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DYNAMIC TEST VEHICLE
INTERNAL DAMPING STUDY

by

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FOREWORD

This report presents the results of The Dynamic Test Vehicle Internal Damping Study, performed by Chrysler Huntsville Operations, under contract No. NAS8-21056 to the Aero-Astrodynamics Laboratory of the George C. Marshall Space Flight Center. Technical coordination for this effort was provided by the Contracting Officer's representative, Mr. James G. Papadopoulos, R-AERO-DD.

This investigation of Saturn Dynamic Test results determines damping characteristics of individual stage and interfaces and develops a method of correlating these distributions to the vehicle's bending mode damping. Fulfillment of this scope of work has resulted in a three-mass model to obtain stage and interstage damping coefficients. This model was also used to correlate these coefficients with the vehicle bending mode damping. The method developed reduces modal damping into individual stages and interstage damping coefficients.

Acknowledgement is extended to James K. Moore, Chrysler Huntsville Operations, for his valuable assistance in the final preparation of Section II, Calculation of Generalized Damping Coefficients, as well as programming and documenting the damping study - phase angle method, Appendix B.

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SUMMARY

Objectives of the Dynamic Test Vehicle Internal Damping Study were (1) to determine the damping characteristics of the individual stages and interfaces by utilizing results of the Saturn dynamic tests, and (2) to develop a method of computing the vehicle's bending mode damping based on the damping characteristics of individual stages and interfaces. The method described in Section II was utilized to obtain the modal damping characteristics of the vehicle. These values were then related to equivalent stage and interstage damping coefficients as described in Section III.

Saturn IB, SA-202, dynamic test data was used for the numerical solutions. This information was used to obtain the percent of generalized damping and the stage and interstage damping coefficients. The methods used to obtain the percent of generalized damping were the phase angle method and the response method. The Saturn IB was modeled as a three-mass system, the three masses representing three stages. This system was used in conjunction with the generalized damping to obtain stage and interstage damping coefficients. FORTRAN IV programs developed for use on the General Electric 415 computer are included. The percent of critical generalized damping, as computed by the various methods is 0.28% to 2.26%.

SECTION I. INTRODUCTION

There has been much dynamic testing of full-scale prototypes of Saturn flight vehicles. For these tests, the vehicle is suspended vertically and excited by electrodynamic shaking at the engine gimbal plane. These tests yield valuable information on the vehicle's dynamic characteristics in studying the modal damping coefficients as used in the normal mode method. However, little analytical study has been expended toward relating vehicle modal damping to damping distributions of individual stages and interfaces. This study fills this need by:

(1) Determining the damping characteristics of the individual stages and interfaces by utilizing results of the Saturn dynamic test, and

(2) Developing a method of computing the vehicle's bending mode damping coefficients based on the damping characteristics of individual stages and interfaces.

The normal mode method is in common use to obtain the response of a structure to transient loading. This method, as given by Hurty and Rubinstein (1) and others, transforms the n simultaneous equations of motion into n independent equations of motion, permitting a solution for response of the system by superposition. Further, for response of the system to frequencies not exceeding the first m resonant frequencies, an adequate response mode can be fabricated using the first m natural modes. Each of the transformed equations of motion will have the form

$$M z'' + C z' + K z = F(x,t)$$

where M, C, and K are the generalized mass, damping, and stiffness and F(x,t) the generalized force. The relation between M, C, and K is

$$K = \omega^2 M$$

$$C = 2\zeta\omega M$$

where ω is the natural frequency and ζ is the percent critical generalized damping.

The first m modes-in this case, the first three-are obtained from the Saturn IB dynamic test for S-202, pitch and yaw bending. The mass value is obtained from these modes and the physical properties of SA-202. The damping coefficient, ζ, is obtained by one of the methods valid for a single degree of freedom system. For this study the phase angle method and the response method were used. In addition to damping by these two methods, ζ has been computed during the dynamic test program. Damping values obtained in these three ways are used to obtain generalized damping, C. Thus, for each test condition three values of generalized damping are obtained corresponding to the first three modes.

A three-mass model corresponding to the three stages is then used to transform the generalized damping coefficients into the stage and interstage damping coefficients. This method establishes the three-point modes corresponding to the three masses.

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SECTION II. CALCULATION OF GENERALIZED DAMPING COEFFICIENTS

A. INTRODUCTION

An indication of the damping in a structural system is the rate of change of phase angle vs. the rate of change of frequency as recorded by an accelerometer during a response sweep. This information recorded during the SA-202 Saturn IB dynamic test is used to obtain the modal damping or generalized damping coefficients for the first three bending frequencies of the assembled structure. First, the damping as indicated at each accelerometer is obtained, and second, these individual damping coefficients are then assembled into a modal damping coefficient using the equations of motion for a simplified structural model.

B. CALCULATION OF THE RATE OF CHANGE OF PHASE ANGLE vs. THE RATE OF CHANGE OF FREQUENCY

For each frequency the phase angle is obtained at each accelerometer location. From the dynamic test data three files at frequencies bounding the resonant frequency are selected and the phase angles at each accelerometer location corresponding to the three frequencies are obtained from the data (figure 1). Thus, for each bending frequency the following information is obtained (ω_2 is the frequency close to the resonant frequency):

Frequency	ω_1	ω_2	ω_3
Phase Angle at Accelerometer Location i	$\theta_{1,1}$	$\theta_{1,2}$	$\theta_{1,3}$
	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$
	$\vdots$	$\vdots$	$\vdots$
	$\theta_{i,1}$	$\theta_{i,2}$	$\theta_{i,3}$

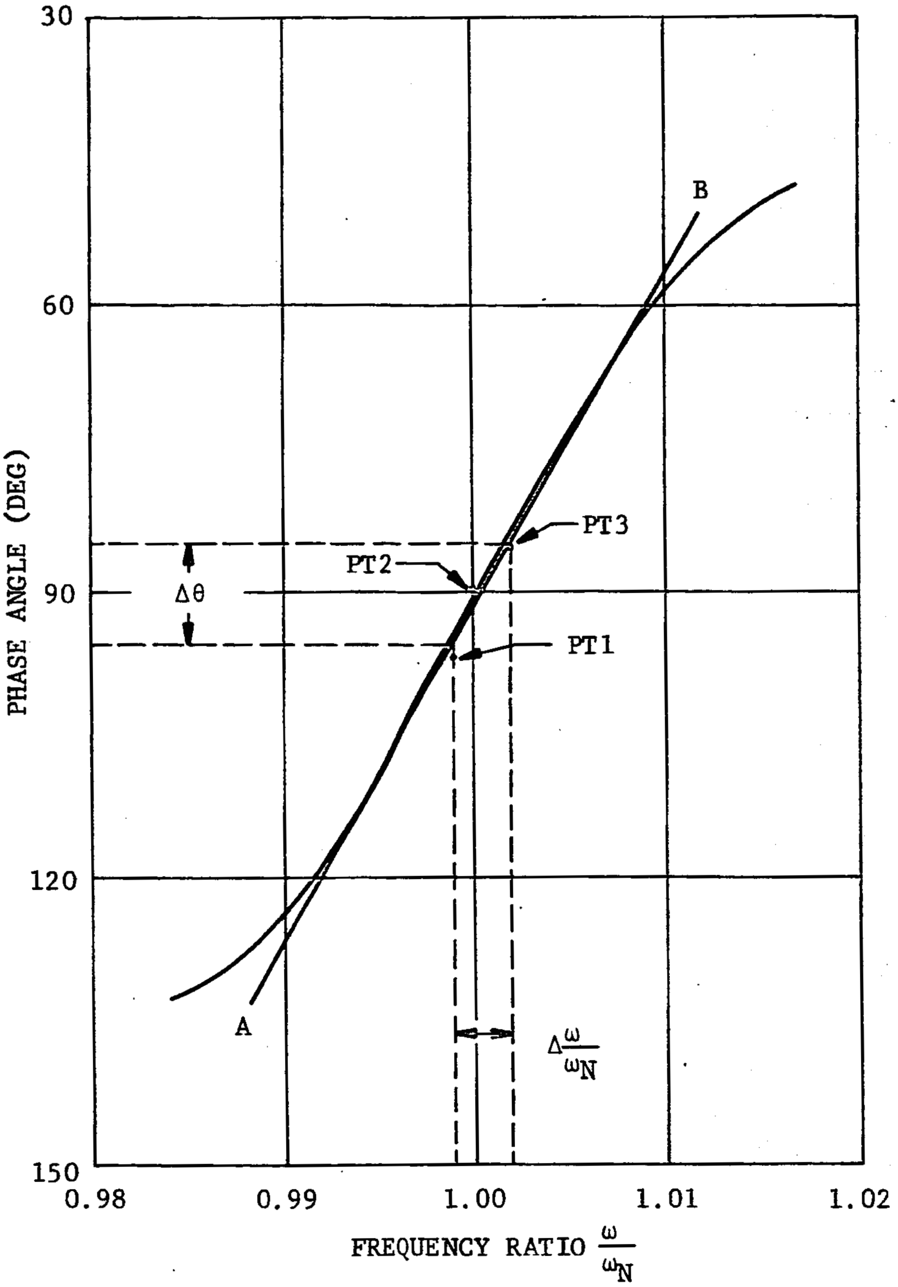
With this data, the rate of change of phase angle vs. rate of change of frequency, $\Delta\theta/\Delta\omega$, may be calculated as the slope from the least square straight line curve fit. The equation of the straight line is

$$a_o + a_1\omega = \theta$$

where a_1 is $\Delta\theta/\Delta\omega$ the desired slope.

To calculate a_o and a_1 the first array must be set up in the form:

$$\begin{matrix} 1 & \omega_1 & \theta_1 \\ 1 & \omega_2 & \theta_2 \\ 1 & \omega_3 & \theta_3 \end{matrix}$$



LEGEND

PT1 - IS FROM FILE 700102
PT2 - IS FROM FILE 700103
PT3 - IS FROM FILE 700104
THE LEAST SQUARE STRAIGHT
LINE CURVE FIT IS THE LINE
$\overline{AB}$

FIGURE 1. TYPICAL PHASE ANGLE vs. FREQUENCY

where the first column is a weight column, which is set to unity as all values are assumed to have equal validity. The elements in each row of the first array are multiplied by the elements of the second column forming a second array.

$$\begin{matrix} \omega_1 & \omega_1^2 & \omega_1\theta_1 \\ \omega_2 & \omega_2^2 & \omega_2\theta_2 \\ \omega_3 & \omega_3^2 & \omega_3\theta_3 \end{matrix}$$

where ω_2 and θ_2 are equal to ω_N and θ_N, respectively.

The normal equations for the solution of a_o and a_1 are

$$B_{11}\, a_o + B_{12}\, a_1 = C_1$$

$$B_{21}\, a_o + B_{22}\, a_1 = C_2$$

where $B_{11} = 3$

$$B_{12} = (\omega_1 + \omega_2 + \omega_3)$$

$$C_1 = (\theta_1 + \theta_2 + \theta_3)$$

$$B_{21} = (\omega_1 + \omega_2 + \omega_3)$$

$$B_{22} = (\omega_1^2 + \omega_2^2 + \omega_3^2)$$

$$C_2 = (\omega_1\theta_1 + \omega_2\theta_2 + \omega_3\theta_3)$$

The solution for a_1 yields the desired $\Delta\theta/\Delta\omega$.

The damping coefficient (ξ) at each accelerometer is calculated by

$$\xi = \frac{57.3}{2\omega_N \frac{\Delta\theta}{\Delta\omega}}$$

C. EQUATIONS OF MOTION

Figure 2 is an illustration of the structural model used to obtain the equation of motion. At each mass an equation of motion will be of the form:

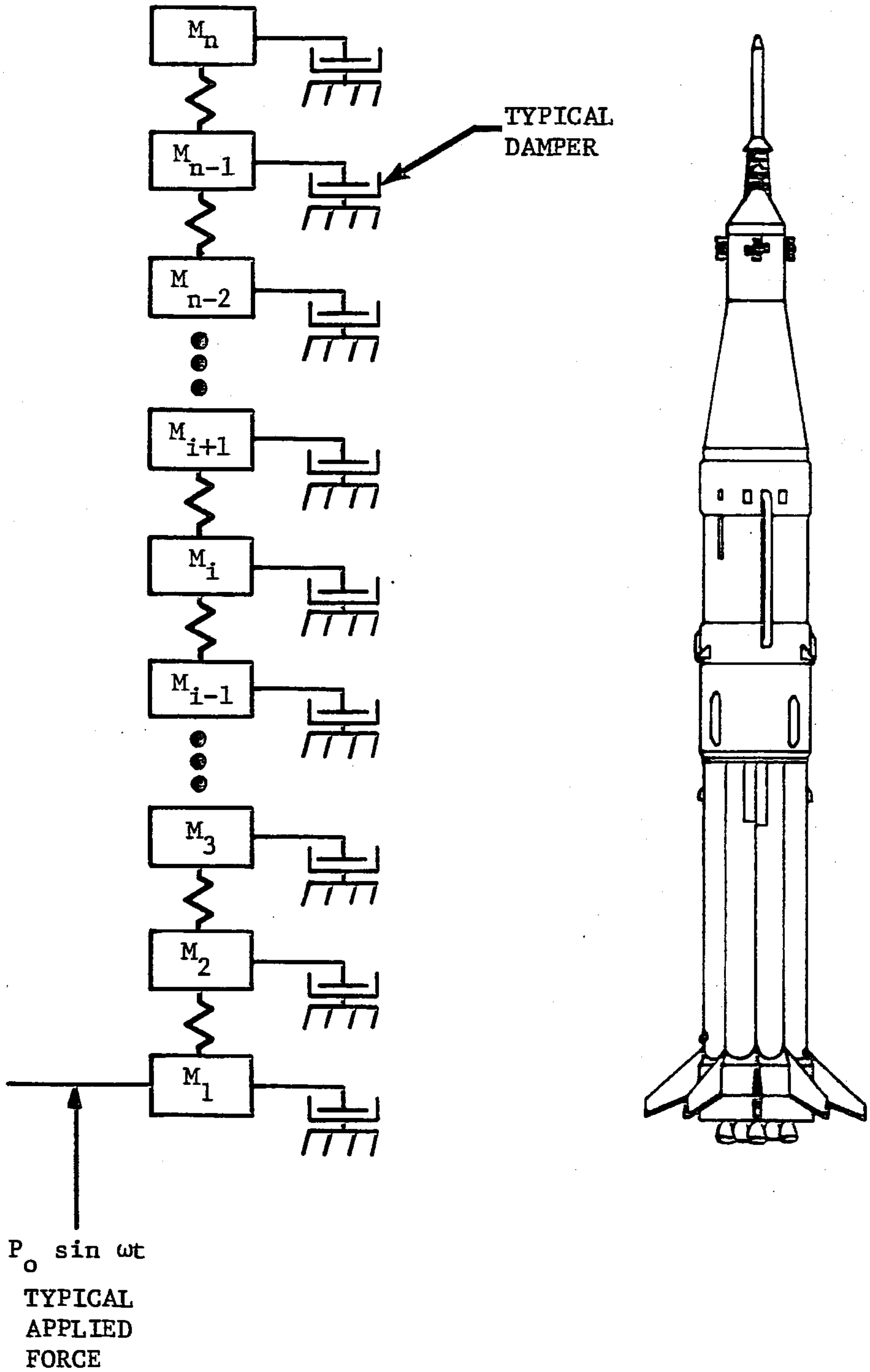


FIGURE 2. STRUCTURAL MODEL OF THE SATURN I-B

$$m\ddot{w} + C\dot{w} + Kw = P_o \sin \omega t$$

$$\ddot{w} + \frac{C}{m}\dot{w} + \frac{K}{m} w = \frac{P_o}{m} \sin \omega t$$

$$\ddot{w} + 2\xi\, \omega_N\, \dot{w} + \frac{K}{m} w = \frac{P_o}{m} \sin \omega t$$

or in matrix form:

$$\left[1\right] \left\{\ddot{w}\right\} + \left[2\xi\, \omega_N\right] \left\{\dot{w}\right\} + \left[\frac{K}{m}\right] \left\{w\right\} = \left\{\frac{P_o}{m}\right\} f(t)$$

where $\left\{\frac{P_o}{m}\right\} = \frac{P_o}{m}\left\{\ \right\}$

$\left\{\ \right\}$ = a matrix with one element equal to unity at the point of applied force and all other elements equal to zero

$f(t) = \sin \omega t$

m = the lumped mass at the point of applied force

The normal mode coordinate transformation is

$\{W\} = [\theta]\ \{z\}$

where $[\theta]$ is the matrix of modal columns.

Making this substitution and premultiplying by $[\theta]^T$ the matrix equation becomes

$$\left[\theta\right]^T \left[1\right] \left[\theta\right] \left\{\ddot{z}\right\} + \left[\theta\right]^T \left[2\xi\, \omega_N\right] \left[\theta\right] \left\{\dot{z}\right\}$$
$$+ \left[\theta\right]^T \left[\frac{K}{m}\right] \left[\theta\right] \left\{z\right\} = \left[\theta\right]^T \left\{\frac{P_o}{m}\right\} f(t)$$

Let

$[M] = [\theta]^T\ [\ 1\ ]\ [\theta]$

$[C] = [\theta]^T [\ 2\xi\ \omega_N\ ]\ [\theta]$

$[K] = \omega^2\ [M]$

Then

$$\left[M\right] \left\{\ddot{z}\right\} + \left[C\right] \left\{\dot{z}\right\} + \left[K\right] \left\{z\right\} = \left[\theta\right]^T \left\{\frac{P_o}{m}\right\} f(t)$$

$$M_j = \sum_{i=1}^{n} \theta_{ij}^2$$

$$C_j = \omega_{Nj} \sum_{i=1}^{n} \left(2\xi_{ij}\right) \theta_{ij}^2$$

$$K_j = \omega^2 M_j$$

$$\zeta = \frac{C_j}{2M_j \, \omega_{Nj}}$$

where

n = number of points

θ_{ij} = the displacement at the i point in the j mode

Note: [M], [C], and [K] are not generalized terms in the usual manner.

SECTION III. STAGE AND INTERSTAGE DAMPING

A. INTRODUCTION

An initial survey of the problem of obtaining the stage and interstage damping indicated that the desired values were a set of damping coefficients which would yield modal damping from values obtained from the phase angle or similar methods. The appropriate damping values would indicate the internal damping in each stage and the damping between the stages. A convenient way to represent the vehicle is by a three-mass system connected by springs and dampers. The three masses represent the three stages: S-IB, S-IVB, and upper stages. To describe the motion in a three-mass system, it is necessary to obtain the three-point modes. Three modes will suffice for identification of the three natural modes of vibration. Following below is a description of how the three-point mode shapes are established and how they are used in conjunction with the modal damping to obtain the stage and interstage damping.

