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# RESEARCH FOR THE DEVELOPMENT OF SOFTWARE TECHNIQUES FOR COMPUTER-DRIVEN DISPLAYS 

By Edward N. Chase, Frank S. Greatorex, Robert M. Metcalfe and James H. Mittica

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# RESEARCH FOR THE DEVELOPMENT <br> OF SOF'TWARE TECHNIQUES FOR COMPUTER-DRIVEN DISPLAYS 

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## I. SUMMARY

The direct use of input data formats for interfacing hardware configurations is inconvenient for the user to program in FORTRAN. The Display Transformation Routines therefore provide a direct software interface of the source data format to the exact display commands, similar to the functions of an assembler.

In addition, the implementation of two-dimensional rotation; scaling and limited scissoring of points, lines and character origins; and the representation of three-dimensional information, including display of "wire frame" points, lines and character origins; and three-space to two-space projections, provide ceitain pseudo-hardware functions that are not otherwise readily available.

## II. INTRODUCTION

Transformation of graphical data structures to display lists suitable for direct use by a display.
III. DISPLAY TRANSFORMATION FOUTINES

## A. General

It is assumed that the reader is generally familiar with the display subsystem described in Section IV.B of the Graphics Reference Manual, especially with subsections 1 through 4.

The routines described in IV.B. 3 allow a user's program to communicate with and control the display. However, all data placed in the display buffer must be in the formats described in IV.B.4. While convenient and efficient for the display hardware, such formats are inconvenient for the user to program, especially in FORTRAN. Therefore, these display transformation routines are provided to transform from a tractable source data format to the exact display commands, similar to the function of an assembler. The source format is convenient for FORTRAN data manipulation and organizes data as it is generally used without forfeiting ultimate control of any display functions.

The transformation routines described transform and move data from the user segment to either the display buffer or to another section of the user segment. In addition to format transformation, certain pseudo-hardware functions (such as two-dimensional transformation) are performed. If an error is detected, an error code is returned. If XFMSER and XFMGEV are not used, error codes are typed on the teletype as errors are detected.

## B. Input Data Formats

The translation routines accept data in a number of formats: global display parameters, poly-items (i.e., multiple vectors, characters, etc.), mixed format or absolute display commands. The poly-items afford an efficient yet traatable coding of information while the mixed format provides more flexibility. The absolute format can be used when the most efficient coding is desired.

Global Parameters. For all parameters, if no value is specified (i.e., value $=0$ ) no change is made to the current setting. If errors in values are detected, the default value is used.

MODE $=$ Not used
CHAN $=$ Output channel control
The IDI (monochrome) display station is
Channel 1. The color display uses
Channel 2 (red), 3 (green) and 4 (blue).
Each channel is controlled by one bit
in the following format:
Bit $=0$ Channel off
Bit $=1$ Channel on


The parameter value is the decimal
equivalent of the bit pattern. Since
the value 0 is reserved to mean "no
change," it is not possible to turn
off all channels.

BRIT - Brightness
$0=$ No change
$1=0 f f$
$2=\operatorname{Dim}$
3 = Bright (default)
$4=$ Extra bright
5 = Increase brightness

```
STRUCT - Line Structure
    0 = No change
    1 = Solid (default)
    2 = Dotted
    3 = Dashed
    4 = Dot-dash
CSIZE - Character Size
    0 = No change
    1 = Smalí
    2 = Medium (default)
    3 = Large
    4 = Extra large
    5 = Increase character size
    6 = Decrease character size
CROT - Character Rotation
    O = No change
    l = Horizontal (default)
    2 = Vertical
LPVIZ - Light Pen Visibility
    0 = No change
    l = Invisible (default)
    2 = Visible
```

```
BLINK - Blink
    0= No change
    1 = No blink (default)
    2 = Blink
```

Poly-Vector. Vectors are drawn from the present beam position to the given end point. This results in a sequence of $N$ connected or "chained" vectors.
$N=$ Number of vectors $\quad N=$ Number of vectors
$X=X$ value of end point $X=X$ value of end points
$Y=Y$ value of end point $\quad Y=Y$ value of end points $Z=Z$ value of end points


The range of $X$ and $Y$ after translation shouid be 0 to 1023. If a value is outside this range, the out-of-range edges are forced to the nearest edge ( 0 to 1023).

