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APRIL 1969

Technical Memorandum

ATTITUDE STABILIZATION OF SPACECRAFT WITH GEOMAGNETIC RATE DAMPING

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This work was supported by the
National Aeronautics and Space
Administration, Office of Space
Science and Applications under
Task I of Contract NOw 62-0604-c.

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Operating under Contract NOw 62-0604-c with the Department of the Navy

ABSTRACT

This paper studies two damping systems which can be used as part of the attitude control systems of geomagnetically, gravitationally, or spin-stabilized spacecraft. The devices are active and passive geomagnetic rate dampers that use the relative motion between the spacecraft and an ambient magnetic field to produce the required torques. The expressions for the torques generated by each device are derived. To investigate the usefulness of the dampers, an analytical study is made of the attitude performance of gravitationally stabilized spacecraft. To verify the assumptions that were made to make the dynamical problem analytically tractable, a digital simulation study is presented. Good agreement is obtained. The characteristics and applicability of each type of damper are also discussed.

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NOMENCLATURE

$\vec{a}$ = spacecraft position vector from center of mass of earth, meters

a_j = coefficients used to express magnetic flux density of the core of an eddy-current rod in terms of the magnetic flux intensity

A = area, $meter^2$

A_m = cross-sectional area of electromagnet, $meter^2$

B_e = magnetic flux density of ambient field, $webers/meter^2$

B_o = flux density of a dipole geomagnetic field in the equatorial plane at a radial distance from the center of the earth of magnitude a, $webers/meter^2$

B_r = magnetic flux density in the permeable core of an eddy-current rod, $webers/meter^2$

e = electromotive force, volts

E = energy, newton-meter

f = active geomagnetic rate damper sensor voltage output per unit time rate of change of magnetic field intensity, $\frac{\text{volt-meter-secs}}{\text{amp-turn}}$

g = amplifier voltage gain of active geomagnetic rate damper

H_e = magnetic intensity of ambient field, amp-turns/meter

H_{er} = component of the magnetic intensity of the ambient field, amp-turns/meter

H_r = magnetic field intensity in the permeable core of an eddy-current rod, amp-turns/meter

I = current, amperes

I_m = current in the active geomagnetic rate damper electromagnet, amperes

I_r, I_p, I_y = mass moments of inertia of the spacecraft about the $\hat{i}$, $\hat{j}$, $\hat{k}$ axes respectively, kilograms-meter^2

k_e = GM where G is universal gravitational constant and M is mass of earth, meters^3/sec^2

ℓ = length of eddy-current rod, meters

$\vec{L}$ = torque, newton-meters

m = mass of passive geomagnetic rate damper exclusive of fittings, kgms

M_e = magnetic moment of the dipole representation of the geomagnetic field, earth, weber-meters

M_r = magnetization of an eddy-current rod, amp-turns/meter

N_m = number of turns of electromagnet winding

$O_s(\hat{i},\hat{j},\hat{k})$ = cartesian frame of reference fixed in the spacecraft

$O_v(\epsilon_\theta, \epsilon_\varphi, \epsilon_r)$ = local vertical cartesian frame of reference

q = number of eddy-current rods along each axis of an orthogonal set of rods

r_c = maximum radius of the permeable core of an eddy-current rod, meters

$\bar{r}_c$ = $\frac{r_c}{\ell}$

r_s = maximum radius of eddy-current rod, meters

$\bar{r}_s$ = $\frac{r_s}{\ell}$

R = resistance, ohms

R_m = resistance of electromagnet of the active damper, ohms

R_o = $.6378166 \times 10^7$ meters, nondimensionalizing constant approximately equal to the mean equatorial radius of the earth

t = time, sec

T = time constant, orbital revolutions

v_c = volume of the core of an eddy-current rod, meters^3

V_s = voltage output of the active damper sensor, volts

α = in-orbit plane angular displacement of spacecraft from local vertical, rads.

α_a = amplitude of oscillatory in-orbit plane angular displacement, degs.

α_s = in-orbit plane bias angle, rads.

β = angle measured from the uniform ambient magnetic flux density vector to the axis of an eddy-current rod, rads.

δ_c = mass density of the core material, kgms/meters^3

δ_s = mass density of the conducting shell, kgms/meters^3

$\hat{\epsilon}$ = unit vector parallel to axis of either an eddy-current rod or a single axis active geomagnetic rate damper

$\hat{\epsilon}_i$ = unit vector parallel to i^{th} axis of a orthogonal triad, $i = 1$ to 3

η = $\dfrac{K B_o^2}{I_p \dot{\theta}}$, nondimensional damper characteristic parameter

θ = angle measured from the geographic pole to spacecraft position vector, rads.

μ = permeability of the core of an eddy-current rod, webers/amp-meter

μ_e = permeability of free space, $4\pi \times 10^{-7}$ webers/amp-meter

ρ = resistivity, ohm-meters

$\vec{\omega}$ = angular velocity of eddy-current rod with respect to uniform ambient magnetic field, rads/sec.

$\hat{}$ = denotes unit vector

$(\dot{\ })$ = denotes time derivative

$(\dot{\ })_s$ = denotes time derivative with respect to the 0_s reference frame

$(\vec{\ })$ = denotes vector quantity

$\langle\ \rangle$ = denotes average value

Introduction

The repeated success of geomagnetically and gravitationally stabilized spacecraft has promoted increased interest in the use of these techniques for future satellite missions. In a geomagnetic stabilization system, a magnetic moment generated within the spacecraft interacts with the geomagnetic field to establish the preferred orientation. It is the interaction of the inertia ellipsoid of the spacecraft with the second derivative of the gravitational potential which establishes the equilibrium orientation of a gravitational stabilization system. Both of these techniques require a damping system to remove excess librational energy that is introduced by disturbing torques and a nonpreferred initial orientation. In this paper two dampers, one active and one passive, are discussed which can be used with geomagnetically or gravitationally stabilized spacecraft. Both of these dampers can also be used to remove undesirable spin momentum induced in a spacecraft by a spin stabilized launch vehicle. In addition, the active damper can also be used in a spin stabilized spacecraft to increase or decrease the spin rate or used in a closed-loop mode to maintain a specified rotation rate with respect to the geomagnetic field.

The two torque producing devices that are studied here are entitled the Passive Geomagnetic Rate Damper and the Active Geomagnetic Rate Damper even though the active system is not restricted to energy dissipation. The passive damper consists of eddy-current rods which have a highly permeable core encased in a shell of conducting material as illustrated in Fig. 1. Angular motion of the device with respect to an ambient magnetic field is

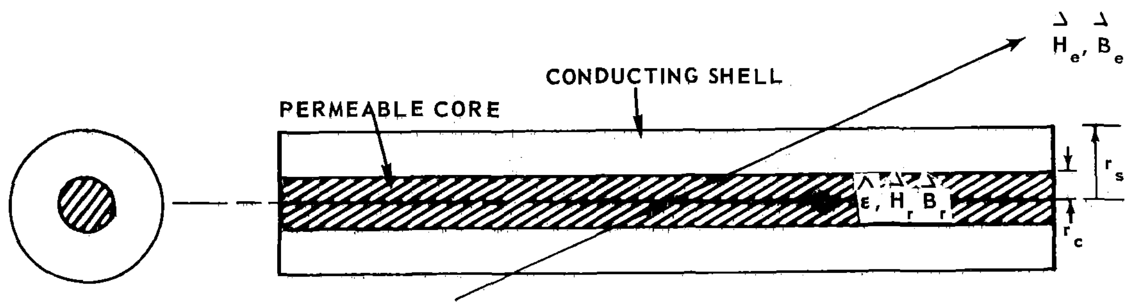

Fig. 1 SINGLE-AXIS PASSIVE GEOMAGNETIC RATE DAMPER

resisted by a dissipation of energy through eddy-current loss in the conducting shell. The active damper illustrated in Fig. 2 consists of a sensor which measures the rate of change of the component of the ambient magnetic field, sensor electronics, and an amplifier that drives an electromagnet whose axis is parallel to the sensor axis. The magnetic moment of the electromagnet is maintained proportional to the time rate of change of the measured field. Angular motion of the device with respect to the ambient magnetic field is resisted by the torque generated by the interaction of the magnetic moment of the electromagnet with the ambient magnetic field.