B. ESTABLISHMENT OF THE THREE-POINT MODES

The property of the mode which is to be retained in establishing the three-point mode is one of orthogonality with respect to mass; that is, $[\theta]^T[\,m\,][\theta]$ will be diagonal matrix, or, in a practical sense, the off-diagonal terms will be small with respect to the diagonal terms. For the first three modes the relation will be

$$[\theta]^T[\,m\,]_{n\times n}\,[\theta]_{n\times 3}$$

where $[\theta]$ represents the modes comprised of n points as measured on the vehicle. It is desired to retain the same relation for the three-point mode.

That is,

$$[\Theta]^T[\,m\,]_{3\times 3}\,[\Theta]_{3\times 3} = [\theta]^T[\,m\,]_{n\times n}\,[\theta]_{n\times 3}$$

where θ is the Saturn IB mode shape as shown in Figure 3 and Θ is the three-point mode shape as shown in Figure 4.

The relationships of the mass-weighted modes can be defined in the following manner.

The generalized mass values of the two modes can be expressed by

$$[Y]^T[Y] = [y]^T[y]$$

where $[y] = [\theta\sqrt{m}]$

$$[Y] = [\Theta\sqrt{m}]$$

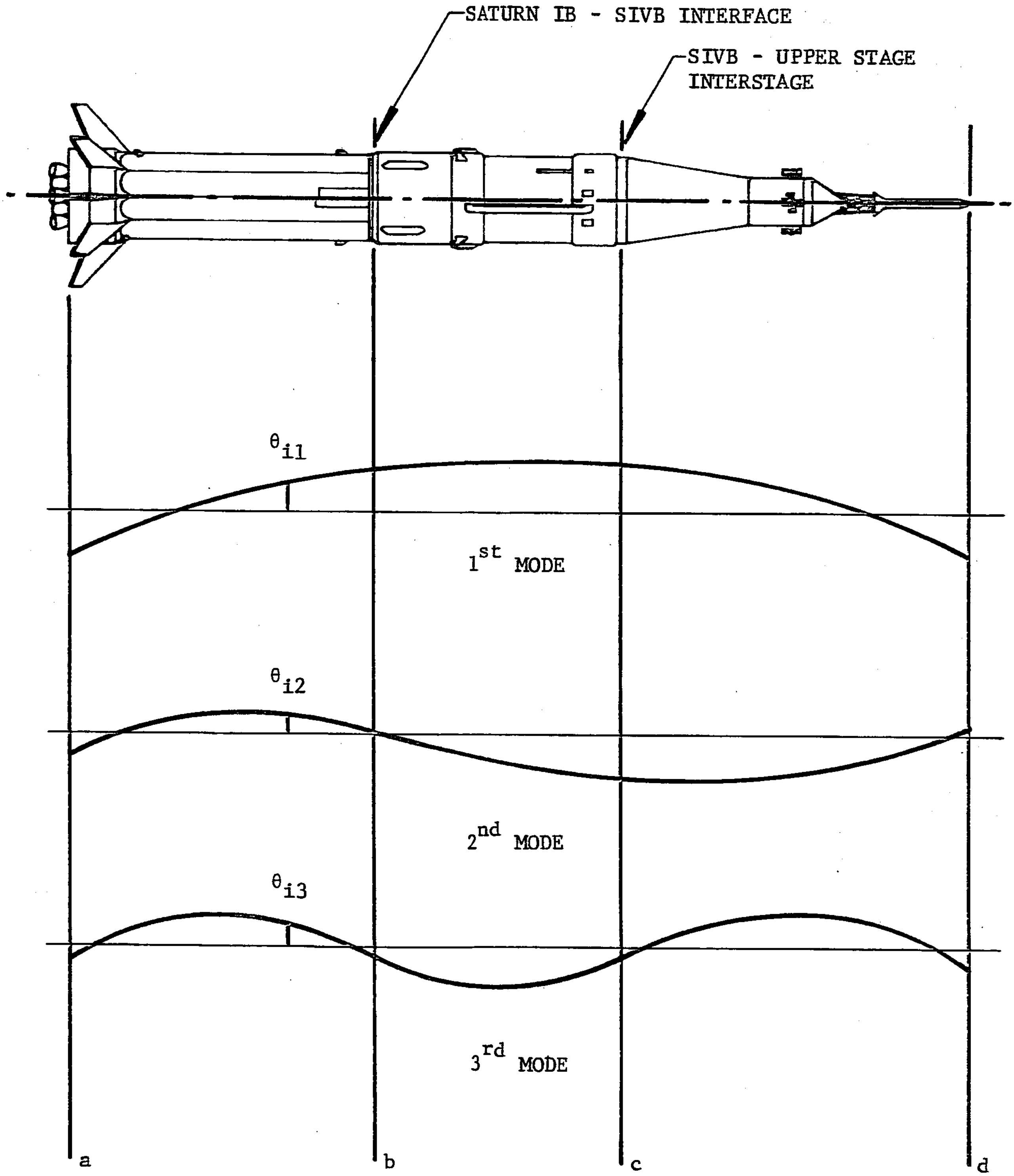


FIGURE 3. FIRST, SECOND, AND THIRD BENDING MODES, SATURN IB DYNAMIC TEST

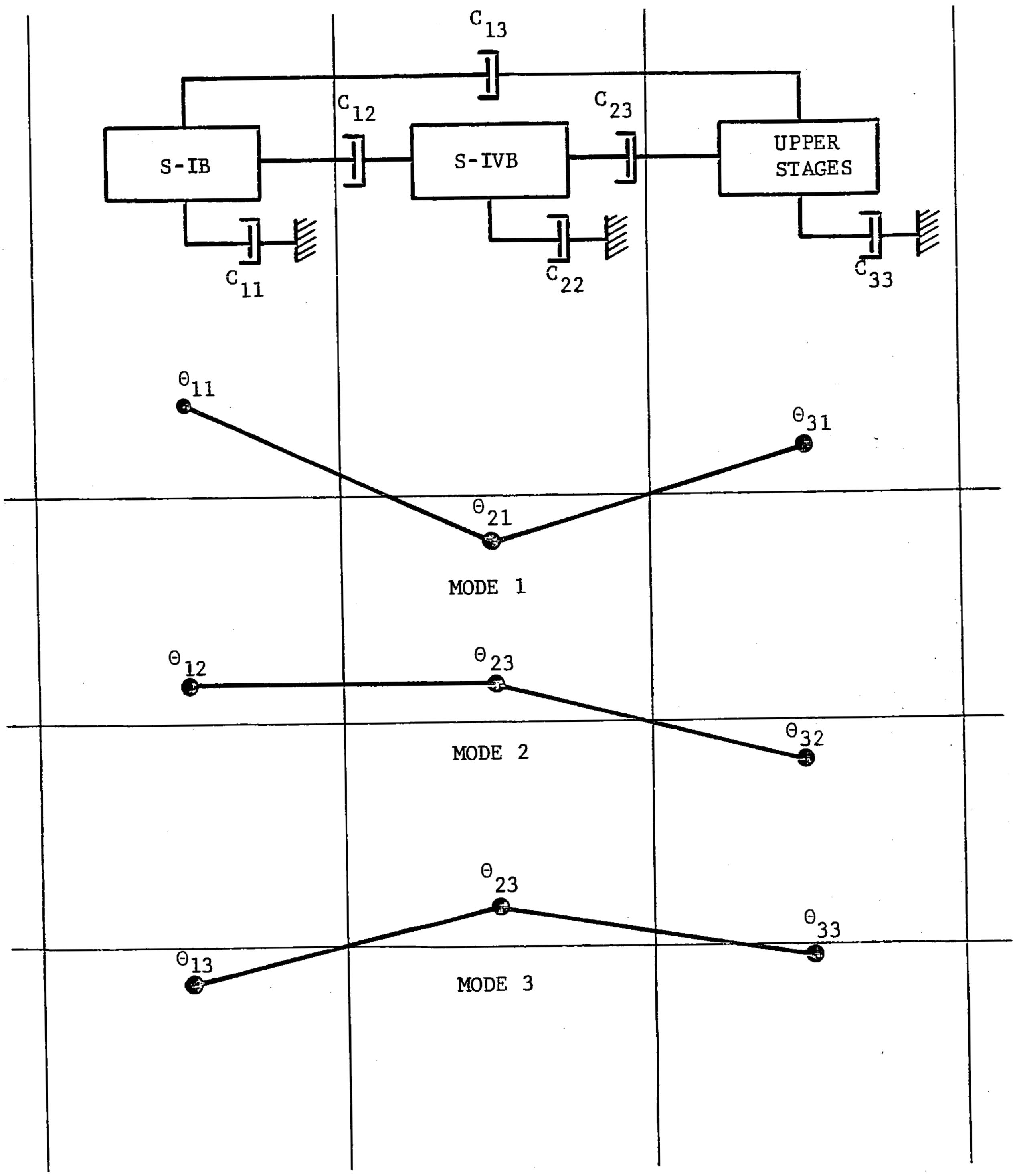


FIGURE 4. FIRST, SECOND, AND THIRD BENDING MODES, THREE-MASS MODEL

The displacement, Y_{1j}, describes the motion of the mass M_1 of the S-IB in the three-point system. This is the portion of the continuous structure between a and b. In like manner, Y_{2i} describes the motion of the S-IVB, or that portion of structure between b and c; and Y_{3i} describes the motion of the upper stages between c and d.

The orthogonal relationship between the ℓ^{th} and the m^{th} mode may be expressed as

$$\sum_{i=1}^{n} y_{i\ell}\, y_{im} = \sum_{i=1}^{n} \theta_{i\ell}\, m_i\, \theta_{im} = 0, \; m \neq \ell$$

for the dynamic test modes, and as

$$\sum_{i=1}^{3} Y_{i\ell}\, Y_{im} = 0 \qquad m \neq \ell$$

for the three-point modes.

For the three-point modes

$$\sum_{i=1}^{3} Y_{i\ell}\, Y_{im} = 0$$

$$= Y_{1\ell}\, Y_{1m} + Y_{2\ell}\, Y_{2m} + Y_{3\ell}\, Y_{3m} = 0$$

where the displacement, $Y_{i\ell}$, refers to the displacement of the i^{th} stage in the ℓ^{th} mode.

The assumption is made that the orthogonal relation

$$\sum_{i=1}^{n} y_{i\ell}\, y_{im}$$

for the physical mode in the region of each stage is equal to this same summation for the quasiphysical modes $Y_{i\ell}$. That is,

$$\sum_{i=1}^{n_k} y_{i\ell}\, y_{im} = Y_{k\ell}\, Y_{km} \qquad \text{No sum on } \ell$$

which is a set of 9 simultaneous equations, where y_{i1} is the displacement and n_k is the number of displacements on the k^{th} stage. These 9 simultaneous equations may be arranged in three sets for k = 1, 2, 3 and solved for the $Y_{k\ell}$'s. As an example, for k = 1,

$$Y_{11}\, Y_{12} = \sum_{i=1}^{n_1} y_{i1}\, y_{i2}$$

$$Y_{12}\, Y_{13} = \sum_{i=1}^{n_1} y_{i2}\, y_{i3}$$

$$Y_{13}\, Y_{11} = \sum_{i=1}^{n_1} y_{i3}\, y_{i1}$$

for which solutions for the Y's are

$$Y_{11} = \frac{(\Sigma\, y_{i1}\, y_{i2})\ (\Sigma\, y_{i3}\, y_{i1})}{\sqrt{\Sigma\, y_{i2}\, y_{i3}}}$$

$$Y_{12} = \frac{\Sigma\, y_{i1}\, y_{i2}}{Y_{11}}$$

$$Y_{13} = \frac{\Sigma\, y_{i3}\, y_{i1}}{Y_{11}}$$

The elements of the matrix [Θ] are obtained by the relation

$$\Theta_{ij} = \frac{Y_{ij}}{\sqrt{m_i}}$$

m_i being the mass of the i^{th} stage, and Θ_{ij} and Y_{ij} being the displacements.

The modes [Y] are established by the three-point mode program. The modes [Θ] are then computed for input to the damping coefficient program.

D. STAGE AND INTERSTAGE DAMPING

The generalized damping for the j^{th} mode is $2\zeta_j \omega_j M_j$ where ζ_j the percent critical damping, M_j is the generalized mass and ω_j the mode frequency. For the three modes generalized damping is:

$$C_1 = 2\zeta_1 \omega_1 M_1$$

$$C_2 = 2\zeta_2 \omega_2 M_2$$

$$C_3 = 2\zeta_3 \omega_3 M_3$$

For the modes used in this study, the three-point modes are normalized to unity at the third mass and the generalized mass is obtained from the dynamic test modes normalized to the center of gravity station for the upper stages. The generalized mass from the three-point mode program was computed using the response at each frequency. The generalized mass for the mode normalized to station ℓ would be the ratio of the generalized mass for the response to the response displacement at station ℓ, squared.

Had the stage and interstage damping been available, the generalized mass could have been obtained using the three-point modes from the damping matrix by the matrix triple product:

$$[\ C_j\ ] = [\Theta]^T[c][\Theta]$$

$$\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} = \begin{bmatrix} \Theta_{11} & \Theta_{21} & \Theta_{31} \\ \Theta_{12} & \Theta_{22} & \Theta_{32} \\ \Theta_{13} & \Theta_{23} & \Theta_{33} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ \Theta_{21} & \Theta_{22} & \Theta_{23} \\ \Theta_{31} & \Theta_{32} & \Theta_{33} \end{bmatrix}$$

where the terms of the matrix [c] represent stage and interstage damping as shown in figure 4.

However, the modal damping matrix has been obtained and it is desired to obtain from it the stage and interstage damping coefficient matrix. Starting with the expression

$$[\ C\ ] = [\Theta]^T[c][\Theta]$$

pre- and postmultiplying by $[\Theta]$ and $[\Theta]^T$ to obtain

$$[\Theta][\ C\ ][\Theta]^T = [\Theta][\Theta]^T[c][\Theta][\Theta]^T$$

then pre- and postmultiplying by $([\Theta][\Theta]^T)^{-1}$ and regrouping

$$([\Theta][\Theta]^T)^{-1}\ [\Theta][\ C\ ][\Theta]^T\ ([\Theta][\Theta]^T)^{-1}$$

$$= ([\Theta][\Theta]^T)^{-1}\ ([\Theta][\Theta]^T)\ [c]\ ([\Theta][\Theta]^T)\ ([\Theta][\Theta]^T)^{-1}$$

is obtained.

But as:

$$([\Theta][\Theta]^T)^{-1}\ ([\Theta][\Theta]^T) = [\ 1\ ]$$

the equation becomes

$$[C] = ([\Theta][\Theta]^T)^{-1}\ [\Theta][\ c\ ][\Theta]^T\ ([\Theta][\Theta]^T)^{-1}$$

From this expression the individual terms of the damping matrix [C] may be obtained.

SECTION IV. PERCENT MODAL DAMPING BY THE RESPONSE METHOD

A. INTRODUCTION

The response of a structure to an applied force can be obtained if the modes and natural frequencies are known. These modes are used to assemble the structural response using the generalized mass and the modal damping. If the modal damping used is correct in each mode, then the actual test response can be obtained. The response of the structure in the lower frequency ranges can be assembled from the first few modes. In this study the first three modes are used and the damping required to obtain the test response is obtained.

To obtain the response of the structural model, the normal mode method as given by Hurty and Rubinstein [1] is the approach used in this analysis. This method is characterized by the fact that the differential equations of motion are decoupled when the displacements are expressed in terms of normal modes. Thus, in a system having n degrees of freedom, n independent differential equations are used rather than n simultaneous differential equations. For a system in which the damping forces are derived from a dissipation function, R, the r^{th} Lagrange equation of the set n is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}_r}\right) - \frac{\partial T}{\partial z_r} + \frac{\partial U}{\partial z_r} + \frac{\partial R}{\partial \dot{z}_r} = N_r$$

where the coordinates z_r ($r = 1, 2, 3, \ldots, n$) are a special set of generalized coordinates defined by

$$w(x,t) = \sum_{i=1}^{n} \theta_1(x)\ z_i(t)$$

where $w(x,t)$ is the deflection of point x at the time t and $\theta_i(x)$ is the displacement configuration of the system vibrating in its i^{th} natural mode. The force, N_r, is the r^{th} generalized applied force in the normal coordinate system. Thus the deflection at any point in the structure is expressed in terms of the normal modes of the structure and the normal coordinates.

B. APPLICATION OF THE NORMAL MODE METHOD TO A DISCRETE MASS SYSTEM

Using the Lagrangian approach, the kinetic energy, strain energy, and dissipation function in terms of the velocities and displacements at the mass points of the structural system are

$$T = \frac{1}{2} \sum_i m_i \dot{w}_i^2$$

$$U = \frac{1}{2} \sum_i \sum_j k_{ij} \; w_i \; w_j$$

$$R = \frac{1}{2} \sum_i \sum_j c_{ij} \; w_i \; w_j$$

where m_i is the mass lumped at the i^{th} point of the system; k_{ij} is the stiffness term relating forces and displacements between point i and j, and c_{ij} is the damping coefficient between i and j. For the elements of the Lagrange equation

$$\frac{\partial T}{\partial \dot{z}} = \sum_{i=1}^{n} \dot{z}_j \sum_{i=1}^{n} m_i \; \theta_{ir} \; \theta_{ir}$$

where M_r, the generalized mass, is defined as

$$M_r = \sum_{i=1}^{n} m_i \; \theta_{ir}^2$$

Thus

$$\frac{\partial T}{\partial \dot{z}} = \dot{z}_j \; M_r$$

$$\frac{\partial U}{\partial z_r} = \omega_r^2 \; M_r \; z_r$$

and

$$\frac{\partial R}{\partial \dot{z}_r} = 2\zeta_r \; \omega_r \; M_r \; \dot{z}_r$$

where ζ_r is the damping factor and ω_r is the natural frequency in the r^{th} mode.