Poly-Dot. Dots are plotted from the present beam position to the given end point.
$N=$ Number of dots $\quad N=$ Number of dots
$X=X$ value of dot coordinates $X=X$ value of dot soordinates
$Y=Y$ value of dot coordinates $Y=Y$ value of dot coordinates $Z=Z$ value of dot coordinates


If either component distance between dots is from -3 to +3 , incremental dots are drawn. If a value is outside this range, it is tested for range 0-1023.

In drawing incrementai dots, if the number of dots drawn is odd, the last dot drawn will be somewhat brighter.

Poly-Circle. Circles are drawn using the radius from the $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ coordinate point.

| $N=$ Number of circles | $N=$ Number of circles |
| :--- | :--- |
| $X=X$ coordinate of center | $X=X$ coordinate of center |
| $Y=Y$ coordinate of center | $Y=Y$ coordinate of center |
| $K \ldots, U S=$Radius magnitude of <br> circle | $Z=Z$ coordinate of center |
|  | RADIUS $=$Radius magnitude of <br> circle |

$\left.\begin{array}{|c|}\hline 40 \\ \hline \mathrm{~N} \\ \hline \mathrm{X} \\ \hline \mathrm{Y} \\ \hline \mathrm{X} \\ \hline \mathrm{Y} \\ \hline \mathrm{O} \\ \hline \mathrm{X} \\ \hline \mathrm{Y} \\ \hline\end{array}\right] \quad$ CIR LE 1


The range of $X$ and $Y$ after translation is 0 to 1023 and the range of RADIUS is 0 to 511 . If a value is outside its range, the out-of-range edges are forced to the nearest edge (0 to 1023).

Poly Character. Characters are drawn from the present beam position $\mathrm{N}=$ Number of characters (odd or even). $C_{1}, C_{2}=$ Characters in system ASCII code. Characters are converted to the codes and functions given in Table IV.B. 4.05 of the Graphics Reference Manual.

| 50 |  |
| :--- | :---: |
| $N$ | Cl |
| C 2 | C 3 |
| C 4 | -- |
| -- | -- |
| $:$ | $:$ |
| -- | -- |
| -- | Cn |

Mixed Format. This format permits execution of a variety of display operations.
$N=$ Number of entries.
Code $=$ Function code.
Data $=$ Value of data relating to the
function. Meanings, ranges and error
conditions are the same as for poly-
items above.

| 100 |
| :--- |
| $N$ |
| Code |
| Data |
| Code |
| Data |
| $:$ |
| Code |
| Data |


| CODE | FUNCTION | DATA |
| :--- | :--- | :--- |
| 110 | Define Beam X | Value of X |
| 111 | Set Beam Y | Value of Y |
| 121 | Define Beam Z | Value of Z |
| 130 | Draw Vector Y | Y end point |
| 140 | Draw circle | Radius of circle |
| 150 | Draw 2 char. | Two 8-bit system <br> ASCII characters |
| 160 | No operation | Number of display <br> commands NOP |
|  | Chain | Pointer to new <br> entity |

Note that code 150 will generate the specified number of NOP's. Code 160 permits processing non-contiguous global parameters or poly-items (but not mixed format) with a return to the next entry.


## C. Program Description

## From User to Display Subroutine

CALL XFMC?D (XBLOK, FROM,TO,NLMWDS). XFMU2D transforms and moves data from the user segment to the display buffer. XBLOK $=$ The name of a Transform Block, an array of 17 words.

FROM = The name of an array (of variable length) in the user segment to be transformed and moved to the display buffer.

