NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-361

Asteroid Belt Meteoroid Hazard Study

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July 1, 1968

TECHNICAL MEMORANDUM 33-361

Prepared Under Contract No. NAS 7-100
National Aeronautics & Space Administration

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Abstract

There is considerable interest in outer planet missions and in the hazard posed by the asteroid belt. Trajectories in the ecliptic plane, through the asteroid belt, require more shielding mass for protection against meteoroids, whereas trajectories out of the ecliptic plane require more propulsion mass. This report gives the System Designer a method for minimizing the shielding mass for a given probability of no meteoroid penetrations of the spacecraft shield.

A model of the asteroid belt is developed based on 1500 numbered asteroids. The meteoroid particle flux is

$$F = \alpha_c m^{-\beta}$$

where

F = particles of mass m or greater, meters^{-2} second^{-1} (m^{-2} s^{-1})

α_c = constant

β = constant

$$\text{The best estimate of } \beta \text{ obtained is } \beta = \frac{1.9}{3} = 0.63$$

A mathematical model is given for the probability, $P(S)$, of successfully traversing the asteroid belt, or the probability $P(0)$, of zero penetrations of the spacecraft shield. The spacecraft is represented by a 26-sided convex polyhedron. The spacecraft trajectory is assumed to be in the form of an elliptical orbit. The meteoroid capability of penetrating the spacecraft shield is included as a function of meteoroid size, density and relative velocity; and shield thickness, density and hardness. The probability, $P(0)$, is calculated as a function of spacecraft size, surface area, shield thickness and shield mass.

Two cases are considered: A, uniform shielding over the entire surface of the spacecraft; and B, optimum shielding so as to maximize $P(0)$ for a given spacecraft shape, size and shielding mass. Calculations are made for a 500- and a 900-day mission spacecraft orbit, for $3\beta = 1.9$ and $3\beta = 3.0$. A computer program is provided for the designer to use, permitting the parametric variation of the spacecraft mission trajectory, the asteroid belt model, the spacecraft shape, size and shielding material. This enables the designer to maximize $P(0)$ and to minimize the shielding mass.

Asteroid Belt Meteoroid Hazard Study

I. Introduction

Considerable interest has developed in recent months in outer planet missions and in the hazard posed by the asteroid belt. In studies relative to this, the System Engineer must compare spacecraft missions which fly out of the ecliptic plane, and thus avoid the asteroid belt, with spacecraft missions which fly in the ecliptic plane and pass through the asteroid belt. Flights out of the ecliptic plane require more propulsion mass, whereas flights in the ecliptic plane and through the asteroid belt require more shielding mass. This study is a theoretical approach to the problem, but it shows the practicing engineer all of the parameters involved in the shielding problem and how they interact.

This report provides the System Designer with a means for making calculations of spacecraft shielding mass for various trajectories, either in or out of the ecliptic plane, for a given probability of successfully traversing the asteroid belt, i.e., no meteoroid penetrations of the spacecraft shield.

II. Discussion

In this study, the spacecraft mission trajectory is assumed to be an elliptical orbit, with position and velocity known as a function of time. The meteoroid particle flux is the generally accepted relation

$$F = \alpha_c m^{-\beta} \qquad (1)$$

where

F = particles, of mass m or greater, meters^{-2} second^{-1} (m^{-2} s^{-1})

α_c = *constant*

β = *constant*

A detailed discussion of various asteroid belt models is given in Appendix A. The Volkoff model is in the ecliptic plane only, from 2 to 4 AU. The asteroid density is the same throughout the belt, and the asteroid velocity is the heliocentric orbital velocity at the mean solar distance of the particles.

The Friedlander and Vickers model is in the form of a doughnut, extending to ± 10 deg ecliptic latitude; inside the doughnut, the asteroid density is constant. The Chestek model is also in the form of a doughnut with constant density inside. For a spacecraft trajectory in the ecliptic plane, Chestek calculates the meteoroid velocity relative to the spacecraft. The Narin model is based on the position of 1563 numbered asteroids as of an April 19, 1973 date. Narin gets a strong clustering of asteroids at certain radii and a gradual fading away with ecliptic latitude and with distance from the center of the belt. The asteroid belt model used in the present report is derived from 1500 numbered asteroids. Each numbered asteroid is replaced by a swarm of meteoroids with a mass distribution given by Eq. (1).

All the meteoroids in the swarm have the same semi-major axis, eccentricity and inclination to the ecliptic as the parent asteroid. However, the longitudes of ascending node, arguments of perihelion, and mean anomalies of the meteoroids are uniformly distributed over the entire range of possible values, from 0 to 2π. This is considered reasonable because of the non-dependence of the asteroid distribution on ecliptic longitude. The meteoroids are assumed to result, in part, from the collision and grinding of the asteroids, and are therefore assumed to have a distribution, in ecliptic latitude and solar distance, similar to that of the asteroids. The model of the asteroid belt used in this report is an improvement over the previous models which have been described in the literature because of its much more detailed and realistic combination of meteoroid space, velocity and mass distributions.

Most of the values of β given in the literature are very close to $\beta = 2/3$. However, one possible value of β given in the literature is $\beta = 1.0$. This report presents calculations for two values of β: a best estimate value of $1.9/3 \approx 0.63$, and a conservative value of 1.0.

A mathematical model is presented in Appendix B for determining the probability of successfully traversing the asteroid belt, or more specifically, the probability of zero meteoroid penetrations of the spacecraft shield. The spacecraft surface is assumed to be that of a convex polyhedron. The probability of zero penetrations $P(0)$, or probability of success $P_I(S)$, is given by Eqs. (B-47 and -53) of Appendix B, or

$$P(0) = P_I(S) = \exp\left\{ - \int_{T_0}^{T_f} \left[\sum_j F_j(T) A_j \right] dT \right\} \tag{2}$$

where

$F_j(T)$ = the effective meteoroid flux on the jth face of the polyhedral spacecraft (destructive impacts $m^{-2}\,s^{-1}$)

A_j = the area of the jth face of the polyhedral spacecraft (m^2)

T = time

T_0 = time at which the spacecraft mission starts

T_f = time at which the spacecraft mission ends

The penetration depth of a high velocity meteoroid in the spacecraft shielding material is given by

$$p_1 = k_1\, d_p \ln\left(1 + \frac{\rho_t V_p^2}{k_2\, h_t}\right) \tag{3}$$

where

p_1 = meteoroid penetration depth, cm

k_1, k_2 = constants

ρ_t = target (spacecraft shielding) density, (g/cm^3)

h_t = target (spacecraft shielding) Brinell hardness, ($kg\text{-}wt/mm^2$)

d_p = projectile (meteoroid) diameter, cm

V_p = projectile (meteoroid) relative velocity component normal to the surface of the target, km/s

The meteoroid space and velocity distributions were obtained by means of a computer using the orbital elements of 1500 numbered asteroids.

Asteroidal meteoroids with masses in the range 1 to 10^{-4} g are of primary interest to this study, since they are large enough to puncture spacecraft structures and numerous enough to be hazardous. The radius R of the smallest meteoroid which can penetrate a shield of thickness t_j, on the jth face of the polyhedral spacecraft, is given by Eq. (B-60) of Appendix B, or

$$R = \frac{t_j}{C_1 \ln(1 + C_2 D^2)} \tag{4}$$

where, from Eqs. (B-34 and -35) of Appendix B,

$$C_1 = 3\,k_1 \approx (1.8 \pm 0.6)\left(\frac{\rho_p}{\rho_t}\right)^{2/3}$$

$$C_2 = \frac{\rho_t}{k_2\, h_t} \approx \frac{\rho_t \left(\frac{\rho_p}{\rho_t}\right)^{2/3}}{(4 \pm 2)\, h_t}$$

ρ_p = projectile (meteoroid) density (g/cm^3)

Also

$$\begin{aligned} D &= -\mathbf{n}_j \cdot \mathbf{W}' \\ &= -\mathbf{n}_j \cdot (\mathbf{U} - \mathbf{V})\, \mathfrak{M}^{-1}(T) \end{aligned}$$

from Eqs. (B-50 and -57) of Appendix B, where

D = component of meteoroid relative velocity normal to spacecraft,

$\mathbf{n}_j$ = outwardly drawn unit vector normal to the jth face of a polyhedral spacecraft, in spacecraft-fixed coordinates

$\mathbf{W}' = (\mathbf{U} - \mathbf{V})\,\mathcal{M}^{-1}(T)$ = velocity of meteoroid relative to the spacecraft in spacecraft-fixed coordinates

$\mathbf{U}$ = velocity of meteoroid

$\mathbf{V}$ = velocity of spacecraft

$\mathcal{M}^{-1}(T)$ = rotation matrix which converts a vector to spacecraft-fixed coordinates from space-fixed coordinates
Example:
$\mathbf{n}(\alpha) = \mathbf{n}(\alpha, T)\,\mathcal{M}^{-1}(T)$

$\mathbf{n}(\alpha)$ = outwardly drawn unit vector normal to spacecraft surface element α, in spacecraft-fixed coordinates

$\mathbf{n}(\alpha, T)$ = outwardly drawn unit vector normal to spacecraft surface element α, at time T, in space-fixed coordinates

The mass M_0 of the smallest meteoroid, of radius R, which can penetrate the shield of thickness t_j is given by

$$M_0 = \frac{4}{3}\pi R^3 \rho' \tag{5}$$

where ρ' = meteoroid density.

The flux $F_j(T)$, in Eq. (2) above, is given in Eq. (B-66) of Appendix B, and is a function of the spacecraft trajectory (position and velocity of the spacecraft as a function of time), meteoroid density distribution, meteoroid relative velocity, self-shadowing effect of a non-convex spacecraft, meteoroid damage function, and spacecraft orientation and surface position as a function of time.

The implementation of the above theoretical approach is given in Appendix C. Three coordinate systems are used: 1) a sun-centered coordinate system, 2) a space-fixed coordinate system with origin at the spacecraft, and 3) a spacecraft-fixed coordinate system. The spacecraft orientation matrix is derived. This is a rotation matrix which converts a vector from spacecraft-fixed coordinates to space-fixed coordinates; for example $\mathbf{n}(\alpha, T) = \mathbf{n}(\alpha)\,\mathcal{M}(T)$. The spacecraft trajectory is developed in terms of the spacecraft mission orbit elements. Expressions are derived giving the velocity of the meteoroids passing through the spacecraft position. The meteoroid density distribution is represented mathematically.

The output of the analytic model, the probability of zero penetrations of the spacecraft shield by meteoroids $P(0)$, or the probability that asteroidal meteoroids do not cause mission failure $P_I(S)$, is given by

$$P(0) = P_I(S) = \exp(-C\, l^2 t^{-3\beta}) \tag{6}$$

$$P(0) = P_I(S) = \exp[-C'\, A_s^{(1+3\beta)}\, W_s^{-3\beta}] \tag{7}$$

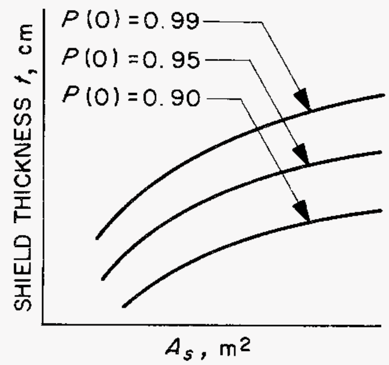
$$P(0) = P_I(S) = \exp[-C''\, l^{2(1+3\beta)}\, W_s^{-3\beta}] \tag{8}$$

$$P(0) = P_I(S) = \exp(-C'''\, A_s\, t^{-3\beta}) \tag{9}$$

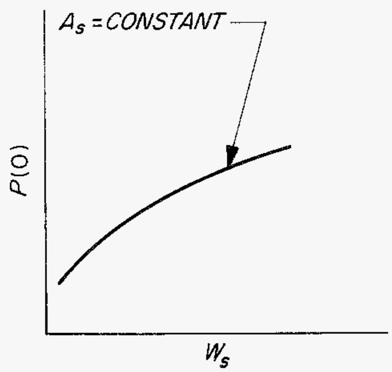
from Eqs. (E-37 through -40) of Appendix E, where C, C', C'', and C''' are constants calculated by the computer, l is the length of an edge of a convex polyhedron, (Fig. F-2, Appendix F), representing the spacecraft surface, and t, A_s, and W_s are the average thickness, total area, and total mass, of the spacecraft shielding. Two cases are considered: *Case A*, where the shielding is of uniform thickness over the entire surface of the spacecraft, and *Case B*, where the shielding is distributed over the faces of a convex polyhedral spacecraft in an optimum manner, so as to maximize the probability of success, for a given spacecraft shape, size l, and shielding mass, W_s.

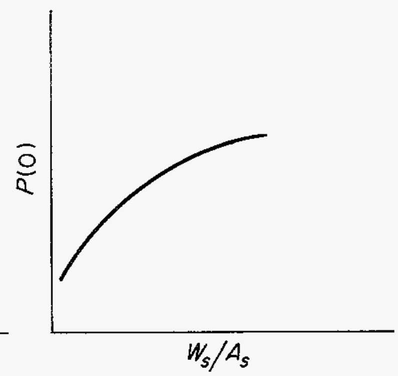
In Appendix F, the results of the calculations of four example cases are given: *Case I* is for a 500-day mission spacecraft orbit and $3\beta = 1.9$; *Case II* is for a 900-day mission and $3\beta = 1.9$; *Case III* is for a 500-day mission and $3\beta = 3.0$; and *Case IV* is for a 900-day mission and $3\beta = 3.0$. The spacecraft mission orbit elements, a (semi-major axis), e (eccentricity), i (inclination of the orbit from the ecliptic plane), and ω (argument of perihelion) were used for the 500- and 900-day mission orbits. Only four orbital elements were needed in the computer program because the asteroid model does not depend on the ecliptic longitude or on the time. The inclination of the 500- and 900-day orbits are about 2 and 4 deg, respectively. The spacecraft was represented by a convex polyhedron with 26 faces, called a rhombicuboctahedron, with l the length of an edge of this body. The meteoroids were assumed to be pure iron ($\rho' = 7.9$ g/cm^3), and the spacecraft shielding material was assumed to be aluminum ($\rho_s = 2.7$ g/cm^3). The expected number of damaging meteoroids/m^2, and a non-dimensional optimum shielding thickness on each of the 26 faces, were calculated for Cases I, II, III, and IV. The optimum shield thickness and the average shield thickness were also calculated. Figure F-6 is a plot of l versus W_s, for $P(0) = 0.99$, for Cases I and II with uniform shielding and with optimum shielding, for $3\beta = 1.9$. For $\beta = 1.0$ the shielding masses become extremely large, making the asteroid belt essentially impenetrable. Thus, use of the proper value of β is very important.

For either optimum or uniform shielding, the computer program can be used to generate curves of



for a particular spacecraft trajectory, meteoroid density and shielding material. Plots can also be made of





The computer program is given in Appendix G together with a description of the program, a simple flow diagram, a description of the input data cards and output, a listing of the program, and a sample problem for the computer user to run.

III. Use of This Report by the Designer

The designer, in using this report, must provide certain inputs to the computer.

A. Computer Inputs

1. Asteroid data. The asteroid data, for the $k = 1$ to 1500 asteroids, consist of w_k, i_k, e_k and a_k,

where

$$w_k = \frac{1}{f} = \text{statistical weight of the } k\text{th asteroid}$$

f = probability of discovery of the asteroid (given in Fig. B-3)

i_k = inclination of the orbit of the kth asteroid to the ecliptic

e_k = eccentricity of the orbit of the kth asteroid

a_k = semi-major axis of the orbit of the kth asteroid

If desired, one can use a subset of these asteroids instead of all 1500. This data is supplied with the computer program, and is listed with it in Appendix G.

2. Damage parameters. The parameters C_1, C_2, ρ_s, h_s, ρ', 3β, ε_r and ε_λ, where C_1 and C_2 are constants,

and

ρ_s = density of spacecraft shielding material, taken as uniform in composition, g/cm³

h_s = Brinell hardness of the shield material, kg/mm²

ρ' = meteoroid density, g/cm³

$3\beta = 1.9$ = a constant relating to the meteoroid mass distribution law

$\varepsilon_r = \varepsilon_\lambda = 0.02$ = averaging parameters used in the meteoroid space distributions to avoid singularities

3. Spacecraft mission orbit parameters. The spacecraft mission orbit parameters are a, e, i, ω, T_P, T_0, ΔT, and N_T

where

a = semi-major axis of spacecraft orbit, AU

e = eccentricity of spacecraft orbit

i = inclination of spacecraft orbit, deg

ω = argument of perihelion of spacecraft orbit, deg

T_P = time of perihelion passage of spacecraft in its orbit, days

T_0 = time at which spacecraft mission starts, days

ΔT = interval between time steps, days

N_T = number of steps into which the mission is divided

4. Spacecraft structure parameters. The spacecraft structure parameters are N_F, $\mathbf{n}_j$, α_j, α_s, l^*, t^*, A_j^*, τ_j^*, $\mathbf{N}_j'$

where

N_F = number of faces of polyhedral spacecraft

$\mathbf{n}_j$ = outwardly drawn unit vector normal to the jth face of a polyhedral spacecraft, in spacecraft fixed coordinates

$\alpha_j = \frac{A_j^*}{l^{*2}}$ = area of jth face of standard spacecraft in m²

$\alpha_s = \sum_{j=1}^{N_F} \alpha_j$

$l^* = 1$ m = length associated with standard spacecraft

$t^* = 1$ cm = average shield thickness of standard spacecraft

A_j^* = area of the jth face of the standard spacecraft, m²

$\tau_j^* = 1$ = ratio of shield thickness on jth face to average shield thickness, for standard spacecraft

$\mathbf{N}_j'$ = a vector indicating the orientation of the jth face of a polyhedral spacecraft = $c\,\mathbf{n}_j$

c = any constant greater than zero

B. Computer Outputs

Following are the computer outputs.

1. Coefficients. The coefficients are C_A, C_A', C_A'', C_A''' for the uniform shielding case and C_B, C_B', C_B'', C_B''' for the optimum shielding case.

From Eqs. (6–9), one can plot

(1) l vs t
(2) A_s vs W_s
(3) l vs W_s
(4) t vs A_s

} for constant $P(0)$

(where t is average shield thickness) for the uniform shielding case and for the optimum shielding case.

2. Parameters. The parameters are τ_j^+, $\sigma[\mathbf{X}(T_i)]$, $n\sigma[\mathbf{X}(T_i)]$, $F_j^*(T_i)$, $\pi_I^*(T_i)$, f_j^*, $(A_j^*)(f_j^*)$, ν_P^*, $P_I^*(S)$, $r(T_i)$, $\lambda(T_i)$, $\eta(T_i)$, $V_1(T_i)$, $V_2(T_i)$, $V_3(T_i)$.

where

τ_j^+ = non-dimensional optimum pattern of shielding thickness on jth face

t_j^* = optimum thickness on jth face of standard spacecraft, in cm

t_j^+ = optimum thickness on jth face = $(t)(\tau_j^+)$, in cm

$\sigma[\mathbf{X}(T_i)]$ = meteoroids (with mass $\geq m_0$) per (AU)³ at the spacecraft position $\mathbf{X}$, at time T_i

$n\sigma[\mathbf{X}(T_i)]$ = number of meteoroid swarms contributing to model at spacecraft position $\mathbf{X}$, at time T_i

$F_j^*(T_i)$ = penetrating meteoroid flux on the jth face of the standard polyhedral spacecraft (meteoroids $m^{-2}\ s^{-1}$) at time T_i

$\pi_I^*(T_i)$ = rate of change of spacecraft state at time T_i, caused by meteoroids; spacecraft failure rate

f_j^* = expected number of penetrating hits/m^2 on the jth face of the standard spacecraft; integrated flux for standard spacecraft

$(A_j^*)(f_j^*)$ = expected number of penetrating hits on the surface face of the standard spacecraft

$$\nu_I^* = \int_{T_0}^{T_f} \pi_I^*(T)\, dT$$

$P_I^*(S)$ = $P(0)$ for standard spacecraft.

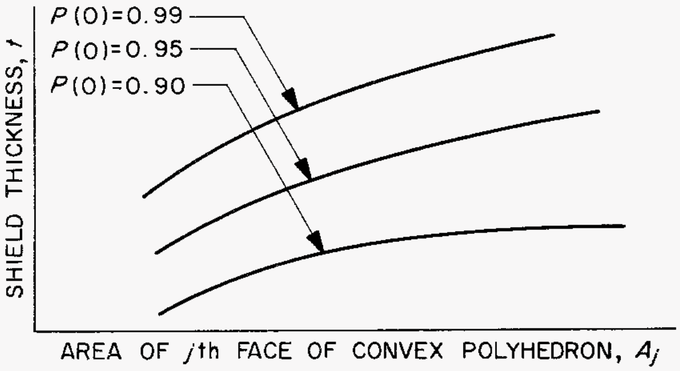
$\lambda(T_i)$ = ecliptic latitude of the spacecraft at time T_i

$r(T_i)$ = radial distance of spacecraft from the sun at time T_i

$\eta(T_i)$ = true anomaly of spacecraft at time T_i

$V_1(T_i), V_2(T_i), V_3(T_i)$ = components of spacecraft velocity in space-fixed coordinates at time T_i

The uniform shielding mass is calculated from $W_s = \rho_s A_s t$. One can thus plot



for constant ρ', ρ_s, h_s, for a particular spacecraft mission trajectory and for the assumed model of the asteroid belt.

The system designer can also obtain the optimum distribution of shielding t_j^+. From C_B'', l and $P(0)$, he can calculate W_s from Eq. (8). From W_s, ρ_s, α_s and l, he can calculate t from

$$t = \frac{W_s}{\rho_s\, \alpha_s\, l^2}$$

From t and τ_j^+ he can calculate $t_j^+ = (\tau_j^+)\, t$, where t_j^+ is the optimum shielding thickness on the jth face of the convex polyhedron.

In conclusion, this report provides the System Engineer with a computer program for the parametric variation of all of the parameters involved in the problem of maximizing the probability, $P(0)$, of zero meteoroid penetrations of the spacecraft shield, and minimizing the meteoroid shield mass. The parameters which can be varied include the spacecraft mission trajectory, details of the asteroid belt model, density of the meteoroids, and spacecraft shape, size and shielding material. The time variation of the probability of zero penetrations $P(0)$ can also be obtained as the asteroid belt is crossed.

Appendix A

Comparison of Asteroid Belt Models

An asteroid belt model consists of a space distribution, a velocity distribution and a mass distribution. The following is a comparison of a number of different models of the asteroid belt. The Volkoff model (Ref. 1) is a simple one. It is defined only in the ecliptic plane. The particle concentration, consisting of asteroidal and cometary matter, is estimated to be 100 times the interplanetary particle concentration. The model extends from 2 AU to 4 AU solar distance. The particle flux is equal to

$$F = \alpha_c m^{-\beta} \quad \text{(A-1)}$$

where

F = particles $m^{-2} s^{-1}$ of mass m or greater

α_c = constant

β = constant

The velocity of the particles is that corresponding to a direct heliocentric circular orbital velocity at the mean solar distance of the particles. The particle flow direction is considered to be isotropic. The average particle density is 0.75 g/cm^3. Figure A-1 shows the Volkoff asteroid belt model. The horizontal line is the edge view of the ecliptic plane, and gives distance from the sun in AU. The ordinate is the ecliptic latitude in degrees. The positions of the earth, Mars and Jupiter are plotted, by the method of Narin (Ref. 2), on a fictitious plane normal to the ecliptic plane. This fictitious plane passes through the sun and the planet, or object of interest. The plane rotates (in longitude) around the sun with, for example, Mars, and the oval shown in Fig. A-1 is the path traced by Mars on this plane as Mars circles the sun. The oval shows the variation of ecliptic latitude and solar distance, and omits properties involved with solar longitude. The Volkoff asteroid belt model is designed to provide a conservative estimate of the asteroidal meteoroid hazard in connection with his basically cometary-meteoroid-oriented general meteoroid hazard analysis.

Figure A-2 shows the Friedlander and Vickers asteroid belt model (Ref. 3). Two models are presented: one is a rectangle in the coordinate system used here and the other an oval. The part below the ecliptic plane is not shown. The rectangular model was presented in the preliminary draft of Ref. 3, and the egg-shaped model in the final draft of the report. The egg-shaped model is an oblate toroid, shown in Fig. A-3, and would be an ellipse in Narin's coordinate system, with ecliptic latitude and solar distance used as polar coordinates. However, we are plotting these as rectangular coordinates, and maximum ecliptic latitude at A in Fig. A-3 appears at A′ in Fig. A-2. In the rectangular model of Fig. A-2, it is assumed that the asteroid density is uniform and constant, and the total asteroid mass is contained in a toroidal "box" extending from 2 AU to 3.5 AU in distance from the sun and from −10 to +10 deg in ecliptic latitude. The oblate toroid

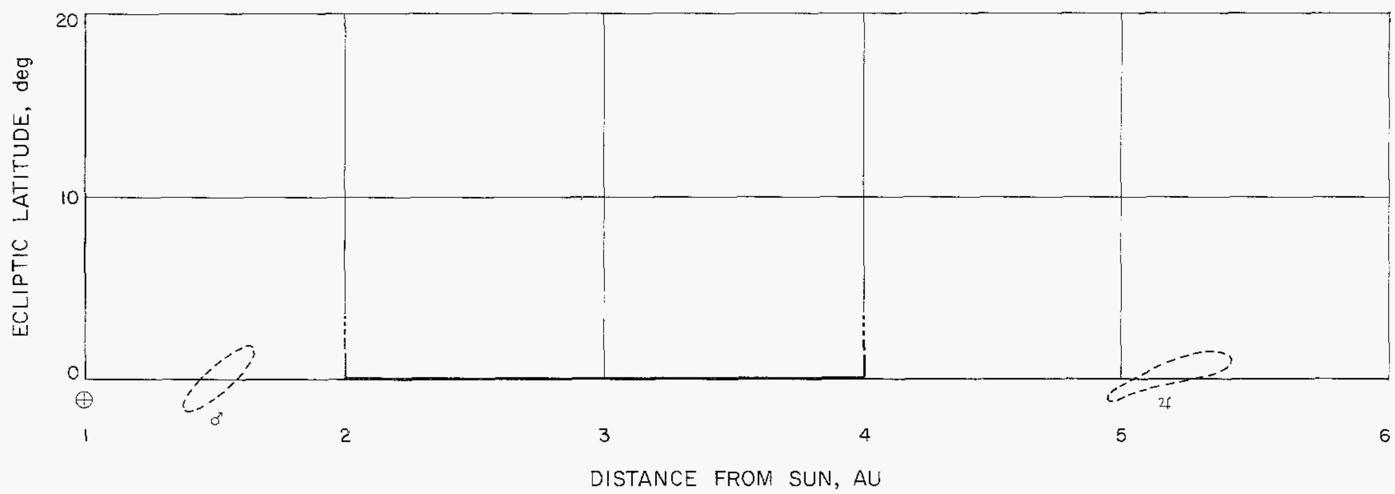


Fig. A-1. Asteroid belt model of Volkoff

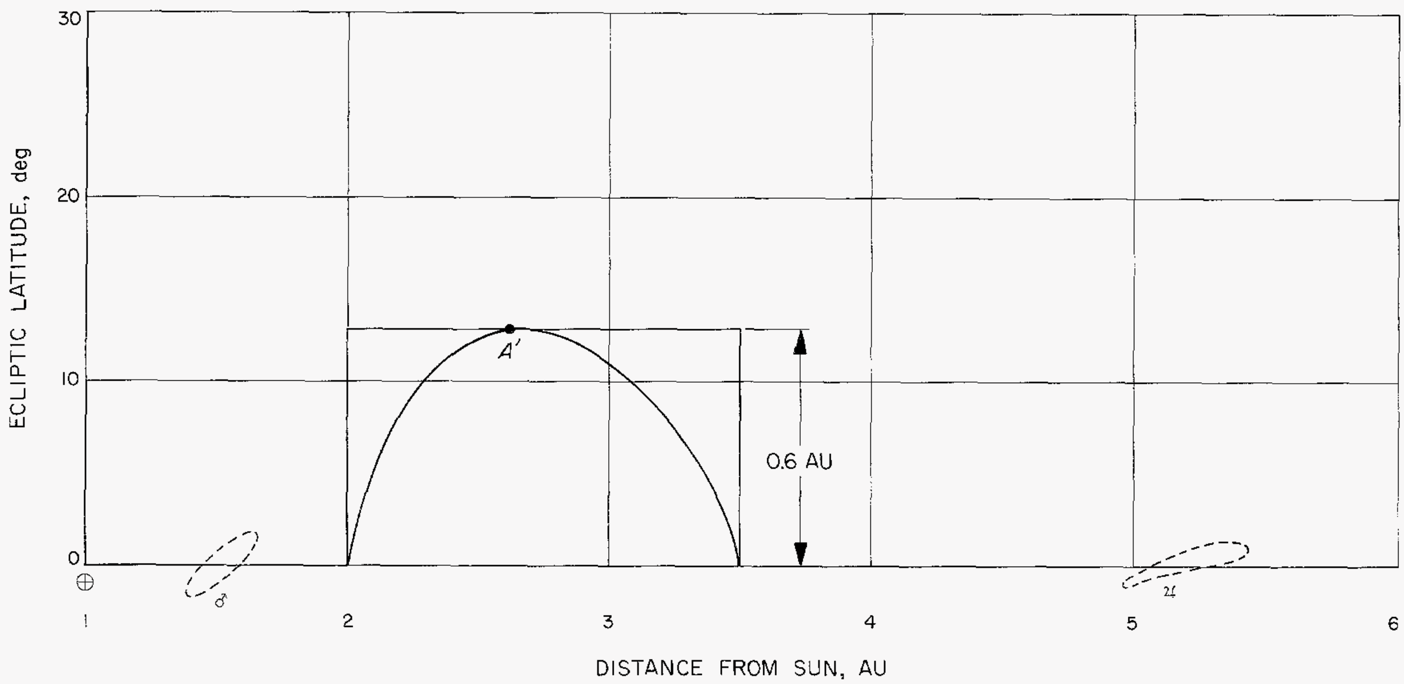


Fig. A-2. Asteroid belt model of Friedlander and Vickers

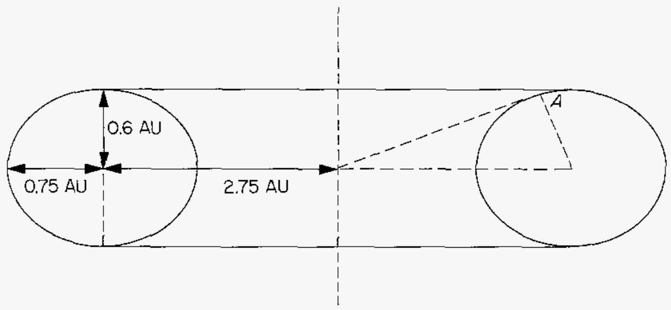


Fig. A-3. Asteroid belt model of Friedlander and Vickers shown as a torus

has similar boundaries extending to a maximum of 0.6 AU from the ecliptic, as shown in Figs. A-2 and A-3, but is slightly more realistic, having rounded corners. The asteroid particles are assumed to have an average density approximately the same as that of the stony meteorites, i.e., 3 g/cm^3, and an average velocity of 20 km/s. This model is more realistic than the Volkoff model since it attempts to produce a three-dimensional meteoroid distribution, rather than one confined to the ecliptic plane only. Figure A-4 shows the Chestek asteroid belt model (Ref. 4). His model, like the Friedlander and Vickers model, is toroidal in shape, is symmetric about the ecliptic plane, is not dependent on ecliptic longitude, and has constant density inside the asteroid belt and zero density outside. Chestek has two toroidal models, each centered at 2.9 AU. The smaller one has a radius of 0.6 AU while the larger has a radius of 0.75 AU. The smaller one has the larger density, since they each include the same asteroid mass. Chestek assumed the spacecraft flight path to be in the ecliptic plane and calculated the velocity of the meteoroids relative to the spacecraft. This was done for various meteoroid orbits in the ecliptic plane (different semimajor axes and eccentricities), and he obtained relative velocities between 6 and 22 km/s. This is much better than the single relative velocity obtainable from the previous models. He also considered particles in orbits inclined as much as 20 deg to the ecliptic and concluded that less than a hemisphere of spacecraft shielding is required. His cumulative mass distribution, Eq. (A-1), is the same as that of Volkoff, and he uses

$$\beta = \frac{2.5 \pm 0.5}{3} = 0.83 \pm 0.17$$

Figure A-5 shows a slightly modified form of Narin's asteroid belt model (Refs. 5 and 6). This model is solely concerned with the space distribution of the known asteroids and contains no velocity distribution or mass distribution. Narin took the orbital elements for 1563 numbered asteroids and, using a computer, generated various graphs and statistics relating to asteroid position. He showed that the distribution of asteroids is essentially independent of

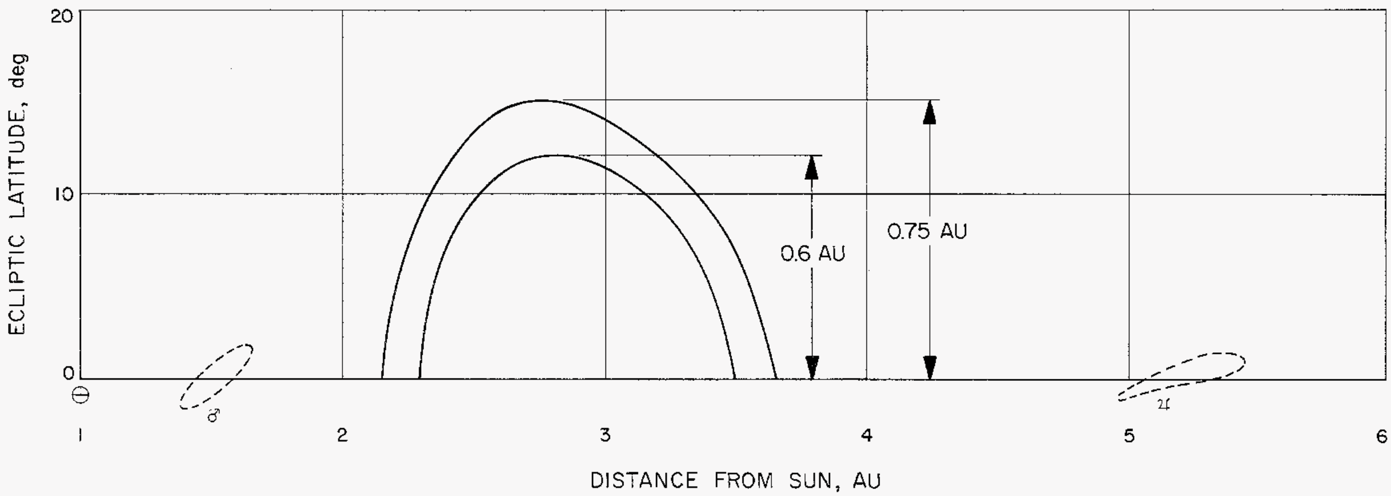


Fig. A-4. Asteroid belt model of Chestek

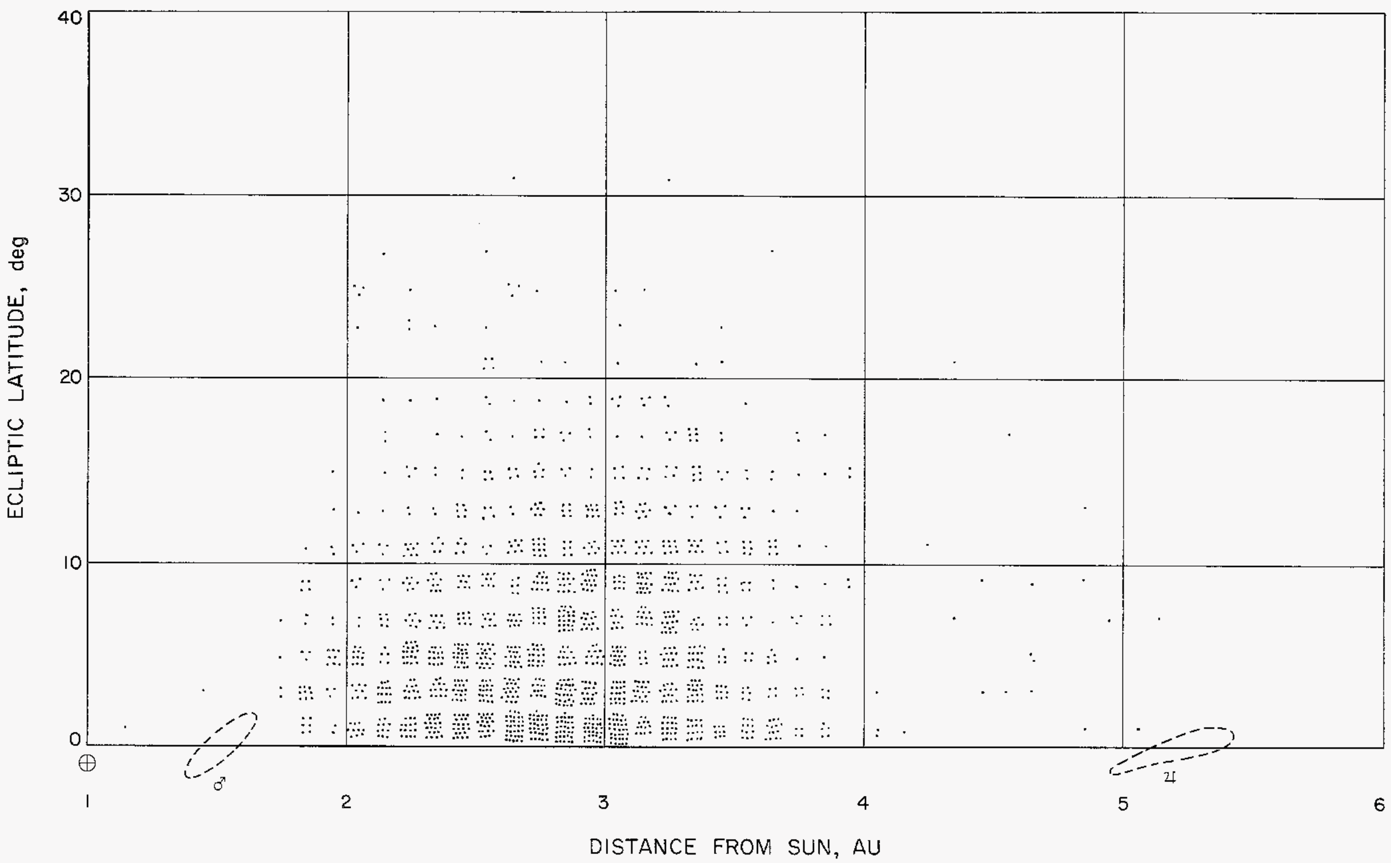


Fig. A-5. Asteroid belt model of Narin

ecliptic longitude. Because of the way in which he constructed his model, it is not independent of the time. The version based on the date April 19, 1973 is the one shown in Fig. A-5. His distribution is not perfectly symmetric about the ecliptic, but has an average ecliptic latitude of −0.10 deg. He found the mean solar distance of the asteroids to be 2.83 AU. His method was to divide the space in the asteroid belt into cells extending through 2 deg of ecliptic latitude and 0.1 AU of solar distance. Narin's figures used ecliptic latitude and solar distance as angular and radial polar coordinates, as mentioned above, whereas they are presented in Fig. A-5 as rectangular coordinates. The number of asteroids in a cell, plus the number in the corresponding cell south of the ecliptic, is shown by the number of dots at the appropriate location in Fig. A-5. Note that the figure illustrates a model, symmetric about the ecliptic, formed by averaging Narin's northern and southern distributions. Figure A-5 is much more realistic than Figs. A-1 through A-4, which have a constant density inside the asteroid belt and zero density outside. There is a very strong clustering in certain areas and a gradual fading away with latitude and with distance from the center of the belt. This space distribution is by far the best available and gives the most revealing picture of what the asteroid belt is like.

Figure A-6 is a sketch of the asteroid belt model used in this report. The numbers on the curves are the expected number of asteroids/$(AU)^3$ with absolute magnitude less than 13.6 at each location. This absolute magnitude corresponds to an asteroid radius of 4.3 km. The meteoroids are assumed to result, in part, from the same sources as

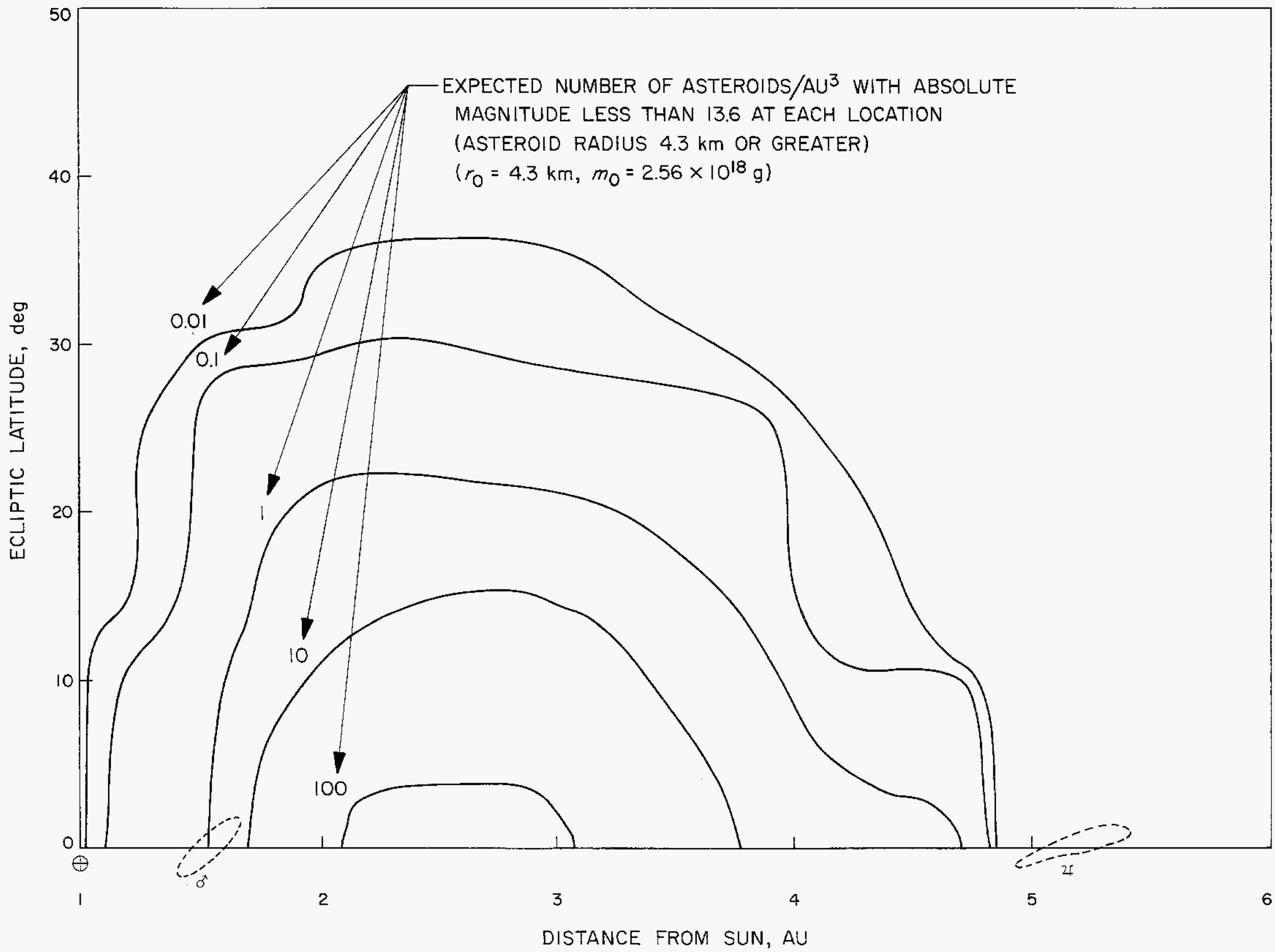


Fig. A-6. Asteroid belt model used in this report

the asteroids, and in part from the collision and consequent grinding of the asteroids, and are therefore assumed to have a distribution, in ecliptic latitude and solar distance, similar to that of the asteroids. Thus, as explained in Section III of Appendix B of this report, the meteoroid space distribution is derived from the numbered asteroid space distribution. Each numbered asteroid is replaced by a swarm of meteoroids with a mass distribution given by Eq. (A-1). All the meteoroids in the swarm have the same semimajor axis, eccentricity and inclination to the ecliptic as their parent asteroid. However, the longitudes of ascending node, arguments of perihelion, and mean anomalies of the meteoroids are uniformly distributed over the entire range of possible values. This is considered to be reasonable because of the non-dependence of the asteroid distribution on ecliptic longitude. This leads to the density distribution shown in Fig. A-6 for the meteoroids as well as the asteroids, with a scale factor for the mass dependence. This model also includes a meteoroid velocity distribution as explained in Appendix B, Section III.

