TECHNICAL NOTE R-244

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LIQUID-GAS INTERFACE RELATIONS IN INTERCONNECTED CONCENTRIC CYLINDER TANKAGE SYSTEMS

by Dr. F. W. Geiger
Dr. J. C. May

September 1967

RESEARCH LABORATORIES

BROWN ENGINEERING COMPANY, INC.

HUNTSVILLE, ALABAMA

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ABSTRACT

A computer program has been developed to graphically and analytically define the static shape of the liquid-gas interface in the annular region between any two concentric cylinders in an axial force field for any Bond number and contact angle. The program also includes as a subroutine a previously reported method of calculating the static shape of the liquid-vapor interface within a single cylindrical tank. Static fluid surface coordinates for both the annular and central regions are given for Bond numbers (with inner cylinder radius as characteristic length) ranging from 10 to 500, for a contact angle of 5 degrees, and for a radius ratio of 1.5.

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LIST OF SYMBOLS

Symbols	Definition
A	A constant, lbm/ft-sec^2
B_o	Bond number $\left(= \rho g R_1^2/T\right)$
g	Effective acceleration of gravity, ft/sec^2
H	Dimensionless modified y-coordinate of surface ($\bar{h}/R_1$)
h	y-coordinate of surface of fluid, ft
$\bar{h}$	Modified y-coordinate of surface, ft
P	Pressure of vapor and gas above liquid, lbm/ft-sec^2
p	Static pressure in liquid, lbm/ft-sec^2
R_1	Radius of inside cylinder, ft
R_2	Radius of outside cylinder, ft
R_{c1}, R_{c2}	Principal radii of curvature of surface at a point, ft
r	Radial distance (from axis of symmetry), ft
s	Nondimensional arc length in H, t-plane
T	Surface tension of liquid, lbm/sec^2
t	Nondimensional radial distance (r/R_1)
y	Distance perpendicular to r-direction, ft
α	Inclination of surface in H, t-plane, radians
ρ	Density of fluid (liquid), lbm/ft^3
θ	Contact angle

Subscripts	Definition
a	Of central region
b	Of annular region
i	Value at end of ith step of integration process
m	Mean value
r	Derivative with respect to r
s	Value at surface
t	Derivative with respect to t
w	Value at inner wall of annulus

INTRODUCTION

For the smooth, reliable, and consistent operation of a vehicle propulsion system, only liquid must be delivered to the engine. Therefore, the liquid phase must always be located at the outlet of the propellant tank. Clodfelter[1], recognizing that the pressure drop across a liquid-gas interface could be utilized for mass transfer, suggested that tanks consisting of several properly sized, interconnected, concentric cylinders be used to position liquid propellants at low gravitational accelerations. In an extension of the concepts advanced by Clodfelter, the present report utilizes the mathematical techniques developed by Bashforth and Adams[2] and Geiger[3, 4] to predict the static shapes of axially symmetric liquid surfaces within concentric cylinder tankage systems at desired Bond numbers and contact angles. The method is used to predict the surface shapes for a radius ratio (outer to inner cylinder) of 1.5, for a contact angle of 5 degrees, and for Bond numbers from 10 to 500 (using the inner radius as the characteristic length).

An alternate analysis of the annular region has been given by Seebold et al[5], who numerically integrated one of the differential equations describing the surface using the Adams predictor-corrector method. Their results are plotted, but the accuracy with which the graphs can be read is not great. The present method can be expected to yield considerably more accurate results.

STATEMENT OF THE PROBLEM

The problem can be defined in consecutive steps as follows: Consider a section of an interconnected concentric cylinder tankage system filled with fluid (Figure 1). Select a y-axis along the axis of symmetry of the system and an r-axis perpendicular to the y-axis in the radial direction. Let the effective acceleration of gravity, g, act in the minus y-direction. Let the mean height of the fluid in the annular region be h_{mb} and the mean height of the fluid in the central region be h_{ma}. Let the height of the liquid at an arbitrary r be h(r). Assume that liquid density, surface tension, contact angle, and pressure above the liquid are constant throughout the system. Find the h- and r-coordinates of the liquid-vapor interface in each region of the system.

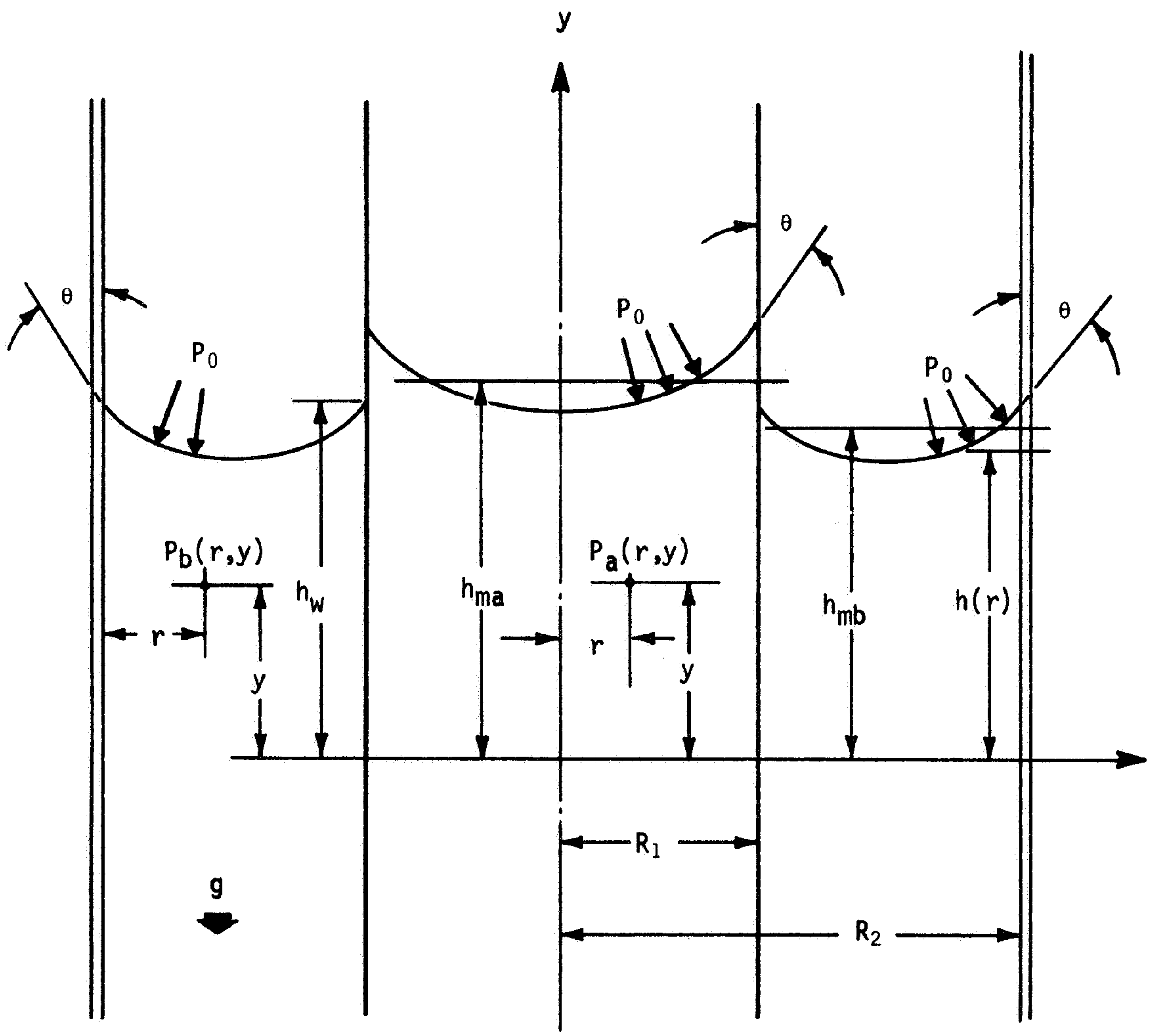

Figure 1. Fluid in Concentric Cylinder Tankage System with Axis Parallel to the Effective Acceleration of Gravity

METHOD OF ANALYSIS

FUNDAMENTALS OF CAPILLARY HYDROSTATICS

The basic equation of capillarity, sometimes referred to as the Young and Laplace equation, is

$$P_0 - P_s = T\left(\frac{1}{R_{c1}} + \frac{1}{R_{c2}}\right) \tag{1}$$

where

$P_0 - P_s$ - the difference in pressure across a liquid-gas interface at any point of the interface

R_{c1}, R_{c2} - the principal radii of curvature of the interface at that point

P_0 - the pressure in the gas at the interface

P_s - the pressure in the liquid at the interface

T - the surface tension of the liquid

Whenever a liquid-gas interface is axisymmetric, R_{c1} and R_{c2} can be expressed explicitly. Thus,

$$\begin{aligned} P_0 - P_s &= T\left[\frac{h_{rr}}{\left(1 + h_r^2\right)^{3/2}} + \frac{h_r}{r\left(1 + h_r^2\right)^{1/2}}\right] \\ &= \frac{T}{r}\frac{d}{dr}\left[\frac{rh_r}{\left(1 + h_r^2\right)^{1/2}}\right] \end{aligned} \tag{2}$$

where h(r) is the ordinate of the liquid-gas interface and the subscript r indicates the derivative with respect to r.

In a previous report, Geiger[4] used the procedure which follows to relate the hydrostatic pressure, $p = p(r, y)$, at any point in a fluid to the pressures, P_0 and P_s, on either side of the liquid-gas interface for a single cylinder of radius R_1. Here the procedure is used for the case of two concentric cylinders of radius R_1 and R_2.