TO $\quad=$ The name of a one-word subscript value; defines the first word in the display buifer (relative to a buffer origin of 1 ) to be filled. If TO $=0$, data is appended to the present contents of the buffer.

```
NUMWDS \(=A\) one-word returned parameter indicating the number of words, in object format transferred to the display buffer.
```


## From User to User Subroutine

CALL XFMU2U (XBLOK,FROM,TO,SPACE, NUMWDS). XFMU2U transforms and moves data from the user segment to another section of the user segment.

XBLOK $=$ The name of a Transform slock, an array of 17 words.

FROM $=$ The name of an array (of variable length) in the user segment to be transformed and moved.

TO $\quad=$ The name of an array in the user segment to be filled. (The array is of SPACE length.)

SPACE $=A$ word defining the number of words available for transformed data in the user segment; i.e., length of the TO block.

NUMWDS = A one-word returned parameter indicating the number of words, in object format, transformed to the user segment.

## Mainline Subroutine

CALL MAIN (XBLOK,WHERE, TO, FROM, SPACE NUMWDS) . MAIN transforms and moves data from the user segment to either the display buffer or to another section of the user segment. This is accomplished by stripping down each input entity table and performing the variols positioning calls to either internal subroutines or subrodtines in the Display Utility Package.

XBLOK $=$ Name ef Transform Block, an array of 17 words.
WHERE $=$ Defines where data from all subsequent utility calle is to be placed. (A one-word parameter.) WHERE $=1$ Append to display buffer.

2 insert in middle of display buffer.
3 Append to utility buffer (internal buffer of maximum length 200 words).

4 Insert in utility buffer.
TO = The name of an array or a one-word subscript value, depending on the type of call. (XFMU2D or XFMU2U)

FROM = The first address in the user segment to be transformed. (An array of unknown length.)

SPACE $=A$ word defining the number of words available for transformed data in the user segment.

NUMWDS $=A$ one-word returned parameter indicating the number of words in object format transformed to the user segment.

## Entity Strip Subroutine

CALL ESTRIP (KEYLOC,KEYTYP,CURKEY,LENGTH). ESTRIP computes the entity length using the keyword type and $N$.

KEYLOC $=$ The first address in the user segment to be transformed. (An array of unknown length replaces FROM.)

KEYTYP $=$ The current entity type. (A returned parameter.)
CURKEY $=$ Not used at this time.
LENGTH $=$ The length of the current entity table. (A returned parameter.)

10-Global parameters Length $=5$
20 - Poly Vector - 2D Length $=2 * N+2$
23 - Poly Vector - 3D Length $=3 * N+2$
30 - Poly Dot - 2D Length $=2 * N+2$
33 - Poly Dot - 3D Length $=3 * N+2$
40 - Poly Circle - 2D Length $=3 * N+2$
43 - Poly Sphere - 3D Length $=4 * N+2$
50 - Poly Character Length $=\mathrm{N} / 2+1$
100 - Mixed Mode Length $=2 * N+2$
200 - Absolute Format Length $=\mathrm{N}+2$

## Node Strip Subroutine

CALL NSTRIP (KEYLOC, XCOORD, YCOORD) . NSTRIP strips the $X$, $Y$ and $Z$ nodes from the entity table.

KEYLOC $=$ The first address in the user segment to be transformed.

XCOORD $=$ The X -coordinate in the entity table. (A returned parameter.)

YCOORD $=$ The Y -coordinate in the entity table. (A returned parameter.)

IZ3D = Although this is not in the calling arguments, the Z -coordinate is returned through COMMON.

## Transform Routine

CALL TRFORM (XCOORD, YCOORD). TRFORM performs the twodimensional rotation, scaling, the representation of threedimensional information, including the display of "wire frame" points, lines and character origins, and three-space to twospace projection.

XCOORD $=$ The X -coordinate in the entity table.
YCOORD $=$ The $Y$-coordinate in the entity table.

## Strip Word Subroutine

CALL STRPWD (KEYLOC,RESULT). STRPWD strips each v.urd in the entity table, one by one, using STPLOC in COMMON as the indexing parameter.

