UNIVERSITY OF HOUSTON

DEPARTMENT OF PHYSICS

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October 1, 1968 - March 1, 1969

SPACE RELATED

TECHNICAL INVESTIGATIONS

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Alvin F. Hildebrandt

This report covers the period from October 1, 1968, through February 28, 1969. The official notice has not been received for continuance of the grant but the opinion is that the grant will continue. In the event it does not a supplement to this report will be submitted and the combined report will consittute a final report. A no cost extension of the present work has been granted. One M.S. thesis has been completed during the last report period. An up dated summary of all publications of work performed under the grant is attached.

In addition to the work outlined a proposal for a "Helium II Momentum Flux Radiometer" and a proposal for a "Thermal Method of Measuring the Quantity of a Cryogenic Fluid in a Tank Under Zero-Gravity Conditions" has been originated under this contract. Copies of the proposals are attached.

Questions regarding the work described in this grant may be directed to Dr. Alvin F. Hildebrandt, Associate Professor, Department of Physics, University of Houston, Houston, Texas 77004, (713) 748-6600, extension 1932 or 1943. The investigators named in the body of the report may be contacted individually regarding their work at the same address.

Atomic Processes and Solid State Physics Studies

1. Phonon Dispersion Relations in Metals, R. H. Walker & G. Hopkins

The frequencies of the normal modes of a vibrating metallic lattice has been studied using a simple model for the ionic screening by the conduction electrons. The screening which is characterized by the density of conduction electrons was treated by means of the frequency-wavelength dependent dielectric constant of the electron system. The short range ion-ion interaction was treated phenomenologically by assuming a Yukawa potential whose range was adjusted to obtain the agreement with sound velocity. Calculations have been carried out for three metals copper, lead and aluminum whose dispersion relations are known from neutron scattering experiments. The agreement between this theory and the experimental results are excellent, particularly in the case of copper. This work has resulted in one M.S. thesis.

2. Many-Body Effects in Metals, R. H. Walker & J. Paulsel

Considerable progress has been made as an approach to the electron gas based directly upon the Random phase approximation of Bohm and Pines.[1] The basic idea in this approach is to introduce dynamical correlations into the electron motion at the outset by the introduction of collective coordinates which describe the plasma oscillations of the gas. The fraction of the degrees of freedom of the system corresponding to individual particle motion are then equivalent to a set of particles interacting with weak short range forces instead of long range coulomb force which is the source of numerous divergence difficulties in the usual perturbation treatments.

The basic hamiltarian for the electron gas is defined by the total kinetic energy of the system and the coulomb interactions between the particles. A suitable canonical transformation(2) transforms this basic hamiltarian into one representing a coupled particle-plasman system in which the particle interactions are of short range. Boththe range of this effective interaction and the coupling is defined by a single parameter to be determined by that value which minimizes the total energy. Further, an analysis of the analytic structure of the single particle electron and plasman propagation will determine the low lying excitation spectrum of the electron gas with their life times.

To this date calculations have been carried out for such a system ignoring the short range interactions between the particles. Expressions have been obtained for the ground state energy and for the propagation using infinite order perturbation theory and selective summation. The formalism for such calculations are now well in hand. The theory will now be modified to include the interparticle interactions. It is hoped that this approach will lead to results which are valid for those electron densities for which currently there is no good theory, namely these densities correspo ding to most metals. This work has resulted in one M.S. thesis granted in February 1969.

3. Response of Crystals to External Fields, R. H. Walker & D. Lo

The scattering operator for a harmonic oscillator driven by an external field with arbitrary time dependence is exactly soluable. This fact is being exploited to investigate certain features of the Mossbauer effect. This approach has been used to calculate the Mossbauer fraction for a crystal in its ground state, the Mossbauer fraction for a crystal at a finite temperature, and the line shape of the emitted x-ray. These results all agree with previous calculations by other authors using the standard methods. The scattering operator approach is also being applied to the problem of Mossbauer emission in an ambient sound wave field, in particular to the line shape of the γ-ray emitted under these conditions. The results of these calculations are different from expressions obtained by other authors using standard techniques.[3][4] In particular a small frequency shift in the central portion of the line is predicted whose magnitude depends upon the intensity of the sound wave. This effect and other predictions of the current theory will be studied to determine the feasibility of an experimental investigation by the Mossbauer group here at the University of Houston. This work will result in one M.S. thesis in June 1969.

(1) Bohm and Pines, Phys. Rev. 92, 609 (1953).
(2) Pines, D., Elementary Excitations in Solids, Benjamin, New York (1963).
(3) Abragam, Proc. Fr. Acad. Sc., 4334 (1960).
(4) Bolef, Phys. Rev. Letters, 5, 5(1960).

The Interaction of Quantum Fluids with Physical Boundaries, Henry E. Corke and Alvin F. Hildebrandt

An earlier report included initial details about a new device to measure the velocity of superfluid helium. It consists of an aluminum block which has .001 cm glass fibers attached around the edge of one face. Aluminum foil about 10^{-4} cm thick is stretched over the fibers like a drumhead to form a capacitor of about 40 pF. When this foil probe is located in a channel through which superfluid helium is flowing the Bernoulli pressure displaces the foil changing the capacity. The foil probe is connected as the capacitance of an L-C tank circuit of a back diode oscillator, so that the Bernoulli pressure is seen as a frequency change. With this technique velocities as small as .05 cm/sec. can be observed with a signal to noise ratio of about 2 to 1.