Substituting into the r^{th} Lagrangian equation, the r^{th} decoupled differential equation of motion becomes

$$M_r \; \ddot{z}_r + 2\zeta_r \; \omega_r \; M_r \; \dot{z}_r + \omega_r^2 \; M_r \; z_r = N_r$$

N_r is the force applied to the decoupled system and is given by

$$N_r = \sum_{j=1}^{m} F(x_j,t)\ \theta_r(x_j)$$

$$= \sum_{j=1}^{m} F(x_j,t)\ \theta_{jr}$$

$F(x_j,t)$ is separable in x and t:

$$F(x_j,t) = \sum_{j=1}^{m} P_o\ p(x_j)\ f(t)$$

where P_o may be a peak amplitude value, $p(x_j)$ is the variation along the structure, and f(t) a time variable. For a harmonic force applied at one point,

$$F(x_j,t) = P_o \sin \omega t$$

where ω is the forcing frequency. Thus, N_r is

$$N_r = (P_o \sin \omega t)\ \theta_{jr}$$

where θ_{jr} is the displacement at j, the point of applied load in the r^{th} mode.

C. MATRIX FORM OF THE NORMAL MODE METHOD

The normalized mode is accomplished by the coordinate transformation

$$\{w\} = [\theta]\{z\}$$

where $\{z\}$ is the normal coordinate, $[\theta]$ is the matrix of mode shapes. The differential equations of motion in the $\{w\}$ coordinate system are

$$[\ m\ ]\{\ddot{w}\} + [c]\{\dot{w}\} + [k]\{w\} = \{F(x_j,t)\}$$

where $F(x_j,t)$ is separable in x and t. The elements of $[\theta]$ are θ_{ij}, which is the displacement at i in the j^{th} mode. The columns of $[\theta]$ are thus the modes for each of the given frequencies and $[\theta]$ will be dimensioned p x q where p is the number of mass points and q is the number of modes used. For acceleration and velocity, $\{\ddot{w}\} = [\theta]\{\ddot{z}\}$ and $\{\dot{w}\} = [\theta]\{\dot{z}\}$. Substituting into the equations of motion for the physical coordinates, the mode equations of motion are obtained:

$$[\,m\,][\theta]\{\ddot{z}\} + [c][\theta]\{\dot{z}\} + [k][\theta]\{z\} = \{F(x_j,t)\}$$

Premultiplying by $[\theta]^T$,

$$[\theta]^T[\,m\,][\theta]\{\ddot{z}\} + [\theta]^T[c][\theta]\{\dot{z}\} + [\theta]^T[k][\theta]\{z\} = [\theta]^T\{F(x_j,t)\}$$

Because of the orthogonal relations between modes, the triple matrix products are diagonal matrices. These triple products are defined as:

$[\,M\,] = [\theta]^T[m][\theta]$, the generalized mass

$[\,C\,] = [\theta]^T[c][\theta]$, the generalized damping

$[\,K\,] = [\theta]^T[k][\theta]$, the generalized stiffness

These generalized constants are related. For example C_j, an element of $[\,C\,]$ which is the generalized damping in the j^{th} mode, is equal to $2\zeta_j\omega_jM_j$, where ζ_j is the modal percent of critical damping; ω_j is the frequency; and M_j is the generalized mass in the j^{th} mode. K_j, the generalized stiffness in the j^{th} mode, is equal to $\omega_j^2\,M_j$.

D. SOLUTION FOR THE NORMAL COORDINATES BY MEANS OF LAPLACE TRANSFORMS

The r^{th} equation for a sinusoidal applied load is

$$M_r\ddot{z}_r + 2\zeta_r\omega_rM_r\dot{z}_r + \omega_r^2M_rz_r = P_o\sin\omega t\,\theta_{jr}$$

where ω_r is the r^{th} natural frequency

ω is the forcing frequency

M_r is the r^{th} generalized mass

ζ_r is the r^{th} damping factor

j is the point of applied load

The Laplace transforms of $\dot{z}_r(t)$ and $\ddot{z}_r(t)$ are

$$\mathcal{L}\dot{z}_r(t) = sz_r(s) - z_r(0)$$

$$\mathcal{L}\ddot{z}_r(t) = s^2z_r(s) - sz_r(0) - \dot{z}_r(0)$$

However the displacements $w(x,t)$ will be measured from $w(x,0)$ therefore:

$$z_r(0) = \dot{z}_r(0) = 0$$

The Laplace transform of the r^{th} equation is

$$(M_r s^2 + 2\zeta_r \omega_r M_r \, s + \omega_r^2 M_r) \; z_r(s) = \frac{P_o \, \theta_{jr} \omega}{s^2 + \omega^2}$$

from which

$$z_r(s) = \frac{P_o \, \theta_{jr}}{M_r} \frac{\omega}{(s^2 + 2\zeta_r \omega_r \, s + \omega_r^2) \; (s^2 + \omega^2)}$$

The steady state portion of z_r is

$$z_r = \frac{P_o \, \theta_{jr}}{M_r} \frac{\sin(\omega t - \psi)}{\sqrt{(\omega_r^2 - \omega^2)^2 + 4\zeta_r \omega_r^2 \omega^2}}$$

where

$$\psi = \tan^{-1} \frac{2\zeta \omega_r \omega}{\omega_r^2 - \omega^2}$$

With the steady state portion of z_r known, the relationship between the physical modes and the normal modes,

$$\{w\} = [\theta]\{z\}$$

may be used to obtain physical displacements. For example, the displacement at point 1 would be:

$$w_1 = \theta_{11} z_1 + \theta_{12} z_2 + \theta_{13} z_3 + \ldots\ldots \; \theta_{1n} z_n$$

E. COMPUTATION OF DAMPING FACTOR

The basis for the normal mode method is that a linear relation holds for the equations of motions. Thus, an increase in one of the parameters of the system-for example, response-must be accompanied by a proportional decrease in some other parameter, such as the damping ratio. The approach used in this study is to obtain response using damping factors as computed by the phase angle method, correct by the ratio of response to dynamic test response, and rerun as a check. The normal mode method is not unique to this study; consequently, the program used to obtain this damping is not included in this report.

SECTION V. DISCUSSION

This report has defined methods to compute generalized damping coefficients and stage and interstage damping coefficients. The methods developed are described in sections II, III, and IV. The results of the methods used are given in Tables 1 and 2.

During the dynamic test damping coefficients were computed which are included in Table 1, column 3. The composite value in Table 1, column 4, is derived from the matrix triple product $[\Theta]^T[C][\Theta]$ where $[C]$ is an average value as indicated in column 4 of Table 2.

The percent of generalized damping as obtained by the phase angle method involves the slope of a curve obtained from experimental information. At best only a few points can be used to obtain the slope during the phase shift. Using the slope of a curve is in essence a differential procedure and results in a roughing of the information, as a differential procedure will magnify any error, whereas an integrating procedure will tend to smooth the data and minimize the effect of error. The phase angle method resulted in, with one exception, the lowest percentages of any of the methods used.

The percent of generalized damping as obtained by the response method is dependent on the force exciting the vehicle and thus accounts for all the energy dissipated. However the modes used were obtained from dynamic test data recorded on the structure and do not consider fluid motion.

The approach used during the dynamic test included use of the half power point and the rate of decay of vibration. Figure 5 illustrates the response for a typical accelerometer. This response indicates that modes other than vehicle bending were excited and would make establishing the half power point with confidence difficult. Also, in allowing the vibration to decay, beating and coupling between modes occured. The typical theoretical response using the first three dynamic test bending modes is also illustrated in Figure 5. Some of these resonant modes represent coupling of the excited vehicle with the test support or coupling with flexible components, for example, engines.

Consideration of the percent of generalized damping as computed by the various methods indicates that for the Saturn IB, SA-202 this percentage would be approximately 1 to 1.5 percent for the first three bending modes. Further refinement of this value based on experimental information would require more precise data on the suspension and fluid motion.

Consideration of the damping coefficients indicates some agreement between yaw and pitch. Some negative values appear but these are small and probably represent values obscured by the noise level. The damping between the SIB and the upper stages denoted by C13 has a small value which is reasonable as the first and third stage have no direct structural connection.

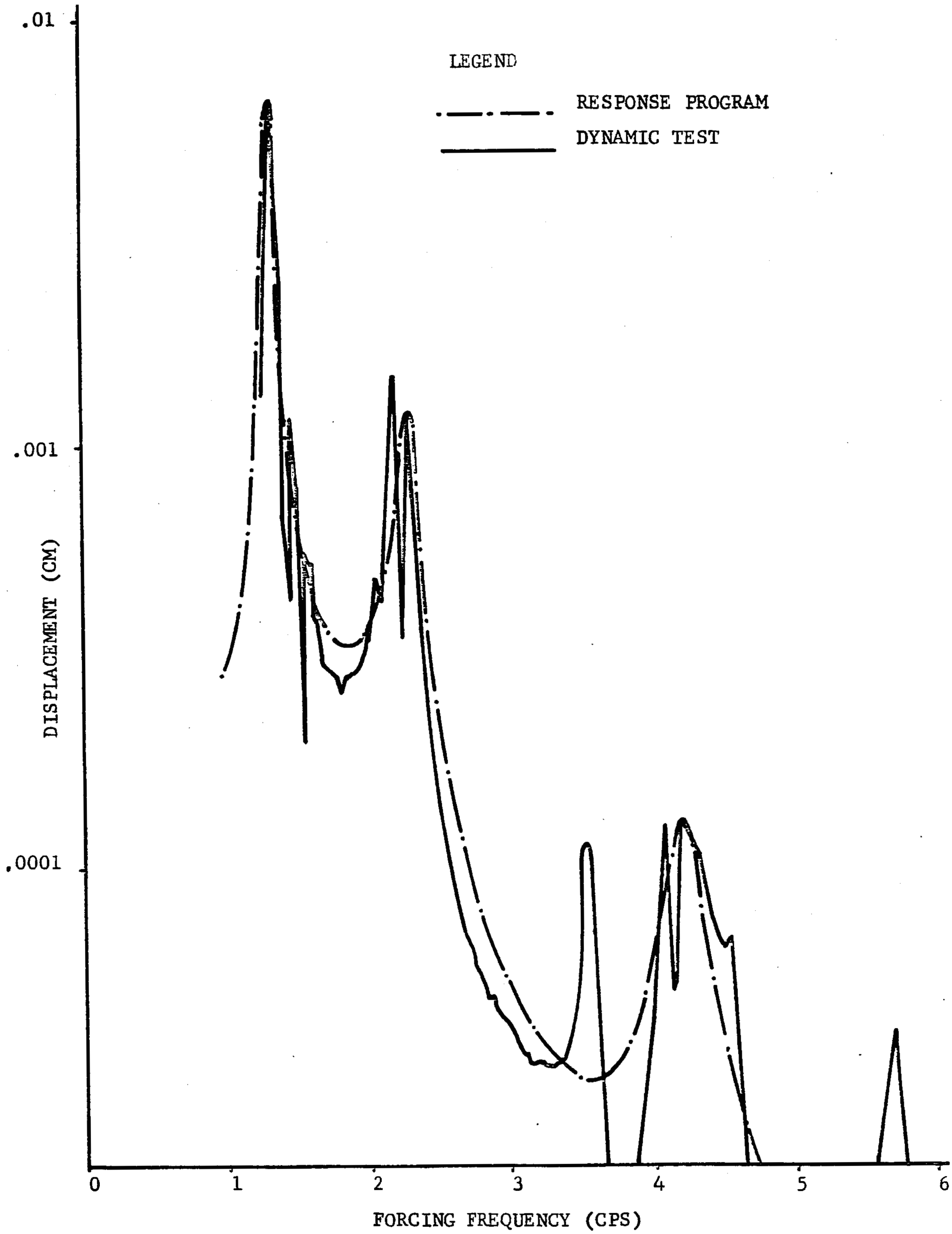


FIGURE 5. TYPICAL ACCELEROMETER RESPONSE VS. FORCING FREQUENCY

TABLE 1. PERCENT GENERALIZED DAMPING, ζ_j, PITCH AND YAW EXCITATION, SATURN IB SA-202

CONDITION		BENDING MODE AND FREQUENCY	PHASE ANGLE METHOD	RESPONSE METHOD	REFERENCE	COMPOSITE + VALUE
PITCH	LIFTOFF	1st @1.35CPS	1.44	1.79	1.67	1.63
		2nd @2.27CPS	.28	1.14	.91	.78
		3rd @4.20CPS	.81	1.10	1.32	1.08
	110SEC	1st @1.64CPS	.54	.73	1.75	1.00
		2nd @2.58CPS	.71	1.75	1.17	1.21
		3rd @6.00CPS	1.71	2.59	1.88	2.06
	147SEC	1st @1.70CPS	2.26	1.19	1.90	1.78
		2nd @2.60CPS	.29	1.14	1.27	.90
		3rd @6.88CPS	.67	.60	.87	.71
YAW	LIFTOFF	1st @1.33CPS	.57	1.21	1.22	1.00
		2nd @2.19CPS	1.18	2.32	1.02	1.50
		3rd @4.14CPS	1.08	1.08	1.54	1.23
	110SEC	1st @1.61CPS	.57	.94	1.85	1.11
		2nd @2.48CPS	1.66	2.84	1.11	1.87
		3rd @6.08CPS	1.72*	2.02	2.00	1.94
	147SEC	1st @1.64CPS	.51	1.10	1.70	1.10
		2nd @2.50CPS	1.00	1.43	1.34	1.26
		3rd @6.91CPS	.98	1.31	1.47	1.25

*AN ADDITIONAL SWEEP THROUGH THIS BENDING MODE HAD A MODAL DAMPING OF 1.17% OF CRITICAL DAMPING

+$\zeta_j = C_j/2M_j\omega_j$ WHERE $[\ C_j\ ] = [\Theta]^T[C][\Theta]$, AND [C] IS FROM TABLE 2, COLUMN 4.

TABLE 2. STAGE AND INTERSTAGE DAMPING COEFFICIENTS, SATURN IB SA-202

CONDITION		DAMPING COEFFICIENT	GENERALIZED DAMPING PHASE ANGLE METHOD	GENERALIZED DAMPING RESPONSE METHOD	GENERALIZED DAMPING REFERENCE	COMPOSITE DAMPING
PITCH	LIFTOFF	C_{11}	159,876	234,509	229,941	208,109
		C_{12}	4,494	-7,472	7,180	1,401
		C_{13}	-2,043	-30,691	-21,205	-17,980
		C_{22}	26,619	44,205	46,657	39,160
		C_{23}	8,041	25,758	22,183	18,661
		C_{33}	10,026	39,214	31,552	26,931
	110SEC	C_{11}	216,048	324,787	277,839	272,891
		C_{12}	44,949	71,014	-1,532	38,144
		C_{13}	13,616	17,673	30,749	20,679
		C_{22}	58,361	87,783	129,681	91,942
		C_{23}	- 102	8,173	-17,235	-3,055
		C_{33}	15,032	33,187	29,786	26,002
	147SEC	C_{11}	517,226	435,861	654,025	535,704
		C_{12}	239,296	270,220	392,560	300,692
		C_{13}	-41,476	18,388	19,512	-1,192
		C_{22}	586,441	447,478	672,968	568,962
		C_{23}	390,635	234,582	369,663	331,627
		C_{33}	363,081	203,483	321,839	296,134
YAW	LIFTOFF	C_{11}	182,394	231,634	277,153	230,394
		C_{12}	57,777	37,273	84,601	59,884
		C_{13}	2,291	-13,018	15,382	1,552
		C_{22}	59,601	74,414	78,384	70,800
		C_{23}	29,713	48,028	30,997	36,246
		C_{33}	30,793	58,025	28,254	39,024
	110SEC	C_{11}	497,199	608,201	676,617	594,006
		C_{12}	137,489	140,875	78,530	118,965
		C_{13}	-45,256	-64,665	-114,022	-74,648
		C_{22}	121,056	166,122	204,169	163,782
		C_{23}	- 291	2,612	57,704	20,008
		C_{33}	36,061	60,141	68,825	55,009
	147SEC	C_{11}	406,340	558,640	647,809	537,586
		C_{12}	247,038	290,871	284,639	274,168
		C_{13}	48,752	23,890	-17,089	18,505
		C_{22}	337,864	536,851	692,043	522,262
		C_{23}	178,741	328,042	468,683	325,176
		C_{33}	142,249	285,573	425,383	284,425

NOTE: UNITS OF DAMPING COEFFICIENTS ARE KG/SEC

REFERENCES

1. W. C. Hurty and M. F. Rubinstein, DYNAMICS OF STRUCTURES, Prentice-Hall, Inc. 1964

2. W. F. Rothe, SATURN DYNAMIC TEST DATA REDUCTION AND ANALYSIS METHODS, Technical Report HSM-R094, Chrysler Corporation, Huntsville Operations, December 31, 1964

3. Dynamic Test Branch, FINAL REPORT OF TOTAL VEHICLE TESTING OF SATURN IB DYNAMIC TEST VEHICLE, SA-200-D, INSA-202, SA-203, SA-206, SA-207 CONFIGURATIONS, HSM-R856 Saturn IB Dynamic Test, Chrysler Corporation, Huntsville Operations, January 31, 1966

4. Structural Dynamics Group, DYNAMIC TEST RESULTS OF SAD-202, Technical Report HSM-R148, Chrysler Corporation, Huntsville Operations, December 27, 1965

5. Unpublished Experimental Data, Saturn IB Dynamic Test

APPENDIX A

DESCRIPTION OF THE DYNAMIC TEST DATA [5]

The data necessary for the performance of this study was taken from the first summary for each test. The data recorded at a given time is called a file and contains the following information:

1. Title, including condition and test date
2. File number
3. Frequency, cps
4. Shaker force in Newtons and pounds
5. Accelerometer and SEL channel number
6. Station for each accelerometer, in.
7. Acceleration at each accelerometer, G's
8. Displacement force ratio at each accelerometer, cm/Newton and in./lb
9. Displacement at each accelerometer, cm and in.
10. Phase angle, degrees

The following information was keypunched for use as input data for the damping programs.

1. Frequency, cps
2. Shaker force, Newtons
3. Accelerometer No.
4. SEL channel No.
5. Acceleration, G's
6. Phase angle, degrees
7. Displacement, cm
8. File number

This information was key punched in FORTRAN IV as used on the General Electric 415 computer. This format is as follows:

Information	Units	Format	Card Columns
Card 0			
Frequency	cps	F 10.5	1-10
Shaker Force	Newtons	F 10.4	11-20
Sequence No.		I 5	66-70
File No.		I 10	71-80

Cards 1 through N

Accelerometer No.		I 5	1-5
SEL channel No.		I 5	6-10
Station	in.	F 10.1	11-20
Acceleration	G's	F 10.5	21-30
Phase Angle	degrees	F 10.5	31-40
Displacement	cm	F 10.6	41-50
Sequence No.		I 5	66-70
File No.		I 5	71-80

Each file key punched consisted of N + 1 cards, the card number being the sequence number running from 0 through N.

