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NORTHWESTERN UNIVERSITY REPORT NUMBER 16

VARIANCE OF SOME SELECTED ATTRIBUTES IN GRANITIC ROCKS

by

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Statistical evaluation of the composition, physical properties,
and surface configuration of terrestrial test sites and their
correlation with remotely-sensed data.

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PREFACE

The paper reproduced in this report was published earlier this year in Russian. Because the work contains material considered to have general interest bearing on the statistical evaluation of the composition and physical properties of terrestrial geological test sites, the paper is reproduced here in English translation.

The original paper appeared on pages 240-252 of "Questions in Mathematical Geology", a volume edited by V. A. Steklova and produced to honor the 50th Anniversary of Dr. Andrew B. Vistelius (Laboratory of Mathematical Geology, Leningrad) by the Academy of Sciences of the U.S.S.R., Lenin Mathematical Institute, Leningrad, U.S.S.R. During the XXIII International Geological Congress held in Prague, August 1968, Dr. Vistelius was elected first President of the newly founded International Assocation for Mathematical Geology.

АКАДЕМИЯ НАУК СССР
ОРДЕНА ЛЕНИНА МАТЕМАТИЧЕСКИЙ ИНСТИТУТ
им. В. А. СТЕКЛОВА
ЛАБОРАТОРИЯ МАТЕМАТИЧЕСКОЙ ГЕОЛОГИИ

ВОПРОСЫ МАТЕМАТИЧЕСКОЙ ГЕОЛОГИИ

ИЗДАТЕЛЬСТВО «НАУКА»
ЛЕНИНГРАДСКОЕ ОТДЕЛЕНИЕ
ЛЕНИНГРАД 1968

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INTRODUCTION

In the past decade considerable attention has been focused on the quantitative nature of granitic rock massifs. Classical petrography emphasized differences between rock species, and there was a tendency to base estimates of the nature of whole plutons (or even batholiths) on single chemical analyses or on isolated petrographic descriptions. However, it is now widely recognized that many (if not most) igneous rock bodies possess considerable compositional variability (Whitten, 1963A). Similarly, current techniques permit a very large number of attributes (200-300) to be measured for a single rock sample. For specimens of a stated size, each attribute may have a dissimilar variance in samples of different size (cf., Baird, et al., 1964,1965; Hahn-Weinheimer and Ackermann, 1963; Hahn-Weinheimer and Johanning, 1963).

Many of the common concepts about petrology and petrography have been based on the assumption that the quantitative nature of a rock unit is known or can be readily determined. For example, that the modal and chemical composition of a granite stock or a dolerite flow are known correctly. However, the continuing problems associated with determining good modes and good chemical analyses emphasize that, although technological advances have materially assisted in developing accurate data, many attributes can not be measured accurately and precisely by every practicing geologist who needs such data.

Experimental petrology has shed valuable light on the petrogenesis of common igneous rock assemblages. Sufficient knowledge is now available to permit erection of simple conceptual process-response models on the basis of experimental and theoretical geochemical data. However, because the significance and magnitude of many variables remain uncertain, or unknown, it is commonly possible to erect several dissimilar process models which lead to dissimilar response models for a particular rock unit (Whitten, 1964; Whitten and Boyer, 1964). The accuracy of a predicted conceptual response model should be susceptible to combined quantitative and qualitative testing on the basis of observed data collected from the actual rock unit concerned. For example, geochemical principles governing the crystallization of a magma could be used to predict the three-dimensional mineralogical and chemical nature and variability of a granite stock. If geology is to be included amongst the physical sciences, it is necessary to remove petrogenetic concepts from the domain of subjective and intuitive judgments, and to use rigorous objective tests of quantitative models. Unfortunately, at the present time, it remains difficult to assess accurately the quantitative nature and variability of many attributes needed to test the conceptual models.

Because many attributes can be measured much more precisely, accurately, and cheaply than others, it might be anticipated that the more difficult attributes could be predicted with sufficient accuracy on the basis of the more-easily-assayed attributes. For example, sequential multivariate regression methods might be applied to a suite of easily-measured attributes, in order to predict the three-dimensional variation of an attribute that can only be measured with difficulty. Vistelius (1962) demonstrated that the P_2O_5

percentage of some granitic rocks of central Tien Shan can be predicted with the aid of a general linear equation, where the independent variables are modal percentages of quartz, potash feldspar, plagioclase, and mafic minerals.

In addition to testing petrogenetic models, and the general desirability of knowing the variance of rock attributes at different levels of sampling, there are other important reasons for investigating the extent to which observed data can be used to predict other attributes. Two obvious cases relate to (a) prospecting and evaluation in economic geology, and (b) the rapidly-developing field of remote sensing of environment on Earth and in Space. In many types of economic geology, the economic 'prize' (the sought-after commodity) occurs in small amounts that are difficult to detect during field exploration and economic development. However, it is commonly realistic to construct a quantitative statistical model to relate other geological variables to the 'prize,' so that, on the basis of the readily-observed attributes, the quantitative three-dimensional distribution of the prize is predicted (cf., Howd, 1964). If sufficient can be learned about the nature and inter-relationships of the common attributes of rocks, such predictions should become relatively simple.