Appendix B

Mathematical Model for Determination of Probability of Successfully Traversing Asteroid Belt

The following is a derivation of an expression for the probability that a spacecraft will successfully traverse the asteroid belt. This is a necessary preliminary to the writing of a computer program for calculating this probability of success.

The analysis begins with utmost generality and proceeds to greater explicitness. An overall probability of success $P(\underline{S})$ is obtained, which is the probability that the spacecraft successfully traverses the asteroid belt. Other information is also generated which is of value in determining: 1) the time variation of the meteoroid hazard as the asteroid belt is crossed, and 2) the pattern of shielding to protect the spacecraft in an optimum manner against meteoroid damage.

I. Spacecraft State and Change of State

A spacecraft *state* is defined as one of the many possible conditions of the spacecraft. If the spacecraft is hit by a meteoroid, damage to one or more essential items may occur and thus cause a change in the state of the spacecraft. Other causes of change of spacecraft state might be random component failures, radiation effects, etc.

The possible states of the spacecraft form a continuum $\underline{S}$ if one assumes a continuous distribution of spacecraft states. Now, if $\underline{R}$ is a portion of $\underline{S}$, the probability that the spacecraft will be in one of the states in $\underline{R}$ at time T is

$$\int_{\underline{R}} P(s, T)\, ds \tag{B-1}$$

where $P(s, T)$ is a probability density function over $\underline{S}$, and $P(s, T)\, ds$ is the probability the spacecraft is in the interval between state s and state $s + ds$. Such integrals reduce to a summation $P(s_1, T) + P(s_2, T) + P(s_3, T) + \cdots$, when the states involved are discrete. Since the spacecraft must be in some state at time T, one can write

$$\int_{\underline{S}} P(s, T)\, ds = 1 \tag{B-2}$$

Assume the spacecraft to be in state s at time T. Let the probability that it changes to state s' during the next infinitesimal time interval dT be defined by $\pi(s, s', T)\, dT$. Here $\pi(s, s', T)$ is the rate of change of spacecraft state, from state s to state s' at time T and is here called the *total transition rate*. Now one can write

$$dP(s, T) = \int_{\underline{S}} [P(s', T)\, \pi(s', s, T)\, dT]\, ds' - P(s, T) \int_{\underline{S}} [\pi(s, s', T)\, dT]\, ds' \tag{B-3}$$

The first integrand $P(s', T)\, \pi(s', s, T)\, dT$ is the probability that the spacecraft is in state s', at time T, multiplied by the probability that it will change from state s' to state s during the next time interval dT.

The integral

$$\int_{\underline{S}} [P(s', T)\, \pi(s', s, T)\, dT]\, ds'$$

is the increase in $P(s, T)$. The second term $\pi(s, s', T)\, dT$ is the probability that a spacecraft in state s will change to state s'. The integral

$$\int_{\underline{S}} [\pi(s, s', T)\, dT]\, ds'$$

is the total probability that the spacecraft will fall out of state s. The product

$$P(s, T) \int_{\underline{S}} [\pi(s, s', T)\, dT]\, ds'$$

is the probability that the spacecraft is in state s, at time T, multiplied by the probability that the spacecraft changes from state s to state s' during the next time interval dT. Equation (B-3) is thus the increase in $P(s, T)$ minus the decrease in $P(s, T)$, or the net change in $P(s, T)$ represented by $dP(s, T)$. Now, dT can be factored out since the integrals are over s', so one gets

$$\frac{dP(s, T)}{dT} = \int_{\underline{S}} [P(s', T)\, \pi(s', s, T)]\, ds' - P(s, T) \int_{\underline{S}} [\pi(s, s', T)]\, ds' \tag{B-4}$$

which is the rate of change of spacecraft state. This equation could be solved if there were a finite number of states, and if one knew the π functions, by straight forward computer techniques as a set of initial value ordinary differential equations.

The term $\pi(s, s', T)\, dT$ can be represented by a sum of components, $\pi(s, s', T)\, dT = \Sigma_i \pi_i(s, s', T)\, dT$ or, cancelling dT on each side of the equation, one gets

$$\pi(s, s', T) = \sum \pi_i(s, s', T) \tag{B-5}$$

where $\pi_i(s, s', T)$ is the rate of change of spacecraft state, from state s to state s', at time T, caused by the ith source and is here called the *ith transition rate*. For example, if s represents success and s' represents failure, then $\pi_i(s, s', T)\, dT$ is the probability that the ith source causes the spacecraft to change from success at time T to failure at time $T + dT$.

Let the Ith transition rate, $\pi_I(s, s', T)$, be due to impact by a certain class of meteoroids which form a set $\underline{M}$. Other classes contribute linearly to the total transition rate $\pi(s, s', T)$. Let μ be one of this class, or a meteoroid type which is an element of $\underline{M}$. Let meteoroids of type μ possess a set of structural properties $\underline{St}'(\mu)$. Typical meteoroid structural properties are shape, size and composition.

Let $\mathbf{X}$ be the three-dimensional vector giving the position of a particular point in space. Let $\mathbf{U}$ be the three-dimensional vector giving the velocity of a meteoroid. Let $d^3\mathbf{X}$ be an element of volume of space and $d^3\mathbf{U}$ an element of volume of velocity phase space. Let

$$\psi(\mu, \mathbf{X}, \mathbf{U}, T)\, d\mu\, d^3\mathbf{X}\, d^3\mathbf{U}$$

be the differential probability that a meteoroid of type μ will pass through location $\mathbf{X}$ with velocity $\mathbf{U}$ at time T, with tolerances $d\mu$, $d^3\mathbf{X}$, $d^3\mathbf{U}$, and dT in meteoroid type, position, velocity and time. Here $d^3\mathbf{X}$ may be thought of as $d^2\mathbf{X} \cdot \mathbf{U}\, dT$. Let the surface of the spacecraft be composed of elements of area which form a set $\underline{A}$. Let $A(\alpha)$ be an area density function, where α is an element of $\underline{A}$, so that

$$\int_{\underline{A}} A(\alpha)\, d\alpha = A_s \tag{B-6}$$

where A_s is the surface area of the spacecraft.

Let $\underline{B}$ be a subset of $\underline{A}$. The total area of the surface elements in $\underline{B}$ is

$$\int_{\underline{B}} A(\alpha)\, d\alpha$$

Now, at element α, the spacecraft possesses a set of structural properties $\underline{St}(\alpha)$. Typical spacecraft structural properties are configuration, thickness and composition. At time T, let the outwardly drawn three-dimensional unit vector, normal to the spacecraft surface at α, be $\mathbf{n}(\alpha, T)$. Let the spacecraft shadowing function $S_h(\alpha, \mathbf{Z}, T)$ be defined as the probability that the line drawn from the spacecraft surface element α in the direction $\mathbf{Z}$, at time T, will penetrate a part of the spacecraft. In Fig. B-1, $\mathbf{n}(\alpha, T) \cdot \mathbf{Z} > 0$, whereas $\mathbf{n}(\alpha, T) \cdot \mathbf{Z}_2 < 0$. Thus,

if

$$\mathbf{n}(\alpha, T) \cdot \mathbf{Z} < 0, \qquad S_h(\alpha, \mathbf{Z}, T) = 1 \tag{B-7}$$

and the line drawn from the element α in the direction $\mathbf{Z}$ penetrates the spacecraft.

Let $\delta(s, s', \alpha, \mu, \mathbf{W}, T)$ be the probability that the spacecraft in state s at time T will change to state s' when hit on surface element α by a meteoroid of type μ moving at a relative velocity $\mathbf{W}$ with respect to the spacecraft, as shown in Fig. B-2. Since the spacecraft must reach some state after being hit by a meteoroid

$$\int_{\underline{S}} \delta(s, s', \alpha, \mu, \mathbf{W}, T)\, ds' = 1 \tag{B-8}$$

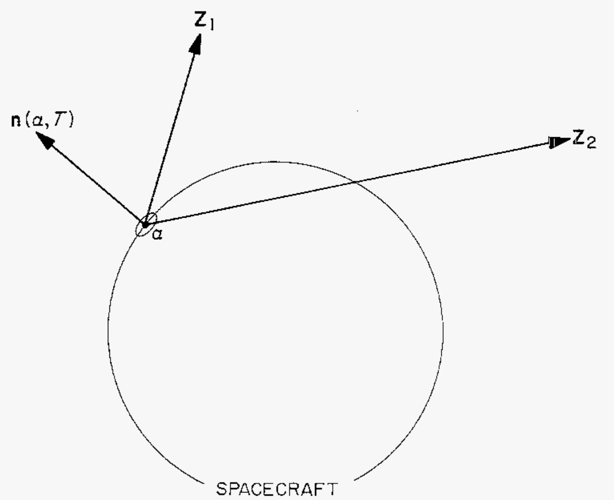


Fig. B-1. Geometry of outward drawn unit vector n (α, T) and unit vector Z originating at surface element α of the spacecraft

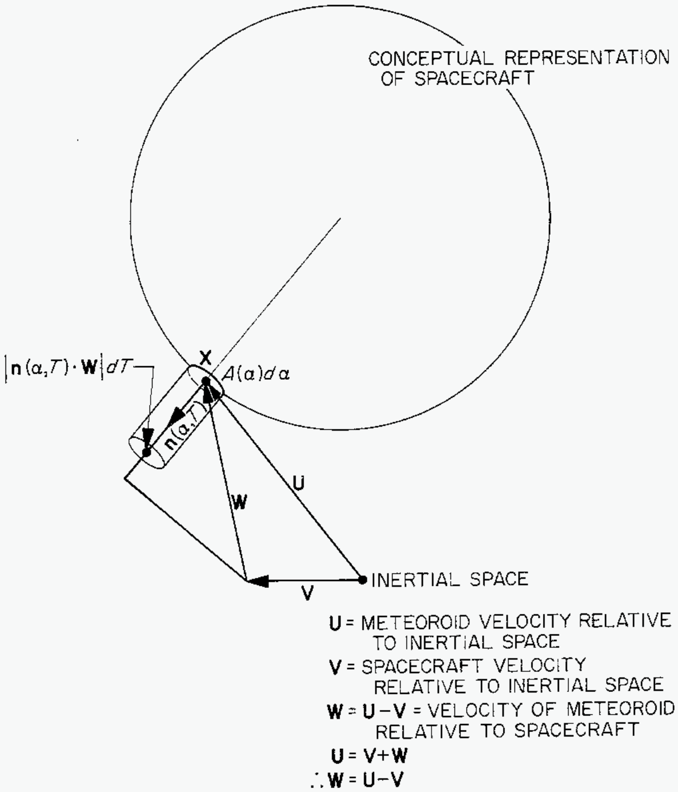


Fig. B-2. Geometry

Let the position and velocity of the spacecraft at time T be $\mathbf{X}(T)$ and $\mathbf{V}(T)$, respectively. Then

$$\mathbf{U} = \mathbf{V} + \mathbf{W}$$

and

$$\mathbf{W} = \mathbf{U} - \mathbf{V} \qquad \text{(B-9)}$$

as shown in Fig. B-2.

The probability that the spacecraft changes from state s to s', between time T and time $T + dT$, caused by a certain set of meteoroids $\underline{M}$, is given by

$$\pi_I(s, s', T)\, dT = \int_{\underline{A}} \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(s, s', \alpha, \mu, \mathbf{W}, T) \cdot [1 - S_h(\alpha, -\mathbf{w}, T)] \cdot \psi[\mu, \mathbf{X}(T), \mathbf{U}, T]\, d\mu\, d^3\mathbf{X}\, d^3\mathbf{U} \qquad \text{(B-10)}$$

Here $\psi(\mu, \mathbf{X}, \mathbf{U}, T)\, d\mu\, d^3\mathbf{X}\, d^3\mathbf{U}$ is the differential probability that the meteoroid of type μ, in the set of meteoroids $\underline{M}$, passes through point $\mathbf{X}$, with velocity $\mathbf{U}$, at time T, with tolerances $d\mu$, $d^3\mathbf{X}$ and $d^3\mathbf{U}$. The quantity $[1 - S_h(\alpha, \mathbf{Z}, T)]$ is the probability that the meteoroid does not hit the spacecraft structure before it reaches point $\mathbf{X}$, and $\delta(s, s', \alpha, \mu, \mathbf{W}, T)$ is the probability the spacecraft goes from state s to s' when hit on surface α by a meteoroid of type μ, moving at relative velocity $\mathbf{W}$ at time T. Here $\mathbf{Z}$ is a unit vector, originating at α, as shown in Fig. B-1, whereas

$$\mathbf{w} = \frac{\mathbf{W}}{|\mathbf{W}|} \text{ is a unit vector directed at } \alpha. \text{ Thus,}$$

$$\mathbf{Z} = -\frac{\mathbf{W}}{|\mathbf{W}|} = -\mathbf{w}$$

Now, let the volume element $d^3\mathbf{X}$ be replaced by the cylindrical volume element shown in Fig. B-2, namely, $|\mathbf{n}(\alpha, T) \cdot \mathbf{W}|\, dT\, A(\alpha)\, d\alpha$ where $|\mathbf{n}(\alpha, T) \cdot \mathbf{W}|\, dT$ is the altitude of the cylinder and $A(\alpha)\, d\alpha$ is the base of the cylinder, or area element of the spacecraft. Thus, Eq. (B-10) becomes

$$\pi_I(s, s', T)\, dT = \int_{\underline{A}} \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(s, s', \alpha, \mu, \mathbf{W}, T)\, [1 - S_h(\alpha, -\mathbf{w}, T)] \cdot |\mathbf{n}(\alpha, T) \cdot \mathbf{W}|\, dT \cdot A(\alpha)\, d\alpha \cdot \psi[\mu, \mathbf{X}(T), \mathbf{U}, T]\, d^3\mathbf{U}\, d\mu$$

and after cancelling out dT on each side and rearranging,

$$\pi_I(s, s', T) = \int_{\underline{A}} \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(s, s', \alpha, \mu, \mathbf{W}, T)\, [1 - S_h(\alpha, -\mathbf{w}, T)] \cdot |\mathbf{n}(\alpha, T) \cdot \mathbf{W}|\, A(\alpha) \cdot \psi[\mu, \mathbf{X}(T), \mathbf{U}, T]\, d^3\mathbf{U}\, d\mu\, d\alpha \qquad \text{(B-11)}$$

Equation (B-11) gives the meteoroid induced transition rate in terms of the meteoroid distribution function ψ, spacecraft surface area distribution $A(\alpha)\,d\alpha$, spacecraft surface normal $\mathbf{n}(\alpha, T)$, spacecraft shadowing function S_h, and spacecraft damage function δ.

Great simplification is introduced by adopting the nearly universally made approximation that the spacecraft has only two states: successful (so far), and failed, S and F_a, respectively, and also assuming that a failed spacecraft never recovers. The set of states is thus $\underline{S} = \{S, F_a\}$

and

$$\pi(F_a, S, T) = \pi_i(F_a, S, T) = 0$$

From Eq. (B-2),

$$P(F_a, T) + P(S, T) = 1$$

or

$$P(F_a, T) = 1 - P(S, T)$$

so that the spacecraft status can be described by a single function $P_s(T) = P(S, T)$, the probability of spacecraft success through time T. Furthermore, there is only one transition with which non-zero probabilities, or rates, are associated, namely, from S to F_a so simpler expressions can be used for the transition rates and probabilities, (now failure rates and probabilities). Thus, one gets

$$\pi(s, s', T) = \pi(T) = \pi(S, F_a, T)$$

$$\pi_i(s, s', T) = \pi_i(T) = \pi_i(S, F_a, T)$$

and

$$\delta(s, s', \alpha, \mu, \mathbf{W}, T) = \delta(S, F_a, \alpha, \mu, \mathbf{W}, T) = \delta(\alpha, \mu, \mathbf{W}, T) \tag{B-12}$$

Thus, if, in Eq. (B-4) one sets $s = S$, and $s' = F_a$,

$$\pi(s', s, T) = \pi(F_a, S, T) = 0$$

$$\pi(s, s', T) = \pi(S, F_a, T) = \pi(T),$$

and Eq. (B-4) becomes

$$\frac{dP(s, T)}{dT} = -P(s, T)\int_{\underline{S}} \pi(T)\,dS'$$

or

$$\frac{dP(s, T)}{P(s, T)} = -\int_{\underline{S}} [\pi(T)\,dT]\,dS' = -\pi(T)\,dT$$

where the integral falls out since there is only one failure state. Thus, one gets

$$\log_e \frac{P_s(T)}{P_s(T')} = -\int_{T'}^{T} \pi(T)\,dT$$

and

$$\frac{P_s(T)}{P_s(T')} = \exp\left(-\int_{T'}^{T} \pi(T)\,dT\right)$$

or

$$P_s(T) = P_s(T')\exp\left(-\int_{T'}^{T} \pi(T)\,dT\right) \tag{B-13}$$

where T and T' are any two times.

We have thus simplified the situation considerably by reducing the spacecraft possibilities to two states, a successful state and a failed state, leading to Eq. (B-13) for $P_s(T)$ which is the probability of being in the successful state at time T. We do not have to do a similar thing for the failed state, since the probability of failure at time T is just one minus the probability of success at time T.

A. Time Dependences

Simplifications can be achieved by assuming that certain functions do not depend on the time. If no attempt is made to allow for individual meteoroid showers in the asteroid belt, the meteoroid distribution function can be simplified:

$$\psi(\mu, \mathbf{X}, \mathbf{U}, T) = \psi(\mu, \mathbf{X}, \mathbf{U}) \tag{B-14}$$

If no major changes in configuration take place during the course of the mission, such as extending fragile instruments on booms, or maneuvering large antennas, further simplifications are possible. In this case, the outwardly drawn unit vector, $\mathbf{n}(\alpha, T)$, normal to the spacecraft surface at α, at time T, can be expressed in the form

$$\mathbf{n}(\alpha, T) = \mathbf{n}(\alpha)\,\mathcal{M}(T) \tag{B-15}$$

where $\mathcal{M}(T)$ is a rotation matrix specifying the orientation of the spacecraft relative to space fixed coordinates at time T. $\mathcal{M}(T)$ is given explicitly in Appendix C, Section II.

Thus, $\mathcal{M}(T)$ operates to convert a vector from spacecraft fixed coordinates to space fixed coordinates at time T. Similarly, one can write

$$\mathbf{n}(\alpha) = \mathbf{n}(\alpha, T)\,\mathcal{M}^{-1}(T) \qquad \text{(B-16)}$$

Thus, $\mathcal{M}^{-1}(T)$ operates to convert a vector from space fixed coordinates to spacecraft fixed coordinates at time T.

One can thus express the shielding and failure function time dependences more explicitly as follows:

$$S_h(\alpha, \mathbf{Z}, T) = S_h[\alpha, \mathbf{Z}\mathcal{M}^{-1}(T)] \qquad \text{(B-17)}$$

$$\delta(\alpha, \mu, \mathbf{W}, T) = \delta[\alpha, \mu, \mathbf{W}\mathcal{M}^{-1}(T)] \qquad \text{(B-18)}$$

Here $\mathbf{Z}$ and $\mathbf{W}$ are relative to space fixed coordinates, whereas $\mathbf{Z}\mathcal{M}^{-1}(T)$ and $\mathbf{W}\mathcal{M}^{-1}(T)$ are relative to spacecraft fixed coordinates.

B. Convex Spacecraft

If the spacecraft can be approximated with sufficient accuracy as being convex, the shadowing function assumes a very simple form:

$$S_h(\alpha, \mathbf{Z}) = H[-\mathbf{n}(\alpha)\cdot\mathbf{Z}] \qquad \text{(B-19)}$$

where $H(x)$ is the Heaviside unit step function

$$H(x) = \begin{cases} 0 \text{ for } x < 0 \\ \frac{1}{2} \text{ for } x = 0 \\ 1 \text{ for } x > 0 \end{cases} \qquad \text{(B-20)}$$

Thus, since $\mathbf{n}(\alpha)$ is an outwardly drawn unit vector normal to the spacecraft surface, and an impacting meteoroid must be coming from outside the spacecraft, $\mathbf{n}(\alpha)\cdot\mathbf{Z}$ will be positive and $x = -\mathbf{n}(\alpha)\cdot\mathbf{Z}$ will be negative, so that $H(x) = 0$ and $S_h(\alpha, \mathbf{Z}) = 0$. If $\mathbf{Z}$ is such that a line drawn from area element α intercepts the spacecraft, then $\mathbf{n}(\alpha)\cdot\mathbf{Z} < 0$ and $-\mathbf{n}(\alpha)\cdot\mathbf{Z} > 0$ and $H(x) = 1 = S_h(\alpha, \mathbf{Z})$.

From Eqs. (B-11 and -15) one has:

$$Q_0 = [1 - S_h(\alpha, -\mathbf{w}, T)]\cdot|\mathbf{n}(\alpha, T)\cdot\mathbf{W}| = [1 - S_h(\alpha, -\mathbf{w}, T)]\,|\mathbf{n}(\alpha)\mathcal{M}(T)\cdot\mathbf{W}| \qquad \text{(B-21)}$$

By use of Eq. (B-17),

$$Q_0 = [1 - S_h\{\alpha, -\mathbf{w}\mathcal{M}^{-1}(T)\}]\cdot|\mathbf{n}(\alpha)\mathcal{M}(T)\cdot\mathbf{W}| \qquad \text{(B-22)}$$

Using Eq. (B-19),

$$Q_0 = (1 - H\{[-\mathbf{n}(\alpha)]\cdot[-\mathbf{w}\mathcal{M}^{-1}(T)]\})\cdot|\mathbf{n}(\alpha)\mathcal{M}(T)\cdot\mathbf{W}| \qquad \text{(B-23)}$$

or

$$Q_0 = (1 - H\{\mathbf{n}(\alpha)\cdot[\mathbf{w}\mathcal{M}^{-1}(T)]\})\cdot|\mathbf{n}(\alpha)\mathcal{M}(T)\cdot\mathbf{W}| \qquad \text{(B-24)}$$

Now, from Eq. (B-20)

$$H(-x) = 1 - H(x)$$

For example:

$$H(-7) = 1 - H(7) \qquad 0 = 1 - 1 = 0$$

$$H(0) = 1 - H(0) \qquad 0.5 = 1 - 0.5 = 0.5$$

$$H(+7) = 1 - H(-7) \qquad 1 = 1 - 0 = 1$$

Thus, Eq. (B-24) can be written

$$Q_0 = H[-\mathbf{n}(\alpha)\cdot\mathbf{w}\mathcal{M}^{-1}(T)]\cdot|\mathbf{n}(\alpha)\mathcal{M}(T)\cdot\mathbf{W}| \qquad \text{(B-25)}$$

Now

$$[\mathbf{n}(\alpha)\mathcal{M}(T)]\cdot\mathbf{W} = \mathbf{n}(\alpha)\cdot\mathbf{W}\mathcal{M}^{-1}(T)] \qquad \text{(B-26)}$$

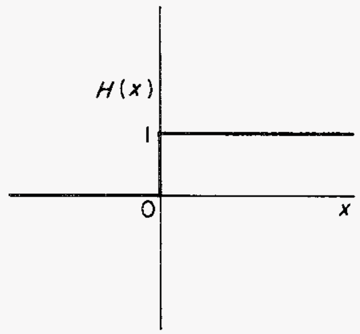
where $\mathbf{n}(\alpha)$ is in the spacecraft fixed coordinate system and $\mathbf{W}$ is in the space-fixed coordinate system. The rotation $\mathcal{M}(T)$ acts on $\mathbf{n}(\alpha)$ rotating it into the space-fixed coordinate system, after which it is projected on $\mathbf{W}$. Similarly the $\mathcal{M}^{-1}(T)$ rotation acts on $\mathbf{W}$ rotating it into the spacecraft fixed coordinate system after which it is projected on $\mathbf{n}(\alpha)$. Thus, Eq. (B-25) becomes

$$Q_0 = H[-\mathbf{n}(\alpha)\cdot\mathbf{w}\mathcal{M}^{-1}(T)]\cdot|\mathbf{n}(\alpha)\cdot\mathbf{W}\mathcal{M}^{-1}(T)| \qquad \text{(B-27)}$$

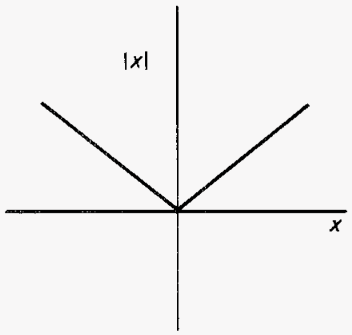
or

$$Q_0 = H[-\mathbf{n}(\alpha)\cdot\mathbf{w}\mathcal{M}^{-1}(T)]\cdot|-\mathbf{n}(\alpha)\cdot\mathbf{W}\mathcal{M}^{-1}(T)| \qquad \text{(B-28)}$$

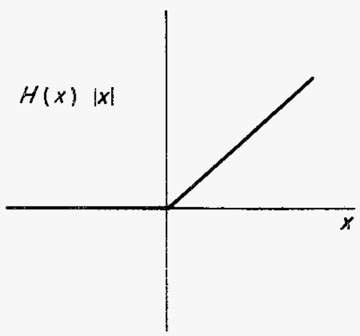
Now, if one plots $H(x)$ versus (x), from Eq. (B-20) one gets



Also, a plot of $|x|$ versus x looks like



The product $H(x)\,|x|$ looks like



Thus,

$$H(x)\,|x| = \max\{0, x\}$$

and for

$$x < 0, \qquad \max\{0, x\} = 0$$
$$x = 0, \qquad \max(0, 0) = 0$$
$$x > 0, \qquad \max(0, x) = x$$

Consequently, Eq. (B-28) becomes

$$Q = \max\{0, -\mathbf{n}(\alpha) \cdot \mathbf{w}\,\mathcal{M}^{-1}(T)\} \qquad \text{(B-29)}$$

Thus, Eq. (B-21) becomes

$$[1 - S_h(\alpha, -\mathbf{w}, T)] \cdot |\mathbf{n}(\alpha, T) \cdot \mathbf{W}| = \max\{0, -\mathbf{n}(\alpha) \cdot \mathbf{w}\,\mathcal{M}^{-1}(T)\} \qquad \text{(B-30)}$$

C. Convex and Polyhedral Spacecraft

A highly useful procedure is to approximate the surface of the spacecraft by a suitably chosen polyhedron, which can be convex or otherwise. In this case $\underline{A}$ becomes a finite set, so the integral over $\underline{A}$ in Eq. (B-11) becomes a sum over the faces of the polyhedron. The jth face of the polyhedral spacecraft surface has area A_j and outwardly drawn normal unit vector $\mathbf{n}_j$. Thus, the integral in Eq. (B-6) reduces to a sum:

$$A_s = \sum_j A_j$$

II. Spacecraft Damage Function Due to Meteoroids

The meteoroid penetration criterion used here is essentially the same as that used by Volkoff (Ref. 1) and is the best available for uniform spacecraft walls. The structural properties $\underline{St}(\alpha)$ of the spacecraft surface at element α, and $\underline{St}'(\mu)$ of the meteoroids of type μ are as follows: The spacecraft surface is assumed to be composed of a single layer of material $M(\alpha)$ of thickness $t(\alpha)$ at element α. For polyhedral spacecraft the material is M_j and the thickness t_j at the jth face. A meteoroid of type μ is assumed to be a sphere of material $M'(\mu)$, mass $m(\mu)$ and radius $r(\mu)$. Material M has density $\rho(M)$ and Brinell hardness $h(M)$.

From Refs. 7 and 8, the penetration depth p_1 of a high-velocity projectile in a semi-infinite target is given by

$$p_1 = k_1\, d_p \log_e\left(1 + \frac{\rho_t V_p^2}{k_2 h_t}\right) \qquad \text{(B-31)}$$

where k_1 and k_2 are constants which depend on the projectile and target materials, d_p is projectile diameter, V_p is projectile velocity normal to surface, ρ_t is target density and h_t is target Brinell hardness.

Here

$$k_1 \approx (0.6 \pm 0.2)\, K^{2/3}$$
$$k_2 \approx (4 \pm 2)\, K^{-2/3}$$

where

$$K = \frac{\rho_p}{\rho_t}$$

ρ_p = projectile density

More accurate values of k_1 and k_2 must be obtained for each pair of materials (projectile and target) by experiment. t_c, the thickness of a plate required to stop a given projectile, is generally taken as 1.5 times the depth of the crater produced by such a projectile in a semi-infinite target.

Let the projectile be a meteoroid of type μ and let the target be element α of the spacecraft surface. Thus, $d_p = 2\,r(\mu)$, $\rho_p = \rho\,[M'(\mu)]$, $\rho_t = \rho[M(\alpha)]$, and $h_t = h\,[M(\alpha)]$. From this,

$$K = \frac{\rho\,[M'(\mu)]}{\rho\,[M(\alpha)]} \qquad \text{(B-32)}$$

The relative velocity with respect to space-fixed coordinates is **W**, and with respect to spacecraft-fixed coordinates it is $\mathbf{W}\mathcal{M}^{-1}(T)$. The normal component of the relative velocity is therefore

$$V_p = \mathbf{n}(\alpha)\cdot\mathbf{W}\mathcal{M}^{-1}(T)$$

Thus, Eq. (B-31) becomes

$$t_c = C_1\,r(\mu)\log_e\{1 + C_2\,[\mathbf{n}(\alpha)\cdot\mathbf{W}\mathcal{M}^{-1}(T)]^2\} \qquad \text{(B-33)}$$

where

$C_1 = C_1\,[M(\alpha), M'(\mu)]$, or, more particularly,

$$C_1 = 2\,(1.5)\,k_1 = 3\,k_1 \approx (1.8 \pm 0.6)\,K^{2/3} \qquad \text{(B-34)}$$

and

$C_2 = C_2\,[M(\alpha), M'(\mu)]$, or more particularly,

$$C_2 = \frac{\rho_t}{k_2\,h_t} = \frac{\rho\,[M(\alpha)]}{k_2\,h\,[M(\alpha)]} \approx \frac{K^{2/3}\rho\,[M(\alpha)]}{(4 \pm 2)\,h\,[M(\alpha)]} \qquad \text{(B-35)}$$

The spacecraft damage function $\delta\,[\alpha, \mu, \mathbf{W}\mathcal{M}^{-1}(T)]$ from Eq. (B-18) is the probability that the meteoroid shield is penetrated, and is given by

$$H\,[t_c - t(\alpha)]$$

where H is the step function mentioned earlier

$$H(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1/2 & \text{for} \quad x = 0 \\ 1 & \text{for} \quad x > 0 \end{cases}$$

and $t(\alpha)$ is the thickness of the meteoroid shield at element α. Thus, one can write

$$\delta\,[\alpha, \mu, \mathbf{W}\mathcal{M}^{-1}(T)] = H\,[C_1\,r(\mu)\log_e\{1 + C_2\,[\mathbf{n}(\alpha)\cdot\mathbf{W}\mathcal{M}^{-1}(T)]^2\} - t(\alpha)] \qquad \text{(B-36)}$$

III. Meteoroid Distribution Function

Let the source of the Ith transition rate be the asteroid belt. Reference 9 states that "the average sized meteorites are usually stones, but extremely large meteorites are usually irons. At a mass of 100 kg the stones outnumber the irons in the ratio 20:1. At a mass of 10^{10} kg the irons outnumber the stones by 10:1. Stones and irons occur in equal numbers at a mass of about 10^6 kg." This refers to conditions at the earth's orbit, where secondary stone meteoroids are dominant in the smaller size ranges. The conservative assumption is made in this report that the asteroidal meteoroids are all iron. With this assumption, it is convenient to identify the meteoroid parameter μ with the mass $m(\mu)$, dropping the μ, or replacing it by m, wherever it appears. When this is done, the mass m, representing the set of meteoroid types M, varies from 0 to ∞. There is now a single meteoroid density

$$\rho\,[M'(\mu)] = \rho(M') = \rho' = 7.9\,\frac{\text{g}}{\text{cm}^3} \qquad \text{(B-37)}$$

A reasonable, simplifying, approximation is that the meteoroid mass and velocity distributions are independent, namely (recalling that the time-dependence of ψ has been removed above [Eq. B-14]):

$$\psi(m, \mathbf{X}, \mathbf{U})\,dm\,d^3\mathbf{X}\,d^3\mathbf{U} = \zeta(m, \mathbf{X})\,dm\cdot d^3\mathbf{X}\cdot\xi(\mathbf{U}, \mathbf{X})\,d^3\mathbf{U} \qquad \text{(B-38)}$$

where $\psi(m, \mathbf{X}, \mathbf{U})\,dm\,d^3\mathbf{X}\,d^3\mathbf{U}$ is the probability that an asteroidal meteoroid of mass m will pass through position **X** with velocity **U** at time T, with tolerances dm, $d^3\mathbf{X}$ and $d^3\mathbf{U}$ in meteoroid mass, position and velocity.

$\xi(\mathbf{U},\mathbf{X})\, d^3\mathbf{X}\, d^3\mathbf{U}$ is the probability that an asteroidal meteoroid of mass $\geqq m_0$ will pass through position $\mathbf{X}$ with velocity $\mathbf{U}$ at time T, with tolerances $d^3\mathbf{X}$ and $d^3\mathbf{U}$ in meteoroid position and velocity. The reference mass m_0 will be discussed below.

Now define:

$$\sigma(\mathbf{X}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \xi(\mathbf{U},\mathbf{X})\, d^3\mathbf{U} \qquad \text{(B-39)}$$

$\sigma(\mathbf{X})$ is a standard meteoroid space-density distribution. That is, for a certain reference mass m_0, $\sigma(\mathbf{X})\, d^3\mathbf{X}$ is the probability that an asteroidal meteoroid of mass $\geqq m_0$ will pass through position $\mathbf{X}$ at time T, with tolerance $d^3\mathbf{X}$ in position or, in other words, the number of asteroidal meteoroids of mass $\geqq m_0$ per unit volume at position $\mathbf{X}$. This reference mass m_0 is chosen as that of a typical iron asteroid of absolute magnitude $G_0 = 13.6$, for reasons discussed later. Such an asteroid would have mass $m_0 = 2.56 \times 10^{18}$ g and radius $r_0 = 4.3$ km.

$\zeta(m,\mathbf{X})\, \sigma(\mathbf{X})\, dm\, d^3\mathbf{X}$ is the probability that an asteroidal meteoroid of mass m will pass through position $\mathbf{X}$ at time T, with tolerances dm and $d^3\mathbf{X}$ in meteoroid mass and position. From the relation of this and the previous definition,

$$\int_{m_0}^{\infty} \zeta(m,\mathbf{X})\, dm = 1 \qquad \text{(B-40)}$$

There seems to be general agreement that the mass distribution is a power law, as in Eq. (A-1), and also that the exponent in this power law is constant throughout the asteroid belt (Ref. 10). Thus, one can choose

$$\zeta(m,\mathbf{X}) = \zeta(m) \qquad \text{(B-41)}$$

and

$$\int_{m}^{\infty} \zeta(m')\, dm' = \left(\frac{m}{m_0}\right)^{-\beta} \qquad \text{(B-42)}$$

so that

$$\zeta(m) = \frac{\beta}{m_0}\left(\frac{m}{m_0}\right)^{-\beta-1} \qquad \text{(B-43)}$$

The total number of asteroidal meteoroids of mass $\geqq m$ for this model is

$$\Phi = \alpha_c'' m^{-\beta} = \left(\frac{m}{m_0}\right)^{-\beta} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \sigma(\mathbf{X})\, d^3\mathbf{X} \qquad \text{(B-44)}$$

From Eqs. (A-1 and B-38),

$$F = \alpha_c' \int_{m}^{\infty} \zeta(m')\, dm' = \alpha_c m^{-\beta} \text{ meteoroids m}^{-2}\text{ s}^{-1}$$

of mass m or greater, where α_c = a constant.

Now, instead of an infinite upper limit on m', assume a finite upper limit M. Then the above equation becomes

$$F' = \alpha_c' \int_{m}^{M} \zeta(m')\, dm' = \alpha_c\left(m^{-\beta} - M^{-\beta}\right)$$

$$= \alpha_c m^{-\beta}\left[1 - \left(\frac{m}{M}\right)^{\beta}\right]$$

or

$$F' = \alpha_c m^{-\beta}\left[1 - \left(\frac{\frac{4}{3}\pi r^3 \rho'}{\frac{4}{3}\pi R^3 \rho'}\right)^{\beta}\right] = \alpha_c m^{-\beta} - \alpha_c m^{-\beta}\left(\frac{r}{R}\right)^{3\beta}$$

and

$$F' = F - F\left(\frac{r}{R}\right)^{3\beta}$$

Thus,

$$\frac{F - F'}{F} = \left(\frac{r}{R}\right)^{3\beta}$$

Now for $3\beta \cong 2$, $r = 1$ cm and $R \cong 100$ km $= 10^7$ cm

$$\frac{F - F'}{F} = \left(\frac{1}{10^7}\right)^2 = \frac{1}{10^{14}} = 10^{-14}$$

Thus, a realistic upper limit, $R = 100$ km, and

$$M = \frac{4}{3}\pi R^3 \rho'$$

and an infinite upper limit on mass M produces for all practical purposes, the same flux of meteoroids F.

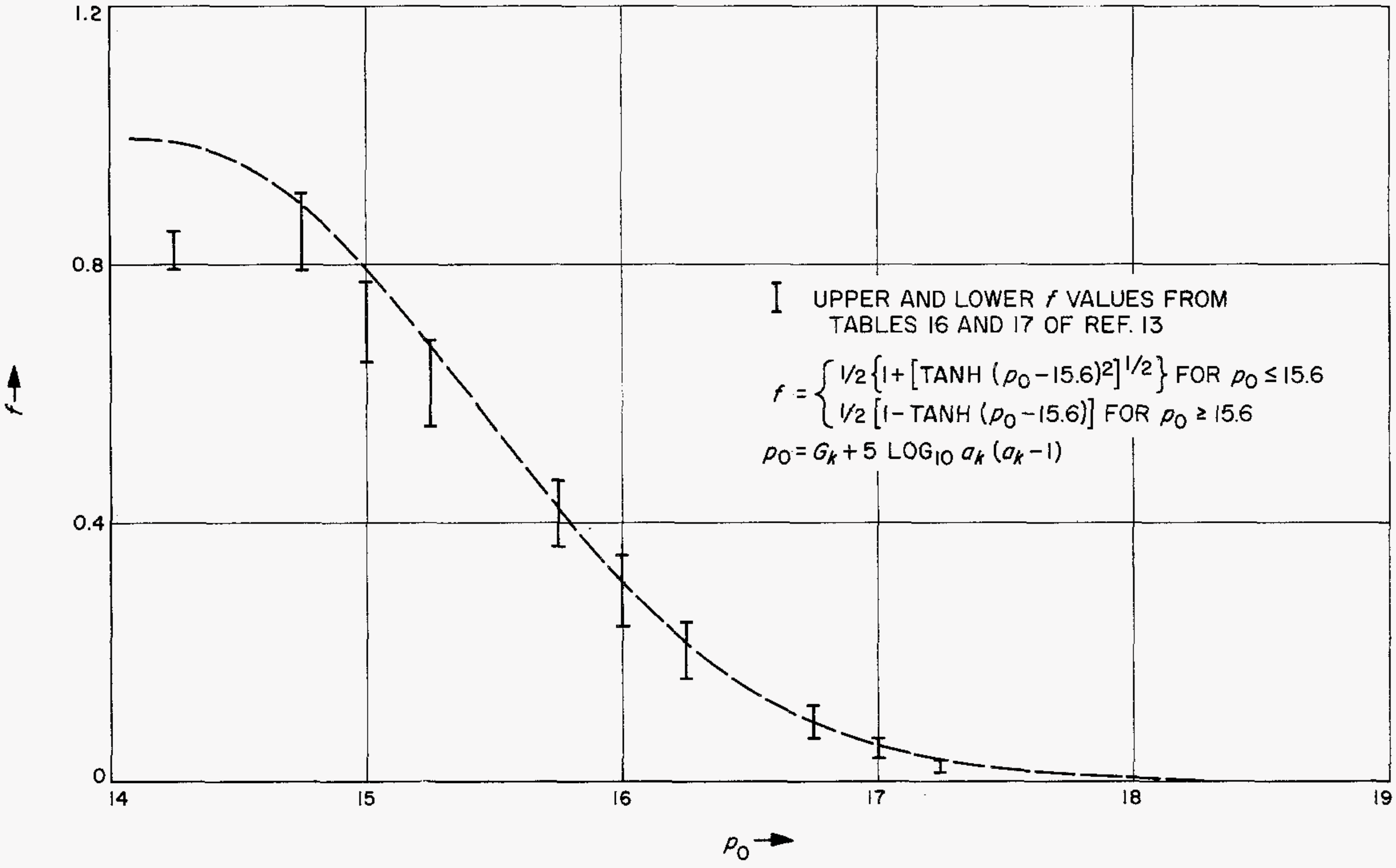


Fig. B-3. Plot of f vs p_0

The meteoroid velocity–space distribution $\xi(\mathbf{U}, \mathbf{X})$ was obtained through the use of the orbital elements and absolute magnitudes of the 1659 numbered asteroids of Ref. 11. Similar data are given in Ref. 12. Each asteroid was assigned a weight equal to the reciprocal of the estimated probability of discovery of an asteroid of similar orbit and magnitude (Ref. 13), thus correcting statistically for observational bias.