The first investigation utilizing the velocity probe is the measurement of critical velocities of superfluid in wide channels. The method used to make the measurement involves two parallel channels through which the superfluid helium is pumped. The amount of fluid flowing through each channel, below any critical velocity, is governed by the fact that the flow is curless, so that $v_s \cdot d = 0$ when there is no circulation present. Therefore by proper design of the channels one can anticipate the velocity in each and produce a critical velocity in one before the other. Since there is no impedence to flow for superfluid helium below critical velocity, when the total volume flow rate is slowly increased, the flow rate in the channel which approaches critical velocity first will stop increasing and the flow rate in the other channel will increase faster. With the velocity probe in series with the first channel that goes critical, the velocity (i.e. frequency) can be seen to remain constant with increasing total volume flow rate. When the second channel finally goes critical the velocity in the first channel will begin to increase again. Thus, with the velocity probe properly calibrated the critical velocity in each channel can be determined.

A controversy exists over the dependence of critical velocity on channel size and temperature. Some measurements show a $d^{-1/4}$ dependence, where d is the diameter or narrowest spacing of the channel, while Feynman's theory predicts a relation

$$v_c = \frac{h}{m} [\ln \frac{4d}{a_o} - \frac{7}{4}]$$

where m is the mass of the helium atom and a_o is the core diameter of the vortices which are presumed to be created producing the criticalness. The results of measuring five rectangular channels of different d match Feynman's theory quite accurately. There should be no temperature dependence but this has not been confirmed yet.

At velocities higher than these first critical velocities there seems to be an indication of abrupt increases in dissipation occurring at velocities near the $d^{-1/4}$ dependence. The conclusions thus far are that there exists a critical velocity of the Feynman type where vortices are first generated and that at higher velocities a much stronger dissipation mechanism takes place which has been mistaken by some for the critical velocity.

Flux Quantization and the Vector Potential, A. F. Hildebrandt and Thomas N. C. Tsien

In an earlier report, it was mentioned that an apparently new electromagnetic induction voltage has been observed in the rotating state of a modified unipolar, and that this voltage can be explained in terms of a non-local vector potential description. Effort has been directed in showing that the new phenomenon is not understood in terms of the available means in the existing literature. The conclusion has been reached that further experiments should be designed to get more data in determining whether the seat of the new emf is of electric origin or magnetic. A report of the preliminary observation is in preparation for publication.

Electrodynamics in Rotating Frames. R. Borochoff, V. Sanders and R. M. Kiehn

A solution to the Maxwell-Einstein equation was obtained, which gave the metric corresponding to a uniform, homogeneous magnetic field in free space. Using the constitutive tensor of E. J. Post, a dielectric constant was obtained corresponding to this field. Although the velocity of a light beam will be reduced by the "effective" dielectric constant, this analysis showed that no optical activity or birefringence resulted from the presence of the magnetic field.

By using the formalism of E. J. Post, it was determined that a normally isotropic optical medium exhibits birefrigence under rotation. The birefringence of the medium produces a splitting of a light beam into two orthogonally polarized parts. When the effect is applied to a Sagnac type experiment, it is found that one polarization state of the beam gives the usual value for the Sagnac fringe shift, a value which is independent of the shape of the light path. However, the other polarization state gives a fringe shift that is dependent on the shape of the path, and which differs in value from the usual result. Though this result is novel, it is third order in v/c and cannot be easily measured.

An attempt was made to determine the principal states of polarization associated with the above rotating frame. Two methods were used. One, an eigenvector formulation, gave the vector states directly. The second method made use of two of the Maxwell equations previously unused in the above work and of the assumption that the electric field is perpendicular to the magnetic field. Although vector states were not obtained explicitly, some rather stringent constraints were obtained which the vectors must satisfy.

The two methods employed gave conflicting results and it was decided that the problem resulted from the assumption that E and B are perpendicular. A further study of this problem is planned.

An investigation was also made to find out if any serious quantitative difference resulted in the calculation of the Sagnac fringe shift when a Lorentz type transformation was applied to the time coordinate. Classically it is assumed that the time is invariant. If the time is multiplied by a Lorentz type factor $1/\sqrt{1 - v^2/c^2}$, it is found that all the previously obtained quantities (velocity, index of refraction, etc.) are simply multiplied by the same quantity. No serious difference thus exists in the non-relativistic approximation.

Apparatus for the construction of a ring laser system has been ordered and partially delivered. An extensive literature search on the applications of ring laser system has been completed.

An Intrinsic Theory of Fluids and Elasticity. J. Pierce and R. M. Kiehn

If a physical system admits description in terms of a metric $g_{\mu\nu}$ and covariant vector field of flow, α, then Cartan's methods of exterior differential forms permit the following theorems to be constructed.

1. $d(\beta * \alpha) = 0, \; N \geqslant 2$
2. $dF = dd\alpha = 0$
3. For $H = Z_o \, {*F}, \; J = H \wedge d\ell nf$

   $dH = J; \; N \geqslant 4$
4. $dJ = 0$
5. $\Psi = d(\alpha \wedge H) - F \wedge J = 0$
6. $\delta\Psi = 0$

An initial attempt to apply these intrinsic theorems to fluid flow problems indicates that theorems 1, 2, 4, and 6 are tentatively related to the familiar theorems of longitudinal waves, vorticity, current conservation, and Eulerian equations of motion, respectively. The interpretation of theorems 3 and 5 are still open, but the units involved in theorem 5 indicate that it is a statement concerning the transport of angular momentum density, and may be related to problems in turbulent transport theory. The functional Ψ is of extreme interest for it is a first integral of the system of equations describing the "conservation" or transport of energy density and momentum density.

A paper entitled "An Intrinsic Transport Theorem" has been accepted for publication as a research note in Physics of Fluids.

The six basic theorems in the above paper may be applied to other representations. In particular, the methods are being applied to problems in hydrodynamics. Two of the theorems developed in the above mentioned paper have no known counterpart in the classical theory of fluids. The interpretation of these theorems in terms of dynamical energy storage mechanisms and the transport of angular momentum in turbulent flow is under investigation. A paper entitled, "An Intrinsic Theory of Continuous Media" has been partially completed. Applying the above theorems to the one-form of action for a fluid that is, by assuming a form for velocity flow vector yields the vorticity momentum flow, an energy flux (Poynting) equation, equation of continuity, and a momentum flux (equation of motion) equation, usual to and augmenting the conventional theory of hydrodynamics.