Although certain facets of remote sensing (e.g., seismic and gravity exploration work, electric logging of oil wells, etc.) have been used extensively for some years, new powerful tools are constantly being developed that hold tremendous promise in many phases of geology. While X-ray fluorescence and infra-red methods for quantitative bulk chemical analysis are well-known, current work with the whole electromagnetic spectrum augurs well for the future of remote sensing of many geological attributes. As additional electromagnetic sensors are used to observe the characteristics of rock - in the laboratory, in the field, down deep drill holes, or on the surface neighboring planets observed from orbiting satellites - a whole plethora of new attributes will be measured (e.g., Colwell 1963; Brewer, 1964; Badgley, et al., 1965). Such attributes will be interesting in their own right, but a major use of the information will be for making predictions about those attributes of rocks with which geologists are currently familiar. For example, when electric logs are made for oil wells, the remotely sensed data can be used to predict lithological characteristics of the penetrated section. The logical use of data collected by remote sensor would be to erect statistical models to predict the nature and variability of the rocks observed (in terms of the well-known traditional attributes). As with methods based on the results of experimental petrology, it is an initial requirement that, whenever possible, such models be tested (and corrected as necessary) on the basis of 'field' studies of the actual rocks.

Unhappily, surprisingly little is known about the quantitative behavior of, and the inter-relationships between, those traditional attributes that have engaged the interest of geologists for decades. Baird, et al., (1964, 1965) showed that much remains to be learned about the necessary sampling methods required to achieve an adequate assessment of the composition and variability of a simple igneous complex. Hence, many uncertainties are involved in making any prediction about the three-dimensional behavior of one traditional attribute on the basis of numerous other traditional attributes. For example, with what degree of confidence can the specific gravity at different points within a granite be predicted from a knowledge of the chemistry and mineralogy of a suite of samples?

December 11, 1968

ERRATA

The National Aeronautics and Space Administration Research Grant NGR-14-007-027 : Northwestern University Report #16, 1968

and

"Questions in Mathematical Geology" edited by V. A. Steklova published in Russian in Leningrad in honor of A. B. Vistelius, 1968

Article "Variance of some selected attributes in granitic rocks" by E. H. Timothy Whitten.

Error on page 9 of the English version (Report #16) and page 242 of the Russian version

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Dr. Felix Chayes has drawn my attention to the fact that erroneous implications may be drawn from the inadequate wording of the following sentences which appear on page 9 of the English and page 242 of the Russian version:

"For example, in arriving at generalizations about granitic rocks, Tuttle (e.g., Tuttle and Bowen, 1958) and Chayes (e.g., 1951) culled from the literature large numbers of published analyses for different granitic masses in the U.S.A. and prepared composite ternary diagrams. Even when a single rock unit is involved, plotting modal or chemical information on a ternary diagram divorces that data from its essential geographical location."

The diagrams referred to in this quotation are:

Tuttle and Bowen, 1958: contoured ternary variation diagram based on 1269 norms calculated for all rocks in Washington's (1917) tables with more than 80 per cent normative albite + orthoclase + quartz.

Chayes, 1951: contoured ternary variation diagram based on 260 modes of calcalkaline granites from New England, Texas, and the southeastern United States.

Chayes' personal letters pointed out that

"(a) the results shown in my graphs were modal analyses made by me on specimens I collected, and (b) I have always been acutely aware of the geographic (and other) contributions to sample variance"

Apology is made for any misunderstanding that has been caused.

E. H. Timothy Whitten

SOME REPRESENTATIVE DATA FOR GRANITIC ROCKS

In an initial attempt to assess the inter-relationships between attributes for, say, granitic rocks, it might be assumed that a miscellaneous set of samples and analyses could be compared without specific regard to their geographical origins. For example, in arriving at generalizations about granitic rocks, Tuttle (e.g., Tuttle and Bowen, 1958) and Chayes (e.g., 1951) culled from the literature large numbers of published analyses for different granitic masses in the U. S. A. and prepared composite ternary diagrams. Even when a single rock unit is involved, plotting modal or chemical information on a ternary diagram divorces that data from its essential geographical location. Similarly, Schmidt equal-area projections and Wulff stereographic projections, although essential tools in the quantitative study of folded rocks, also divorce the observations from their geographical sites.

