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SOLAR X-RAYS SCATTERED BY VENUS, MARS AND THE MOON*

by

E. D. Loh and G. P. Garmire

Technical Report

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Introduction

In the range 4Å to 50Å, the important scattering processes for the light elements are K or L photoionization with fluorescence, Rayleigh scattering, and Compton scattering. Roughly $\sigma_{PI} = (100 \text{ to } 1000) \times \sigma_R$ and $\sigma_R = Z\sigma_c$. Since the loss of energy per Compton collision is less than .01 percent and the magnitude of σ_{PI} precludes a significant contribution from multiple Compton collisions, the Compton scattering will be treated as an addition to Rayleigh scattering. The differential Compton cross section has a $\frac{1}{2}(1 + \cos^2\varphi)$ dependence, whereas the differential Rayleigh cross section is a function of $\frac{\sin\varphi}{\lambda}$.

The planetary atmospheres will be considered plane-parallel layers. The atmospheres are assumed to be thick for x-rays. A spherical coordinate system is defined to describe the surface of the planet, Figure 1(a). The polar angle φ is the angle between the sun and a surface element of the planet; and the aziumthal angle θ is measured in the plane of the sun, observer, and planet. Another coordinate z describes the depth in the atmosphere, Figure 1(b).

$\rho(z)$	=	density of the atmosphere at z
$R(z)$	=	$\int_{-\infty}^{z} \rho(z)\, dz$
$\chi(\lambda)$	=	photoionization cross section/unit mass
$\sigma(\lambda)$	=	combined Rayleigh and Compton total cross section/unit mass
$K(\lambda)$	=	$\chi(\lambda) + \sigma(\lambda)$
$G_+(\theta, \varphi)$	=	$\frac{1}{\cos\varphi}$ A geometric factor that reflects the fact that photons which enter the atmosphere at φ traverse longer path lengths than photons which travel

perpendicular to the layers of the atmosphere, see Figure 1b.

$G_-(\theta,\varphi)$ = $1/\cos\varphi\,(\cos\beta + \tan\varphi\sin\beta\cos\theta)$ The geometric factor for emergent photons.

α = fluorescence photons emitted/photons captured by photoionization, the fluorescence yield.

Ω = solid angle of the observer.

$F_o(\lambda)\cos\varphi$ = flux from the sun above the atmosphere at φ.

The emergent radiation from each surface element $dA = r^2 \sin\varphi\, d\theta d\varphi$ is calculated. The flux at z which has not suffered any scattering is

$$F(z,\lambda) = F_o(\lambda)\cos\varphi\, \exp(-K(\lambda)\, R(z)\, G_+).$$

The contribution dE_{R1} by photons that Rayleigh (or Compton) scatter at z and scatter no more is calculated. Similarly the contribution dE_{Rn} by photons that Rayleigh scatter at z and Rayleigh scatter n-1 times more is calculated. Then E_{Rn} is found by integrating over all z

$$E_{Rn}(\lambda) = \int_{z=-\infty}^{z=\infty} dE_{Rn}.$$

The total energy from dA is found by summation over the number of scatterings

$$E_R(\lambda) = \sum_{n=1}^{\infty} E_{Rn}.$$

The total flux from the entire planet is

$$I_R(\lambda) = \int_A E_R dA.$$

The emergent line flux $I_L(\lambda_K)$ at λ_K, the K limit, is the integral of contributions from photons that were absorbed from all wavelengths $\lambda < \lambda_K$. $dE_{Kn}(\lambda)$ is the contribution by photons that K scatter once and Rayleigh scatter n-1 times.

$$E_{Ln}(\lambda_K) = \int_{z=-\infty}^{z=\infty} \int_{\lambda=o}^{\lambda=\lambda_K} d\,E_{Ln}(\lambda)d\lambda$$

$$E_L(\lambda_K) = \sum_{n=1}^{\infty} E_{Ln}(\lambda_K)$$

$$I_L(\lambda_K) = \int_A E_L(\lambda_K)\,dA.$$

In reality, the E_{L1} and E_{R1} terms are the only significant ones. Instead of an exact calculation for E_{L2}, E_{L3},... and E_{R2}, E_{R3}..., an upper bound for these terms will be found.

