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ON THE APPLICATION OF DIMENSIONALITY ANALYIS TO QUESTIONS OF INTERPLANETARY GAS MOTION DURING SOLAR FLARES

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SUMMARY

The author considers the motion processes outside the seat of a flare in hydrodynamic approximation. The relevant basic gas parameters are ascertained and, neglecting some of them, the π-theorem of dimensionality is used. Only a spherically-symmetrical model medium motion is considered. After elaborate discussions formulas are derived and converted. The results are applied to the simplest case of gas motion dependence on coordinates and time. The influence of the magnetic field on interplanetary gas motion is considered.

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As is well known [1 - 3], solar chromospheric flares occur and take place within a comparatively small volume, the area occupied by a flare being about 0.1 percent of solar disk area with flare height of the order of 10^9 cm. The development time of processes in the seat of the flare has, as a rule, an order of magnitude $3 \cdot 10^2 - 4 \cdot 10^3$ sec. The total energy E_0, liberated in the chromosphere during flares, varies by order of magnitude within the range $10^{29} - 10^{34}$ ergs.

We shall consider the processes of medium's motion outside the seat of the flare, where the details of the process of occurrence and development of the flare no longer have any substantial significance.

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* The fundamental results of this paper have been reported by the author at the 3rd All-Union Assembly on Theoretical and Applied Mechanics (Moscow, 25 January- 1 February 1968)

** Paper presented by Academician L. I. Sedov on 10 September 1968.

From the works [1, 4, 5], devoted to theoretical and experimental questions of interplanetary gas' parameter determination, it is possible to conclude that for gas density ρ_1 and quiet solar wind velocity v_1 inside a certain solid angle $\kappa < 2\pi$ originating from the Sun, and in the region between the Sun and the Earth's orbit, one may assume the following approximate dependences:

$$\rho_1 = Ar^{-\omega}, \quad v_1 = Br^{\omega-2}. \tag{1}$$

Here r is the distance from the Sun; A and B are quantities which will be considered as constant and independent of the angles θ, ϕ in a spherical system of coordinates with center in the Sun, and of time t. The concrete values of the quantities A,B and ω may be found, for example, on the basis of data of rocket and radar measurements.

2. To describe the motion of the medium we shall make use of the hydrodynamic approximation. If we neglect the influence of initial gas pressure p_1, of electromagnetic forces and also of viscosity and heat conductivity, the system of the basic characteristic parameters during gas motion has the form

$$r,\ \theta,\ \phi,\ t,\ E_0,\ A,\ B,\ \omega,\ \gamma,\ g_{\odot},\ R_{\odot},\ l,\ \Omega, \tag{2}$$

where γ is the effective adiabate exponent of the gas; Ω is the mean angular velocity of Sun's rotation; $g_{\odot}$ is the gravitation acceleration on the surface of the Sun; $R_{\odot}$ is the radius of the Sun; l is the characteristic linear dimension of the seat of the flare. The quantities A, B and ω may assume various values, depending upon the time of the year and on the state of interplanetary medium, while the energy E_0 is different for various flares (here and further we understand by energy E_0 the part of flare's total energy which participates in the motion of the gas). This is why it makes sense to ascertain those basic dimensionless parameters, which characterize the considered phenomenon and to examine the question of conversion of functions characterizing the motion of gas at variation of medium's parameters and energy E_0.

Since the gravitation and the Sun's proper rotation feebly influence the propagation of perturbations during flares, parameters Ω and $g_{\odot}$ may be disregarded during rough estimates of characteristics of moving gas. From dimensional parameters E_0, A, B it is possible to form with length and time dimensionalities the following quantities: $r^* = (E_0/AB^2)^{1/(\omega-1)}$ for the kinetic characteristic of length and $t^* = (r^*)^{3-\omega}/B$ for the kinetic characteristic time. Then, on the basis of the π-theorem of dimensionality [6], we may write for any dimensionless characteristic of flow (for example, density $g = \rho/\rho_1$):

$$g = g(r/r^*, t/t^*, \theta, \phi, \omega, \alpha_1, \alpha_2), \qquad (3)$$

where $\alpha_1 = R_\odot/r^*$, $\alpha_2 = l/r^*$ ($\alpha_2 << \alpha_1$).

It follows from formula (3) that for fixed γ, ω, α_1, α_2 the dimensionless functions of the form (3) will describe the class of flows for various parameters E_0 , A and B. At characteristic dimensions of the region of motion much greater than $R_\odot$, the influence of the finiteness of the radius of the Sun on the motion of gas in a certain region of the leading edge of perturbations may be disregarded, that is, the influence of parameter α_1 and, consequently of α_2 may be neglected.

If we consider the solar wind velocity inside the considered solid angle as directed along the radius, and the conditions of energy liberation as corresponding to spherical symmetry of the flow, the gas flow inside the considered solid angle κ may be considered a spherically symmetrical.