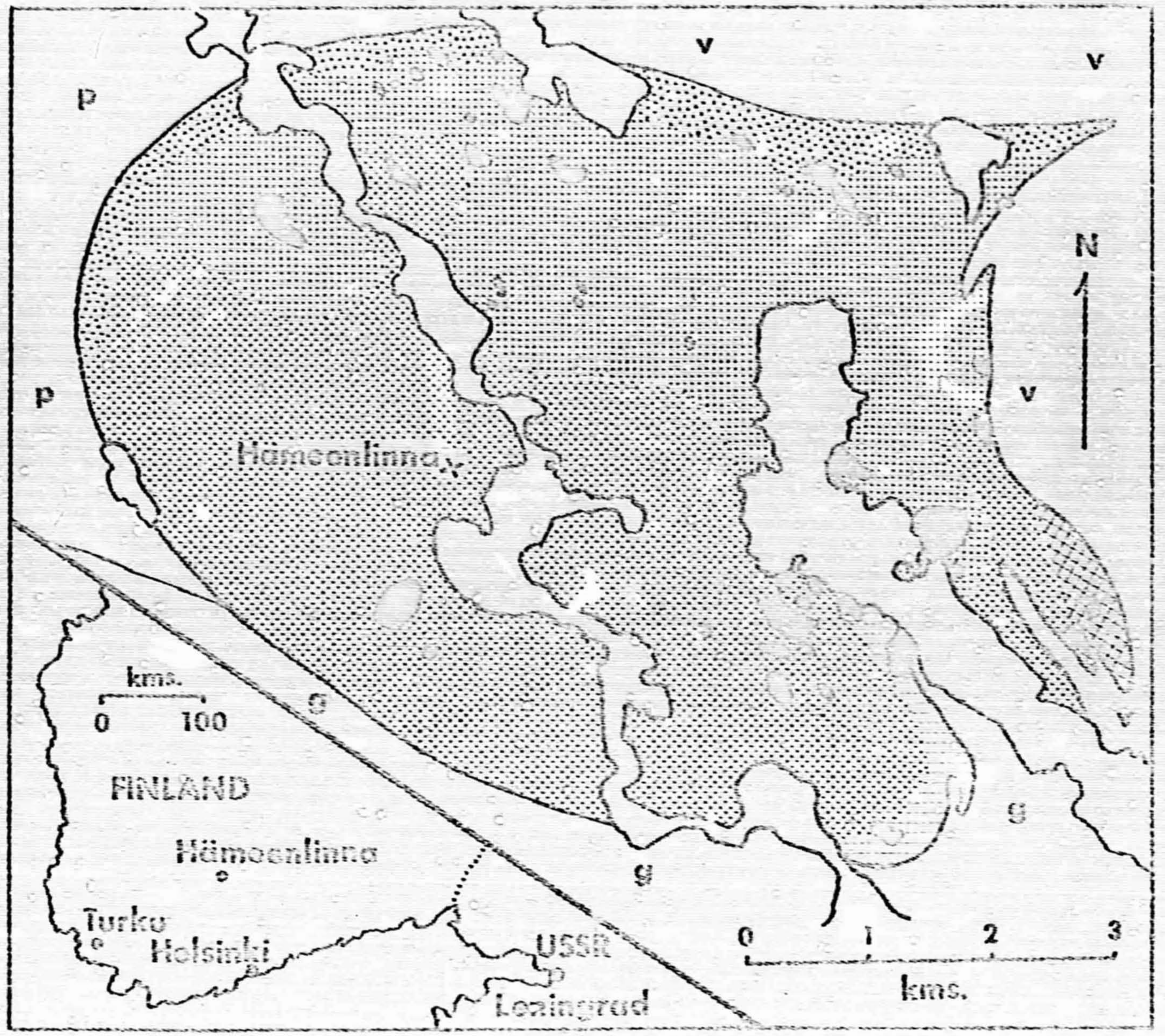
Such techniques can divert attention from the sampling problems inherent in the collection of all geological data. It is not necessary to emphasize that no amount of mathematical manipulation can rectify inadequate sampling - special care must be given to defining the target and sampled populations of interest before the samples are selected and analyzed (Whitten, 1961). Recently, Baird, et al. (1964, 1965) described their extensive sampling studies of the granitic rocks in Rattlesnake Mountain Pluton, S. California, U. S. A. On the basis of 863 X-ray spectrographic analyses of 465 rocks specifically collected according to a stratified sampling plan, Baird, et al. (1964) used analysis of variance techniques to evaluate the components of the standard deviation associated with each of four levels of sampling. Their results confirmed that "..without estimates of smaller scale variability it is not possible to judge the significance of la ge scale variations. The collection of at least two specimens at each locality permits an estimate of the small scale chemical variability....The figures presented suggest that the variability at the scale of the outcrop (even with cores 30 cm. in length) may require the collection of more than two specimens per locality." Being unique, these data can not be compared with results for other rock units, but, intuitively, it might be anticipated that a large number of granitic massifs will prove to possess variance at the outcrop level that is as large, or larger, than that for the Rattlesnake Mountain pluton.

Hence, it becomes even more important to discriminate between the sampled and the target populations when evaluating a series of chemical analyses for a granite mass. In the sequel some relationships shown by small groups of analyses for two granites are discussed. The limitations of such small sets of analyses must be borne in mind clearly.