A by-product of the study of fluids has been an investigation of contemporary theories of elasticity in terms of the language of differential forms. One graduate student has spent the fall of 1968 in casting the theory of classical elasticity into the format of Steinberg and Abraham, a mathematical description of the "presence" of matter in geometric terms. A theory for the construction of a metric, or connection, to describe in geometric terms a material body based on the symmetry, homogeneity and elastic response of the body has recently been initiated by Noll[1] and Wang[2]. The student is now developing and expanding this work as part of his Ph.D. thesis. A contemporary statement of the equations of classical elasticity theory in the language of forms is expected, and when this work is combined with the preceeding work in electro-dynamics, theoretical analyses of the propagation of electromagnetic waves in accelerated, stressed, elastic media will be made.

<u>Translations of Fundamental Works on Differential Forms.</u> J. Pierce and R. M. Kiehn.

The following works have been translated and submitted for publication as a text for generating interest in the study of differential forms as applied in physics:

a) E. Cartan, <u>Lecons sur les Invariants Integranx</u>, Hermann, 1929.

b) J. Klein, "Espaces variationnels en mecanique," <u>Annals de l'Institut Fourier, Grenoble</u>, 12:1-124, 1962.

The following works are in advanced stages of translation.

a) F. Gallissot, "Application des Forms Exterieures du 2^{e} Ordre a la Dynamique Newtonienne et Relativiste," <u>Annales de l'Institut Fourier, Grenoble</u> 3, 1951, 278-285.

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[1]W. Noll, "Materially Uniform Simple Bodies with Inhomogeneities," Archives of Rational Mechanics and Analysis, 27:1-32, 1967-68.

[2]C. C. Wang, "On the Geometric Structures of Simple Bodies, a Mathematical Foundation for the Theory of Continuous Distributions of Dislocations," <u>Ibid</u> 27:33-94, 1967-68.

b) ___________, "Les Formes Exterieures en Mecanique," ibid 4, 1952, 145-297.

c) J. Klein, "Les Systemes Dynamiques Abstracts," ibid 13, 1962, 191-202.

d) A. Lichnerowicz, "Les Relations Integrales d'Invariance et Leurs Applications a la Dynamique," Bulletin des Sciences Mathematiques, Series II, 70, 1946, 82-95.

e) ___________, Problemes Globaux en Mecanique Relativiste, Paris, Hermann, 1939.

f) J. M. Souriau, Geometrie et Relativite, Hermann, 1968.

The following works are in the preliminary stages of translation.

a) V. Arnold, "Sur la Geometrie differentielle des groupes de Lie de dimension infinie et ses applications a L'hydrodynamique des fluides parfaits," Annales de L'Institut Fourier, Grenoble, 16:1, 319-361, 1966.

b) J. Vallant, "Characteristiques multiples et bicharacteristiques des systems d'equations aux derivees partielles lineaires et a coefficients constants," ibid., 16:2, 1-29, 1966.

Plasma Investigation, Dr. Melvin Eisner

The plasma portion of the Space Related Studies project has conducted its studies in the following areas: Theoretical and Design Studies of Flash Photoionization for Application to Epithermal Neutral Particle Studies, and Investigation of Stripping Cross Sections for H Atoms in the 50-500 e.v. range.

Although these projects have not as yet been carried to a definitive stage, yielding results appropriate for publication, significant progress has been made in areas important for the ultimate success of these studies. Summaries of the present status of the work follows.

Theoretical Studies and Design Studies on Flash Photoioniser for Application to Epithermal Neutral Particle Studies

The detection of low energy neutrals is usually accomplished by first converting them into ions. A difficulty encountered in the low energy regime is the perturbation of the neutral energy during ionization. Photoionization seems to offer the possibility of achieving significant ionization with a minimum energy perturbation, however, one requires intense pulsed sources with suitable spectra. The properties of the Plasma Focus Device (PFD) appear promising for application to this problem and these studies are aimed towards investigating the possible utility of the PFD as a flash photoioniser.

1. Theoretical Studies

The development of a plasma focus in a coaxial discharge may be viewed dynamically as consisting of three successive stages. In the first stage a $(\vec{J} \times \vec{B})$ force accelerates the plasma sheath axially down to the end of the central cylindrical electrode. In the following stage a small portion of the accelerated plasma pinches radially right beyond the end of the central electrode. A one dimensional snow plow model was used to study the dynamics of the discharge in the first two stages. The radial equation of motion

$$(\frac{d}{dt}\ [m_o + \pi\rho_o(R_c^2 - R^2)]\ \frac{dR}{dt} = -2\pi R\frac{\mu_o}{2}\ (\frac{I}{2\ R})^2)\ \text{is}$$

is solved numerically to give the radius R and velocity $\dot{R}$ of the sheath as a function of time. In about 175 n.s., the plasma pinches radially from the initial 3 cm radius of the central electrode down to a radius less than 1 mm. During this time, its density increases from $10^{13}/cm^3$ to about $2 \times 10^{18}/cm^3$ and its velocity increases from essential zero to about $2.5 \times 10^7 cm/sec$, corresponding to an electron energy in the ev range and ion energy in the kev range.

In the third stage, the high density electrons and ions collide with each other. Thermalization and Bremsstrahlung radiation occur. The intensity of the Bremsstrahlung spectrum radiated by an electron due to its interaction with an unscreened ion was calculated and it was found that intensity increased with increasing frequency. Photoionization cross sections were calculated, and found to be increasing roughly proportional to the 7/2 power of wavelength up to a cut off wavelength. For a ground state H-atom the cut-off wavelength is about 900 A. Practically all atoms within the region of the pinch would be ionized by the plasma sheath. Thus the total intensity of radiation would be increased significantly by the presence of a small fraction of impurity (high z) atoms in this region.