The use of eddy-current rods to despin orbiting earth satellites has been studied and proposed for use with geomagnetically and gravitationally stabilized spacecraft in [1]. A study of their applicability for the GEOS-II spacecraft is presented in [2] in which an averaging technique is used that gives neither an explicit expression for the torque nor a measure of the induced perturbations. The active damper can be used as an alternative to the passive damper, and an active geomagnetic rate damper was proposed by F. F. Mobley in [3]. However, the system discussed here differs from that proposal in that a phase lag need not be explicitly introduced by the instrumentation. This should simplify the implementation of such a system. The active damper discussed here was conceived by the author as a result of an attempt to improve the damping characteristics of the passive geomagnetic rate damper. The technique was formulated independently by F. F. Mobley and B. E. Tossman of the Laboratory for use as a nutation damper on a spin stabilized spacecraft. This author is unaware of any discussion of the active geomagnetic rate damper in the literature but the conceptual simplicity of the device implies that it may have been proposed elsewhere.

The analysis is initiated by the derivation of the torque exerted on

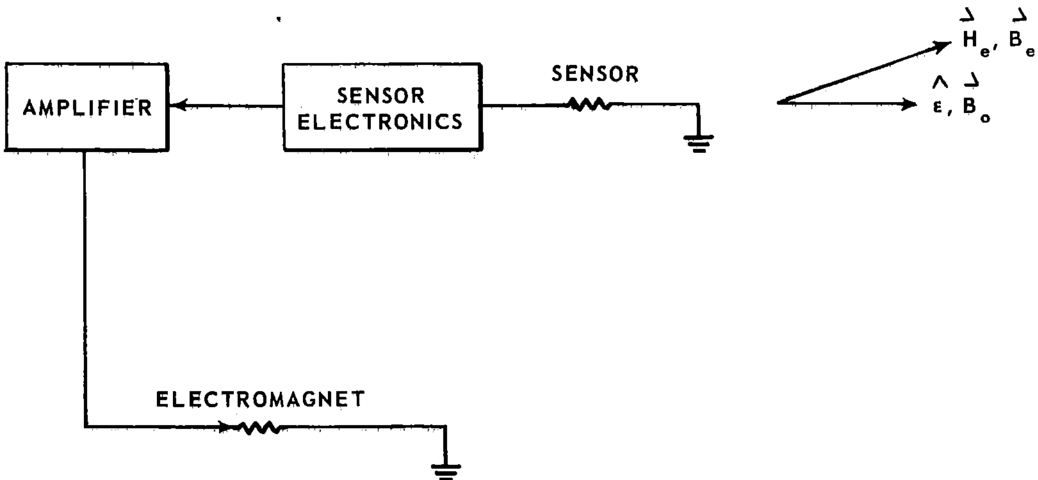

Fig. 2 SINGLE-AXIS ACTIVE GEOMAGNETIC RATE DAMPER

each type of single axis damper by an ambient magnetic field. For a magnetically linear core material it is shown that the torque expressions for a set of three mutually orthogonal dampers of either type are mathematically equivalent. The attitude performance of a gravitationally stabilized spacecraft is then investigated. Analytical expressions are developed for the transient and steady-state performance of a satellite in a circular and geographically and geomagnetically polar orbit. These results are then verified by a digital simulation of the nonlinear differential equations which do not include the approximations that were required to make the equations analytically tractable. A discussion of the relative merits of the passive and active dampers concludes the paper.

Damper Torque Expressions

Passive geomagnetic rate damper

If an eddy-current rod of the type illustrated in Fig. 1 is inserted into a magnetic field of intensity $\overline{H}_e$, a change in the component of the field along the rod will introduce an electromotive force in the conducting shell. The electromotive force is given by Faraday's law to be

$$e = -\frac{d}{dt} \int_{A_{CORE}} B_r \, d\,A \tag{1}$$

For a magnetic flux density that is uniform over the core

$$e = -\dot{B}_r A_{CORE} = -\pi r_c^2 \dot{B}_r \tag{2}$$

The current in an elemental ring of the conducting shell is given by Ohm's law to be

$$dI = \frac{e}{R} \tag{3}$$

if the resistance is assumed to be much greater than the inductance. The resistance, which is equal to the resistivity times the length of the conducting path divided by the cross-sectional area of the conducting path, is given by

$$R = \rho \frac{2\pi r}{\ell \, dr} \tag{4}$$

where

$$r_c \leq r \leq r_s$$

Then the current in the conducting shell is given by substituting Eqs. (2) and (4) into Eq. (3) so that

$$I = \int_{r_c}^{r_s} dI = - \frac{\ell \, r_c^{\,2} \, \dot{B}_r}{2\rho} \log_e \frac{r_s}{r_c} \tag{5}$$

or

$$I = - k \, \ell \, \dot{B}_r \tag{6a}$$

if

$$k \equiv \frac{r_c^{\,2}}{2\rho} \log_e \frac{r_s}{r_c} \tag{6b}$$

If the conducting shell constitutes a standard solenoid (one in which the length to diameter ratio is very large) the magnetic field intensity is uniform over the cross-section and is given by I/ℓ which, by Eq. (6a), is equal to $(- k \, \dot{B}_r)$. With this approximation the sum of the ambient and induced magnetic field intensities in the rod is given by

$$H_r = H_{er} - k \, \dot{B}_r \tag{7a}$$

where

$$H_{er} \equiv \vec{H}_e \cdot \hat{\epsilon} \tag{7b}$$

If the magnetic flux density of the rod is related to the magnetic flux intensity of the rod by the power series expansion

$$B_r \equiv \sum_{j=0}^{n} a_j H_r^{\,j} \tag{8}$$

then Eq. (7a) can be written as

$$H_r = H_{er} - k \dot{H}_r \sum_{j=0}^{n} j a_j H_r^{\,j-1} \tag{9}$$

Eq. (9) is a first-order nonlinear differential equation for H_r. In the applications of interest here

$$H_{er} \gg k \dot{H}_r \sum_{j=0}^{n} j a_j H_r^{\,j-1} \tag{10}$$

so that an approximate solution to Eq. (9) is

$$H_r = H_{er} - k \dot{H}_{er} \sum_{j=0}^{n} j a_j H_{er}^{\,j-1} + \ldots \tag{11}$$

The torque exerted on the eddy-current rod is given by

$$\vec{L} = v_c \vec{M}_r \times \vec{B}_e \tag{12}$$

Since the core is ferromagnetic,

$$M_r = \frac{B_r}{\mu_e} - H_{er} \approx \frac{B_r}{\mu_e} \tag{13}$$

so that

$$\vec{L} = \frac{v_c}{\mu_e} B_r \hat{\epsilon} \times \vec{B}_e \tag{14a}$$

where

$$B_r = \sum_{i=0}^{n} a_i [(\vec{H}_e \cdot \hat{\epsilon}) - k \frac{d}{dt} (\vec{H}_e \cdot \hat{\epsilon}) \sum_{j=0}^{n} ja_j (\vec{H}_e \cdot \hat{\epsilon})^{j-1}]^i \tag{14b}$$

Eqs. (14) give the desired result, the torque exerted on an eddy-current rod because of relative motion with respect to an ambient magnetic field.