THE MALSBURG GRANITE AND THE AULANKO GRANODIORITE

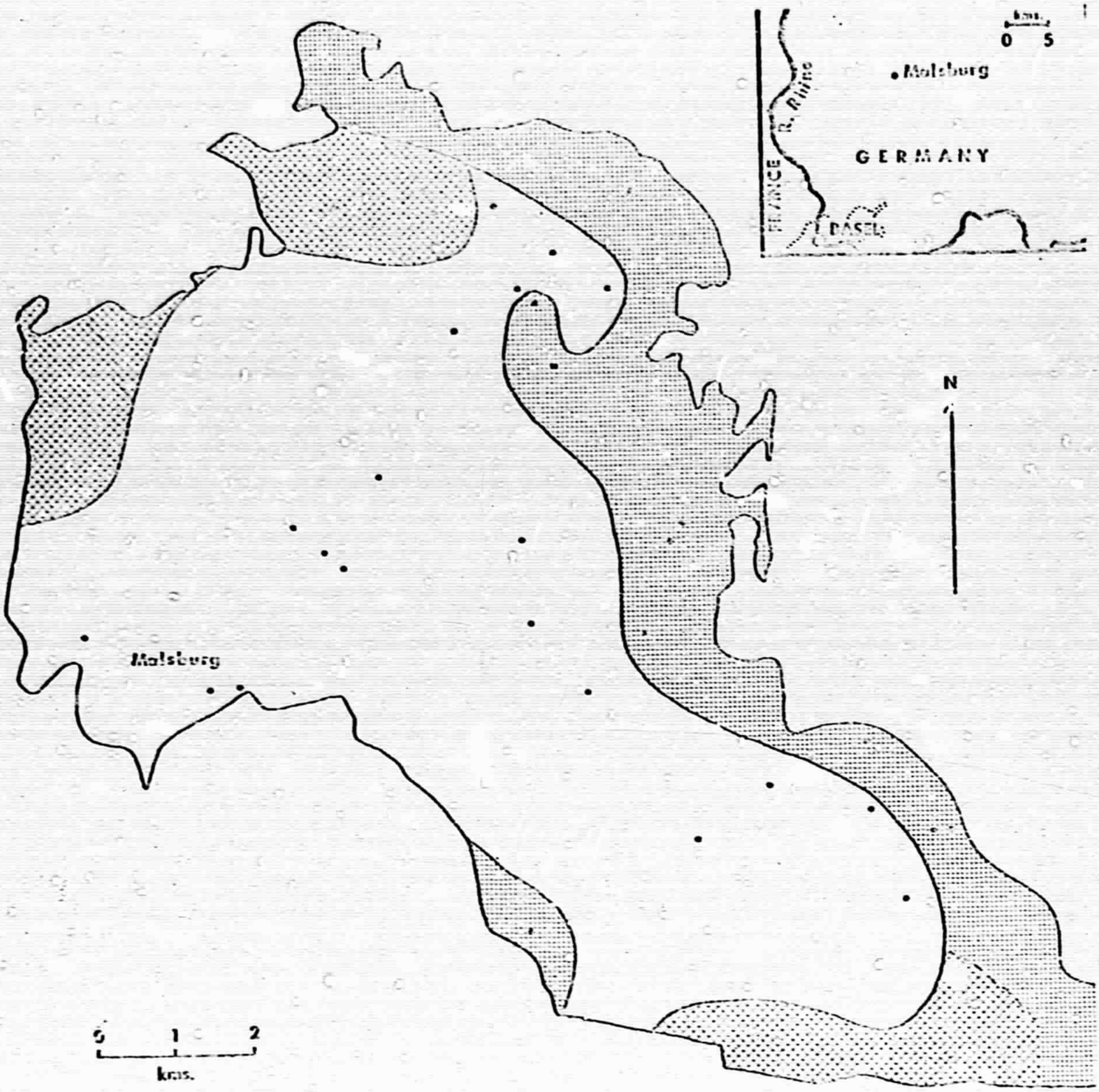
Two granitic complexes that have received extensive recent petrologic study were chosen for examination during 1962. First, the Aulanko Granodiorite (Fig. 1), a small Svecofennidic post-tectonic unit in central southern Finland, which was described and mapped by Simonen (1948). Second, the Malsburg Granite which is rather poorly exposed in the southern Black Forest, Germany (Fig. 2); this granite has been the subject of several petrographical and chemical studies (*e.g.*, Mehnert and Willgallis, 1961; Rein, 1961; Whitten, 1962; Hahn-Weinheimer and Ackermann, 1963).

More detailed results will be incorporated in a subsequent paper. At this time it is intended to expose certain differences between these two masses revealed when sequential multivariate analysis is used in an attempt to predict



1. The Aulanko Granodiorite, Finland, showing the petrographic variations recognized and mapped by Simonen (1948). Inset is location map.
Irregular dot pattern (north margin): granitized granodiorite without hornblende
Horizontal dot pattern: granitized granodiorite with hornblende
Oblique dot pattern: granodiorite
Horizontal ruling: gneissose granodiorite
Double oblique ruling: marginal facies of granodiorite
Solid black: Actual outcrops of Aulanko Granodiorite mass seen by Simonen.

The envelope rocks are intermediate and basic metavolcanic rocks (v), uralite porphyrite (p), and microcline granite (g).



2. The Malsburg Granite, southern Black Forest, Germany, showing the petrographic variations recognized by Mehnert (1960) and Mehnert and Willgallis (1961). "Central" zone unshaded; "marginal" zone has dot pattern or has double oblique ruling where rich in feldspar megacrysts. Dots are locations of samples used in present study.

P_2O_5 percentage, TiO_2 percentage, or specific gravity on the basis of the ten major oxides analyzed in samples collected from well-scattered outcrops throughout the granites.

SPECIFIC GRAVITY

The Aulanko Granodiorite is poorly exposed and outcrops are few. Two samples (one to two meters apart) were collected from each available outcrop (Fig. 1); the two samples at each outcrop were arbitrarily assigned to sampled populations A and B. The degree 3 trend surfaces (Fig. 3) for specific gravity of the sampled populations A and B account for 68.03 and 64.42 per cent of the sum of squares, respectively. The 36 samples comprising population A were chemically analyzed. Then sequential multivariate analysis was used to determine whether a general linear equation could provide a reasonable basis for predicting specific gravity when

$$SiO_2,\ Al_2O_3,\ Fe_2O_3,\ FeO,\ MgO,\ CaO,\ Na_2O,\ K_2O,\ P_2O_5,\ \text{and}\ TiO_2$$

are considered independent variables.