Files used to establish percent generalized damping by the phase angle method were punched for each mode as follows:

Test Date	Test Name	
April 2, 1965	SA-202, pitch at liftoff (rerun), Saturn IB dynamic test files:	
	1st Bending Mode:	0700110 0700111 0700112
	2nd Bending Mode:	0700065 0700067 0700070
	3rd Bending Mode:	0700102 0700103 0700104
April 4, 1965	SA-202, pitch at 110 seconds (rerun), Saturn IB dynamic test files:	
	1st Bending Mode:	0712260 0712261 0712262
	2nd Bending Mode:	0712301 0712302 0712303
	3rd Bending Mode:	0712461 0712462 0712463

Test Date	Test Name		
March 19, 1965	SA-202, pitch at 147 seconds, Saturn IB dynamic test files:		
	1st Bending Mode:	0022423 0022424	
	2nd Bending Mode:	0022732 0022733 0022734	
	3rd Bending Mode:	0022564 0022565	
April 8, 1965	SA-202, yaw at liftoff (rerun), Saturn IB dynamic test files:		
	1st Bending Mode:	0602043 0602044 0602045	
	2nd Bending Mode:	0602132 0602133 0602134	
	3rd Bending Mode:	0602207 0602210 0602211	
April 9, 1967	SA-202, yaw at 110 seconds (rerun), Saturn IB dynamic test files:		
	1st Bending Mode:	0612245 0612246 0612247	
	2nd Bending Mode:	0612034 0612035 0612036	
	3rd Bending Mode:	Set 1: 0612300 0612301 0612302	Set 2: 0612135 0612136 0612137
March 16, 1965	SA-202, yaw at 147 seconds, Saturn IB dynamic test files:		
	1st Bending Mode:	0122015 0122017 0122020	

Test Date	Test Name	
March 16, 1965 (Cont'd.)	2nd Bending Mode:	0122067
		0122070
		0122072
	3rd Bending Mode:	0122200
		0122201
		0122202

Files used to establish three-point modes were as follows:

Test Condition	Bending Mode	File
Pitch Liftoff	1st	0700111
	2nd	0700005
	3rd	0700103
Pitch 110 sec	1st	0712225
	2nd	0712227
	3rd	0712463
Pitch 147 sec	1st	0022423
	2nd	0022733
	3rd	0022646
Yaw Liftoff	1st	0602044
	2nd	0602010
	3rd	0602016
Yaw 110 sec	1st	0612246
	2nd	0612035
	3rd	0612301
Yaw 147 sec	1st	0122020
	2nd	0122070
	3rd	0122201

The accelerometers used to establish pitch modes were 1 through 23. The accelerometers used to establish yaw modes were 1, 82 through 85, 8 through 22, 66, and 23. Files used were from the same tests as were files for the phase angle method.

APPENDIX B

DAMPING STUDY - PHASE ANGLE METHOD

INPUT AND OUTPUT

SYMBOLS

TEXT	PROGRAM	DEFINITION	INPUT FORMAT
	NCASE	Number of cases to run	I5
		Title card	80H
	NP	Number of accelerometers	I5
	NJ	Number of modes per flight time	I5
	NOS	Point number to which modes are normalized	I5
	NIF	Number of accelerometers in the file	I5
	LP	Accelerometers that are to be used (24 values per card)	24I3
	KMI	If 0, add 360 to phase angle of THETA1 If 1, subtract 360 from phase angle of THETA1	I5
	KM2	If 0, add 360 to phase angle of THETA2 If 1, subtract 360 from phase angle of THETA2	I5
	KM3	If 0, add 360 to phase angle of THETA3 If 1, subtract 360 from phase angle of THETA3	I5
	NTC1	Number of phase angles to change in THETA1	I5
	NTC2	Number of phase angles to change in THETA2	I5
	NTC3	Number of phase angles to change in THETA3	I5
	NTH1	The accelerometer numbers to be changed for THETA1 (24 values per card) If NTC1 = 0, do not include this card in data deck.	24I3

Input & Output (Cont'd.)

SYMBOLS

TEXT	PROGRAM	DEFINITION	INPUT FORMAT
	NTH2	Same as NTH1, except for THETA2 If NTC2 = 0, do not include this card in data deck.	24I3
	NTH3	Same as NTH1, except for THETA3 If NTC3 = 0, do not include this card in data deck.	24I3
ω_1	F1	Frequency	F10.5
	NAC	Accelerometer No.	I5
θ_1	THETA1	Phase angle	F10.4
ω_n	F2	Natural frequency	F10.5
	SF2	Shaker force at natural frequency	F10.4
	NAC	Accelerometer No.	I5
θ_2	THETA2	Phase angle	F10.4
θ_{ij}	DX	Displacement	F10.6
ω_3	F3	Frequency	F10.5
	NAC	Accelerometer No.	I5
θ_3	THETA3	Phase angle	F10.4
i	ACCEL	Accelerometer No.	N/A
$\Delta\theta/\Delta\omega$	Slope	Slope at each accelerometer location	N/A
ξ	XI	Damping coefficient at each accelerometer location	N/A
Cj	C(J)	Generalized damping coefficient	N/A
ζ	ZETA(J)	Percent critical damping	N/A

INTERNAL PROGRAM SYMBOLS

SYMBOLS

TEXT	PROGRAM	DEFINITION
$\Delta\theta$	TOP	Change in phase angle
$\Delta\omega$	BOT	Change in frequency
$\Delta\theta/\Delta\omega$	A1	Slope
ϕ_{ij}	PHI(I,J)	Normalized displacements
ξ	EPS	Point damping
M_j	GM(J)	Generalized mass

ChO-420-107

PROGRAM DAMPING STUDY-PHASE ANGLE METHOD

CODED BY ____________

DATE ____________

PAGE ____ OF ____

IDENTIFICATION 73 — 80

FORTRAN CODING FORM

1–5	6	7–10	11–15	16–20	21–25	26–30	31–40	41–50	51–68	69–70	71–72
NCASE											
		TITLE CARD									
NP		NJ	NOS	NIF							
LP(I)	LP(2)									LP(I-1)	LP(I)
LP(I+1)										LP(N-1)	LP(N)
KM1		KM2	KM3	NTC1	NTC2	NTC3					
NTH1 (I)	NTH1 (2)									NTH1 (I-1)	NTH1 (I)
NTH1 (I+1)										NTH1 (N-1)	NTH1 (N)
NTH2 (I)	NTH2 (2)									NTH2 (I-1)	NTH2 (I)
NTH2 (I+1)										NTH2 (N-1)	NTH2 (N)
NTH3 (I)	NTH3 (2)									NTH3 (I-1)	NTH3 (I)
NTH3 (I+1)										NTH3 (N-1)	NTH3 (N)
F1											
NAC							THETA 1 (I)				
NAC							THETA 1 (NIF)				
F2			SF2								
NAC							THETA 2 (I)	DX (I)			
NAC							THETA 2 (NIF)	DX (NIF)			
F3											
NAC							THETA 3 (I)				
NAC							THETA 3 (NIF)				

```
C      PROGRAM NUMBER 823-0048B
C      DAMPING STUDY PROGRAM - PHASE ANGLE METHOD
       DIMENSION F1(5),F2(5),F3(5),CAP(25),SF2(5),C(5),CAS(5)
       DIMENSION NA2(25,5),TH1(25,5),TH2(25,5),TH3(25,5),GM(5)
       DIMENSION X2(25,5), ZETA(5),PHI(25,5),CM(25,5),A(25,25)
       DIMENSION B(25,25),IT(25),LP(25),NTH1(25),NTH2(25),NTH3(25)
*************************************************************************
C      READ AND PRINT INPUT DATA
*************************************************************************
       READ 101,NCASE
       NC=1
   400 READ 100
       READ 101,NP,NJ,NOS,NIF
       READ 113,(LP(K),K=1,NP)
       PRINT 100
       PRINT 198
       PRINT 104,NP,NJ,NOS,NIF
       PRINT 108
       PRINT 109,(LP(K),K=1,NP)
       DO 200 J=1,NJ
       DO 456 I=1,NP
       NTH1(I)=0
       NTH2(I)=0
   456 NTH3(I)=0
       READ 101,KM1,KM2,KM3,NTC1,NTC2,NTC3
       PRINT 125,KM1,KM2,KM3,NTC1,NTC2,NTC3
       IF(NTC1)99,451,450
   450 READ 113,(NTH1(I),I=1,NTC1)
       PRINT 122
       PRINT 109,(NTH1(I),I=1,NTC1)
   451 IF(NTC2)99,453,452
   452 READ 113,(NTH2(I),I=1,NTC2)
       PRINT 123
       PRINT 109,(NTH2(I),I=1,NTC2)
   453 IF(NTC3)99,455,454
   454 READ 113,(NTH3(I),I=1,NTC3)
       PRINT 124
       PRINT 109,(NTH3(I),I=1,NTC3)
   455 READ 102,F1(J)
       K1=0
       K1=K1+1
       DO 201 I=1,NIF
       READ 103,NAC,THETA
       K=0
   403 K=K+1
       IF(NAC-LP(K))401,402,401
   401 IF(K-NP)403,201,201
   402 TH1(K,J)=THETA
       IF (NTC1)99,201,404
   404 IF(NAC-NTH1(K1))201,405,201
   405 IF(KM1)99,406,407
   406 TH1(K,J)=TH1(K,J)+360.0
       K1=K1+1
       GO TO 201
```

```
  407 TH1(K,J)=TH1(K,J)-360.0
      K1=K1+1
  201 CONTINUE
      K1=0
      K1=K1+1
      READ 102,F2(J),SF2(J)
      DO 202 I=1,NIF
      READ 114,NAC,THETA,DX
      K=0
  413 K=K+1
      IF(NAC-LP(K))411,412,411
  411 IF(K-NP)413,202,202
  412 NA2(K,J)=NAC
      TH2(K,J)=THETA
      X2(K,J)=DX
      IF(NTC2)99,202,414
  414 IF(NAC-NTH2(K1))202,415,202
  415 IF(KM2)99,416,417
  416 TH2(K,J)=TH2(K,J)+360.0
      K1=K1+1
      GO TO 202
  417 TH2(K,J)=TH2(K,J)-360.0
      K1=K1+1
  202 CONTINUE
      READ 102,F3(J)
      K1=0
      K1=K1+1
      DO 203 I=1,NIF
      READ 103,NAC,THETA
      K=0
  423 K=K+1
      IF(NAC-LP(K))421,422,421
  421 IF(K-NP)423,203,203
  422 TH3(K,J)=THETA
      IF(NTC3)99,203,424
  424 IF(NAC-NTH3(K1))203,425,203
  425 IF(KM3)99,426,427
  426 TH3(K,J)=TH3(K,J)+360.0
      K1=K1+1
      GO TO 203
  427 TH3(K,J)=TH3(K,J)-360.0
      K1=K1+1
  203 CONTINUE
      PRINT 198
      PRINT 105,J,F1(J),F2(J),F3(J),SF2(J)
      PRINT 106
      DO 210 I=1,NP
  210 PRINT 107,I,NA2(I,J),TH1(I,J),TH2(I,J),TH3(I,J),X2(I,J)
  200 CONTINUE
      PRINT 199
      MM=25
*****************************************************************
C *** NORMALIZE DISPLACEMENTS ***
*****************************************************************
```

```
      DO 205 J=1,NJ
      XNOS = X2(NOS,J)
      DO 205 I=1,NP
  205 PHI(I,J)=X2(I,J)/XNOS
***********************************************************************
C     CALCULATION OF THE DAMPING COEFFICIENTS
***********************************************************************
      DO 10 J=1,NJ
      EPHI = 0.0
      PRINT 121,J
      DO 11 I=1,NP
      TOP=3.0*(F1(J)*TH1(I,J)+F2(J)*TH2(I,J)+F3(J)*TH3(I,J))-
     $(F1(J)+F2(J)+F3(J))*(TH1(I,J)+TH2(I,J)+TH3(I,J))
      BOT=3.0*(F1(J)**2+F2(J)**2+F3(J)**2)-(F1(J)+F2(J)+F3(J))**2
      A1=TOP/BOT
      EPS=57.3/(2.0*F2(J)*A1)
      PRINT 111,NA2(I,J) ,A1,EPS
      EPHI = EPHI + EPS * PHI(I,J)**2
   11 CONTINUE
      C(J)=12.56636*F2(J)*EPHI
      PRINT 110,J,C(J)
      PRINT 198
   10 CONTINUE
      PRINT 198
      DO 351 J=1,NJ
      GM(J)=0.0
      ZETA(J)=0.0
      DO 350 I=1,NP
  350 GM(J)=GM(J)+PHI(I,J)**2
      ZETA(J)=C(J)/(12.56636*F2(J)*GM(J))
      PRINT 120,J,ZETA(J)
  351 CONTINUE
      NC=NC+1
      PRINT 199
      IF(NC-NCASE)400,400,99
   99 STOP
***********************************************************************
  100 FORMAT(80H
     $                                        )
  101 FORMAT(9I5)
  102 FORMAT(F10.5,F10.4)
  103 FORMAT(I5,25X,F10.4 )
  104 FORMAT(1X,15HNO. OF ACCEL. =I4,2X,14HNO. OF MODES =I4,2X,24HNORMAL
     $IZED TO POINT  NO.,I4,2X,22HNO. OF ACCEL. IN FILE,I4,/)
  105 FORMAT(1X,5HMODE ,7X,2HF1,9X,2HF2,9X,2HF3,9X,3HSF2,/,I5,1X,4F11.4
     $,/)
  106 FORMAT(1X,5HPOINT,2X,3HNAC,3X,6HTHETA1,5X,6HTHETA2,5X,6HTHETA3,7X,
     $3HDX2,)
  107 FORMAT(1X,2I5,4F11.5)
  108 FORMAT(27HLIST OF ACCELEROMETERS USED,/)
  109 FORMAT(1X,25I5)
  110 FORMAT(/,1X,4HMODE,I3,2X,31HGENERALIZED DAMPING COEFFICIENT,F13.6)
  111 FORMAT(I6,5E15.6)
  113 FORMAT(24I3)
```

```
  114 FORMAT(I5,25X,F10.4,F10.6)
  120 FORMAT(1X,4HMODE,I3,2X,6HZETA =E13.6,/)
  121 FORMAT(16X,4HMODEI3,//,1X,5HACCEL,5X,5HSLOPE,12X,2HYI)
  122 FORMAT(1X,43HLIST OF ACCEL. TO CHANGE PHASE ANGLE THETA1,/)
  123 FORMAT(1X,43HLIST OF ACCEL. TO CHANGE PHASE ANGLE THETA2,/)
  124 FORMAT(1X,43HLIST OF ACCEL. TO CHANGE PHASE ANGLE THETA3,/)
  125 FORMAT(1X,/,32HCONTROLS FOR PHASE ANGLE CHANGES,/,
     $5H  KM1,5H  KM2,5H  KM3,5H NTC1,5H NTC2,5H NTC3,/,6I5,/)
  198 FORMAT(/)
  199 FORMAT(1H1)
      END
```

LIFTOFF PITCH 3RD BENDING * FILE NOS. 700102, 700103, 700104

NO. OF ACCEL. = 22 NO. OF MODES = 1 NORMALIZED TO POINT NO. 22 NO. OF ACCEL. IN FILE, 88

LIST OF ACCELEROMETERS USED

1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23

CONTROLS FOR PHASE ANGLE CHANGES

KM1	KM2	KM3	NTC1	NTC2	NTC3
0	1	0	0	10	0

LIST OF ACCEL. TO CHANGE PHASE ANGLE THETA2

3 4 5 6 7 17 18 19 20 21

MODE	F1	F2	F3	SF2
1	4.1990	4.2041	4.2129	4738.5265

POINT	NAC	THETA1	THETA2	THETA3	DX2
1	1	-99.12850	-102.29860	-110.39060	-.04856
2	3	-263.03640	-269.32920	-274.89050	0.02428
3	4	-270.86010	-274.28710	-282.51520	0.06178
4	5	-270.80210	-274.36460	-262.62410	0.08619
5	6	-272.97090	-276.78860	-285.10690	0.05909
6	7	-273.98880	-277.64230	-285.75500	0.01873
7	8	-89.17660	-92.08190	-98.51170	-.01393
8	9	-92.33740	-96.89070	-104.36960	-.04282
9	10	-95.29660	-98.86980	-106.65540	-.10059
10	11	-94.27380	-97.85610	-105.94130	-.07308
11	12	-96.38590	-100.22160	-108.16270	-.08276
12	13	-95.24660	-98.60130	-106.65700	-.08386
13	14	-95.11620	-98.64620	-107.12030	-.06772
14	15	-95.54680	-98.79630	-107.08250	-.05500
15	16	-97.00980	-100.90700	-109.02610	-.03113
16	17	-268.24870	-274.03490	-281.30820	0.02068
17	18	-273.37070	-277.71860	-285.70540	0.04357
18	19	-272.42190	-276.42460	-284.10700	0.05956
19	20	-273.62770	-277.64550	-285.39630	0.08707
20	21	-274.24640	-278.46710	-286.24950	0.05025
21	22	-95.07650	-98.98920	-106.05150	-.04031
22	23	-100.43500	-104.03730	-111.58560	-.18760

MODE 1

ACCEL	SLOPE	XI
1	-.824114E+03	-.826915E-02
3	-.831079E+03	-.819986E-02
4	-.851073E+03	-.800721E-02
5	-.862198E+03	-.790390E-02
6	-.883124E+03	-.771662E-02
7	-.856802E+03	-.795368E-02
8	-.679690E+03	-.100262E-01
9	-.866122E+03	-.786810E-02
10	-.826568E+03	-.824461E-02
11	-.850114E+03	-.801625E-02
12	-.855374E+03	-.796696E-02
13	-.833209E+03	-.817889E-02
14	-.876553E+03	-.777446E-02
15	-.844106E+03	-.807331E-02
16	-.872972E+03	-.780635E-02
17	-.929689E+03	-.733012E-02
18	-.891831E+03	-.764128E-02
19	-.846292E+03	-.805245E-02
20	-.852506E+03	-.799376E-02
21	-.867982E+03	-.785123E-02
22	-.792979E+03	-.859384E-02
23	-.810254E+03	-.841061E-02

MODE 1 GENERALIZED DAMPING COEFFICIENT -.135447E+01

MODE 1 ZETA = -.814458E-02

APPENDIX C

THREE-POINT MODE PROGRAM

The purpose of this program is to obtain the three-point mode described in Section III. The program is written in FORTRAN IV as used on the General Electric 415 computer in the Chrysler Corporation Huntsville Facility. This program generates modes for the input mass points from the dynamic test modes, computes generalized mass, and computes the mass weighted three-point modes. Included are input data description and format, internal data description, FORTRAN listing, and sample output.