In the case of a magnetically linear core material, that is, one in which

$$a_0 = 0$$

$$a_1 = \mu$$

$$a_2 = a_3 = \ldots = 0$$

Eqs. (14) reduce to

$$\vec{L} = K_1 (\vec{H}_e \cdot \hat{\epsilon}) \hat{\epsilon} \times \vec{B}_e - K_2 \frac{d}{dt} (\vec{H}_e \cdot \hat{\epsilon}) \hat{\epsilon} \times \vec{B}_e \tag{16a}$$

where

$$K_1 \equiv v_c \frac{\mu}{\mu_e} \tag{16b}$$

$$K_2 \equiv v_c \frac{\mu^2}{\mu_e} \; k \tag{16c}$$

The validity of Eqs. (16) can be established since the average rate of energy dissipated by an eddy-current rod rotating in a uniform magnetic field has been derived in [1]. For rotation of an eddy-current rod about an axis normal to the ambient field as illustrated in Fig. 3,

$$\vec{H}_e \cdot \hat{\epsilon} = H_e \text{ Cos } \beta \tag{17}$$

so that the torque exerted on the rod as given by Eq. (16a) is

$$\vec{L} = (K_1 H_e \text{ Cos } \beta) \; \hat{\epsilon} \times \vec{B}_e + (K_2 H_e \dot{\beta} \text{ Sin } \beta) \; \hat{\epsilon} \times \vec{B}_e \tag{18}$$

The rate of energy dissipation is given by

$$\dot{E} = \vec{L} \cdot \vec{\omega} \tag{19a}$$

which becomes, after substitution of Eq. (18) and $\vec{\omega} = \dot{\beta} \hat{\omega}$,

$$\dot{E} = -K_1 B_e H_e \dot{\beta} \text{ Sin } \beta \text{ Cos } \beta - K_2 B_e H_e \dot{\beta}^2 \text{ Sin}^2 \beta \tag{19b}$$

The average rate of change of energy over one complete revolution, assuming

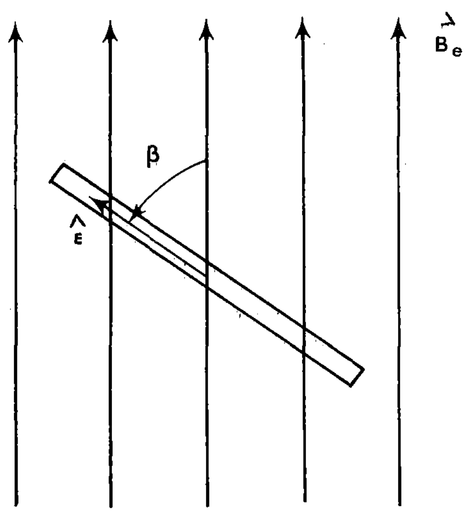

Fig. 3 GEOMETRICAL CONFIGURATION OF SINGLE-AXIS PASSIVE GEOMAGNETIC RATE DAMPER IN UNIFORM MAGNETIC ENVIRONMENT

that the angular velocity is a constant, is given by

$$\langle \dot{E} \rangle = \frac{\dot{\beta}}{2\pi} \int_0^{2\pi/\dot{\beta}} \dot{E} \, d t \tag{20a}$$

$$= - \frac{\pi r_c^4 \, \ell \, \dot{\beta}^2 \, \mu^2 \, H_e^2}{4\rho} \log_e \frac{r_s}{r_c} \tag{20b}$$

For a thin conducting shell, Eq. (20b) can be approximated by

$$\langle \dot{E} \rangle \approx - \frac{\pi r_c^3 \, \ell \, \dot{\beta}^2 \, \mu^2 \, H_e^2 \, (r_s - r_c)}{4\rho} \tag{21}$$

where the negative sign indicates that energy is being dissipated. Eq. (21) agrees with Eq. (38) of [1] if one takes into account a typographical error in the latter equation which makes it too small by a factor of ten.

In an actual application, any number of eddy-current rods can be employed. The total number, their relative directions and their characteristics are best determined by the individual spacecraft configuration and mission objective. The first term of the energy expression as given by Eq. (19b), which follows directly from the first term of the torque expression can justly be characterized adversely as primarily providing disturbing torques while the second term can be characterized as primarily providing damping torques.

When using the passive geomagnetic rate damper with a geomagnetic stabilization system, Eq. (16a) indicates that by placing the eddy-current rods either parallel, normal or symmetrical with respect to the axis which is to be stabilized, it is possible to obtain a disturbance free system. When used with a gravitational stabilization system, a disturbance free system cannot be realized. Except for a geostationary trajectory, when the spacecraft is stabilized there will still be relative motion between it and the geomagnetic field. However, it is possible to minimize the perturbing effect of the damper. If the same number of rods of identical characteristics are placed in the spacecraft along each axis of an orthogonal triad, denoted by $\hat{\epsilon}_i$ (i = 1 to 3), then

$$\sum_{i=1}^{3} (\vec{H}_e \cdot \hat{\epsilon}_i) \hat{\epsilon}_i = \vec{H}_e \tag{22}$$

so that the torque expression given by Eq. (16a) becomes

$$\vec{L} = - q K_2 \frac{d}{dt} (\vec{H}_e)_s \times \vec{B}_e \tag{23}$$

The subscript s denotes that the derivative is taken with respect to the frame of reference fixed in the spacecraft. This particular selection of rod orientation has succeeded in completely eliminating that portion of the eddy-current rod torque which generates a perturbation without damping. Of course, it is not necessary to use rods with the same characteristics as long as the sum of the K_1's of the rods along each axis is identical.

Active geomagnetic rate damper

The sensor output of the active geomagnetic rate damper illustrated in Fig. 2 is a voltage that is proportional to the rate of change of the component of the ambient magnetic field intensity along the axis of the sensor and is given by

$$V_s = -f \frac{d}{dt} (\vec{H}_e \cdot \hat{\epsilon}) \tag{24}$$

The voltage output of the linear amplifier is $g\, V_s$ and the current which passes through the electromagnet is

$$I_m = \frac{g\, V_s}{R_m} \tag{25}$$

The torque exerted on the electromagnet by the ambient field is

$$\vec{L} = N_m I_m A_m \hat{\epsilon} \times \vec{B}_e$$

or

$$\vec{L} = -K_3 \frac{d}{dt} (\vec{H}_e \cdot \hat{\epsilon})_s\, \hat{\epsilon} \times \vec{B}_e \tag{26a}$$

where

$$K_3 \equiv \frac{N_m\, g\, f\, A_m}{R_m} \tag{26b}$$

A comparison of the torque expressions for the two single-axis dampers as given by Eqs. (16) and (26) is informative. The term in the torque expression for the passive damper, whose function was shown to be detrimental, is absent in the torque expression for the active damper. The damping terms of each expression have the same mathematical form.

If three single-axis active geomagnetic rate dampers are oriented orthogonally the torque expression becomes

$$\vec{L} = -K_3 \frac{d}{dt} (\vec{H}_e)_s \times \vec{B}_e \tag{27}$$

It is interesting to note that the torque expressions for mutually orthogonal dampers of either type, as given by Eqs. (23) and (27), are mathematically equivalent. Consequently, it is possible to generalize the torque expression and write

$$\vec{L} = -K \frac{d}{dt} (\vec{B}_e)_s \times \vec{B}_e \tag{28a}$$

where

$$K \equiv \begin{cases} \dfrac{K_3}{\mu_e}, & \text{active geomagnetic rate damper} \\ \\ \dfrac{qK_2}{\mu_e}, & \text{passive geomagnetic rate damper} \end{cases} \tag{28b}$$

This makes it possible to investigate the performance of both damper systems in terms of the single parameter K.