The general linear equation may be written as:

$$\underline{Y} = \underline{a}_0 + \underline{a}_1\underline{X}_1 + \underline{a}_2\underline{X}_2 \pm \underline{a}_3\underline{X}_3 + \ldots\ldots\ldots\ \underline{a}_{10}\underline{X}_{10} \qquad \text{(i)}$$

where $\underline{Y}$ is specific gravity and $\underline{X}_1$, ... $\underline{X}_{10}$ are the ten analyzed oxides. Solving the equation by the method of least squares and using all ten independent variables, permits 94.83 per cent of the total variability of specific gravity to be predicted*. However, for this hornblende granodiorite, it is instructive to examine the strongest equations for prediction when only a

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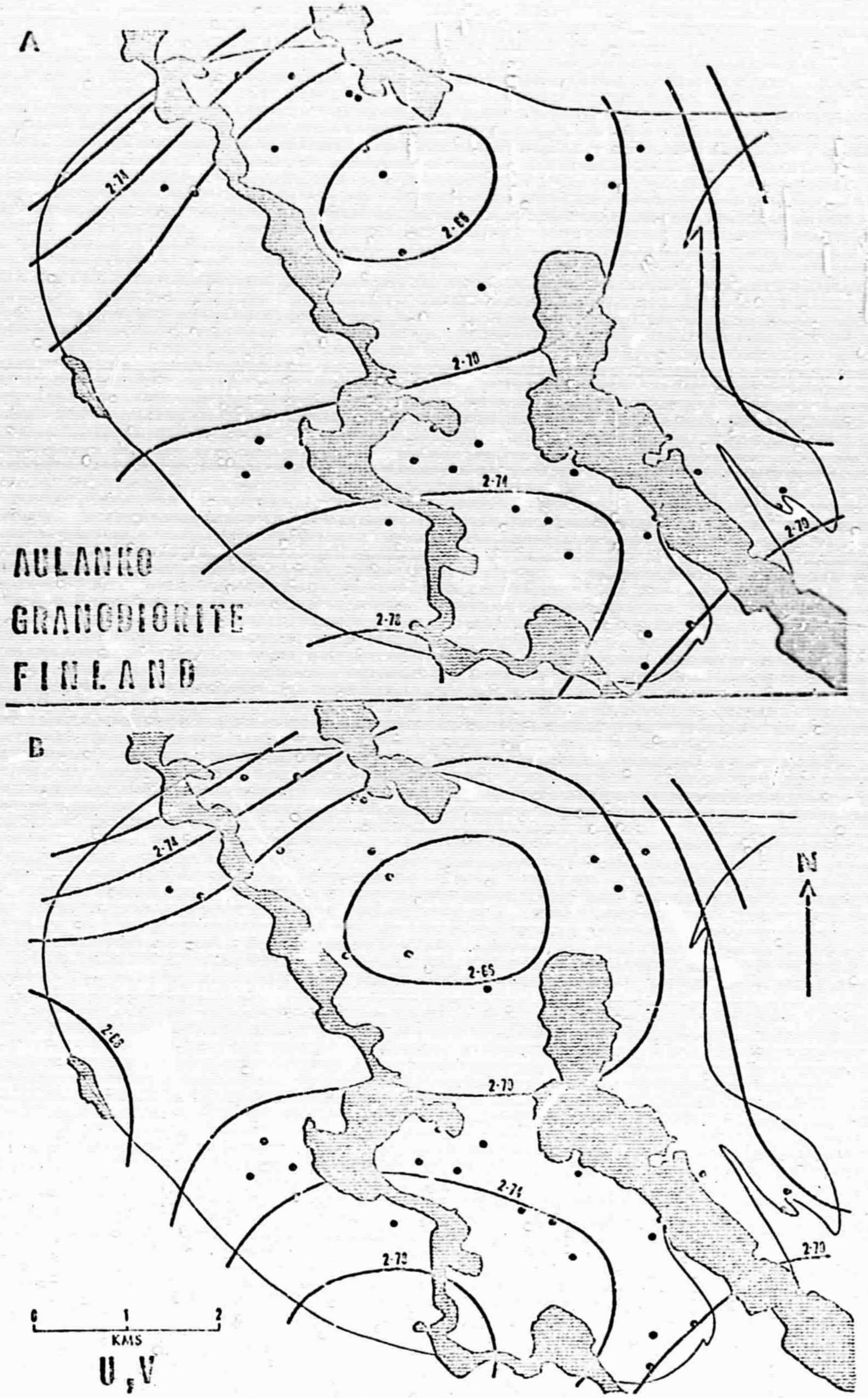
* These and similar calculations were made with the aid of the FORTRAN computer program prepared by Krumbein, et al. (1964).



3. Degree 3 trend surfaces for specific gravity of samples collected by Whitten from the Aulanko Granodiorite, Finland. A and B relate to sampled populations A and B, respectively. Dots show sample locations.