Let us introduce the denotations: $x = r/r^*$, $y = t/t^*$. For a spherically symmetrical model of flow, functions of the form (3) will depend only on two parameters x, y, i. e.

$$g = g(x, y, \gamma, \omega) \qquad (4)$$

It is natural that for a global consideration of the process of propagation of perturbations at flares, one must take into account the dependence of the initial density and solar wind velocity components on the angles θ and ϕ and, possibly on time t. In this case, the solution of the hydrodynamic problem sought for will depend on a large number of variables, and the functions entering into this solution will have the form (3).

In the following we shall consider only a spherically-symmetrical model of medium's motion. If the motion of the quiet solar wind is considered isothermic and if we set up the question on accounting for the initial pressure p_1, we shall have for the one-dimensional model:

$$p_1 = Cr^{-\omega} \qquad (5)$$

In this case we have a new dimensional constant C, which allow us to form a new characteristic length $r^0 = (E_0/C)^{1/(3-\omega)}$. Still another dimensionless parameter

$\alpha_3 = r^0/r^*$ will enter into formulas of the form (4). The estimates conducted by us of the total initial energy of the solar wind in a volume inside a certain solid angle show that at distances of the order of one astronomical unit, the total initial kinetic energy of the quiet solar wind too is by one order greater than the total initial thermal energy of the gas (and greater than the initial energy of the magnetic field). We shall consider that parameter l may be neglected during rough qualitative analysis of the phenomenon under consideration.

To represent the order of magnitude of the kinetic characteristic length r^*, we shall refer r^* to a certain radius r_a, and, on the basis of formulas for ρ_1 and v_1 we eliminate AB^2 from the expression for r^*. We shall then have

$$r^*/r_a = (E_0/k^*)^{1(\omega-1)}, \quad k^* = \rho_1(r_a)[v_1(r_a)]^2 r_a^{\ 3}.$$

Plotted in Fig.1 are the dependences of r^*/r_a on E_0 for the case when $r_a = 1.5\cdot 10^{13}$ cm (astronomical unit), $k^* = (1.5)^3\cdot 10^{31}$ $g\cdot cm^2 \cdot sec$ (dashed lines), $k^* = 5(1.5)^3\cdot 10^{32}$ $g\cdot cm^2/sec$ (solid lines), and the energy E_0 of the flare varies from 10^{28} to 10^{35} ergs.

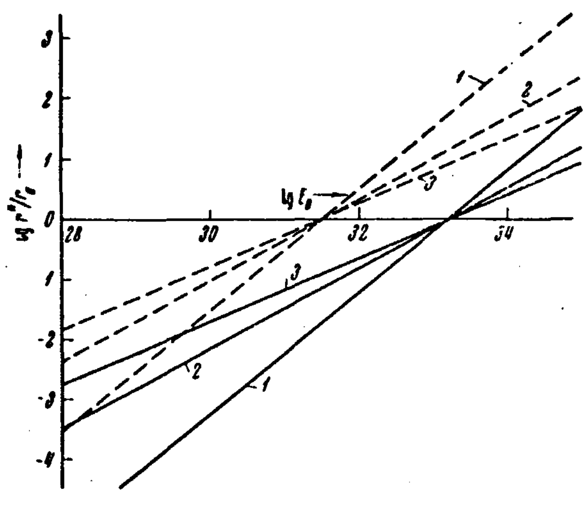

Fig.1

In Fig.1 curves 1 correspond to $\omega = 2$, curves 2 – to $\omega = 2.5$, curves 3 – to $\omega = 2.9$. The calculations performed indicate that for great values of E_0, the quantity r^* may be greater than the astronomical unit.

3. The dependences of the sought for functions on dimensionless parameters may be determined theoretically or experimentally, in the course of rocket measurements during solar flares. At theoretical, and more particularly, at experimental determination of gas parameters' dependence on coordinates and time for density, pressure and gas velocity, and the time of perturbation arrival at the given point may be found only for some fixed $E_0 = E_{01}$, $A = A_1$, $B = B_1$.

This is why we shall examine the question of conversion of the obtained data to the case of other values of these constants E_{02}, A_2, B_2. One may adopt the following procedure. We shall find the dimensionless time $\underline{y}$ and the dimensionless coordinate $\underline{x}$ for the state E_{01}, A_1, B_1, and then from formulas $r = r^*x$, $t = t^*y$ we shall find the value of the coordinate and the time for the state E_{02}, A_2, B_2.