For example, if there is only a 0.5 probability of discovering asteroids of type k, there should be twice as many asteroids of this type as have been discovered, and the weighting factor is $w_k = 1/0.5 = 2$. For most of the asteroids $w_k \simeq 1$. The parameter f is defined as $f = 1/w_k$, where w_k is the statistical weight and f is the probability of discovery of the asteroid. Thus, one has $w_k = 1/f$. Figure B-3 is a plot of f versus p_0 and was obtained from Figs. 16 and 17 of Ref. 13. Here p_0 is the mean opposition magnitude, and is equal to

$$p_0 = G_k + 5 \log_{10} [a_k (a_k - 1)]$$

where G_k = absolute magnitude of the kth asteroid and a_k = semi-major axis of orbit of kth asteroid. The dashed curve was used in the computer program. The following equation represents the dashed curve

$$f = \begin{cases} \frac{1}{2}\{1 + [\tanh (p_0 - 15.6)^2]^{1/2}\} & \text{for } p_0 \leqq 15.6 \\ \frac{1}{2}[1 - \tanh(p_0 - 15.6)] & \text{for } p_0 \geqq 15.6 \end{cases}$$

The 14 Trojan asteroids were removed from consideration because the statistics of their orbits are very different from those of the rest of the asteroid belt (owing to the major influence of Jupiter). The reference asteroid of mass m_0, is the minimum size asteroid considered, and corresponds to an absolute magnitude of $G_0 = 13.6$. All asteroids with absolute magnitudes greater than $G_0 = 13.6$ were omitted, since smaller asteroids often had such low probabilities of discovery that the weighting method became unreasonable. Hidalgo was also omitted, because owing to its unique orbit, the probability of discovery equation given in Fig. B-3 above could not reasonably be extended to it. As a result of these restrictions, only

1500 asteroids were finally used. The kth asteroid in this group has statistical weight w_k, absolute magnitude G_k, semi-major axis a_k, eccentricity e_k, and inclination i_k. In the model used here, each of the 1500 asteroids is replaced by an appropriately weighted "swarm" of meteoroids. The kth meteoroid swarm, which replaces the kth asteroid, contains $w_k(m/m_0)^{-\beta}$ meteoroids of mass greater than m.

The kth meteoroid swarm has a space density $\sigma_k(\mathbf{X})$, and the overall meteoroid space density is the sum of these.

$$\sigma(\mathbf{X}) = \sum_k \sigma_k(\mathbf{X}), \qquad \text{(B-45)}$$

thus replacing the integrals of Eq. (B-39) by a sum. The meteoroids in each swarm all have the same semi-major axis, eccentricity, and inclination to the ecliptic as their "parent" asteroid, but their longitudes of ascending node, arguments of perihelion and mean anomalies were all uniformly and independently distributed in the interval 0 to 2π. This appears to be reasonable on the following grounds: If a particular asteroid were fragmented into many pieces, these pieces would continue to move in the same orbit with the same a, e and i but would be gradually spread, more or less uniformly, around the orbit due to external perturbing forces from Jupiter, Mars and other asteroids or meteoroids. The "swarm" model thus gives a better meteoroid density distribution than has been used in the past. These assumptions produce an explicit form for $\sigma_k(\mathbf{X})$ which is given in Appendix C, Section V of this report. The integration over velocity in Eq. (B-10) is also reduced to a sum. The kth meteoroid swarm contributes four velocities for those positions, $\mathbf{X}$ where $\sigma_k(\mathbf{X}) \neq 0$. This is shown as follows: Figure B-4 shows the ecliptic plane, spacecraft, solar distance, ecliptic latitude and longitude of the spacecraft. Figure B-5 is a perspective view of the spacecraft and ecliptic plane. Meteoroids in orbits with inclinations less than the latitude of the spacecraft cannot impact the spacecraft. If the meteoroid inclination i is greater than or equal to the spacecraft latitude the meteoroid can impact the spacecraft. Figure B-6 shows the spacecraft and the two possible orbital planes of meteoroids, Plane No. 1 and Plane No. 2, which can impact the spacecraft, each plane having the same inclination where $i > \lambda$. If i were equal to λ, the two planes would merge into a single plane. Figure B-7 shows the two meteoroid orbits in Plane No. 1, with semi-major axis a and eccentricity e which pass through the spacecraft. Figure B-8 shows the same thing for Plane No. 2. There are thus two possible planes (Planes No. 1 and No. 2) with inclination i, and two meteoroid orbits in each plane, with semi-major axis a and eccentricity e, for a total of four meteoroid orbits, with the same a, e, i, which can impact the spacecraft. In Fig. B-7 the spacecraft position is at $m = 1$, which in Fig. B-8, is at $m = 2$. When the angle θ, in Figs. B-7 and -8 (the argument of the latitude), is greater than $\pi/2$, m is taken as 1, and when it is less than $\pi/2$, is taken as 2.

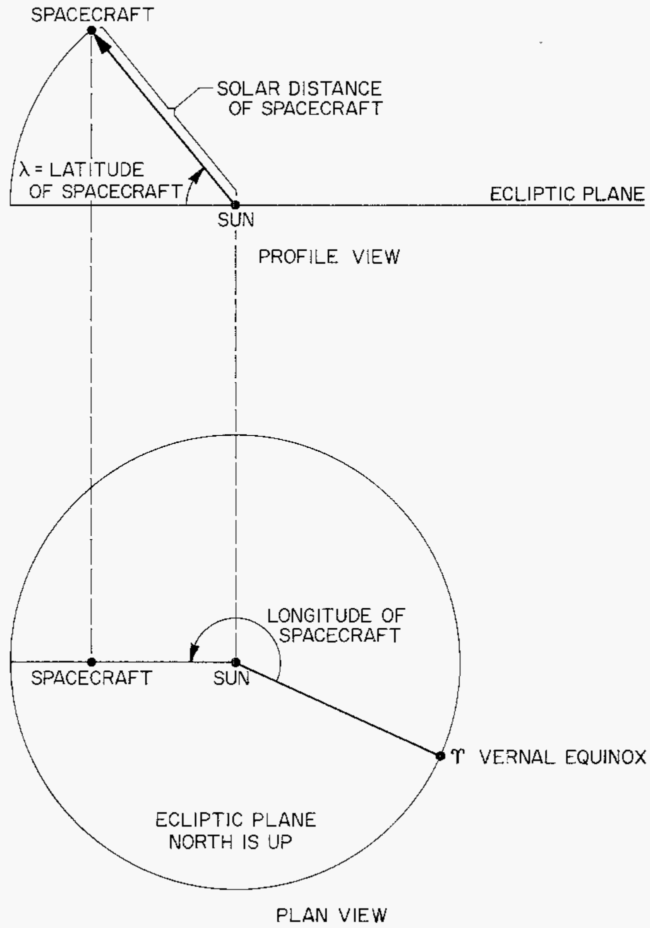


Fig. B-4. Ecliptic plane, solar distance, latitude and longitude of spacecraft

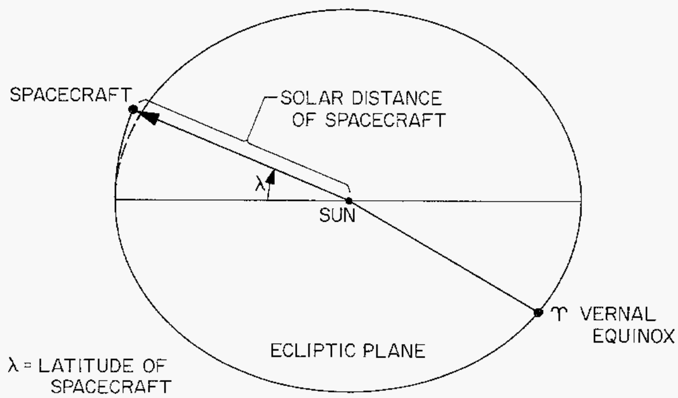


Fig. B-5. Perspective view of spacecraft and ecliptic plane

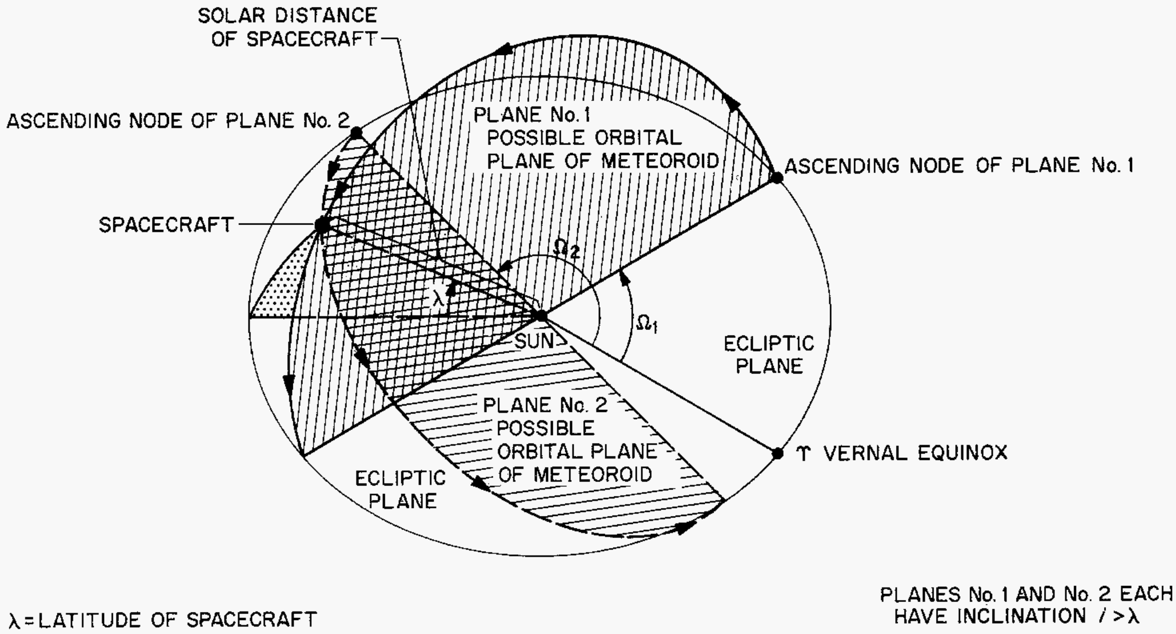


Fig. B-6. The two possible orbital planes of meteoroids with inclination *i*, where $i > \lambda$, which can impact the spacecraft

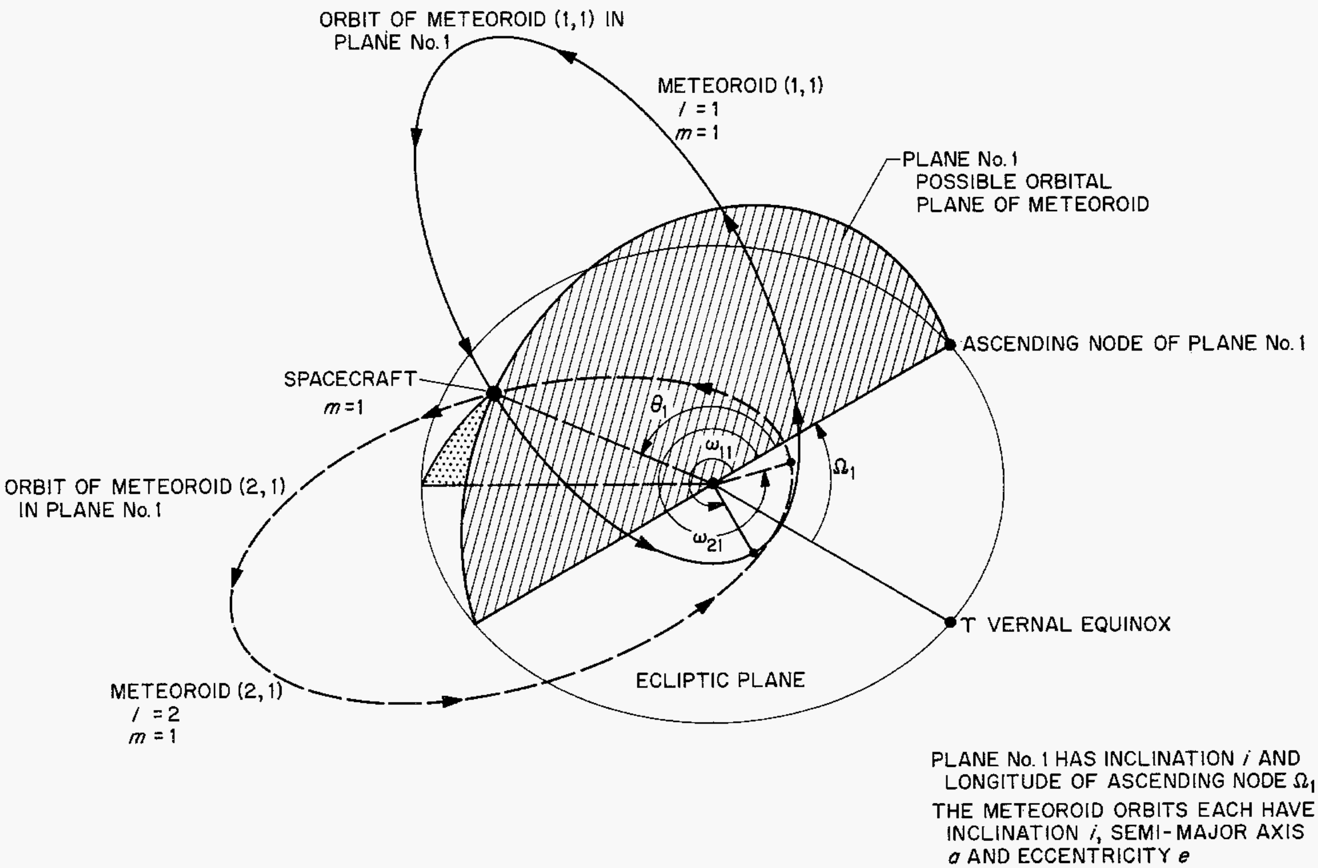


Fig. B-7. The two meteoroid orbits in plane No. 1, with semi-major axis *a* and eccentricity e, which pass through the spacecraft

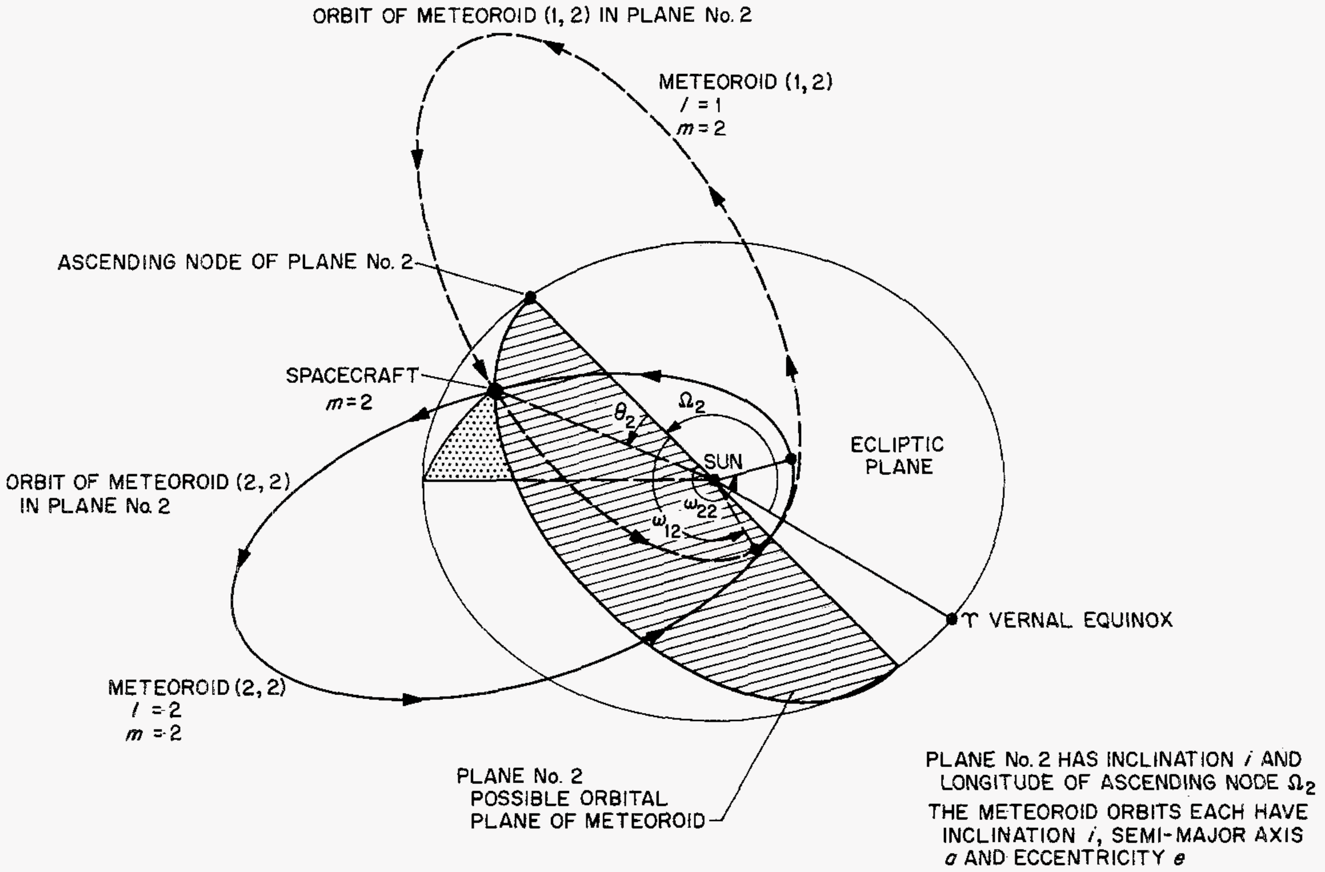


Fig. B-8. The two meteoroid orbits in plane No. 2, with semi-major axis *a* and eccentricity e, which pass through the spacecraft

When the meteoroid is moving in toward the sun, prior to impacting the spacecraft, l is taken as 1, whereas if it is moving outwards from the sun, l is taken as 2. The kth meteoroid swarm contributes four velocities: $\mathbf{U}_k^{(1,1)}(\mathbf{X})$, $\mathbf{U}_k^{(1,2)}(\mathbf{X})$, $\mathbf{U}_k^{(2,1)}(\mathbf{X})$ and $\mathbf{U}_k^{(2,2)}(\mathbf{X})$, or $\mathbf{U}_k^{(l,m)}(\mathbf{X})$ for those positions $\mathbf{X}$ where $\sigma_k(\mathbf{X}) \neq 0$. These four components of each meteoroid swarm at each location are present in equal quantities.

IV. Probability of Spacecraft Successfully Traversing the Asteroid Belt

Next, the probability of success $P_s(T)$, over the whole mission, is determined. Only meteoroids are considered, and all other factors contributing to spacecraft failure are ignored. The mission time interval is from time T_0 at the beginning, to time T_f at the end of the mission. The overall probability of success of the mission $P(S)$ is then the probability that the spacecraft is in the success state S at time T_f. Thus, in Eq. (B-13), $T = T_f$, $T' = T_0$,

and

$$P(S) = P_s(T_f) = P_s(T_0)\exp\left(-\int_{T_0}^{T_f} \pi(T)\,dT\right) \tag{B-46}$$

Thus, $\pi(T)$ must be evaluated. Since only spacecraft failure caused by meteoroid impact is considered, then

$$\pi(T) = \pi_I(T), \qquad P(S) = P_I(S)$$

and

$$P(S) = P_s(T_f) = P_s(T_0)\exp\left(-\int_{T_0}^{T_f} \pi_I(T)\,dT\right) \tag{B-47}$$

Equations (B-11 and -12) combine to produce the general meteoroid induced failure rate equation:

$$\pi_I(T) = \int_{\underline{A}} \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\alpha, \mu, \mathbf{W}, T)\,[1 - S_h(\alpha, -\mathbf{w}, T)] \cdot |\mathbf{n}(\alpha, T) \cdot \mathbf{W}|\, A(\alpha) \cdot \psi[\mu, \mathbf{X}(T), \mathbf{U}, T]\, d^3\mathbf{U}\, d\mu\, d\alpha \tag{B-48}$$

For spacecraft with reasonably constant configurations, Eqs. (B-15 and -18) can be combined with Eq. (B-48) to yield

$$\pi_I(T) = \int_{\underline{A}} \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\alpha, \mu, \mathbf{W}')\,[1 - S_h(\alpha, -\mathbf{w}')]\, |\mathbf{n}(\alpha) \cdot \mathbf{W}|\, A(\alpha) \cdot \psi[\mu, \mathbf{X}(T), \mathbf{U}, T]\, d^3\mathbf{U}\, d\mu\, d\alpha \tag{B-49}$$

where

$$\mathbf{W}' = \mathbf{W}\,\mathcal{M}^{-1}(T) = [\mathbf{U} - \mathbf{V}(T)]\,\mathcal{M}^{-1}(T)$$

$$\mathbf{w}' = \mathbf{w}\,\mathcal{M}^{-1}(T) = \frac{\mathbf{W}'}{|\mathbf{W}'|} \tag{B-50}$$

Note that $\mathbf{W}$ is in space fixed coordinates while $\mathbf{W}'$ and $\mathbf{w}'$ are in spacecraft fixed coordinates. Eq. (B-30) may be written

$$[1 - S_h(\alpha, -\mathbf{w}')]\, |\mathbf{n} \cdot \mathbf{W}'| = \max\{0, -\mathbf{n} \cdot \mathbf{W}'\} \tag{B-51}$$

For a general convex spacecraft, Eq. (B-49) becomes by use of Eq. (B-51)

$$\pi_I(T) = \iint_{\Omega'} \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\mathbf{n}, \mu, \mathbf{W}') \max\{0, -\mathbf{n} \cdot \mathbf{W}'\}\, A(n) \cdot \psi[\mu, \mathbf{X}(T), \mathbf{U}, T]\, d^3\mathbf{U}\, d\mu\, d^2\mathbf{n} \tag{B-52}$$

where it has been convenient to identify the spacecraft surface parameter α with the outwardly drawn normal unit vector $\mathbf{n}(\alpha)$ of the corresponding surface element, dropping the α, or replacing it by $\mathbf{n}$, wherever it appears. In this case $\underline{A}$ becomes Ω', the surface of the unit sphere, and $d\alpha$ becomes $d^2\mathbf{n}$.

For a general polyhedral spacecraft, Eq. (B-49) may be written

$$\pi_I(T) = \sum_j [F_j(T)]\, A_j \tag{B-53}$$

where

$$F_j(T) = \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta_j(\mu, \mathbf{W}')\,[1 - S_h(j, -\mathbf{w}')]\, |\mathbf{n}_j \cdot \mathbf{W}'| \cdot \psi[\mu, \mathbf{X}(T), \mathbf{U}, T]\, d^3\mathbf{U}\, d\mu \tag{B-54}$$

Equation (B-53) thus gives $\pi_I(T)$ as a sum, over the faces of the polyhedral surface, of the flux of destructive meteoroids incident on the jth face, at time T, multiplied by the area of the jth face. The flux F_j is given in Eq. (B-54). Note that α in Eq. (B-49) is replaced by j in Eq. (B-54). For a spacecraft which is both convex and polyhedral, as will be assumed hereafter, Eq. (B-54) becomes

$$F_j(T) = \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta_j(\mu, \mathbf{W}') \max\{0, -\mathbf{n}_j \cdot \mathbf{W}'\}\, \psi[\mu, \mathbf{X}(T), \mathbf{U}, T]\, d^3\mathbf{U}\, d\mu \tag{B-55}$$

If one inserts the damage function of Eq. (B-36) into Eq. (B-55) one gets

$$F_j(T) = \int_{\underline{M}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H\left[C_1 r(\mu) \ln(1 + C_2 D^2) - t_j\right] \max\{0, D\}\, \psi\left[\mu, \mathbf{X}(T), \mathbf{U}, T\right] d^3\mathbf{U}\, d\mu \qquad \text{(B-56)}$$

where

$$D = -\mathbf{n}_j \cdot \mathbf{W}'$$

$$C_1 = C_1(M_j, M') \qquad \text{(B-57)}$$

$$C_2 = C_2(M_j, M')$$

The inclusion of the meteoroid distribution model is most conveniently done in two steps. *First,* utilizing Eqs. (B-14, -38, and -39), and replacing μ with m, Eq. (B-56) becomes

$$F_j(T) = \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H\left[C_1 r \ln(1 + C_2 D^2) - t_j\right] \max\{0, D\}\, \zeta(m)\, \xi\left[\mathbf{U}, \mathbf{X}(T)\right] d^3\mathbf{U}\, dm \qquad \text{(B-58)}$$

where r is found from

$$m = \frac{4}{3} \pi r^3 \rho' \qquad \text{(B-59)}$$

Also, one can write

$$\int_0^{\infty} H\left[C_1 r \ln(1 + C_2 D^2) - t_j\right] \zeta(m)\, dm = \int_0^{M_0} 0\, dm + \int_{M_0}^{\infty} \zeta(m)\, dm$$

since

$$H(x) = 0 \qquad x < 0$$

$$H(x) = 1 \qquad x > 0$$

and at meteoroid size $r = R$, the shield is penetrated, or

$$C_1 R \ln(1 + C_2 D^2) - t_j = 0$$

or

$$R = \frac{t_j}{C_1 \ln(1 + C_2 D^2)} \qquad \text{(B-60)}$$

so that

$$H\left[C_1 r \ln(1 + C_2 D^2) - t_j\right] = \begin{cases} 1 \text{ for } m > M_0 \\ 0 \text{ for } m < M_0 \end{cases}$$

where

$$M_0 = \frac{4}{3}\pi R^3 \rho'$$

so that

$$\int_0^\infty H\left[C_1 r \ln(1 + C_2 D^2) - t_j\right] \zeta(m)\, dm = \int_{4/3\,\pi R^3 \rho'}^\infty \zeta(m)\, dm \qquad \text{(B-61)}$$

Now, from Eqs. (B-40 and -44)

$$\int_{M_0}^\infty \zeta(m)\, dm = \left(\frac{M_0}{m_0}\right)^{-\beta} = \left(\frac{R}{r_0}\right)^{-3\beta}$$

Thus, Eq. (B-58) becomes

$$F_j(T) = \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \left(\frac{R}{r_0}\right)^{-3\beta} \max\{0, D\}\, \xi\left[\mathbf{U}, \mathbf{X}(T)\right] d^3\mathbf{U} \qquad \text{(B-62)}$$

Second, since the *k*th swarm has four meteoroid orbits with the same a_k, e_k and i_k which can impact the spacecraft, and each of these four components is present in equal quantity, Eq. (B-39) becomes

$$\frac{1}{4}\sigma_k\left[\mathbf{X}(T)\right] = \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \xi(\mathbf{U}_k, \mathbf{X})\, d^3\mathbf{U} \qquad \text{(B-63)}$$

From Eq. (B-60)

$$\left(\frac{R}{r_0}\right)^{-3\beta} = \left(\frac{r_0}{R}\right)^{3\beta} = \left[\frac{r_0}{t_j} C_1 \ln(1 + C_2 D^2)\right]^{3\beta} = \left(\frac{C_1 r_0}{t_j}\right)^{3\beta} \left[\ln(1 + C_2 D^2)\right]^{3\beta} \qquad \text{(B-64)}$$

Now, one can write

$$\int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty f(\mathbf{U}, \mathbf{X})\, \xi(\mathbf{U}, \mathbf{X})\, d^3\mathbf{U} = \sum_{k=1}^{1500} \sum_{l,m=1}^{2} f\left[\mathbf{U}_k^{(l,m)}(\mathbf{X}), \mathbf{X}\right] \cdot \frac{1}{4}\sigma_k(\mathbf{X}) \qquad \text{(B-65)}$$

Thus, Eq. (B-62) becomes

$$F_j(T) = \left(\frac{C_1 r_0}{t_j}\right)^{3\beta} \sum_k \sum_{l,m} \left(\ln\left\{1 + C_2\left[D_k^{(l,m)}\right]^2\right\}\right)^{3\beta} \max\left\{0, D_k^{(l,m)}\right\} \frac{1}{4}\sigma_k\left[\mathbf{X}(T)\right] \qquad \text{(B-66)}$$

where

$$D_k^{(l,m)} = -\mathbf{n}_j \cdot \left\{\mathbf{U}_k^{(l,m)}\left[\mathbf{X}(T)\right] - \mathbf{V}(T)\right\} \mathcal{M}^{-1}(T) \qquad \text{(B-67)}$$

where $\mathbf{n}_j$ is in spacecraft-fixed coordinates and $\mathbf{U}_k^{(l,m)}[\mathbf{X}(T)]$ and $\mathbf{V}(T)$ are in space-fixed coordinates. This may also be written as

$$D_k^{(l,m)} = -\mathbf{n}_j \mathcal{M}(T) \cdot \left\{\mathbf{U}_k^{(l,m)}\left[\mathbf{X}(T)\right] - \mathbf{V}(T)\right\} \qquad \text{(B-68)}$$

where $\mathbf{n}_j \mathcal{M}(T)$ is now in space fixed coordinates as shown in Eq. (B-15).

In Eq. (B-56) the summations are over k, a number related to the numbered asteroids, and k ranges from 1 to 1500, as well as over l and m, which each can take on values of 1 and 2 and refer to the four different velocity streams for each meteoroid swarm. There are a total of k swarms.

Equation (B-66) gives $F_j(T)$, the expected number of destructive hits per unit area per unit time at the spacecraft position at time T. Inputs required are $\mathbf{X}(T)$ and $\mathbf{V}(T)$, the position and velocity of the spacecraft as a function of time. These are not explicitly analyzed in this derivation, and are assumed to be two known functions of the time. Eqs. (B-66 to -68) include the asteroid belt model represented by $\sigma_k[\mathbf{X}(T)]$, $\mathbf{U}_k^{(l,m)}[\mathbf{X}(T)]$; the spacecraft position and velocity represented by $\mathbf{X}(T)$ and $\mathbf{V}(T)$; the shadowing effect of the spacecraft represented by max $\{0, D_k^{(l,m)}\}$; the meteoroid damage function, represented by $(C_1 r_0/t_j)$ and the logarithmic term; the spacecraft surface position represented by $\mathbf{n}_j$; and the orientation of the spacecraft at time T, relative to some system of space coordinates, represented by the matrix $\mathcal{M}(T)$.

From $F_j(T)$ in Eq. (B-66) and areas A_j, one obtains $\pi_I(T)$ in Eq. (B-53), and the probability of successfully traversing the asteroid belt $P(S)$ from Eq. (B-47).

Appendix C

Implementation

I. Coordinate Systems

A. Sun-Centered Coordinate System

The sun-centered coordinate system, shown in Fig. C-1, is the standard ecliptic coordinate system. Here $\mathbf{e}$ is a three-dimensional unit vector. The basis vectors are:

$\mathbf{e}_N$ is in the direction of the earth's angular momentum vector, i.e., it points to the ecliptic north.

$\mathbf{e}_\Upsilon$ is in the direction of the vernal equinox, Υ in the ecliptic.

$\mathbf{e}_- = \mathbf{e}_N \times \mathbf{e}_\Upsilon$ and thus lies in the ecliptic.

These three unit vectors are the basis for the sun-centered coordinate system. For any spacecraft position $\mathbf{X}$ in Fig. C-2, the solar distance is

$$r = r(\mathbf{X}) = |\mathbf{X}| = (\mathbf{X} \cdot \mathbf{X})^{1/2} \tag{C-1}$$

the unit vector in the direction $\mathbf{X}$ is

$$\mathbf{e}_X = \frac{\mathbf{X}}{r} \tag{C-2}$$

and

$$\cos\left(\frac{\pi}{2} - \lambda\right) = \mathbf{e}_X \cdot \mathbf{e}_N = \sin\lambda$$

so that the ecliptic latitude λ is given by

$$\lambda = \lambda(\mathbf{X}) = \sin^{-1}(\mathbf{e}_X \cdot \mathbf{e}_N) \tag{C-3}$$

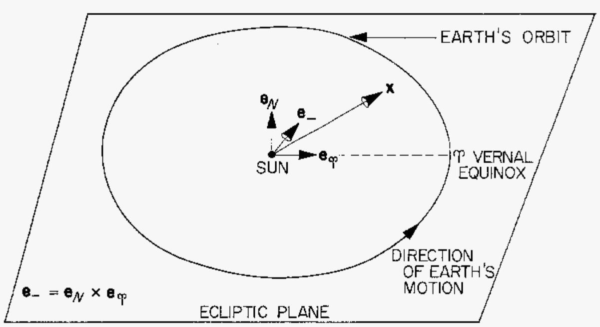


Fig. C-1. Sun-centered coordinate system

From Fig. C-2, one can represent the spacecraft position $\mathbf{X}$ by

$$\mathbf{X} = X_N \mathbf{e}_N, + X_\Upsilon \mathbf{e}_\Upsilon + X_- \mathbf{e}_- \tag{C-4}$$

where X_N, X_Υ and X_- are the components of $\mathbf{X}$ on $\mathbf{e}_N$, $\mathbf{e}_\Upsilon$ and $\mathbf{e}_-$, respectively. Thus, from Fig. C-2,

$$\mathbf{Y} + X_N \mathbf{e}_N = \mathbf{X}$$

and

$$\mathbf{Y} = \mathbf{X} - X_N \mathbf{e}_N$$

Now,

$$\mathbf{X} = X\mathbf{e}_X, \qquad X_N = (X\mathbf{e}_X) \cdot \mathbf{e}_N = X\mathbf{e}_X \cdot \mathbf{e}_N$$

so that

$$\mathbf{e}_Y = \frac{\mathbf{Y}}{|\mathbf{Y}|} = \frac{X\mathbf{e}_X - (X\mathbf{e}_X \cdot \mathbf{e}_N)\mathbf{e}_N}{|X\mathbf{e}_X - (X\mathbf{e}_X \cdot \mathbf{e}_N)\mathbf{e}_N|} = \frac{\mathbf{e}_X - (\mathbf{e}_X \cdot \mathbf{e}_N)\mathbf{e}_N}{|\mathbf{e}_X - (\mathbf{e}_X \cdot \mathbf{e}_N)\mathbf{e}_N|}$$

Thus, the ecliptic longitude Λ is given by

$$\cos\Lambda = \mathbf{e}_Y \cdot \mathbf{e}_\Upsilon$$

$$\sin\Lambda = \cos\left(\frac{\pi}{2} - \Lambda\right) = \mathbf{e}_Y \cdot \mathbf{e}_- \tag{C-5}$$

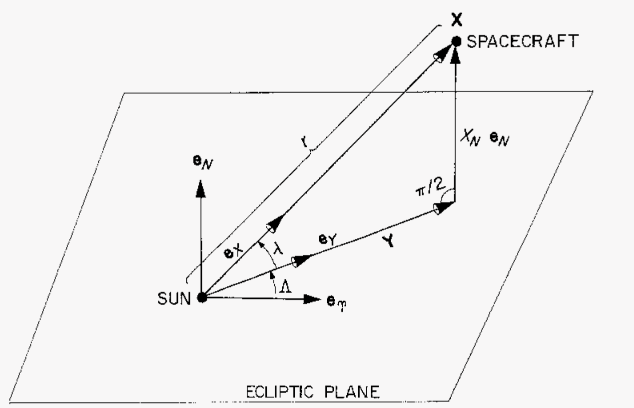


Fig. C-2. Spacecraft coordinates r, λ, Λ in sun-centered coordinate system

and

$$\tan \Lambda = \frac{\mathbf{e}_Y \cdot \mathbf{e}_{-}}{\mathbf{e}_Y \cdot \mathbf{e}_{\Upsilon}}$$

Here

$$r = |\mathbf{X}| > 0$$

$$-\frac{\pi}{2} \leq \lambda \leq \frac{\pi}{2}$$

$$0 \leq \Lambda \leq 2\pi$$

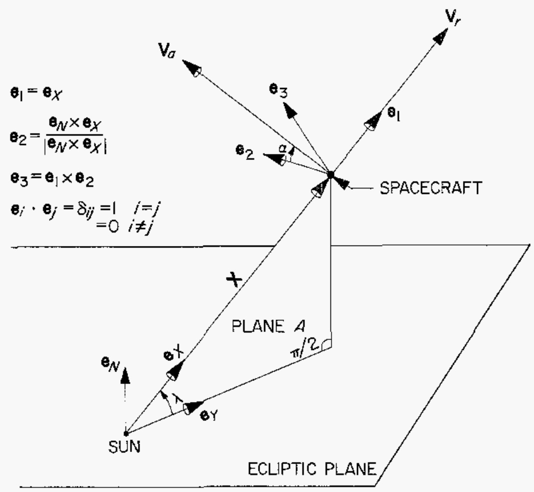


Fig. C-3. Space-fixed coordinate system

B. Space-Fixed Coordinate System

Figure C-3 shows the basis vectors in the space-fixed coordinate system, with origin at the spacecraft at position **X**. The position of the sun is indicated. Here $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are functions of **X**, and are given by the expressions

$$\left.\begin{aligned} &\mathbf{e}_1 = \mathbf{e}_1(\mathbf{X}) = \mathbf{e}_X \text{ and is in the direction of } \mathbf{X} \\ &\mathbf{e}_2 = \mathbf{e}_2(\mathbf{X}) = \frac{\mathbf{e}_N \times \mathbf{e}_X}{|\mathbf{e}_N \times \mathbf{e}_X|} \text{ and is in the direction of } \Lambda(\mathbf{X}) \\ &\mathbf{e}_3 = \mathbf{e}_3(\mathbf{X}) = \mathbf{e}_1 \times \mathbf{e}_2 \text{ and is in the direction of } \lambda(\mathbf{X}) \end{aligned}\right\} \tag{C-6}$$

In this coordinate system the components of the meteoroid velocity vector **U** are U_1, U_2 and U_3, and the components of the spacecraft velocity vector **V** are V_1, V_2, and V_3, or

$$\left.\begin{aligned} \mathbf{U} &= U_1 \mathbf{e}_1 + U_2 \mathbf{e}_2 + U_3 \mathbf{e}_3 \\ \mathbf{V} &= V_1 \mathbf{e}_1 + V_2 \mathbf{e}_2 + V_3 \mathbf{e}_3 \end{aligned}\right\} \tag{C-7}$$

C. Spacecraft-Fixed Coordinate System

The spacecraft-fixed coordinate system has origin at $\mathbf{X}(T)$ at time T. Its basis vectors are the orthogonal unit vectors $\mathbf{e}_1'(T)$, $\mathbf{e}_2'(T)$ and $\mathbf{e}_3'(T)$ shown in Fig. C-4.

Here

$$\begin{aligned} \mathbf{e}_1' &= \mathbf{e}_1'(T) = \mathbf{e}_2 = \frac{\mathbf{e}_N \times \mathbf{e}_X}{|\mathbf{e}_N \times \mathbf{e}_X|} \\ \mathbf{e}_2' &= \mathbf{e}_2'(T) = \mathbf{e}_Y = \mathbf{e}_1' \times \mathbf{e}_3' \\ \mathbf{e}_3' &= \mathbf{e}_3'(T) = \mathbf{e}_N \end{aligned} \tag{C-8}$$

II. Spacecraft Orientation Matrix

The spacecraft orientation matrix $\mathcal{M}(T)$ at time T, is a rotation matrix which converts a vector from spacecraft-fixed coordinates to space-fixed coordinates, as shown in Eq. (B-15).

Here $\mathcal{M}(T)$ is given by $\mathcal{M}_{ij}(T) = \mathbf{e}_i'(T) \cdot \mathbf{e}_j[\mathbf{X}(T)]$, or

$$\mathcal{M}(T) = \begin{Bmatrix} \mathbf{e}_1'(T) \cdot \mathbf{e}_1[\mathbf{X}(T)] & \mathbf{e}_1'(T) \cdot \mathbf{e}_2[\mathbf{X}(T)] & \mathbf{e}_1'(T) \cdot \mathbf{e}_3[\mathbf{X}(T)] \\ \mathbf{e}_2'(T) \cdot \mathbf{e}_1[\mathbf{X}(T)] & \mathbf{e}_2'(T) \cdot \mathbf{e}_2[\mathbf{X}(T)] & \mathbf{e}_2'(T) \cdot \mathbf{e}_3[\mathbf{X}(T)] \\ \mathbf{e}_3'(T) \cdot \mathbf{e}_1[\mathbf{X}(T)] & \mathbf{e}_3'(T) \cdot \mathbf{e}_2[\mathbf{X}(T)] & \mathbf{e}_3'(T) \cdot \mathbf{e}_3[\mathbf{X}(T)] \end{Bmatrix} \tag{C-9}$$

Now $\mathbf{e}_1$, and $\mathbf{e}_3$ in Fig. C-3 are both in Plane A. Also, $\mathbf{e}_2'$ and $\mathbf{e}_3'$ in Fig. C-4 are in Plane A. This is shown in Fig. C-5. From Fig. C-4 one can see that $\mathbf{e}_1$ makes an angle λ with $\mathbf{e}_2'$. Thus, one can write

$$\begin{aligned} \mathbf{e}_1 &= \mathbf{e}_2' \cos\lambda + \mathbf{e}_3' \sin\lambda \\ \mathbf{e}_2 &= \mathbf{e}_1' \\ \mathbf{e}_3 &= -\mathbf{e}_2' \sin\lambda + \mathbf{e}_3' \cos\lambda \end{aligned}$$

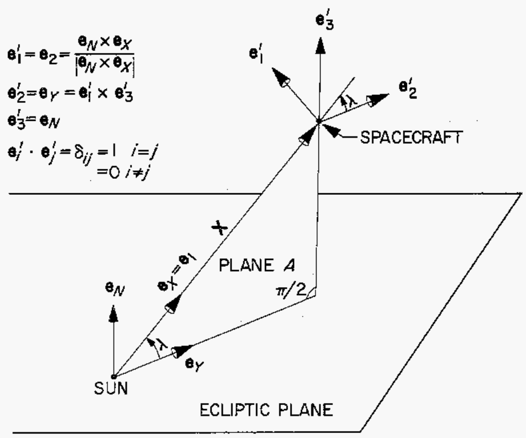


Fig. C-4. Spacecraft-fixed coordinates

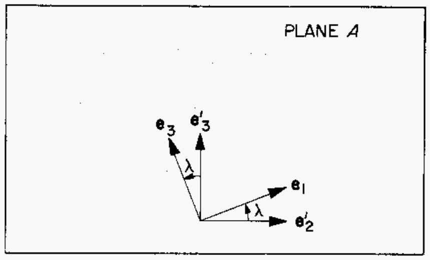


Fig. C-5. Unit vectors in Plane A

Thus, Eq. (C-9) becomes

$$\mathcal{M}(T) = \begin{Bmatrix} 0 & 1 & 0 \\ \cos\lambda\,[\mathbf{X}(T)] & 0 & -\sin\lambda\,[\mathbf{X}(T)] \\ \sin\lambda\,[\mathbf{X}(T)] & 0 & \cos\lambda\,[\mathbf{X}(T)] \end{Bmatrix} \qquad \text{(C-10)}$$

III. Spacecraft Position and Velocity

The parameters of an elliptic orbit about the sun are:

a semi-major axis

e eccentricity

i inclination to the ecliptic

Ω longitude of the ascending node

ω argument of perihelion

η true anomaly

E eccentric anomaly

M mean anomaly

μ_s mean motion where $\mu_s = \left(\frac{\Gamma M_\odot}{a^3}\right)^{1/2}$ (from p. 342 of Ref. 14)

T_P time of perihelion passage

p semi-latus rectum, where $p = a(1 - e^2)$

Here Γ is the Newton constant of gravitation and $M_\odot$ is the mass of the sun. One can find M for a given value of T from

$$M(T) = \mu(T - T_P) \qquad \text{(from p. 335 of Ref. 14).} \qquad \text{(C-11)}$$

With this value of M, one can find $E(T)$ from

$$M(T) = E(T) - e\sin E(T) \qquad \text{(C-12)}$$

(from p. 335 of Ref. 14).

With this value of E one can get $\eta(T)$ from

$$\eta(T) = 2\tan^{-1}\left[\left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{1}{2}E(T)\right] \qquad \text{(C-13)}$$

(from p. 341 of Ref. 14).