Thermalization and radiation processes have different dependences on charge, mass, and the relative velocity of the two colliding species of particles. The screening effect due to the orbital electrons of the high z ion is important to Bremsstrahlung radiation. Considering the Thomas-Fermi potential

$$V(r) = \frac{2Ze^2}{r} \exp\left(-\frac{r}{a}\right),$$

and define the maximum impact parameter due to screening by the atomic electrons,

$$b_{max}^{(s)} \quad a \quad 1.4 \frac{a_o}{Z^{1/3}},$$

where a_o is the Bohr radius of H-atom. Beyond $b_{max}^{(s)}$ the potential is regarded as a single or doubly charged ion and is cut off completely at the Debye radius.

Since the temperatures of ions and electrons as a function of time are dependent on the rate of thermalization and the intensity radiation (energy lost), three coupled equations of the following type have to be solved to obtain the ion and electron temperatures and the spectrum of radiation.

$$\frac{dT_e}{dt} = -\frac{2}{3}(Q_{e1} + Q_{e2}) - (R_{e1} + R_{e2}),$$

where'

$$Q_{e1} = \frac{e}{2} \frac{(T_e - T_1)}{\tau_{e1}} \text{ with } \tau_{e1} = \frac{3mm^*}{8\sqrt{2\pi}n^* L(Z_e Z^* e^*)^2} \left(\frac{T_e}{m_e} + \frac{T_1}{m_1}\right)^{3/2}$$

is the collisional energy transfer from electron to one species of ions and R_{e1} and R_{e2} are radiation lost of electron due to its interaction with two different species of ion respectively.

$$R_{e1} = -\frac{4\pi^3}{3} \frac{Z^e Z^4 e^6}{m_e c^2} n_e n_1 \frac{2\sqrt{\pi}}{3} \left[\frac{12}{\sqrt{\frac{m_e}{2T_e}}} + \frac{6\sqrt{\frac{m_e}{2T_e}}}{\frac{m_1}{2T_1}} \right]$$

$$R_{e2} = -\frac{16}{3} \frac{e^2}{c} \left(\frac{Z^2 e^2}{m_e c^2}\right)^2 n_e n_2 c^2 \iint \left\{ \frac{Z^2 \gamma^2}{a} \ln \left(\frac{2aM\mu}{\hbar}\right) + \right.$$

$$\left. \frac{M\mu}{\hbar} \left(\ln \frac{2\gamma^2}{\hbar} + 1\right) - \frac{\gamma^2}{a} \left(\ln \frac{2Mpa}{\hbar} + 1\right) \right\}$$

$$\left(\frac{b_e}{\sqrt{\pi}}\right)^3 \left(\frac{b_1}{\sqrt{\pi}}\right)^3 e^{-b_e^2 v_e^2} e^{-b_1^2 v_1^2} d^3 v_e d^3 v_1$$

where $b = \sqrt{\frac{m}{2T}}$, $u = |\vec{V}_e - \vec{V}_1|$

By solving the above equations, it should be possible to determine the optimum mixtures of ions and the physical conditions that would yield maximum ionizing radiations. Substantial progress has been made toward the solution and it is anticipated that significant design information will be forthcoming.

2. Design Studies

A small PFD has been constructed and tested. The 12 kilojoule capacitor storage bank is switched into a coaxial plasma accelerator through four low inductance ignition switches. The ambient pressure of 2 mm. of Helium in the accelerator has been adjusted so that the formation of the focus coincides with the first current maximum in the ringing of the energy storage bank and accelerator system.

Diagnostic studies of the radiation spectrum emanating from the focus have been carried out using a proportional counter with various absorbing foils. The initial yield is in the range of 10^{-10}, which is too small for a practical ionizing device but useful for model studies. Attempts to unfold the energy spectrum through the use of foils has proved to be difficult because of the high level of background radio frequency noise, originating from the high voltages and currents inherent in the energy discharge system. The analysis is further hampered by the relatively poor reproducibility of the spatial position of the focus. Improved reproducibility seems to be possible if a relatively weak axial magnetic field is incorporated into the accelerator.

The operation of the prototype PFD has indicated that it can be used to study the effect on ionization shaping the radiation spectrum. It is hoped that the electron-ion thermalization rate can be retarded, by the use of some heavy ion component, and thus lower the hard x-ray content of the pulse. Although the total energy radiated may be reduced, the ionizing quality of the radiation pulse may be significently increased. An increase in the ionization efficiency by a factor of 10^4 seems a reasonable expectation.

Stripping Cross Section for H Atoms in the Epithermal (50-500 e.v.) Range

These studies are aimed at investigating the stripping cross section for lost energy neutral hydrogen atoms in collision with various neutral target gases. The basic scheme is to first produce a reasonably monoenergetic neutral beam of known intensity. A duoplasmatron source has been developed to provide an ion beam of given energy and the ion beam is neutralized in a charge exchange cell. The charged particles are filtered from the beam and the neutral beam is passed through the stripping cell. The ions resulting from the stripping reaction are then analyzed and detected. The major problems involve obtaining sufficient energy in the initial lost energy ion beam and the efficient detection of the final loss energy neutrals. Considerable progress has been made in both these areas. Magnetic focusing has been used with the duoplasmatron source to obtain increased beam confinement and enhanced neutral production. An extremely efficient low energy ion detector has been developed and calibrated. This detector appears to be a significant improvement in the state of the art and a paper describing its construction and operation is being prepared for publication.

Using known information on the charge exchange reaction and the analysis of the original ion beam a calibrated neutral beam has been obtained. This beam has been used to obtain upper limits for the stripping cross section. Using the improved ion detection and improved stripping cell configurations, measurements of some accuracy will be made in the near future.