First the pressure, p, at any point, (r, y), is expressed as follows (see Figure 1):

$$p + \rho g y = P_0 + A \tag{3}$$

where A is a constant to be determined. Thus,

$$P_s + \rho g h = P_0 + A \tag{4}$$

P_s is now eliminated from Equations 2 and 4, yielding

$$\frac{T}{r}\frac{d}{dr}\left[\frac{r\,h_r}{\left(1 + h_r^2\right)^{1/2}}\right] = \rho g h - A \tag{5}$$

Equation 5 is integrated after multiplying it by r to give

$$T\left[\frac{r\,h_r}{\left(1 + h_r^2\right)^{1/2}}\right]_{R_1}^{R_2} = \rho g \int_{R_1}^{R_2} h_r\, dr - A \int_{R_1}^{R_2} r\, dr \tag{6}$$

where R_1 and R_2 are any two radii.

For the annular region between concentric cylinders of radii R_1 and R_2 (see Figure 1),

$$h_r = -\cot\theta \quad \text{at } R_1$$

$$h_r = +\cot\theta \quad \text{at } R_2$$

where θ is the contact angle between the liquid-gas interface and the tank wall (measured in the fluid). Therefore,

$$T\left[\frac{r\,h_r}{\left(1+h_r^2\right)^{1/2}}\right]_{R_1}^{R_2} = T\left(R_2 + R_1\right)\cos\theta \qquad (7)$$

Now

$$\int_{R_1}^{R_2} r\,dr = \frac{R_2^2 - R_1^2}{2} \qquad (8)$$

Let V be the volume of fluid above the r-axis. Then

$$V = 2\pi\int_{R_1}^{R_2} h\,r\,dr = \pi\left(R_2^2 - R_1^2\right)h_{mb} \qquad (9)$$

where h_{mb} is the average height of the interface above the r-axis. The above results are then substituted into Equation 6, giving

$$T\left(R_2 + R_1\right)\cos\theta = \rho g\left(\frac{R_2^2 - R_1^2}{2}\right)h_{mb} - A\left(\frac{R_2^2 - R_1^2}{2}\right) \qquad (10)$$

from which it follows that

$$A = \rho g h_{mb} - \frac{2T\cos\theta}{\left(R_2 - R_1\right)} \qquad (11)$$

The pressure at any point in the fluid is now written

$$P + \rho g y = P_0 + \rho g h_{mb} - \frac{2T\cos\theta}{R_2 - R_1} \qquad (12)$$

which defines the pressure at all points in the annular region. For a single cylindrical tank, the expression for the pressure at any point can be written[4]

$$P + \rho g y = P_0 + \rho g h_{ma} - \frac{2T \cos \theta}{R_1} \tag{13}$$

where R_1 is the radius of the cylinder. This completes the procedure.

Equations 12 and 13 therefore define respectively the pressure in the annular and central regions of any concentric cylinder system. At the same y, p is the same and

$$h_{mb} - h_{ma} = \frac{2T \cos \theta}{\rho g} \left(\frac{1}{R_2 - R_1} - \frac{1}{R_1} \right) \tag{14}$$

when

$$R_2 = 2R_1 , \quad h_{mb} - h_{ma} = 0$$

$$R_2 > 2R_1 , \quad h_{mb} - h_{ma} < 0$$

$$R_2 < 2R_1 , \quad h_{mb} - h_{ma} > 0$$

Thus the relative importance of the radii in a concentric cylinder tankage system in positioning the fluid in one region with respect to its position in the other becomes apparent.

NONDIMENSIONAL FORMS OF THE GENERAL DIFFERENTIAL EQUATION FOR AXISYMMETRIC SURFACES IN ANNULAR REGIONS

The general differential equation of capillary hydrostatics for axisymmetric surfaces can be obtained from Equations 5 and 11. Thus

$$\frac{T}{r} \frac{d}{dr} \left[\frac{r h_r}{\left(1 + h_r^2\right)^{1/2}} \right] = \rho g h - \rho g h_{mb} + \frac{2T \cos \theta}{R_2 - R_1} \tag{15}$$

Letting

$$\bar{h} = h - h_{mb} + \frac{2T \cos \theta}{\rho g (R_2 - R_1)} ,$$

one obtains

$$\frac{T}{r}\frac{d}{dr}\left[\frac{r\,\bar{h}_r}{\left(1+h_r^{\,2}\right)^{1/2}}\right] = \rho\, g\, \bar{h} \quad . \tag{16}$$

The $\bar{h}$ and r coordinates are made nondimensional here by dividing by the radius of the inside cylinder, R_1. Actually, this amounts to taking the quantity R_1 as the unit of length. For the sake of simplicity, the following transformations have been made:

$$\frac{r}{R_1} = t \;, \qquad \frac{\bar{h}}{R_1} = H \;, \qquad B_o = \frac{\rho\, g\, R_1^2}{T}$$

where the Bond number, B_o, is the ratio of the body forces to capillary forces in the prevailing force field. The dimensionless differential equation of the equilibrium surface profile can now be written

$$\frac{1}{t}\frac{t}{dt}\left[\frac{t\,H_t}{\left(1+H_t^{\,2}\right)^{1/2}}\right] = B_o H \tag{17}$$

For the special case of zero Bond number, the reader is referred to a report by Clodfelter[1] in which numerical solutions for the shape of the liquid-vapor interface in the annular region between concentric cylinders are given.

The nonlinearity of Equation 17 renders general closed-form solution impossible. However, various schemes have been devised by Geiger[3], Seebold et al[5], and Bashforth and Adams[2] to integrate this equation or an equivalent equation numerically. The technique which Geiger proposed will be used in this report. This involves further transformations.

Let

$$H_t = \tan\alpha$$

where α is the angle between the H-to-t (or the h-to-r) curve and the t (or r) axis. Equation 17 then can be written

$$\frac{1}{t}\frac{d}{dt}\left(t \sin \alpha\right) = B_o\left(H_w + \int_1^t \tan \alpha \, dt\right)$$

or

$$\frac{\sin \alpha}{t} + \cos \alpha \frac{d\alpha}{dt} = B_o\left(H_w + \int_1^t \tan \alpha \, dt\right) \qquad (18)$$

where H_w is the undetermined value of H at the wall at which $t = 1$ or $r = R_1$. The boundary conditions are

$$\text{at } t = 1: \qquad \alpha = -\left(\frac{\pi}{2} - \theta\right)$$

$$\text{at } t = \frac{R_2}{R_1}: \qquad \alpha = \frac{\pi}{2} - \theta$$

for any contact angle. Although Equation 18 can be integrated numerically, it is difficult to use for low contact angles (where α is large). This difficulty can be avoided by changing the independent variable from t to s, the arc length in the H-to-t plane. This technique was first used by Bashforth and Adams[2] although they used a surface curvature as characteristic length (i.e., in place of R_1). Thus,

$$\frac{dt}{ds} = \cos \alpha \quad ,$$

$$\int_1^t dt = \int_0^s \cos \alpha \, ds$$

where $s = 0$ at $t = 1$, and

$$t = 1 + \int_0^s \cos \alpha \, ds \quad .$$

Also,

$$\cos \alpha \frac{d\alpha}{dt} = \cos \alpha \frac{d\alpha}{ds}\frac{ds}{dt} = \frac{d\alpha}{ds}$$

Now, Equation 18 becomes

$$\frac{\sin\alpha}{1 + \int_0^s \cos\alpha \, ds} + \frac{d\alpha}{ds} = B_o\left(H_w + \int_0^s \sin\alpha \, ds\right) \qquad (19)$$

where $\frac{d\alpha}{ds}$ is the curvature of the surface in the H-to-t plane.

NUMERICAL INTEGRATION OF THE DIFFERENTIAL EQUATION

Equation 19 is to be integrated numerically to obtain the shape of the interface in the annular region between concentric cylinders. A subinterval of s is chosen of length $\Delta s = \frac{1}{n}$, where n is a large number; e.g., 10,000. Starting at s = 0, a forward difference scheme is used to calculate the values of α, t, and $(h - h_{ma})/R_1$ at the end of each successive subinterval.

Since $\left.\frac{d\alpha}{ds}\right|_{s=0}$ and H_w are unknown (they are interrelated through Equation 20), an iterative procedure must be employed to determine that value of $\left.\frac{d\alpha}{ds}\right|_{s=0}$ which satisfies the boundary conditions. A minimum possible value of $\left.\frac{d\alpha}{ds}\right|_{s=0}$ is easily determined. From Equation 19 it follows that

$$\left.\frac{d\alpha}{ds}\right|_{s=0} = B_oH_w - \sin\alpha\Big|_{s=0}$$

$$= B_oH_w - \sin\left[\theta - \frac{\pi}{2}\right]$$

$$= B_oH_w + \cos\theta$$

$$= B_o \frac{h_w - h_{mb}}{R_1} + \frac{2R_1 \cos\theta}{R_2 - R_1} + \cos\theta$$

$$= B_o \frac{h_w - h_{mb}}{R_1} + \frac{R_1 + R_2}{R_2 - R_1}\cos\theta \qquad (20)$$

Since $h_w - h_{mb}$ is positive for acute values of θ and is negative for obtuse values of θ, it is clear that

$$\left| \left. \frac{d\alpha}{ds} \right|_{s=0} \right| > \left| \frac{R_1 + R_2}{R_2 - R_1} \cos\theta \right|$$

or that the latter is the minimum sought. Unfortunately, no convenient maximum value for $\left| \left. \frac{d\alpha}{ds} \right|_{s=0} \right|$ has been found.