Calculation of K Fluorescence Flux

Only one sequence of events is possible in the E_{L1} scattering process.

$$dE_{L1} = \left[F_o(\lambda)\cos\varphi\ \exp\ (-K(\lambda)\ R(z)\ G_+)\right]\ \left[\kappa(\lambda)\ dR(z)\ G_+\right]$$
$$\times \left[\alpha\frac{\Omega}{4\pi}\right]\ \left[\exp\ (-K(\lambda_K)\ R(z)\ G_-)\right]$$

The first factor is the incident flux at z; the second factor is the probability that a photon is absorbed by photoionization; the third, the probability that a fluorescence photon is emitted into the solid angle of the observer; the fourth, the probability that the photon escapes the atmosphere.

$$E_{L1} = \int_{z=-\infty}^{z=\infty}\int_{\lambda=0}^{\lambda=\lambda_K} dE_{L1}\ (\lambda)\ d\lambda$$

$$= \int_{\lambda=o}^{\lambda=\lambda_K} \frac{F_o(\lambda)\ d\lambda\ \cos\varphi\ \frac{\alpha}{4\pi}}{\frac{K(\lambda)}{\kappa(\lambda)} + \frac{K(\lambda_K)}{\kappa(\lambda)}\ \frac{G_-}{G_+}}$$

$$= \frac{\alpha\Omega}{4\pi}\int_{\lambda=o}^{\lambda=\lambda_K} \frac{F_o(\lambda)\ d\lambda}{1 + \frac{\kappa(\lambda_K)}{\kappa(\lambda)}\ \frac{G_-}{G_+}}\ \cos\varphi \quad \text{since } K(\lambda) \simeq \kappa(\lambda).$$

Two sequences of events are possible in the E_{L2} process.

(1) The photon is K captured at R(z), and the fluorescence photon is Rayleigh scattered at $R(z_1)$.

(2) The photon is Rayleigh scattered at R(z), and the photon is K processed at $R(z_1)$.

$$E_{L2}^{(1)} = \int_{o}^{\lambda_K} \int_{z=-\infty}^{z=\infty} \int_{z_1=z}^{z=\infty} \int_{\omega} \left[F_o(\lambda) \cos\varphi \exp(-K(\lambda)\, R(z)\, G_+)\right]$$

$$\times \left[\chi(\lambda)\, d\, R(z)\, G_+\right] \left[\alpha \frac{d\omega}{4\pi}\right] \left[\exp(-K(\lambda_K)\, (R(z_1) - R(z))\, G_1)\right]$$

$$\times \left[\frac{d\sigma(\lambda_K)}{d\omega} \Omega\, d\, R(z_1)\, G_1\right] \left[\exp(-K(\lambda_K)\, R(z_1)\, G_-)\right]$$

G_1 is the geometric factor for the path that the photon traversed in going from z to z_1. The sequence of the bracketed expressions follows the sequence of the events in the scattering process. The ω integral is taken over all directions of the first K emission.

$$E_{L2}^{(1)} = \int_{o}^{\lambda_K} \frac{F_o(\lambda) \cos\varphi\, \Omega\, \alpha/4\pi}{1 + \frac{\chi(\lambda_K)}{K(\lambda)} \frac{G_-}{G_+}} \quad \frac{1}{\chi(\lambda_K)} \left(\int_{\omega} \frac{G_1 \frac{d\sigma(\lambda_K)}{d\omega} d\omega}{(G_1 + G_-)}\right) d\lambda$$

$\frac{d\sigma}{d\omega} \leq \frac{\sigma_o}{4\pi}$, where $\frac{\sigma_o}{4\pi}$ is the differential Rayleigh cross section for scattering with infintesimal change in direction.