Thermal Method of Measuring the Quantity of a Cryogenic Fluid in a Tank Under Zero-Gravity Conditions

A. F. Hildebrandt and Nathan H. Wells, Jr.

Technical Note No. 1

Under NASA Grant NGR 44-005-022

Thermal Method of Measuring the Quantity of a Cryogenic Fluid in a Tank Under Zero-Gravity Conditions*

by A. F. Hildebrandt and Nathan H. Wells, Jr.

Abstract

Discussions with Mr. Richard Ferguson of the Manned Spacecraft Center concerning quantitative measurements of cryogenic fluids in zero-gravity fields have led to the incorporated proposal of using total specific heat measurements.

*Work supported by NASA Grant NGR 44-005-022

A liquid in a tank under conditions of zero-gravity tends to float around thus making an ordinary depth gauge useless. Weighing the tank is not feasible unless some means of producing an artifical gravity is provided for. Also capacitive techniques have been found to be useless since a thin film of liquid coats the inside of the tank and anything else in the tank.

It is herein proposed that this problem be solved by putting a known quantity of heat into the tank and measuring the resulting change in temperature and pressure of the tank and its contents. Then from a knowledge of such parameters as the specific heats of the liquid, vapor, and tank structural material, the latent heat of vaporization of the liquid, densities of liquid and vapor, and the volume of the tank - the mass of liquid in the tank could be calculated.

An expression for the mass of liquid in the tank will now be derived which gives the liquid mass in terms of the system parameters listed above. In deriving this expression the following assumptions will be made:

1. The vapor is an ideal gas.

2. Tank pressure equals vapor pressure (i.e. only one vapor in the tank, not a mixture).

3. The process of injecting heat into the tank is an essentially constant volume, non-flow process.

4. Tank and its contents are all in thermal equilibrium with each other when measurements are made.

5. The liquid is at its boiling point so that all processes take place along the vapor pressure curve. (A very accurate assumption).

6. All parameters are assumed constant over the range of pressure change to be encountered in this process. (For an input of 100 BTU to liquid oxygen and with a pressure change of 3 psi, this assumption would introduce no more than about -6% error.)

7. Total boil off is small enough so that the volumes of liquid and gas and also their masses are approximately constant during the process (within about ±1 lb. for a heat input of 100 BTU to liquid oxygen).

The equation of state of an ideal gas for a const. volume process is,

$$\frac{P_1}{M_{1gas}T_1} = \frac{P_2}{M_{2gas}T_2}$$

and, $\Delta M = M_{2gas} - M_{1gas} = \frac{Q_{BO}}{\ell}$ therefore,

(1) $$Q_{BO} = M_{1gas}\ell\left(\frac{P_2T_1}{P_1T_2} - 1\right) *$$

Now the total heat absorbed by the tank and its contents is,

(2) $$Q_{Total} = Q_{BO} + Q_{TR}$$

where Q_{BO} is given by (1) and Q_{TR} is given by

(3) $$Q_{TR} = M_{liq}Cp_{liq}\Delta T + M_{gas}Cp_{gas}\Delta T + M_{tank}Cp_{tank}\Delta T$$

*Derivation of Q_{BO} taken from: "Liq. Hydrogen Storage Parameters For a Lunar Voyage" by Charles C. Love, Jr. in Ballistic Missile and Space Technology Vol. 4 edited by D. P. LeGalley.

and $\Delta T = T_2 - T_1$

Symbols

P = tank pressure

T = temperature

M_{gas} = mass of vapor

M_{liq} = mass of liquid

Q_{BO} = heat absorbed by the liquid that boils off

ℓ = latent heat of vaporization

Q_{TR} = heat absorbed in raising the temperature

M_{tank} = mass of the tank

ρ_{gas}, ρ_{liq} = densities of the vapor and liquid

V_{tank} = tank volume

subscripts 1 and 2 refer to values before and after the heat is added respectively.

The three terms in (3) are the heats to raise the temperatures of the liquid, vapor, and tank material respectively by an amount ΔT. The masses of liquid and vapor in the tank are related to the tank's volume by the expression

$$\frac{M_{liq}}{\rho_{liq}} + \frac{M_{gas}}{\rho_{gas}} = V_{tank}$$

(4) Thus $M_{gas} = \rho_{gas}(V_{tank} - \frac{M_{liq}}{\rho_{liq}})$

Substituting (1), (3), and (4) into (2), Q_{Total} is given as,

$$Q_{Total} = \rho_{gas}\ell(V_{tank} - \frac{M_{liq}}{\rho_{liq}})(\frac{P_2 T_1}{P_1 T_2} - 1) + M_{liq}Cp_{liq}\Delta T$$

$$+ \rho_{gas}(V_{tank} - \frac{M_{liq}}{\rho_{liq}})Cp_{gas}\Delta T + M_{tank}Cp_{tank}\Delta T$$

Hence M_{liq} is given by,

$$(5)\quad M_{liq} = \frac{Q_{total} - \ell V_{tank}\rho_{gas}\left(\frac{P_2T_1}{P_1T_2} - 1\right) - V_{tank}\rho_{gas}Cp_{gas}\Delta T - M_{tank}Cp_{tank}\Delta T}{Cp_{liq}\Delta T - \frac{\rho_{gas}}{\rho_{liq}}Cp_{gas}\Delta T - \ell\frac{\rho_{gas}}{\rho_{liq}}\left(\frac{P_2T_1}{P_1T_2} - 1\right)}$$

The total heat input Q_{Total} is the sum of the known quantity put in plus the heat leak during the time this process is occuring.

Using eqn. (5) the mass of liquid in the tank can be calculated for various values of P_1 and P_2. The corresponding values of T_1 and T_2 can be obtained from the vapor pressure curve. With this data a table or graph can be prepared which gives the mass of liquid M_{liq}, for given values of the initial pressure, P_1 and the change in pressure, $\Delta P = P_2 - P_1$.