As an initial or trial value of $\left. \frac{d\alpha}{ds} \right|_{s=0}$, an arbitrary value is selected somewhat larger in magnitude than the minimum possible value. The corresponding value of H_w is found from Equation 20. Then values of α, H, t, and $\frac{d\alpha}{ds}$ at the end of each subinterval, Δs, are calculated. In making these calculations, the following approximations are made:

$$\alpha_{i+1} = \alpha_i + \left. \frac{d\alpha}{ds} \right|_i \Delta s$$

$$H_{i+1} = H_i + \left(\frac{\sin\alpha_i + \sin\alpha_{i+1}}{2} \right) \Delta s$$

$$t_{i+1} = t_i + \left(\frac{\cos\alpha_i + \cos\alpha_{i+1}}{2} \right) \Delta s \ ; \qquad (21)$$

and $\left. \frac{d\alpha}{ds} \right|_{i+1}$ is calculated from

$$\left. \frac{d\alpha}{ds} \right|_{i+1} = B_o H_{i+1} - \frac{\sin\alpha_{i+1}}{t_{i+1}} \ . \qquad (22)$$

When $t = 1 + \int_0^s \cos\alpha \, ds$ becomes equal to $\frac{R_2}{R_1}$, α(s) must equal $\frac{\pi}{2} - \theta$ or be very close to it. If it is not, a new value of $\left. \frac{d\alpha}{ds} \right|_{s=0}$ must be selected and the procedure repeated. In the iterative technique used to obtain $\left. \frac{d\alpha}{ds} \right|_{s=0}$ for acute contact angles, it should be noted that:

1. If $1 + \int_0^s \cos\alpha \, ds < \frac{R_2}{R_1}$ for all s, $\left.\frac{d\alpha}{ds}\right|_{s=0}$ is too large.

2. If at $1 + \int_0^s \cos\alpha \, ds = \frac{R_2}{R_1}$, $\frac{\pi}{2} - \alpha(s) < \theta$, $\left.\frac{d\alpha}{ds}\right|_{s=0}$ is too large.

3. If at $1 + \int_0^s \cos\alpha \, ds = \frac{R_2}{R_1}$, $\frac{\pi}{2} - \alpha(s) > \theta$, $\left.\frac{d\alpha}{ds}\right|_{s=0}$ is too small. Similar conditions can be written for obtuse contact angles.

Once $\left.\frac{d\alpha}{ds}\right|_{s=0}$ is known, the surface coordinates $\frac{h - h_{ma}}{R_1}$ and $t\left(\frac{r}{R_1}\right)$ are also known. In fact,

$$t = 1 + \int_0^s \cos\alpha \, ds$$

$$\begin{aligned}
\frac{h - h_{ma}}{R_1} &= \frac{h - h_{mb}}{R_1} + \frac{h_{mb} - h_{ma}}{R_1} \\
&= \frac{1}{B_o}\left(\left.\frac{d\alpha}{ds}\right|_{s=0} - \cos\theta\right) - \frac{2}{B_o} \frac{R_1}{R_2 - R_1} \cos\theta \\
&\quad + \int_0^s \sin\alpha \, ds + \frac{2}{B_o} \cos\theta \left(\frac{R_1}{R_2 - R_1} - 1\right) \\
&= \frac{1}{B_o}\left(\left.\frac{d\alpha}{ds}\right|_{s=0} - 3\cos\theta\right) + \int_0^s \sin\alpha \, ds \, .
\end{aligned} \tag{23}$$

A listing of the FORTRAN IV computer program which was used to obtain surface profiles $\left(\text{values of } \frac{h - h_{ma}}{R_1}\right.$ and the corresponding values of $\left. t = \frac{r}{R_1}\right)$ for both the central and annular regions of concentric cylinder tankage systems is given in the Appendix.

RESULTS OF CALCULATIONS

Computer calculations were made for $\frac{R_2}{R_1} = 1.5$, for seven Bond numbers between 10 and 500, and for a contact angle of 5 degrees. For these calculations, an arbitrary value of Δs, the increment of arc, was selected for the central region and the value of Δs for the annular region was chosen such that the error in the contact angle was less than 0.0005 radians or 0.0286 degrees.

The results of the calculations are plotted in Figures 2 through 8. Selected results are given in Tables 1 through 7. In the tables the last point is that point for the minimum ordinate in the annular region for which the results were printed by the computer. Results were printed for every 50th calculated point in this region. The results show the expected trends.

Seebold et al[5] numerically integrated one of the differential equations describing the surface in the annular region using the Adams predictor-corrector method. They used the larger radius as characteristic length (inner radius is used in this report), and their maximum Bond number was 30 (the corresponding Bond number of this report would be 13.3). They plotted height at the outer wall, maximum depression, and height at the inner wall against radius ratio for various Bond numbers for contact angles of 0, 5, and 15 degrees.

When the results of the present study are compared with those just discussed (and this can only be done for the lowest Bond number of this report, 10, and for the one radius ratio), a difference in $\frac{h - h_{mb}}{R_2}$ appears which is of the order of magnitude of ±0.01; and this number is too large to be explained on the basis of errors in interpolation and in curve reading, estimated as ±0.004. Thus the agreement between the two sets of results cannot be said to be good.

To what this is attributable is not known. It is believed, however, that the present results should be correct to within about 0.0001 in $(h - h_{ma})/R_1$ or to within 0.00007 in $(h - h_{mb})/R_2$.