For the conversion of time we shall find

$$y = t_{(1)} / t_1^*, \quad t_{(2)} = t_2^* y = (t_2^* / t_1^*) t_{(1)}, \tag{6}$$

where

$$t_i^* = (r_i^*)^{3-\omega} / B_i, \qquad r_i^* = (E_{0i}/A_i B_i^2)^{1/(\omega-1)} \qquad (i = 1, 2).$$

For the conversion of distances (Euler or Lagrange coordinate of gas particle), we have

$$x = r_{(1)} / r_1^*, \quad r_{(2)} = r_2^* x = (r_2^* / r_1^*) r_{(1)}. \tag{7}$$

Here $t_{(i)}$, $r_{(i)}$ $(i = 1, 2)$ are the dimensionless values of time and the coordinates corresponding to the flare for parameters E_{0i}, A_i, B_i. Analogous formulas may be also written for the conversion of velocity, density and pressure of the gas and of other quantities. Thus, for the density ρ we have

$$g = \rho_{(1)}/\rho_{11}, \; \rho_{(2)} = g\rho_{12} = (\rho_{12}/\rho_{11})\rho_{(1)}. \tag{8}$$

Formulas (6) - (8) yield (in the assumed model flow) the similarity laws for the process of gas motion induced by the flare. For a more complex model of gas flow formulas (6) - (8) will take place only at fixed values of additional dimensionless constant parameters, for example, a_j $(j = 1,2,3)$.

4. Let us bring forth a simplest example of velocity, density and pressure of gas dependences on coordinate $\underline{r}$ and time $\underline{t}$. We shall simulate the flare phenomenon by a spherically-symmetrical point explosion in gas, neglecting the dimensions of the Sun, the motion of interplanetary gas in solar wind, the initial pressure, the influence of gravitation and of magnetic fields. We shall consider that the initial density of the gas is

$$\rho_1 = Ar^{-\omega_1}, \quad \omega_1 = (7 - \gamma) / (\gamma + 1) \tag{9}$$

In this case the system of determining parameters is reduced to constants E_0, A and γ and the motion of gas is "automodel" [6], whereupon the solution

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of the hydrodynamic problem has the form:

$$\rho = \frac{\gamma+1}{\gamma-1}\rho_1\frac{r}{r_2}, \quad p = \frac{2\delta^2\rho_1}{\gamma+1}\frac{r_2^2}{t^2}\left(\frac{r}{r_2}\right)^3, \quad v = \frac{2\delta}{\gamma+1}\frac{r}{t},$$
$$r_2 = \left(\frac{E_0}{\alpha A}\right)^{\delta/2} t^\delta, \quad \delta = \frac{2}{(5-\omega_1)}, \quad \alpha = \frac{8\pi(\gamma+1)}{3(\gamma-1)(3\gamma-1)^2}. \tag{10}$$

Here r_2 is the radius of the shock wave, which is the leading front of the perturbation.

The solution (10) was obtained by L. I. Sedov [6]. In the particular case $\omega_1 = 2$, $\gamma = 5/3$, it was utilizied in the work [1] to describe the motion of gas during flares. Note that if E_0 actually is the energy of the flare liberated inside the solid angle κ, when utilizing solution (10), in the formula for $\underline{\alpha}$ in the descritpion of motion inside the solid angle κ, we should write $\underline{2\kappa}$ instead of $\underline{2\pi}$. If we now bring the formula for $r_2(t)$ to a dimensionless form, by relating radius r_2 to r^*, and the time $\underline{t}$ to t^*, we shall have

$$x_2 = \alpha^{-\delta/2} y^\delta. \tag{11}$$

Let r_a be the astronomical unit. Then, for the time of shock wave arrival to Earth's orbit we shall have

$$t_a = r_a^{1/\delta}(\alpha A/E_0)^{1/2} \tag{12}$$

or in dimensionless form: $x_{2a} = r_a/r^*, \quad y_a = \alpha^{1/2} x_{2a}^{1/\delta}.$

Note that formulas (11), (12) are valid not only for the dependence of $\omega = \omega_1$ on γ, indicated in relations (9), but also for any automodel motions of the considered type [6]. If the quantities A, γ, ω for two different flares are identical, it follows from formula (12) that the ratio of the squares of times of perturbation's arrival at a certain point of space is inversely proportional to the ratio of energies of solar flares, i. e. $t^2_{(2)}/t^2_{(1)} = E_{01}/E_{02}$. Formula (12) provides the possibility to determine the quantity E_0 according to known values t_a, r_a, A. Thus, if $\omega = 2$, $t_a = 10^5$ sec, $A = 10^{-23} r_a$ g/cm, we find $E_0 \sim 10^{33}$ ergs. If in the considered example we took into account the motion of the gas in solar wind ($v_1 \neq 0$, $P_1 \neq 0$), the problem would no longer be automodel, and for its solution we would have to utilize the approximate analytical and numerical methods.

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5. When taking into account the influence of magnetic field on the motion of interplanetary plasma, the problem of determination of the parameters of flow is substantially complicated even in the approximation of standard magnetohydrodynamics. However, if we neglect the influence of the field on the motion of the medium and if we limit ourselves to the determination of only the deformation of the field, it will be easy to conduct here not only the analysis of the dimensionality of the phenomenon, but also point on the basis of the results of works [1, 7] to the analytical dependences for the distribution of magnetic fields in the outer space.

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*** THE END ***

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