Damper Performance with Gravitational Stabilization

The attitude performance of a gravitationally stabilized spacecraft with an orthogonal set of dampers of either type is studied here. To make the equations of motion analytically tractable, so that preliminary design formulae can be obtained, several restrictions and simplifying assumptions are made. The spacecraft is taken to be on a circular and geographically polar orbit. The geomagnetic field is approximated by a dipole aligned with the geographic poles so that it coincides with the daily mean position of the dipole component of the true field. Consequently, the geomagnetic field at the spacecraft will lie in the orbital plane. For librational motion in the orbital plane the time derivative of the geomagnetic flux density vector will also be in the orbital plane. It follows from Eq. (28) that the damper system will introduce a torque about an axis normal to the orbital plane. This means that the in-orbit plane libration of gravitationally stabilized spacecraft which is uncoupled from the out-of-orbit plane motion in the presence of gravitational forces remains uncoupled in the presence of the damper torques.

The large spacecraft inertia required for gravitational stabilization is usually obtained after the spacecraft is in orbit by the extension of tubular stabilizing booms. Extension of the booms usually occurs when the period of the attitude motion is of the order of the orbital period. The large change in inertia which accompanies boom extension will reduce the angular velocity of the spacecraft in inertial space to near zero except possibly for the component about the longitudinal axis in a dumbbell configuration.

Since a gravitationally stabilized spacecraft in its equilibrium position has an angular velocity normal to the orbital plane of one revolution per orbital revolution, in-orbit plane motion about the equilibrium orientation will result. Whether there is any out-of-orbit plane motion depends on the particular orientation of the spacecraft at the time of boom extension. This can be made small by proper selection of the instant at which the booms are extended or by a technique employing geomagnetic stabilization prior to boom extension. For these reasons the analytic study is restricted to the in-orbit plane librational motion of the spacecraft in order to characterize the transient and steady-state performance.

The components of the geomagnetic field in the 0_v reference system, as illustrated in Figs. 4 and 5 are given by

$$\vec{B}_e = -B_o \left[\mathrm{Sin}\theta \; \hat{\epsilon}_\theta + 2 \; \mathrm{Cos}\theta \; \hat{\epsilon}_r\right] \tag{29a}$$

where

$$B_o \equiv \frac{M_e}{a^3} \tag{29b}$$

For small in-orbit plane libration amplitudes, the geomagnetic field in the spacecraft frame of reference 0_s is given by

$$(\vec{B}_e)_s \approx \begin{pmatrix} 1 & 0 & -\alpha \\ 0 & 1 & 0 \\ \alpha & 0 & 1 \end{pmatrix} (\vec{B}_e)_v$$

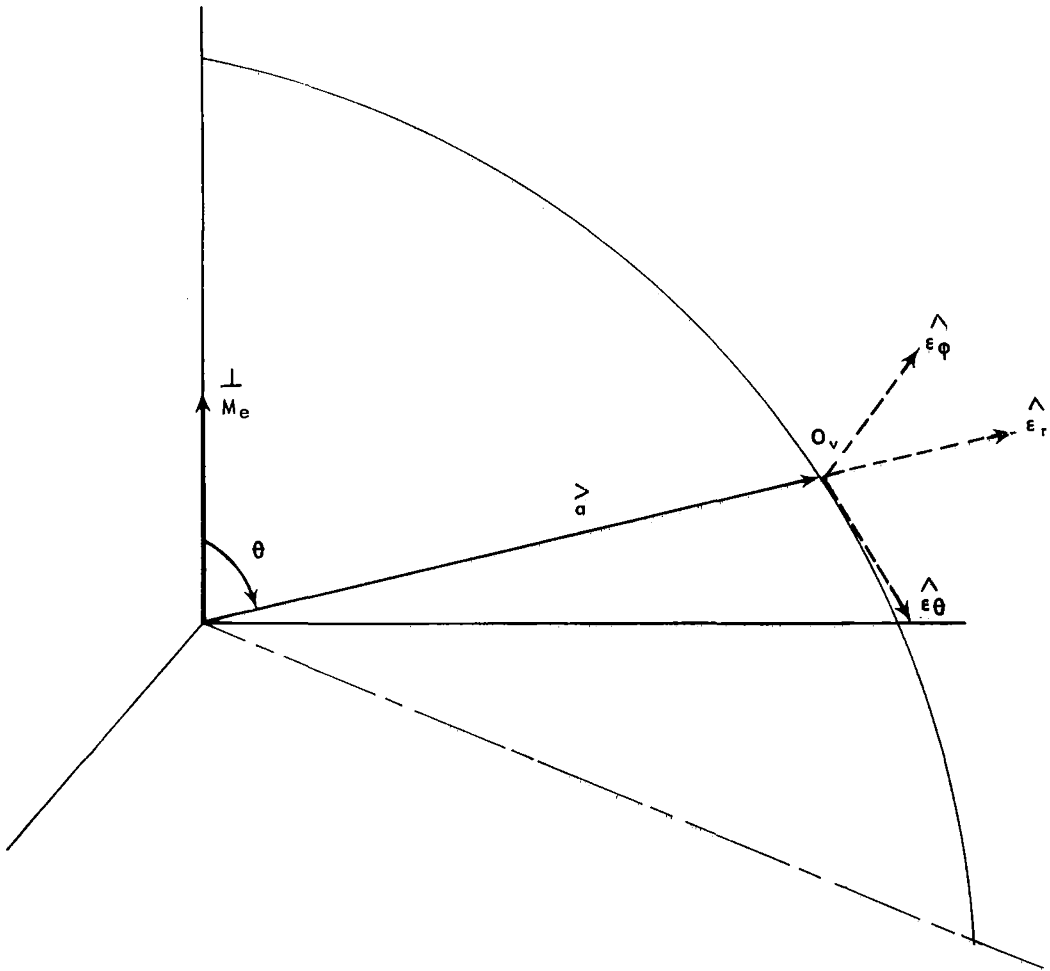

Fig. 4 ORBIT CONFIGURATION AND LOCAL VERTICAL REFERENCE SYSTEM

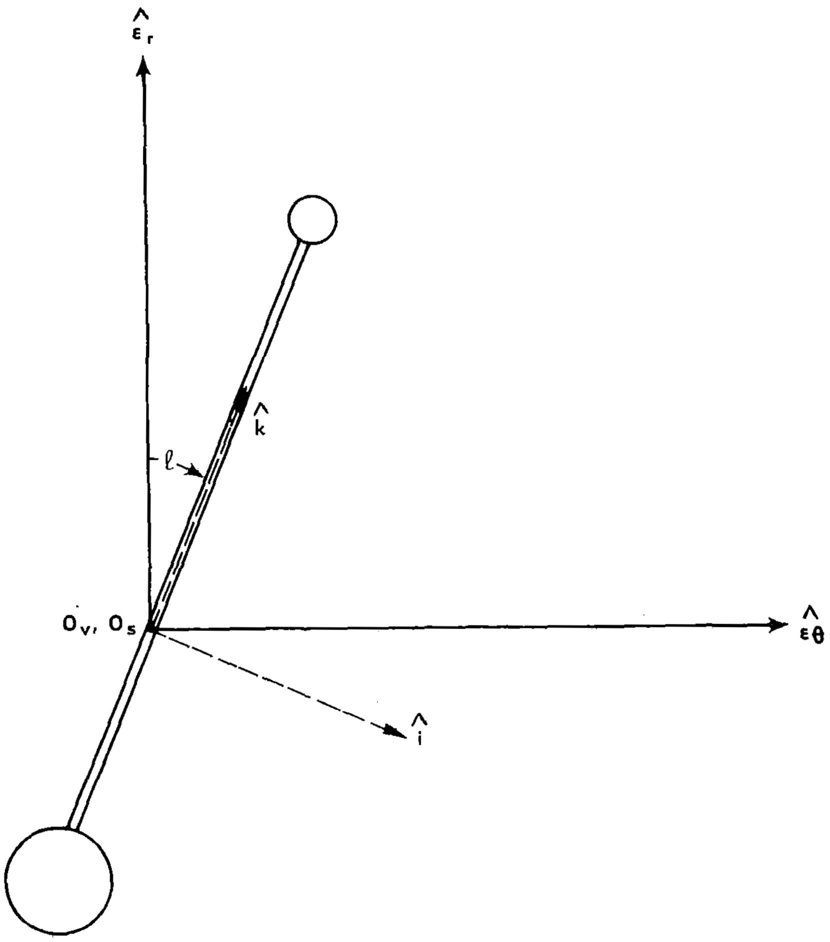

Fig. 5 SPACECRAFT REFERENCE FRAMES

so that

$$\vec{B}_e = -B_o \left[(\mathrm{Sin}\theta - 2\alpha\,\mathrm{Cos}\theta)\,\hat{i} + (\alpha\,\mathrm{Sin}\theta + 2\,\mathrm{Cos}\theta)\,\hat{k} \right] \tag{30}$$