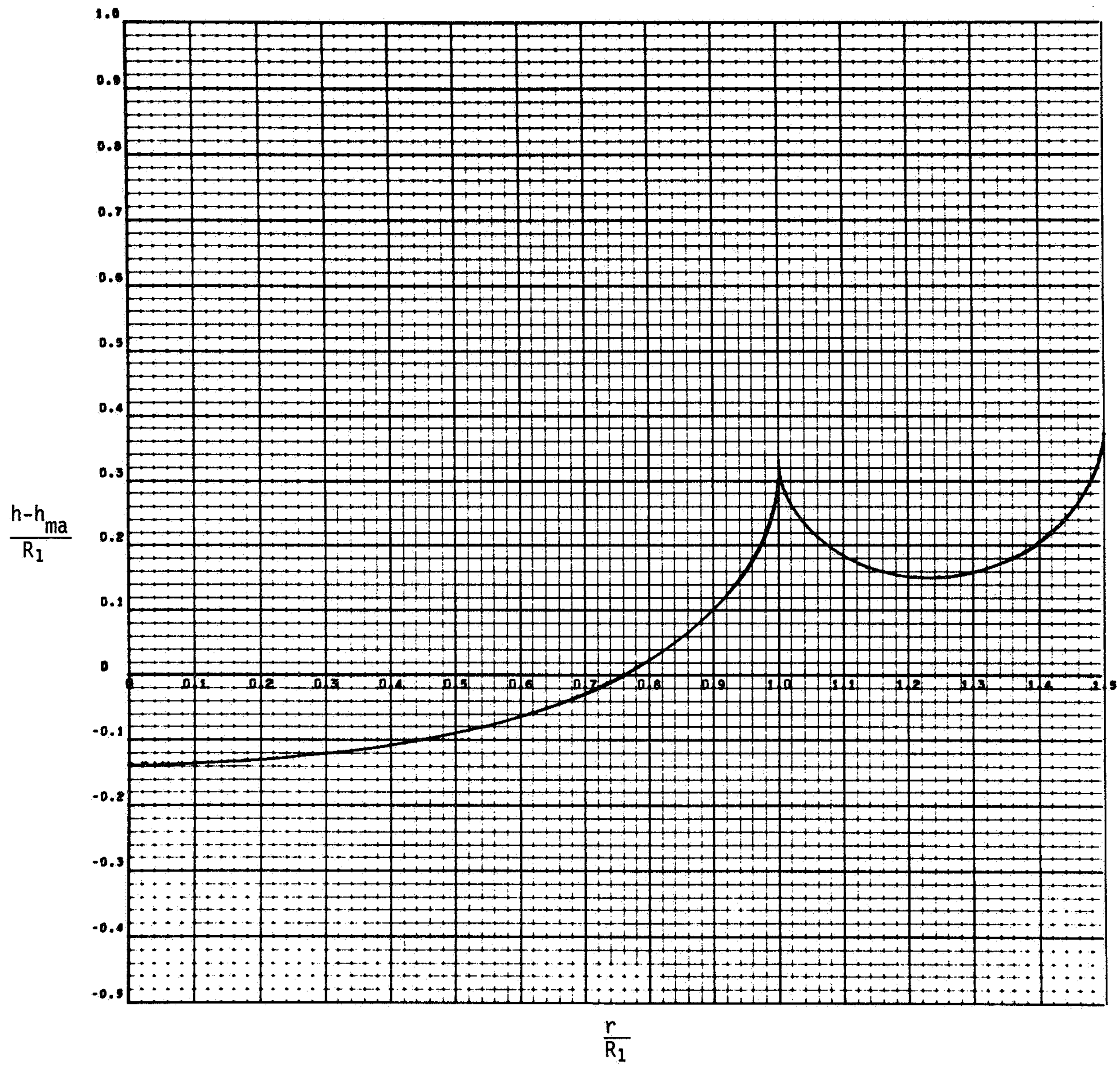

Figure 2. Static Surfaces $\frac{R_2}{R_1}$ = 1.5, B_o = 10, θ=5°

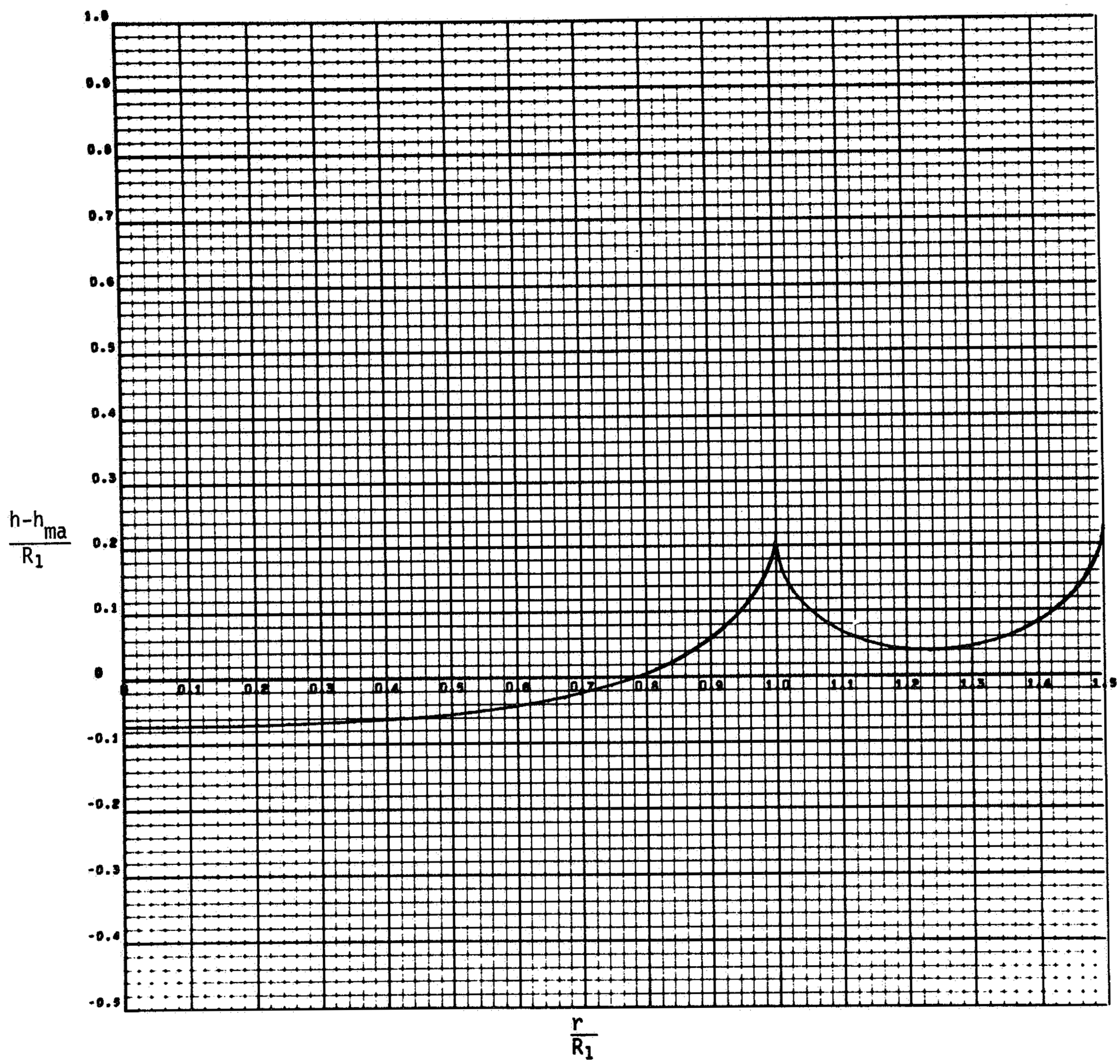

Figure 3. Static Surfaces $\frac{R_2}{R_1}$ = 1.5, B_o = 25, θ=5°

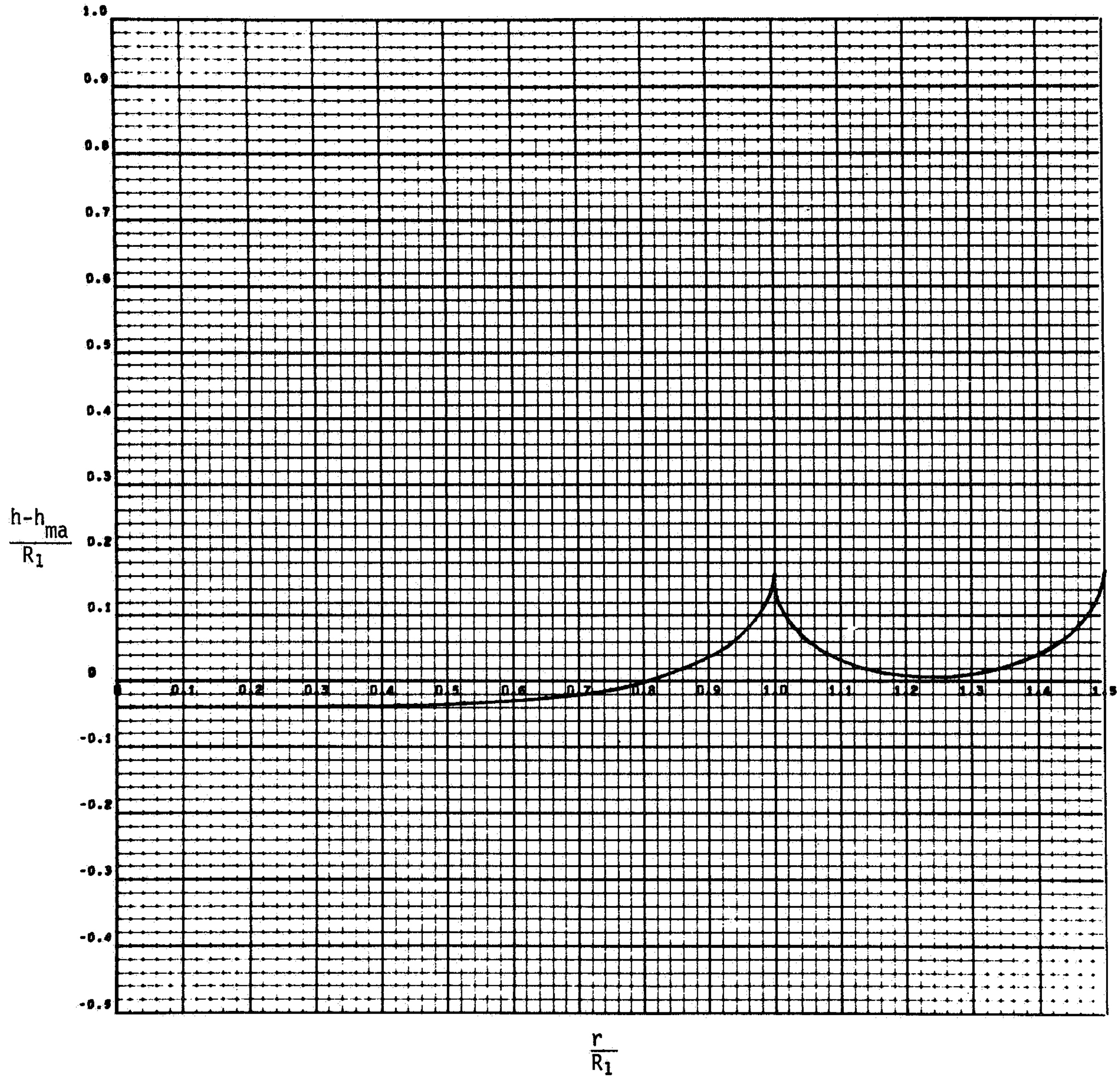

Figure 4. Static Surfaces $\frac{R_2}{R_1} = 1.5$, $B_o = 50$, $\theta=5°$

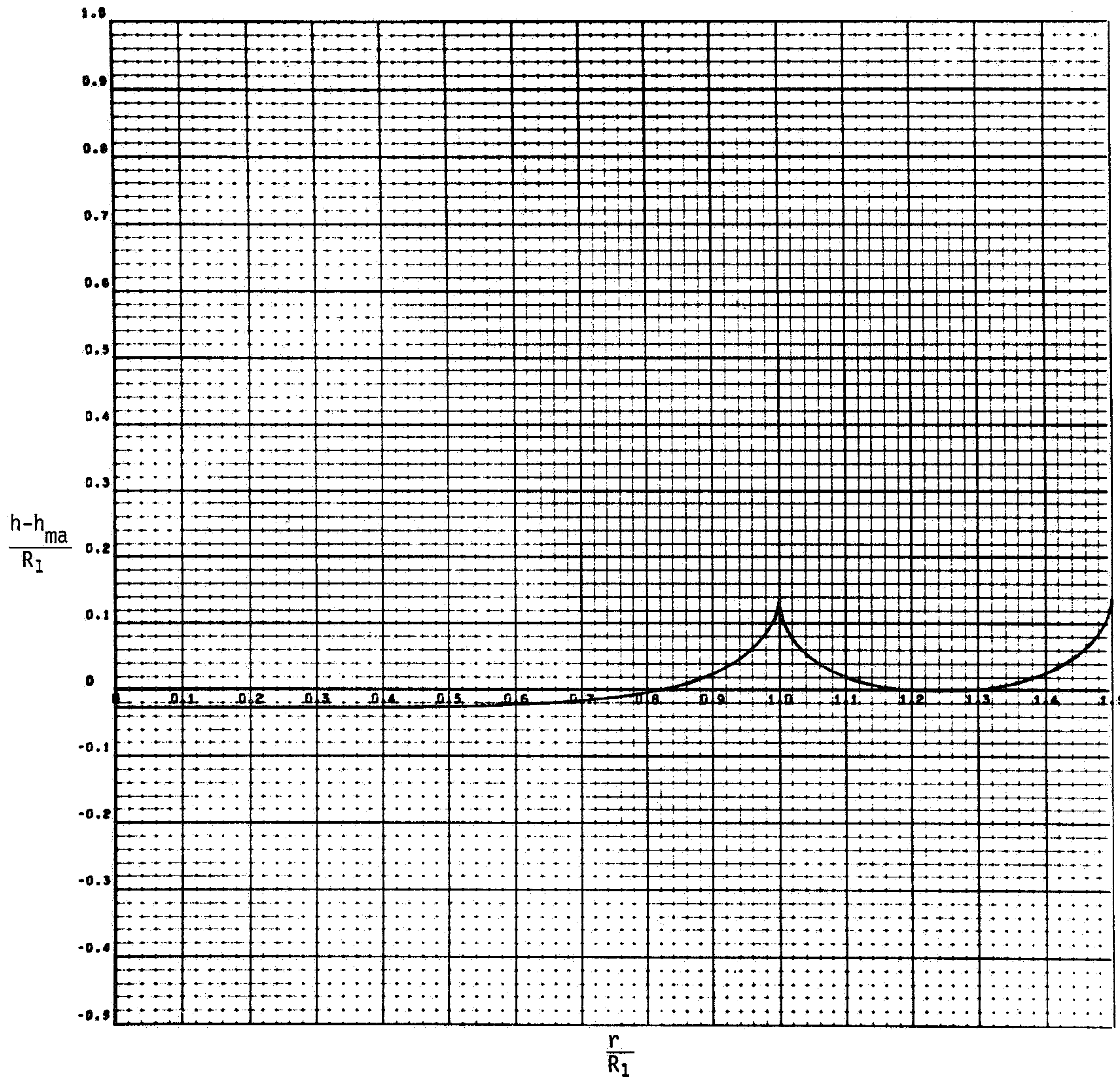

Figure 5. Static Surfaces $\frac{R_2}{R_1}$ = 1.5, B_o = 75, θ=5°

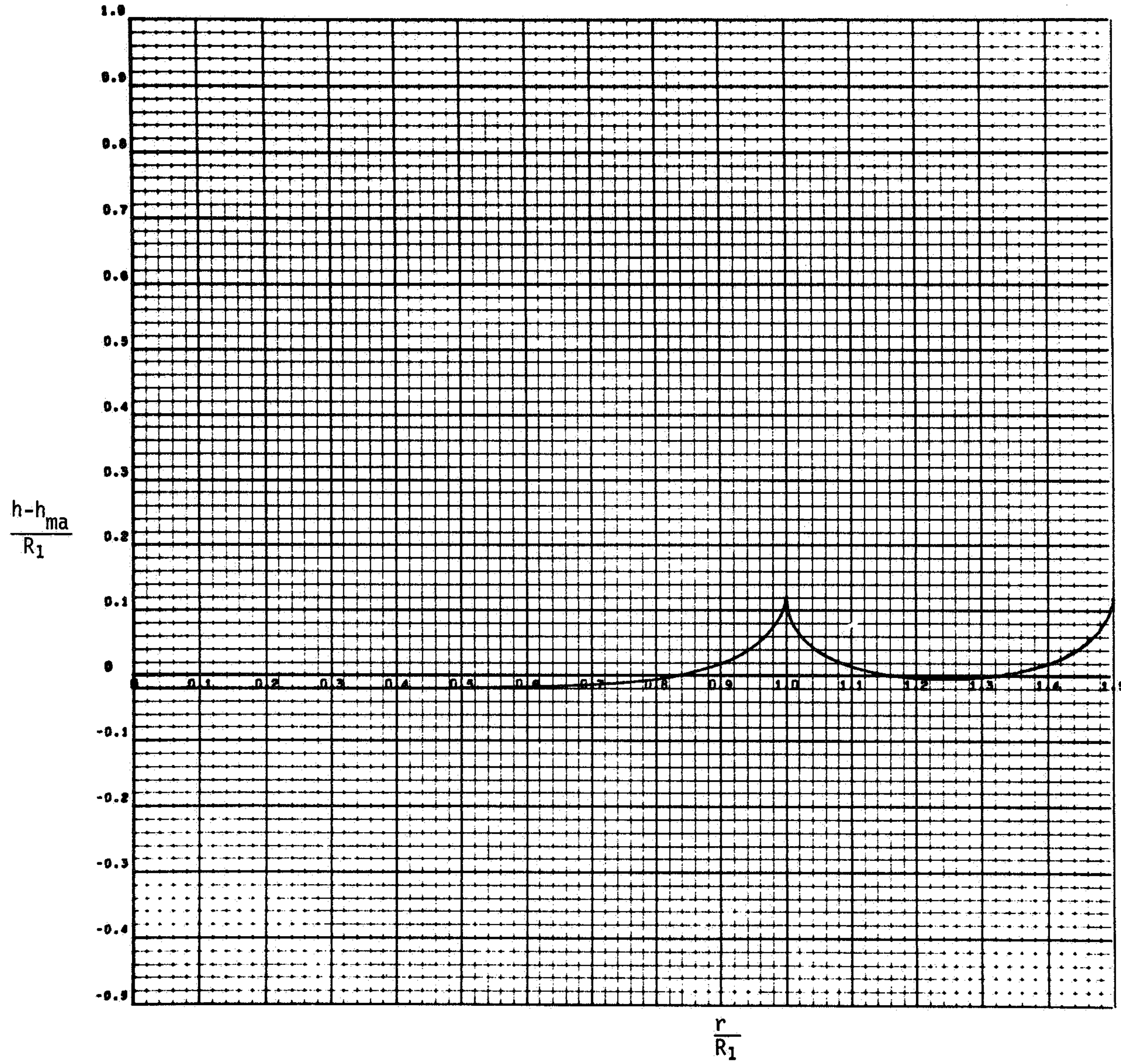

Figure 6. Static Surfaces $\frac{R_2}{R_1} = 1.5$, $B_o = 100$, $\theta=5°$

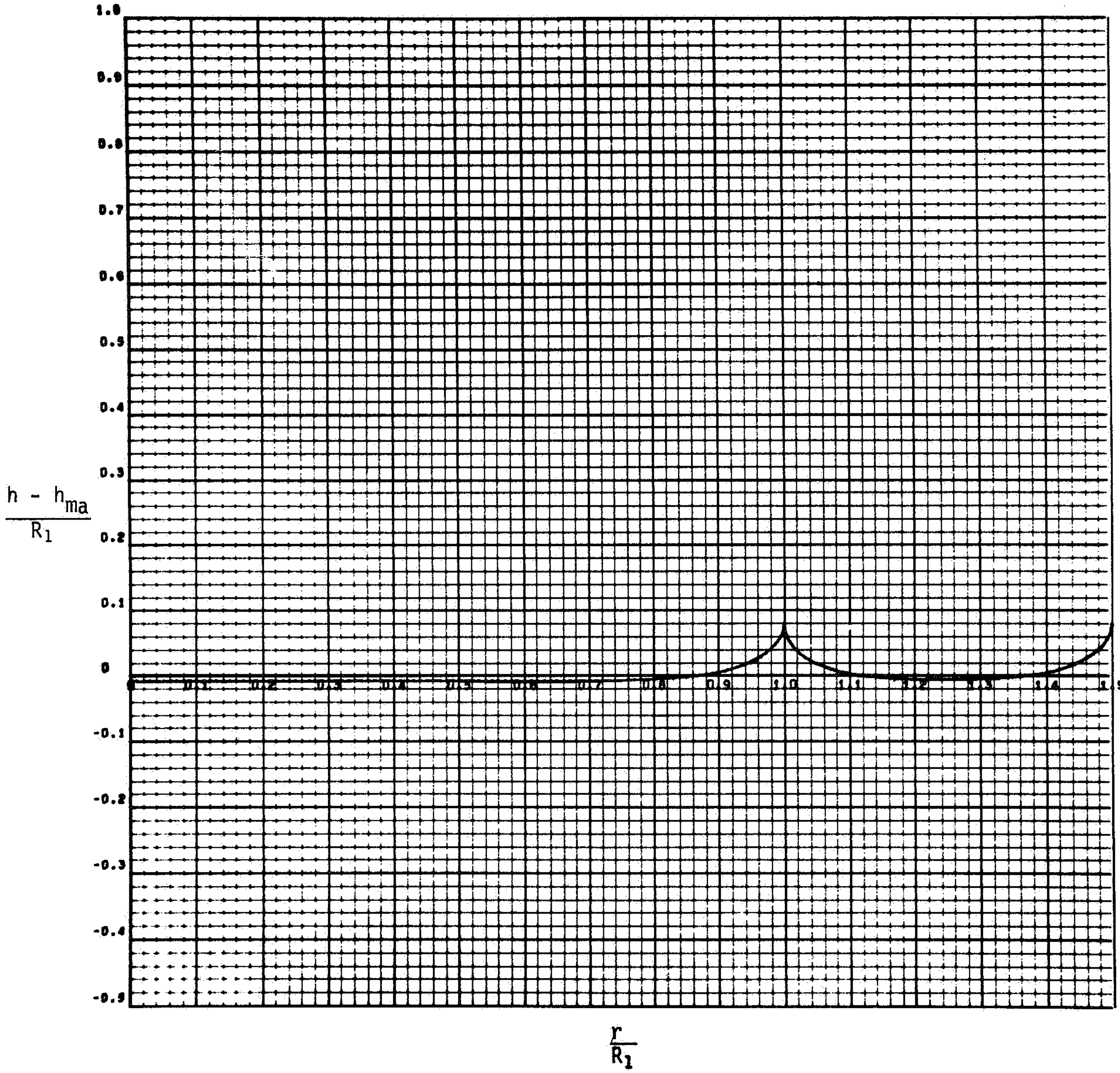

Figure 7. Static Surfaces $\frac{R_2}{R_1} = 1.5$, $B_o = 250$, $\theta=5°$

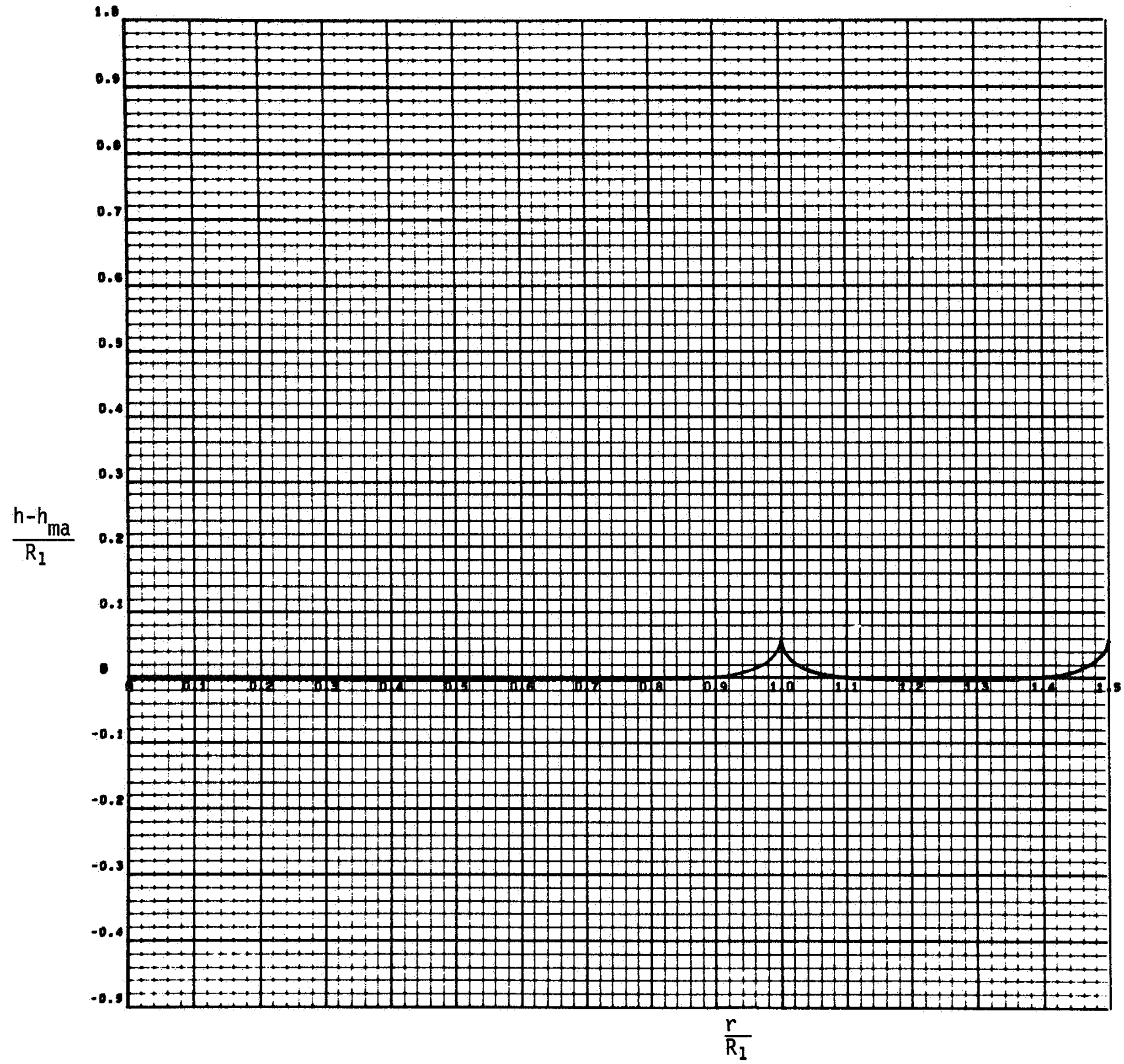

Figure 8. Static Surfaces $\frac{R_2}{R_1}$ = 1.5, B_o = 500, θ=5°

TABLE 1

COORDINATES OF SURFACES

$\frac{R_2}{R_1} = 1.5$ $B_o = 10$ $\theta = 5°$

$\Delta s = 2.5 \times 10^{-4}$ (annular region)

Central Region t	Central Region $\frac{h - h_{ma}}{R_1}$	Annular Region t	Annular Region $\frac{h - h_{ma}}{R_1}$
0.00000	-0.13601	1.00000	0.32979
0.12497	-0.13352	1.01174	0.28164
0.24971	-0.12575	1.03670	0.23849
0.37392	-0.11179	1.07185	0.20310
0.49698	-0.09004	1.11417	0.17667
0.61775	-0.05801	1.16105	0.15953
0.73398	-0.01227	1.21034	0.15159
0.84124	0.05154	1.26027	0.15254
0.93125	0.13774	1.30930	0.16203
0.98961	0.24746	1.35600	0.17970
1.00002	0.30111	1.39894	0.20519
		1.43657	0.23801
		1.46717	0.27744