Then

$$\int_{\omega} G_1 \frac{\frac{d\sigma(\lambda_K)}{d\omega}}{G_1 + G_-} d\omega \leq \int_{\omega} G_1 \frac{\frac{\sigma_o}{4\pi}\, d\omega}{G_1 + G_-} .$$

If γ is the angle between the z-axis and the direction the photon takes,

$$\int_{\omega} \frac{G_1 \; \sigma_o/4\pi}{G_1 + G_-} \, d\omega = \int_0^{\pi} \frac{\frac{1}{|\cos\alpha|} \; \sigma_o/4\pi \quad 2\pi \sin\gamma \, d\gamma}{\frac{1}{|\cos\alpha|} + G_-} = \sigma_o \frac{1}{G_-} \ln(1 + G_-).$$

The maximum value for the function $\frac{1}{G_-} \ln(1 + G_-)$ is 1.

Hence

$$E_{L2}^{(1)} \leq \frac{\alpha}{4\pi} \int_0^{\lambda_K} \frac{F_o(\lambda) \cos\varphi}{1 + \frac{\chi(\lambda_K) \; G_-}{\chi(\lambda) \; G_+}} \frac{\sigma_o(\lambda_K)}{\chi(\lambda_K)} \, d\lambda = E_{L1} \frac{\sigma_o(\lambda_K)}{\chi(\lambda_K)} .$$

By a similar calculation,

$$E_{L2}^{(2)} \leq \frac{\alpha \Omega}{4\pi} \int_0^{\lambda_K} \frac{F_o(\lambda) \cos\varphi}{1 + \frac{\chi(\lambda_K)}{\chi(\lambda)} \frac{G_-}{G_+}} \frac{\sigma(\lambda)}{\chi(\lambda)} \, d\lambda$$

Let Q = maximum of $\frac{\sigma(\lambda)}{\chi(\lambda_K)}$ for the pertinent range of λ. Then

$$E_{L2}^{(2)} \leq E_{L1} \; Q \text{ and } E_{L2} = E_{L2}^{(1)} + E_{L2}^{(2)} \leq 2 \; E_{L1} \; Q.$$

It can be shown by similar calculation that

$$E_n \leq n \quad E_{L1} \quad Q^n$$

Then

$$\frac{E_L - E_{L1}}{E_{L1}} \leq \frac{2\,Q - Q^2}{(1 - Q)^2} \lesssim .01.$$

This estimate is very generous for the elements with $Z \geq 6$ in the range $\lambda > 5\text{Å}$. Thus multiple Rayleigh scattering contributes insignificantly to line formation.

$$I_L\,(\lambda_K) = \int_A \int_o^{\lambda_K} \frac{F_o(\lambda)\; d\lambda\; \cos\varphi\; \alpha\; \frac{\Omega}{4\pi}}{1 + \frac{\chi(\lambda_K)}{\chi(\lambda)}\,\frac{G_-}{G_+}} \quad d\,A.$$

Calculation of Rayleigh and Compton Scattered Fluxes

All the Rayleigh calculations are similar to the K calculations.

$$E_{R1} = \int_{z=-\infty}^{z=\infty} \left[F_o'(\lambda) \exp(-K(\lambda) R(z) G_+)\right] \cdot \left[\exp(-K(\lambda) R(z) G_-)\right] \cdot \left[\frac{d\sigma(\lambda)}{d\omega} \Omega \, dR(z) \, G_+\right].$$

where

$\frac{d\sigma(\lambda)}{d\omega}$ is evaluated at $\frac{\sin\beta}{\lambda}$.