The derivative of $\vec{B}_e$ in O_s is

$$(\dot{\vec{B}}_e)_s = -B_o\left[(\dot{\theta}\,\mathrm{Cos}\theta - 2\dot{\alpha}\,\mathrm{Cos}\theta + 2\alpha\dot{\theta}\,\mathrm{Sin}\theta)\,\hat{i} \right.$$

$$\left. + (\dot{\alpha}\,\mathrm{Sin}\theta + \alpha\dot{\theta}\,\mathrm{Cos}\theta - 2\dot{\theta}\,\mathrm{Sin}\theta)\,\hat{k} \right] \tag{31}$$

The damper torque, as given by Eqs. (28), (30) and (31), is

$$\vec{L} = K\,B_o^{\,2}\left[2\dot{\theta} - \dot{\alpha}\,(1 + 3\,\mathrm{Cos}^2\theta)\right]\hat{j} \tag{32}$$

The differential equation for the in-orbit spacecraft motion is given in [4] to be

$$I_p(\ddot{\theta} + \ddot{\alpha}) - \frac{3k_e}{2a^3}\,(I_y - I_r)\,\mathrm{Sin}\,2\alpha = L \tag{33}$$

so that for a small amplitude libration

$$I_p\,\ddot{\alpha} + K\,B_o^{\,2}\,(1 + 3\,\mathrm{Cos}^2\theta)\,\dot{\alpha} + 3\,\dot{\theta}^2\,(I_r - I_y)\,\alpha = 2K\,B_o^{\,2}\,\dot{\theta} \tag{35}$$

By defining

$$\sigma \equiv \frac{I_r - I_y}{I_p} \qquad (36a)$$

$$(\ )' \equiv \frac{d}{d\theta}(\ ) = \frac{1}{\dot{\theta}} \frac{d}{dt}(\ ) \qquad (36b)$$

$$\eta \equiv \frac{K B_o^2}{I_p \dot{\theta}}\ , \qquad (36c)$$

$$P(\theta) \equiv \frac{\eta}{2}(1 + 3 \cos^2\theta) \qquad (36d)$$

$$R \equiv (3\sigma)^{1/2} \qquad (36e)$$

$$Q \equiv 2\eta \qquad (36f)$$

Eq. (35) can be rewritten as

$$\alpha'' + 2P(\theta)\ \alpha' + R^2\alpha = Q \qquad (37)$$

Eq. (37) is a second-order linear nonhomogeneous differential equation with a periodic coefficient. The parameter denoted by η is a nondimensional representation of the damper characteristics. Eq. (37) can be reduced to a homogeneous form by defining

$$\alpha \equiv Z + \frac{Q}{R^2} \qquad (38a)$$

$$= Z + \frac{2\eta}{3\sigma} \qquad (38b)$$

so that

$$Z'' + 2P(\theta)\, Z' + R^2 Z = 0 \tag{39}$$

The removal of the first derivative term may be accomplished by changing the dependent variable from Z to X through the definition

$$Z \equiv X\, e^{-\int_0^{\theta} P(\tau)\, d\tau} \tag{40}$$

If this change of variable is introduced in Eq. (39), then

$$X'' + S(\theta)\, X = 0 \tag{41}$$

where

$$S(\theta) \equiv R^2 - P^2(\theta) - P'(\theta) \tag{42a}$$

$$= 3\sigma + \frac{3}{2}\,\eta \,\mathrm{Sin}\, 2\theta - \frac{\eta^2}{32}\,(59 + 60\,\mathrm{Cos}\, 2\theta + 9\,\mathrm{Cos}\, 4\theta) \tag{42b}$$

Eq. (41), in which $S(\theta)$ is expressible as a Fourier series as in the case of interest, is Hill's equation which has received considerable attention.

No simple method exists for obtaining a general solution to Hill's equation. However, a simple criterion for boundness due to Liapunoff does exist and is given in [5]:

If $S(\theta)$ is real and continuous with period π and

$$S(\theta) \neq 0 \tag{43a}$$

$$\int_0^\pi S(\theta)\, d\theta \geq 0 \tag{43b}$$

$$\int_0^\pi |S(\theta)|\, d\theta \leq \frac{4}{\pi} \tag{43c}$$

then all solutions of Eq. (41) are bounded for $-\infty < \theta < \infty$.

For the $S(\theta)$ given by Eq. (42b), the stability criteria are that

$$\sigma \geq \frac{59}{96}\eta^2 \tag{44a}$$

$$\int_0^\pi \left|3\sigma + \frac{3}{2}\eta \text{ Sin } 2\theta - \frac{\eta^2}{32}(59 + 60 \text{ Cos } 2\theta + 9 \text{ Cos } 4\theta)\right| d\theta \leq \frac{4}{\pi} \tag{44b}$$

The integration can be easily performed only for specific values of σ and η .

A simplified stability criterion can be obtained when

$$-1 \ll \eta \ll 1 \tag{45}$$

In this case, the periodic coefficient given by Eq. (42b) can be approximated by

$$S(\theta) \approx 3\sigma + \frac{3}{2}\eta \text{ Sin } 2\theta \tag{46}$$

The stability criteria given by Eqs. (43) now give

$$\sigma > 0 \tag{47a}$$

$$\int_0^{\pi} \left|3\sigma + \frac{3}{2}\,\eta \text{ Sin } 2\theta\right| \, d\theta \leq \frac{4}{\pi} \tag{47b}$$

If the independent variable is changed by the transformation

$$\gamma = \theta + \frac{\pi}{4}$$

the differential equation of motion given by Eq. (41) can be rewritten using Eq. (46) as

$$\frac{d^2 X}{d\gamma^2} + (u - 2v \text{ Cos } 2\gamma)\, X = 0 \tag{48a}$$

where

$$u \equiv 3\sigma \tag{48b}$$

$$v \equiv \frac{3}{4}\,\eta \tag{48c}$$

Eq. (48a) has the form of the well-studied Mathieu equation which arises in several different types of physical problems. Exact locations of boundaries between stable and unstable solution as a function of u and v are readily available, e.g. [5]. For

$$\sigma > 0 \tag{49a}$$

$$-2\sigma < \eta < 2\sigma \tag{49b}$$

the solutions to the Mathieu equation are essentially stable with only narrow regions of instabilities.

An approximate solution for the libration amplitude of the spacecraft is possible when the periodic coefficient $S(\theta)$ as given by Eq. (42) has relatively small variations about a large mean value. This solution of Eq. (41) is known as the WKBJ approximation and is given by

$$X = [G(\theta)]^{-\frac{1}{2}} [A \text{ Cos } \varphi (\theta) + B \text{ Sin } \varphi (\theta)] \tag{50a}$$

where

$$\varphi(\theta) \equiv \int_0^\theta G(\theta) \, d\theta \tag{50b}$$

$$G^2(\theta) = S(\theta) \tag{50c}$$

if

$$|G^2| >> |\frac{1}{2} \frac{G''}{2G} - \frac{3}{4} (\frac{G'}{G})^2| \tag{50d}$$

The primes denote differentiation with respect to the independent variable θ and the A and B are the constants of integration that must be determined from the initial conditions. For $S(\theta)$ given by Eqs. (42), the librational motion is given by

$$\alpha = e^{-\frac{5}{4}\eta\left(\theta + \frac{3}{10}\sin 2\theta\right)}[G(\theta)]^{-\frac{1}{2}}[A \operatorname{Cos} \varphi(\theta) + B \operatorname{Sin} \varphi(\theta)] + \frac{2\eta}{3\sigma} \tag{51}$$

The time constant of the motion, which is the time required for the libration amplitude to reduce to e^{-1} of its initial value, is obtained directly from Eq. (51) to be

$$T = \frac{2}{5\pi\,\eta} \tag{52}$$

where T is in orbital revolutions.