		1.48888	0.32237
		1.49980	0.37104
		1.50001	0.37328
		1.23533	0.15098

$$\frac{h_{mb} - h_{ma}}{R_1} = 0.19924$$

TABLE 2

COORDINATES OF SURFACES

$\frac{R_2}{R_1} = 1.5$ $B_o = 25$ $\theta = 5°$

$\Delta s = 2.5 \times 10^{-4}$ (annular region)

Central Region

t	$\frac{h - h_{ma}}{R_1}$
0.00000	-0.07220
0.12500	-0.07145
0.24997	-0.06897
0.37487	-0.06401
0.49954	-0.05505
0.62351	-0.03925
0.74534	-0.01168
0.86066	0.03590
0.95657	0.11483
1.00000	0.21716

$$\frac{h_{mb} - h_{ma}}{R_1} = 0.07970$$

Annular Region

t	$\frac{h - h_{ma}}{R_1}$
1.00000	0.19996
1.01345	0.15235
1.04195	0.11150
1.08053	0.07990
1.12523	0.05769
1.17332	0.04426
1.22298	0.03885
1.27289	0.04094
1.32195	0.05032
1.36900	0.06708
1.41254	0.09152
1.45051	0.12391
1.48007	0.16407
1.49755	0.21072
1.50001	0.22753
1.23547	0.03868

TABLE 3

COORDINATES OF SURFACES

$\frac{R_2}{R_1} = 1.5$ $B_o = 50$ $\theta = 5°$

$\Delta s = 1.25 \times 10^{-4}$ (annular region)

Central Region t	Central Region $\frac{h - h_{ma}}{R_1}$	Annular Region t	Annular Region $\frac{h - h_{ma}}{R_1}$
0.00000	-0.03907	1.00000	0.14712
0.12500	-0.03891	1.01575	0.10025
0.25000	-0.03834	1.04849	0.06278
0.37499	-0.03698	1.09065	0.03614
0.49995	-0.03395	1.13753	0.01897
0.62476	-0.02723	1.18658	0.00949
0.74880	-0.01220	1.23645	0.00644
0.86894	0.02136	1.28635	0.00924
0.96986	0.09290	1.33554	0.01797
1.00000	0.16392	1.38307	0.03333
		1.42727	0.05653
		1.46519	0.08890
		1.49186	0.13091