DESCRIPTION OF INPUT DATA SYMBOLS, THREE-POINT MODE PROGRAM

FORTRAN SYMBOL	DESCRIPTION	FORMAT	COLUMN
NCASE	Number of cases to run three point modes for.	I3	1-3
NNFAC	Total number of accelerometers in files used.	I3	1-3
NPRO	Total number or discrete masses in mass property file.	I3	4-6
NTIP	Point number in the mass property file of station to normalize on.	I3	7-9
NNFMS	Subset of NNFAC, number of accelerometers used to establish mode.	I3	10-12
KT	1, for liftoff. 2, for 110 sec. 3, for 147 sec.	I3	13-15
FREQ(J)	Frequency of the j^{th} mode from the j^{th} file, j = 1 through 3 for the three modes.	F10.5	1-10
NAC	Accelerometer number from file NNFAC total.	I5	6-10
STANF(J,NAC)	Station in the j^{th} file for the NAC^{th} accelerometer.	F10.1	11-20
DISP(J,NAC)	Displacement in the j^{th} file as recorded at the NAC^{th} accelerometer.	F10.6	41-50
SSTA(J,I)	Station for the i^{th} condition for the j^{th} mass point. j = 1 through NPRO, i = KT, in mass property file.	F8.3	1-80
SMASS(J,I)	Concentrated mass related to SSTA(J,I), in mass property file.	F8.2	1-80
SINER(J,I)	Concentrated rotational inertia related to SSTA(J,I), in mass property file.	E10.5	1-80

FORTRAN SYMBOL	DESCRIPTION	FORMAT	COLUMN
NACC(J)	Subset of NAC, accelerometers used to establish mode, j = 1 through NNFMS.	I3	1-78
SXI	Interface station between first and second stages.	E15.8	1-15
SX2	Interface station between second and third stages.	E15.8	16-30

DESCRIPTION OF INTERNAL DATA SYMBOLS, THREE-POINT MODE PROGRAM

SNMFD(1,J)	Subset of STANF(J,NAC), stations of accelerometers selected to establish 1st bending mode, j = 1 through NNEMS.
SDISP(1,J)	Subset of DISP(J,NAC), displacements relating to stations SNMFD(1,J).
SNMFD(2,J) SNMFD(3,J) SDISP(2,J) SDISP(3,J)	Same as SNMFD(1,J) and SDISP(1,J), but for 2nd and 3rd bending modes.
NACC(J)	Subset of NAC, accelerometer numbers for accelerometers used to establish SNMFD and SDISP.
K KK KKK	Constants for station check K = 1, KK = 2, KKK = 3
SMOD(I,J)	Same as SNMFD but for mass property Stations i = 1 through 3, j = 1 through NPRO.
DELTA(I,J)	Mode slope associated with SMOD.
GM1	Generalized mass, 1st mode.
GM2	Generalized mass, 2nd mode.
GM3	Generalized mass, 3rd mode.
X(1,J) X(2,J) X(3,J)	Mass weighted displacement for the 1st, 2nd, and 3rd modes.
C112	$\sum_{a}^{b} y_{i1} y_{i2}$
C212	$\sum_{a}^{b} y_{i1} y_{i2}$
C312	$\sum_{c}^{d} y_{i1} y_{i2}$
C123	$\sum_{a}^{b} y_{i2} y_{i3}$

C223	$\sum_{b}^{c} y_{i2}\, y_{i3}$
C323	$\sum_{c}^{d} y_{i2}\, y_{i3}$
C131	$\sum_{u}^{b} y_{i3}\, y_{i1}$
C231	$\sum_{b}^{c} y_{i3}\, y_{i1}$
C331	$\sum_{c}^{d} y_{i3}\, y_{i1}$
X11	Y_{11}
X21	Y_{12}
X31	Y_{13}
X12	Y_{21}
X22	Y_{22}
X32	Y_{23}
X13	Y_{31}
X23	Y_{32}
X33	Y_{33}
O12	$\sum_{a}^{d} y_{i1}\, y_{i2}$
O23	$\sum_{a}^{d} y_{i2}\, y_{i3}$
O31	$\sum_{a}^{d} y_{i3}\, y_{i1}$

INPUT FORMAT, THREE-POINT MODE PROGRAM

Card or File No.		Column
1	Initial title card	1-80
2	NCASE	1-3
3	1st case title card	1-80
4	NNFAC	1-3
	NPRO	4-6
	NTIP	7-9
	NNFMS	10-12
	KT	13-15
5	Dynamic test file for 1st bending mode.	
6	Dynamic test file for 2nd bending mode.	
7	Dynamic test file for 3rd bending mode.	

Card or File No.		Column
8	Mass properties file for initial case.	
9	File of accelerometers used to establish modes.	
10	SX1	1-15
	SX2	16-30
	NCASE total sets of cards and files 3 through 10.	

INPUT FORMAT, FILES

File No.	
5	Dynamic Test Data, see Appendix A
6	
7	
8	Mass properties file

Card No.		Column
1	Title Card	1-80
Set 2	SSTA(J,I), 10 per card for NPRO total	1-80
Set 3	SMASS(J,I), 10 per card for NPRO total	1-80
Set 4	SINER(J,I), 8 per card for NPRO total	1-80

9 Accelerometer file

Card No.		Column
Set 1	NACC(1), 26 per card for NNFMS total	1-78

THREE POINT MODES

```
      DIMENSION FREQ(3),SHAPNF(3,140),STANF(3,140),SMOD(3,140)
      DIMENSION SSTA(3,140),SMASS(3,140),SINER(3,140),DISP(3,140)
      DIMENSION SNFMD(3,30),SDISP(3,30),DELTA(3,140),NACC(30),X(3,140)
      READ 101
      PRINT 121
      READ 102,NCASE
      DO 1000  NN=1,NCASE
      READ 101
      PRINT 101
      PRINT 120
      READ 102,NNFAC,NPRO,NTIP,NNFMS,KT
      DO 20 J=1,3
      READ 103,FREQ(J)
      DO 20 K=1,NNFAC
   20 READ 104,NAC,STANF(J,NAC),DISP(J,NAC)
      READ 105
      PRINT 135,FREQ(1)
      PRINT 120
      PRINT 136,FREQ(2)
      PRINT 120
      PRINT 137,FREQ(3)
      PRINT 120
      J=KT
      L=KT
      READ 106,(SSTA(J,I),I=1,NPRO)
      READ 107,(SMASS(J,I),I=1,NPRO)
      READ 108,(SINER(J,I),I=1,NPRO)
      PRINT 138
      PRINT 120
      PRINT 139
      PRINT 120
      PRINT 149,(I,SSTA(J,I),SMASS(J,I),SINER(J,I),I=1,NPRO)
      PRINT 120
C     ESTABLISH MODES FROM FILES
      READ 109,(NACC(I),I=1,NNFMS)
      PRINT 129
      PRINT 120
      PRINT 130
      PRINT 120
      DO 50 J=1,NNFMS
      JJJ=NACC(J)
      SNFMD(1,J)=STANF(1,JJJ)
      SNFMD(2,J)=STANF(2,JJJ)
      SNFMD(3,J)=STANF(3,JJJ)
      SDISP(1,J)=DISP(1,JJJ)
      SDISP(2,J)=DISP(2,JJJ)
      SDISP(3,J)=DISP(3,JJJ)
      PRINT 128, J,JJJ,(SNFMD(I,J),I=1,3),(SDISP(I,J),I=1,3)
   50 CONTINUE
      PRINT 120
C     STATION CHECK
      DO 71 J=1,NNFMS
      K=1
      KK=2
```

```
      KKK=3
      IF(SNFMD(1,J)-SNFMD(2,J))60,61,60
   60 PRINT 110,K,KK,J
   61 IF(SNFMD(1,J)-SNFMD(3,J))70,71,70
   70 PRINT 110,K,KKK,J
   71 CONTINUE
C     ESTABLISH MODE SHAPE AT MASS PROPERTIES STATIONS
      DO 90 I=1,3
      DO 90 J=1,NPRO
      IF(SSTA(L,J)-SNFMD(I,1))72,72,74
   72 DELTA(I,J)=(SDISP(I,2)-SDISP(I,1))/(SNFMD(I,2)-SNFMD(I,1))
      SMOD(I,J)=SDISP(I,1)-DELTA(I,J)*(SNFMD(I,1)-SSTA(L,J))
      GO TO 90
   74 IF(SSTA(L,J)-SNFMD(I,NNFMS))77,75,75
   75 DELTA(I,J)=(SDISP(I,NNFMS)-SDISP(I,NNFMS-1))
      DELTA(I,J)=DELTA(I,J)/(SNFMD(I,NNFMS)-SNFMD(I,NNFMS-1))
      SMOD(I,J)=SDISP(I,NNFMS)-DELTA(I,J)*(SNFMD(I,NNFMS)-SSTA(L,J))
      GO TO 90
   77 DO 80 K=1,NNFMS
      IF(SSTA(L,J)-SNFMD(I,K))83,82,80
   82 DELTA(I,J)=(SDISP(I,K+1)-SDISP(I,K-1))/(SNFMD(I,K+1)-SNFMD(I,K-1))
      SMOD(I,J)=SDISP(I,K)
      GO TO 90
   83 DELTA(I,J)=(SDISP(I,K)-SDISP(I,K-1))/(SNFMD(I,K)-SNFMD(I,K-1))
      SMOD(I,J)=SDISP(I,K-1)+DELTA(I,J)*(SSTA(L,J)-SNFMD(I,K-1))
      GO TO 90
   80 CONTINUE
   90 CONTINUE
      PRINT 127
      PRINT 120
      PRINT 131
      PRINT 120
      PRINT 126,(I,SSTA(L,I),(SMOD(J,I),DELTA(J,I),J=1,3),I=1,NPRO)
      READ 112,SX1,SX2
      PRINT 120
      PRINT 133
      PRINT 120
      PRINT 117,SX1,SX2
      PRINT 120
      GM1=0.0
      GM2=0.0
      GM3=0.0
      C112=0.0
      C212=0.0
      C312=0.0
      C123=0.0
      C223=0.0
      C323=0.0
      C131=0.0
      C231=0.0
      C331=0.0
C     GENERALIZED MASS AND MASS WEIGHTED DISPLACEMENTS
      DO 600 J=1,NPRO
      X(1,J)=SMOD(1,J)*SQRT(SMASS(L,J)+DELTA(1,J)**2*SINER(L,J)/SMOD(1,J
```

```
     $)**2)
      X(2,J)=SMOD(2,J)*SQRT(SMASS(L,J)+DELTA(2,J)**2*SINER(L,J)/SMOD(2,J
     $)**2)
      X(3,J)=SMOD(3,J)*SQRT(SMASS(L,J)+DELTA(3,J)**2*SINER(L,J)/SMOD(3,J
     $)**2)
      GM1=GM1+SMASS(L,J)*SMOD(1,J)**2+SINER(L,J)*DELTA(1,J)**2
      GM2=GM2+SMASS(L,J)*SMOD(2,J)**2+SINER(L,J)*DELTA(2,J)**2
      GM3=GM3+SMASS(L,J)*(SMOD(3,J))**2+SINER(L,J)*DELTA(3,J)**2
  600 CONTINUE
      PRINT 125
      PRINT 120
      PRINT 132
      PRINT 120
      PRINT 117,GM1,GM2,GM3
      PRINT 120
C     NORMALIZED DISPLACEMENTS
      DO  605 J=1,NPRO
      X(1,J)=X(1,J)/SQRT(GM1)
      X(2,J)=X(2,J)/SQRT(GM2)
  605 X(3,J)=X(3,J)/SQRT(GM3)
      PRINT 122
      PRINT 120
      PRINT  140
      PRINT 120
      DO 606 J=1,NPRO
  606 PRINT 126,J,SSTA(L,J),X(1,J),X(2,J),X(3,J)
      DO  400  J=1,NPRO
      IF(SSTA(L,J)-SX1)320,330,330
  330 IF(SSTA(L,J)-SX2)340,350,350
  320 C112=C112+X(1,J)*X(2,J)
      C123=C123+X(2,J)*X(3,J)
      C131=C131+X(3,J)*X(1,J)
      GO TO 400
  340 C212=C212+X(1,J)*X(2,J)
      C223=C223+X(2,J)*X(3,J)
      C231=C231+X(3,J)*X(1,J)
      GO TO 400
  350 C312=C312+X(1,J)*X(2,J)
      C323=C323+X(2,J)*X(3,J)
      C331=C331+X(3,J)*X(1,J)
  400 CONTINUE
      X11=SQRT(C131*C112/C123)
      X21=C112/X11
      X31=C131/X11
      X12=SQRT(C231*C212/C223)
      X22=C212/X12
      X32=C231/X12
      X13=SQRT(C331*C312/C323)
      X23=C312/X13
      X33=C331/X13
      PRINT 120
      PRINT 141
      PRINT 120
      PRINT 142
```

```
          PRINT 143,X11,X12,X13
          PRINT 144,X21,X22,X23
          PRINT 145,X31,X32,X33
          PRINT 120
C         ORTHOGONAL CHECK
          O12=C112+C212+C312
          O23=C123+C223+C323
          O31=C131+C231+C331
          PRINT 123
          PRINT 120
          PRINT 146,O12
          PRINT 120
          PRINT 147,O23
          PRINT 120
          PRINT 148,O31
          PRINT 121
     1000 CONTINUE
      101 FORMAT (80H
         $                                                                              )
      102 FORMAT (26I3)
      103 FORMAT (F10.5)
      104 FORMAT  (I5,5H     ,F10.1,20H                    ,F10.6)
      105 FORMAT (80H
         $                                                                              )
      106 FORMAT (10F8.3)
      107 FORMAT (10F8.2)
      108 FORMAT (8E10.5)
      109 FORMAT (26I3)
      110 FORMAT (26I3)
      112 FORMAT (5E15.8)
      117 FORMAT (6(3H   ,E15.8))
      120 FORMAT (1H0)
      121 FORMAT (1H1)
      122 FORMAT (50H   MASS  WEIGHTED  MODES  FOLLOW                        )
      123 FORMAT (50H   ORTHOGONALITY  CHECK                                 )
      125 FORMAT (80H   GENERALIZED  MASS  FOR  MODES  AND  MASS  PROPERTIES
         $,  MODES  NOT  NORMALIZED                                             )
      126 FORMAT (I5,7(3H   ,E15.8))
      127 FORMAT (80H   MODE SHAPE AT MASS PROP. STA.  FOLLOW
         $                                                                      )
      128 FORMAT (2I5,6(3H   ,E15.8))
      129 FORMAT (80H   SELECTED ACCELEROMETERS STATIONS AND DISPLACEMENTS
         $                                                                      )
      130 FORMAT (116H   PT   ACC   STA   MODE 1       STA  MODE 2       STA
         $MODE 3        DISP  MODE 1      DISP  MODE 2      DISP  MODE 3      )
      131 FORMAT (130H   PT   STATION            DISP  MODE 1      SLOPE  MOD
         $E 1      DISP  MODE 2      SLOPE  MODE 2     DISP  MODE 3      SLOP
         ,E  MODE 3         )
      132 FORMAT (80H   MODE 1             MODE 2             MODE 3
         $                                                                      )
      133 FORMAT  (40H   INTERFACE  STATIONS                                  )
      135 FORMAT (35H   FIRST   BENDING  FREQUENCY  IS  ,E15.8)
      136 FORMAT (35H   SECOND  BENDING  FREQUENCY  IS  ,E15.8)
      137 FORMAT (35H   THIRD   BENDING  FREQUENCY  IS  ,E15.8)
```

```
  138 FORMAT (22H   MASS  PROPERTIES    )
  139 FORMAT (100H   PT   STATION              MASS                 ROTATORY
     $INERTIA                                                              )
  140 FORMAT (100H   PT   STATION              DISP  MODE  1       DISP  MODE
     $2       DISP  MODE  3                                                )
  141 FORMAT (50H   THREE  POINT  MODES,  MASS  WEIGHTED                   )
  142 FORMAT (80H               DISP  PT  1       DISP  PT  2        DISP  P
     $T  3                                                                 )
  143 FORMAT (12H   MODE  1   ,(5(E15.8,3H   )))
  144 FORMAT (12H   MODE  2   ,(5(E15.8,3H   )))
  145 FORMAT (12H   MODE  3   ,(5(E15.8,3H   )))
  146 FORMAT   (8H   012= ,E15.8)
  147 FORMAT   (8H   023= ,E15.8)
  148 FORMAT   (8H   031= ,E15.8)
  149 FORMAT (I5,3(3H   ,E15.8))
      END
```

THREE POINT MODES, SAD-202, YAW LIFTOFF USING FILES 0602044, 0602010, 0602016

FIRST BENDING FREQUENCY IS 0.13273900E+01

SECOND BENDING FREQUENCY IS 0.22905900E+01

THIRD BENDING FREQUENCY IS 0.41364300E+01

MASS PROPERTIES

PT	STATION	MASS	ROTATORY INERTIA
1	0.00000000E+00	0.48540000E+02	0.12336000E+07
2	0.12000000E+02	0.84417000E+03	0.12953000E+08
3	0.33000000E+02	0.12448000E+04	0.37624000E+08
4	0.53000000E+02	0.39820000E+04	0.13271000E+09
5	0.72000000E+02	0.28747000E+04	0.93753000E+08
6	0.92000000E+02	0.52501000E+04	0.13734000E+09
7	0.11100000E+03	0.37317000E+04	0.23395000E+09
8	0.12700000E+03	0.27733000E+04	0.92622000E+08
9	0.14900000E+03	0.18045000E+04	0.66408000E+08
10	0.16900000E+03	0.16174000E+04	0.54072000E+08
11	0.18900000E+03	0.27871000E+04	0.86660000E+08
12	0.21100000E+03	0.20690000E+04	0.44718000E+08
13	0.22900000E+03	0.21057000E+04	0.40431000E+08
14	0.25100000E+03	0.15999000E+04	0.82542000E+07
15	0.27000000E+03	0.25894000E+04	0.92774000E+07
16	0.29000000E+03	0.29135000E+04	0.14297000E+08
17	0.31000000E+03	0.29168000E+04	0.13938000E+08
18	0.33000000E+03	0.29211000E+04	0.14041000E+08
19	0.35000000E+03	0.29063000E+04	0.13835000E+08
20	0.37000000E+03	0.29120000E+04	0.13835000E+08
21	0.39000000E+03	0.29082000E+04	0.13835000E+08
22	0.41000000E+03	0.29039000E+04	0.13835000E+08
23	0.43000000E+03	0.29073000E+04	0.14246000E+08
24	0.45000000E+03	0.29030000E+04	0.14555000E+08
25	0.47000000E+03	0.29039000E+04	0.14246000E+08
26	0.49000000E+03	0.29077000E+04	0.14246000E+08
27	0.51000000E+03	0.29016000E+04	0.14144000E+08
28	0.53000000E+03	0.29025000E+04	0.14143000E+08
29	0.55000000E+03	0.29058000E+04	0.14246000E+08
30	0.57000000E+03	0.29011000E+04	0.14863000E+08
31	0.59000000E+03	0.29049000E+04	0.15377000E+08
32	0.61000000E+03	0.29058000E+04	0.14349000E+08
33	0.63000000E+03	0.28978000E+04	0.14144000E+08
34	0.65000000E+03	0.29025000E+04	0.14246000E+08
35	0.67000000E+03	0.29044000E+04	0.14247000E+08
36	0.69000000E+03	0.28978000E+04	0.14144000E+08
37	0.71000000E+03	0.29039000E+04	0.14247000E+08
38	0.73000000E+03	0.29044000E+04	0.14247000E+08
39	0.75000000E+03	0.29011000E+04	0.14453000E+08
40	0.77000000E+03	0.29097000E+04	0.14452000E+08
41	0.79000000E+03	0.29087000E+04	0.14351000E+08
42	0.81000000E+03	0.29087000E+04	0.14453000E+08