It also follows directly from Eq. (51) that the motion of the geomagnetic field relative to the local vertical frame of reference introduces a steady-state bias of magnitude

$$\alpha_s = \frac{2\eta}{3\sigma} \tag{53}$$

The time constant and steady-state bias angle given by Eqs. (52) and (53) are illustrated in Figs. 6 and 7 as functions of the damper parameter η .

Comparison of Analytical and Simulation Results

The formulae developed in the preceeding section for the time constant and steady-state bias angle are particularly useful for preliminary design. To establish confidence in the formulae it is necessary to obtain a measure

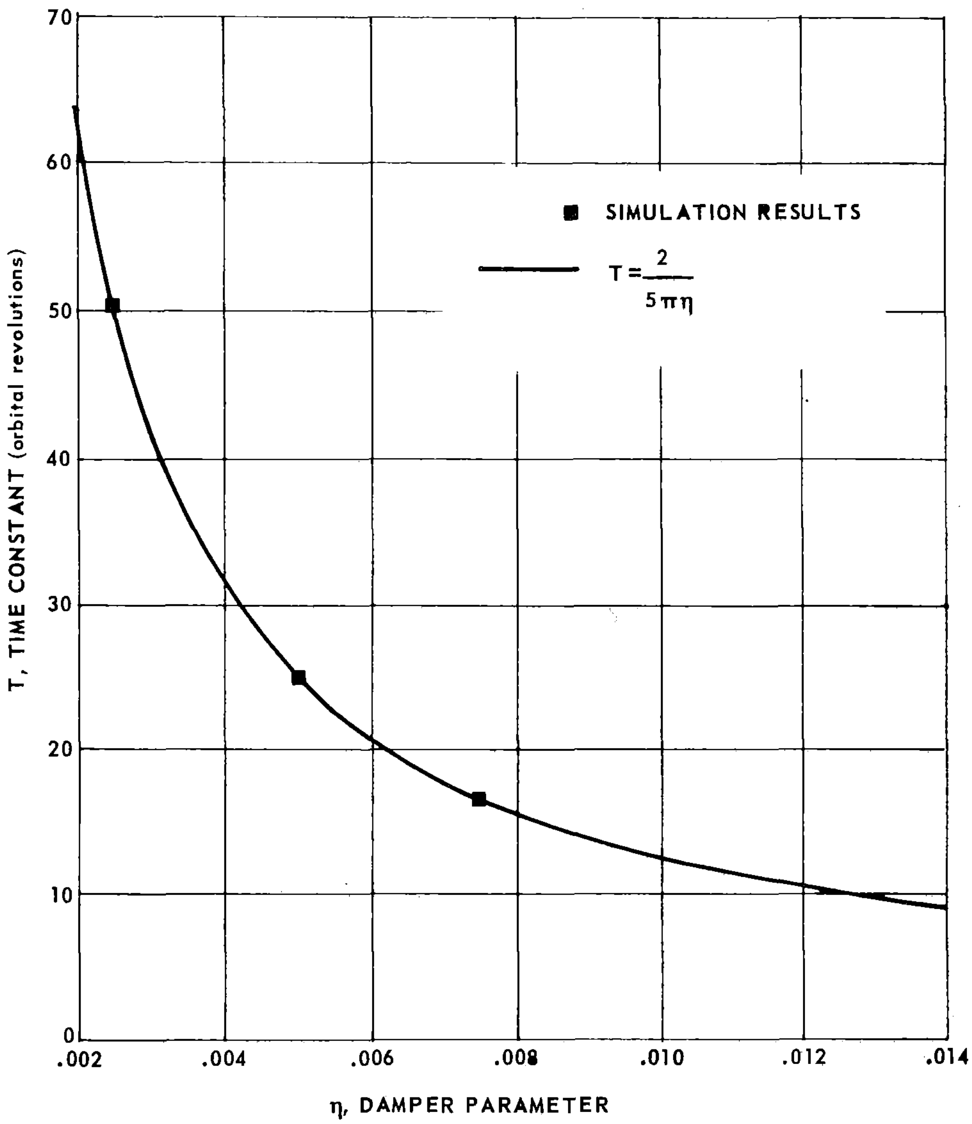

Fig. 6 TIME-CONSTANT VERSUS DAMPER PARAMETER OF A GRAVITATIONALLY STABILIZED SPACECRAFT IN CIRCULAR POLAR ORBIT

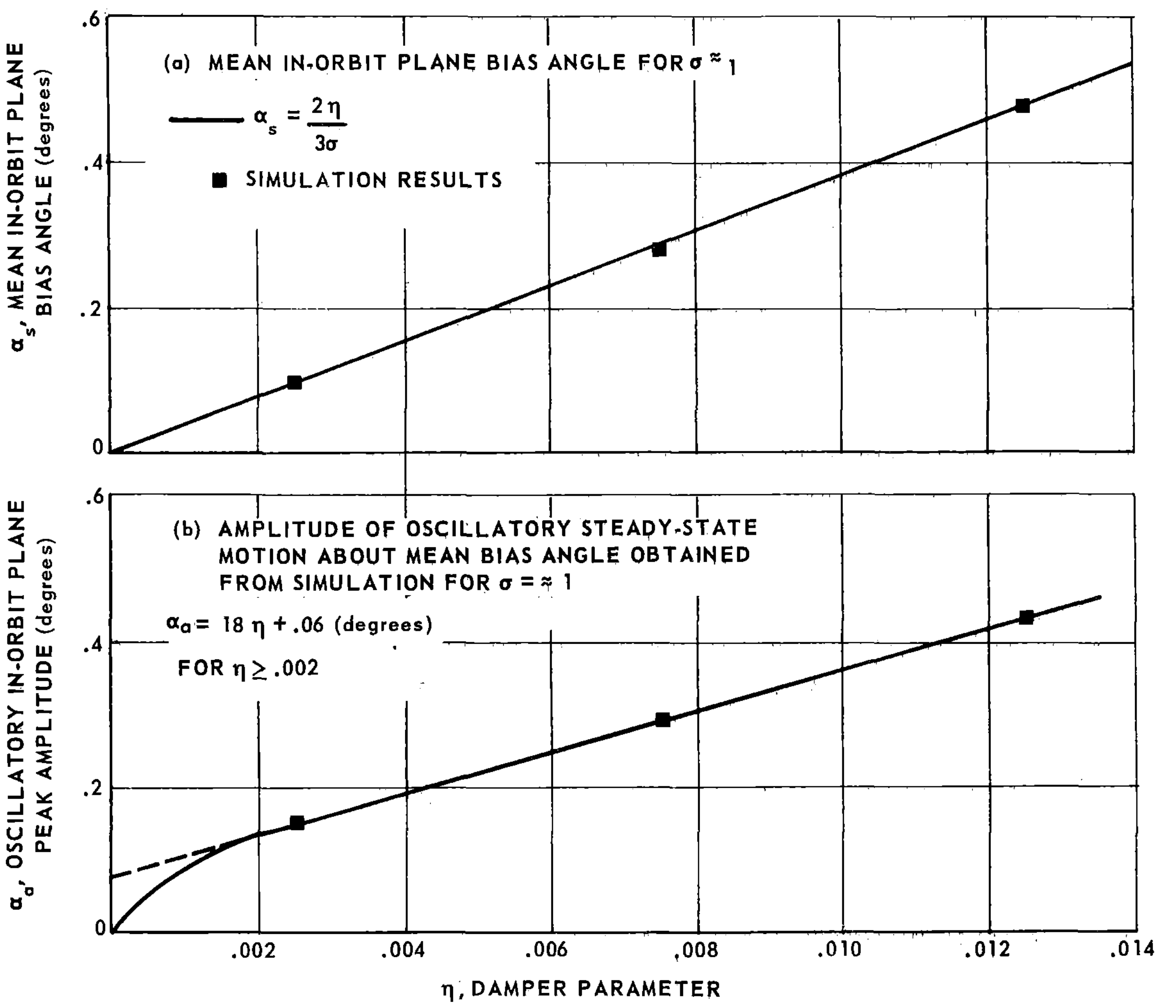

Fig. 7 IN-ORBIT PLANE STEADY-STATE BEHAVIOR VERSUS DAMPER PARAMETER OF A GRAVITATIONALLY STABILIZED SPACECRAFT IN CIRCULAR POLAR ORBIT