This method will now be applied to the specific case of the liquid oxygen tank carried on the Apollo spacecraft. This tank is constructed with 188 pounds of Inconel X-750 and carries 1200 pounds of liquid oxygen when full. From this fact the volume of the tank, V_{tank}, can be estimated from the density of liquid oxygen to be about 20 ft.^{3} Its pressure operating range is 100 to 150 psi. The following table was calculated* for P_1 = 50, 100, 150, and 200 psi and for ΔP = 1,2, and 3 psi. The values of Cp_{liq}, Cp_{gas}, and ℓ were calculated from some data on the properties of liquid oxygen furnished by MSC. The specific heats were calculated using the relation,

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*Calculations made using a slide rule.

$Cp = (\frac{\partial h}{\partial T})_p$, where h is the specific enthalpy. The latent heat of transformation, was calculated from, $\ell = \Delta h = h_{gas} - h_{liq}$ (at the point of transformation).

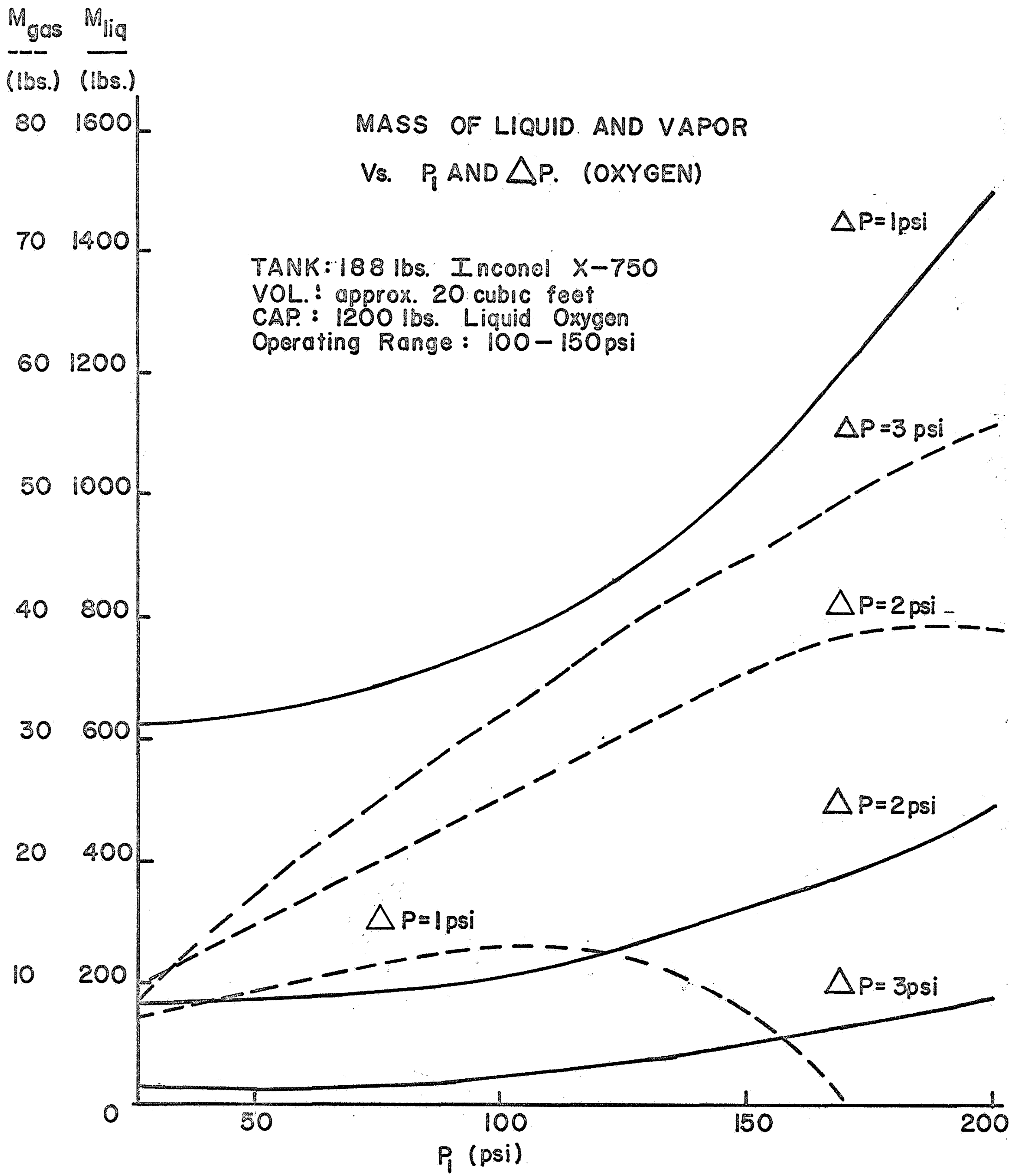
The values of these quantities at pressures of 50, 100, 150, and 200 psi are given in the following table. Using this data and setting Q_{Total} equal to 100 BTU, values of M_{liq} and M_{gas} (in parentheses) were calculated and are displayed in the second table.

Data for eqn. (5)

P_1 (psi)	T_1 (°R)	Cp_{liq} $(\frac{BTU}{lbm\,°R})$	Cp_{gas} $(\frac{BTU}{lbm\,°R})$	ℓ $(\frac{BTU}{lbm})$	ρ_{liq} $(\frac{lbm}{ft^3})$	ρ_{gas} $(\frac{lbm}{ft^3})$	Cp_{tank} $(\frac{BTU}{lbm\,°R})$
50	186.8	.437	.250	87.12	66.8	.865	.0668
100	204.4	.443	.278	81.12	63.3	1.67	.0699
150	216.4	.460	.300	76.31	60.6	2.48	.0721
200	225.7	.480	.330	71.95	58.4	3.32	.0740