The approximations that were used in the previous section are also valid.

$$\frac{E_R - E_{R1}}{E_{R1}} \leq \frac{2Q - Q^2}{(1-Q)^2} \simeq .01.$$

Hence

$$I_R(\lambda)\, d\lambda = \int_A \frac{F_o(\lambda)\cos\varphi}{1 + \frac{G_-}{G_+}} \quad \frac{\left.\frac{d\sigma}{d\omega}\right|\beta}{\chi(\lambda)} \quad dA \;\; d\lambda.$$

The assumption of a one constituent atmosphere was implicit in the calculation. However the results can be modified easily for a multiconstituent atmsophere in which the relative amount of each constituent is a constant of depth. If $\rho_i(z)$, $\chi_i(\lambda)$, and $\sigma_i(\lambda)$ are the descriptive variables of the i-th constituent, and if

$$\frac{\rho_i(z)}{\sum_j \rho_j(z)} \quad \text{is independent of } z,$$

$$I_R \, d\lambda = \int_A \frac{F_o(\lambda) \cos\varphi_\Omega}{1 + \frac{G_-}{G_+}} \frac{\sum \rho_i \frac{d\sigma_i}{d\omega}}{\sum \rho_i \chi_i} \, dA$$

and

$$I_L(\lambda_{K_i}) = \int_A \int_o^\lambda \frac{F_o(\lambda) \cos\varphi \, \alpha_i \frac{\Omega}{4\pi}}{1 + \frac{\sum \rho_j \chi_j(\lambda_{K_i})}{\sum \rho_j \chi_j(\lambda)} \frac{G_-}{G_+}} \frac{\rho_i \chi_i(\lambda)}{\sum \rho_j \chi_j(\lambda)} \, d\lambda \, dA.$$

Numerical Results

Absolute fluxes were computed from theoretical calculations of Mandel'štam and from a solar spectrum of Neupert, et al. The spectrum was normalized by Mandel'štam's fluxes for an electron temperature of 2×10^6 °K. For wavelengths outside Neupert's spectrum, Mandel'štam's fluxes were smoothed and used directly. The expected fluxes from the moon, venus, and mars are presented in table 1. Because the surface of the moon is rough, the calculations for atmospheres does not apply for the moon. The ratio G_-/G_+ is complicated. However, when $\beta = 0$, $G_-/G_+ = 1$ for rough surfaces as well as for smooth atmospheres, and the calculations are precise. The compositions of the moon, mars, and venus were taken from Turkevich et al, Chamberlain and McElroy, and Kliore et al, respectively. The fluxes in table 1 are for full moon ($\beta = 0$) and full mars ($\beta = 0$). $\beta = 96.5^{\circ}$ for venus. Figure 2 shows the continuous reflected spectrum of the moon and the sun spectrum of Mandel'štam. The reflected spectrum reproduces the sun spectrum at a reduced intensity and exhibits the fluroescence lines of its constituents.

Table 1

Moon

Element	Relative Abundance	Fluorescence Line (Å)	Flux ($ergs/cm^2/sec$)
O	58	23.62	1.38×10^{-10}
Na	1	11.9, 11.6	1.39×10^{-12}
Mg	3	9.89, 9.52	1.84×10^{-12}
Al	6.5	8.34, 7.96	1.45×10^{-12}
Si	18.5	7.13, 6.75	7.76×10^{-13}
K	2.5	47.2, 47.7	7.20×10^{-12}
Ca	4.5	40.5, 36.0 41.0, 36.3	1.32×10^{-11}
Fe	6	15.7, 19.8, 17.3 20.2, 17.6	1.59×10^{-11}

Element	Flux counts/sec/cm^2			
	Mars	Venus		
	44 percent CO_2*	70 percent CO_2*	83.3 percent CO_2*	95 percent CO_2*
C	6.59×10^{-6}	4.11×10^{-5}	4.57×10^{-5}	4.87×10^{-5}
N	6.50×10^{-6}	1.85×10^{-5}	1.06×10^{-5}	0.388×10^{-5}
O	5.38×10^{-6}	3.66×10^{-5}	4.07×10^{-5}	4.87×10^{-5}

(*remainder N_2)