The solar distance of the spacecraft is

$$r[\mathbf{X}(T)] = \frac{p}{1 + e\cos\eta(T)} \qquad \text{(C-14)}$$

(from p. 336 of Ref. 14).

The spacecraft velocity vector $\mathbf{V}$ may be represented by

$$\mathbf{V} = \mathbf{V}_r + \mathbf{V}_a \qquad \text{(C-15)}$$

where V_r is the radial velocity, or component of $\mathbf{V}$ along the radius r, and V_a is the azimuthal velocity, or component of $\mathbf{V}$ perpendicular to the radius r. The vectors $\mathbf{V}_r$ and $\mathbf{V}_a$ are shown in Fig. C-6. Figure C-6 shows Plane A, containing $\mathbf{X}$, in the plane of the paper, and perpendicular to the ecliptic. Plane B is perpendicular to $\mathbf{X}$. Figure C-7 shows Plane B, in the plane of the paper, with vectors $\mathbf{e}_1$, $\mathbf{e}_X$, $\mathbf{X}$ and $\mathbf{V}_r$ directed out of the paper. Vector $\mathbf{V}_a$ is in Plane B, which also contains $\mathbf{e}_2$ and $\mathbf{e}_3$, and is at angle α from $\mathbf{e}_2$. This is also shown in Fig. C-3. The components of $\mathbf{V}$ in Eq. (C-7) are thus,

$$\begin{aligned} V_1 &= V_r \\ V_2 &= V_a\cos\alpha \\ V_3 &= V_a\sin\alpha \end{aligned} \qquad \text{(C-16)}$$

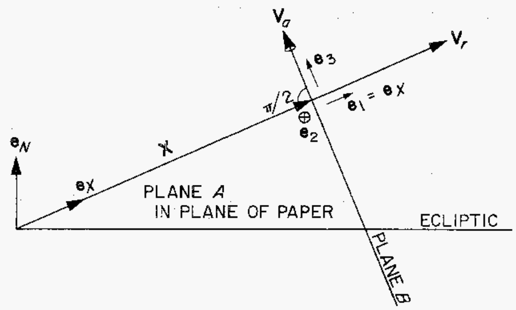


Fig. C-6. Plane *A* contains X, and is in plane of paper and perpendicular to ecliptic; Plane *B* is perpendicular to X

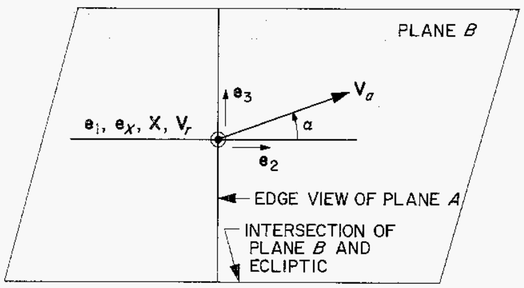


Fig. C-7. Plane *B* with vectors e_1, e_X, X and V_r directed out of the paper

where

$$V_r = V_r(T), \qquad V_a = V_a(T), \qquad \alpha = \alpha(T)$$

Figure C-8 shows $\eta + \omega$ is measured from the ascending node to the spacecraft in the direction of orbital motion.

Figure C-9 gives the relations on the celestial sphere. Since **V** is at an angle α with the ecliptic, as shown in Fig. C-7, the upper angle in the spherical triangle, in Fig. C-9 is $(\pi/2) - \alpha$.

Now, from the law of sines, for spherical triangles, from Fig. C-9, one has

$$\frac{\sin\lambda}{\sin i} = \frac{\sin(\eta + \omega)}{\sin\dfrac{\pi}{2}} = \frac{\sin(\Lambda - \Omega)}{\cos\alpha} \tag{C-17}$$

and therefore

$$\sin\lambda = \sin i \sin(\eta + \omega) \tag{C-18}$$

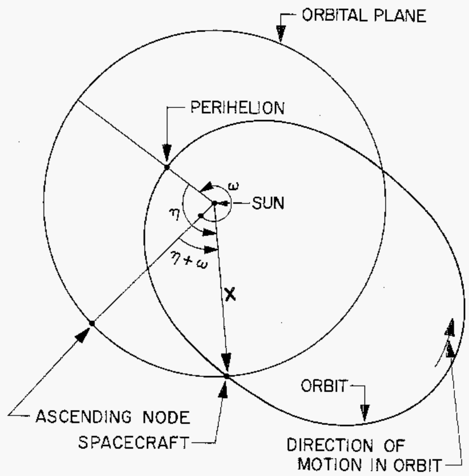


Fig. C-8. Plot showing $\omega + \eta$ measured from ascending node to spacecraft in the orbit plane and in the direction of spacecraft motion

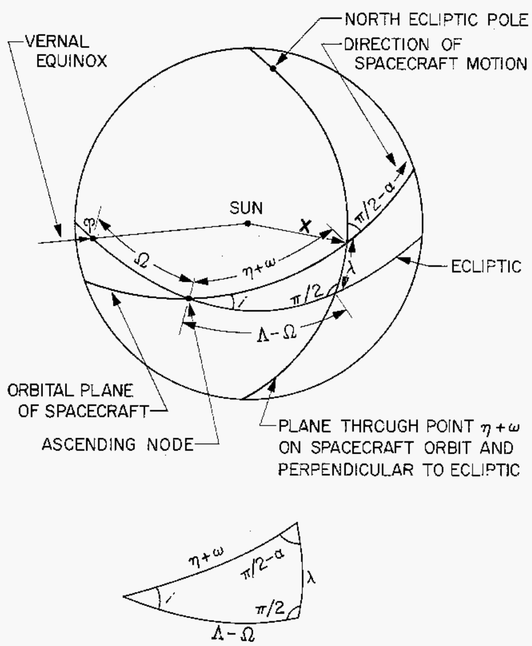


Fig. C-9. Relations on celestial sphere and spherical triangle

The ecliptic latitude of the spacecraft at time T is thus given by

$$\lambda[\mathbf{X}(T)] = \sin^{-1}[\sin i \sin\{\eta(T) + \omega\}] \tag{C-19}$$

From Ref. 15, Napier's Rule for right spherical triangles: "Take the five parts, *excluding the right angle,* and consider them to be in a circular arrangement (as shown in

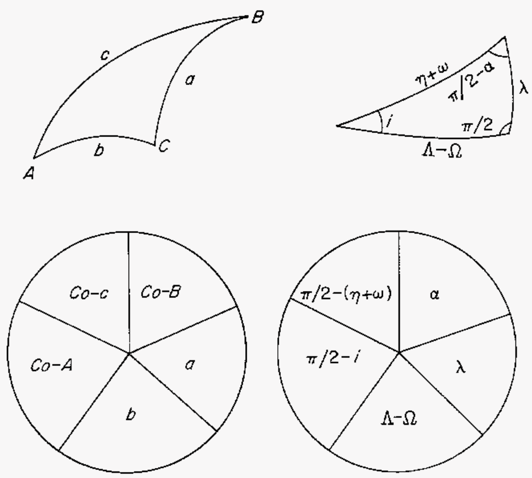


Fig. C-10. Circular arrangement of spherical triangle for application of the Napier rule

Fig. C-10). Attach a *Co* to the two angles and the hypotenuse, meaning 'Complement of *A*,' etc. Then, the sine of the middle part equals the product of the tangents of the adjacent parts." Thus, one has

$$\sin a = \tan b \tan (Co - B) \qquad \text{(C-20)}$$

or

$$\sin \alpha = \tan \lambda \tan \left[\frac{\pi}{2} - (\eta + \omega)\right]$$

and

$$\sin \alpha = \tan \lambda \cot (\eta + \omega)$$

or

$$\sin \alpha = \frac{\tan \lambda}{\tan (\eta + \omega)} \qquad \text{(C-21)}$$

Similarly,

$$\sin (\Lambda - \Omega) = \tan \lambda \tan \left(\frac{\pi}{2} - i\right) = \tan \lambda \cot i$$

or

$$\sin (\Lambda - \Omega) = \frac{\tan \lambda}{\tan i} \qquad \text{(C-22)}$$

By use of elementary trigonometry and Eqs. (C-18 and -22) one gets

$$\sin (\Lambda - \Omega) = \frac{\cos i \sin (\eta + \omega)}{[1 - \sin^2 i \sin^2 (\eta + \omega)]^{1/2}} \qquad \text{(C-23)}$$

From Eq. (C-17), one can write

$$\cos \alpha = \frac{\sin (\Lambda - \Omega) \sin i}{\sin \lambda} = \frac{\dfrac{\tan \lambda}{\tan i} (\sin i)}{\sin \lambda} = \frac{\dfrac{\left(\dfrac{\sin \lambda}{\cos \lambda}\right)}{\left(\dfrac{\sin i}{\cos i}\right)} \sin i}{\sin \lambda}$$

or

$$\cos \alpha = \frac{\cos i}{\cos \lambda} \qquad \text{(C-24)}$$

The azimuthal velocity $V_a = |\mathbf{V}_a|$ is given by

$$V_a = V \cos \theta' = \frac{V (Kp)^{1/2}}{rV} = \frac{(Kp)^{1/2}}{r} \qquad \text{(C-25)}$$

from Ref. 14, p. 342, and

$$|\mathbf{V}_a| = \frac{1}{r} (p)^{1/2} (\Gamma M_\odot)^{1/2} \qquad \text{(C-26)}$$

where θ' is the angle between $\mathbf{V}_a$ and $\mathbf{V}$, $V = |\mathbf{V}|$, and $K = \Gamma M_\odot$

The radial velocity $V_r = |\mathbf{V}_r|$ is given by

$$V_r = V \sin \theta' = (V^2 - V_a^2)^{1/2}$$

and

$$V = \left(\frac{2K}{r} - \frac{K}{a}\right)^{1/2}$$

from Ref. 14, p. 339.

Thus, using Eq. (C-25) and $p = a - ae^2$, one has

$$V_r = \left(\frac{2K}{r} - \frac{K}{a} - \frac{Kp}{r^2}\right)^{1/2} = \left(\frac{2K}{r} - \frac{K}{a} - \frac{K(a - ae^2)}{r^2}\right)^{1/2}$$

$$V_r = \left(\frac{2Kar}{ar^2} - \frac{Kr^2}{ar^2} - \frac{Ka^2}{ar^2} + \frac{Ka^2e^2}{ar^2}\right)^{1/2}$$

$$V_r = \frac{(K)^{1/2}}{r} \left(\frac{2ar - r^2 - a^2 + a^2e^2}{a}\right)^{1/2}$$

$$V_r = \frac{1}{r} \left(\frac{a^2e^2 - (a - r)^2}{a}\right)^{1/2} (\Gamma M_\odot)^{1/2} \qquad \text{(C-27)}$$

IV. Meteoroid Velocity

The velocity $\mathbf{U}(\mathbf{X})$ is the meteoroid velocity. The kth meteoroid stream has velocity

$$\mathbf{U}_k^{(l,m)}(\mathbf{X}) = \sum_{i=1}^{3} \mathbf{U}_{k,i}^{(l,m)}(\mathbf{X})\,\mathbf{e}_i \qquad \text{(C-28)}$$

and in terms of radial and azimuthal components it has the form

$$\mathbf{U}_k^{(l,m)}(\mathbf{X}) = \mathbf{U}_{k,r}^{(l,m)}(\mathbf{X}) + \mathbf{U}_{k,a}^{(l,m)}(\mathbf{X}) \qquad \text{(C-29)}$$

The indices $l = 2$ is positive (out from the sun along r); $l = 1$ is negative (in toward the sun along r); $m = 2$ is positive (toward the North from the ecliptic plane); and $m = 1$ is negative (toward the South from the ecliptic plane).

The radial component is

$$U_{k,r}^{(l,m)}(\mathbf{X}) = (2l-3)\frac{1}{r(\mathbf{X})}\left(\frac{a_k^2 e_k^2 - [a_k - r(\mathbf{X})]^2}{a_k}\right)^{1/2}(\Gamma M_\odot)^{1/2} \qquad \text{(C-30)}$$

and the azimuthal component is

$$U_{k,a}^{(l,m)}(\mathbf{X}) = \frac{1}{r(\mathbf{X})}(p_k)^{1/2}(\Gamma M_\odot)^{1/2} \qquad \text{(C-31)}$$

The angle α, Eq. (C-24), for the kth meteoroid stream passing through the spacecraft position is

$$\alpha_k^{(l,m)}(\mathbf{X}) = (2m-3)\cos^{-1}\left[\frac{\cos i_k}{\cos\lambda(\mathbf{X})}\right] \qquad \text{(C-32)}$$

Thus, one can write

$$\begin{aligned} U_{k,1}^{(l,m)}(\mathbf{X}) &= U_{k,r}^{(l,m)}(\mathbf{X}) \\ U_{k,2}^{(l,m)}(\mathbf{X}) &= U_{k,a}^{(l,m)}(\mathbf{X})\cos[\alpha_k^{(l,m)}(\mathbf{X})] \\ U_{k,3}^{(l,m)}(\mathbf{X}) &= U_{k,a}^{(l,m)}(\mathbf{X})\sin[\alpha_k^{(l,m)}(\mathbf{X})] \end{aligned} \qquad \text{(C-33)}$$

There are meteoroids with all possible sign combinations. Each sign is equally likely.

The meteoroid velocity, $U_k^{(l,m)}(\mathbf{X})$, of the kth stream is defined only for $|\lambda(\mathbf{X})| \leq i_k$ and $|a_k - r(\mathbf{X})| \leq a_k e_k$.

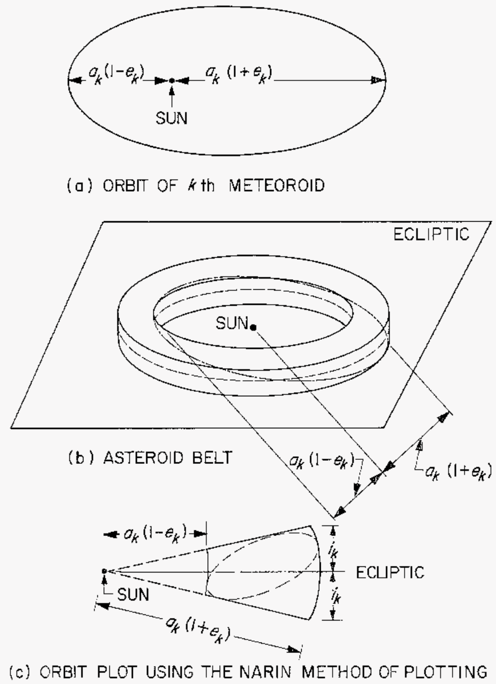


Fig. C-11. Asteroid and meteoroid orbits

V. Meteoroid Density Distribution

From the averaging over the longitude of ascending node Ω, it is clear that the meteoroid density distribution $\sigma_k(\mathbf{X})$ is independent of $\Lambda(\mathbf{X})$. There has also been averaging over the mean anomaly M and the argument of perihelion ω. With this averaging over Ω, M and ω, the density distribution is spread over a finite volume of space as shown in Fig. C-11b.

A. Distribution Over Orbital Parameters

The distribution with respect to M, ω, Ω, each being taken from 0 to 2π, may be represented by a cube of edge 2π as shown in Fig. C-12, in M-ω-Ω space, of volume Vol_1.

The density, by assumption, is independent of asteroid belt longitude. The meteoroid density in the cube is thus a constant. Since the integral of the density over the M-ω-Ω space is w_k, the probability of finding the meteoroid, the density itself must be $w_k/(2\pi)^3$ in the cube, and

$$\int\int\int \frac{w}{(2\pi)^3}\,dM\,d\omega\,d\Omega = \frac{w}{(2\pi)^3}(2\pi)^3 = w$$

B. Distribution Over Spherical Space Coordinates

The spherical coordinates (r, λ, Λ) are represented in rectangular coordinates in Fig. C-13. The regime is a rectangular solid in (r, λ, Λ) space of volume Vol_2. There is a

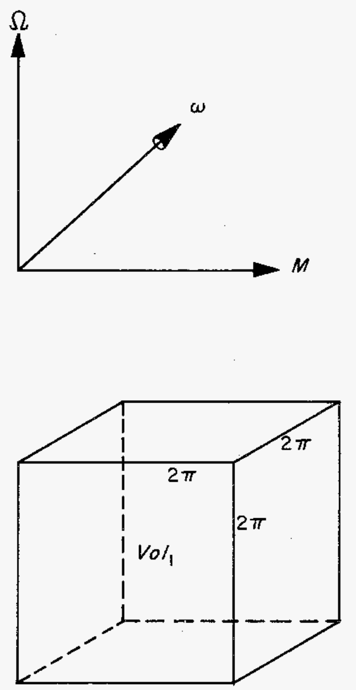


Fig. C-12. Density distribution over M, ω, Ω

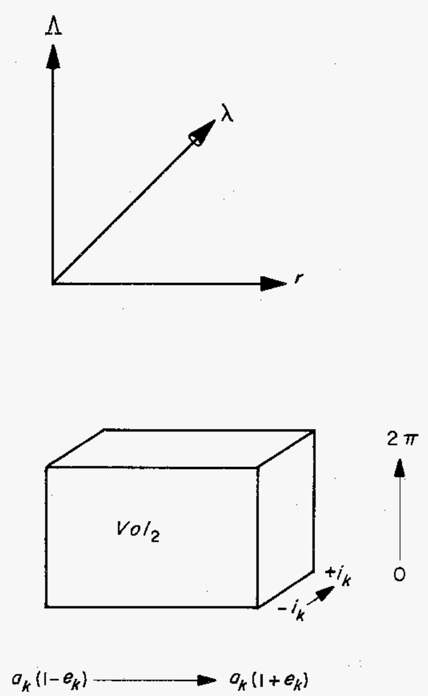


Fig. C-13. Spherical space coordinates taken as rectangular coordinates

non-uniform meteoroid distribution in this space. Here

$$\sigma_k(\mathbf{X}) = \sigma_k[r(\mathbf{X}), \lambda(\mathbf{X}), \Lambda(\mathbf{X})] = \sigma_k(r, \lambda, \Lambda)\, r^2 \cos\lambda$$

and

$$\iiint \sigma_k(\mathbf{X})\, d^3\mathbf{X} = \iiint \sigma_k(r, \lambda, \Lambda)\, dr\,(r d\lambda)\,(r \cos\lambda\, d\Lambda) \tag{C-34}$$

Now, one has

$$dM\, d\omega\, d\Omega = 4\,|J|\, dr\, d\lambda\, d\Lambda \tag{C-35}$$

and

$$\iiint_{Vol_1} \frac{w_k}{(2\pi)^3}\, dM\, d\omega\, d\Omega = \iiint_{Vol_2} \frac{w_k}{(2\pi)^3} 4\,|J|\, dr\, d\lambda\, d\Lambda$$

$$= \iiint_{Vol_2} \sigma_k(r, \lambda, \Lambda)\, r^2 \cos\lambda\, dr\, d\lambda\, d\Lambda$$

$$= \iiint \sigma_k(\mathbf{X})\, d^3\mathbf{X} \tag{C-36}$$

The mapping here is not 1 to 1 but 4 to 1, owing to the four meteoroid streams at the spacecraft position—thus, the factor of 4. Here J is the Jacobian

$$J = \begin{vmatrix} \frac{\partial M}{\partial r} & \frac{\partial M}{\partial \lambda} & \frac{\partial M}{\partial \Lambda} \\ \frac{\partial \omega}{\partial r} & \frac{\partial \omega}{\partial \lambda} & \frac{\partial \omega}{\partial \Lambda} \\ \frac{\partial \Omega}{\partial r} & \frac{\partial \Omega}{\partial \lambda} & \frac{\partial \Omega}{\partial \Lambda} \end{vmatrix} \tag{C-37}$$

The meteoroid density in r, λ, Λ space is given by

$$\sigma_k(r, \lambda, \Lambda) = \frac{4\, w_k}{(2\pi)^3} |J| \frac{1}{r^2 \cos\lambda} \tag{C-38}$$

Now

$$M = E - e_k \sin E \tag{C-39}$$

from Ref. 14, p. 335, and

$$E = \cos^{-1}\left(\frac{a_k - r}{a_k e_k}\right), \text{ and, } \cos E = \frac{a_k - r}{a_k e_k} \tag{C-40}$$

from Ref. 14, p. 333. Thus,

$$\frac{\partial E}{\partial r} = \frac{\dfrac{1}{a_k e_k}}{\left[1 - \left(\dfrac{a_k - r}{a_k e_k}\right)^2\right]^{1/2}} = \frac{1}{[a_k^2 e_k^2 - (a_k - r)^2]^{1/2}}$$

and

$$\frac{\partial M}{\partial r} = \frac{\partial E}{\partial r}(1 - e_k \cos E)$$

$$= \frac{1}{[a_k^2 e_k^2 - (a_k - r)^2]^{1/2}}\left[1 - \left(\frac{a_k - r}{a_k}\right)\right]$$

or

$$\frac{\partial M}{\partial r} = \frac{r}{a_k\,[a_k^2 e_k^2 - (a_k - r)^2]^{1/2}} \tag{C-41}$$

Also, from Eq. (C-39)

$$\frac{\partial M}{\partial \lambda} = 0 \tag{C-42}$$

$$\frac{\partial M}{\partial \Lambda} = 0 \tag{C-43}$$

Now, from Eq. (C-18),

$$\sin(\eta + \omega) = \frac{\sin \lambda}{\sin i_k}$$

so that

$$\omega = \sin^{-1}\left(\frac{\sin \lambda}{\sin i_k}\right) - \eta$$

and

$$\eta = \cos^{-1}\left(\frac{p_k - r}{r e_k}\right)$$

from Ref. 14, p. 34, and

$$\frac{\partial \omega}{\partial r} = -\frac{\partial \eta}{\partial r} = \frac{\left[\dfrac{r e_k(-1) - (p_k - r)\,e_k}{r^2 e_k^2}\right]}{\left[1 - \left(\dfrac{p_k - r}{r e_k}\right)^2\right]^{1/2}}$$

$$= \frac{-\dfrac{p_k}{r}}{[r^2 e_k^2 - (p_k - r)^2]^{1/2}} \tag{C-44}$$

Also,

$$\frac{\partial \omega}{\partial \lambda} = \frac{\left(\dfrac{\cos \lambda}{\sin i_k}\right)}{\left[1 - \left(\dfrac{\sin \lambda}{\sin i_k}\right)^2\right]^{1/2}} = \frac{\cos \lambda}{(\sin^2 i_k - \sin^2 \lambda)^{1/2}} \tag{C-45}$$

and

$$\frac{\partial \omega}{\partial \Lambda} = 0 \tag{C-46}$$

From Eq. (C-22) one gets

$$\sin(\Lambda - \Omega) = \frac{\tan \lambda}{\tan i_k}$$

so that

$$\Omega = \Lambda - \sin^{-1}\left(\frac{\tan \lambda}{\tan i_k}\right)$$

Thus,

$$\frac{\partial \Omega}{\partial r} = 0 \tag{C-47}$$

$$\frac{\partial \Omega}{\partial \lambda} = \frac{-\dfrac{\sec^2 \lambda}{\tan i_k}}{\left[1 - \left(\dfrac{\tan \lambda}{\tan i_k}\right)^2\right]^{1/2}} = \frac{-\sec^2 \lambda}{(\tan^2 i_k - \tan^2 \lambda)^{1/2}} \tag{C-48}$$

$$\frac{\partial \Omega}{\partial \Lambda} = 1 \tag{C-49}$$

Consequently, Eq. (C-37) becomes

$$J = \begin{vmatrix} \dfrac{r/a_k}{[a_k^2 e_k^2 - (a_k - r)^2]^{1/2}} & 0 & 0 \\ \dfrac{-p_k/r}{[r^2 e_k^2 - (p_k - r)^2]^{1/2}} & \dfrac{\cos \lambda}{(\sin^2 i_k - \sin^2 \lambda)^{1/2}} & 0 \\ 0 & \dfrac{-\sec^2 \lambda}{(\tan^2 i_k - \tan^2 \lambda)^{1/2}} & 1 \end{vmatrix} = \left\{\frac{r/a_k}{[a_k^2 e_k^2 - (a_k - r)^2]^{1/2}}\right\}\left[\frac{\cos \lambda}{(\sin^2 i_k - \sin^2 \lambda)^{1/2}}\right] \tag{C-50}$$

Now, if one defines σ_k^* and ρ_k^* as follows,

$$\sigma_k^*(r) = E_k(r) - e_k \sin E_k(r), E_k(r) = \cos^{-1}\left(\frac{a_k - r}{a_k e_k}\right) \tag{C-51}$$

which is similar in form to M above in Eq. (C-39), and

$$\rho_k^*(\lambda) = \sin^{-1}\left(\frac{\sin\lambda}{\sin i_k}\right) \tag{C-52}$$

which is equal to $\omega + \eta$ above, then

$$\frac{d\sigma_k^*(r)}{dr} = \frac{r}{a_k[a_k^2 e_k^2 - (a_k - r)^2]^{1/2}} \tag{C-53}$$

which is valid from

$$a_k(1 - e_k) \text{ to } a_k(1 + e_k)$$

and

$$\frac{d\rho_k^*(\lambda)}{d\lambda} = \frac{\cos\lambda}{(\sin^2 i_k - \sin^2\lambda)^{1/2}} \tag{C-54}$$

which is valid from $-i_k$ to $+i_k$.

Thus, Eq. (C-38) above becomes

$$\sigma_k(r,\lambda,\Lambda) = \frac{4w_k \dfrac{d\sigma_k^*(r)}{dr}\dfrac{d\rho_k^*(\lambda)}{d\lambda}}{(2\pi)^3 r^2 \cos\lambda} = \frac{4w_k}{(2\pi)^3 r^2\cos\lambda}\left\{\frac{r}{a_k[a_k^2e_k^2 - (a_k - r)^2]^{1/2}}\right\}\left[\frac{\cos\lambda}{(\sin^2 i_k - \sin^2\lambda)^{1/2}}\right] \tag{C-55}$$

Thus, the meteoroid density $\sigma_k(r,\lambda,\Lambda) = \sigma_k(\mathbf{X})$ becomes infinite at $r = a_k(1 \pm e_k)$ and $\lambda = \pm i_k$. The problem of these singularities is handled by using smeared out versions of

$$\frac{d\rho_k^*(\lambda)}{d\lambda}$$

as shown in Fig. C-14, and similarly for $\dfrac{d\sigma_k^*(r)}{dr}$.

One can write

$$\frac{d\rho_k^*(\lambda)}{d\lambda} = \frac{\Delta\rho_k^*(\lambda)}{\Delta\lambda} = \frac{\rho_k^*(\lambda + \varepsilon_\lambda) - \rho_k^*(\lambda - \varepsilon_\lambda)}{2\varepsilon_\lambda} \tag{C-56}$$

A similar effect occurs for $\dfrac{d\sigma_k^*(r)}{dr}$.

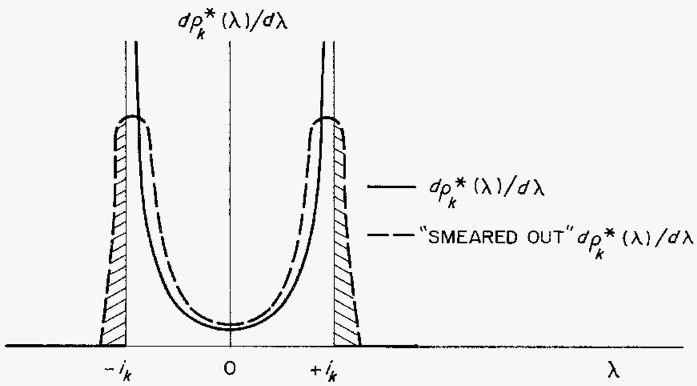


Fig. C-14. Plot of $\dfrac{d\rho_k^*(\lambda)}{d\lambda}$ and "smeared out" $\dfrac{d\rho_k^*(\lambda)}{d\lambda}$

The definitions of $\sigma_k^*(r)$ and $\rho_k^*(\lambda)$ are extended outside of their normal ranges as follows:

$$\sigma_k^*(r) = E_k(r) - e_k \sin E_k(r)$$

$$E_k(r) = \begin{cases} \pi & \text{for } r \geqq a_k(1+e_k) \\ \cos^{-1}\left(\dfrac{a_k - r}{a_k e_k}\right) & \text{for } a_k(1-e_k) \leqq r \leqq a_k(1+e_k) \\ 0 & \text{for } r \leqq a_k(1-e_k) \end{cases}$$

$$\frac{d\sigma_k^*(r)}{dr} = \frac{\dfrac{r}{a_k}}{[a_k^2 e_k^2 - (a_k - r)^2]^{1/2}} H\,[a_k^2 e_k^2 - (a_k - r)^2] \qquad \text{(C-57)}$$

and

$$\rho_k^*(\lambda) = \begin{cases} +\dfrac{\pi}{2} & \text{for } \lambda \geqq +i_k \\ \sin^{-1}\left(\dfrac{\sin\lambda}{\sin i_k}\right) & \text{for } -i_k \leqq \lambda \leqq +i_k \\ -\dfrac{\pi}{2} & \text{for } \lambda \leqq -i_k \end{cases}$$

and

$$\frac{d\rho_k^*(\lambda)}{d\lambda} = \frac{\cos\lambda}{(\sin^2 i_k - \sin^2\lambda)^{1/2}} H\,(\sin^2 i_k - \sin^2\lambda) \qquad \text{(C-58)}$$

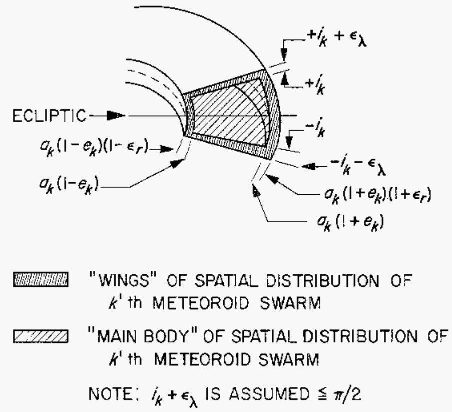


Fig. C-15. "Extended body" including "main body" and "wings"

The "extended body" is shown in Fig. C-15. A "smeared-out" value of σ_k, called $< \sigma_k >$, equal to σ_k averaged over the volume $r^2 \cos\lambda \, dr \, d\lambda \, d\Lambda$ was obtained as follows:

$$< \sigma_k > (r, \lambda, \Lambda) = \frac{\displaystyle\int_{r(1-\varepsilon_r)}^{r(1+\varepsilon_r)} \int_{\lambda-\varepsilon_\lambda}^{\lambda+\varepsilon_\lambda} \int_{\Lambda-\varepsilon_\Lambda}^{\Lambda+\varepsilon_\Lambda} \sigma_k(r', \lambda', \Lambda') \, r'^2 \cos\lambda' \, dr' \, d\lambda' \, d\Lambda'}{\displaystyle\int_{r(1-\varepsilon_r)}^{r(1+\varepsilon_r)} \int_{\lambda-\varepsilon_\lambda}^{\lambda+\varepsilon_\lambda} \int_{\Lambda-\varepsilon_\Lambda}^{\Lambda+\varepsilon_\Lambda} r'^2 \cos\lambda' \, dr' \, d\lambda' \, d\Lambda'} \qquad \text{(C-59)}$$

$$< \sigma_k > = \frac{\displaystyle\int_{r(1-\varepsilon_r)}^{r(1+\varepsilon_r)} \int_{\lambda-\varepsilon_\lambda}^{\lambda+\varepsilon_\lambda} \int_{\Lambda-\varepsilon_\Lambda}^{\Lambda+\varepsilon_\Lambda} \frac{w_k}{2\pi^3} \frac{d\rho_k^*(\lambda')}{d\lambda'} \frac{d\sigma_k^*(r')}{dr'} \frac{r'^2\cos\lambda'}{r'^2\cos\lambda'} dr' \, d\lambda' \, d\Lambda'}{\displaystyle\int_{r(1-\varepsilon_r)}^{r(1+\varepsilon_r)} r'^2 \, dr' \int_{\lambda-\varepsilon_\lambda}^{\lambda+\varepsilon_\lambda} \cos\lambda' \, d\lambda' \int_{\Lambda-\varepsilon_\Lambda}^{\Lambda+\varepsilon_\Lambda} d\Lambda'}$$

$$< \sigma_k > = \frac{\dfrac{w_k}{2\pi^3} \displaystyle\int_{r(1-\varepsilon_r)}^{r(1+\varepsilon_r)} \frac{d\sigma_k^*(r')}{dr'} dr' \int_{\lambda-\varepsilon_\lambda}^{\lambda+\varepsilon_\lambda} \frac{d\rho_k^*(\lambda')}{d\lambda'} d\lambda' \int_{\Lambda-\varepsilon_\Lambda}^{\Lambda+\varepsilon_\Lambda} d\Lambda'}{\left| \begin{matrix} r(1+\varepsilon_r) \\ \\ r(1-\varepsilon_r) \end{matrix} \right. \dfrac{1}{3} r'^3 \left| \begin{matrix} \lambda+\varepsilon_\lambda \\ \\ \lambda-\varepsilon_\lambda \end{matrix} \right. \sin\lambda' \displaystyle\int_{\Lambda-\varepsilon_\Lambda}^{\Lambda+\varepsilon_\Lambda} d\Lambda'}$$

$$< \sigma_k > = \frac{w_k}{2\pi^3} \cdot \frac{[\sigma_k^*(r(1+\varepsilon_r)) - \sigma_k^*(r(1-\varepsilon_r))] \, [\rho_k^*(\lambda+\varepsilon_\lambda) - \rho_k^*(\lambda-\varepsilon_\lambda)]}{r^3 \dfrac{1}{3} [(1+\varepsilon_r)^3 - (1-\varepsilon_r)^3] \, [\sin(\lambda+\varepsilon_\lambda) - \sin(\lambda-\varepsilon_\lambda)]} \qquad \text{(C-60)}$$

Now,

$$\frac{1}{3}[(1+\varepsilon_r)^3 - (1-\varepsilon_r)^3] = \frac{1}{3}[(1 + 3\varepsilon_r + 3\varepsilon_r^2 + \varepsilon_r^3) - (1 - 3\varepsilon_r + 3\varepsilon_r^2 - \varepsilon_r^3)] = \frac{2}{3}(3\varepsilon_r + \varepsilon_r^3) = \frac{2}{3}\varepsilon_r(3 + \varepsilon_r^2),$$

and

$$\sin(\lambda + \varepsilon_\lambda) - \sin(\lambda - \varepsilon_\lambda) = (\sin\lambda\cos\varepsilon_\lambda + \cos\lambda\sin\varepsilon_\lambda) - (\sin\lambda\cos\varepsilon_\lambda - \cos\lambda\sin\varepsilon_\lambda) = 2\cos\lambda\sin\varepsilon_\lambda.$$

Thus, one has

$$< \sigma_k > (r, \lambda, \Lambda) = \frac{3w_k}{(2\pi r)^3} \frac{[\sigma_k^*(r(1+\varepsilon_r)) - \sigma_k^*(r(1-\varepsilon_r))]\ [\rho_k^*(\lambda+\varepsilon_\lambda) - \rho_k^*(\lambda - \varepsilon_\lambda)]}{\cos\lambda\ \varepsilon_r (3 + \varepsilon_r^2)\sin\varepsilon_\lambda} \tag{C-61}$$

Now $\sigma_k(\mathbf{X}) \neq 0$ within the "main body," defined by $|\lambda| \leqq i_k$, and $|r/a_k - 1| \leqq e_k$, but $\sigma_k = 0$ outside the "main body" (see Fig. C-15). Similarly one has $< \sigma_k > \neq 0$ within the "extended body," (the "main body" and the "wings"), defined by $|\lambda| \leqq i_k + \varepsilon_\lambda$, and

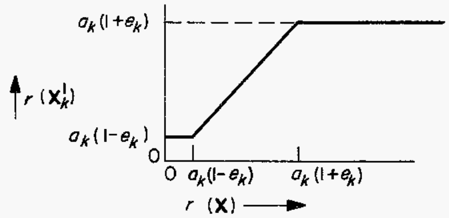
$$(1 - e_k)(1 - \varepsilon_r) \leqq \frac{r}{a_k} \leqq (1 + e_k)(1 + \varepsilon_r)$$

or

$$\left| \frac{r}{a_k} - (1 + e_k \varepsilon_r) \right| \leqq e_k + \varepsilon_r,$$

and $<\sigma_k> = 0$ outside the extended body.

If $\sigma_k(\mathbf{X})$ were used, $\mathbf{U}_k^{(l,m)}(\mathbf{X})$ would only have to be defined where $\sigma_k(\mathbf{X}) \neq 0$, that is, in the "main body." For the use of $<\sigma_k>(\mathbf{X})$, however, it is necessary to extend the definitions of $\mathbf{U}_k^{(l,m)}(\mathbf{X})$ to the entire region where $<\sigma_k>(\mathbf{X}) \neq 0$, that is, the extended body. To achieve this, $\mathbf{U}_k^{(l,m)}(\mathbf{X})$ is replaced everywhere by $\mathbf{U}_k^{(l,m)}(\mathbf{X}_k')$



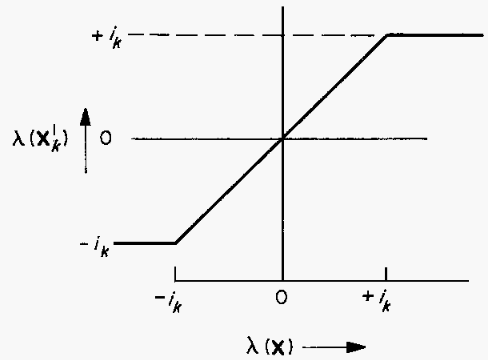


Fig. C-16. Plot of $r(\mathbf{X}_k')$ vs $r(\mathbf{X})$ and $\lambda(\mathbf{X}_k')$ vs $\lambda(\mathbf{X})$

where

$$r(\mathbf{X}_k') = \begin{cases} a_k(1+e_k) & \\ r(\mathbf{X}) & \text{for} \\ a_k(1-e_k) & \end{cases} \quad \begin{array}{l} r(\mathbf{X}) \geqq a_k(1+e_k) \\ a_k(1-e_k) \leqq r(\mathbf{X}) \leqq a_k(1+e_k) \\ r(\mathbf{X}) \leqq a_k(1-e_k) \end{array} \tag{C-62}$$

$$\lambda(\mathbf{X}_k') = \begin{cases} +i_k & \\ \lambda(\mathbf{X}) & \text{for} \\ -i_k & \end{cases} \quad \begin{array}{l} \lambda(\mathbf{X}) \geqq +i_k \\ -i_k \leqq \lambda(\mathbf{X}) \leqq +i_k, \text{and } \Lambda(\mathbf{X}_k') = \Lambda(\mathbf{X}). \\ \lambda(\mathbf{X}) \leqq -i_k \end{array}$$

The quantities $r(\mathbf{X}')$ and $\lambda(\mathbf{X}')$ are plotted in Fig. C-16.

The quantities $r(\mathbf{X}_k')$, $\lambda(\mathbf{X}_k')$ and $\Lambda(\mathbf{X}_k')$ may be represented as follows:

$$\begin{aligned} r(\mathbf{X}_k') &= \max[a_k(1-e_k),\quad \min\{a_k(1+e_k),\quad r(\mathbf{X})\}] \\ \lambda(\mathbf{X}_k') &= \max[-i_k,\quad \min\{+i_k, \lambda(\mathbf{X})\}] \\ \Lambda(\mathbf{X}_k') &= \Lambda(\mathbf{X}) \end{aligned} \tag{C-63}$$

Figure C-16 shows that in the main body

$$r(\mathbf{X}'_k) = r(\mathbf{X})$$

$$\lambda(\mathbf{X}'_k) = \lambda(\mathbf{X}_k).$$

Thus, in the "wings" the velocity pattern is taken as that at the nearest boundary.

In the computer program

$$\varepsilon_r = \varepsilon_\lambda = 0.02$$

Appendix D

The Value of β

There are many values of β in the literature but those that apply to the asteroid belt meteoroids are almost all clustered closely around $\beta = 2/3$. Consider the following four functions:

(1) The cumulative mass distribution function $N_1(m)$ = the number of meteoroids of mass greater than m

(2) $N_2(r)$ = the number of meteoroids of radius greater than r

(3) $N_1'(m) = \dfrac{dN_1(m)}{dm}$ = the mass frequency distribution function

(4) $N_2'(r) = \dfrac{dN_2(r)}{dr}$ = the radius frequency distribution function

If $N_1 \propto m^{-\beta}$, then $N_2 \propto r^{-3\beta}$ (since $m \propto r^3$)
$N_1' \propto m^{-(\beta+1)}$ and $N_2' \propto r^{-(3\beta+1)}$.