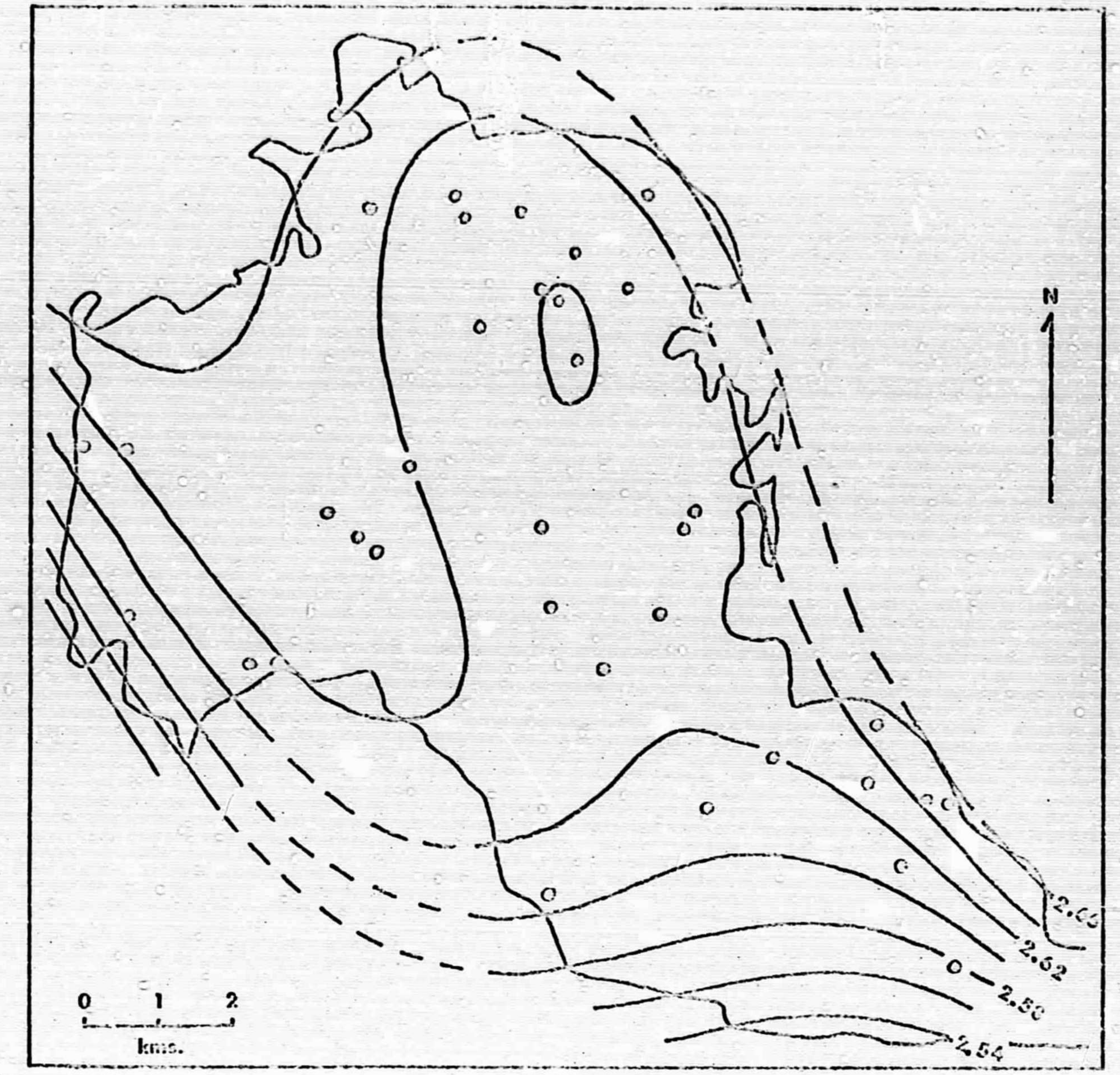
few independent attributes are included. Table I shows that

$$(\text{sp. gr.}) = \underline{a}_0 + \underline{a}_1\,(\text{CaO}) \qquad \text{(ii)}$$

accounts for 90.12 per cent of the total sum of squares. Hence, for many purposes, inclusion of the other nine oxides is redundant; that is, the inclusion of one or two oxides as independent variables additional to CaO adds little to the predictive power of the equation. Notice too that the degree 3 trend surface based on geographic coordinates only accounts for 68.03 per cent of the variability.

Figure 4 shows the degree 3 trend surface for specific gravity based on samples collected from sites in the Malsburg Granite shown in Figure 2. These samples were analyzed for the same ten oxides as for the Aulanko samples, but now, using all ten oxides as independent variables in equation (i) only 49.77 per cent of the total sum of squares is accounted for. Table I shows that CaO again provides the strongest equation when only one independent variable is used, but, for this granite, an equation like (ii) accounts for only 37.76 per cent of the total variability; use of both FeO and CaO as independent variables permits 43.11 per cent of the total variability to be predicted.

The reason for this dissimilarity between the Aulanko and Malsburg complexes is not immediately obvious. Although the Aulanko granodiorite is considerably more mafic than the Malsburg Granite, the variances of both sets of specific gravity data are comparable (Table II). The means and variances for all of the analyzed oxides are shown in Table II. For the Malsburg complex the data are considered together, but they are also divided into two groups - (a) the central and (b) the porphyritic marginal zones mapped by Mehnert (1960) and Mehnert and Willgallis (1961). On the basis of a study of



4. Degree 3 trend surface for specific gravity of samples collected by Whitten from the Malsburg Granite, Germany. Dots show sample locations. Despite the relief of the area, $\underline{W}$ was assumed constant and the surface computed accounts for only 25.85 per cent of the total variability.

Table II. Data for the samples collected from the Aulanko Granodiorite and the Malsburg Granite.