of the effects of the assumptions that were made to make the dynamical solution analytically tractable. This is accomplished by a comparison of the analytical results with those obtained from a full scale digital simulation of the nonlinear differential equations of motion. The simulation used for this purpose is a general purpose Digital Attitude Simulation developed at the Applied Physics Laboratory and is described in [6]. As used for the current study, the simulation is operated in an abbreviated mode in that the majority of the optional capabilities are not exercised. The center of mass of the spacecraft is constrained to a circular and geographically polar orbit. Only torques that arise from the interaction of the inertia ellipsoid with the Newtonian component of the geopotential and the interaction of the damper with a realistic model of the geomagnetic field are considered. The geomagnetic field is represented in a rotating earth-fixed coordinate system by the eight leading terms of the spherical harmonic expansion discussed in [7]. The model has an rms residual of 9 percent when compared with satellite measured geomagnetic data. The three second-order nonlinear differential equations are integrated numerically by the fourth-order Runge-Kutta-Gill method. The characteristics of the spacecraft and orbit that are used in the simulation are given in Table I.

The results of the digital simulation are superimposed on Figs. 6 and 7 so that a direct comparison can be made with the analytical result. Excellent agreement is obtained. To determine the transient response from the simulation, the initial conditions were selected so that the spacecraft was aligned with the local vertical at the geographic north pole with zero angular velocity in inertial space. This is typical of the conditions that

Table 1. Spacecraft and orbit characteristics for digital simulation

Mass properties

I_p = 500 kgm-meter^2

I_r = 500 kgm-meter^2

I_y = 20 kgm-meter^2

m = 70 kgms

Orbit characteristics

semimajor axis	=	1.2 earth radii
eccentricity	=	0
inclination	=	90 degrees
argument of ascending node	=	0
argument of perigee	=	0
mean anomaly	=	90 degrees
epoch		67 80 0.0 year-day-sec
mean motion		.943 x 10^{-3} rads/sec

prevail when the mass distribution of a gravitationally stabilized spacecraft is achieved by extendible members. The time constant is computed from the libration history over 39 orbital revolutions of the satellite which corresponds to approximately three days. To determine the steady-state dynamic behavior from the simulation, the initial conditions were selected so that the spacecraft was aligned with the local vertical at the geographic north pole but with an in-orbit plane angular velocity equal to the orbital mean motion. In the absence of perturbations the spacecraft would remain aligned with the local vertical. The mean in-orbit plane bias angle is computed from averaging the peak libration amplitudes over the last 13 orbital revolutions out of a total of 50 revolutions. Fig. 7b represents the amplitude of the envelope of the oscillatory libration about the mean bias angle that is present in the simulation results. For a specific value of η addition of the ordinates of Figs. 7a and 7b gives the maximum deviation from the local vertical.

Damper System Characteristics

The choice between the active or passive damper will in general be determined by the particular stabilization requirements and spacecraft constraints. Mass, volume, power requirement, reliability, and flexibility are the more important damper characteristics that will influence the choice.

The active damper can be implemented in several ways. The function of the sensor is to determine the time rate of change of the geomagnetic flux density. This can be accomplished by time differentiating the output of a vector magnetometer or by detecting the magnitude of the current induced in a coil of conducting material. The function of the electromagnetic is to generate a dipole moment which interacts with the geomagnetic field to produce the damping torques. Either an air or iron core electromagnet can be used in either a continuous or discontinuous mode. In the continuous mode the output of the amplifier electronics is fed continuously into the electromagnet so that it has good fidelity with the sensor output. The discontinuous mode utilizes a chargeable electromagnet so that pulsed charging of the electromagnet results in a dipole moment that is a step approximation to the desired function. This would have the advantage of minimizing power consumption.

It is obvious that the active damper can be packaged into a relatively small unit and can be made extremely flexible by incorporating the ability to change the amplifier gain by ground command. The virtue of the passive damper is its extreme reliability. However, for at least some applications it appears to require an inordinate amount of mass and volume. The mass of the passive damper, exclusive of fittings, can be obtained from Eq. (36c) which can be rewritten as

$$\eta = \frac{\pi q \ell^5 M_e^{\,2}}{2k_e^{\,1/2} \rho \, I_p a^{9/2}} \left(\frac{\mu}{\mu_e}\right)^2 \left(\bar{r}_c^{\,4} \log_e \frac{\bar{r}_s}{\bar{r}_c}\right) \tag{54}$$

For given values of η , I_p , $\bar{r}_c$, μ , a , and ρ , the problem is one of determining the number of rods required along each axis from

$$\ell = \left[\frac{2\rho\eta \, I_p \, k_e^{\,1/2} a^{9/2}}{\pi q \, M_e^{\,2} (\mu/\mu_e)^2}\right]^{1/5} \left[\bar{r}_c^{\,4} \log_e \frac{\bar{r}_s}{\bar{r}_c}\right]^{-1/5} \tag{55}$$

subject to a constraint on the length of the rods. The total mass, exclusive of fittings, is then given by

$$m = 3q\pi \ell^3 [\bar{r}_c^{\,2} \delta_c + (\bar{r}_s^{\,2} - \bar{r}_c^{\,2}) \delta_s] \qquad (56)$$

Repeating this calculation for several values of $\bar{r}_s$ results in the determination of a damper system of minimum mass.

It is instructive to compute the mass of the passive damper for a typical application. Consider the spacecraft and orbit whose characteristics are described in Table 1 and a mutually orthogonal set of eddy-current rods that have a core material of mu metal whose properties are given for

$$\bar{r}_c = .002$$

in [1] to be

$$(\frac{\mu}{\mu_e}) = 10^4$$

$$\delta_c = 7.86 \times 10^3 \text{ kgm/meter}^3$$

For a conducting shell of copper

$$\rho = 1.7 \times 10^{-8} \text{ ohm-meters}$$

$$\delta_s = 8.9 \times 10^3 \text{ kgm/meter}^3$$

The mass of the passive damper exclusive of fittings for an η of .0025, which is equivalent to a time constant of approximately 50 orbital revolutions, as a function of the nondimensional maximum radius of the conducting shell is given in Fig. 8. For a maximum allowable length of one meter a damper mass of 45 kgms is required while for a maximum allowable length of 0.5 meters a mass of 57 kgms is required. Consequently, unless the permeability of the core material can be increased appreciably the passive damper appears to have limited application for missions in which the emphasis is on the transient performance. For gravitational stabilization it is possible, by multiple boom extensions, to capture the spacecraft so that its initial libration energy is small. In addition, digital simulation studies as well as experimental evidence accumulated at the Laboratory indicate that some gravitationally stabilized spacecraft will maintain good stabilization in the absence of any damping mechanism for extended periods of time. Consequently, in applications in which the steady-state performance is of prime importance, so that a damping parameter of appreciably smaller magnitude can be utilized, the passive damper may be competitive.