Piotrowski (Ref. 16) gave $dN_2 \propto r^{-3}\,dr$, or $N_2' = (dN_2/dr) \propto r^{-3}$, and $3\beta + 1 = 3$, therefore

$$\beta = \frac{3-1}{3} = \frac{2}{3}.$$

In the computer program in this report, the value $3\beta = 1.9$ is used. This value, $3\beta = 1.9$ comes from Anders (Ref. 10). Anders states that the value of β does not depend on position in the asteroid belt.[1]

Another approach indicating $3\beta = 1.9$ is as follows:

Let $N_3(G, a)$ be the number of asteroids with absolute magnitude less than or equal to G and semi-major axis greater than or equal to a. Let

$$N_3'(G, a) = \frac{\partial N_3(G, a)}{\partial a} \qquad \text{(D-1)}$$

Kuiper (Ref. 13) takes

$$\log_{10} N_3'(G, a) = C(a) + b(a)\,G, \qquad \text{(D-2)}$$

where C and b are functions of a. Anders (Ref. 10) states that b is a constant. Let $N_4(p_0)$ be the number of asteroids with mean opposition magnitude $\leqq p_0$ and semi-major axis > 1 AU. Let a be in AU. Only asteroids with $a > 1$ AU are considered because p_0 (Eq. D-3) becomes singular for $a = 1$. (*Note: in the present report the asteroid belt model* contains no meteoroids with $a \leqq 1$ AU) Kuiper, Ref. 13, p. 318, gives p_0 = mean asteroid magnitude at opposition for $a \geqq 1$ AU as

$$p_0 = G + 5 \log_{10} a\,(a-1) \qquad \text{(D-3)}$$

or

$$G = G(p_0, a) = p_0 - 5 \log_{10} a\,(a-1) \qquad \text{(D-4)}$$

From Eqs. (D-2 and -4) and Anders (Ref. 10) one can write

$$\log_{10} N_3'[G(p_0, a), a] = C(a) + bG$$

so that

$$N_3'[G(p_0, a), a] = 10^{C(a)+bG} \qquad \text{(D-5)}$$

or

$$N_3'[G(p_0, a)\, a] = 10^{C(a)+b[p_0 - 5\log_{10} a\,(a-1)]} \qquad \text{(D-6)}$$

Now, define $N_4(p_0)$ as

$$N_4(p_0) = \int_1^\infty N_3'[G(p_0, a), a]\, da$$

$$= \int_1^\infty 10^{C(a)+b[p_0 - 5\log_{10} a\,(a-1)]}\, da$$

so that

$$N_4(p_0) = \left[\int_1^\infty 10^{C(a) - 5b\log_{10} a\,(a-1)}\, da\right] 10^{bp_0} \qquad \text{(D-7)}$$

Thus

$$\log_{10}[N_4(p_0)] = d + bp_0 \qquad \text{(D-8)}$$

[1]Anders obtained this value from "recent unpublished work by C. J. and I. Van Houten, based on data for 2179 asteroids."

where

$$d = \log \int_1^\infty 10^{C(a) - 5b \log_{10} a(a-1)}\, da \qquad \text{(D-9)}$$

Define $N_3(G)$ to be the total number of meteoroids with absolute magnitude $\leq G$. Thus,

$$N_3(G) = \int_0^\infty N_3'(G, a)\, da = \int_0^\infty 10^{C(a) + bG}\, da$$

$$= \left[\int_0^\infty 10^{C(a)}\, da\right] 10^{bG} \qquad \text{(D-10)}$$

Thus, Eq. (D-10) can be written

$$\log_{10} N_3(G) = d^* + bG,$$

where

$$d^* = \log \int_0^\infty 10^{C(a)}\, da \qquad \text{(D-11)}$$

and

$$N_3(G) = 10^{d^* + bG} \qquad \text{(D-12)}$$

Kiang, Ref. 17, gives

$$\log_{10} N_4(p_0) = -2.63 + 0.375\, p_0 \qquad \text{(D-13)}$$

Thus, comparing Eqs. (D-8 and -13), one gets

$$b = 0.375 \qquad \text{(D-14)}$$

Ref. 18, p. 153 gives

$$\log_{10} r = 2.95 - \frac{1}{2}\log_{10}(0.16) - 0.2G = 3.35 - \frac{1}{5}G \qquad \text{(D-15)}$$

where r is the radius of the asteroid in km. Now in this report, the minimum asteroid absolute magnitude considered (for the reference asteroid of mass m_0, and radius r_0) is $G_0 = 13.6$. With this value of G_0, Eq. (D-15) gives

$$\log_{10} r_0 = 3.35 - \frac{1}{5}(13.6) = 3.35 - 2.72 = 0.63$$

so that

$$r_0 = 4.3\,\text{km} \qquad \text{(D-16)}$$

From Eq. (D-15) one can write

$$G = G(r) = 16.75 - 5\log_{10} r \qquad \text{(D-17)}$$

Now, from Eqs. (D-12) and (D-17)

$$N_2(r) = N_3[G(r)] = 10^{d^* + bG(r)} = 10^{d^* + b(16.75 - 5\log_{10} r)}$$

or

$$N_2(r) = (10^{d^* + 16.75b})\,[10^{\log_{10}(r^{-5b})}] = 10^{d^* + 16.75b}\, r^{-5b} \qquad \text{(D-18)}$$

But

$$N_2(r) \propto r^{-3\beta}$$

so that

$$3\beta = 5b$$

and using Eq. (D-14)

$$3\beta = 5(0.375) = 1.875 \qquad \text{(D-19)}$$

in agreement with previous results of

$$3\beta = 1.9$$

Hartmann, Ref. 19, from an analysis of lunar cratering, states that β is approximately 0.7 to 0.8. The Hartmann value is an example of the extremes in the scattering of β values about $\beta = 2/3$.

Appendix E

Analytic Model Output

A computer was used to carry out the calculations. The reason the computer was needed was because the arithmetic calculations involved in evaluating $P_I(S)$ were too extensive to be done without a computer. However, from a single computer run one can obtain information on a whole family of related spacecraft, rather than on only a single spacecraft. These spacecraft must have the same trajectories, $\mathbf{X}(T)$ and $\mathbf{V}(T)$, the same attitude $\mathcal{M}(T)$ as a function of time, and the same set of outwardly drawn unit vectors $\mathbf{n}_j$, normal to the polyhedral spacecraft surface. For the jth surface the area is A_j and the spacecraft surface thickness t_j, where j ranges from 1 to N_F.

An analytic expression is derived below for $P_I(S)$ which applies to all of the spacecraft in such a family. The computer supplies the values of certain coefficients which appear in this analytic expression. One first selects one of the spacecraft in this family as the "standard" or reference spacecraft. All of the properties of the reference are designated by an asterisk, for example, the total area is A_s^*. The parameters α_j', τ_j' and α_s' are defined as follows:

$$\alpha_j' = \frac{A_j}{A_j^*} \tag{E-1}$$

$$\tau_j' = \frac{t_j}{t_j^*} \tag{E-2}$$

$$\alpha_s' = \frac{A_s}{A_s^*} \tag{E-3}$$

From Eq. (B-66) one sees that

$$F_j \propto t_j^{-3\beta}$$

and

$$F_j(T)\, t_j^{3\beta} = F_j^*(T)\,(t_j^*)^{3\beta},$$

so that

$$F_j(T) = F_j^*(T)\left(\frac{t_j^*}{t_j}\right)^{3\beta} = F_j^*(T)\left(\frac{t_j}{t_j^*}\right)^{-3\beta} \tag{E-4}$$

and from Eq. (E-2),

$$F_j(T) = F_j^*(T)\,(\tau_j')^{-3\beta}. \tag{E-5}$$

One can define

$$f_j = \int_{T_0}^{T_f} F_j(T)\, dT \qquad \text{and} \qquad f_j^* = \int_{T_0}^{T_f} F_j^*(T)\, dT \tag{E-6}$$

as the effective meteoroid flux integral, or meteoroids/m². Thus, from Eqs. (E-5 and -6)

$$f_j = f_j^*\,(\tau_j')^{-3\beta}$$

and from Eqs. (B-47 and -53), if $P_s(T_0) = 1$

$$P_I(S) = \exp\left(-\int_{T_0}^{T_f} \pi_I(T)\, dT\right) = \exp\left(-\int_{T_0}^{T_f} \sum_j \,[F_j(T)]\, A_j\, dT\right)$$

or

$$P_I(S) = \exp\left(-\sum_j A_j \int_{T_0}^{T_f} F_j(T)\, dT\right)$$

and, from Eq. (E-6)

$$P_I(S) = \exp\left(-\sum_j A_j f_j\right). \tag{E-8}$$

Now, from $A_j = A_j^* \alpha_j'$, and Eq. (E-7), one can write

$$P_I(S) = \exp\left[-\sum_{j=1}^{N_F} A_j^* f_j^* \alpha_j' (\tau_j')^{-3\beta}\right]. \tag{E-9}$$

What the computer calculates is f_j^* using Eqs. (E-6 and B-66), with * denoting the standard spacecraft, and with $F_j^*(T)$, $f_j^*(T)$ and t_j^* in place of $F_j(T)$, $f_j(T)$ and t_j. One particularly simple form for the A_j terms in the family is obtained by assuming that the spacecraft are all of the same shape but have different sizes. Let l be a typical length parameter associated with this shape. From Eqs. (E-1 and -3) one can write

$$\alpha'_j = \alpha'_s = \left(\frac{l}{l^*}\right)^2 \tag{E-10}$$

Now, define

$$\alpha_j = \frac{A_j}{l^2} = \frac{A_j^*}{l^{*2}} = \alpha_j^* \tag{E-11}$$

$$\alpha_s = \frac{A_s}{l^2} = \frac{A_s^*}{l^{*2}} = \sum_{j=1}^{N_F} \alpha_j^* = \alpha_s^* \tag{E-12}$$

Now, from Eqs. (E-1 and -9) one gets

$$P_I(S) = \exp\left[-\sum_{j=1}^{N_F} A_j^* f_j^* \frac{A_j}{A_j^*} (\tau'_j)^{-3\beta}\right]$$

and from Eq. (E-11) $A_j = \alpha_j l^2$, so that

$$P_I(S) = \exp\left[-l^2 \sum_{j=1}^{N_F} \alpha_j f_j^* (\tau'_j)^{-3\beta}\right] \tag{E-13}$$

It is assumed that all of the spacecraft shielding has the same density, or

$$\rho_j = \rho_s. \tag{E-14}$$

The mass W_j of the shielding on the jth face is given by

$$W_j = \rho_s A_j t_j \tag{E-15}$$

and

$$W_j^* = \rho_s A_j^* t_j^*. \tag{E-16}$$

The total mass W_s of shielding on the spacecraft is given by

$$W_s = \sum_{j=1}^{N} W_j \tag{E-17}$$

$$W_s^* = \sum_{j=1}^{N} W_j^* \tag{E-18}$$

One now defines t, the average thickness of the spacecraft surface by

$$t = \frac{W_s}{\rho_s A_s} \tag{E-19}$$

and

$$t^* = \frac{W_s^*}{\rho_s A_s^*}. \tag{E-20}$$

Now, define

$$\tau_j = \frac{t_j}{t}, \qquad \text{or} \qquad t_j = \tau_j t \tag{E-21}$$

$$\tau_j^* = \frac{t_j^*}{t^*}, \qquad \text{or} \qquad t_j^* = \tau_j^* t^* \tag{E-22}$$

and Eq. (E-2) can then be written as

$$\tau'_j = \frac{t_j}{t_j^*} = \frac{\tau_j t}{\tau_j^* t^*}. \tag{E-23}$$

If one now substitutes Eq. (E-23) into Eq. (E-13) one gets

$$P_I(S) = \exp\left\{-l^2 \sum_{j=1}^{N_F} \left[\alpha_j f_j^* \tau_j^{-3\beta} t^{-3\beta} (t_j^* t^*)^{3\beta}\right]\right\}$$

or

$$P_I(S) = \exp\left\{-l^2 t^{-3\beta} \sum_{j=1}^{N_F} \left[f_j^* (\tau_j^* t^*)^{3\beta}\right] (\alpha_j \tau_j^{-3\beta})\right\} \tag{E-24}$$

and

$$P_I(S) = \exp(-C\, l^2 t^{-3\beta}) \tag{E-25}$$

where

$$C = \sum_{j=1}^{N_F} C_j \alpha_j \tau_j^{-3\beta} \tag{E-26}$$

and

$$C_j = f_j^* (\tau_j^* t^*)^{3\beta} \tag{E-27}$$

One can express $P_I(S)$ in terms of A_s and W_s instead of l and t as follows: Eq. (E-25) becomes, by use of Eq. (E-19) and $l^2 = A_s/\alpha_s$,

$$P_I(S) = \exp\left[-C\left(\frac{A_s}{\alpha_s}\right) W_s^{-3\beta} \rho_s^{3\beta} A_s^{3\beta}\right] = \exp\left[-\left(\frac{C\rho_s^{3\beta}}{\alpha_s}\right) A_s^{1+3\beta} W_s^{-3\beta}\right]$$

$$P_I(S) = \exp(-C' A_s^{1+3\beta} W_s^{-3\beta}) \tag{E-28}$$

where

$$C' = C\frac{\rho_s^{3\beta}}{\alpha_s} = \frac{\rho_s^{3\beta}}{\alpha_s} \sum_{j=1}^{N_F} [f_j^* (\tau_j^* t^*)^{3\beta}] (\alpha_j \tau_j^{-3\beta}) \tag{E-29}$$

from Eq. (E-24). Now, using Eq. (E-20),

$$\rho_s = \frac{W_s^*}{A_s^* t^*},$$

$$C' = \frac{1}{\alpha_s} \frac{(W_s^*)^{3\beta}}{(A_s^*)^{3\beta} (t^*)^{3\beta}} \sum_{j=1}^{N_F} f_j^* \tau_j^{*3\beta} t^{*3\beta} \alpha_j \tau_j^{-3\beta}$$

or

$$C' = \sum_{j=1}^{N_F} \frac{f_j^*}{\alpha_s} \left(\frac{\tau_j^* W_s^*}{A_s^*}\right)^{3\beta} \alpha_j \tau_j^{-3\beta} \tag{E-30}$$

and

$$C' = \sum_{j=1}^{N_F} C_j' \alpha_j \tau_j^{-3\beta} \tag{E-31}$$

where

$$C_j' = \frac{f_j^*}{\alpha_s} \left(\frac{\tau_j^* W_s^*}{A_s^*}\right)^{3\beta} \tag{E-32}$$

One can also express $P_I(S)$ in terms of l and W_s as follows: From Eq. (E-12),

$$A_s = \alpha_s l^2,$$

so that Eq. (E-28) can be written

$$P_I(S) = \exp[-C' \alpha_s^{1+3\beta} l^{2(1+3\beta)} W_s^{-3\beta}]$$

or

$$P_I(S) = \exp[-C'' l^{2(1+3\beta)} W_s^{-3\beta}] \tag{E-33}$$

where

$$C'' = C' \alpha_s^{1+3\beta}$$

and from Eq. (E-29)

$$C'' = \left(\frac{C\rho_s^{3\beta}}{\alpha_s}\right) \alpha_s^{1+3\beta} = C(\alpha_s \rho_s)^{3\beta} \tag{E-34}$$

In addition $P_I(S)$ can be expressed in terms of A_s and t as follows:

from Eq. (E-12)

$$l^2 = \frac{A_s}{\alpha_s}$$

and Eq. (E-25) can be written

$$P_I(S) = \exp\left(-C\frac{A_s}{\alpha_s} t^{-3\beta}\right)$$

or

$$P_I(S) = \exp(-C''' A_s t^{-3\beta}) \tag{E-35}$$

where

$$C''' = \frac{C}{\alpha_s} = C' \rho_s^{-3\beta} \tag{E-36}$$

from Eq. (E-29). Thus $P_I(S)$ can be written, from Eqs. (E-25, -28, -33, and -35)

$$P_I(S) = \exp(-C l^2 t^{-3\beta}) \tag{E-37}$$

$$P_I(S) = \exp(-C''' A_s t^{-3\beta}) \tag{E-38}$$

$$P_I(S) = \exp[-C' A_s^{(1+3\beta)} W_s^{-3\beta}] \tag{E-39}$$

$$P_I(S) = \exp[-C'' l^{2(1+3\beta)} W_s^{-3\beta}] \tag{E-40}$$

where C, C', C'', C''' are defined in Eqs. (E-26, -29, -34, and -36).

Two special forms of $P_I(S)$ are derived as follows:

Case A: In this case the shielding is assumed to be of uniform thickness over the entire surface of the spacecraft. Thus, $t_j = t$ and τ_j, from Eq. (E-21) is

$$\tau_j = \frac{t_j}{t} = \frac{t}{t} = 1 \qquad \text{(E-41)}$$

Thus, Eq. (E-26) becomes

$$C = C_A = \sum_{j=1}^{N_F} C_j \alpha_j \qquad \text{(E-42)}$$

and Eq. (E-25) is therefore

$$P_I(S) = \exp(-C_A l^2 t^{-3\beta}). \qquad \text{(E-43)}$$

Similarly, Eqs. (E-28 and -29) become

$$P_I(S) = \exp(-C'_A A_s^{1+3\beta} W_s^{-3\beta}) \qquad \text{(E-44)}$$

and

$$C'_A = C_A \frac{(\rho_s)^{3\beta}}{\alpha_s} \qquad \text{(E-45)}$$

Also, Eq. (E-40) becomes

$$P_I(S) = \exp[-C''_A l^{2(1+3\beta)} W_s^{-3\beta}] \qquad \text{(E-46)}$$

and from Eq. (E-34)

$$C''_A = C_A (\alpha_s \rho_s)^{3\beta} \qquad \text{(E-47)}$$

Eq. (E-35) becomes

$$P_I(S) = \exp(-C'''_A A_s t^{-3\beta}) \qquad \text{(E-48)}$$

where

$$C'''_A = \frac{C_A}{\alpha_s} \qquad \text{(E-49)}$$

Case B: The shielding is distributed over the faces of the spacecraft so as to maximize $P_I(S)$, for a fixed l and W_s. Maximizing $P_I(S)$ in these circumstances is equivalent to minimizing

$$C = \sum_{j=1}^{N_F} C_j \alpha_j \tau_j^{-3\beta} \qquad \text{(E-50)}$$

from Eq. (E-26), subject to the constraint $W_s = \sum_{j=1}^{N_F} W_j$

or

$$W_s = \rho_s A_s t = \sum_{j=1}^{N_F} \rho_s A_j t_j = \rho_s \sum_{j=1}^{N_F} (\alpha_j l^2)(\tau_j t) = \textit{constant}$$

or

$$\alpha_s = \frac{A_s}{l^2} = \sum_{j=1}^{N} \alpha_j \tau_j = \textit{constant} \qquad \text{(E-51)}$$

Now, using Lagrange's "Method of Multipliers" for constrained maxima and minima, Ref. 20, p. 163, if q is the Lagrange multiplier,

$$\frac{\partial C}{\partial \tau_j} - q \frac{\partial \alpha_s}{\partial \tau_j} = 0$$

and

$$-3\beta \tau_j^{-3\beta-1} C_j \alpha_j - q \alpha_j = 0 \qquad \text{(E-52)}$$

If Eqs. (E-52 and -53) are solved for

$$q, \tau_1, \tau_2, \tau_3, \cdots, \tau_{N_F} \qquad \text{(E-53)}$$

these values of τ_j in Eq. (E-53) are the optimum pattern of τ_j, or the best pattern of shielding for a given meteoroid flux and a fixed l and W_s. Thus, from Eq. (E-52)

$$\frac{-3\beta C_j}{\tau_j^{(1+3\beta)}} = q$$

$$\tau_j = \left(\frac{-3\beta C_j}{q}\right)^{\frac{1}{1+3\beta}} = Q C_j^{\frac{1}{1+3\beta}} \qquad \text{(E-54)}$$

where

$$Q = \left(\frac{-3\beta}{q}\right)^{\frac{1}{1+3\beta}} \qquad \text{(E-55)}$$

Now, combining Eqs. (E-51 and -54) one gets

$$\alpha_s = \sum_{j=1}^{N_F} \alpha_j \tau_j = \sum_{j=1}^{N_F} \alpha_j Q C_j^{\frac{1}{1+3\beta}}$$

so that

$$Q = \frac{\alpha_s}{\sum_{i=1}^{N_F} \alpha_i C_i^{\frac{1}{1+3\beta}}} \qquad \text{(E-56)}$$

and

$$\tau_j = \tau_j^+ = \frac{\alpha_s C_j^{\frac{1}{1+3\beta}}}{\sum_{i=1}^{N_F} \alpha_i C_i^{\frac{1}{1+3\beta}}} = \left(\frac{t_j}{t}\right)^+ \qquad \text{(E-57)}$$

Here τ_j^+ is the optimal pattern of thicknesses τ_j. If one combines Eqs. (E-50 and -57) one gets

$$C_B = \sum_{j=1}^{N_F} C_j \alpha_j \tau_j^{-3\beta} = \sum_{j=1}^{N_F} C_j \alpha_j \left(\frac{\sum_{i=1}^{N_F} \alpha_i C_i^{\frac{1}{1+3\beta}}}{\alpha_s C_j^{\frac{1}{1+3\beta}}}\right)^{3\beta}$$

or

$$C_B = \sum_{i=1}^{N_F} \left(\alpha_i C_i^{\frac{1}{1+3\beta}}\right)^{3\beta} \alpha_s^{-3\beta} \sum_{j=1}^{N_F} \alpha_j C_j^{\left(1-\frac{3\beta}{1+3\beta}\right)}$$

and

$$\left.\begin{aligned} C_B &= \sum_{j=1}^{N_F} \left(\alpha_j C_j^{\frac{1}{1+3\beta}}\right)^{1+3\beta} \alpha_s^{-3\beta} \\ C_B' &= C_B \frac{\rho_s^{3\beta}}{\alpha_s} && \text{from Eq. (E-29)} \\ C_B'' &= C_B (\alpha_s \rho_s)^{3\beta} && \text{from Eq. (E-34)} \\ C_B''' &= \frac{C_B}{\alpha_s} && \text{from Eq. (E-36)} \end{aligned}\right\} \qquad \text{(E-58)}$$

The optimum distribution of shielding is that which gives maximum probability of success for a given weight of shielding and a given spacecraft size.

Appendix F

Example Cases Calculated

Example cases were calculated for typical short and long duration missions to Jupiter. The mission orbits were taken from actual matched conic trajectory data. The flight times were 512 and 904 days, but these have been rounded off here to 500 and 900 days, respectively, as shown in Fig. F-1. The perihelion, of the two mission orbits shown, actually would not be oriented in the same direction. However, since the longitude is of no interest here, they have been rotated to fit on the paper and to better show their relationship. The calculations were made for $3\beta = 1.9$ and $3\beta = 3.0$. Four cases were run as shown in Table F-1.

Table F-1. Four example cases

3β value	500-day mission spacecraft orbit	900-day mission spacecraft orbit
1.9	Case I	Case II
3.0	Case III	Case IV

Case I is for $3\beta = 1.9$ and the 500-day spacecraft mission orbit; Case II for $3\beta = 1.9$ and the 900-day mission orbit, etc. The mission orbit elements used, a, e, i, and ω, are shown in Table F-2.

Table F-2. Mission orbit elements

Orbital elements	500-day mission orbit: May 18, 1974[a] October 12, 1975[b]	900-day mission orbit: May 30, 1974[a] November 19, 1976[b]
a	4.5731 AU	3.0135 AU
e	0.77887	0.66562
i	2.1304 deg	4.3296 deg
ω	177.92 deg	170.09 deg

[a]Launch date
[b]Arrival date

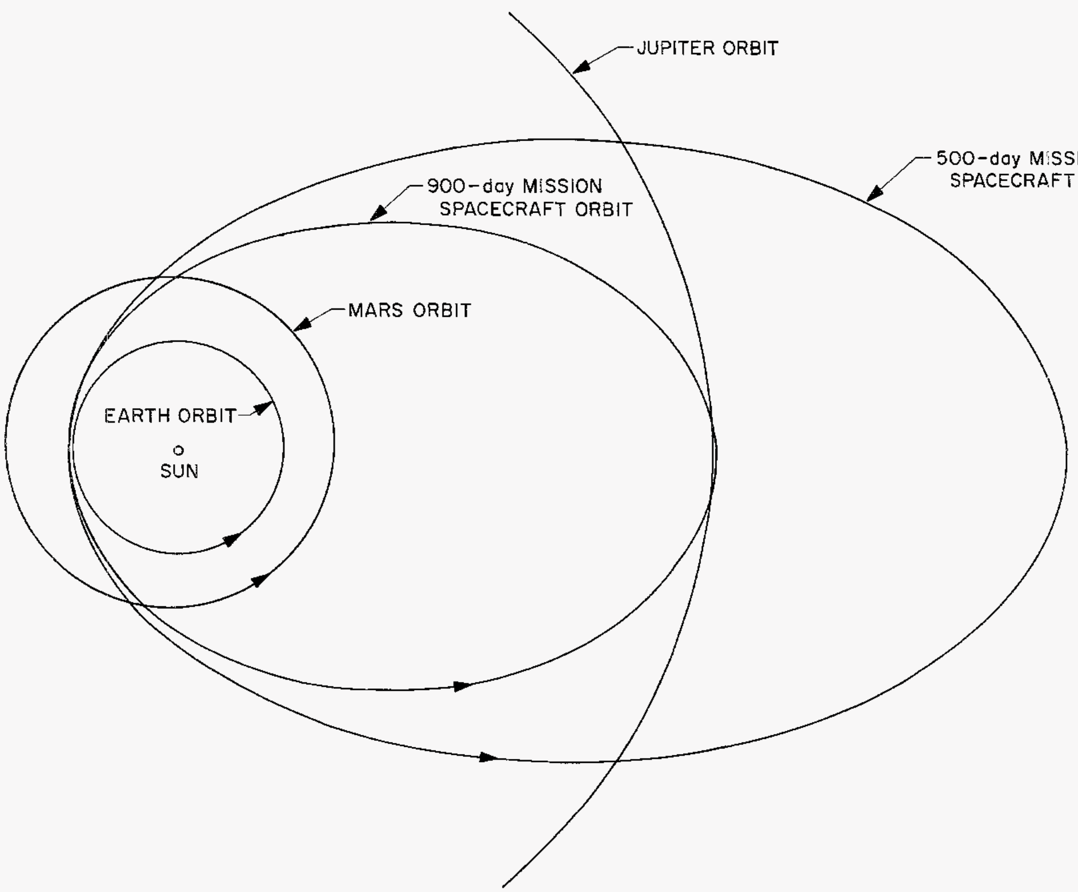


Fig. F-1. 500-day and 900-day missions to Jupiter

These four orbital parameters were the only ones needed in the computer program because the asteroid model does not depend on the ecliptic longitude or on the time. The inclination of these orbits is about 2 to 4 deg, as shown in the table.

Figure F-2a shows the shape of the family of spacecraft used, namely, a rhombicuboctahedron. In this family, l is the length of an edge. There are $N_F = 26$ faces. Figure F-2b is numbered from $j = 1$ to $j = 26$, the upper number being for the northern face and the lower number being for the southern face. Table F-3 gives values of α_j and $\mathbf{n}_j$ for the various faces shown in Fig. F-2b. The directions of $\mathbf{e}'_1$, $\mathbf{e}'_2$ and $\mathbf{e}'_3$ are shown on Fig. F-2b, and $\mathbf{e}'_3$ is in the direction of $\mathbf{e}_N$. For the reference, or standard spacecraft, $l^* = 1$ m, $t^* = 1$ cm, $t^*_j = 1$ and $\tau^*_j = 1$. For this shape

$$\alpha_s = \sum_{j=1}^{26} \alpha_j = 18 + 2\,(3)^{1/2} = 21.464, \quad \text{and}$$

$$A^*_s = 21.464\ \text{m}^2$$

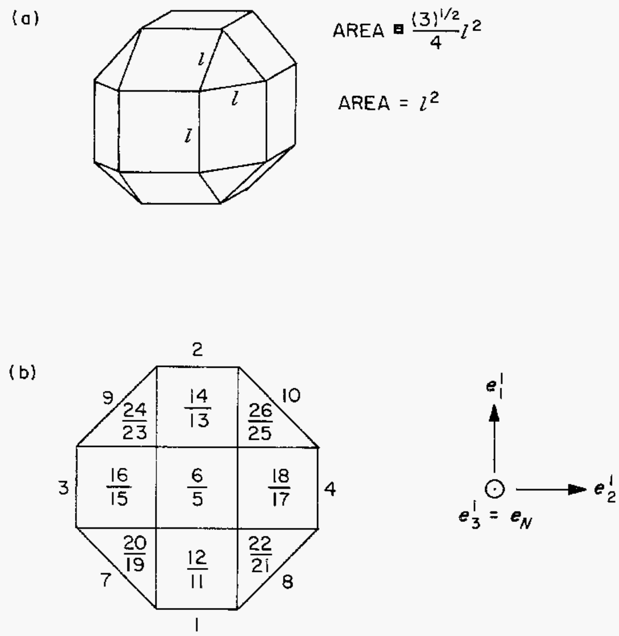


Fig. F-2. Convex polyhedral spacecraft shape

The spacecraft shielding material is assumed to be aluminum, so that $\rho_s = 2.7$ g/cm³. With this value of ρ_s, the reference spacecraft mass, W^*_s, in the uniform shielding case is,

$$W^*_s = t^*\rho_s l^{*2}\alpha_s = (1\ \text{cm})\left(2.7\frac{\text{g}}{\text{cm}^3}\right)(100\ \text{cm})^2\,(21.46)$$

$$= 575{,}000\ \text{g} = 575\ \text{kg}$$

The constants k_1, k_2 and h_t in Eq. (B-31) are taken from Ref. 7, p. 429 for an iron projectile impacting a 2024 aluminum target and are $k_1 = 0.672$, $k_2 = 0.765$, $h_t = 120$ kg/mm².

From Eq. (B-34) one gets

$$C_1 = 3\,k_1 = 2.016,$$

$$C_2 = \frac{\rho_t}{k_2 h_t} = \frac{2.7\dfrac{\text{g}}{\text{cm}^3}}{0.765\left(120\dfrac{\text{kg}}{\text{mm}^2}\right)\left(10^3\dfrac{\text{g}}{\text{kg}}\right)\left(10^2\dfrac{\text{mm}^2}{\text{cm}^2}\right)\left(980\dfrac{\text{cm}}{\text{s}^2}\right)\left(\dfrac{\text{m}^2}{10^4\ \text{cm}^2}\right)\left(\dfrac{\text{km}^2}{10^6\ \text{m}^2}\right)}$$

or

$$C_2 = \frac{2.7}{0.765\,(1.20)\,(0.980)}\,\frac{\text{s}^2}{\text{km}^2} = 3.00\,\frac{\text{s}^2}{\text{km}^2}$$

The computer input includes the following:

(1) The spacecraft trajectory, $\mathbf{X}(T)$, $\mathbf{V}(T)$ which is given in terms of a, e, i, ω presented above.

(2) The spacecraft attitude represented by the rotation matrix $\mathcal{M}(T)$ given in Eq. (C-10).

(3) The spacecraft shielding material characteristics and the asteroid material characteristics, represented by C_1, C_2 and ρ_s.

(4) The spacecraft size and geometry represented by l, α_j, $\mathbf{n}_j$.

(5) The asteroid belt model parameters, for the 1500 asteroids used, namely

$$w_k, a_k, e_k, i_k, \rho', \beta, \varepsilon_r = \varepsilon_\lambda = 0.02$$

(Note: G_k is used to obtain w_k.)

Table F-3. Values of α_j and $\mathbf{n}_j$

j	α_j	$\mathbf{n}_j$
1	1	(−1, 0, 0)
2	1	(1, 0, 0)
3	1	(0, −1, 0)
4	1	(0, 1, 0)
5	1	(0, 0, −1)
6	1	(0, 0, 1)
7	1	0.707 (−1, −1, 0)
8	1	0.707 (−1, 1, 0)
9	1	0.707 (1, −1, 0)
10	1	0.707 (1, 1, 0)
11	1	0.707 (−1, 0, −1)
12	1	0.707 (−1, 0, 1)
13	1	0.707 (1, 0, −1)
14	1	0.707 (1, 0, 1)
15	1	0.707 (0, −1, −1)
16	1	0.707 (0, −1, 1)
17	1	0.707 (0, 1, −1)
18	1	0.707 (0, 1, 1)
19	$\frac{1}{4}(3)^{1/2}$	0.577 (−1, −1, −1)
20	$\frac{1}{4}(3)^{1/2}$	0.577 (−1, −1, 1)
21	$\frac{1}{4}(3)^{1/2}$	0.577 (−1, 1, −1)
22	$\frac{1}{4}(3)^{1/2}$	0.577 (−1, 1, 1)
23	$\frac{1}{4}(3)^{1/2}$	0.577 (1, −1, −1)
24	$\frac{1}{4}(3)^{1/2}$	0.577 (1, −1, 1)
25	$\frac{1}{4}(3)^{1/2}$	0.577 (1, 1, −1)
26	$\frac{1}{4}(3)^{1/2}$	0.577 (1, 1, 1)

Total: $\alpha_s = \sum_{j=1}^{N_F} \alpha_j = 18 + 2(3)^{1/2} = 21.464$

(6) The times T_0 = initial time, T_f = final time at end of mission, T_P = time of perihelion passage, ΔT = interval between time steps, and N_T = number of steps into which the mission is divided.[2]

The computer does the following: It takes a collection of points in time $T_i = T_0 + i\Delta T$, where i ranges from 0 to N_T. For time T_i it computes and outputs $F_j^*(T_i)$ for $j = 1$ to 26, using Eq. (B-66) and others. From $F_j^*(T_i)$ it computes and outputs $\pi_I^*(T_i)$, essentially from Eq. (B-53) as follows:

$$\pi_I^*(T_i) = \sum_{j=1}^{26} F_j^*(T_i)\, A_j^* = \sum_{j=1}^{26} \alpha_j l^{*2} F_j^*(T_i)$$

$$= l^{*2} \sum_{j=1}^{26} \alpha_j F_j^*(T_i)$$

[2]Note that $T_f = T_0 + N_T\,\Delta T$.

Figures F-3 and -4 give $\pi_I^*(T)$ versus T for the 500- and 900-day missions and $3\beta = 1.9$ and $3\beta = 3.0$.

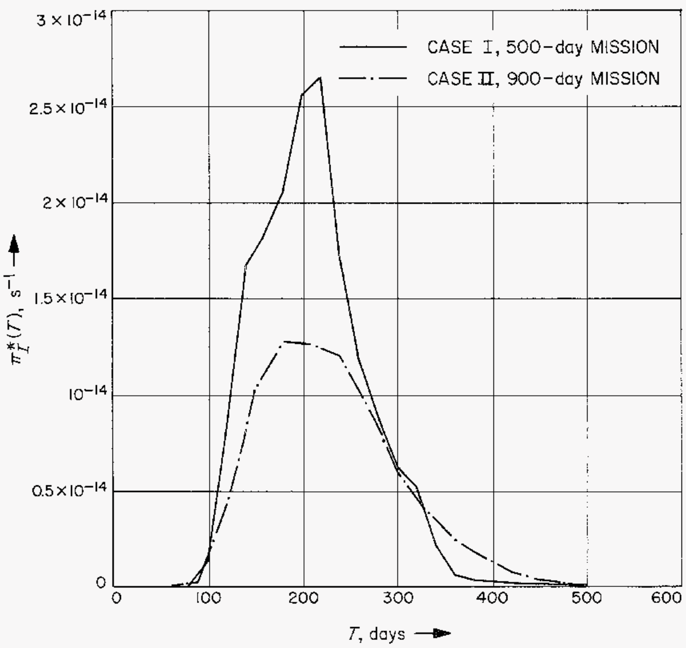


Fig. F-3. π_I^* (T) vs T for Cases I and II ($3\beta = 1.9$)

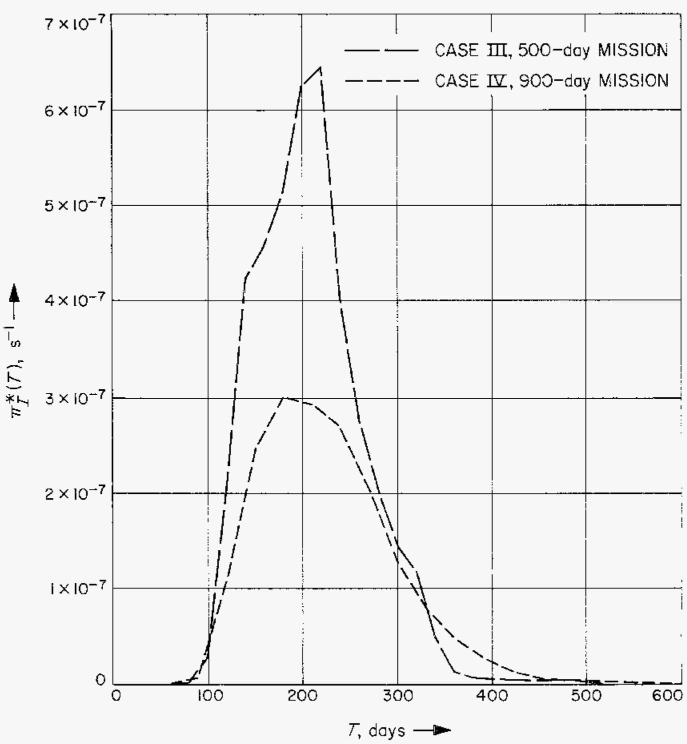


Fig. F-4. π_I^* (T) vs T for Cases III and IV ($3\beta = 3.0$)

The computer calculates only starred (*) quantities. An equation with a * applies to the reference or standard spacecraft while an equation without a * applies to any spacecraft in the family. The reference or standard spacecraft is also a member of the family. The computer next calculates f_j^* from Eq. (E-6) and v_I^* from

$$v_I^* = \int_{T_0}^{T_f} \pi_I^*(T)\, dT$$

The computer calculates the integrals approximately by use of the trapezoidal rule:

$$\int_{T_0}^{T_f} f(T)\, dT \cong \sum_{i=0}^{N_F} f(T_i)\, \Delta T - \frac{1}{2}\left[f(T_0) + f(T_{N_T})\right] \Delta T$$

The values of all the output quantities are punched on cards as well as printed. This enables the user to perform any desired additional computer analysis on the data. The total expected number of destructive impacts on the jth face of the spacecraft is

$$A_j f_j = (\alpha_j l^2)\left(\frac{t_j^*}{t_j}\right)^{3\beta} (f_j^*) = \left(\frac{l^2}{t^{3\beta}}\right)\left(\frac{\alpha_j}{\tau_j^{3\beta}}\right)(\tau_j^{*3\beta})(t^{*3\beta})(f_j^*)$$

using $t_j = t\tau_j$.

The computer calculates $F_j^*(T_i)$, $\pi_I^*(T_i)$, f_j^*, v_I^* and τ_j^+. The expected number of destructive meteoroids per m², f_j^*, and non-dimensional optimum shielding thicknesses, τ_j^+, for Cases I, II, III and IV are listed in Table F-4, and is shown in Fig. F-5 for Case I. From the data in Table F-4, plots similar to that given in Fig. F-5 can be drawn. The direction of motion of the spacecraft, shown in the same figure, results in large values of f_j^* on the front faces: $j = 17$, 18, 21, 22, 25, and 26 and small values of f_j^* on the rear faces: $j = 15$, 16, 19, 20, 23 and 24. The required thicknesses

$$\tau_j^+ = \frac{t_j^+}{t}$$

Table F-4. Expected numbers of destructive meteoroids / m², f_j^*, and non-dimensional optimum-shielding thicknesses τ_j^+, for Cases I, II, III, and IV

j	Case I		Case II		Case III		Case IV	
	$f_j^* \times 10^9$ [m⁻²]	τ_j^+	$f_j^* \times 10^9$ [m⁻²]	τ_j^+	f_j^* [m⁻²]	τ_j^+	f_j^* [m⁻²]	τ_j^+
1	4.40	1.129	4.90	1.289	0.0622	1.148	0.0720	1.272
2	0.00453	0.1054	0.001496	0.0791	0.000070	0.210	0.000018	0.1602
3	0	0	0	0	0	0	0	0
4	69.7	2.93	45.3	2.78	1.893	2.70	1.153	2.55
5	1.093	0.699	1.348	0.826	0.01629	0.822	0.0205	0.930
6	0.695	0.598	0.779	0.684	0.01008	0.729	0.01127	0.800
7	0	0	0.001781	0.0840	0	0	9.6×10^{-6}	0.1369
8	53.8	2.68	38.4	2.62	1.381	2.49	0.934	2.42
9	0	0	0	0	0	0	0	0
10	27.6	2.13	15.01	1.897	0.615	2.04	0.305	1.825
11	3.40	1.033	3.93	1.195	0.0491	1.082	0.0604	1.218
12	2.60	0.941	2.88	1.074	0.0357	1.000	0.0426	1.116
13	0.0990	0.305	0.0948	0.331	0.001462	0.450	0.001449	0.479
14	0.0638	0.262	0.0516	0.268	0.000953	0.404	0.000759	0.408
15	0.000188	0.0352	0.001040	0.0698	2.3×10^{-6}	0.0900	0.000015	0.1530
16	0.000056	0.0232	0.000300	0.0455	6×10^{-7}	0.0641	3.1×10^{-6}	0.1034
17	41.6	2.45	27.6	2.34	1.016	2.31	0.637	2.19
18	38.9	2.39	24.6	2.25	0.937	2.26	0.551	2.12
19	0.000055	0.0230	0.00253	0.0949	5×10^{-7}	0.0598	0.000021	0.1661
20	0.000006	0.01067	0.00456	0.1162	3×10^{-8}	0.0309	0.000034	0.1880
21	40.0	2.42	29.0	2.38	0.966	2.28	0.668	2.22
22	37.9	2.37	26.5	2.31	0.903	2.24	0.597	2.16
23	0.000241	0.0383	0.000668	0.0599	2.9×10^{-6}	0.0953	9.1×10^{-6}	0.1351
24	0.000089	0.0272	0.000196	0.0393	1.1×10^{-6}	0.0740	2.1×10^{-6}	0.0931
25	20.6	1.924	11.86	1.749	0.435	1.868	0.233	1.707
26	18.9	1.867	10.00	1.649	0.391	1.818	0.1884	1.619

FROM C_B'', l AND W_s ONE CAN CALCULATE $P_I(S)$ FROM Eq. (F-1)
ALSO $t = W_s/\rho_s l^2 a_s$, SO THAT $t_j^+ = (\tau_j^+)t$

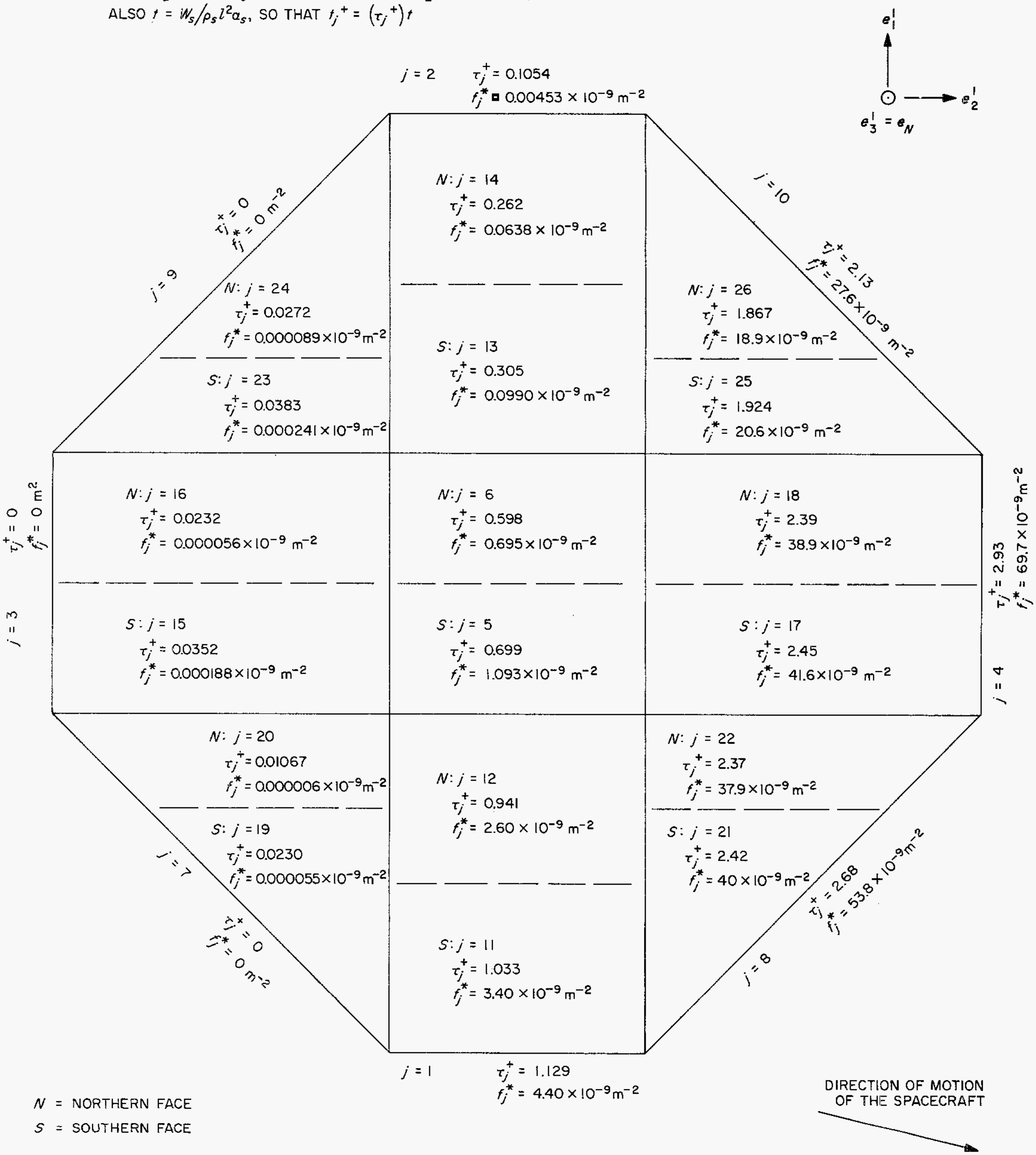


Fig. F-5. Non-dimensional optimum shielding $\tau_j^+ = \dfrac{t_j^+}{t}$ and number of damaging meteoroids/m², f_j^*, for Case I

(where t_j^+ is the thickness on the jth face and t is the average thickness of the spacecraft surface) are larger on the front faces than on the rear faces. The computed values of C_A, C'_A, C''_A, C'''_A, C_B, C'_B, C''_B, and C'''_B are listed in Table F-5 for Cases I, II, III and IV. The equation for $P_I(S)$ in terms of l and W_s, namely

$$P_I(S) = \exp\left[-C'' l^{2(1+3\beta)} W_s^{-3\beta}\right] \qquad \text{(F-1)}$$

is plotted in Fig. F-6 for $P_I(S) = 0.99$ and $3\beta = 1.9$ and 3.0, for Cases I, II, III and IV for uniform shielding and for optimum shielding.