		1.50000	0.16404
		1.23645	0.00644

$$\frac{h_{mb} - h_{ma}}{R_1} = 0.03985$$

TABLE 4

COORDINATES OF SURFACES

$\frac{R_2}{R_1} = 1.5$ $B_o = 75$ $\theta = 5°$

$\Delta s = 2.5 \times 10^{-4}$ (annular region)

Central Region

t	$\frac{h - h_{ma}}{R_1}$
0.00000	-0.02642
0.12500	-0.02638
0.25000	-0.02620
0.37500	-0.02572
0.49999	-0.02443
0.62494	-0.02098
0.74955	-0.01160
0.87168	0.01395
0.97564	0.08028
1.00002	0.13795

$$\frac{h_{mb} - h_{ma}}{R_1} = 0.02657$$

Annular Region

t	$\frac{h - h_{ma}}{R_1}$
1.00000	0.12440
1.01746	0.07841
1.05312	0.04374
1.09744	0.02087
1.14545	0.00711
1.19492	0.00005
1.24487	-0.00177
1.29476	0.00111
1.34411	0.00901
1.39203	0.02310
1.43671	0.04534
1.47431	0.07801
1.49747	0.12186
1.50002	0.13663
1.24487	-0.00177

TABLE 5

COORDINATES OF SURFACES

$\frac{R_2}{R_1} = 1.5$ $B_o = 100$ $\theta = 5°$

$\Delta s = 2.50 \times 10^{-4}$ (annular region)

Central Region

t	$\frac{h - h_{ma}}{R_1}$
0.00000	-0.01989
0.12500	-0.01987
0.25000	-0.01981
0.37500	-0.01961
0.49999	-0.01899
0.62498	-0.01702
0.74980	-0.01073
0.87295	0.00956
0.97903	0.07186
1.00002	0.12159

$$\frac{h_{mb} - h_{ma}}{R_1} = 0.01992$$

Annular Region

t	$\frac{h - h_{ma}}{R_1}$
1.00000	0.11056
1.01892	0.06530
1.05677	0.03306
1.10251	0.01315
1.15118	0.00188
1.20086	-0.00355
1.25083	-0.00467
1.30075	-0.00193
1.35021	0.00518
1.39845	0.01818
1.44348	0.03966
1.48059	0.07279
1.49983	0.11828
1.50000	0.12002
1.23834	-0.00475