Summary and Conclusions

This paper studies the applicability of both active and passive geomagnetic rate damping to spacecraft attitude control systems. Torque expressions which can be used in digital simulation studies are derived for each type of damper. It is shown that a triad of mutually orthogonal single-axis passive dampers with a magnetically linear core material has the same mathematical form for the torque expression as a triad of mutually orthogonal

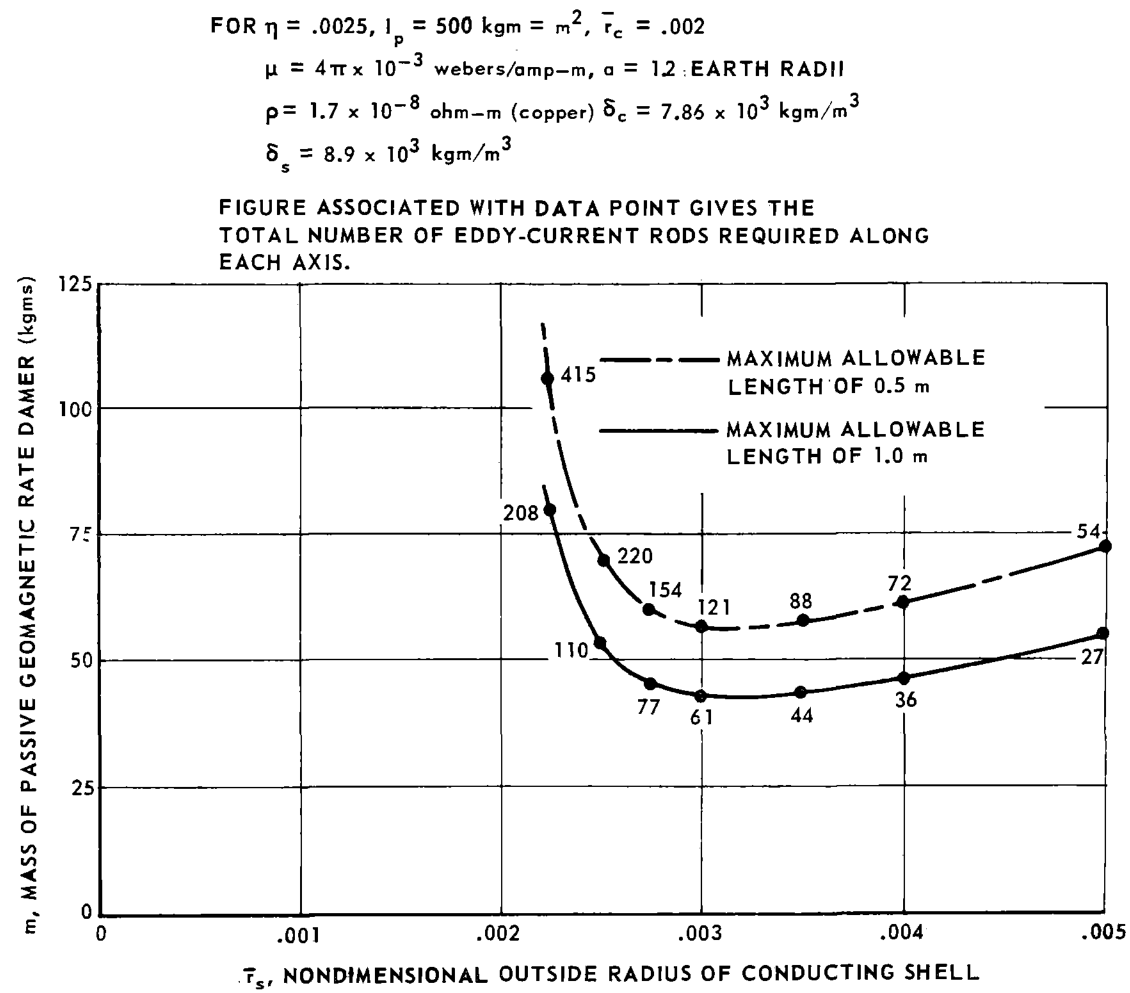

Fig. 8 MASS OF PASSIVE GEOMAGNETIC RATE DAMPER VERSUS NONDIMENSIONAL RADIUS OF CONDUCTING SHELL

single-axis active dampers.

Design formulae are derived which characterize the transient and steady-state motion of gravitationally stabilized spacecraft in geographically polar and circular orbits. The validity of the assumptions that are necessary to make the dynamical solution analytically tractable is established by agreement between the analytical results and those of a detailed digital simulation study.

It is further shown that for a typical application that the passive damper requires an inordinate amount of mass to provide good transient performance so that its usefulness appears to be limited to applications in which only a small amount of damping is required. The active damper has good potential to complement the attitude stabilization systems of geomagnetic and gravitationally stabilized spacecraft. Its lack of moving parts implies that good reliability should be attained. Since the characteristics of the active damper can be changed by ground command it is an extremely flexible device. It can also be used in a closed loop feedback mode to maintain a specified spacecraft rotation rate with respect to the geomagnetic field.

REFERENCES

1. Fischell, R. E., "Magnetic damping of the angular motions of earth satellites," APL/JHU Report CM-983, November 1960.

2. Communications and Systems, Inc., "Analyses of two alternate damping systems for the GEOS gravity gradient stabilized satellite," Report No. R-4035-30-1, April 1968.

3. "Design definition-Department of Defense gravity-gradient experiment Phase I," APL/JHU Report SDO-1271, June 1965.

4. Pisacane, V. L., Pardoe, P. P. and Hook, B. J., "Stabilization system analysis and performance of the GEOS-A gravity-gradient satellite (Explorer XXIX)," J. Spacecraft and Rockets, Vol. 4, No. 12, Dec. 1967, pp. 1623-1630.

5. Kaplan, W., Operational methods for linear systems, Addison-Wesley, Reading, Mass., 1962, p. 486.

6. Pisacane, V. L., Pardoe, P. P. and Whisnant, J. M., "Simulation of the attitude stabilization of the DODGE spacecraft with time-lag magnetic damping," APL/JHU Report TG-949, October 1967, also Proceeding XVIIIth International Astronautical Congress, Belgrade, Sept. 1967.

7. Cain, J. L., Daniels, W. E., Hendricks, S. J., and Jensen, D. L., "An evaluation of the main geomagnetic field," J. Geophysical Research, Vol. 70, No. 15, 1965, pp. 1940-1962.

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* The Johns Hopkins University Applied Physics Lab. 8621 Georgia Ave. Silver Spring, Md. 20910	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Attitude Stabilization of Spacecraft with Geomagnetic Rate Damping

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Memorandum

5. AUTHOR(S) *(First name, middle initial, last name)*

V. L. Pisacane

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
April 1969	36	7

8a. CONTRACT OR GRANT NO. NOw 62-0604-c b. PROJECT NO. c. Task Assignment I42 d.	9a. ORIGINATOR'S REPORT NUMBER(S) TG-1058 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES This work was supported by NASA, Office of Space Science and Applications under Task I of Contract NOw 62-0604-c.	12. SPONSORING MILITARY ACTIVITY National Aeronautics and Space Administration

13. ABSTRACT

This paper studies two damping systems which can be used as part of the attitude control systems of geomagnetically, gravitationally, or spin-stabilized spacecraft. The devices are active and passive geomagnetic rate dampers that use the relative motion between the spacecraft and an ambient magnetic field to produce the required torques. The expressions for the torques generated by each device are derived. To investigate the usefulness of the dampers, an analytical study is made of the attitude performance of gravitationally stabilized spacecraft. To verify the assumptions that were made to make the dynamical problem analytically tractable, a digital simulation study is presented. Good agreement is obtained. The characteristics and applicability of each type of damper are also discussed.

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14. KEY WORDS

Spacecraft attitude control

Spacecraft stabilization

Spacecraft damping systems

Satellite attitude control