significantly higher than tracking by radio waves. At the zenith angle $\psi = 70^\circ$, and through the altitude range of 0 to 9 km, the standard error of the mean refractive index based on the U. S. Standard Atmosphere is only ± 1 mm, compared to ± 5 mm when the radio waves are used. The largest standard error for the total altitude range from 0 to 36 km (Fairbanks, February 3, 1966), was ± 4.2 cm, while the difference between distances based on the standard and local atmospheres was +3.8 cm. This suggests that properly selected and computed standard atmosphere values guarantee sufficient accuracy to most geodetic and geophysical applications when light waves are utilized.

In the continuation of this research, emphasis should be put to investigate the influence of the location of the tracking station to the surrounding sounding stations by a proper interpolation of the data area-wise. Also the interpolation time-wise between the 00.00 and 12.00 Greenwich-time soundings should be made, together with a proper error analysis. The results of this analysis should then be applied to radio-wave tracking.

In the case of the light-wave tracking, an empirical standard atmosphere should be built up based on a great number of sounding values covering large areas of national and international tracking sites both for daytime and nighttime. The parabolic constants through four different altitude ranges of this adjusted standard atmosphere should be used and the standard errors analyzed.

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