$M_{liq.}$ (lbs.) and M_{gas} (lbs.) in parentheses

P_1 (psi)	ΔP = 1 psi		ΔP = 2psi		ΔP = 3 psi	
50	647	(9)	184	(15)	33	(17)
100	758	(13)	212	(25)	46	(32)
150	1029	(8)	324	(36)	102	(45)
200	1516		491	(39)	183	(56)



MASS OF LIQUID AND VAPOR
Vs. P_1 AND ΔP. (OXYGEN)
TANK: 188 lbs. Inconel X-750
VOL.: approx. 20 cubic feet
CAP.: 1200 lbs. Liquid Oxygen
Operating Range: 100-150psi
M_{gas} (lbs.)
M_{liq} (lbs.)
80 1600
70 1400
60 1200
50 1000
40 800
30 600
20 400
10 200
0
50
100
150
200
P_1 (psi)
ΔP=1psi
ΔP=3 psi
ΔP=2 psi
ΔP=2psi
ΔP=1psi
ΔP=3psi

In practice such a table as this or the accompanying graph would be calculated at many more points than was done here. For example ΔP might be given at every .1 psi in the range to be used and P_1 at every pound. Also the calculations should be done on a computer or with a calculator.

To use this table, first the initial pressure P_1 would be measured. Then the known quantity of heat, Q_{Total} would be put in. After allowing time for equilibrium to be reached, P_2 would be measured and ΔP calculated. Then with the values of P_1 and ΔP, M_{liq} and M_{gas} could be read from the table or graph.

Finally it should be emphasized that these results are not exact due to the approximations made to start with. However, the data for a table such as the one presented here could be obtained experimentally. Except for the effects of convection, which would affect the time required to reach equilibrium, this process of injecting heat could be used with or without gravity. Therefore the best way to obtain this data would be to perform the measurements on earth where a conventional depth gauge could be used to directly obtain the mass of liquid, M_{liq} for various values of P_1 and ΔP.

Helium II Momentum Flux Radiometer

Alvin F. Hildebrandt

Technical Note No. 2

Under NASA Grant NGR 44-005-022

HELIUM II MOMENTUM FLUX RADIOMETER

Alvin F. Hildebrandt

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The flow of heat in Helium II is accompanied by motion of two fluids as first described by Landau.[1] Since the two fluids have momentum, the two fluid model predicts that a flux of heat is accompanied by a momentum flux of the fluids. This momentum was first observed by Kapitza[2] who showed that a container with a small nozzle immersed in Helium II experienced reaction forces when a source of heat was present in the container. He found when either electrical energy or radiant energy was supplied to that container that the counterflowing normal and superfluid exerted large forces on the container. The reaction force varied linearly with power for large power levels whereas the reaction force variation was square law for small powers. It was not clear precisely what effect the throat shape had on the force law.

The reaction forces here are quite large because one is dealing with a small velocity but large momentum. In the case of an electromagnetic radiometer where the velocity of energy transput is large a small momentum is observed. Here the radiant energy is converted to a large momentum in the counterflowing superfluid.

The experiment by Hall[3] showed that the pressure on the heat source surface was given by

$$P = \rho_n V_n^2 + \rho_s V_s^2 = \frac{W^2}{C_2^2 \rho C T}$$

where ρ_n and ρ_s are the density of the normal fluid and superfluid respectively, V_n and V_s the velocity of the two components, W the power dissipated, C_2 the velocity of second sound, ρ the total fluid density, C the specific heat of helium and T the temperature in °K.

The magnitude of this force in the region of 1.5°K is 10 dyne cm^2/watt. This quantity is readily measurable with either a tension fiber or a physical pendulum and optical levers. We have shown in earlier work[4] that forces of this magnitude are easily measured by having the pendulum as part of an oscillator circuit and measuring the resonant frequency. More recent work has shown that use of a capacitive diaphragm is just as sensitive and extremely rugged. These forces can be calibrated absolutely in terms of an applied D.C. voltage.

The ability to accurately measure forces in liquid helium offers an attractive possibility of the development of a very accurate radiometer. In Fig. 1 is shown a schematic of a proposed solar radiometer.

The Sun's energy is absorbed on the absorbing surface and converted to heat. This causes an amount of superfluid to be converted to normal fluid with a consequent reaction force due to the counterflow of normal and superfluid. The reaction force can be measured quite accurately as a capacitance charge and in fact be absolutely calibrated in terms of a known steady voltage. The precise flow pattern for a particular nozzle should be known but can always be accounted for with a force calibration.

The proposed instrument should be quite rugged but very simple in concept and design.

References

[1] Landau, L. D. and Lifshitz, E. M., Fluid Mechanics (Pergamon Press, 1959).

[2] Kapitza, P. L., Zh. eksp. teor. Fiz. 11, 581 (1941); J. Phys. Moscow 5, 59 (1941).

[3] Hall, H. E., Proc. phys. Soc. A67, 485 (1954).

[4] Corke, H. E. and Hildebrandt, A. F., Phys. of Fluids, 11, No. 3, (1968).