Attribute	Aulanko Granodiorite		Malsburg Granite			
				Variances		
	Mean %	Variance	Mean % of all analyses	All analyses	Central area analyses only	Marginal area analyses only
SiO_2	67.41	3.47	66.09	2.31	1.44	2.39
Al_2O_3	14.76	1.18	15.61	0.83	0.79	0.79
Fe_2O_3	1.98	0.60	1.72	0.57	0.54	0.60
FeO	2.07	0.86	1.54	0.53	0.33	0.62
MgO	1.73	0.56	2.06	0.60	0.35	0.71
CaO	3.53	1.11	1.96	0.66	0.66	0.64
Na_2O	3.93	0.45	3.83	0.34	0.35	0.34
K_2O	2.88	0.44	4.48	0.36	0.36	0.33
TiO_2	0.39	0.12	0.41	0.12	0.06	0.14
P_2O_5	0.16	0.05	0.23	0.06	0.03	0.06
Specific Gravity	2.71	0.03	2.63	0.03	0.02	0.04

Orders of magnitude of variances

Aulanko Granodiorite			Malsburg Granite (whole mass)*		
>1.0	>0.1	>0.01	>1.0	>0.1	>0.01
SiO_2 Al_2O_3 CaO	Fe_2O_3 FeO MgO Na_2O K_2O TiO_2	P_2O_5	SiO_2	Al_2O_3 CaO Fe_2O_3 FeO MgO Na_2O K_2O TiO_2	P_2O_5

* The same ranking applies to both the central and the marginal areas considered separately, except that TiO_2 should be transferred to the third column for the central area data.

some 25 suites of volcanic rocks Chayes (1962A, 1962B, 1964) suggested that the variance of SiO_2 is commonly an order of magnitude greater than that for Al_2O_3, MgO, CaO, or (FeO + Fe_2O_3), and that the variance of Na_2O, K_2O, and TiO_2 is a magnitude smaller still. Table II shows that the Malsburg analyses conform to this pattern, but that for the Aulanko rocks Al_2O_3 and CaO have larger variances (1.18 and 1.11, respectively).

TiO_2

Multiple linear regression was used to study the contribution of specific gravity and the other nine oxides for predicting the variability of TiO_2 percentage. Using all ten independent variables 91.25 per cent and 80.89 per cent of the total sum of squares are accounted for at Aulanko and Malsburg, respectively (Table III). For both sets of samples a large proportion of the total variability is accounted for, but one independent variable can account for a very large proportion of the total variability; the single variable is MgO for Aulanko and SiO_2 for Malsburg. While SiO_2 and MgO are the strongest pair of variables for Aulanko, there are eight pairs of attributes that are slightly stronger at Malsburg.

At Aulanko all sample sites are at approximately the same elevation above sea level so that three-dimensional sampling was impracticable. Trend surfaces account for only a small proportion of the total variability of TiO_2; there is a strong linear correlation between TiO_2 and MgO at Aulanko, and their degree 3 trend surfaces account for similar proportions of the total variability (Table IV). While lack of relief precludes evaluation of the vertical variability of these oxides, the Malsburg samples have a vertical range of 620 meters and the percentage of the total sum of squares

Table III. Best predictions of TiO_2 percentage based on 9 oxides, specific gravity, and spatial coordinates for the Aulanko and Malsburg granitic masses.

Number of independent variables taken at a time	Aulanko Granodiorite, Finland		Malsburg Granite, Germany	
	Independent variables	Sum of squares reduction % *	Independent variables	Sum of squares reduction % **
1	MgO	80.73	SiO_2	75.53
	SiO_2	66.19	MgO	47.31
	FeO	61.16	P_2O_5	47.23
2	MgO, SiO_2	84.98	SiO_2, V	81.69
	MgO, CaO	83.98	SiO_2, W	79.28
	MgO, Sp. Gr.	83.79	SiO_2, U	77.27
	MgO, Na_2O	82.13	SiO_2, Fe_2O_3	76.90
	MgO, Fe_2O_3	81.79	SiO_2, CaO	76.87
3	MgO, SiO_2, Al_2O_3	88.29	SiO_2, U, V	83.43
	MgO, SiO_2, P_2O_5	86.63	SiO_2, V, Na_2O	82.48
	MgO, P_2O_5, Sp. Gr.	86.45	SiO_2, V, Fe_2O_3	82.26
	MgO, P_2O_5, CaO	86.41	SiO_2, V, W	82.24
4	MgO, SiO_2, Al_2O_3, P_2O_5	89.88	SiO_2, U, V, CaO	84.27
	MgO, SiO_2, Al_2O_3, FeO	89.03	SiO_2, U, V, Na_2O	84.11
	MgO, SiO_2, Al_2O_3, Fe_2O_3	88.86	SiO_2, U, V, Al_2O_3	83.63

* Maximum with all nine oxides and specific gravity as independent variables - 91.25 per cent.
Maximum with all nine oxides, specific gravity, U, and V - 91.57 per cent.

** Maximum with all nine oxides and specific gravity as independent variables - 80.89 per cent.
Maximum with all nine oxides, specific gravity, U, V, and W - 86.97 per cent.

Percentage of total sums of squares accounted for by trend surfaces for TiO_2 percentage

Degree	Independent variables	Aulanko Granodiorite	Malsburg Granite
1	U, V	21.87	13.16
2	U, V	22.75	21.24
	U, V, W	n.d.	28.01
3	U, V	47.17	29.26
	U, V, W	n.d.	55.39

n.d. = not determined.

Table IV. Percentages of total sums of squares accounted for by some trend surfaces for the Aulanko and Malsburg granitic masses.