43	0.83000000E+03	0.29177000E+04	0.14350000E+08
44	0.85000000E+03	0.29097000E+04	0.14145000E+08
45	0.87100000E+03	0.12182000E+03	0.12336000E+07
46	0.89000000E+03	0.20034000E+03	0.22116000E+07
47	0.90800000E+03	0.63555000E+03	0.12216000E+08
48	0.93200000E+03	0.62623000E+03	0.18401000E+08
49	0.94900000E+03	0.17597000E+04	0.42559000E+08
50	0.96600000E+03	0.80307000E+03	0.26975000E+08
51	0.99100000E+03	0.45840000E+03	0.17465000E+08
52	0.10110000E+04	0.46191000E+03	0.16917000E+08
53	0.10310000E+04	0.48738000E+03	0.17191000E+08
54	0.10510000E+04	0.52382000E+03	0.16734000E+08
55	0.10720000E+04	0.65686000E+03	0.18754000E+08
56	0.10920000E+04	0.76663000E+03	0.12527000E+08
57	0.11070000E+04	0.48957000E+03	0.10059000E+08
58	0.11290000E+04	0.37322000E+03	0.71324000E+07
59	0.11530000E+04	0.23588000E+04	0.84859000E+07
60	0.11710000E+04	0.68827000E+04	0.98840000E+07
61	0.11910000E+04	0.11117000E+05	0.42343000E+08
62	0.12100000E+04	0.14103000E+05	0.44651000E+08
63	0.12300000E+04	0.16441000E+05	0.54267000E+08
64	0.12500000E+04	0.17345000E+05	0.10335000E+08
65	0.12700000E+04	0.17864000E+05	0.15139000E+09
66	0.12860000E+04	0.64435000E+04	0.24970000E+08
67	0.13090000E+04	0.35038000E+03	0.14722000E+08
68	0.13290000E+04	0.19715000E+03	0.13350000E+08
69	0.13490000E+04	0.19451000E+03	0.10333000E+08
70	0.13690000E+04	0.19319000E+03	0.10333000E+08
71	0.13890000E+04	0.19276000E+03	0.10516000E+08
72	0.14090000E+04	0.19451000E+03	0.10790000E+08
73	0.14300000E+04	0.23754000E+03	0.11796000E+08
74	0.14480000E+04	0.25818000E+03	0.12345000E+08
75	0.14690000E+04	0.27881000E+03	0.12985000E+08
76	0.14890000E+04	0.28408000E+03	0.99671000E+07
77	0.15090000E+04	0.26125000E+03	0.95099000E+07
78	0.15280000E+04	0.30560000E+03	0.97842000E+07
79	0.15460000E+04	0.25686000E+03	0.27981000E+08
80	0.15680000E+04	0.23842000E+03	0.10607000E+08
81	0.15880000E+04	0.20198000E+03	0.88698000E+07
82	0.16080000E+04	0.27135000E+03	0.12619000E+08
83	0.16290000E+04	0.37234000E+03	0.16642000E+08
84	0.16490000E+04	0.69857000E+03	0.32187000E+08
85	0.16710000E+04	0.83488000E+03	0.42631000E+08
86	0.16890000E+04	0.93499000E+03	0.46365000E+08
87	0.17040000E+04	0.16880000E+03	0.18125000E+08
88	0.17250000E+04	0.11569000E+03	0.11865000E+08
89	0.17450000E+04	0.11916000E+03	0.11585000E+08
90	0.17680000E+04	0.13703000E+03	0.12581000E+08
91	0.17820000E+04	0.31022000E+03	0.27436000E+08
92	0.18050000E+04	0.92270000E+02	0.76610000E+07
93	0.18250000E+04	0.10318000E+03	0.80970000E+07
94	0.18450000E+04	0.95020000E+02	0.70381000E+07
95	0.18650000E+04	0.84690000E+02	0.58859000E+07
96	0.18850000E+04	0.82270000E+02	0.53876000E+07
97	0.19050000E+04	0.79910000E+02	0.51073000E+07
98	0.19250000E+04	0.90390000E+02	0.40796000E+07
99	0.19450000E+04	0.89810000E+02	0.38616000E+07
100	0.19650000E+04	0.86960000E+02	0.34879000E+07
101	0.19850000E+04	0.84160000E+02	0.31454000E+07
102	0.20050000E+04	0.96080000E+02	0.33011000E+07
103	0.20290000E+04	0.57376000E+03	0.18062000E+08
104	0.20450000E+04	0.20241000E+04	0.51063000E+08
105	0.20650000E+04	0.19836000E+04	0.50575000E+08
106	0.20850000E+04	0.19389000E+04	0.49423000E+08
107	0.21050000E+04	0.19443000E+04	0.49578000E+08
108	0.21250000E+04	0.20145000E+04	0.51353000E+08

109	0.21420000E+04	0.92251000E+03	0.23512000E+08
110	0.21650000E+04	0.68877000E+03	0.17564000E+08
111	0.21820000E+04	0.50485000E+03	0.12862000E+08
112	0.22070000E+04	0.10650000E+04	0.27156000E+08
113	0.22250000E+04	0.17493000E+04	0.44596000E+08
114	0.22440000E+04	0.12906000E+04	0.25661000E+08
115	0.22640000E+04	0.76588000E+03	0.11460000E+08
116	0.22860000E+04	0.63522000E+03	0.68201000E+07
117	0.23030000E+04	0.23456000E+03	0.16941000E+07
118	0.23210000E+04	0.65670000E+02	0.28931000E+06
119	0.23440000E+04	0.33320000E+02	0.75676000E+05
120	0.23660000E+04	0.33560000E+02	0.10464000E+06
121	0.23890000E+04	0.72910000E+02	0.20741000E+06
122	0.24050000E+04	0.19956000E+03	0.51478000E+06
123	0.24260000E+04	0.27029000E+03	0.62938000E+06
124	0.24450000E+04	0.30457000E+03	0.71004000E+06
125	0.24650000E+04	0.30457000E+03	0.31765000E+06
126	0.24850000E+04	0.30457000E+03	0.16599000E+06
127	0.25050000E+04	0.30457000E+03	0.16599000E+06
128	0.25250000E+04	0.30457000E+03	0.16599000E+06
129	0.25450000E+04	0.29762000E+03	0.16225000E+06
130	0.25620000E+04	0.13230000E+03	0.72250000E+05
131	0.25880000E+04	0.85940000E+02	0.46713000E+05
132	0.26060000E+04	0.13708000E+03	0.74741000E+05
133	0.26250000E+04	0.20091000E+03	0.10962000E+06
134	0.26460000E+04	0.53397000E+03	0.29118000E+06
135	0.26620000E+04	0.55620000E+02	0.31142000E+05
136	0.26800000E+04	0.52100000E+01	0.18685000E+04

SELECTED ACCELEROMETERS STATIONS AND DISPLACEMENTS

PT	ACC	STA MODE 1	STA MODE 2	STA MODE 3	DISP MODE 1	DISP MODE 2	DISP MODE 3
1	1	0.11800000E+03	0.11800000E+03	0.11800000E+03	0.11440060E+01	0.60230000E-01	0.52561000E-01
2	82	0.33800000E+03	0.33800000E+03	0.33800000E+03	0.64062100E+00	0.25094000E-01	-.25955000E-01
3	83	0.45700000E+03	0.45700000E+03	0.45700000E+03	-.15741000E-01	-.39890000E-02	0.12580000E-02
4	84	0.61600000E+03	0.61600000E+03	0.61600000E+03	-.27142100E+00	-.28865000E-01	-.80429000E-01
5	85	0.73500000E+03	0.73500000E+03	0.73500000E+03	-.68819300E+00	-.41133000E-01	-.57758000E-01
6	8	0.94400000E+03	0.94400000E+03	0.94400000E+03	-.11482560E+01	-.69065000E-01	0.16296000E-01
7	9	0.10860000E+04	0.10860000E+04	0.10860000E+04	-.11202370E+01	-.27913000E-01	0.54245000E-01
8	10	0.10980000E+04	0.10980000E+04	0.10980000E+04	-.11801840E+01	-.27156000E-01	0.11014000E+00
9	11	0.12000000E+04	0.12000000E+04	0.12000000E+04	-.95652200E+00	0.15702000E-01	0.76316000E-01
10	12	0.13600000E+04	0.13600000E+04	0.13600000E+04	-.85710200E+00	0.48924000E-01	0.10791800E+00
11	13	0.14550000E+04	0.14550000E+04	0.14550000E+04	-.54603600E+00	0.58143000E-01	0.84237000E-01
12	14	0.16200000E+04	0.16200000E+04	0.16200000E+04	-.23293000E+00	0.96115000E-01	0.69083000E-01
13	15	0.16810000E+04	0.16810000E+04	0.16810000E+04	-.10317300E+00	0.11753100E+00	0.63412000E-01
14	16	0.18500000E+04	0.18500000E+04	0.18500000E+04	0.41310500E+00	0.15678200E+00	0.18831000E-01
15	17	0.20000000E+04	0.20000000E+04	0.20000000E+04	0.78366800E+00	0.18374000E+00	-.18316000E-01
16	18	0.21200000E+04	0.21200000E+04	0.21200000E+04	0.11356460E+01	0.18236900E+00	-.43169000E-01
17	19	0.22000000E+04	0.22000000E+04	0.22000000E+04	0.14038250E+01	0.18344400E+00	-.73540000E-01
18	20	0.22950000E+04	0.22950000E+04	0.22950000E+04	0.20079290E+01	0.11659700E+00	-.93019000E-01
19	21	0.24270000E+04	0.24270000E+04	0.24270000E+04	0.35702580E+01	-.23300900E+00	-.56574000E-01
20	22	0.25460000E+04	0.25460000E+04	0.25460000E+04	0.51045460E+01	-.69778800E+00	0.46134000E-01
21	66	0.26360000E+04	0.26360000E+04	0.26360000E+04	0.60221370E+01	-.10552070E+01	0.13309200E+00
22	23	0.26850000E+04	0.26850000E+04	0.26850000E+04	0.63962480E+01	-.13393190E+01	0.17086100E+00

67	0.13090000E+04	-.88879212E+00	0.62137500E-03	0.38334487E-01	0.20763750E-03	0.97844862E-01	0.19751250E-03
68	0.13290000E+04	-.87636462E+00	0.62137500E-03	0.42487237E-01	0.20763750E-03	0.10179511E+00	0.19751250E-03
69	0.13490000E+04	-.86393712E+00	0.62137500E-03	0.46639987E-01	0.20763750E-03	0.10574536E+00	0.19751250E-03
70	0.13690000E+04	-.82763259E+00	0.32743789E-02	0.49797379E-01	0.97042105E-04	0.10567453E+00	-.24927368E-03
71	0.13890000E+04	-.76214501E+00	0.32743789E-02	0.51738221E-01	0.97042105E-04	0.10068906E+00	-.24927368E-03
72	0.14090000E+04	-.69665743E+00	0.32743789E-02	0.53679063E-01	0.97042105E-04	0.95703589E-01	-.24927368E-03
73	0.14300000E+04	-.62789547E+00	0.32743789E-02	0.55716947E-01	0.97042105E-04	0.90468842E-01	-.24927368E-03
74	0.14480000E+04	-.56895665E+00	0.32743789E-02	0.57463705E-01	0.97042105E-04	0.85981916E-01	-.24927368E-03
75	0.14690000E+04	-.51946943E+00	0.18976121E-02	0.61364867E-01	0.23013333E-03	0.82951206E-01	-.91842424E-04
76	0.14890000E+04	-.48151719E+00	0.18976121E-02	0.65967533E-01	0.23013333E-03	0.81114358E-01	-.91842424E-04
77	0.15090000E+04	-.44356495E+00	0.18976121E-02	0.70570200E-01	0.23013333E-03	0.79277509E-01	-.91842424E-04
78	0.15280000E+04	-.40751032E+00	0.18976121E-02	0.74942733E-01	0.23013333E-03	0.77532503E-01	-.91842424E-04
79	0.15460000E+04	-.37335330E+00	0.18976121E-02	0.79085133E-01	0.23013333E-03	0.75879339E-01	-.91842424E-04
80	0.15680000E+04	-.33160583E+00	0.18976121E-02	0.84148067E-01	0.23013333E-03	0.73858806E-01	-.91842424E-04
81	0.15880000E+04	-.29365359E+00	0.18976121E-02	0.88750733E-01	0.23013333E-03	0.72021958E-01	-.91842424E-04
82	0.16080000E+04	-.25570135E+00	0.18976121E-02	0.93353400E-01	0.23013333E-03	0.70185109E-01	-.91842424E-04
83	0.16290000E+04	-.21378552E+00	0.21271639E-02	0.99274738E-01	0.35108197E-03	0.68246295E-01	-.92967213E-04
84	0.16490000E+04	-.17124225E+00	0.21271639E-02	0.10629637E+00	0.35108197E-03	0.66386951E-01	-.92967213E-04
85	0.16710000E+04	-.12444463E+00	0.21271639E-02	0.11402018E+00	0.35108197E-03	0.64341672E-01	-.92967213E-04
86	0.16890000E+04	-.78733805E-01	0.30548994E-02	0.11938903E+00	0.23225444E-03	0.61301657E-01	-.26379290E-03
87	0.17040000E+04	-.32910314E-01	0.30548994E-02	0.12287285E+00	0.23225444E-03	0.57344763E-01	-.26379290E-03
88	0.17250000E+04	0.31242574E-01	0.30548994E-02	0.12775020E+00	0.23225444E-03	0.51805112E-01	-.26379290E-03
89	0.17450000E+04	0.92340562E-01	0.30548994E-02	0.13239528E+00	0.23225444E-03	0.46529254E-01	-.26379290E-03
90	0.17680000E+04	0.16260325E+00	0.30548994E-02	0.13773714E+00	0.23225444E-03	0.40462018E-01	-.26379290E-03
91	0.17820000E+04	0.20537184E+00	0.30548994E-02	0.14098870E+00	0.23225444E-03	0.36768917E-01	-.26379290E-03
92	0.18050000E+04	0.27563453E+00	0.30548994E-02	0.14633055E+00	0.23225444E-03	0.30701680E-01	-.26379290E-03
93	0.18250000E+04	0.33673251E+00	0.30548994E-02	0.15097564E+00	0.23225444E-03	0.25425822E-01	-.26379290E-03
94	0.18450000E+04	0.39783050E+00	0.30548994E-02	0.15562073E+00	0.23225444E-03	0.20149964E-01	-.26379290E-03
95	0.18650000E+04	0.45016130E+00	0.24704200E-02	0.15947780E+00	0.17972000E-03	0.15116300E-01	-.24764667E-03
96	0.18850000E+04	0.49956970E+00	0.24704200E-02	0.16307220E+00	0.17972000E-03	0.10163366E-01	-.24764667E-03
97	0.19050000E+04	0.54897810E+00	0.24704200E-02	0.16666660E+00	0.17972000E-03	0.52104333E-02	-.24764667E-03
98	0.19250000E+04	0.59838650E+00	0.24704200E-02	0.17026100E+00	0.17972000E-03	0.25750000E-03	-.24764667E-03
99	0.19450000E+04	0.64779490E+00	0.24704200E-02	0.17385540E+00	0.17972000E-03	-.46954333E-02	-.24764667E-03
100	0.19650000E+04	0.69720330E+00	0.24704200E-02	0.17744980E+00	0.17972000E-03	-.96483667E-02	-.24764667E-03
101	0.19850000E+04	0.74661170E+00	0.24704200E-02	0.18104420E+00	0.17972000E-03	-.14601300E-01	-.24764667E-03
102	0.20050000E+04	0.79833375E+00	0.29331500E-02	0.18368287E+00	-.11425000E-04	-.19351542E-01	-.20710833E-03
103	0.20290000E+04	0.86872935E+00	0.29331500E-02	0.18340867E+00	-.11425000E-04	-.24322142E-01	-.20710833E-03
104	0.20450000E+04	0.91565975E+00	0.29331500E-02	0.18322587E+00	-.11425000E-04	-.27635875E-01	-.20710833E-03
105	0.20650000E+04	0.97432275E+00	0.29331500E-02	0.18299737E+00	-.11425000E-04	-.31778042E-01	-.20710833E-03
106	0.20850000E+04	0.10329857E+01	0.29331500E-02	0.18276887E+00	-.11425000E-04	-.35920208E-01	-.20710833E-03
107	0.21050000E+04	0.10916487E+01	0.29331500E-02	0.18254037E+00	-.11425000E-04	-.40062375E-01	-.20710833E-03
108	0.21250000E+04	0.11524871E+01	0.33522375E-02	0.18243619E+00	0.13437500E-04	-.45067187E-01	-.37963750E-03
109	0.21420000E+04	0.12093952E+01	0.33522375E-02	0.18266462E+00	0.13437500E-04	-.51521025E-01	-.37963750E-03
110	0.21650000E+04	0.12864967E+01	0.33522375E-02	0.18297369E+00	0.13437500E-04	-.60252687E-01	-.37963750E-03
111	0.21820000E+04	0.13434847E+01	0.33522375E-02	0.18320212E+00	0.13437500E-04	-.66706525E-01	-.37963750E-03
112	0.22070000E+04	0.14483379E+01	0.63589895E-02	0.17851843E+00	-.70365263E-03	-.74075221E-01	-.20503158E-03
113	0.22250000E+04	0.15627997E+01	0.63589895E-02	0.16585268E+00	-.70365263E-03	-.78665789E-01	-.20503158E-03
114	0.22440000E+04	0.16836205E+01	0.63589895E-02	0.15248328E+00	-.70365263E-03	-.82561389E-01	-.20503158E-03
115	0.22640000E+04	0.18108003E+01	0.63589895E-02	0.13841023E+00	-.70365263E-03	-.86662021E-01	-.20503158E-03
116	0.22860000E+04	0.19506981E+01	0.63589895E-02	0.12292987E+00	-.70365263E-03	-.91172716E-01	-.20503158E-03
117	0.23030000E+04	0.21026156E+01	0.11835825E-01	0.95408758E-01	-.26485303E-02	-.90809273E-01	0.27609091E-03
118	0.23210000E+04	0.23156605E+01	0.11835825E-01	0.47735212E-01	-.26485303E-02	-.85830636E-01	0.27609091E-03
119	0.23440000E+04	0.25878845E+01	0.11835825E-01	-.13180985E-01	-.26485303E-02	-.79489545E-01	0.27609091E-03
120	0.23660000E+04	0.28482726E+01	0.11835825E-01	-.71448652E-01	-.26485303E-02	-.73415545E-01	0.27609091E-03
121	0.23890000E+04	0.31204966E+01	0.11835825E-01	-.13236485E+00	-.26485303E-02	-.67065455E-01	0.27609091E-03
122	0.24050000E+04	0.33098698E+01	0.11835825E-01	-.17474133E+00	-.26485303E-02	-.62648000E-01	0.27609091E-03
123	0.24260000E+04	0.35584222E+01	0.11835825E-01	-.23036047E+00	-.26485303E-02	-.56850091E-01	0.27609091E-03
124	0.24450000E+04	0.38023352E+01	0.12893176E-01	-.30331171E+00	-.39057059E-02	-.41038336E-01	0.86309244E-03
125	0.24650000E+04	0.40601987E+01	0.12893176E-01	-.38142582E+00	-.39057059E-02	-.23776487E-01	0.86309244E-03
126	0.24850000E+04	0.43180622E+01	0.12893176E-01	-.45953994E+00	-.39057059E-02	-.65146387E-02	0.86309244E-03
127	0.25050000E+04	0.45759258E+01	0.12893176E-01	-.53765406E+00	-.39057059E-02	0.10747210E-01	0.86309244E-03
128	0.25250000E+04	0.48337893E+01	0.12893176E-01	-.61576818E+00	-.39057059E-02	0.28009059E-01	0.86309244E-03
129	0.25450000E+04	0.50916528E+01	0.12893176E-01	-.69388229E+00	-.39057059E-02	0.45270908E-01	0.86309244E-03
130	0.25620000E+04	0.52676733E+01	0.10195455E-01	-.76132916E+00	-.39713222E-02	0.61593700E-01	0.96620000E-03
131	0.25880000E+04	0.55327551E+01	0.10195455E-01	-.86458353E+00	-.39713222E-02	0.86714400E-01	0.96620000E-03
132	0.26060000E+04	0.57162733E+01	0.10195455E-01	-.93606733E+00	-.39713222E-02	0.10410600E+00	0.96620000E-03
133	0.26250000E+04	0.59099870E+01	0.10195455E-01	-.10115224E+01	-.39713222E-02	0.12246380E+00	0.96620000E-03
134	0.26460000E+04	0.60984862E+01	0.76349184E-02	-.11131890E+01	-.57982041E-02	0.14079996E+00	0.77079592E-03
135	0.26620000E+04	0.62206449E+01	0.76349184E-02	-.12059603E+01	-.57982041E-02	0.15313269E+00	0.77079592E-03
136	0.26800000E+04	0.63580734E+01	0.76349184E-02	-.13103280E+01	-.57982041E-02	0.16700702E+00	0.77079592E-03