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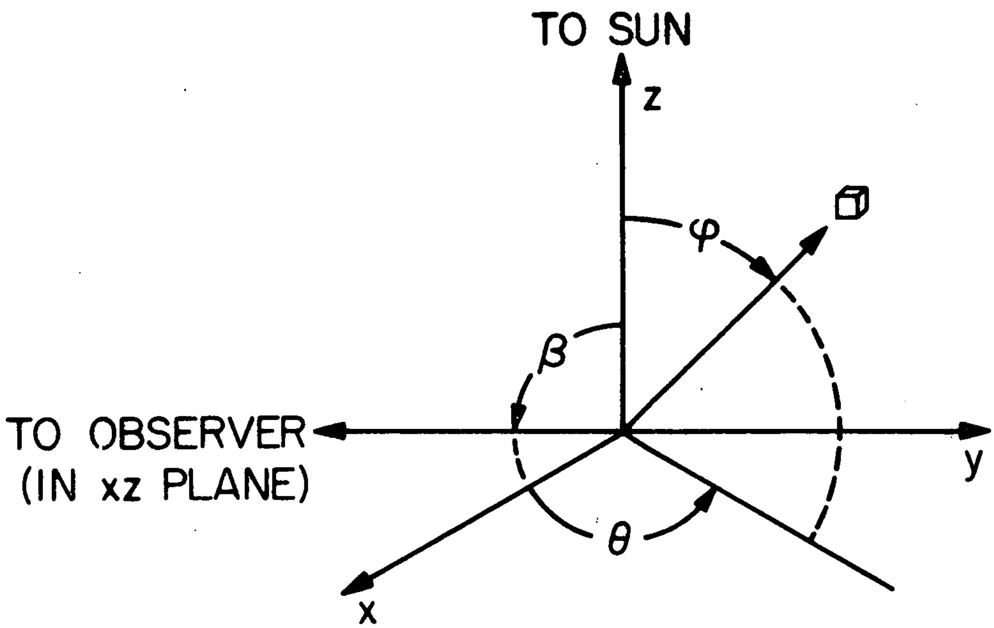