From C''_B, l and W_s one can calculate $P_I(S)$, from Eq. (F-1).

Table F-5. Computed values of C_A, C'_A, C''_A, C'''_A, C_B, C'_B, C''_B and C'''_B for Cases I, II, III and IV

Constant	$3\beta = 1.9$		$3\beta = 3.0$	
	500-day mission	900-day mission	500-day mission	900-day mission
	Case I	Case II	Case III	Case IV
C_A	2.9476×10^{-7}	1.9828×10^{-7}	7.1850	4.5204
C'_A	7.2003×10^{-6}	4.8435×10^{-6}	6.5888×10^{3}	4.1453×10^{3}
C''_A	5.2398×10^{-2}	3.5247×10^{-2}	1.3985×10^{9}	8.7985×10^{8}
C'''_A	1.3733×10^{-8}	9.2377×10^{-9}	0.33475	0.21060
C_B	6.6404×10^{-8}	5.0315×10^{-8}	0.76736	0.58934
C'_B	1.6221×10^{-6}	1.2291×10^{-6}	7.0369×10^{2}	5.4043×10^{2}
C''_B	1.1804×10^{-2}	8.9442×10^{-3}	1.4936×10^{8}	1.1471×10^{8}
C'''_B	3.0937×10^{-9}	2.3442×10^{-9}	0.035751	0.027457

Also

$$t = \frac{W_s}{\rho_s l^2 \alpha_s} \qquad \text{so that } t_j^+ = (\tau_j^+)\, t$$

The equation is thus

$$0.99 = \exp(-C'' l^{5.8} W_s^{-1.9}), \qquad \text{for } 3\beta = 1.9$$

$$0.99 = \exp(-C'' l^{8} W_s^{-3}) \quad , \qquad \text{for } 3\beta = 3.0$$

For $3\beta = 3.0$, in Cases III and IV, the C''_A and C''_B values are much larger than the corresponding values for $3\beta = 1.9$ in Cases I and II and lead to much larger shielding masses W_s. For example, for Case III (500-day mission, $3\beta = 3.0$), with $P_I(S) = 0.99$ and $l = 1$ m, with optimum shielding $C''_B = 1.4936 \times 10^8$

and

$$0.99 = \exp(C''_B l^8 W_s^{-3}) = \exp(-C''_B W_s^{-3}),$$

so that

$$W_s = 5297 \text{ kg}$$

Thus, $3\beta = 3.0$ or $\beta = 1.0$ gives very large shielding masses, making the asteroid belt nearly impenetrable. Therefore, the shielding mass is extremely sensitive to β and use of the proper value of β is very important. As indicated earlier, the authors' best estimate is

$$\beta = \frac{1.9}{3} = 0.633$$

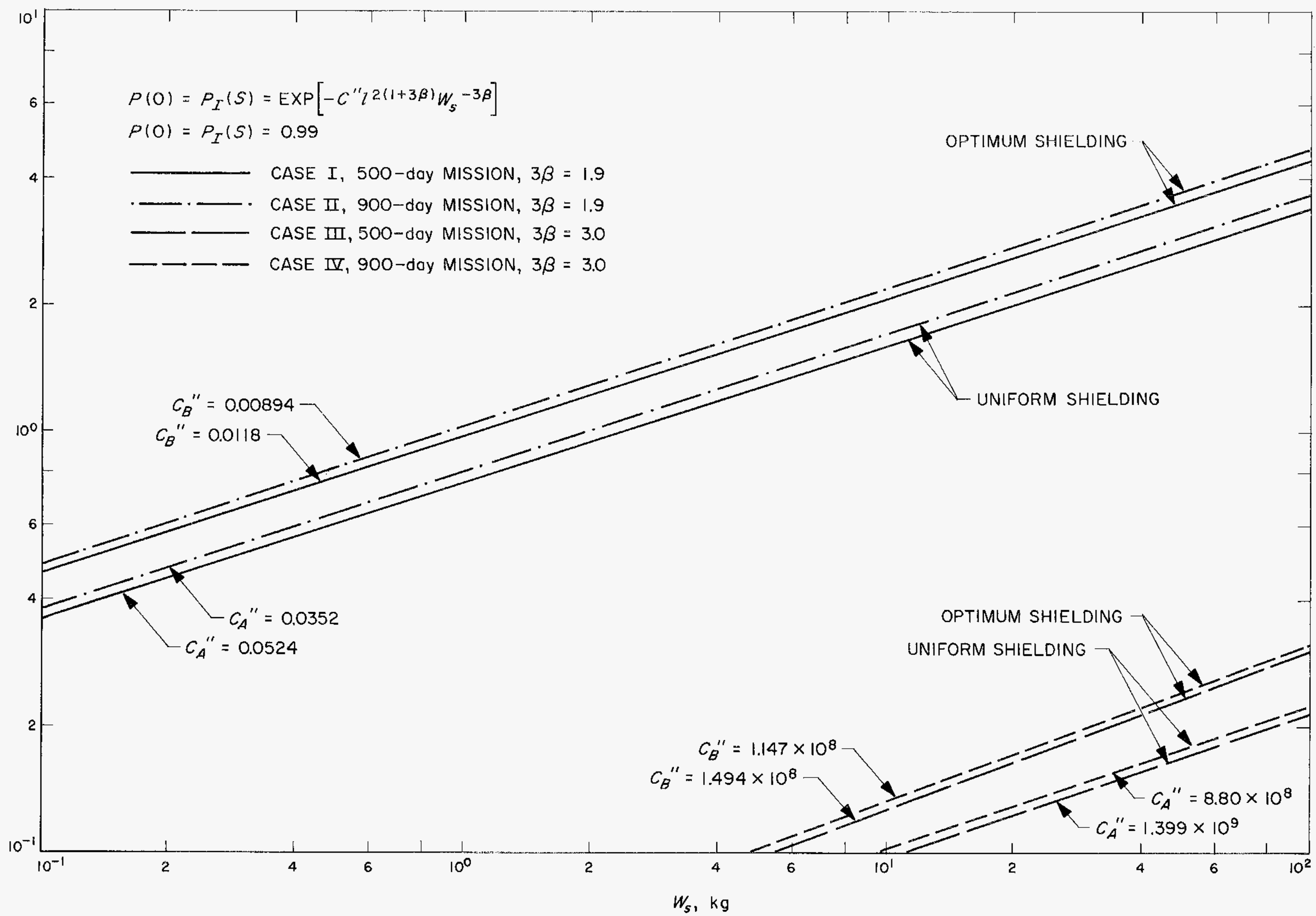


Fig. F-6. l **vs** W_s **for** $P(0) = P_I(S) = 0.99$ **for Cases I and II with uniform shielding and with optimum shielding**

Appendix G

Computer Program

I. Description of Computer Program Including Simple Flow Diagram

The program is called ASTEFF (*Ast*eroid Belt *Eff*ects Program) and is written in FORTRAN IV. The flow chart is shown in Fig. G-1. The asteroid belt data is read first. A sequence of cases can be run with one pass on the computer. Each case is represented by an appropriate collection of input data. Properties which do not change from one case to another do not have to be repeated in the input. Each case has a label card, a parametric card and an orbit card and may or may not have a collection of structure cards. In each case one takes a set of user supplied points in time and runs through them in sequence. At each time point the following items are computed and printed: spacecraft location $\mathbf{X}(T)$, velocity $\mathbf{V}(T)$, meteoroid space density σ, flux of hazardous meteoroids on the *j*th face of the reference spacecraft $F_j^*(T)$, and the failure rate, due to meteoroids, of the reference spacecraft $\pi_I^*(T)$. The following items are then computed and printed for the reference spacecraft: the integrated flux (T), the f_j^* number of destructive impacts on the *j*th face $A_j^* f_j^*$, the dimensionless ratio τ_j^+ related to the optimum thickness, the failure rate integral ν_P^*, the probability of success $P_I^*(S)$, and the eight coefficients discussed in Appendix E: C_A, C'_A, C''_A, C'''_A, C_B, C'_B, C''_B, and C'''_B.

II. Description of Input Data Cards and Output

The description below refers to the use of the ASTEFF program on the JPL 7040-7090 Direct Couple Operating System, but the program should run on any large computer system equipped with a FORTRAN IV compiler with at most, very minor modifications.

Section III of this Appendix G presents a listing of the FORTRAN IV ASTEFF Decks and the Asteroid Deck Cards.

A. Card Deck

The card deck presented to the machine is as follows:

```
$JOB Card                                                          $JOB Card
$EXECUTE IBJOB                                                     $EXECUTE IBJOB
$IBJOB                                                             $IBJOB
$IBFTC DJAA . .   ⎫                                              ⎧ $IBLDR DJAA . .
      .3          ⎪                                              ⎪       .3
      .           ⎪                                              ⎪       .
      .           ⎪                                              ⎪       .
      END         ⎪                                              ⎪ $DKEND DJAA . .
$IBFTC DJAB . .   ⎪                                              ⎪ $IBLDR DJAB . .
      .           ⎪                                              ⎪       .
      .           ⎬  FORTRAN IV        or    Binary (machine     ⎨       .
      .           ⎪  ASTEFF Decks                  language)     ⎪       .
      END         ⎪                          ASTEFF Decks        ⎪ $DKEND DJAB . .
  .4              ⎪                                              ⎪    .4
  .               ⎪                                              ⎪    .
  .               ⎪                                              ⎪    .
$IBTFC DJAZ       ⎪                                              ⎪ $IBLDR DJAZ . .
      .           ⎪                                              ⎪       .
      .           ⎪                                              ⎪       .
      .           ⎪                                              ⎪       .
    END           ⎭                                              ⎩ $DKEND DJAZ . .
```

[3]Denotes a number of cards.

[4]Denotes the start of, and continuation of, a number of cards in the series.

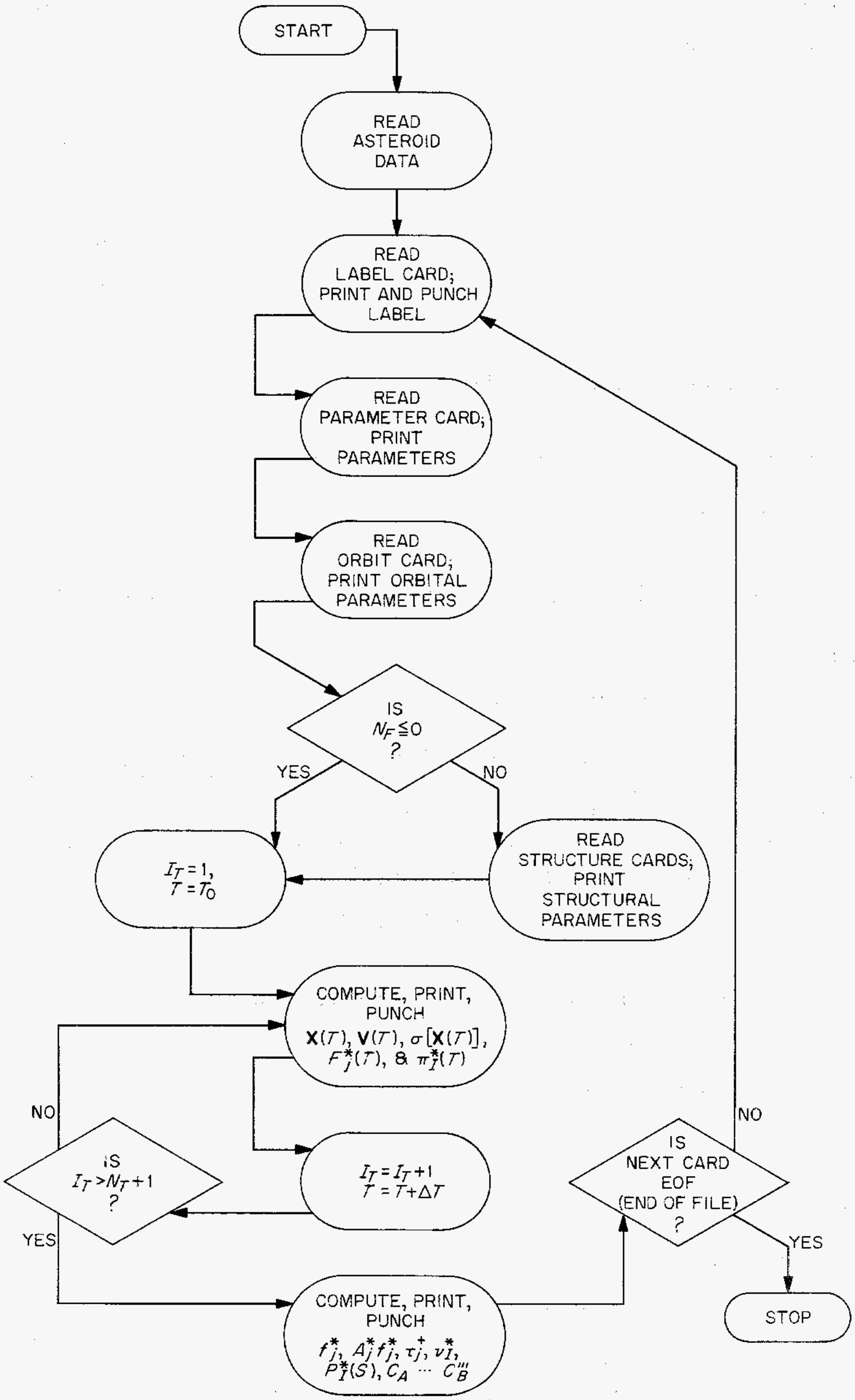


Fig. G-1. ASTEFF flow chart

$DATA

Asteroid quantity card

$[(N_{AST} + 2)/3]$ asteroid data cards

First case data block

Second case data block

.
.
.

Last case data block

EOF card

$DATA

Asteroid quantity card

$[(N_{AST} + 2)/3]$ asteroid data cards

First case data block

Second case data block

.
.
.

Last case data block

EOF card

B. Asteroid Quantity Card Format

The format for the asteroid quantity card is as follows:

1–5	6
N_{AST}	

FORMAT ($I5$)

N_{AST} is the number of asteroids for which data is to be read from the asteroid data cards following.

C. Asteroid Data Card Format

The format for the nth asteroid data card is as follows:

1–4	5–10	11–16	17–23		26–29	30–35	36–41	42–48		51–54	55–60	61–66	67–73	
w_k	i_k	e_k	a_k		w_k	i_k	e_k	a_k		w_k	i_k	e_k	a_k	
$(3n-2)$'th asteroid data set					$(3n-1)$'th asteroid data set					$3n$'th asteroid data set				

FORMAT [3 ($F4.2$, $2F6.5$, $F7.5$, $2X$)]

w_k is, as the reciprocal of a probability, dimensionless

i_k is in rad

e_k is dimensionless

a_k is in AU

D. Case Block Structure

A case block has the following structure:

Case data block:

Label card:

Parameter card: $C_1, C_2, \rho_s, h_s, \rho', 3\beta, \varepsilon_r, \varepsilon_\lambda$

Orbit card: $a, e, i, \omega, T_P, T_0, \Delta T, N_T, N_F$

$[(N_F + 1)/2]$ structure cards: $\mathbf{n}_j$, α_j

The formats for the label card, parameter card, orbit card and structure cards are given below.

E. Label Card Format

The format for the label card is as follows:

1 – 72	73 – 76	77 – 80
Label	N_{A1}	N_{A2}

FORMAT (12*A*6, 2*I*4)

The label is any message; it is used to identify a case, and is reproduced in both forms of output. The N_{A1}th through N_{A2}th asteroid data sets are used in each case. If N_{A1} is left blank or given as ≤ 0, $> N_{AST}$, or $> N_{A2}$, it is taken as $= 1$. If N_{A2} is left blank or given as ≤ 0 or $> N_{AST}$, it is taken as $= N_{AST}$. N_A is the number of asteroid data sets used, $N_A = N_{A2} - N_{A1} + 1$. It is printed out, but not input or punched.

F. Parameter Card Format

The format for the parameter card is as follows:

1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80
C_1	C_2	ρ_S	h_s	ρ'	3β	ε_r	ε_λ

FORMAT (8*E*10.5)

C_1 is dimensionless	$C_1 > 0$
C_2 is in $(\text{km/s})^{-2}$	$C_2 > 0$
ρ_S is in g/cm^3	$\rho_S > 0$
h_s is in kg/mm^2	$h_s > 0$
ρ' is in g/cm^3	$\rho' > 0$
3β is dimensionless	$3\beta > 0$
ε_r is dimensionless	$\varepsilon_r > 0$
ε_λ is dimensionless	$\varepsilon_\lambda > 0$

If C_1 and/or C_2 is left blank (or given as ≤ 0), it will be computed approximately from ρ_S, h_s, and ρ', using Eqs. (B-32, -34, -37, and E-14).

If ρ_S is left blank (or given as ≤ 0), it is taken as 2.7 g/cm^3 and h_s is taken as 120 kg-wt/mm^2. This is also done if C_2 and h_s are left blank (or given as ≤ 0). If ρ_S, ρ' and either C_1 or C_2 are left blank (or given as ≤ 0), C_1 and C_2 are taken as 2.016 and 3.00 $(\text{km/s})^{-2}$, the experimental values for iron projectiles hitting aluminum targets.

If ρ', 3β, ε_r, and/or ε_λ are left blank (or given as ≤ 0), they will be taken as 7.9 g/cm^3, 1.9, 0.02, and 0.02, respectively.

G. Mission Orbit Card

The format for the spacecraft mission orbit card is as follows:

1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–75	76–80
a	e	i	ω	T_P	T_0	ΔT	N_T	N_F

FORMAT (7E10.5, 2I5)

a is in AU	$a > 0$
e is dimensionless	$0 \leqq e < 1$
i is in deg	$0 \leqq i \leqq 180$ deg
ω is in deg	-180 deg $\leqq \omega \leqq 360$ deg
T_P is in days	
T_0 is in days	
ΔT is in days	
N_T is dimensionless	$N_T \geqq 0$
N_F is dimensionless	

Leaving a quantity blank is equivalent to giving it a value of zero.

$N_T + 1$ times are considered by the program: $T_i = T_0 + i\,\Delta T$, $i = 0, \cdots, N_T$. If ΔT is zero, N_T is automatically taken as zero, and vice versa. If $N_T < 0$, it is taken as $= 0$.

If $N_F < 0$, it is taken as $= 0$. If $N_F = 0$, no structure cards are read, and the values of N_F, $\mathbf{n}_j$, and α_j are retained from the previous case: Thus, N_F must not be $\leqq 0$ for the first case; if it is, cases will be rejected by the program until it encounters one with $N_F > 0$.

H. n^{th} Structure Card

The format for the nth structure card is as follows:

1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80
$\mathbf{N}'_j \cdot \mathbf{e}'_1$	$\mathbf{N}'_j \cdot \mathbf{e}'_2$	$\mathbf{N}'_j \cdot \mathbf{e}'_3$	α_j	$\mathbf{N}'_j \cdot \mathbf{e}'_1$	$\mathbf{N}'_j \cdot \mathbf{e}'_2$	$\mathbf{N}'_j \cdot \mathbf{e}'_3$	α_j
$(2n-1)$'th face				$2n$'th face			

FORMAT (8E10.5)

$\mathbf{N}'_j$ is any vector parallel to $\mathbf{n}_j$; that is, $\mathbf{N}'_j = c\,\mathbf{n}_j$ where $0 < c$.

Thus $\mathbf{n}_j = \mathbf{N}'_j / |\mathbf{N}'_j|$.

l^* is taken $= 1$ m, and $t^* = 1$ cm. Thus, $\alpha_j = A^*_j/(1\,\text{m})^2 =$ the area of the jth face of the reference spacecraft in m². τ^*_j is taken $= 1$.

I. Printer Output

Each case starts on a new page. The output from the printer takes the following form[5]:

Label ⟶

$N_{A1} =$ ______ $N_{A2} =$ ______ $N_A =$ ______

$C_1 =$	______	$C_2 =$	______	$\rho_S =$	______	$h_s =$	______
$\rho' =$	______	$3\beta =$	______	$\varepsilon_r =$	______	$\varepsilon_\lambda =$	______
$a =$	______	$e =$	______	$i =$	______	$\omega =$	______
$T_P =$	______	$T_0 =$	______	$T_F =$	______	$\Delta T =$	______

$N_T =$

j	$\mathbf{n}_j \cdot \mathbf{e}'_1$	$\mathbf{n}_j \cdot \mathbf{e}'_2$	$\mathbf{n}_j \cdot \mathbf{e}'_3$	α_j
1	______	______	______	______
2	______	______	______	______
⋮	⋮	⋮	⋮	⋮
N_F	______	______	______	______

First time-step (T_0) printer block

Second time-step (T_1) printer block

⋮

($N_T + 1$)th time-step (T_{N_T}) printer block

j	f_j^*	$f_j^*/\Sigma f_j^*$	$A_j^* f_j^*$	$A_j^* f_j^*/v_I^*$	τ_j^+
1	______	______	______	______	______
2	______	______	______	______	______
⋮	⋮	⋮	⋮	⋮	⋮
N_F	______	______	______	______	______

$v_I^* =$ ______ $P_I^*(S) =$ ______

$C_A =$	______	$C_B =$	______
$C'_A =$	______	$C'_B =$	______
$C''_A =$	______	$C''_B =$	______
$C'''_A =$	______	$C'''_B =$	______

[5]Section V of this same Appendix G presents the original printout of this sample problem. In this case the label would be SHORT JUPITER MISSION and C_1 is printed out as C1 as the printer does not differentiate for symbols, subscripts, or superscripts.

$(i + 1)$'th time-step (T_i) printer block:

$(T_i) =$ ______ $i =$ ______ $\eta(T_i) =$ ______

$r(T_i) =$ ______ $\lambda(T_i) =$ ______

$V_1(T_i) =$ ______ $V_2(T_i) =$ ______ $V_3(T_i) =$ ______

$n\,\sigma[\mathbf{X}(T_i)] =$ ______ $\sigma[\mathbf{X}(T_i)] =$ ______

$j\,F_j^*(T_i)$

1 ______ 2 ______ 3 ______ 4 ______ 5 ______

⋮ ⋮ ⋮ ⋮ ⋮

⋮

$N_F =$ ______

$\pi_I^*(T_i) =$ ______

J. Punched Card Output

The punched output is as follows:[6]

First case output card block

Second case output card block

⋮

Last case output card block

Each case output card block has the following structure:

Label card (same format as input label card)

First time-step (T_0) card block

Second time-step (T_1) card block

⋮

$(N_T + 1)$th time-step (T_{N_T}) card block

$[(N_F + 4)/5]$ integrated flux cards

$[(N_F + 4)/5]$ optimum shielding cards

Probability of success card

Uniform shielding coefficient card

Optimum shielding coefficient card

Case end card

[6]Section VI of this same Appendix G presents an original sample problem punched output for comparison.

nth Time-step (T_{n-1}) card block:

Position card

Velocity card

Density card

$[(N_F + 4)/5]$ flux cards

Failure rate card

All output data cards[6] except the label and case end cards use FORMAT [5 (E14.7, 1X)]:

1 — 14	16 — 29	31 — 44	46 — 59	61 — 74	
First datum	Second datum (if any)	Third datum (if any)	Fourth datum (if any)	Fifth datum (if any)	

The case end card has asterisks (*) in all 80 columns.

K. Card Data

The data on each card is as follows:

Card	First datum	Second datum	Third datum	Fourth datum	Fifth datum
Position card of $(n+1)'$th time-step (T_n) card block	T_n	$\eta(T_n)$	$r(T_n)$	$\lambda(T_n)$	
Velocity card of $(n+1)'$th time-step (T_n) card block	$V_1(T_n)$	$V_2(T_n)$	$V_3(T_n)$		
Density card of $(n+1)'$th · · · card block	$\sigma(\mathbf{X}(T_n))$	$\sigma(\mathbf{X}(T_n))$			
i'th flux card of $(n+1)'$th time-step (T_n) card block	$F^*_{5i-4}(T_n)$	$F^*_{5i-3}(T_n)$	$F^*_{5i-2}(T_n)$	$F^*_{5i-1}(T_n)$	$F^*_{5i}(T_n)$
Failure rate card of $(n+1)'$th time-step (T_n) card block	$\pi^*_I(T_n)$				
n'th integrated flux card	f^*_{5n-4}	f^*_{5n-3}	f^*_{5n-2}	f^*_{5n-1}	f^*_{5n}
n'th optimum shielding card	τ^+_{5n-4}	τ^+_{5n-3}	τ^+_{5n-2}	τ^+_{5n-1}	τ^+_{5n}
Probability of success card	v^*_I	$P^*_I(S)$			
Uniform shielding coefficient card	C_A	C'_A	C''_A	C'''_A	
Optimum shielding coefficient card	C_B	C'_B	C''_B	C'''_B	

T_i in days

η in deg

r in AU

λ in deg

σ in (meteoroids with mass $\geqq m_0$) AU^{-3}

n_σ dimensionless

V_1, V_2, and V_3 in km s^{-1}

F_j^* in (penetrating meteoroids) m^{-2} s^{-1}

π_I^* in (penetrating meteoroids) s^{-1}

f_j^* in (penetrating meteoroids) m^{-2}

ν_I^* dimensionless [expected number of penetrating meteoroids]

$P_I^*(S)$ dimensionless [probability of zero penetrating meteoroids]

C	in m^{-2} $cm^{3\beta}$	i.e.	l	in m
C'	in $m^{-2(1+3\beta)}$ $kg^{3\beta}$		A_s	in m^2
C''	in $m^{-2(1+3\beta)}$ $kg^{3\beta}$		t	in cm
C'''	in m^{-2} $cm^{3\beta}$		W_s	in kg

$A_j^* f_j^*$ dimensionless [number of meteoroids expected to penetrate jth face]

III. Listing

The following is a listing of the FORTRAN decks.

```
$JOB
$EXECUTE       IBJOB
$IBJOB


$IBFTC DJAA..
C**** ASTEFF MAIN PROGRAM
      REAL I,ISC,LSC
      COMMON /CAST/NAST,NA,NA1,NA2,DS(1500,4)
      DIMENSION VSC(3),U(2,2,3),UC(3)
      COMMON /CPI/HALFPI,PI,TWOPI,DEGREE,RADIAN
      COMMON /CG/GMC,VC
      COMMON /CCETC/C1,C2,RHOSC,HSC,RHOAST,BETA3,EPSR,EPSL,C
      DIMENSION DG(100),F(100),FI(100)
      COMMON /CSC/NF,ENF(3,100),ENFR(3,100),AREA(100),AS
      CALL ASTDAT
      LOGICAL FIRST
      FIRST=.TRUE.
      NF=0
    1 CALL CASE(ASC,ESC,ISC,OMSC,PERSC,TPSC,T,DT,NT,FIRST,$1)
      DO 2 JF=1,NF
    2 FI(JF)=0.
      FNU=0.
      DO 7 IT=1,NT
      DEN=0.
      NDEN=0
      CALL XVSC(ASC,ESC,ISC,OMSC,PERSC,TPSC,T,RSC,LSC,SL,CL,VSC,IT)
      CALL ROT(SL,CL)
      DO 3 JF=1,NF
      F(JF)=0.
    3 DG(JF)=DOT(VSC,JF)
      DO 6 KA=NA1,NA2
      A =DS(KA,4)
      E =DS(KA,3)
      I =DS(KA,2)
      IF(ABS(LSC).GE.(I+EPSL)) GO TO 6
      IF(RSC.GE.(A*(1.+E)*(1.+EPSR))) GO TO 6
      IF(RSC.LE.(A*(1.-E)*(1.-EPSR))) GO TO 6
      CALL UAST(A,E,I,RSC,LSC,U)
      WT=DS(KA,1)*.25*SIGMA(A,E,I,RSC,LSC)
      DEN=DEN+WT
      NDEN=NDEN+1
      DO 5 L=1,2
      DO 5 M=1,2
      DO 4 N=1,3
    4 UC(N)=U(L,M,N)
      DO 5 JF=1,NF
      D=DG(JF)-DOT(UC,JF)
      IF(D.GT.0.) F(JF)=F(JF)+WT*D*ALOG(1.+C2*D*D)**BETA3
    5 CONTINUE
    6 CONTINUE
      CALL INTER(F,DEN,NDEN,FPI,FI,FNU,IT,NT)
    7 T=T+DT
      CALL POST(FI,FNU,DT,BETA3)
      GO TO 1
      END


$IBFTC DJAB..
      SUBROUTINE ASTDAT
C**** READS ASTEROID BELT MODEL DATA
      COMMON /CAST/NAST,NA,NA1,NA2,DS(1500,4)
      READ (5,1) NAST
    1 FORMAT(I5)
      READ (5,2) ((DS(K,J),J=1,4),K=1,NAST)
    2 FORMAT(3(F4.2,2F6.5,F7.5,2X))
      RETURN
      END
```

```
$IBFTC DJAC..
      SUBROUTINE CASE(ASC,ESC,ISC,OMSC,PERSC,TPSC,T,DT,NT,FIRST,*)
C**** READS DATA SPECIFYING A CASE TO BE ANALYSED
      REAL ISC
      LOGICAL FIRST
      DIMENSION LABEL(12)
      COMMON /CPI/HALFPI,PI,TWOPI,DEGREE,RADIAN
      COMMON /CG/GMC,VC
      COMMON /CCP/CP,CU,CC,DAY,CRHO
      COMMON /CAST/NAST,NA,NA1,NA2,DS(1500,4)
      COMMON /CCETC/C1,C2,RHOSC,HSC,RHOAST,BETA3,EPSR,EPSL,C
      COMMON /CSC/NF,ENF(3,100),ENFR(3,100),AREA(100),AS
      LOGICAL OK
      OK=.TRUE.
      READ (5,1) LABEL,NA1,NA2
    1 FORMAT(12A6,2I4)
      IF(NA1.LE.0) NA1=1
      IF(NA1.GT.NAST) NA1=1
      IF(NA1.GT.NA2) NA1=1
      IF(NA2.LE.0) NA2=NAST
      IF(NA2.GT.NAST) NA2=NAST
      NA=NA2-NA1+1
      WRITE (6,2) LABEL,NA1,NA2,NA
    2 FORMAT(1H1,12A6//25X,5HNA1 =,I5,5X,5HNA2 =,I5,5X,4HNA =,I5//)
      PUNCH 1, LABEL,NA1,NA2
      READ (5,3) C1,C2,RHOSC,HSC,RHOAST,BETA3,EPSR,EPSL
    3 FORMAT(8E10.5)
      IF((C1.LE.0. .OR. C2.LE.0.) .AND. RHOSC.LE.0. .AND. RHOAST.LE.0.)
     *     CALL ALFE
      IF(C2.LE.0. .AND. RHOSC.GT.0. .AND. HSC.LE.0.) RHOSC=0.
      IF(RHOAST.LE.0.) RHOAST=7.9
      IF(RHOSC.LE.0.) HSC=120.
      IF(RHOSC.LE.0.) RHOSC=2.7
      FAC=(RHOAST/RHOSC)**(2./3.)
      DATA GS/.00980665/
      IF(C1.LE.0.) C1=1.8*FAC
      IF(C2.LE.0.) C2=RHOSC*FAC/(4.*HSC*GS)
      IF(BETA3.LE.0.) BETA3=1.9
      IF(EPSR.LE.0.) EPSR=.02
      IF(EPSL.LE.0.) EPSL=.02
      ASTN=NA
      C=3./((TWOPI**3)*EPSR*(3.+EPSR*EPSR)*SIN(EPSL)) *1500./ASTN
      CP=CU*(CC*C1)**BETA3
      WRITE (6,4) C1,C2,RHOSC,HSC,RHOAST,BETA3,EPSR,EPSL
    4 FORMAT(8X,4HC1 =,E14.7,7X,4HC2 =,E14.7,4X,7HRHOSC =,E14.7,
     *       6X,5HHSC =,E14.7/  4X,8HRHOAST =,E14.7,
     *       4X,7HBETA3 =,E14.7,5X,6HEPSR =,E14.7,5X,6HEPSL =,E14.7//)
      READ (5,5) ASC,ESC,ISC,OMSC,TPSC,T,DT,NT,NFI
    5 FORMAT(7E10.5,2I5)
      IF(ASC.LE.0.) OK=.FALSE.
      IF(ESC.LT.0. .OR. ESC.GE.1.) OK=.FALSE.
      IF(ISC.LT.0. .OR. ISC.GT.180.) OK=.FALSE.
      IF(OMSC.LE.(-180.) .OR. OMSC.GE.360.) OK=.FALSE.
      IF(DT.EQ.0.) NT=0
      IF(NT.LT.0) NT=0
      IF(NT.EQ.0) DT=0.
      IF(NFI.GT.0) NF=NFI
      IF(FIRST .AND. NFI.LE.0) OK=.FALSE.
      T0=T




      ANT=NT
      TF=T0+ANT*DT
      WRITE (6,6) ASC,ESC,ISC,OMSC,TPSC,T0,TF,DT,NT
    6 FORMAT(7X,5HASC =,E14.7,6X,5HESC =,E14.7,6X,5HISC =,E14.7,
     *       5X,6HOMSC =,E14.7/  6X,6HTPSC =,E14.7,
     *       7X,4HT0 =,E14.7,7X,4HTF =,E14.7,7X,4HDT =,E14.7/
     *       46X,4HNT =,I5//)
      IF(NFI.GT.0) CALL GEOMIN
      IF(NFI.GT.0) FIRST=.FALSE.
      IF(.NOT.FIRST) CALL GEOMOU
      ISC=ISC*DEGREE
      OMSC=OMSC*DEGREE
      PERSC=TWOPI*SQRT(ASC**3)/GMC
      NT=NT+1
      IF(.NOT.OK) RETURN 1
      RETURN
      END
```

```
$IBFTC DJAD..
      SUBROUTINE ALFE
C**** ESTABLISHES PARAMETERS FOR ALUMINUM SPACECRAFT, IRON METEOROIDS
      COMMON /CCETC/C1,C2,RHOSC,HSC,RHOAST,BETA3,EPSR,EPSL,C
      C1=2.016
      C2=3.00
      RHOSC=2.7
      HSC=120.
      RHOAST=7.9
      RETURN
      END
```

```
$IBFTC DJAE..
      SUBROUTINE GEOMIN
C**** READS DATA SPECIFYING SPACECRAFT GEOMETRY
      COMMON /CSC/NF,ENF(3,100),ENFR(3,100),AREA(100),AS
      READ (5,1) ((ENF(I,J),I=1,3),AREA(J),J=1,NF)
    1 FORMAT(8E10.5)
      DO 4 J=1,NF
      ENFL=0.
      DO 2 I=1,3
    2 ENFL=ENFL+ENF(I,J)**2
      IF(ENFL.EQ.0.) GO TO 4
      ENFL=SQRT(ENFL)
      DO 3 I=1,3
    3 ENF(I,J)=ENF(I,J)/ENFL
    4 CONTINUE
      AS=0.
      DO 5 J=1,NF
    5 AS=AS+AREA(J)
      RETURN
      END
```

```
$IBFTC DJAF..
      SUBROUTINE GEOMOU
C**** OUTPUTS DATA INPUT BY GEOMIN
      COMMON /CSC/NF,ENF(3,100),ENFR(3,100),AREA(100),AS
      WRITE (6,1)
    1 FORMAT(12X,1HJ,7X,8HENF(1,J),8X,8HENF(2,J),8X,8HENF(3,J),10X,
     *       7HAREA(J))
      WRITE (6,2) (J,(ENF(I,J),I=1,3),AREA(J),J=1,NF)
    2 FORMAT(8X,I5,4X,E14.7,2X,E14.7,2X,E14.7,4X,E14.7)
      WRITE (6,3) AS
    3 FORMAT(1H0,62X,4HAS =,E14.7///)
      RETURN
      END
```

```
$IBFTC DJAG..
      SUBROUTINE XVSC(A,E,I,OM,PER,TP,T,R,L,SL,CL,V,IT)
C**** COMPUTES SPACECRAFT POSITION AND VELOCITY AT SPECIFIED TIME
      REAL I,L
      DIMENSION V(3)
      COMMON /CPI/HALFPI,PI,TWOPI,DEGREE,RADIAN
      COMMON /CG/GMC,VC
      TD=T-TP
      IF(TD.GT.0.) TD=AMOD(TD,PER)
      IF(TD.LT.0.) TD=-AMOD(-TD,PER)
      ETA=ETAF(E,TWOPI*TD/PER)
      EO=ETA+OM
      P=A*(1.-E*E)
      R=P/(1.+E*COS(ETA))
      SL=SIN(I)*SIN(EO)
      CL=SQRT(1.-SL*SL)
      L=ASIN(SL)
      VT=VC*SQRT(2./R-1./A)
      VA=VC*SQRT(P)/R
      VP=SQRT(AMAX1(0.,VT*VT-VA*VA))
      IF(ETA.GT.PI) VP=-VP
      CA=COS(I)/CL
      SA=SQRT(1.-CA*CA)
      IF(EO.LE.HALFPI) EO=EO+TWOPI
      IF(EO.GT.(TWOPI+HALFPI)) EO=EO-TWOPI
      IF(EO.LT.(PI+HALFPI)) SA=-SA
      V(1)=VP
      V(2)=VA*CA
      V(3)=VA*SA
      ETAO=ETA*RADIAN
      REAL LO
      LO=L*RADIAN
      WRITE (6,1) T,IT,ETAO,R,LO,V
    1 FORMAT(1X,3HT =,E14.7,23X,I5,21X,5HETA =,E14.7/
     *        14X,3HR =,E14.7,31X,5HLAT =,E14.7/
     *        13X,4HVR =,E14.7,4X,7HVLONG =,E14.7,5X,6HVLAT =,E14.7)
      PUNCH 2, T,ETAO,R,LO
      PUNCH 2, V
    2 FORMAT(5(E14.7,1X))
      RETURN
      END
```

```
$IBFTC DJAH..
      FUNCTION ETAF(E,M)
C**** INVERTS THE RELATIONS  M=EE-E*SIN(EE) AND
C     TAN(EE/2)*SQRT(1.+E)=TAN(ETA/2)*SQRT(1.-E)
      REAL M,MT
      LOGICAL K
      COMMON /CPI/HALFPI,PI,TWOPI,DEGREE,RADIAN
      PIEPS=.000 000 3
      K=.FALSE.
      IF(M.LE.PI) GO TO 1
      K=.TRUE.
      M=TWOPI-M
    1 IF(ABS(PI-M).GT.PIEPS) GO TO 2
      ETAF=PI
      RETURN
    2 EE=0.
      DE=2.
    3 IF((1.+DE).EQ.1.) GO TO 5
    4 ET=EE+DE
      DE=.5*DE
      IF(ET.GE.PI) GO TO 4
      MT=ET-E*SIN(ET)
      IF(MT.GT.M) GO TO 3
      EE=ET
      IF(MT.NE.M) GO TO 3
    5 S=TAN(.5*EE)
      Q=S*SQRT((1.+E)/(1.-E))
      ETAF=2.*ATAN(Q)
      IF(K) ETAF=TWOPI-ETAF
      RETURN
      END
```

```
$IBFTC DJAI..
      SUBROUTINE ROT(SL,CL)
C**** ROTATES SPACECRAFT-FIXED VECTORS TO SPACE-FIXED CO-ORDINATES
      COMMON /CSC/NF,ENF(3,100),ENFR(3,100),AREA(100),AS
      DO 1 IF=1,NF
      X=ENF(1,IF)
      Y=ENF(2,IF)
      Z=ENF(3,IF)
      ENFR(1,IF)=CL*Y+SL*Z
      ENFR(2,IF)=X
    1 ENFR(3,IF)=CL*Z-SL*Y
      RETURN
      END
```

```
$IBFTC DJAJ..
      FUNCTION DOT(X,J)
C**** TAKES DOT PRODUCT OF DIRECTION AND VELOCITY VECTORS
      DIMENSION X(3)
      COMMON /CSC/NF,ENF(3,100),ENFR(3,100),AREA(100),AS
      DOT=0.
      DO 1 I=1,3
    1 DOT=DOT+X(I)*ENFR(I,J)
      RETURN
      END
```

```
$IBFTC DJAK..
      SUBROUTINE UAST(A,E,I,RI,LI,U)
C**** COMPUTES METEOROID SWARM VELOCITIES AT SPECIFIED POSITION
      REAL I,LI,LA
      DIMENSION U(2,2,3),UC(3)
      COMMON /CG/GMC,VC
      RA=RI
      LA=LI
      RMIN=A*(1.-E)
      IF(RA.LT.RMIN) RA=RMIN
      RMAX=A*(1.+E)
      IF(RA.GT.RMAX) RA=RMAX
      IF(ABS(LA).GT.I) LA=SIGN(I,LA)
      UT=VC*SQRT(2./RA-1./A)
      UA=VC*SQRT(A*(1.-E*E))/RA
      UP=SQRT(AMAX1(0.,UT*UT-UA*UA))
      SL=SIN(LA)
      CL=SQRT(1.-SL*SL)
      CA=COS(I)/CL
      SA=SQRT(1.-CA*CA)
      UC(1)=UP
      UC(2)=UA*CA
      UC(3)=UA*SA
      DO 1 L=1,2
      DO 1 M=1,2
      U(L,M,1)=SIGN(UC(1),(-1.)**L)
      U(L,M,2)=UC(2)
    1 U(L,M,3)=SIGN(UC(3),(-1.)**M)
      RETURN
      END
```

```
$IBFTC DJAL..
      FUNCTION SIGMA(A,E,I,R,L)
C**** COMPUTES METEOROID SWARM SPACE DENSITY AT SPECIFIED LOCATION
      REAL I,L
      COMMON /CCETC/C1,C2,RHOSC,HSC,RHOAST,BETA3,EPSR,EPSL,C
      SIGMA=(SSTAR(R*(1.+EPSR),A,E)-SSTAR(R*(1.-EPSR),A,E))
     *     *(RSTAR(L+EPSL,I)-RSTAR(L-EPSL,I))
     *     *C/(R**3*COS(L))
      RETURN
      END
```

```
$IBFTC DJAM..
      FUNCTION RSTAR(L,I)
C**** EVALUATES LATITUDE-DEPENDENCE OF METEOROID SWARM SPACE DENSITY
      REAL L,I
      RSTAR=ASIN(SIN(L)/SIN(I))
      RETURN
      END
```

```
$IBFTC DJAN..
      FUNCTION ASIN(X)
C**** THIS FUNCTION IS USED BECAUSE THE FORTRAN IV LIBRARY FUNCTION
C****    ARSIN HAS ARSIN(X)=0., AND MAY HALT EXECUTION OF THE PROGRAM,
C****    WHEN ABS(X) IS GREATER THAN 1.
      COMMON /CPI/HALFPI,PI,TWOPI,DEGREE,RADIAN
      IF(ABS(X)-1.)2,1,1
    1 ASIN=SIGN(HALFPI,X)
      RETURN
    2 ASIN=ARSIN(X)
      RETURN
      END
```

```
$IBFTC DJAO..
      FUNCTION SSTAR(R,A,E)
C**** EVALUATES RADIAL DEPENDENCE OF METEOROID SWARM SPACE DENSITY
      X=ACOS((A-R)/(A*E))
      SSTAR=X-E*SIN(X)
      RETURN
      END
```

```
$IBFTC DJAP..
      FUNCTION ACOS(X)
C**** THIS FUNCTION IS USED BECAUSE THE FORTRAN IV LIBRARY FUNCTION
C****    ARCOS HAS ARCOS(X)=0., AND MAY HALT EXECUTION OF THE PROGRAM,
C****    WHEN ABS(X) IS GREATER THAN 1.
      COMMON /CPI/HALFPI,PI,TWOPI,DEGREE,RADIAN
      ACOS=HALFPI-ASIN(X)
      RETURN
      END
```

```
$IBFTC DJAQ..
      SUBROUTINE INTER(F,DEN,NDEN,FPI,FI,FNU,IT,NT)
C**** EVALUATES, OUTPUTS INTERMEDIATE DATA DURING A CASE
      DIMENSION F(100),FI(100)
      COMMON /CCP/CP,CU,CC,DAY,CRHO
      COMMON /CSC/NF,ENF(3,100),ENFR(3,100),AREA(100),AS
      DEN=4.*DEN
      WRITE (6,1) DEN,NDEN
    1 FORMAT(28X,5HDEN =,E14.7,6X,6HNDEN =,I5/4X,7HJ   F(J))
      DO 2 J=1,NF
    2 F(J)=F(J)*CP
      WRITE (6,3) (J,F(J),J=1,NF)
    3 FORMAT(5(I5,1X,E14.7,1X))
      FPI=0.
      DO 4 J=1,NF
    4 FPI=FPI+AREA(J)*F(J)
      WRITE (6,5) FPI
    5 FORMAT(13X,5HFPI =,E14.7//)
      DO 6 J=1,NF
      FI(J)=FI(J)+F(J)
    6 IF(IT.EQ.1 .OR. IT.EQ.NT) FI(J)=FI(J)-.5*F(J)
      FNU=FNU+FPI
      IF(IT.EQ.1 .OR. IT.EQ.NT) FNU=FNU-.5*FPI
      PUNCH 7, DEN,NDEN
      PUNCH 7, (F(J),J=1,NF)
      PUNCH 7, FPI
    7 FORMAT(5(E14.7,1X))
      RETURN
      END
```

```
$IBFTC DJAR..
      SUBROUTINE POST(FI,FNU,DT,BETA3)
C**** EVALUATES, OUTPUTS FINAL DATA FOR A CASE
      COMMON /CCP/CP,CU,CC,DAY,CRHO
      COMMON /CCETC/C1,C2,RHOSC,HSC,RHOAST,BETA3,EPSR,EPSL,C
      COMMON /CSC/NF,ENF(3,100),ENFR(3,100),AREA(100),AS
      DIMENSION FI(100),FIF(100),AFI(100),AFIF(100),TAU(100), X(100)
      DIMENSION CA(4),CB(4)
      FIS=0.
      AFIS=0.
      DO 1 J=1,NF
      FI(J)=FI(J)*DT*DAY
      AFI(J)=AREA(J)*FI(J)
      FIS=FIS+FI(J)
    1 AFIS=AFIS+AFI(J)
      IF(FIS.EQ.0.) FIS=1.
      IF(AFIS.EQ.0.) AFIS=1.
      DO 2 J=1,NF
      FIF(J)=FI(J)/FIS
    2 AFIF(J)=AFI(J)/AFIS
      AXS=0.
      DO 3 J=1,NF
      X(J)=FI(J)**(1./(BETA3+1.))
    3 AXS=AXS+AREA(J)*X(J)
      XX=AS/AXS
      DO 4 J=1,NF
    4 TAU(J)=X(J)*XX
      CO=AXS**(BETA3+1.)/AS**BETA3
      FNU=FNU*DT*DAY
      PS=EXP(-FNU)
      CHECK=.05-ALOG10(AMAX1(1.E-10,ABS(FNU-AFIS)*2./(FNU+AFIS)))
      RHO=CRHO*RHOSC
      CA(1)=FNU
      CA(2)=FNU*RHO**BETA3/AS
      CA(3)=FNU*(RHO*AS)**BETA3
      CA(4)=FNU/AS
      CB(1)=CO
      CB(2)=CO*RHO**BETA3/AS
      CB(3)=CO*(RHO*AS)**BETA3
      CB(4)=CO/AS
      WRITE (6,5)
    5 FORMAT(1H0,3X,1HJ,5X,5HFI(J),10X,6HFIF(J),
     *                   6X,6HAFI(J),9X,7HAFIF(J),5X,6HTAU(J))
      WRITE (6,6) (J,FI(J),FIF(J),AFI(J),AFIF(J),TAU(J),J=1,NF)
    6 FORMAT(I5,2X,E14.7,F11.7,2X,E14.7,F11.7,2X,E14.7)
      WRITE (6,7) FNU,PS,CHECK
    7 FORMAT(1H0,10X,5HFNU =,E14.7,11X,6HP(S) =,E14.7,10X,F5.1)
      WRITE (6,8) (CA(I),CB(I),I=1,4)
    8 FORMAT(1H0,13X,6HCA    =,E14.7,4X,6HCB    =,E14.7/
     *           14X,6HCA-   =,E14.7,4X,6HCB-   =,E14.7/
     *           14X,6HCA--  =,E14.7,4X,6HCB--  =,E14.7/
     *           14X,6HCA---=,E14.7,4X,6HCB---=,E14.7)
      PUNCH 9, (FI(J),J=1,NF)
      PUNCH 9, (TAU(J),J=1,NF)
      PUNCH 9, FNU,PS
      PUNCH 9, CA
      PUNCH 9, CB
    9 FORMAT(5(E14.7,1X))
      DATA ASTRSK/4H****/
      PUNCH 10, (ASTRSK,I=1,20)
   10 FORMAT(20A4)
      RETURN
      END
```

```
$IBFTC DJAZ..
      BLOCK DATA
      COMMON /CPI/HALFPI,PI,TWOPI,DEGREE,RADIAN
      COMMON /CG/GMC,VC
      COMMON /CCP/CP,CU,CC,DAY,CRHO
      DATA HALFPI/1.5707963/,PI/3.1415927/,TWOPI/6.2831853/,
     *         DEGREE/.017453293/,RADIAN/57.295780/
      DATA GMC/.017202099/,VC/29.784696/
      DATA CU/2.9869199E-31/,CC/4.3E+5/,DAY/86400./,CRHO/10./
C**** AU = ASTRONOMICAL UNIT
C****    = 149 597 892. KILOMETERS (ASTRONOMICAL JOURNAL, APRIL 1967)
C**** GMC = GAUSSIAN GRAVITATIONAL CONSTANT FOR THE SUN
C****      = .017 202 098 95 (ASTRONOMICAL UNITS)**1.5/DAY
C**** VC = EMOS = EARTH MEAN ORBITAL SPEED = GMC/AU**.5
C****    = .017 202 098 95 AU/DAY  = 29.784 696 08 KILOMETERS/SECOND
C**** CU = ((1 KILOMETER/SECOND)/(1 METER/SECOND))/(1 AU/1 METER)**
C****    = 2.986 919 914 E-31
C**** CC = R0/T*
C****    R0 = RADIUS OF STANDARD ASTEROID (4.3 KILOMETERS)
C****    T* = THICKNESS OF REFERENCE SPACECRAFT SHIELDING (1 CENTIMETER)
C**** DAY = 1 DAY/1 SECOND  = 86 400.
C**** CRHO =
C****    (1 GRAM/CENTIMETER**3)/(1 KILOGRAM/(1 CENTIMETER*1 METER**2))
      END
```