MODE SHAPE AT MASS PROP. STA. FOLLOW

PT	STATION	DISP MODE 1	SLOPE MODE 1	DISP MODE 2	SLOPE MODE 2	DISP MODE 3	SLOPE MODE 3
1	0.00000000E+00	0.14140034E+01	-.22881136E-02	0.79075673E-01	-.15970909E-03	0.94674127E-01	-.35689091E-03
2	0.12000000E+02	0.13865460E+01	-.22881136E-02	0.77159164E-01	-.15970909E-03	0.90391436E-01	-.35689091E-03
3	0.33000000E+02	0.13384957E+01	-.22881136E-02	0.73805273E-01	-.15970909E-03	0.82896727E-01	-.35689091E-03
4	0.53000000E+02	0.12927334E+01	-.22881136E-02	0.70611091E-01	-.15970909E-03	0.75758909E-01	-.35689091E-03
5	0.72000000E+02	0.12492592E+01	-.22881136E-02	0.67576618E-01	-.15970909E-03	0.68977982E-01	-.35689091E-03
6	0.92000000E+02	0.12034969E+01	-.22881136E-02	0.64382436E-01	-.15970909E-03	0.61840164E-01	-.35689091E-03
7	0.11100000E+03	0.11600228E+01	-.22881136E-02	0.61347964E-01	-.15970909E-03	0.55059236E-01	-.35689091E-03
8	0.12700000E+03	0.11234129E+01	-.22881136E-02	0.58792618E-01	-.15970909E-03	0.49348982E-01	-.35689091E-03
9	0.14900000E+03	0.10730744E+01	-.22881136E-02	0.55279018E-01	-.15970909E-03	0.41497382E-01	-.35689091E-03
10	0.16900000E+03	0.10273122E+01	-.22881136E-02	0.52084836E-01	-.15970909E-03	0.34359564E-01	-.35689091E-03
11	0.18900000E+03	0.98154993E+00	-.22881136E-02	0.48890655E-01	-.15970909E-03	0.27221745E-01	-.35689091E-03
12	0.21100000E+03	0.93121143E+00	-.22881136E-02	0.45377055E-01	-.15970909E-03	0.19370145E-01	-.35689091E-03
13	0.22900000E+03	0.89002539E+00	-.22881136E-02	0.42502291E-01	-.15970909E-03	0.12946109E-01	-.35689091E-03
14	0.25100000E+03	0.83968689E+00	-.22881136E-02	0.38988691E-01	-.15970909E-03	0.50945091E-02	-.35689091E-03
15	0.27000000E+03	0.79621273E+00	-.22881136E-02	0.35954218E-01	-.15970909E-03	-.16864182E-02	-.35689091E-03
16	0.29000000E+03	0.75045045E+00	-.22881136E-02	0.32760036E-01	-.15970909E-03	-.88242364E-02	-.35689091E-03
17	0.31000000E+03	0.70468818E+00	-.22881136E-02	0.29565855E-01	-.15970909E-03	-.15962055E-01	-.35689091E-03
18	0.33000000E+03	0.65892591E+00	-.22881136E-02	0.26371673E-01	-.15970909E-03	-.23099873E-01	-.35689091E-03
19	0.35000000E+03	0.57443324E+00	-.55156471E-02	0.22161261E-01	-.24439496E-03	-.23210832E-01	0.22868067E-03
20	0.37000000E+03	0.46412029E+00	-.55156471E-02	0.17273361E-01	-.24439496E-03	-.18637218E-01	0.22868067E-03
21	0.39000000E+03	0.35380735E+00	-.55156471E-02	0.12385462E-01	-.24439496E-03	-.14063685E-01	0.22868067E-03
22	0.41000000E+03	0.24349441E+00	-.55156471E-02	0.74975630E-02	-.24439496E-03	-.94899916E-02	0.22868067E-03
23	0.43000000E+03	0.13318147E+00	-.55156471E-02	0.26096639E-02	-.24439496E-03	-.49163782E-02	0.22868067E-03
24	0.45000000E+03	0.22868529E-01	-.55156471E-02	-.22782353E-02	-.24439496E-03	-.34276471E-03	0.22868067E-03
25	0.47000000E+03	-.36645654E-01	-.16080503E-02	-.60228868E-02	-.15645283E-03	-.54208113E-02	-.51375472E-03
26	0.49000000E+03	-.68806660E-01	-.16080503E-02	-.91519434E-02	-.15645283E-03	-.15695906E-01	-.51375472E-03
27	0.51000000E+03	-.10096766E+00	-.16080503E-02	-.12281000E-01	-.15645283E-03	-.25971000E-01	-.51375472E-03
28	0.53000000E+03	-.13312867E+00	-.16080503E-02	-.15410057E-01	-.15645283E-03	-.36246094E-01	-.51375472E-03
29	0.55000000E+03	-.16528968E+00	-.16080503E-02	-.18539113E-01	-.15645283E-03	-.46521189E-01	-.51375472E-03
30	0.57000000E+03	-.19745069E+00	-.16080503E-02	-.21668170E-01	-.15645283E-03	-.56796283E-01	-.51375472E-03
31	0.59000000E+03	-.22961169E+00	-.16080503E-02	-.24797226E-01	-.15645283E-03	-.67071377E-01	-.51375472E-03
32	0.61000000E+03	-.26177270E+00	-.16080503E-02	-.27926283E-01	-.15645283E-03	-.77346472E-01	-.51375472E-03
33	0.63000000E+03	-.32045300E+00	-.35022857E-02	-.30308294E-01	-.10309243E-03	-.77761824E-01	0.19051261E-03
34	0.65000000E+03	-.39049871E+00	-.35022857E-02	-.32370143E-01	-.10309243E-03	-.73951571E-01	0.19051261E-03
35	0.67000000E+03	-.46054443E+00	-.35022857E-02	-.34431992E-01	-.10309243E-03	-.70141319E-01	0.19051261E-03
36	0.69000000E+03	-.53059014E+00	-.35022857E-02	-.36493840E-01	-.10309243E-03	-.66331067E-01	0.19051261E-03
37	0.71000000E+03	-.60063586E+00	-.35022857E-02	-.38555689E-01	-.10309243E-03	-.62520815E-01	0.19051261E-03
38	0.73000000E+03	-.67068157E+00	-.35022857E-02	-.40617538E-01	-.10309243E-03	-.58710563E-01	0.19051261E-03
39	0.75000000E+03	-.72121188E+00	-.22012584E-02	-.43137689E-01	-.13364593E-03	-.52443120E-01	0.35432536E-03
40	0.77000000E+03	-.76523704E+00	-.22012584E-02	-.45810608E-01	-.13364593E-03	-.45356612E-01	0.35432536E-03
41	0.79000000E+03	-.80926221E+00	-.22012584E-02	-.48483526E-01	-.13364593E-03	-.38270105E-01	0.35432536E-03
42	0.81000000E+03	-.85328738E+00	-.22012584E-02	-.51156445E-01	-.13364593E-03	-.31183598E-01	0.35432536E-03
43	0.83000000E+03	-.89731255E+00	-.22012584E-02	-.53829364E-01	-.13364593E-03	-.24097091E-01	0.35432536E-03
44	0.85000000E+03	-.94133771E+00	-.22012584E-02	-.56502282E-01	-.13364593E-03	-.17010584E-01	0.35432536E-03
45	0.87100000E+03	-.98756414E+00	-.22012584E-02	-.59308847E-01	-.13364593E-03	-.95697512E-02	0.35432536E-03
46	0.89000000E+03	-.10293880E+01	-.22012584E-02	-.61840120E-01	-.13364593E-03	-.28375694E-02	0.35432536E-03
47	0.90800000E+03	-.10690107E+01	-.22012584E-02	-.64253746E-01	-.13364593E-03	0.35402871E-02	0.35432536E-03
48	0.93200000E+03	-.11218409E+01	-.22012584E-02	-.67461249E-01	-.13364593E-03	0.12044095E-01	0.35432536E-03
49	0.94900000E+03	-.11472694E+01	0.19731690E-03	-.67615986E-01	0.28980282E-03	0.17632232E-01	0.26724648E-03
50	0.96600000E+03	-.11439150E+01	0.19731690E-03	-.62689338E-01	0.28980282E-03	0.22175423E-01	0.26724648E-03
51	0.99100000E+03	-.11389821E+01	0.19731690E-03	-.55444268E-01	0.28980282E-03	0.28856585E-01	0.26724648E-03
52	0.10110000E+04	-.11350357E+01	0.19731690E-03	-.49648211E-01	0.28980282E-03	0.34201514E-01	0.26724648E-03
53	0.10310000E+04	-.11310894E+01	0.19731690E-03	-.43852155E-01	0.28980282E-03	0.39546444E-01	0.26724648E-03
54	0.10510000E+04	-.11271430E+01	0.19731690E-03	-.38056099E-01	0.28980282E-03	0.44891373E-01	0.26724648E-03
55	0.10720000E+04	-.11229994E+01	0.19731690E-03	-.31970239E-01	0.28980282E-03	0.50503549E-01	0.26724648E-03
56	0.10920000E+04	-.11502105E+01	-.49955833E-02	-.27534500E-01	0.63083333E-04	0.82192500E-01	0.46579167E-02
57	0.11070000E+04	-.11604491E+01	0.21927647E-02	-.23374412E-01	0.42017647E-03	0.10715552E+00	-.33160784E-03
58	0.11290000E+04	-.11122082E+01	0.21927647E-02	-.14130529E-01	0.42017647E-03	0.99860157E-01	-.33160784E-03
59	0.11530000E+04	-.10595819E+01	0.21927647E-02	-.40462941E-02	0.42017647E-03	0.91901569E-01	-.33160784E-03
60	0.11710000E+04	-.10201121E+01	0.21927647E-02	0.35168824E-02	0.42017647E-03	0.85932627E-01	-.33160784E-03
61	0.11910000E+04	-.97625688E+00	0.21927647E-02	0.11920411E-01	0.42017647E-03	0.79300471E-01	-.33160784E-03
62	0.12100000E+04	-.95030825E+00	0.62137500E-03	0.17778375E-01	0.20763750E-03	0.78291125E-01	0.19751250E-03
63	0.12300000E+04	-.93788075E+00	0.62137500E-03	0.21931125E-01	0.20763750E-03	0.82241375E-01	0.19751250E-03
64	0.12500000E+04	-.92545325E+00	0.62137500E-03	0.26083875E-01	0.20763750E-03	0.86191625E-01	0.19751250E-03
65	0.12700000E+04	-.91302575E+00	0.62137500E-03	0.30236625E-01	0.20763750E-03	0.90141875E-01	0.19751250E-03
66	0.12860000E+04	-.90308375E+00	0.62137500E-03	0.33558825E-01	0.20763750E-03	0.93302075E-01	0.19751250E-03

INTERFACE STATIONS

0.10421000E+04 0.16029000E+04

GENERALIZED MASS FOR MODES AND MASS PROPERTIES, MODES NOT NORMALIZED

MODE 1	MODE 2	MODE 3
0.29957631E+06	0.28863986E+04	0.16221133E+04

MASS WEIGHTED MODES FOLLOW

PT	STATION	DISP MODE 1	DISP MODE2	DISP MODE 3
1	0.00000000E+00	0.18588169E-01	0.10772934E-01	0.19107021E-01
2	0.12000000E+02	0.75125037E-01	0.43077431E-01	0.72589188E-01
3	0.33000000E+02	0.90010349E-01	0.51784856E-01	0.90706840E-01
4	0.53000000E+02	0.15662854E+00	0.89728515E-01	0.15655624E+00
5	0.72000000E+02	0.12889629E+00	0.73325183E-01	0.12567291E+00
6	0.92000000E+02	0.16668414E+00	0.93558697E-01	0.15218910E+00
7	0.11100000E+03	0.14439811E+00	0.83265645E-01	0.15919868E+00
8	0.12700000E+03	0.11533454E+00	0.64339950E-01	0.10694142E+00
9	0.14900000E+03	0.89980900E-01	0.49972306E-01	0.84440063E-01
10	0.16900000E+03	0.81503835E-01	0.44698688E-01	0.73640855E-01
11	0.18900000E+03	0.10236127E+00	0.55442544E-01	0.89877203E-01
12	0.21100000E+03	0.82282667E-01	0.43256661E-01	0.63165685E-01
13	0.22900000E+03	0.79211817E-01	0.40928399E-01	0.58243274E-01
14	0.25100000E+03	0.62527836E-01	0.30257681E-01	0.25956359E-01
15	0.27000000E+03	0.75111565E-01	0.35237419E-01	-.27074292E-01
16	0.29000000E+03	0.75676743E-01	0.34779863E-01	-.35531440E-01
17	0.31000000E+03	0.71263923E-01	0.31725641E-01	-.39402802E-01
18	0.33000000E+03	0.66925377E-01	0.28773357E-01	-.45425072E-01
19	0.35000000E+03	0.67868674E-01	0.27942771E-01	-.37566934E-01
20	0.37000000E+03	0.59150736E-01	0.24234401E-01	-.32704345E-01
21	0.39000000E+03	0.51187595E-01	0.20996407E-01	-.28295242E-01
22	0.41000000E+03	0.44493558E-01	0.18516076E-01	-.24642393E-01
23	0.43000000E+03	0.40234750E-01	0.17368228E-01	-.22418604E-01
24	0.45000000E+03	0.38511646E-01	-.17504574E-01	-.21666674E-01
25	0.47000000E+03	-.11661181E-01	-.12542140E-01	-.48689408E-01
26	0.49000000E+03	-.12996837E-01	-.14324328E-01	-.52532519E-01
27	0.51000000E+03	-.14860210E-01	-.16479152E-01	-.59228151E-01
28	0.53000000E+03	-.17140354E-01	-.18940202E-01	-.68206169E-01
29	0.55000000E+03	-.19696910E-01	-.21605997E-01	-.78708113E-01
30	0.57000000E+03	-.22490885E-01	-.24452882E-01	-.90486051E-01
31	0.59000000E+03	-.25376226E-01	-.27372344E-01	-.10275299E+00
32	0.61000000E+03	-.28080710E-01	-.30113170E-01	-.11424370E+00
33	0.63000000E+03	-.39654006E-01	-.31213795E-01	-.10544605E+00
34	0.65000000E+03	-.45395104E-01	-.33258483E-01	-.10052025E+00
35	0.67000000E+03	-.51377648E-01	-.35290438E-01	-.95539039E-01
36	0.69000000E+03	-.57465795E-01	-.37271180E-01	-.90423724E-01
37	0.71000000E+03	-.63877578E-01	-.39344806E-01	-.85535893E-01
38	0.73000000E+03	-.70315663E-01	-.41382757E-01	-.80563767E-01
39	0.75000000E+03	-.72600803E-01	-.44269339E-01	-.77700771E-01