TABLE 6

COORDINATES OF SURFACES

$\frac{R_2}{R_1} = 1.5$ $B_o = 250$ $\theta = 5°$

$\Delta s = 1.0 \times 10^{-4}$ (annular region)

Central Region

t	$\frac{h - h_{ma}}{R_1}$
0.00000	-0.00797
0.10000	-0.00797
0.20000	-0.00797
0.30000	-0.00797
0.40000	-0.00796
0.49999	-0.00794
0.59999	-0.00783
0.69998	-0.00734
0.79995	-0.00513
0.89933	0.00501
0.98693	0.04898
1.00000	0.07995

$$\frac{h_{mb} - h_{ma}}{R_1} = 0.00797$$

Annular Region

t	$\frac{h - h_{ma}}{R_1}$
1.00000	0.07520
1.01730	0.04010
1.05027	0.01788
1.08829	0.00569
1.12776	-0.00069
1.16762	-0.00391
1.20759	-0.00538
1.24758	-0.00575
1.28757	-0.00521
1.32753	-0.00356
1.36738	-0.00016
1.40684	0.00629
1.44496	0.01820
1.47867	0.03937
1.49908	0.07305
1.50001	0.07918
1.24258	-0.00576

TABLE 7

COORDINATES OF SURFACES

$\frac{R_2}{R_1} = 1.5$ $\quad B_o = 500$ $\quad \theta = 5°$

$\Delta s = 1.0 \times 10^{-4}$ (annular region)

Central Region t	Central Region $\frac{h - h_{ma}}{R_1}$	Annular Region t	Annular Region $\frac{h - h_{ma}}{R_1}$
0.00000	-0.00398	1.00000	0.05526
0.10000	-0.00398	1.02102	0.02279
0.20000	-0.00398	1.05752	0.00697
0.30000	-0.00398	1.09694	0.00043
0.40000	-0.00398	1.13684	-0.00220
0.49999	-0.00398	1.17682	-0.00324
0.59999	-0.00398	1.21682	-0.00362
0.69998	-0.00392	1.25681	-0.00367
0.79998	-0.00345	1.29681	-0.00346
0.89985	0.00075	1.33680	-0.00281
0.99082	0.03591	1.37676	-0.00120
1.00001	0.05772	1.41656	0.00269
		1.45538	0.01201
		1.49870	0.05040
		1.50001	0.05727
		1.24181	-0.00368

$$\frac{h_{mb} - h_{ma}}{R_1} = 0.00398$$

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APPENDIX

FORTRAN IV COMPUTER PROGRAM FOR INTERFACE CONFIGURATIONS IN CONCENTRIC CYLINDER TANKAGE SYSTEMS

Cross-Reference Between Symbols (Input and Output) Main Program

Algebraic Symbol	FORTRAN Symbol	Description
α	ALPHA	Local angle of inclination of surface
$\alpha_{i-1} + \left.\frac{d\alpha}{ds}\right\|_{i-1} \frac{\Delta s}{2}$	ALPME	
$1 + \int_0^s \cos\alpha \, ds$	ANTCOS	$t = \frac{r}{R_1}$
$\int_0^s \sin\alpha \, ds$	ANTSIN	$H - H_w$
B_0	BO	Bond number
$\frac{d\alpha}{ds}$	DADS	Local curvature of surface
$\left.\frac{d\alpha}{ds}\right\|_{s=0}$	DADSO	$\frac{d\alpha}{ds}$ at inner wall
Δs	DS	Increment of arc
$\frac{h_{mb} - h_{ma}}{R_1}$	DIFF	Nondimensional difference between mean heights
h_{ma}	HM	Mean height in annular region
H_w	HR1	Value of H at inner wall
R_1	R1	Inner radius

Algebraic Symbol	FORTRAN Symbol	Description
R_2	R2	Outer radius
$\frac{R_2}{R_1}$	OLIM	Ratio of outer to inner radius
π	PI	3. 14159
θ	THET	Contact angle in degrees
θ	THETA	Contact angle in radians
θ	XXX	Contact angle in radians (for subroutine)

FORTRAN IV LISTING OF MAIN PROGRAM

```
      DIMENSION ALPHA(2),X(2000),Y(2000),K(2000),BCDX(12),BCDY(12)
      DIMENSION ARRAY(8)
      DIMENSION XX(2000),YY(2000)
      EQUIVALENCE (P1,PI)
      DATA BCD/6H      /
      CALL CAMRAV(935)
      DO 1000 JJ = 1,12
      BCDX(JJ) = BCD
 1000 BCDY(JJ) = BCD
      CALL BC (BCDX(6), 6H  R/R1)
      CALL BC  (BCDY(6), 6H (H-HM)
      CALL BC  (BCDY(7), 6H)/R1  )
      PI=3.1415927
      READ(5,5) THET , R1, R2
      OLIM= R2/R1
      XXX = THET
      XXX = XXX * PI/180.
      THETA=THET /57.29578
      READ(5,96) KK
   96 FORMAT(I4)
      DO 488 N = 1,KK
      READ(5,5) BO
      IF(BO - 100.) 401,401,402
  401 DS = .00025
      GO TO 403
```

FORTRAN IV LISTING OF MAIN PROGRAM

```
  402 DS = .00010
  403 CONTINUE
      DX = 100.
      DY = (R2+R1)/(R2-R1)*COS(THETA)
      CALL FRED (B0, XXX, DS, XX, YY, JJ)
  105 HM = 0.0
      KOUNT=0
    5 FORMAT(3F10.0)
C**   B0=RHO*G*R1**2/T
      WRITE(6,1011) B0, DS
 1011 FORMAT(1H ,10X,3HB0=,1X,E15.8, E15.8)
      DX = DX + 1.
      DY = DY - 1.
      DADS0 = DX
      DALPDS = DADS0
      IF(DS.LT. .001)LL=50
      IF(DS.LT..0001)LL=100
      IF(DS.LT..00001)LL=1000
   10 HR1 = (DALPDS - SIN(P1/2.-THETA))/B0
      KOUNT=KOUNT + 1
      IF(KOUNT - 100) 50,30,30
   50 CONTINUE
      ANTCOS=1.0
      ANTSIN=0.0
      X(1)=1.0
```

FORTRAN IV LISTING OF MAIN PROGRAM

```
      Y(1)=0.0
      K(1) = 1
      IK = 2
      II = 1
      I = 1
      ALPHA(1)= -(PI/2.-THETA)
      WRITE(6,6) DALPDS, HR1
    6 FORMAT(1H0, 6HDALPDS, 2X, F10.5, 3HHR1, 2X, F10.5/)
   20 CONTINUE
      I=2
      ALPHA(I)= ALPHA(I-1) + DALPDS*DS
      ALPME = ALPHA(I-1) + DALPDS*DS/2.
      ANTSIN = ANTSIN + DS/6.*(SIN(ALPHA(I))+4.*SIN(ALPME)+SIN(ALPHA(I-1
     1)))
      SAVE= ANTCOS
      ANTCOS = ANTCOS + DS/6.*(COS(ALPHA(I))+4.*COS(ALPME)+COS(ALPHA(I-1
     1)))
      IF(II - II/LL*LL) 90,90,91
   90 IK = IK + 1
   91 CONTINUE
      X(IK) = ANTCOS
      Y(IK) = ANTSIN
      K(IK) = II
      II=II + 1
      IF(SAVE-ANTCOS) 51,51,22
```

FORTRAN IV LISTING OF MAIN PROGRAM

```
 51 CONTINUE
    DALPDS = B0*HR1 + B0*ANTSIN - SIN(ALPHA(1))/ANTCOS
    IF(DALPDS) 23,23,29
 29 CONTINUE
    ALPHA(1)= ALPHA(2)
    IF(ANTCOS - OLIM)   20,52,52
 52 CONTINUE
    WRITE(6,8) ANTCOS
  8 FORMAT(1H0,6HANTCOS, F10.5)
    WRITE(6,7) ALPHA(1)
  7 FORMAT(1H0,6HALPHA , F10.5)
    IF(ABS(ALPHA(1)+THETA-PI/2.) -.0005)   30,53,53
 53 IF(ALPHA(I) - (PI/2.-THETA)) 23,30,22
 22 DX = DX - (DX - DY)/2.
    IF(ABS(DADSO-DX)-.00000001) 101,101,103
101 DS = DS/2.
    GO TO 105
103 DADSO = DX
    DALPDS = DX
    WRITE(6,75) ALPHA(2),ALPHA(1),X(IK),SAVE,Y(IK)
 75 FORMAT(2X,10HALPHA(2) =,F10.5,2X,10HALPHA(1) =,F10.5,2X,7HT(II) =,
   1F10.5,2X,9HT(II-1) =,F10.5,2X,7HH(II) =,F10.5/)
    GO TO 10
 23 DX = DX + (DX - DY)/2.
    DY = DY + 2./3.*(DX-DY)
```

```
      IF(ABS(DADSO-DX)-.00000001) 102,102,104
  102 DS = DS/2.
      GO TO 105
  104 DADSO = DX
      DALPDS = DX
      WRITE(6,75) ALPHA(2),ALPHA(1),X(IK),SAVE,Y(IK)
      GO TO 10
   30 CONTINUE
      DIFF = 2./BO * (R1/(R2-R1) - 1.) * COS(THETA)
      DO 666 I=1,IK
      Y(I)=                     HM/R1 + 1./BO*(DADSO-COS(THETA)-2.*R1*COS(THE
     1TA)/(R2-R1)) + Y(I) + DIFF
  666 CONTINUE
      ALPHA(2) = ALPHA(2) * 57.29578
      WRITE(6,66) BO, ALPHA(2), THET , DS
   66 FORMAT(1X,10HBOND NO. =,F10.0,3X,7HALPHA =,F10.5,3X,7HTHETA =,F10.
     25,3X,9HARC LG. =,F10.6/)
      WRITE(6,67)
   67 FORMAT(      5X,7HPT. NO.,7X,4HR/R1,7X,9H(H-HM)/R1/)
      WRITE(6,68) (K(J), X(J), Y(J), J = 1,IK)
   68 FORMAT(I10, 4X, F10.5, 4X, F10.5)
      YMIN = 1000.
      YMAX = -1000.
      DO 4000 II = 1,IK
      IF(Y(II).LT.YMIN) YMIN=Y(II)
```

```
      IF(Y(II).GT.YMAX) YMAX=Y(II)
 4000 CONTINUE
      YB = - .5
      YT = 1.0
      XL =  XX(1)
      XR = OLIM
      CALL  SCOUTV
      WRITE(16,1600)
     1BO,
     1THET ,
     1OLIM      ,
     1Y(1)      ,
     1Y(IK)      ,
     1YMIN       ,
     1DS,
     1DIFF
 1600 FORMAT(1H1,5X,12HBOND NO. =   ,F10.5,//,5X,31HACTUAL CONTACT ANGLE
     1,THETA =  ,F10.5,//,5X,9HR2/R1 =  ,F10.5,//,5X,21HLEFT WALL HEIGHT
     2  =  ,F10.5,//,5X,21HRIGHT WALL HEIGHT  =  ,F10.5,  //
     35X,12H MINIMUM =  ,F10.5,//5X,6HDS =  ,F10.5
     4,//5X,55HMEAN HEIGHT(ANNULAR REG.) - MEAN HEIGHT(CENTRAL REG.) =,
     5F10.5)
      CALL QUIK3L (-1,XL,XR,YB,YT,42,BCDX,BCDY,JJ,XX,YY)
      CALL QUIK3L( 0,1.0,OLIM,YB,YT,42,BCDX,BCDY,IK,X,Y)
  488 CONTINUE
```