IV. Sample Problem Input

The following is a sample problem with input, including asteroid belt data.

```
$DATA
 150
 100 18513  7590 276747   100 60734 23402 277181   100 22677 25848 266832    1
 100 12448  8888 236166   100  9315 18534 257898   100 25754 20267 242591    2
 100  9601 22978 238590   100 10287 15693 220160   100  9741 12197 238626    3
 100  6655  9961 315083   100  8083 10212 245145   100 14624 22071 233317    4
 100 28850  8530 257633   100 15933 16443 258779   100 20450 18628 264250    5
 100  5390 13530 292282   100  9762 13779 246924   100 17701 21882 229542    6
 100  2716 15761 244183   100  1218 14334 240879   100  5372 16117 243612    7
 100 23953 10302 290915   100 17741 23579 262467   100  1340 12083 313800    8
 100 37648 25533 240068   100  6217  8808 265556   100  2770 17217 234716    9
 100 16418 15369 277507   100 10620  7360 255439   100  3672 12700 236573   10
 100 45932 21271 316876   100  9510  8197 258763   100  3323 34003 286217   11
 100  9619 10814 268682   100 14020 21518 300585   100 32364 30056 274877   12
 100  5372 17528 264275   100 12163 15547 273858   100 18111 11201 277019   13
 100  7432  4700 226706   100 27704 27031 276241   100 14868 22539 244097   14
 100  6051 16816 220323   100  6477 15052 242189   100 11523  8058 272136   15
 100  4040 17038 252454   100  8716 13501 287733   100 11369  7219 311162   16
 100  5507 22408 309891   100  4929 28678 264952   100 17371  6561 236572   17
 100 13011 11062 309557   100  8985 20677 261644   100 20616 19966 270959   18
 100 12559 14263 275982   100 14043 23767 259730   100 26470 10048 315777   19
 100  8809  4449 269928   100 15060 11672 271405   100  6267 18347 239330   20
 100 31748 16090 298907   100  3871 16932 313441   100 10093 12748 239461   21
 100  2290 12443 268181   100  6185 12123 342048   100  5327 17217 264674   22
 100 10489 18484 242138   100 13902 18453 278394   100 14888 16983 297828   23
 100 20279 18150 261443   100 40621 17308 275587   100  9437 12029 226635   24
 100  4150  4278 266574   100  6999 23847 277858   100  8709 30616 267155   25
 100  3634 20351 336639   100  4241 13186 266942   100 15181 20887 261861   26
 100  8055 19384 244400   100 15041 20000 229599   100 13762 21179 285348   27
 100  4958 22337 276202   100  8725  8514 243087   100 16301 23633 236246   28
 100 20851 19151 265436   100  8390 21940 310073   100 18947  9725 347866   29
 100  9137 16537 276631   100 28044 18093 255205   100  3915 17478 313684   30
 100  3684 10746 259007   100 17335  7176 319987   100 14949 14130 275411   31
 100 14017 10278 315335   100 22628 14928 306889   100 28035 13355 305504   32
 100 20577 25745 266876   100 27213 18937 268684   116 24244 19708 266380   33
 100 11198 16832 308707   100 17771 13961 258295   100  8959 25550 265911   34
 100  9440  7879 270174   100  4979 16887 313987   100 37488 17549 237355   35
 100  8072 17427 316714   100 17317  6995 348946   100  7671  9227 321292   36
 100 14022 29826 269604   100 10448  8009 273246   100  8610 10295 259310   37
 100  4538 12748 243393   100  8795  8571 237619   100  8610 13945 267519   38
 100 20230 19223 237934   100  6231 14315 276591   100 26073  2237 299264   39
 100 13586 16183 243804   100 10051  8076 258067   100 12146  5085 312089   40
 100 13237 12610 345377   100  2827  6018 321191   100 11195 12194 269361   41
 100  5123  7722 263037   100  8098  7983 274287   100  5123 10635 243912   42
 100 14413  6431 275576   100 10908 12500 275147   100 21358 20552 287409   43
 100 40064 20778 312195   100  8671  6956 243089   100 43914 38276 261226   44
 100 12498 13933 306162   100 20274 11469 256420   100  4019 20738 242810   45
 100 16696  8484 228692   100 23293 21521 312116   100  5536 16608 244804   46
 100 19160 17287 278289   100  5568 21542 273224   100 20796 21475 266531   47
 100  3903 13317 241871   100 20024  7313 275993   100  8390 23489 265440   48
 100 22052 14482 267369   100 22845  6613 271755   100  3355  2009 313856   49
 100 44213 18615 277022   101  1614  6525 217480   100  3793 12511 298246   50
```

```
  **  SHORT JUPITER MISSION  --  BETA=1.9/3  **          (CASE I)

  4.5731    .77887    2.1304    177.92              180.      20.         2   26
-1.                           1.        +1.                             1.
          -1.                 1.                  +1.                   1.
                    -1.       1.                            +1.         1.
-1.       -1.                 1.        -1.       +1.                   1.
+1.       -1.                 1.        +1.       +1.                   1.
-1.                 -1.       1.        -1.                 +1.         1.
+1.                 -1.       1.        +1.                 +1.         1.
          -1.       -1.       1.                  -1.       +1.         1.
          +1.       -1.       1.                  +1.       +1.         1.
-1.       -1.       -1.        .43301270-1.       -1.       +1.          .43301270
-1.       +1.       -1.        .43301270-1.       +1.       +1.          .43301270
+1.       -1.       -1.        .43301270+1.       -1.       +1.          .43301270
+1.       +1.       -1.        .43301270+1.       +1.       +1.          .43301270
  **  LONG JUPITER MISSION  --  BETA=1.9/3  **           (CASE II)

  3.0135    .66562    4.3296    170.09              210.      30.         2
  **  SHORT JUPITER MISSION  --  BETA=1  **              (CASE III)
                                                     3.0
  4.5731    .77887    2.1304    177.92              180.      20.         2
  **  LONG JUPITER MISSION  --  BETA=1  **               (CASE IV)
                                                     3.0
  3.0135    .66562    4.3296    170.09              210.      30.         2
$EOF
```

NOTE THE TWO BLANK CARDS - THE ONLY ONES USED - IMMEDIATELY FOLLOWING THE
CASE I AND CASE II LABEL CARDS.

V. Sample Problem Printed Output

The following is the sample problem printed output.

```
**  SHORT JUPITER MISSION  --  BETA=1.9/3  **            (CASE I)

                        NA1 =     1      NA2 =   150      NA =   150

      C1 = 0.2016000E 01        C2 = 0.3000000E 01    RHOSC = 0.2700000E 01       HSC = 0.1200000E 03
  RHOAST = 0.7900000E 01     BETA3 = 0.1900000E 01     EPSR = 0.2000000E-01      EPSL = 0.2000000E-01

     ASC = 0.4573100E 01       ESC = 0.7788700E 00      ISC = 0.2130400E 01      OMSC = 0.1779200E 03
   TPSC =-0.0000000E-38         TO = 0.1800000E 03       TF = 0.2200000E 03        DT = 0.2000000E 02
                                     NT =     2

       J        ENF(1,J)         ENF(2,J)         ENF(3,J)           AREA(J)
       1     -0.1000000E 01   -0.0000000E-38   -0.0000000E-38      0.1000000E 01
       2      0.1000000E 01   -0.0000000E-38   -0.0000000E-38      0.1000000E 01
       3     -0.0000000E-38   -0.1000000E 01   -0.0000000E-38      0.1000000E 01
       4     -0.0000000E-38    0.1000000E 01   -0.0000000E-38      0.1000000E 01
       5     -0.0000000E-38   -0.0000000E-38   -0.1000000E 01      0.1000000E 01
       6     -0.0000000E-38   -0.0000000E-38    0.1000000E 01      0.1000000E 01
       7     -0.7071068E 00   -0.7071068E 00   -0.0000000E-38      0.1000000E 01
       8     -0.7071068E 00    0.7071068E 00   -0.0000000E-38      0.1000000E 01
       9      0.7071068E 00   -0.7071068E 00   -0.0000000E-38      0.1000000E 01
      10      0.7071068E 00    0.7071068E 00   -0.0000000E-38      0.1000000E 01
      11     -0.7071068E 00   -0.0000000E-38   -0.7071068E 00      0.1000000E 01
      12     -0.7071068E 00   -0.0000000E-38    0.7071068E 00      0.1000000E 01
      13      0.7071068E 00   -0.0000000E-38   -0.7071068E 00      0.1000000E 01
      14      0.7071068E 00   -0.0000000E-38    0.7071068E 00      0.1000000E 01
      15     -0.0000000E-38   -0.7071068E 00   -0.7071068E 00      0.1000000E 01
      16     -0.0000000E-38   -0.7071068E 00    0.7071068E 00      0.1000000E 01
      17     -0.0000000E-38    0.7071068E 00   -0.7071068E 00      0.1000000E 01
      18     -0.0000000E-38    0.7071068E 00    0.7071068E 00      0.1000000E 01
      19     -0.5773503E 00   -0.5773503E 00   -0.5773503E 00      0.4330127E 00
      20     -0.5773503E 00   -0.5773503E 00    0.5773503E 00      0.4330127E 00
      21     -0.5773503E 00    0.5773503E 00   -0.5773503E 00      0.4330127E 00
      22     -0.5773503E 00    0.5773503E 00    0.5773503E 00      0.4330127E 00
      23      0.5773503E 00   -0.5773503E 00   -0.5773503E 00      0.4330127E 00
      24      0.5773503E 00   -0.5773503E 00    0.5773503E 00      0.4330127E 00
      25      0.5773503E 00    0.5773503E 00   -0.5773503E 00      0.4330127E 00
      26      0.5773503E 00    0.5773503E 00    0.5773503E 00      0.4330127E 00

                                                             AS = 0.2146410E 02

T = 0.1800000E 03                              1                       ETA = 0.1111522E 03
          R = 0.2502104E 01                                          LAT =-0.2013408E 01
         VR = 0.1613110E 02     VLONG = 0.1596456E 02               VLAT = 0.1940643E 00
                         DEN = 0.1211546E 03        NDEN =   116
  J   F(J)
  1   0.1849182E-15      2  0.1856295E-23      3  0.0000000E-38      4  0.4136114E-14      5  0.5085370E-16
  6   0.2729158E-16      7  0.0000000E-38      8  0.3068372E-14      9  0.0000000E-38     10  0.1749400E-14
 11   0.1523649E-15     12  0.1031682E-15     13  0.3922503E-17     14  0.2531968E-17     15  0.0000000E-38
 16   0.0000000E-38     17  0.2480924E-14     18  0.2303091E-14     19  0.0000000E-38     20  0.0000000E-38
 21   0.2290398E-14     22  0.2148279E-14     23  0.0000000E-38     24  0.0000000E-38     25  0.1312630E-14
 26   0.1195368E-14
          FPI = 0.1727095E-13

T = 0.2000000E 03                              2                       ETA = 0.1150851E 03
          R = 0.2685750E 01                                          LAT =-0.1960901E 01
         VR = 0.1566503E 02     VLONG = 0.1487246E 02               VLAT = 0.2162220E 00
                         DEN = 0.1290913E 03        NDEN =   123
  J   F(J)
  1   0.2099917E-15      2  0.1135755E-23      3  0.0000000E-38      4  0.4203142E-14      5  0.3926071E-16
  6   0.2254627E-16      7  0.0000000E-38      8  0.3158675E-14      9  0.0000000E-38     10  0.1736095E-14
 11   0.1553686E-15     12  0.1092334E-15     13  0.2831270E-17     14  0.1863290E-17     15  0.0000000E-38
 16   0.0000000E-38     17  0.2502927E-14     18  0.2342530E-14     19  0.0000000E-38     20  0.0000000E-38
 21   0.2344649E-14     22  0.2215986E-14     23  0.0000000E-38     24  0.0000000E-38     25  0.1291815E-14
 26   0.1186931E-14
          FPI = 0.1753261E-13
```

T = 0.2200000E 03 3 ETA = 0.1185211E 03

R = 0.2863991E 01 LAT =-0.1907457E 01

VR = 0.1519737E 02 VLONG = 0.1394644E 02 VLAT = 0.2310191E 00

DEN = 0.8988587E 02 NDEN = 112

J	F(J)	J	F(J)	J	F(J)	J	F(J)	J	F(J)
1	0.1453715E-15	2	0.7874782E-24	3	0.0000000E-38	4	0.2789971E-14	5	0.3021185E-16
6	0.1861206E-16	7	0.0000000E-38	8	0.2110169E-14	9	0.0000000E-38	10	0.1135647E-14
11	0.1086411E-15	12	0.8094324E-16	13	0.1897863E-17	14	0.1265224E-17	15	0.0000000E-38
16	0.0000000E-38	17	0.1653639E-14	18	0.1559359E-14	19	0.0000000E-38	20	0.0000000E-38
21	0.1560695E-14	22	0.1484871E-14	23	0.0000000E-38	24	0.0000000E-38	25	0.8413287E-15
26	0.7801077E-15								

FPI = 0.1165660E-13

J	FI(J)	FIF(J)	AFI(J)	AFIF(J)	TAU(J)
1	0.6482359E-09	0.0095503	0.6482359E-09	0.0117243	0.1073778E 01
2	0.4246805E-17	0.0000000	0.4246805E-17	0.0000000	0.1617928E-02
3	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
4	0.1324717E-07	0.1951675	0.1324717E-07	0.2395954	0.3039297E 01
5	0.1378831E-09	0.0020314	0.1378831E-09	0.0024938	0.6296712E 00
6	0.7862070E-10	0.0011583	0.7862070E-10	0.0014220	0.5187819E 00
7	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
8	0.9932450E-08	0.1463325	0.9932450E-08	0.1796436	0.2751989E 01
9	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
10	0.5492652E-08	0.0809220	0.5492652E-08	0.0993431	0.2243527E 01
11	0.4939861E-09	0.0072778	0.4939861E-09	0.0089345	0.9777290E 00
12	0.3478276E-09	0.0051245	0.3478276E-09	0.0062910	0.8663309E 00
13	0.9921231E-11	0.0001462	0.9921231E-11	0.0001794	0.2540941E 00
14	0.6500538E-11	0.0000958	0.6500538E-11	0.0001176	0.2196234E 00
15	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
16	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
17	0.7897321E-08	0.1163494	0.7897321E-08	0.1428352	0.2542786E 01
18	0.7385049E-08	0.1088023	0.7385049E-08	0.1335700	0.2484656E 01
19	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
20	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
21	0.7378897E-08	0.1087116	0.3195156E-08	0.0577893	0.2483942E 01
22	0.6968266E-08	0.1026619	0.3017348E-08	0.0545734	0.2435380E 01
23	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
24	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
25	0.4093277E-08	0.0603053	0.1772441E-08	0.0320573	0.2027184E 01
26	0.3757828E-08	0.0553632	0.1627187E-08	0.0294302	0.1968286E 01

FNU = 0.5528974E-07 P(S) = 0.9999999E 00 8.1

CA = 0.5528974E-07 CB = 0.1131863E-07

CA' = 0.1350588E-05 CB' = 0.2764854E-06

CA'' = 0.9828558E-02 CB'' = 0.2012051E-02

CA'''= 0.2575917E-08 CB'''= 0.5273284E-09

** LONG JUPITER MISSION -- BETA=1.9/3 ** (CASE II)

NA1 = 1 NA2 = 150 NA = 150

C1 = 0.2016000E 01 C2 = 0.3000000E 01 RHOSC = 0.2700000E 01 HSC = 0.1200000E 03

RHOAST = 0.7900000E 01 BETA3 = 0.1900000E 01 EPSR = 0.2000000E-01 EPSL = 0.2000000E-01

ASC = 0.3013500E 01 ESC = 0.6656200E 00 ISC = 0.4329600E 01 OMSC = 0.1700900E 03

TPSC =-0.0000000E-38 TO = 0.2100000E 03 TF = 0.2700000E 03 DT = 0.3000000E 02

NT = 2

J	ENF(1,J)	ENF(2,J)	ENF(3,J)	AREA(J)
1	-0.1000000E 01	-0.0000000E-38	-0.0000000E-38	0.1000000E 01
2	0.1000000E 01	-0.0000000E-38	-0.0000000E-38	0.1000000E 01
3	-0.0000000E-38	-0.1000000E 01	-0.0000000E-38	0.1000000E 01
4	-0.0000000E-38	0.1000000E 01	-0.0000000E-38	0.1000000E 01
5	-0.0000000E-38	-0.0000000E-38	-0.1000000E 01	0.1000000E 01
6	-0.0000000E-38	-0.0000000E-38	0.1000000E 01	0.1000000E 01
7	-0.7071068E 00	-0.7071068E 00	-0.0000000E-38	0.1000000E 01
8	-0.7071068E 00	0.7071068E 00	-0.0000000E-38	0.1000000E 01
9	0.7071068E 00	-0.7071068E 00	-0.0000000E-38	0.1000000E 01
10	0.7071068E 00	0.7071068E 00	-0.0000000E-38	0.1000000E 01

11	-0.7071068E 00	-0.0000000E-38	-0.7071068E 00	0.1000000E 01
12	-0.7071068E 00	-0.0000000E-38	0.7071068E 00	0.1000000E 01
13	0.7071068E 00	-0.0000000E-38	-0.7071068E 00	0.1000000E 01
14	0.7071068E 00	-0.0000000E-38	0.7071068E 00	0.1000000E 01
15	-0.0000000E-38	-0.7071068E 00	-0.7071068E 00	0.1000000E 01
16	-0.0000000E-38	-0.7071068E 00	0.7071068E 00	0.1000000E 01
17	-0.0000000E-38	0.7071068E 00	-0.7071068E 00	0.1000000E 01
18	-0.0000000E-38	0.7071068E 00	0.7071068E 00	0.1000000E 01
19	-0.5773503E 00	-0.5773503E 00	-0.5773503E 00	0.4330127E 00
20	-0.5773503E 00	-0.5773503E 00	0.5773503E 00	0.4330127E 00
21	-0.5773503E 00	0.5773503E 00	-0.5773503E 00	0.4330127E 00
22	-0.5773503E 00	0.5773503E 00	0.5773503E 00	0.4330127E 00
23	0.5773503E 00	-0.5773503E 00	-0.5773503E 00	0.4330127E 00
24	0.5773503E 00	-0.5773503E 00	0.5773503E 00	0.4330127E 00
25	0.5773503E 00	0.5773503E 00	-0.5773503E 00	0.4330127E 00
26	0.5773503E 00	0.5773503E 00	0.5773503E 00	0.4330127E 00

AS = 0.2146410E 02

T = 0.2100000E 03 1 ETA = 0.1209315E 03
R = 0.2551244E 01 LAT =-0.4040952E 01
VR = 0.1312662E 02 VLONG = 0.1511906E 02 VLAT = 0.4106112E 00
DEN = 0.1085456E 03 NDEN = 106

J	F(J)	J	F(J)	J	F(J)	J	F(J)	J	F(J)
1	0.2395207E-15	2	0.0000000E-38	3	0.0000000E-38	4	0.2676346E-14	5	0.4831894E-16
6	0.2008363E-16	7	0.0000000E-38	8	0.2201890E-14	9	0.0000000E-38	10	0.9273736E-15
11	0.1869351E-15	12	0.1145161E-15	13	0.2626289E-17	14	0.1528392E-17	15	0.1481220E-19
16	0.6325745E-21	17	0.1630825E-14	18	0.1433756E-14	19	0.2215644E-19	20	0.1810658E-20
21	0.1667863E-14	22	0.1504412E-14	23	0.4840155E-20	24	0.8860210E-23	25	0.7215725E-15
26	0.6023501E-15								

FPI = 0.1143066E-13

T = 0.2400000E 03 2 ETA = 0.1263439E 03
R = 0.2771720E 01 LAT =-0.3876202E 01
VR = 0.1232616E 02 VLONG = 0.1391365E 02 VLAT = 0.4689396E 00
DEN = 0.8031670E 02 NDEN = 103

J	F(J)	J	F(J)	J	F(J)	J	F(J)	J	F(J)
1	0.1676024E-15	2	0.0000000E-38	3	0.0000000E-38	4	0.1794042E-14	5	0.3232776E-16
6	0.1750183E-16	7	0.0000000E-38	8	0.1490448E-14	9	0.0000000E-38	10	0.6046552E-15
11	0.1249575E-15	12	0.8752027E-16	13	0.1967489E-17	14	0.1258864E-17	15	0.8127251E-20
16	0.4702295E-21	17	0.1074043E-14	18	0.9739916E-15	19	0.1970092E-19	20	0.2105463E-20
21	0.1113580E-14	22	0.1030275E-14	23	0.2560355E-20	24	0.2095400E-22	25	0.4612316E-15
26	0.4015061E-15								

FPI = 0.7672226E-14

T = 0.2700000E 03 3 ETA = 0.1309803E 03
R = 0.2978540E 01 LAT =-0.3707513E 01
VR = 0.1155276E 02 VLONG = 0.1294500E 02 VLAT = 0.5058029E 00
DEN = 0.5304787E 02 NDEN = 88

J	F(J)	J	F(J)	J	F(J)	J	F(J)	J	F(J)
1	0.1311463E-15	2	0.0000000E-38	3	0.0000000E-38	4	0.1067624E-14	5	0.2346360E-16
6	0.1594421E-16	7	0.0000000E-38	8	0.9342365E-15	9	0.0000000E-38	10	0.3225918E-15
11	0.9270656E-16	12	0.7521975E-16	13	0.1189147E-17	14	0.9009406E-18	15	0.3647702E-20
16	0.1307949E-21	17	0.6297591E-15	18	0.5880604E-15	19	0.1384669E-19	20	0.3007863E-20
21	0.6899551E-15	22	0.6544597E-15	23	0.1003173E-20	24	0.2021780E-23	25	0.2435721E-15
26	0.2201631E-15								

FPI = 0.4665806E-14

J	FI(J)	FIF(J)	AFI(J)	AFIF(J)	TAU(J)
1	0.9148099E-09	0.0183702	0.9148099E-09	0.0224507	0.1293840E 01
2	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
3	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
4	0.9502341E-08	0.1908154	0.9502341E-08	0.2332010	0.2899999E 01
5	0.1768237E-09	0.0035508	0.1768237E-09	0.0043395	0.7340846E 00
6	0.9205682E-10	0.0018486	0.9205682E-10	0.0022592	0.5861284E 00
7	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
8	0.7927660E-08	0.1591944	0.7927660E-08	0.1945561	0.2724363E 01
9	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
10	0.3187221E-08	0.0640022	0.3187221E-08	0.0782190	0.1989781E 01
11	0.6863055E-09	0.0137816	0.6863055E-09	0.0168429	0.1171768E 01
12	0.4727502E-09	0.0094932	0.4727502E-09	0.0116020	0.1030431E 01
13	0.1004454E-10	0.0002017	0.1004454E-10	0.0002465	0.2730393E 00
14	0.6411392E-11	0.0001287	0.6411392E-11	0.0001573	0.2338792E 00
15	0.4498987E-13	0.0000009	0.4498987E-13	0.0000011	0.4229499E-01

16	0.2208162E-14	0.0000000	0.2208162E-14	0.0000001	0.1495825E-01
17	0.5713635E-08	0.1147348	0.5713635E-08	0.1402208	0.2433432E 01
18	0.5144860E-08	0.1033133	0.5144860E-08	0.1262622	0.2347016E 01
19	0.9772485E-13	0.0000020	0.4231610E-13	0.0000010	0.5526600E-01
20	0.1170216E-13	0.0000002	0.5067186E-14	0.0000001	0.2658380E-01
21	0.5942131E-08	0.1193232	0.2573018E-08	0.0631455	0.2466559E 01
22	0.5468370E-08	0.1098097	0.2367874E-08	0.0581110	0.2396893E 01
23	0.1420939E-13	0.0000003	0.6152847E-14	0.0000002	0.2842426E-01
24	0.6841582E-16	0.0000000	0.2962492E-16	0.0000000	0.4514184E-02
25	0.2446340E-08	0.0491247	0.1059296E-08	0.0259966	0.1816294E 01
26	0.2106681E-08	0.0423040	0.9122196E-09	0.0223872	0.1725046E 01

FNU = 0.4074742E-07   P(S) = 0.1000000E 01   10.1

CA = 0.4074742E-07   CB = 0.9302285E-08
CA' = 0.9953564E-06   CB' = 0.2272313E-06
CA'' = 0.7243449E-02   CB'' = 0.1653617E-02
CA'''= 0.1898399E-08   CB'''= 0.4333881E-09

** SHORT JUPITER MISSION -- BETA=1 **   (CASE III)

NA1 = 1   NA2 = 150   NA = 150

C1 = 0.2016000E 01   C2 = 0.3000000E 01   RHOSC = 0.2700000E 01   HSC = 0.1200000E 03
RHOAST = 0.7900000E 01   BETA3 = 0.3000000E 01   EPSR = 0.2000000E-01   EPSL = 0.2000000E-01

ASC = 0.4573100E 01   ESC = 0.7788700E 00   ISC = 0.2130400E 01   OMSC = 0.1779200E 03
TPSC =-0.0000000E-38   T0 = 0.1800000E 03   TF = 0.2200000E 03   DT = 0.2000000E 02
NT = 2

J	ENF(1,J)	ENF(2,J)	ENF(3,J)	AREA(J)
1	-0.1000000E 01	-0.0000000E-38	-0.0000000E-38	0.1000000E 01
2	0.1000000E 01	-0.0000000E-38	-0.0000000E-38	0.1000000E 01
3	-0.0000000E-38	-0.1000000E 01	-0.0000000E-38	0.1000000E 01
4	-0.0000000E-38	0.1000000E 01	-0.0000000E-38	0.1000000E 01
5	-0.0000000E-38	-0.0000000E-38	-0.1000000E 01	0.1000000E 01
6	-0.0000000E-38	-0.0000000E-38	0.1000000E 01	0.1000000E 01
7	-0.7071068E 00	-0.7071068E 00	-0.0000000E-38	0.1000000E 01
8	-0.7071068E 00	0.7071068E 00	-0.0000000E-38	0.1000000E 01
9	0.7071068E 00	-0.7071068E 00	-0.0000000E-38	0.1000000E 01
10	0.7071068E 00	0.7071068E 00	-0.0000000E-38	0.1000000E 01
11	-0.7071068E 00	-0.0000000E-38	-0.7071068E 00	0.1000000E 01
12	-0.7071068E 00	-0.0000000E-38	0.7071068E 00	0.1000000E 01
13	0.7071068E 00	-0.0000000E-38	-0.7071068E 00	0.1000000E 01
14	0.7071068E 00	-0.0000000E-38	0.7071068E 00	0.1000000E 01
15	-0.0000000E-38	-0.7071068E 00	-0.7071068E 00	0.1000000E 01
16	-0.0000000E-38	-0.7071068E 00	0.7071068E 00	0.1000000E 01
17	-0.0000000E-38	0.7071068E 00	-0.7071068E 00	0.1000000E 01
18	-0.0000000E-38	0.7071068E 00	0.7071068E 00	0.1000000E 01
19	-0.5773503E 00	-0.5773503E 00	-0.5773503E 00	0.4330127E 00
20	-0.5773503E 00	-0.5773503E 00	0.5773503E 00	0.4330127E 00
21	-0.5773503E 00	0.5773503E 00	-0.5773503E 00	0.4330127E 00
22	-0.5773503E 00	0.5773503E 00	0.5773503E 00	0.4330127E 00
23	0.5773503E 00	-0.5773503E 00	-0.5773503E 00	0.4330127E 00
24	0.5773503E 00	-0.5773503E 00	0.5773503E 00	0.4330127E 00
25	0.5773503E 00	0.5773503E 00	-0.5773503E 00	0.4330127E 00
26	0.5773503E 00	0.5773503E 00	0.5773503E 00	0.4330127E 00

AS = 0.2146410E 02

T = 0.1800000E 03   1   ETA = 0.1111522E 03
R = 0.2502104E 01   LAT =-0.2013408E 01
VR = 0.1613110E 02   VLUNG = 0.1596456E 02   VLAT = 0.1940643E 00
DEN = 0.1211546E 03   NDEN = 116

J	F(J)								
1	0.2402165E-08	2	0.2405121E-18	3	0.0000000E-38	4	0.1142221E-06	5	0.7022925E-09
6	0.3814752E-09	7	0.0000000E-38	8	0.7942508E-07	9	0.0000000E-38	10	0.4019007E-07
11	0.2104848E-08	12	0.1348515E-08	13	0.5450210E-10	14	0.3632679E-10	15	0.0000000E-38

```
 16  0.0000000E-38    17  0.6164019E-07    18  0.5632573E-07    19  0.0000000E-38    20  0.0000000E-38
 21  0.5576346E-07    22  0.5158015E-07    23  0.0000000E-38    24  0.0000000E-38    25  0.2838820E-07
 26  0.2532675E-07
          FPI = 0.4285737E-06

T = 0.2000000E 03                           2                       ETA = 0.1150851E 03
           R = 0.2685750E 01                                  LAT =-0.1960901E 01
          VR = 0.1566503E 02     VLONG = 0.1487246E 02       VLAT = 0.2162220E 00
                        DEN = 0.1290913E 03       NDEN =    123
  J  F(J)
  1  0.2806445E-08     2  0.1263765E-18     3  0.0000000E-38     4  0.1148001E-06     5  0.5286038E-09
  6  0.3070850E-09     7  0.0000000E-38     8  0.8108653E-07     9  0.0000000E-38    10  0.3917697E-07
 11  0.2081612E-08    12  0.1387968E-08    13  0.3746436E-10    14  0.2542316E-10    15  0.0000000E-38
 16  0.0000000E-38    17  0.6131333E-07    18  0.5656737E-07    19  0.0000000E-38    20  0.0000000E-38
 21  0.5648774E-07    22  0.5272789E-07    23  0.0000000E-38    24  0.0000000E-38    25  0.2734110E-07
 26  0.2464667E-07
          FPI = 0.4299220E-06

T = 0.2200000E 03                           3                       ETA = 0.1185211E 03
           R = 0.2863991E 01                                  LAT =-0.1907457E 01
          VR = 0.1519737E 02     VLONG = 0.1394644E 02       VLAT = 0.2310191E 00
                        DEN = 0.8988587E 02       NDEN =    112
  J  F(J)
  1  0.1928899E-08     2  0.7628982E-19     3  0.0000000E-38     4  0.7542844E-07     5  0.3985639E-09
  6  0.2430184E-09     7  0.0000000E-38     8  0.5367445E-07     9  0.0000000E-38    10  0.2525249E-07
 11  0.1464727E-08    12  0.1041707E-08    13  0.2406846E-10    14  0.1683594E-10    15  0.0000000E-38
 16  0.0000000E-38    17  0.4005855E-07    18  0.3730038E-07    19  0.0000000E-38    20  0.0000000E-38
 21  0.3723243E-07    22  0.3503812E-07    23  0.0000000E-38    24  0.0000000E-38    25  0.1754396E-07
 26  0.1599580F-07
          FPI = 0.2826493E-06


  J      FI(J)          FIF(J)        AFI(J)         AFIF(J)       TAU(J)
  1    0.8591576E-02  0.0052194    0.8591576E-02  0.0063294    0.1115412E 01
  2    0.4920954E-12  0.0000000    0.4920954E-12  0.0000000    0.3068524E-02
  3   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
  4    0.3622327E 00  0.2200585    0.3622327E 00  0.2668573    0.2842262E 01
  5    0.1864567E-02  0.0011327    0.1864567E-02  0.0013736    0.7613103E 00
  6    0.1070205E-02  0.0006502    0.1070205E-02  0.0007884    0.6626496E 00
  7   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
  8    0.2551155E 00  0.1549842    0.2551155E 00  0.1879440    0.2603763E 01
  9   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
 10    0.1242402E 00  0.0754766    0.1242402E 00  0.0915279    0.2175117E 01
 11    0.6681138E-02  0.0040588    0.6681138E-02  0.0049220    0.1047442E 01
 12    0.4463560E-02  0.0027116    0.4463560E-02  0.0032883    0.9469729E 00
 13    0.1326234E-03  0.0000806    0.1326234E-03  0.0000977    0.3931626E 00
 14    0.8986381E-04  0.0000546    0.8986381E-04  0.0000662    0.3567085E 00
 15   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
 16   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
 17    0.1938171E 00  0.1177451    0.1938171E 00  0.1427854    0.2430890E 01
 18    0.1786414E 00  0.1085257    0.1786414E 00  0.1316054    0.2381841E 01
 19   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
 20   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
 21    0.1779593E 00  0.1081113    0.7705862E-01  0.0567692    0.2379564E 01
 22    0.1659520E 00  0.1008168    0.7185931E-01  0.0529389    0.2338368E 01
 23   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
 24   -0.0000000E-38 -0.0000000   -0.0000000E-38 -0.0000000   -0.0000000E-38
 25    0.8693081E-01  0.0528110    0.3764214E-01  0.0277310    0.1989347E 01
 26    0.7829214E-01  0.0475629    0.3390149E-01  0.0249753    0.1937969E 01

       FNU = 0.1357402E 01          P(S) = 0.2573285E 00          10.J

          CA    = 0.1357402E 01    CB    = 0.1191362E 00
          CA'   = 0.1244764E 04    CB'   = 0.1092503E 03
          CA''  = 0.2642030E 09    CB''  = 0.2318853E 08
          CA''' = 0.6324057E-01    CB''' = 0.5550489E-02
```

** LONG JUPITER MISSION -- BETA=1 ** (CASE IV)

NA1 = 1 NA2 = 150 NA = 150

C1 = 0.2016000E 01 C2 = 0.3000000E 01 RHOSC = 0.2700000E 01 HSC = 0.1200000E 03
RHOAST = 0.7900000E 01 BETA3 = 0.3000000E 01 EPSR = 0.2000000E-01 EPSL = 0.2000000E-01

ASC = 0.3013500E 01 ESC = 0.6656200E 00 ISC = 0.4329600E 01 OMSC = 0.1700900E 03
TPSC =-0.0000000E-38 TO = 0.2100000E 03 TF = 0.2700000E 03 DT = 0.3000000E 02
NT = 2

J	ENF(1,J)	ENF(2,J)	ENF(3,J)	AREA(J)
1	-0.1000000E 01	-0.0000000E-38	-0.0000000E-38	0.1000000E 01
2	0.1000000E 01	-0.0000000E-38	-0.0000000E-38	0.1000000E 01
3	-0.0000000E-38	-0.1000000E 01	-0.0000000E-38	0.1000000E 01
4	-0.0000000E-38	0.1000000E 01	-0.0000000E-38	0.1000000E 01
5	-0.0000000E-38	-0.0000000E-38	-0.1000000E 01	0.1000000E 01
6	-0.0000000E-38	-0.0000000E-38	0.1000000E 01	0.1000000E 01
7	-0.7071068E 00	-0.7071068E 00	-0.0000000E-38	0.1000000E 01
8	-0.7071068E 00	0.7071068E 00	-0.0000000E-38	0.1000000E 01
9	0.7071068E 00	-0.7071068E 00	-0.0000000E-38	0.1000000E 01
10	0.7071068E 00	0.7071068E 00	-0.0000000E-38	0.1000000E 01
11	-0.7071068E 00	-0.0000000E-38	-0.7071068E 00	0.1000000E 01
12	-0.7071068E 00	-0.0000000E-38	0.7071068E 00	0.1000000E 01
13	0.7071068E 00	-0.0000000E-38	-0.7071068E 00	0.1000000E 01
14	0.7071068E 00	-0.0000000E-38	0.7071068E 00	0.1000000E 01
15	-0.0000000E-38	-0.7071068E 00	-0.7071068E 00	0.1000000E 01
16	-0.0000000E-38	-0.7071068E 00	0.7071068E 00	0.1000000E 01
17	-0.0000000E-38	0.7071068E 00	-0.7071068E 00	0.1000000E 01
18	-0.0000000E-38	0.7071068E 00	0.7071068E 00	0.1000000E 01
19	-0.5773503E 00	-0.5773503E 00	-0.5773503E 00	0.4330127E 00
20	-0.5773503E 00	-0.5773503E 00	0.5773503E 00	0.4330127E 00
21	-0.5773503E 00	0.5773503E 00	-0.5773503E 00	0.4330127E 00
22	-0.5773503E 00	0.5773503E 00	0.5773503E 00	0.4330127E 00
23	0.5773503E 00	-0.5773503E 00	-0.5773503E 00	0.4330127E 00
24	0.5773503E 00	-0.5773503E 00	0.5773503E 00	0.4330127E 00
25	0.5773503E 00	0.5773503E 00	-0.5773503E 00	0.4330127E 00
26	0.5773503E 00	0.5773503E 00	0.5773503E 00	0.4330127E 00