40	0.77000000E+03	-.76950659E-01	-.46957252E-01	-.69344973E-01
41	0.79000000E+03	-.81184028E-01	-.49574382E-01	-.61130829E-01
42	0.81000000E+03	-.85458555E-01	-.52217218E-01	-.53500564E-01
43	0.83000000E+03	-.89855407E-01	-.54934707E-01	-.46423025E-01
44	0.85000000E+03	-.93996761E-01	-.57496180E-01	-.40172428E-01
45	0.87100000E+03	-.20409386E-01	-.12493630E-01	-.10117034E-01
46	0.89000000E+03	-.27283696E-01	-.16708843E-01	-.13121178E-01
47	0.90800000E+03	-.51205478E-01	-.31379114E-01	0.30828407E-01
48	0.93200000E+03	-.54115028E-01	-.33185105E-01	0.38473122E-01
49	0.94900000E+03	-.87960217E-01	-.63447808E-01	0.47022473E-01
50	0.96600000E+03	-.59256179E-01	-.43339432E-01	0.37830499E-01
51	0.99100000E+03	-.44579339E-01	-.31565554E-01	0.31690543E-01
52	0.10110000E+04	-.44593823E-01	-.29777501E-01	0.32831988E-01
53	0.10310000E+04	-.45646767E-01	-.28721343E-01	0.35025777E-01
54	0.10510000E+04	-.47155119E-01	-.27381373E-01	0.37249923E-01
55	0.10720000E+04	-.52608210E-01	-.27897764E-01	0.43111171E-01
56	0.10920000E+04	-.66551682E-01	-.14786347E-01	0.41321262E+00
57	0.11070000E+04	-.48601861E-01	-.26607044E-01	0.64400148E-01
58	0.11290000E+04	-.40688714E-01	-.21495944E-01	0.52705822E-01
59	0.11530000E+04	-.94742857E-01	-.23074333E-01	0.11338821E+00
60	0.11710000E+04	-.15513497E+00	0.25180421E-01	0.17889230E+00
61	0.11910000E+04	-.18986161E+00	0.56010890E-01	0.21440274E+00
62	0.12100000E+04	-.20632899E+00	0.47024108E-01	0.23316309E+00
63	0.12300000E+04	-.21987324E+00	0.59583660E-01	0.26430722E+00
64	0.12500000E+04	-.22271334E+00	0.65137195E-01	0.28228643E+00
65	0.12700000E+04	-.22339288E+00	0.88992191E-01	0.30516547E+00
66	0.12860000E+04	-.13256633E+00	0.53731271E-01	0.18756434E+00
67	0.13090000E+04	-.30706511E-01	0.19957080E-01	0.49213621E-01
68	0.13290000E+04	-.22861183E-01	0.17963954E-01	0.39755238E-01
69	0.13490000E+04	-.22314451E-01	0.17347349E-01	0.39866835E-01
70	0.13690000E+04	-.28487433E-01	0.14131063E-01	0.41542718E-01
71	0.13890000E+04	-.27388151E-01	0.14597088E-01	0.40094761E-01
72	0.14090000E+04	-.26481728E-01	0.15145278E-01	0.38879517E-01
73	0.14300000E+04	-.27106801E-01	0.17145395E-01	0.40625185E-01
74	0.14480000E+04	-.26847650E-01	0.18320428E-01	0.40614843E-01
75	0.14690000E+04	-.20179783E-01	0.24535626E-01	0.35358405E-01
76	0.14890000E+04	-.18430157E-01	0.24722036E-01	0.34700159E-01
77	0.15090000E+04	-.16908236E-01	0.25005009E-01	0.32583311E-01
78	0.15280000E+04	-.16941389E-01	0.27823893E-01	0.34400322E-01
79	0.15460000E+04	-.21350661E-01	0.32710776E-01	0.32514978E-01
80	0.15680000E+04	-.14663267E-01	0.27919781E-01	0.29273823E-01
81	0.15880000E+04	-.12835702E-01	0.26719497E-01	0.26306100E-01
82	0.16080000E+04	-.14522539E-01	0.32416410E-01	0.29826846E-01
83	0.16290000E+04	-.17554700E-01	0.44519712E-01	0.34025981E-01
84	0.16490000E+04	-.23548555E-01	0.64102015E-01	0.45491638E-01
85	0.16710000E+04	-.26211850E-01	0.74705132E-01	0.48557916E-01
86	0.16890000E+04	-.38258480E-01	0.74051982E-01	0.64459762E-01
87	0.17040000E+04	-.23774794E-01	0.34952260E-01	0.33462510E-01
88	0.17250000E+04	0.19235275E-01	0.29595026E-01	0.26465110E-01
89	0.17450000E+04	0.19086328E-01	0.30661692E-01	0.25612901E-01
90	0.17680000E+04	0.20100189E-01	0.33701331E-01	0.26038663E-01
91	0.17820000E+04	0.29972708E-01	0.51469720E-01	0.37888317E-01
92	0.18050000E+04	0.16188142E-01	0.28769305E-01	0.19551592E-01
93	0.18250000E+04	0.17067259E-01	0.31082527E-01	0.19709707E-01
94	0.18450000E+04	0.16414966E-01	0.30475876E-01	0.18047433E-01
95	0.18650000E+04	0.13311473E-01	0.28497375E-01	0.15312224E-01
96	0.18850000E+04	0.13352658E-01	0.28604999E-01	0.14454524E-01
97	0.19050000E+04	0.13580734E-01	0.28743333E-01	0.13943968E-01
98	0.19250000E+04	0.13825617E-01	0.30878153E-01	0.12419550E-01
99	0.19450000E+04	0.14299369E-01	0.31363698E-01	-.12133427E-01
100	0.19650000E+04	0.14565605E-01	0.31427666E-01	-.11698763E-01
101	0.19850000E+04	0.14855192E-01	0.31478423E-01	-.11400993E-01
102	0.20050000E+04	0.17297656E-01	0.33514742E-01	-.10462921E-01
103	0.20290000E+04	0.44318452E-01	0.81777415E-01	-.26208031E-01
104	0.20450000E+04	0.84447362E-01	0.15344259E+00	-.47992483E-01
105	0.20650000E+04	0.87966565E-01	0.15171039E+00	-.50717370E-01

106	0.20850000E+04	0.91244242E-01	0.14980400E+00	-.53377355E-01
107	0.21050000E+04	0.95698040E-01	0.14982494E+00	-.56875137E-01
108	0.21250000E+04	0.10419565E+00	0.15242159E+00	-.84172854E-01
109	0.21420000E+04	0.73389261E-01	0.10327418E+00	-.59988573E-01
110	0.21650000E+04	0.66813968E-01	0.89387638E-01	-.55696215E-01
111	0.21820000E+04	0.59364958E-01	0.76623731E-01	-.50276095E-01
112	0.22070000E+04	0.10546476E+00	0.12812880E+00	-.66290434E-01
113	0.22250000E+04	0.14241138E+00	0.15595056E+00	-.88483043E-01
114	0.22440000E+04	0.12520111E+00	0.12164769E+00	-.78027746E-01
115	0.22640000E+04	0.99648233E-01	0.83958726E-01	-.61991769E-01
116	0.22860000E+04	0.94811129E-01	0.67049263E-01	-.58582504E-01
117	0.23030000E+04	0.65220427E-01	0.69691054E-01	-.35665666E-01
118	0.23210000E+04	0.36204282E-01	0.27476220E-01	-.17660717E-01
119	0.23440000E+04	0.27933296E-01	-.13635187E-01	-.11547594E-01
120	0.23660000E+04	0.30947549E-01	-.17710376E-01	-.10790181E-01
121	0.23890000E+04	0.49667617E-01	-.30767276E-01	-.14557147E-01
122	0.24050000E+04	0.86824251E-01	-.57984050E-01	-.22517424E-01
123	0.24260000E+04	0.10825347E+00	-.80615108E-01	-.23835001E-01
124	0.24450000E+04	0.12285255E+00	-.11601759E+00	-.25343465E-01
125	0.24650000E+04	0.13013943E+00	-.13050011E+00	-.15875167E-01
126	0.24850000E+04	0.13801658E+00	-.15218555E+00	-.91758831E-02
127	0.25050000E+04	0.14621984E+00	-.17714356E+00	0.98952031E-02
128	0.25250000E+04	0.15442510E+00	-.20220522E+00	0.14950860E-01
129	0.25450000E+04	0.16076587E+00	-.22472807E+00	0.21225878E-01
130	0.25620000E+04	0.11081251E+00	-.16420152E+00	0.18734930E-01
131	0.25880000E+04	0.93796324E-01	-.15003847E+00	0.20621905E-01
132	0.26060000E+04	0.12238342E+00	-.20499188E+00	0.30966216E-01
133	0.26250000E+04	0.15317429E+00	-.26798888E+00	0.43824849E-01
134	0.26460000E+04	0.25758007E+00	-.48232326E+00	0.81440496E-01
135	0.26620000E+04	0.84796952E-01	-.16848563E+00	0.28556279E-01
136	0.26800000E+04	0.26521796E-01	-.55865061E-01	0.95009184E-02

THREE POINT MODES, MASS WEIGHTED

	DISP PT 1	DISP PT 2	DISP PT 3
MODE 1	0.50599498E+00	0.55407526E+00	0.26984763E+00
MODE 2	0.31892669E+00	-.14861435E+00	-.86068698E+00
MODE 3	0.37049647E+00	-.71157076E+00	-.18731789E+00

ORTHOGONALITY CHECK

012= -.15322257E+00

023= 0.62688772E-01

031= -.25734169E+00

APPENDIX D

DAMPING COEFFICIENT PROGRAM

The purpose of this program is to obtain the damping coefficients as described in Section IV. The program is written in FORTRAN IV as used on the General Electric 415 located in the Chrysler Corporation Huntsville Facility. This program generates damping coefficients from the input generalized damping and three-point modes. Included are input data description and format, internal data description, FORTRAN IV listing, and sample output.

DESCRIPTION OF INPUT DATA SYMBOLS, DAMPING COEFFICIENT PROGRAM.

FORTRAN SYMBOL	DESCRIPTION	FORMAT	COLUMN
IJ	Total number of cases	I3	1-3
M	Number of points in mode, 3 for three-point mode	I3	1-3
NN	Point to normalize on, 1 displacement of 1st stage, 2 for 2nd stage, 3 for 3rd stage.	I3	4-6
GDAMP(J)	Generalized damping, $2\zeta_j\omega_j M_j$ for the j^{th} mode, j = 1 through 3 for three-point mode.	E15.8	1-45
FREQ(J)	Frequency, ω_j, for the j^{th} mode.	E15.8	1-45
THETA(I,J)	Displacement Θ_{ij} of the j^{th} point in the i^{th} mode, Θ_{ij}		

DESCRIPTION OF INTERNAL DATA SYMBOLS, DAMPING COEFFICIENT PROGRAM

BB	Θ_{ij}, where the i^{th} mode is being normalized to the j^{th} point. NN is jmax.
TCTT(I,J)	$[\Theta][C][\Theta]^T$
TTT(I,J)	$[\Theta][\Theta]^T$
A(I,J)	$([\Theta][\Theta]^T)^{-1}$
B(I)	Temporary calculation for forming the inverse of $[\Theta][\Theta]^T$
E(I,J)	$([\Theta][\Theta]^T)^{-1}([\Theta][C][\Theta]^T)$
G(I,J)	$[C] = ([\Theta][\Theta]^T)^{-1}([\Theta][C][\Theta]^T)([\Theta][\Theta]^T)^{-1}$

DAMPING COEFFICIENTS FOR THREE POINT MODES

```
      DIMENSION A(5,5),E(5,5),G(5,5),IAC(5),FREQ(5),GDAMP(5),THETA(5,5)
      DIMENSION TCTT(5,5),TTT(5,5),B(5),C(5)
      READ 113
      PRINT 113
      PRINT 112
      READ 107,IJ
      DO 50 JI=1,IJ
      READ 113
      PRINT 113
      PRINT 111
      PRINT 127
      READ 107,M,NN
      IF(NN)200,201,200
  201 NN=1
  200 CONTINUE
      N=M
      READ 102,(GDAMP(J),J=1,M)
      READ 102,(FREQ(J),J=1,M)
      DO 9 J=1,M
    9 READ 102,(THETA(I,J),I=1,M)
      DO 11 J=1,M
      PRINT 111
      PRINT 110,J,FREQ(J),(THETA(I,J),I=1,M)
      BB=THETA(NN,J)
      DO 10 I=1,N
      THETA(I,J)=THETA(I,J)/BB
   10 CONTINUE
   11 PRINT 126,NN,(THETA(I,J),I=1,N)
      PRINT 111
      PRINT 116
      PRINT 111
      PRINT 117
      PRINT 111
      PRINT 128,(GDAMP(J),J=1,M)
      PRINT 111
      DO 20 I=1,N
      DO 20 J=1,N
      TCTT(I,J)=0.0
      TTT(I,J)=0.0
      DO 20 K=1,M
      TCTT(I,J) =TCTT(I,J)+THETA(I,K)*THETA(J,K)*GDAMP(K)
      TTT(I,J)=TTT(I,J)+THETA(I,K)*THETA(J,K)
   20  A(I,J)=TTT(I,J)
      NN=N-1
      A(1,1)=1.0/A(1,1)
      DO 1110 M=1,NN
      K=M+1
      DO 1160 I=1,M
      B(I)=0.0
      DO 1160 J=1,M
 1160 B(I)=B(I)+A(I,J)*A(J,K)
      D=0.0
      DO 1170 I=1,M
 1170 D=D+A(K,I)*B(I)
```

```
      D=-D+A(K,K)
      A(K,K)=1.0/D
      DO 1180 I=1,M
 1180 A(I,K)=-B(I)*A(K,K)
      DO 1190 J=1,M
      C(J)=0.0
      DO 1190 I=1,M
 1190 C(J)=C(J)+A(K,I)*A(I,J)
      DO 1100 J=1,M
 1100 A(K,J)=-C(J)*A(K,K)
      DO 1110 I=1,M
      DO 1110 J=1,M
 1110 A(I,J)=A(I,J)-B(I)*A(K,J)
      DO 30 I=1,N
      DO 30 J=1,N
      G(I,J)=0.0
      DO 30 K=1,N
   30 G(I,J)=G(I,J)+A(I,K)*TCTT(K,J)
      DO 40 I=1,N
      DO 40 J=1,N
      E(I,J)=0.0
      DO 40 K=1,N
   40 E(I,J)=E(I,J)+G(I,K)*A(K,J)
      PRINT 106
      PRINT 111
      PRINT 120, E(1,1)
      PRINT 111
      PRINT 121, E(1,2)
      PRINT 111
      PRINT 122, E(1,3)
      PRINT 111
      PRINT 123, E(2,2)
      PRINT 111
      PRINT 124, E(2,3)
      PRINT 111
      PRINT 125, E(3,3)
      PRINT 111
      PRINT 111
      PRINT 111
      PRINT 111
   50 PRINT 112
  102 FORMAT (5E15.8)
  106 FORMAT (40H   DAMPING COEFFICIENTS                                     )
  107 FORMAT  (25I3)
  110 FORMAT (10H    MODE   ,1I3,16H,  FREQUENCY,   ,1E15.8,3(3H   ,E15.8
     $))
  111 FORMAT  (1H0)
  112 FORMAT  (1H1)
  113 FORMAT (80H
     $                                                                   )
  116 FORMAT  (80H   GENERALIZED DAMPING COEFFICIENTS
     $                                                                        )
  117 FORMAT  (50H   MODE  1              MODE  2             MODE  3         )
  120 FORMAT (9H   C11=  ,E15.8)
```

```
      121 FORMAT (9H    C12=  ,E15.8)
      122 FORMAT (9H    C13=  ,E15.8)
      123 FORMAT (9H    C22=  ,E15.8)
      124 FORMAT (9H    C23=  ,F15.8)
      125 FORMAT (9H    C33=  ,F15.8)
      126 FORMAT (24H   NORMALIZED  TO  POINT,I3,17H                    ,3(3H
     $ ,F15.8))
      127 FORMAT (48H                                             ,18HDIS
     $P  PT  1      ,18HDISP  PT  2      ,18HDISP  PT  3      )
      128 FORMAT (5(3H   ,F15.8))
          END
```

DAMPING COEFFICIENTS FOR THREE POINT MODES, YAW 147 SECONDS

		DISP PT 1	DISP PT 2	DISP PT 3
MODE 1, FREQUENCY,	0.16400000E+01	0.10060400E+01	-.71022000E+00	-.18303000E+01
NORMALIZED TO POINT 3		-.54965853E+00	0.38803475E+00	0.10000000E+01
MODE 2, FREQUENCY,	0.25000000E+01	0.10086900E+01	-.22029000E+01	0.25044700E+01
NORMALIZED TO POINT 3		0.40275587E+00	-.87958730E+00	0.10000000E+01
MODE 3, FREQUENCY,	0.69100000E+01	0.14498600E+01	-.17991900E+00	0.78715000E+00
NORMALIZED TO POINT 3		0.18419107E+01	-.22857016E+00	0.10000000E+01

GENERALIZED DAMPING COEFFICIENTS

MODE 1	MODE 2	MODE 3
0.29562900E+06	0.19361000E+05	0.14283390E+07

DAMPING COEFFICIENTS

C11= 0.40634001E+06

C12= 0.24703839E+06

C13= 0.48752038E+05

C22= 0.33786358E+06

C23= 0.17874135E+06

C33= 0.14224948E+06

FORTRAN CODING FORM

CARD NO.	Columns 1–5	Columns 6–15	Columns 16–30	Columns 31–45
1		INITIAL TITLE CARD		
2	I,J			
3		1st SET TITLE CARD		
4	M	NN		
5	GDAMP(1)		GDAMP(2)	GDAMP(3)
6	FREQ(1)		FREQ(2)	FREQ(3)
7	THETA(1,1)		THETA(2,1)	THETA(3,1)
8	THETA(1,2)		THETA(2,2)	THETA(3,2)
9	THETA(1,3)		THETA(2,3)	THETA(3,3)

J TOTAL SETS OF CARDS 3 THROUGH 9

APPROVALS

TECHNICAL REPORT HSM-R-107-67

DYNAMIC TEST VEHICLE INTERNAL DAMPING STUDY

December 1967

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ABSTRACT

The structural damping of the Saturn vehicle is determined in terms of damping characteristics of individual stages and interfaces. Using these damping characteristics of the individual stages and interfaces, an analytical method was developed to compute the overall vehicle modal damping coefficients for various bending modes. This approach can also be used to obtain the modal damping of clustered arrangements knowing the individual damping characteristics of upper stages, payloads, and interfaces.