Dependent variables	Independent variables	Aulanko Granodiorite*			Malsburg Granite**		
		Degree 1	Degree 2	Degree 3	Degree 1	Degree 2	Degree 3
TiO_2	U,V U,V,W	21.87 n.c.	22.75 n.c.	47.17 n.c.	13.15 n.c.	21.24 28.01	29.26 55.39
P_2O_5	U,V U,V,W	27.85 n.c.	31.01 n.c.	42.25 n.c.	10.59 10.67	11.73 17.51	19.14 48.52
MgO'	U,V	25.06	31.80	49.10	10.86	23.04	28.18
FeO	U,V	60.98	66.45	76.32"	18.35	19.11	25.13

* 36 samples n.c. = not computed
** 37 samples
' MgO is strongest single independent variable for predicting TiO_2 at Aulanko.
" Strongest degree 3 trend component amongst the ten oxides and specific gravity at Aulanko.

for TiO_2 is almost doubled when the vertical dimension is included in the degree 3 trend component* (Table IV).

P_2O_5

When equation (i) is used, the dependent variable is divorced from its essential spatial coordinates. The regression equation can be extended easily to include U, V, and W, thus:

$$Y = a_0 + a_1X_1 + a_2X_2 + \ldots\ldots + a_{10}X_{10} + a_{11}U + a_{12}V + a_{13}W \qquad \text{(iii)},$$

where U, V, and W are orthogonal spatial coordinates with U and V being horizontal map coordinates and W the vertical direction. Only linear regression is considered in this paper, but equation (iii) could be extended readily to include independent terms that are powers of U, V, W, or X_n. Table IV shows that, for the Aulanko samples, P_2O_5 percentage can be predicted on the basis of U and V almost as efficiently as the MgO or TiO_2 percentages. By contrast, for the Malsburg rocks, only 19.14 per cent of the total sum of squares of P_2O_5 is predicted from U and V with the degree 3 polynomial; inclusion of W as an independent variable increases the percentage to 48.52.

At Aulanko 80.65 per cent of the total variability of P_2O_5 percentage can be predicted on the basis of the nine other oxides, U, and V; the strongest single variable is MgO (60.53 per cent) while SiO_2 is third (46.54 per cent). Table V shows that over 70 per cent of the variability can be accounted for by four combinations of three variables - all of which include MgO and two of which include the spatial coordinate V.

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* These trend surface components were calculated by the use of computer programs made available by Whitten (1963B), Whitten, *et al.* (1965), and Peikert (1963).

Table V. Best predictions of P_2O_5 percentage based on 9 oxides, specific gravity, and spatial coordinates for the Aulanko and Malsburg granitic masses.

Number of independent variables taken at a time	Aulanko Granodiorite, Finland		Malsburg Granite, Germany	
	Independent variables	Sum of squares reduction % *	Independent variables	Sum of squares reduction % **
1	MgO	60.53	SiO_2	56.34
	Sp. Gr.	52.91	TiO_2	47.23
	SiO_2	46.54	Al_2O_3	42.32
	FeO	44.29	MgO	27.51
2	MgO, Sp. Gr.	67.09	SiO_2, Al_2O_3	62.57
	MgO, $\underline{V}$	66.32	Al_2O_3, TiO_2	59.90
	MgO, CaO	65.99	SiO_2, Na_2O	59.37
	SiO_2, Na_2O	65.06	SiO_2, Sp. Gr.	58.18
	MgO, $\underline{U}$	63.14	SiO_2, $\underline{V}$	58.13
3	MgO, Sp. Gr., TiO_2	72.48	Al_2O_3, Fe_2O_3, FeO	65.54
	MgO, Sp. Gr., $\underline{V}$	71.15	SiO_2, Al_2O_3, Na_2O	65.42
	MgO, CaO, TiO_2	71.14	SiO_2, Al_2O_3, Fe_2O_3	64.60
	MgO, CaO, $\underline{V}$	70.73	Al_2O_3, Na_2O, TiO_2	64.24
	MgO, Sp. Gr., Na_2O	69.22	SiO_2, Al_2O_3, Sp. Gr.	63.99

* Maximum with all nine oxides, specific gravity, $\underline{U}$, and $\underline{V}$ as independent variables - 80.65 per cent.

** Maximum with all nine oxides, specific gravity, $\underline{U}$, $\underline{V}$, and $\underline{W}$ as independent variables - 75.09 per cent.

At Malsburg, use of the nine other oxides, U, V, and W together only permits 75.09 per cent of the variability of P_2O_5 to be predicted. Unlike the Aulanko samples, SiO_2 is the strongest single attribute (only 56.34 per cent) and MgO is the fourth in rank order and accounts for only 27.51 per cent of the P_2O_5 variability. Hence, there is a marked difference between the P_2O_5: MgO correlations in the two suites of granitic samples. Inclusion of linear powers of U, V, and W is not useful for prediction of P_2O_5, although Table IV suggests that inclusion of the degree 3 powers of U, V, and W (including the cross product terms) would significantly increase the success of predictions.

CONCLUDING REMARKS

In a subsequent paper, it is proposed to evaluate the significance of the full chemical analyses in terms of the detailed petrography. In this preliminary account, it has been intended to demonstrate that predictive models developed for one rock unit do not necessarily apply to another, broadly-analogous, rock unit. Clearly, one aim of petrography must be to erect generalized conceptual models. Petrogenetic concepts lead to the erection of process-response models and it would be very significant if petrographers could discover what general rules are involved in the behavior and inter-relationship of common attributes of granitic complexes. It is regrettable that, at the present time, extraordinarily little is known about the variance of chemical attributes in individual granitic complexes, or about the inter-relationships of common attributes.

Clearly, there is an urgent need to acquire more information on these subjects, and this paper is offered in the hope of stimulating more geologists to apply themselves to obtaining and publishing this much-needed data.

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