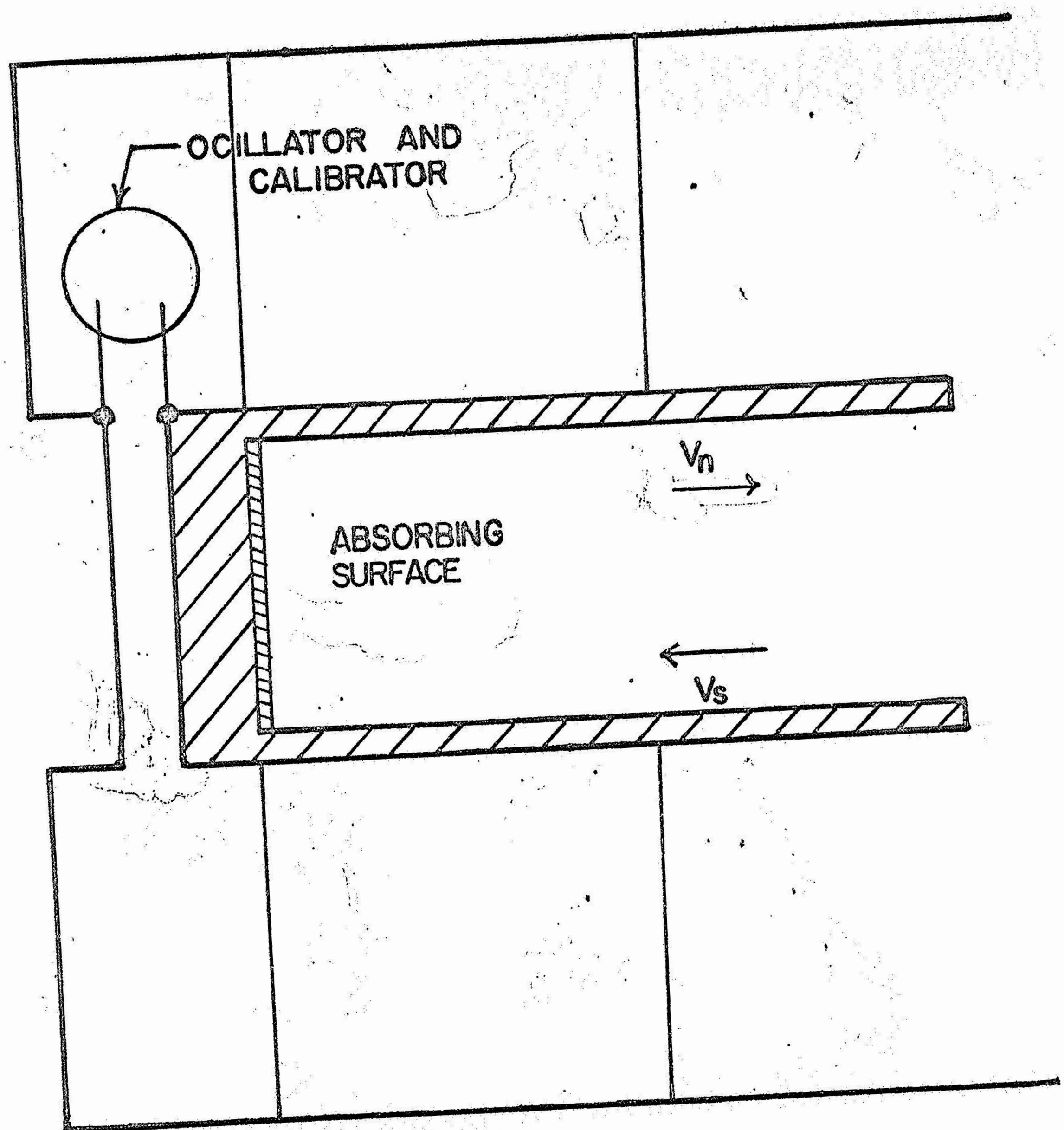

FIG. I

Appendix I

Papers Prepared Under
Space Related Technical Investigations Grant
NGR 44-005-022

1. "Pressure Studies of Pure Superfluid Flow", A. F. Hildebrandt and H. E. Corke presented at A.P.S. meeting June 1967, Toronto.

2. "Pressure Measurements in Subcritical Helium Flow", A. F. Hildebrandt and H. E. Corke, Phy. Fluids, 11, 3, March 1968.

3. "Low Field Superconductivity", presented at University of Minnesota January 1968 by A. F. Hildebrandt.

4. "Entrainment of Second Sound in Counterflowing Normal and Superfluid He II", E. M. Johnson and A. F. Hildebrandt, accepted for publication in Physical Review.

5. "Non-Local Electromagnetic Induction Effects", A. F. Hildebrandt and Thomas N. C. Tsien (submitted for publication).

6. "Proposed Hydromagnetic Model for Comets", N. S. Kovar and J. W. Kern presented at 122nd meeting of American Astronomical Society, July 1966.

7. "Optical Pumping and the D Line Ratio of Comet 1962 III", N. S. Kovar and R. P. Kovar, Solar Physics, April 1968.

8. "I(6300/,6363)/I(5577) Ratio of [01] in the Spectra of Comets", N. S. Kovar and R. P. Kovar published abstract in the Astronomical Journal April 1966 - full paper to be submitted to Astronomical Journal.

9. "Spin-Exchange Cross Sections of Alkali Atoms", R. H. Walker and C. K. Chang presented at Austin meeting of A.P.S. February 1967.

10. "Spin-Exchange Cross Sections in Atomic Hydrogen", R. H. Walker and B. M. Mayes presented at Austin meeting of A.P.S. February 1967.

11. "Spin Dependent Scattering of Monovalent Atoms" to be published in Feb. 5, 1968 edition of Physical Review.

12. "Exterior Forms and Electrodynamics", R. M. Kiehn presented at the Austin Meeting of the A.P.S. February 1967, Bull. A.P.S. 12 2, p. 198 (1967).

13. "Absolute Invariants," John F. Pierce and R. M. Kiehn, presented at the Austin meeting of the A.P.S., February 1967. Bull. A.P.S. 12, 2, p. 198 (1967).

14. "A Mechanized Approach to Piecewise Potential Problems", J. P. Shores and R. M. Kiehn, Bull. A.P.S. 11, 5, p. 749 (1966).

15. "The Relativistic Two State", John F. Pierce and R. M. Kiehn, Bull. A.P.S. 11, 5, p. 774 (1966).

16. Translation of E. Cartan, Lecons sur les Invariants Integraux Hermann, 1929. submitted for publication.

17. Translation of J. Klein, "Espaces variationnels en mecanique", Annals de l"Institut Fourier, Grenoble, 12:1-124, 1962, submitted for publication.

18. "An Intrinsic Transport Theorem" R. M. Kiehn and J. Pierce, accepted for publication in Physics of Fluids.