FORTRAN IV LISTING OF MAIN PROGRAM

```
STOP
END
```

CROSS-REFERENCE BETWEEN SYMBOLS
(INPUT AND OUTPUT)
SUBROUTINE FRED

Algebraic Symbol	FORTRAN Symbol	Description
a	A	Radius of tank
α	ALPHA	Local angle of inclination of surface
B_o	BO	Bond number
B_oH_o	BOHO	
$\sin \alpha_1$	SALP1	
$\sin \alpha_2$	SALP2	
$\cos \alpha_1$	CALP1	
$\cos \alpha_2$	CALP2	
$\left.\frac{d\alpha}{ds}\right\|_{s=0}$	DADSO	Curvature on axis of symmetry
$\frac{d\alpha}{ds}$	DALPDS	Curvature at general point
H_o	HO	Value of H on axis of symmetry
h_m	HM	Mean height of surface
$\frac{\Delta s}{2}$	HALFDS	Half of increment of arc length
Δs	DS	Increment of arc length
	KOUNT	Number of iterations
	LL	Integer determining what data is printed (every 10 or every 10^2)
θ	THETA	Contact angle in radians

Algebraic Symbol	FORTRAN Symbol	Description
$\theta - \frac{\pi}{2}$	THET	
$\int_o^s \cos \alpha \, ds$	TCOS	$t\left(= \frac{r}{a}\right)$
$\int_o^s \sin \alpha \, ds$	TSIN	$H - H_o$
	XALP	α at wall
	YI	Storage location for surface heights
	XI	Storage location for radial distances

FORTRAN IV PROGRAM LISTING OF SUBROUTINE FRED

```
      SUBROUTINE FRED(BO, THETA, DS, X, Y, II)
C*
C*
C*
C****     CALCULATE THE APPROXIMATIONS OF AXIALLY SYMMETRIC
C***       STATIC SURFACES AT ZERO CONTACT ANGLE
C*
C*
C*
      DIMENSION ALPHA(2    )
      DIMENSION X(2000),Y(2000),Z(10),W(10)
C
C     CALL CAMRAV(935)
C
      HM=0.0
      A=1.0
      PI=3.1415927
    4 CONTINUE
    1 CONTINUE
  256 FORMAT(I2,2F10.0)
      IF(L.EQ.1)GO TO 9999
      IF(DS.GT.0.00005)LL=100
      IF(DS.GT.0.0005)LL=10
    9 FORMAT(1H ,2X,7H  RHO= ,F10.5,/,3X,7H    G= ,F10.5,/,
     13X,7H    T= ,F10.5,/,3X,7H    A= ,F10.5,/,3X,7H   HM= ,F10.5,/,
```

```
     23X,7HTHETA= ,F10.5,/,
     23X,7H    DS= ,F10.5)
      KOUNT = 0
      DX=.5
    5 FORMAT(5F10.0)
 1011 FORMAT(1H ,10X,3HB0=,1X,E16.8)
      WRITE(6,1011)B0
      WRITE(6,1012)THETA
 1012 FORMAT(1H ,10X,6HTHETA=,1X,E16.8,4H DEG )
      WRITE(6,1013)DS
 1013 FORMAT(1H ,10X,3HDS=,1X,E15.8)
      HALFDS=DS/2.
      THET=THETA-PI/2.
      DALPDS   =.5
      DADS0=DALPDS
   10 H0=2.*DALPDS   /B0
      B0H0=B0*H0
      KOUNT = KOUNT+1
      IF (KOUNT.GT.100) GO TO 30
      TCOS=0.0
      TSIN=0.0
      X(1)=0.0
      Y(1)=0.0
      DALP=0.0
      IK=1
```

FORTRAN IV PROGRAM LISTING OF SUBROUTINE FRED

```
      II=1
      SALP1=0.0
      CALP1=1.0
    6 FORMAT(1H0,6HDALPDS,2X,F10.5,I4)
      ALPHA(1)=0.0
   20 CONTINUE
      ALPHA(2)=ALPHA(1)+DALPDS*DS
      SALP2=SIN(ALPHA(2))
      TSIN= TSIN+(SALP2+SALP1)*HALFDS
       SAVE=TCOS
      CALP2=COS(ALPHA(2))
      TCOS=TCOS+(CALP2+CALP1)*HALFDS
      SALP1=SALP2
      CALP1=CALP2
      IJ=MOD(IK,LL)
      IF(IJ.NE.0)GO TO 80
      II=II+1
      X(II)=TCOS
      Y(II)=TSIN
   80 IK=IK+1
      DALPDS=B0H0+B0*TSIN-SALP2/TCOS
      ALPHA(1)=ALPHA(2)
      IF(SAVE.GT.TCOS.AND.TCOS+.000001.LT.1.)GO TO 22
      IF(TCOS+.000001.LT.1.0)GO TO 20
    8 FORMAT(1H0,6HINTCOS,F10.5)
```

```
      XALP=ALPHA(2)*57.29578
      WRITE(6,660)XALP
    7 FORMAT(1H0,6HALPHA ,F10.5)
      IF(ABS(SAVE1-ALPHA(2)).LT..000001) GO TO 30
      SAVE1 =ALPHA(2)
      IF(ABS(ALPHA(2)+THET).LT..0002)GO TO 30
      IF(ALPHA(2).LT.(0.0-THET    ))GO TO 23
   22 CONTINUE
      DX=DX/2.
      DADS0=DADS0-DX
      DALPDS=DADS0
      GO TO 10
   23 DX=DX/2.
      DADS0=DADS0+DX
      DALPDS=DADS0
      GO TO 10
   30 CONTINUE
      II=II+1
      X(II)=TCOS
      Y(II)=TSIN
       DO 666 I=1,II
      Y(I)=HM/A+2./B0*(DADS0-COS(THETA))+Y(I)
  666 CONTINUE
   65 FORMAT(1H0,//,5X,5E15.8,/,5X,5E15.8)
  660 FORMAT(1H0,10X,6HALPHA=,1X,E16.8,4H DEG)
```

FORTRAN IV PROGRAM LISTING OF SUBROUTINE FRED

```
      WRITE(6,66)(X(J),J=1,II)
      WRITE(6,66)(Y(J),J=1,II)
   66 FORMAT(1H0,10F10.5)
   67 FORMAT(1H1,5X,3HIK=,I8)
      RETURN
 9999 CONTINUE
      STOP
      END
```

FORTRAN IV PROGRAM LISTING OF SUBROUTINE BC

```
SUBROUTINE BC (A,B)
A = B
RETURN
END
```

FORTRAN IV PROGRAM LISTING OF SUBROUTINE RPRIN

```
      SUBROUTINE RPRINT(BCD,X)
      WRITE(6,1401)BCD,X
 1401 FORMAT (1H ,12H*** TEST ***,10X,A6,1X,2H= E15.8)
      RETURN
      END
```

SAMPLE DATA

```
    5.0         2.          3.
7
    10.
    25.
    50.
    75.
   100.
   250.
   500.
```

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13. ABSTRACT

A computer program has been developed to graphically and analytically define the static shape of the liquid-gas interface in the annular region between any two concentric cylinders in an axial force field for any Bond number and contact angle. The program also includes as a subroutine a previously reported method of calculating the static shape of the liquid-vapor interface within a single cylindrical tank. Static fluid surface coordinates for both the annular and central regions are given for Bond numbers (with inner cylinder radius as characteristic length) ranging from 10 to 500, for a contact angle of 5 degrees, and for a radius ratio of 1.5.

14. KEY WORDS

hydrostatics

fluid mechanics

free surface

low g

Bond number

surface tension