Figure 1a

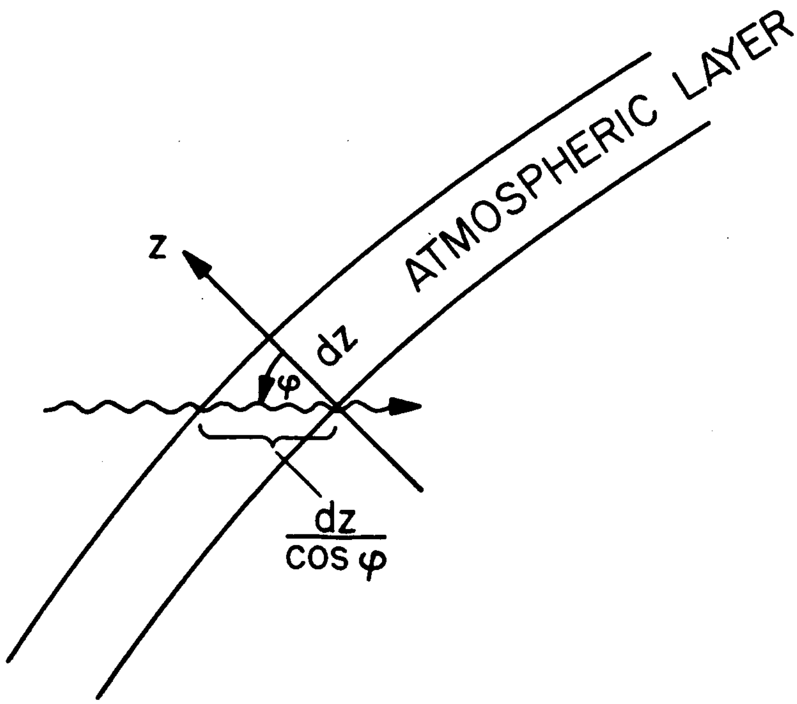
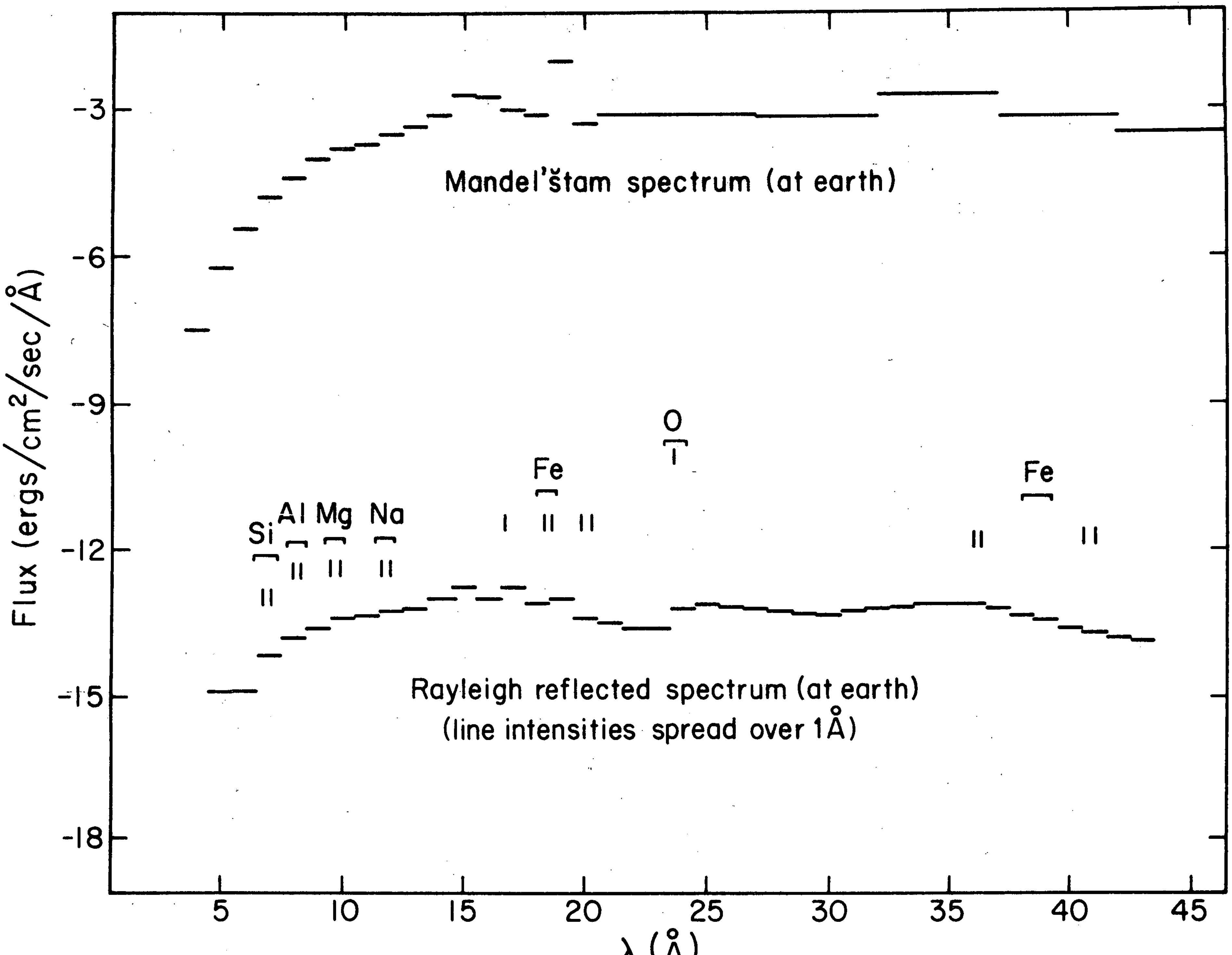


Figure 1b


Mandel'štam spectrum (at earth)
Rayleigh reflected spectrum (at earth)
(line intensities spread over 1Å)
Si
Al
Mg
Na
Fe
O
Fe
Flux (ergs/cm²/sec/Å)
λ (Å)
-3
-6
-9
-12
-15
-18
5
10
15
20
25
30
35
40
45