AS = 0.2146410E 02

T = 0.2100000E 03 1 ETA = 0.1209315E 03
R = 0.2551244E 01 LAT =-0.4040952E 01
VR = 0.1312662E 02 VLONG = 0.1511906E 02 VLAT = 0.4106112E 00
DEN = 0.1085456E 03 NDEN = 106

J	F(J)								
1	0.3415291E-08	2	0.0000000E-38	3	0.0000000E-38	4	0.6904582E-07	5	0.6637029E-09
6	0.2888663E-09	7	0.0000000E-38	8	0.5429970E-07	9	0.0000000E-38	10	0.1893931E-07
11	0.2709444E-08	12	0.1535126E-08	13	0.3737212E-10	14	0.2155003E-10	15	0.1313150E-12
16	0.1576081E-14	17	0.3789608E-07	18	0.3240396E-07	19	0.1553654E-12	20	0.7097765E-14
21	0.3877805E-07	22	0.3418969E-07	23	0.3148191E-13	24	0.4427075E-17	25	0.1400982E-07
26	0.1124528E-07								

FPI = 0.2637881E-06

T = 0.2400000E 03 2 ETA = 0.1263439E 03
R = 0.2771720E 01 LAT =-0.3876202E 01
VR = 0.1232616E 02 VLONG = 0.1391365E 02 VLAT = 0.4689396E 00
DEN = 0.8031670E 02 NDEN = 103

J	F(J)								
1	0.2349608E-08	2	0.0000000E-38	3	0.0000000E-38	4	0.4535306E-07	5	0.4413288E-09
6	0.2390171E-09	7	0.0000000E-38	8	0.3605786E-07	9	0.0000000E-38	10	0.1201401E-07
11	0.1765199E-08	12	0.1163692E-08	13	0.2616913E-10	14	0.1675584E-10	15	0.6457178E-13
16	0.1265577E-14	17	0.2436058E-07	18	0.2162725E-07	19	0.1397642E-12	20	0.6943074E-14
21	0.2532298E-07	22	0.2302559E-07	23	0.1487880E-13	24	0.1657812E-16	25	0.8690366E-08
26	0.7342538E-08								

FPI = 0.1732926E-06

T = 0.2700000E 03 3 ETA = 0.1309803E 03

R = 0.2978540E 01 LAT =-0.3707513E 01

VR = 0.1155276E 02 VLONG = 0.1294500E 02 VLAT = 0.5058029E 00

DEN = 0.5304787E 02 NDEN = 88

J	F(J)								
1	0.1936293E-08	2	0.0000000E-38	3	0.0000000E-38	4	0.2637867E-07	5	0.3215007E-09
6	0.2140780E-09	7	0.0000000E-38	8	0.2233488E-07	9	0.0000000E-38	10	0.6109618E-08
11	0.1361134E-08	12	0.1063441E-08	13	0.1616553E-10	14	0.1217336E-10	15	0.2383341E-13
16	0.2323666E-15	17	0.1393347E-07	18	0.1282784E-07	19	0.9774263E-13	20	0.1349471E-13
21	0.1547718E-07	22	0.1451443E-07	23	0.4178400E-14	24	0.5629412E-18	25	0.4397669E-08
26	0.3900303E-08								

FPI = 0.1030892E-06

J	FI(J)	FIF(J)	AFI(J)	AFIF(J)	TAU(J)
1	0.1302584E-01	0.0116346	0.1302584E-01	0.0140874	0.1301318E 01
2	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
3	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
4	0.2412252E 00	0.2154612	0.2412252E 00	0.2608835	0.2699528E 01
5	0.2420748E-02	0.0021622	0.2420748E-02	0.0026180	0.8544163E 00
6	0.1271348E-02	0.0011356	0.1271348E-02	0.0013750	0.7273580E 00
7	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
8	0.1927804E 00	0.1721905	0.1927804E 00	0.2084907	0.2552395E 01
9	-0.0000000E-38	-0.0000000	-0.0000000E-38	-0.0000000	-0.0000000E-38
10	0.6360371E-01	0.0568105	0.6360371E-01	0.0687870	0.1934429E 01
11	0.9850864E-02	0.0087987	0.9850864E-02	0.0106536	0.1213531E 01
12	0.6384033E-02	0.0057022	0.6384033E-02	0.0069043	0.1088820E 01
13	0.1372152E-03	0.0001226	0.1372152E-03	0.0001484	0.4169008E 00
14	0.8713665E-04	0.0000778	0.8713665E-04	0.0000942	0.3721623E 00
15	0.3684424E-06	0.0000003	0.3684424E-06	0.0000004	0.9490182E-01
16	0.5624125E-08	0.0000000	0.5624125E-08	0.0000000	0.3335768E-01
17	0.1303137E 00	0.1163955	0.1303137E 00	0.1409334	0.2314353E 01
18	0.1146782E 00	0.1024300	0.1146782E 00	0.1240237	0.2241570E 01
19	0.6902968E-06	0.0000006	0.2989073E-06	0.0000003	0.1110302E 00
20	0.4468429E-07	0.0000000	0.1934887E-07	0.0000000	0.5600423E-01
21	0.1359519E 00	0.1214316	0.5886891E-01	0.0636663	0.2338991E 01
22	0.1228029E 00	0.1096869	0.5317520E-01	0.0575086	0.2280259E 01
23	0.8478161E-07	0.0000001	0.3671151E-07	0.0000000	0.6572908E-01
24	0.4943756E-10	0.0000000	0.2140709E-10	0.0000000	0.1021401E-01
25	0.4638153E-01	0.0414277	0.2008379E-01	0.0217205	0.1787592E 01
26	0.3866053E-01	0.0345314	0.1674050E-01	0.0181047	0.1708044E 01

FNU = 0.9246475E 00 P(S) = 0.3966712E 00 8.1

CA = 0.9246475E 00 CB = 0.9749542E-01

CA' = 0.8479198E 03 CB' = 0.8940520E 02

CA'' = 0.1799722E 09 CB'' = 0.1897639E 08

CA'''= 0.4307879E-01 CB'''= 0.4542255E-02

VI. Sample Problem Punched Output

The following is the sample problem punched output.

```
 **  SHORT JUPITER MISSION  --  BETA=1.9/3  **            (CASE I)        1 150
 0.1800000E 03  0.1111522E 03  0.2502104E 01 -0.2013408E 01
 0.1613110E 02  0.1596456E 02  0.1940643E 00
 0.1211546E 03  0.0000003E-38
 0.1849182E-15  0.1856295E-23  0.0000000E-38  0.4136114E-14  0.5085370E-16
 0.2729158E-16  0.0000000E-38  0.3068372E-14  0.0000000E-38  0.1749400E-14
 0.1523649E-15  0.1031682E-15  0.3922503E-17  0.2531968E-17  0.0000000E-38
 0.0000000E-38  0.2480924E-14  0.2303091E-14  0.0000000E-38  0.0000000E-38
 0.2290398E-14  0.2148279E-14  0.0000000E-38  0.0000000E-38  0.1312630E-14
 0.1195368E-14
 0.1727095E-13
 0.2000000E 03  0.1150851E 03  0.2685750E 01 -0.1960901E 01
 0.1566503E 02  0.1487246E 02  0.2162220E 00
 0.1290913E 03  0.0000003E-38
 0.2099917E-15  0.1135755E-23  0.0000000E-38  0.4203142E-14  0.3926071E-16
 0.2254627E-16  0.0000000E-38  0.3158675E-14  0.0000000E-38  0.1736095E-14
 0.1553686E-15  0.1092334E-15  0.2831270E-17  0.1863290E-17  0.0000000E-38
 0.0000000E-38  0.2502927E-14  0.2342530E-14  0.0000000E-38  0.0000000E-38
 0.2344649E-14  0.2215986E-14  0.0000000E-38  0.0000000E-38  0.1291815E-14
 0.1186931E-14
 0.1753261E-13
 0.2200000E 03  0.1185211E 03  0.2863991E 01 -0.1907457E 01
 0.1519737E 02  0.1394644E 02  0.2310191E 00
 0.8988587E 02  0.0000002E-38
 0.1453715E-15  0.7874782E-24  0.0000000E-38  0.2789971E-14  0.3021185E-16
 0.1861206E-16  0.0000000E-38  0.2110169E-14  0.0000000E-38  0.1135647E-14
 0.1086411E-15  0.8094324E-16  0.1897863E-17  0.1265224E-17  0.0000000E-38
 0.0000000E-38  0.1653639E-14  0.1559359E-14  0.0000000E-38  0.0000000E-38
 0.1560695E-14  0.1484871E-14  0.0000000E-38  0.0000000E-38  0.8413287E-15
 0.7801077E-15
 0.1165660E-13
 0.6482359E-09  0.4246805E-17 -0.0000000E-38  0.1324717E-07  0.1378831E-09
 0.7862070E-10 -0.0000000E-38  0.9932450E-08 -0.0000000E-38  0.5492652E-08
 0.4939861E-09  0.3478276E-09  0.9921231E-11  0.6500538E-11 -0.0000000E-38
-0.0000000E-38  0.7897321E-08  0.7385049E-08 -0.0000000E-38 -0.0000000E-38
 0.7378897E-08  0.6968266E-08 -0.0000000E-38 -0.0000000E-38  0.4093277E-08
 0.3757828E-08
 0.1073778E 01  0.1617928E-02 -0.0000000E-38  0.3039297E 01  0.6296712E 00
 0.5187819E 00 -0.0000000E-38  0.2751989E 01 -0.0000000E-38  0.2243527E 01
 0.9777290E 00  0.8663309E 00  0.2540941E 00  0.2196234E 00 -0.0000000E-38
-0.0000000E-38  0.2542786E 01  0.2484656E 01 -0.0000000E-38 -0.0000000E-38
 0.2483942E 01  0.2435380E 01 -0.0000000E-38 -0.0000000E-38  0.2027184E 01
 0.1968286E 01
 0.5528974E-07  0.9999999E 00
 0.5528974E-07  0.1350588E-05  0.9828558E-02  0.2575917E-08
 0.1131863E-07  0.2764854E-06  0.2012051E-02  0.5273284E-09
********************************************************************************
```

```
0.2100000E 03  0.1209315E 03  0.2551244E 01 -0.4040952E 01
0.1312662E 02  0.1511906E 02  0.4106112E 00
0.1085456E 03  0.0000002E-38
0.2395207E-15  0.0000000E-38  0.0000000E-38  0.2676346E-14  0.4831894E-16
0.2008363E-16  0.0000000E-38  0.2201890E-14  0.0000000E-38  0.9273736E-15
0.1869351E-15  0.1145161E-15  0.2626289E-17  0.1528392E-17  0.1481220E-19
0.6325745E-21  0.1630825E-14  0.1433756E-14  0.2215644E-19  0.1810658E-20
0.1667863E-14  0.1504412E-14  0.4840155E-20  0.8860210E-23  0.7215725E-15
0.6023501E-15
0.1143066E-13
0.2400000E 03  0.1263439E 03  0.2771720E 01 -0.3876202E 01
0.1232616E 02  0.1391365E 02  0.4689396E 00
0.8031670E 02  0.0000002E-38
0.1676024E-15  0.0000000E-38  0.0000000E-38  0.1794042E-14  0.3232776E-16
0.1750183E-16  0.0000000E-38  0.1490448E-14  0.0000000E-38  0.6046552E-15
0.1249575E-15  0.8752027E-16  0.1967489E-17  0.1258864E-17  0.8127251E-20
0.4702295E-21  0.1074043E-14  0.9739916E-15  0.1970092E-19  0.2105463E-20
0.1113580E-14  0.1030275E-14  0.2560355E-20  0.2095400E-22  0.4612316E-15
0.4015061E-15
0.7672226E-14
0.2700000E 03  0.1309803E 03  0.2978540E 01 -0.3707513E 01
0.1155276E 02  0.1294500E 02  0.5058029E 00
0.5304787E 02  0.0000002E-38
0.1311463E-15  0.0000000E-38  0.0000000E-38  0.1067624E-14  0.2346360E-16
0.1594421E-16  0.0000000E-38  0.9342365E-15  0.0000000E-38  0.3225918E-15
0.9270656E-16  0.7521975E-16  0.1189147E-17  0.9009406E-18  0.3647702E-20
0.1307949E-21  0.6297591E-15  0.5880604E-15  0.1384669E-19  0.3007863E-20
0.6899551E-15  0.6544597E-15  0.1003173E-20  0.2021780E-23  0.2435721E-15
0.2201631E-15
0.4665806E-14
0.9148099E-09 -0.0000000E-38 -0.0000000E-38  0.9502341E-08  0.1768237E-09
0.9205682E-10 -0.0000000E-38  0.7927660E-08 -0.0000000E-38  0.3187221E-08
0.6863055E-09  0.4727502E-09  0.1004454E-10  0.6411392E-11  0.4498987E-13
0.2208162E-14  0.5713635E-08  0.5144860E-08  0.9772485E-13  0.1170216E-13
0.5942131E-08  0.5468370E-08  0.1420939E-13  0.6841582E-16  0.2446340E-08
0.2106681E-08
0.1293840E 01 -0.0000000E-38 -0.0000000E-38  0.2899999E 01  0.7340846E 00
0.5861284E 00 -0.0000000E-38  0.2724363E 01 -0.0000000E-38  0.1989781E 01
0.1171768E 01  0.1030431E 01  0.2730393E 00  0.2338792E 00  0.4229499E-01
0.1495825E-01  0.2433432E 01  0.2347016E 01  0.5526600E-01  0.2658380E-01
0.2466559E 01  0.2396893E 01  0.2842426E-01  0.4514184E-02  0.1816294E 01
0.1725046E 01
0.4074742E-07  0.1000000E 01
0.4074742E-07  0.9953564E-06  0.7243449E-02  0.1898399E-08
0.9302285E-08  0.2272313E-06  0.1653617E-02  0.4333881E-09
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```
 **  SHORT JUPITER MISSION  --  BETA=1  **                    (CASE III)      1 150
 0.1800000E 03  0.1111522E 03  0.2502104E 01 -0.2013408E 01
 0.1613110E 02  0.1596456E 02  0.1940643E 00
 0.1211546E 03  0.0000003E-38
 0.2402165E-08  0.2405121E-18  0.0000000E-38  0.1142221E-06  0.7022925E-09
 0.3814752E-09  0.0000000E-38  0.7942508E-07  0.0000000E-38  0.4019007E-07
 0.2104848E-08  0.1348515E-08  0.5450210E-10  0.3632679E-10  0.0000000E-38
 0.0000000E-38  0.6164019E-07  0.5632573E-07  0.0000000E-38  0.0000000E-38
 0.5576346E-07  0.5158015E-07  0.0000000E-38  0.0000000E-38  0.2838820E-07
 0.2532675E-07
 0.4285737E-06
 0.2000000E 03  0.1150851E 03  0.2685750E 01 -0.1960901E 01
 0.1566503E 02  0.1487246E 02  0.2162220E 00
 0.1290913E 03  0.0000003E-38
 0.2806445E-08  0.1263765E-18  0.0000000E-38  0.1148001E-06  0.5286038E-09
 0.3070850E-09  0.0000000E-38  0.8108653E-07  0.0000000E-38  0.3917697E-07
 0.2081612E-08  0.1387968E-08  0.3746436E-10  0.2542316E-10  0.0000000E-38
 0.0000000E-38  0.6131333E-07  0.5656737E-07  0.0000000E-38  0.0000000E-38
 0.5648774E-07  0.5272789E-07  0.0000000E-38  0.0000000E-38  0.2734110E-07
 0.2464667E-07
 0.4299220E-06
 0.2200000E 03  0.1185211E 03  0.2863991E 01 -0.1907457E 01
 0.1519737E 02  0.1394644E 02  0.2310191E 00
 0.8988587E 02  0.0000002E-38
 0.1928899E-08  0.7628982E-19  0.0000000E-38  0.7542844E-07  0.3985639E-09
 0.2430184E-09  0.0000000E-38  0.5367445E-07  0.0000000E-38  0.2525249E-07
 0.1464727E-08  0.1041707E-08  0.2406846E-10  0.1683594E-10  0.0000000E-38
 0.0000000E-38  0.4005855E-07  0.3730038E-07  0.0000000E-38  0.0000000E-38
 0.3723243E-07  0.3503812E-07  0.0000000E-38  0.0000000E-38  0.1754396E-07
 0.1599580E-07
 0.2826493E-06
 0.8591576E-02  0.4920954E-12 -0.0000000E-38  0.3622327E 00  0.1864567E-02
 0.1070205E-02 -0.0000000E-38  0.2551155E 00 -0.0000000E-38  0.1242402E 00
 0.6681138E-02  0.4463560E-02  0.1326234E-03  0.8986381E-04 -0.0000000E-38
-0.0000000E-38  0.1938171E 00  0.1786414E 00 -0.0000000E-38 -0.0000000E-38
 0.1779593E 00  0.1659520E 00 -0.0000000E-38 -0.0000000E-38  0.8693081E-01
 0.7829214E-01
 0.1115412E 01  0.3068524E-02 -0.0000000E-38  0.2842262E 01  0.7613103E 00
 0.6626496E 00 -0.0000000E-38  0.2603763E 01 -0.0000000E-38  0.2175117E 01
 0.1047442E 01  0.9469729E 00  0.3931626E 00  0.3567085E 00 -0.0000000E-38
-0.0000000E-38  0.2430890E 01  0.2381841E 01 -0.0000000E-38 -0.0000000E-38
 0.2379564E 01  0.2338368E 01 -0.0000000E-38 -0.0000000E-38  0.1989347E 01
 0.1937969E 01
 0.1357402E 01  0.2573285E 00
 0.1357402E 01  0.1244764E 04  0.2642030E 09  0.6324057E-01
 0.1191362E 00  0.1092503E 03  0.2318853E 08  0.5550489E-02
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** LONG JUPITER MISSION -- BETA=1 ** (CASE IV) 1 150

0.2100000E 03	0.1209315E 03	0.2551244E 01	-0.4040952E 01	
0.1312662E 02	0.1511906E 02	0.4106112E 00		
0.1085456E 03	0.0000002E-38			
0.3415291E-08	0.0000000E-38	0.0000000E-38	0.6904582E-07	0.6637029E-09
0.2888663E-09	0.0000000E-38	0.5429970E-07	0.0000000E-38	0.1893931E-07
0.2709444E-08	0.1535126E-08	0.3737212E-10	0.2155003E-10	0.1313150E-12
0.1576081E-14	0.3789608E-07	0.3240396E-07	0.1553654E-12	0.7097765E-14
0.3877805E-07	0.3418969E-07	0.3148191E-13	0.4427075E-17	0.1400982E-07
0.1124528E-07				
0.2637881E-06				
0.2400000E 03	0.1263439E 03	0.2771720E 01	-0.3876202E 01	
0.1232616E 02	0.1391365E 02	0.4689396E 00		
0.8031670E 02	0.0000002E-38			
0.2349608E-08	0.0000000E-38	0.0000000E-38	0.4535306E-07	0.4413288E-09
0.2390171E-09	0.0000000E-38	0.3605786E-07	0.0000000E-38	0.1201401E-07
0.1765199E-08	0.1163692E-08	0.2616913E-10	0.1675584E-10	0.6457178E-13
0.1265577E-14	0.2436058E-07	0.2162725E-07	0.1397642E-12	0.6943074E-14
0.2532298E-07	0.2302559E-07	0.1487880E-13	0.1657812E-16	0.8690366E-08
0.7342538E-08				
0.1732926E-06				
0.2700000E 03	0.1309803E 03	0.2978540E 01	-0.3707513E 01	
0.1155276E 02	0.1294500E 02	0.5058029E 00		
0.5304787E 02	0.0000002E-38			
0.1936293E-08	0.0000000E-38	0.0000000E-38	0.2637867E-07	0.3215007E-09
0.2140780E-09	0.0000000E-38	0.2233488E-07	0.0000000E-38	0.6109618E-08
0.1361134E-08	0.1063441E-08	0.1616553E-10	0.1217336E-10	0.2383341E-13
0.2323666E-15	0.1393347E-07	0.1282784E-07	0.9774263E-13	0.1349471E-13
0.1547718E-07	0.1451443E-07	0.4178400E-14	0.5629412E-18	0.4397669E-08
0.3900303E-08				
0.1030892E-06				
0.1302584E-01	-0.0000000E-38	-0.0000000E-38	0.2412252E 00	0.2420748E-02
0.1271348E-02	-0.0000000E-38	0.1927804E 00	-0.0000000E-38	0.6360371E-01
0.9850864E-02	0.6384033E-02	0.1372152E-03	0.8713665E-04	0.3684424E-06
0.5624125E-08	0.1303137E 00	0.1146782E 00	0.6902968E-06	0.4468429E-07
0.1359519E 00	0.1228029E 00	0.8478161E-07	0.4943756E-10	0.4638153E-01
0.3866053E-01				
0.1301318E 01	-0.0000000E-38	-0.0000000E-38	0.2699528E 01	0.8544163E 00
0.7273580E 00	-0.0000000E-38	0.2552395E 01	-0.0000000E-38	0.1934429E 01
0.1213531E 01	0.1088820E 01	0.4169008E 00	0.3721623E 00	0.9490182E-01
0.3335768E-01	0.2314353E 01	0.2241570E 01	0.1110302E 00	0.5600423E-01
0.2338991E 01	0.2280259E 01	0.6572908E-01	0.1021401E-01	0.1787592E 01
0.1708044E 01				
0.9246475E 00	0.3966712E 00			
0.9246475E 00	0.8479198E 03	0.1799722E 09	0.4307879E-01	
0.9749542E-01	0.8940520E 02	0.1897639E 08	0.4542255E-02	

****

Nomenclature

$\underline{A}$ the set of spacecraft surface elements

$A(\alpha)$ area density function over $\underline{A}$

A_j area of the jth face of a polyhedral spacecraft

A_j^* A_j for standard spacecraft

A_s surface area of the spacecraft

A_s^* A_s for standard spacecraft

a semi-major axis of spacecraft orbit

a_k semi-major axis of the orbit of the kth asteroid

$\underline{B}$ a subset of $\underline{A}$; a set of spacecraft surface elements

b a constant

$b(a)$ a function of a

C coefficient in equation for $P_I(S)$ in terms of l and t

C' coefficient in equation for $P_I(S)$ in terms of A_s and W_s

C'' coefficient in equation for $P_I(S)$ in terms of l and W_s

C''' coefficient in equation for $P_I(S)$ in terms of A_s and t

C_1, C_2 *constants* in the meteoroid damage function

C_j $f_j^* (\tau_j^* t^*)^{3\beta}$

C_j' $\dfrac{f_j^*}{\alpha_s} \left(\dfrac{\tau_j^* W_s^*}{A_s^*} \right)^{3\beta}$

C_A, C_A', C_A'', C_A''' are C, C', C'', C''' respectively, for uniformly distributed spacecraft shielding

C_A' $C_A \dfrac{\rho_s^{3\beta}}{\alpha_s}$

C_A'' $C_A (\alpha_s \rho_s)^{3\beta}$

C_A''' $\dfrac{C_A}{\alpha_s}$

C_B, C_B', C_B'', C_B''' are C, C', C'', C''', respectively, for optimum distribution of spacecraft shielding

C_B' $C_B \dfrac{\rho_s^{3\beta}}{\alpha_s}$

C_B'' $C_B (\alpha_s \rho_s)^{3\beta}$

C_B''' $\dfrac{C_B}{\alpha_s}$

$C(a)$ a function of a

$D = -\mathbf{n}_j \cdot \mathbf{W}'$ component of meteoroid relative velocity normal to spacecraft

Nomenclature (contd)

$D_k^{(l,m)}$ — the four $(l, m = 1, 2)$ components of relative velocity normal to the spacecraft of the kth meteoroid swarm

d_p — projectile diameter

d, d^* — constants

$E(T)$ — eccentric anomaly of spacecraft mission orbit at time T

$E_k(r)$ — $\cos^{-1}[(a_k - r)/a_k e_k]$

e — eccentricity of spacecraft orbit

$\mathbf{e}$ — a three-dimensional unit vector

$\mathbf{e}_1(\mathbf{X})$ — $\mathbf{e}_X$

$\mathbf{e}_2(\mathbf{X})$ — $\dfrac{\mathbf{e}_N \times \mathbf{e}_X}{|\mathbf{e}_N \times \mathbf{e}_X|}$

$\mathbf{e}_3(\mathbf{X})$ — $\mathbf{e}_1 \times \mathbf{e}_2$

(basis vectors for the space-fixed coordinate system)

$\mathbf{e}_1'(T)$ — $\mathbf{e}_2 = \dfrac{\mathbf{e}_N \times \mathbf{e}_X}{|\mathbf{e}_N \times \mathbf{e}_X|}$

$\mathbf{e}_2'(T)$ — $\mathbf{e}_Y = \mathbf{e}_1 \times \mathbf{e}_3'$

$\mathbf{e}_3'(T)$ — $\mathbf{e}_N$

(basis vectors for the spacecraft-fixed coordinate system)

e_k — eccentricity of the orbit of the kth asteroid

$\mathbf{e}_N$ — unit vector in the direction of ecliptic North

$\mathbf{e}_{\Upsilon}$ — unit vector in the direction of the vernal equinox

$\mathbf{e}_{\perp}$ — unit vector equal to $\mathbf{e}_N \times \mathbf{e}_{\Upsilon}$

$\mathbf{e}_X$ — unit vector in the direction of $\mathbf{X}$

$\mathbf{e}_Y$ — unit vector in the direction of $\mathbf{Y}$

F_a — failure; a spacecraft state

F, F' — meteoroid flux (meteoroids $m^{-2}\, s^{-1}$)

$F_j(T)$ — effective meteoroid flux on the jth face of a polyhedral spacecraft (destructive impacts $m^{-2}\, s^{-1}$)

f — probability of discovery of an asteroid

$f_j = \int_{T_0}^{T_f} F_j(T)\, dT$ — expected number of penetrating meteoroid impacts/m^2 on the jth face of a polyhedral spacecraft; integrated flux

$f_j^* = \int_{T_0}^{T_f} F_j^*(T)\, dT$ — expected number of penetrating meteoroid impacts/m^2 on the jth face of the standard spacecraft; integrated flux for standard spacecraft

G_k — absolute magnitude of the kth asteroid

G_0 — reference meteoroid absolute magnitude ($G_0 = 13.6$)

$G(p_0, a)$ — $p_0 - 5\log_{10}[a(a-1)]$, a, in AU

Nomenclature (contd)

$H(x)$	the unit step function; $H(x) = 0$ for $x < 0$, ½ for $x = 0$, 1 for $x > 0$
$h(M)$	Brinell hardness of material M
$h_t = h_s$	target Brinell hardness
I	a particular value that i can assume
i	inclination to the ecliptic of spacecraft orbit
i_k	inclination of the orbit of the kth asteroid to the ecliptic
J	Jacobian $\partial(M, \omega, \Omega)/\partial(r, \lambda, \Lambda)$
$K = \Gamma M_\odot$	gravitational field constant of the sun
k_1, k_2	constants in the projectile penetration function
k	a number ranging from 1 to 1500, used to label asteroid and meteoroid-swarm properties
l	a length parameter associated with a spacecraft
l^*	l for standard spacecraft
$\ln(x)$	natural logarithm
$\log(x)$	10-based logarithm
$\underline{M}$	set of meteoroid types
M	material
M'	meteroid material; iron
$M(T)$	mean anomaly of the spacecraft at time T
$M'(\mu)$	material composition of meteoroid of type μ
$M(\alpha)$	material composition of spacecraft surface element α
M_j	material composition of the jth face of a polyhedral spacecraft
M_0	mass of asteroidal meteoroid of radius R
$M_\odot$	mass of the sun = 1.989×10^{33} g
$\mathcal{M}(T)$	spacecraft orientation matrix at time T; rotation matrix which converts a vector from spacecraft-fixed coordinates to space-fixed coordinates. Example: $\mathbf{n}(\alpha, T) = \mathbf{n}(\alpha)\mathcal{M}(T)$
$\mathcal{M}^{-1}(T)$	rotation matrix which converts a vector to spacecraft-fixed coordinates from space-fixed coordinates. Example: $\mathbf{n}(\alpha) = \mathbf{n}(\alpha, T)\mathcal{M}^{-1}(T)$
$m(\mu)$	mass of meteoroid of type μ
m_0	reference meteoroid mass = 2.56×10^{18} g (absolute magnitude 13.6)
N_F	number of faces of polyhedral spacecraft
N_T	number of steps into which the mission is divided
$N_1(m)$	number of meteoroids of mass $\geqq m$

Nomenclature (contd)

$N_1'(m)$	$\dfrac{dN_1}{dm}$
$N_2(r)$	number of meteoroids of radius $\geqq r$
$N_2'(r)$	$dN_2(r)/dr$
$N_3(G)$	the number of meteoroids with absolute magnitude $\leqq G$
$N_3(G,a)$	number of meteoroids with absolute magnitude $\leqq G$ and semi-major axis $\geqq a$
$N_3'(G,a)$	$\partial N_3(G,a)/\partial a$
$N_4(p_0)$	number of meteoroids with mean opposition magnitude $\leqq p_0$
N_j'	any vector parallel to $\mathbf{n}_j$, that is $\mathbf{N}_j' = c\,\mathbf{n}_j$ where $c > 0$
$\mathbf{n}(\alpha, T)$	outwardly directed unit vector normal to the spacecraft surface element α, at time T, in space-fixed coordinates
$\mathbf{n} = \mathbf{n}(\alpha)$	outwardly directed unit vector normal to spacecraft surface element α, in spacecraft-fixed coordinates
$\mathbf{n}_j$	outwardly directed unit vector normal to the jth face of a polyhedral spacecraft, in spacecraft-fixed coordinates
$P(0)$	probability of no meteoroid penetrations of spacecraft shield
$P(S)$	the mission probability of success
$P_I(S)$	probability that spacecraft does not fail because of meteoroid impact
$P(s,T)$	spacecraft status at time T; a probability density function over S
$P_s(T)$	probability of spacecraft success through time T
$P(F_a, T)$	probability of failure state at time T
p	semi-latus rectum of spacecraft orbit $p = a(1 - e^2)$
p_0	$G_k + 5\log_{10}[a_k(a_k - 1)]$ = mean asteroid magnitude at opposition
p_1	meteoroid penetration depth
Q	$\left(-\dfrac{3\beta}{q}\right)^{\frac{1}{1+3\beta}}$
q	Lagrange multiplier
$\underline{R}$	portion of set $\underline{S}$
R	radius of meteoroid which just penetrates the shielding of the jth face of a polyhedral spacecraft
$r = r(\mathbf{X}) = \lvert\mathbf{X}\rvert$	radial distance from sun to spacecraft
$r(\mu)$	radius of meteoroid of type μ
r_0	reference meteoroid radius = 4.3 km (absolute magnitude 13.6)
$\underline{S}$	continuum of possible spacecraft states
S	success (so far); a spacecraft state

Nomenclature (contd)

$S_h(\alpha, \mathbf{Z}, T)$	spacecraft shadowing function; probability that the line drawn from the spacecraft surface element α in the direction $\mathbf{Z}$, at time T, will penetrate a part of the spacecraft
$S_h(\alpha, \mathbf{Z})$	spacecraft shadowing function; probability that the line drawn from the spacecraft surface element α in the direction $\mathbf{Z}$ will penetrate a part of the spacecraft
$S_h(j, \mathbf{Z})$	polyhedral spacecraft shadowing function; probability that the line drawn from the jth face in the direction $\mathbf{Z}$ will penetrate a part of the spacecraft
$\underline{St}(\alpha)$	set of structural properties of the spacecraft surface at α
$\underline{St}'(\mu)$	set of structural properties of meteoroids of type μ
s, s'	spacecraft states; elements of $\underline{S}$
T	time
T_0	the time at which the spacecraft mission starts
T_f	the time the spacecraft mission ends
T_P	time of perihelion passage of spacecraft in its orbit
$t = \frac{W_s}{\rho_S \mathbf{A}_s}$	the average thickness of the spacecraft surface
$t(\alpha)$	thickness of spacecraft surface element α
t_c	thickness of plate required to stop projectile
t_j	thickness of jth face of polyhedral spacecraft
t_j^*	t_j for standard spacecraft
$\mathbf{U}$	a three-dimensional vector giving the velocity of a meteoroid
$\mathbf{U}_k^{(l,m)}(\mathbf{X})$	the four $(l, m = 1, 2)$ velocity functions of the kth meteoroid swarm: $\mathbf{U}_k^{(1,1)}(\mathbf{X})$, $\mathbf{U}_k^{(1,2)}(\mathbf{X})$, $\mathbf{U}_k^{(2,1)}(\mathbf{X})$, $\mathbf{U}_k^{(2,2)}(\mathbf{X})$
$U_{k,i}^{(l,m)}(\mathbf{X}) = \mathbf{U}_k^{(l,m)}(\mathbf{X}) \cdot \mathbf{e}_i$	component of $\mathbf{U}_k^{(l,m)}(\mathbf{X})$ along $\mathbf{e}_i$
$U_{k,a}^{(l,m)}(\mathbf{X}) = \lvert \mathbf{U}_{k,a}^{(l,m)}(\mathbf{X}) \rvert$	azimuthal velocity functions of kth meteoroid swarm
$U_{k,r}^{(l,m)}(\mathbf{X}) = \lvert \mathbf{U}_{k,r}^{(l,m)}(\mathbf{X}) \rvert$	radial velocity functions of kth meteoroid swarm
$\mathbf{V}(T)$	a three-dimensional vector giving the spacecraft velocity at time T.
$V(T) = \lvert \mathbf{V}(T) \rvert$	magnitude of velocity vector $\mathbf{V}$
$\mathbf{V}_a(T)$	azimuthal velocity of spacecraft at time T; component of $\mathbf{V}$ perpendicular to $\mathbf{X}$
$V_a(T) = \lvert \mathbf{V}_a(T) \rvert$	magnitude of azimuthal velocity vector
V_p	projectile relative velocity normal to the surface of the target
$\mathbf{V}_r(T)$	radial velocity of spacecraft at time T
$V_r(T) = \lvert \mathbf{V}_r(T) \rvert$	magnitude of radial velocity at time T
$\mathbf{W}$	velocity (three-dimensional vector) of a meteoroid with respect to the spacecraft
$\mathbf{W}' = \mathbf{W} \mathcal{M}^{-1}(T)$	velocity of meteoroid relative to the spacecraft in spacecraft-fixed coordinates
W_j	the mass of the jth face of a polyhedral spacecraft

Nomenclature (contd)

W_j^*	W_j for standard spacecraft
W_s	shielding mass of spacecraft
W_s^*	W_s for standard spacecraft
$w_k = \frac{1}{f}$	statistical weight of the kth asteroid
$\mathbf{w}$	$\frac{\mathbf{W}}{\lvert\mathbf{W}\rvert}$ unit vector in direction of $\mathbf{W}$
$\mathbf{w}'$	$\frac{\mathbf{W}'}{\lvert\mathbf{W}'\rvert}$ unit vector in direction of $\mathbf{W}'$
$\mathbf{X}$	a three-dimensional vector giving the position of a point in space
$\mathbf{X}(T)$	a three-dimensional vector giving the spacecraft position at time T
X_N	component of $\mathbf{X}$ in the $\mathbf{e}_N$ direction
$X_{♈}$	component of $\mathbf{X}$ in the $\mathbf{e}_{♈}$ direction
X_{-}	component of $\mathbf{X}$ in the $\mathbf{e}_{-}$ direction
$\mathbf{X}_k'$	modified version of $\mathbf{X}$ used as argument of $\mathbf{U}_k^{(l,m)}$ in connection with $\langle\sigma_k\rangle$
Y	component of $\mathbf{X}$ in the $\mathbf{e}_Y$ direction; $Y = r\cos\lambda$
$\mathbf{Y}$	projection of $\mathbf{X}$ on the ecliptic plane
$\mathbf{Z}$	a three-dimensional unit vector originating at surface element α of the spacecraft
*	a superscript to A_j, A_s, F_s, f_j, π_I, ν_I, P_I, W_j, W_s, t, t_j, and τ_j referring to the standard spacecraft
α	an element of A; a spacecraft surface element
$\alpha_c, \alpha_c', \alpha_c''$	constants
$\alpha_j = \frac{A_j}{l^2} = \frac{A_j^*}{l^{*2}}$	α_j^*
α_j'	$\frac{A_j}{A_j^*}$
$\alpha_s = \frac{A_s}{l^2} = \sum_{j=1}^{N_F} \alpha_j$	α_s^*
α_s'	$\frac{A_s}{A_s^*}$
$\alpha(T)$	angle between $\mathbf{V}_a$ and $\mathbf{e}_2$ at time T
$\alpha_k^{(l,m)}(\mathbf{X})$	angle between $\mathbf{U}_{k,a}^{(l,m)}(\mathbf{X})$ and $\mathbf{e}_2$
β	the exponent in the meteoroid mass distribution law
Γ	universal gravitation constant $= 6.668 \times 10^{-8}$ dynes cm^2 g^{-2}
ΔT	interval between time steps
$\delta(s, s', \alpha, \mu, \mathbf{W}, T)$	probability that the spacecraft, in state s at time T, will change to state s' when hit on surface α by a meteoroid of type μ moving at a relative velocity $\mathbf{W}$ with respect to the spacecraft

Nomenclature (contd)

$\delta(\alpha, \mu, \mathbf{W}, T)$	probability that the spacecraft fails at time T when hit on surface α by a meteoroid of type μ moving at relative velocity $\mathbf{W}$ with respect to the spacecraft
$\delta(\alpha, \mu, \mathbf{W}')$	probability the spacecraft fails when hit on surface α by a meteoroid of type μ moving at relative velocity $\mathbf{W}'$ with respect to the spacecraft
$\delta_j(\mu, \mathbf{W}')$	probability the polyhedral spacecraft fails when hit on surface j by a meteoroid of type μ moving at relative velocity $\mathbf{W}'$ with respect to the spacecraft
δ_{ij}	$\begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$
ε_r, ε_λ, ε_Λ	averaging parameters: $\lim_{\varepsilon_r, \varepsilon_\lambda \to 0} \langle \sigma_k \rangle (r, \lambda, \Lambda) = \sigma_k(r, \lambda, \Lambda)$
$\zeta(m, \mathbf{X}), \zeta(m)$	meteoroid mass distribution functions
$\eta(T)$	true anomaly of spacecraft at time T
θ	argument of the latitude (angle measured at the sun from the ascending node of the meteoroid orbit plane to the spacecraft)
θ'	angle between $\mathbf{V}$ and $\mathbf{V}_a$
$\Lambda, \Lambda(\mathbf{X}), \Lambda(T)$	ecliptic longitude of spacecraft
$\lambda, \lambda(\mathbf{X})$	ecliptic latitude of spacecraft
μ	an element of $\underline{M}$; a meteoroid type
μ_s	mean motion in the spacecraft orbit; $\mu_s = \left(\frac{\Gamma M_\odot}{a^3}\right)^{1/2}$
ν_i	$\int_{T_0}^{T_f} \pi_i(T)\, dT =$ ith partial failure rate integral
$\xi(\mathbf{U}, \mathbf{X})\, d^3\mathbf{X}\, d^3\mathbf{U}$	the probability that an asteroidal meteoroid of mass $\geqq m_0$ will pass through position $\mathbf{X}$ with velocity $\mathbf{U}$ at time T, with tolerances $d^3\mathbf{X}$ and $d^3\mathbf{U}$ in meteoroid position and velocity
$\pi(s, s', T)$	rate of change of spacecraft state, from state s to state s' at time T; total transition rate
$\pi_i(s, s', T)$	rate of change of spacecraft state, from state s to state s', at time T, caused by ith source; ith transition rate
$\pi_I(s, s', T)$	rate of change of spacecraft state, from state s to state s', at time T, caused by a certain class of meteoroids $\underline{M}$; Ith transition rate
$\pi(T)$	total failure rate at time T
$\pi_i(T)$	the ith failure rate at time T
$\rho(M)$	density of material M
ρ'	meteoroid density; 7.9 g cm^{-3}
ρ_j	density of material of spacecraft surface j

Nomenclature (contd)

$\rho_k^*(\lambda)$	$\sin^{-1}\left(\frac{\sin\lambda}{\sin i_k}\right)$
ρ_p	density of projectile
ρ_s	density of the spacecraft surface material, taken as uniform in composition
ρ_t	target density
$\sigma(\mathbf{X})$	number of asteroidal meteoroids of mass $\geqq m_0$ per unit volume at **X**
$\sigma_k, \sigma_k(\mathbf{X}), \sigma_k(r, \lambda, \Lambda)$	number of meteoroids per unit volume at **X**, or at (r, λ, Λ), the meteoroids having mass $\geqq m_0$ and coming from the kth swarm
$<\sigma_k>(r, \lambda, \Lambda)$	averaged version of $\sigma_k(r, \lambda, \Lambda)$, used to avoid singularities
$\sigma_k^*(r)$	$E_k(r) - e_k \sin E_k(r)$
τ_j	$\frac{t_j}{t}$
τ_j^*	$\frac{t_j^*}{t^*}$
τ_j'	$\frac{t_j}{t_j^*}$
τ_j^+	$\frac{t_j^+}{t}$ optimal pattern of thicknesses τ_j
♈	vernal equinox
Φ	$\alpha_c'' m^{-\beta}$ = the total number of asteroidal meteoroids with mass $\geqq m$
$\psi(\mu, \mathbf{X}, \mathbf{U}, T)\, d\mu\, d^3\mathbf{X}\, d^3\mathbf{U}$ $= \psi(\mu, \mathbf{X}, \mathbf{U})\, d\mu\, d^3\mathbf{X}\, d^3\mathbf{U}$	the probability that a meteoroid of type μ will pass through position **X** with velocity **U** at time T with tolerances $d\mu$, $d^3\mathbf{X}$, and $d^3\mathbf{U}$ in meteoroid type, position, and velocity
$\psi(m, \mathbf{X}, \mathbf{U})\, dm\, d^3\mathbf{X}\, d^3\mathbf{U}$	the probability that an iron meteoroid of mass m will pass through position **X** with velocity **U** at time T with tolerances dm, $d^3\mathbf{X}$, and $d^3\mathbf{U}$ in meteoroid mass, position, and velocity
Ω	longitude of the ascending node of spacecraft orbit
Ω'	the surface of the unit sphere
ω	argument of perihelion of spacecraft orbit

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