KEYLOC $=$ The first address in the user segment to be transformed.

RESULT $=A$ returned parameter of one entity word.

## Move Buffer Subroutine

CALL MOVBUF (WHERE, TO, WRDLEN, SPACE, MBFERR,NUMWDS). MOVBUF checks to see which buffer is being used (the display or utility). If the utility buffer is used, it is moved back to the user when transformation is completed or the buffer is full. In effect, if WHERE is equal to 3 or 4.

WHERE $=$ Defines where data from all subsequent utility calls is to be placed. WHERE $=1$ Append to display buffer.

2 Insert in middle of display buffer.
3 Append to utility buffer (internal buffer of maximum length 200 words).

4 Insert in utility buffer.
TO $\quad=$ The name of an array in the user scgment to be filled. (The array is of SPACE length.)

WRDLEN $=$ The number of object words being appended to or inserted in the utility buffer.

SPACE $=A$ word defining the nurier of words available for transformed data in the ussi segment.

MBFERR $=A$ one-word returned parameter indicating the overflow condition of the user buffer.

NUMWDS $=A$ one-word returned parameter indicating the number of words, in object format, transformed to the user segment.

## Error Return Address Subroutine

CALL XFMSER (N). XFMSER allows the user to set an address to which control ie returned following all display transformation errors. The call should be preceded by an ASSIGN statement.
$\mathrm{N}=$ The statement number to which control will be returned.

## Get Error Value Subroutine

CALL XFMGEV (N). XFMGEV returns the code of the most recent: transformation error.
$\mathrm{N}=$ Error code (a returned parameter).
The following is a list of the error code conditions resulting from calls to XFMU2D or XFMU2U:

## XFMU2D ERROR CODES

$1=$ No error.
$2=$ No console assigned to user. No action taken.
3 = TO is out-of-range; negative or larger than present buffer. No action taken.

4 = Length of the array to be transformed is out-of-range; N in data format negative or length greater than 6000. No action taken.

5 = Zero scale word or view area edges exceed permissible scope coordinates. Out-of-range edges are forced to the nearest edge ( 0 or 1023); for zero scale, nothing is drawn.
$6=$ Illegal keyword found.
7 = Mixed format recursion.

## XFMUZU ERROR CODES

$1=$ No error.
$2=$ TO is out-of-range of user's memory bound. No action taken.

3 = SPACE filled before transformation completed. Remaining source data is not transformed.

4 = Length of the array to be transformed is out-of-range; N in data format negative or length greater than 6000. No action taken.

```
5 = Zero scale word o: view area exceed permissible scope coordinates. Out-of-range edges are forced to the nearest edge ( 0 or 1023); for zero scale, nothing is drawn.
6 = Illegal keyword is found.
7 = Mixed format recursion.
```


## Line Scissor Routine

CALL SCISOR (IX,IY, IF). SCISOR calculates what the image of a vector drawn from the current beam position to the specified vector head will be on the viewing frame. It then returns a plot or no-plot flag and appropriately altered X,Y for a modified vector plot. The actual internal beam position is automatically updated. Five cases are considered and dealt with in the following manner:

Case 1. The vector is found to be entirely within the viewing frame. The $X, Y$ values are left unchanged and the plot flag is set to 1.

Case 2. The vector is found to be entirely outside the viewing frame. The no-plot signal is set to 2.

Case 3. The vector is found to be entering the viewing frame. The beam is then reset to the frame entry point and the plot flag is set to 1 . The $\mathrm{X}, \mathrm{Y}$ values are left unchanged.

Case 4. The vector is found to be leaving the viewing frame. The plot flag is set to 1 and the $\mathrm{X}, \mathrm{Y}$ values are set to the exit point.

Case 5. The vector is found to cross over the viewing frame. The beam is then reset to an entry point and the $X, Y$ values are changed to be an exit point. The plot flag is set to 1 .

IX $=$ The X-coordinate input value.
$I Y=$ The $Y$-coordinate input value.
IF = The plot or : J-plot flag. (A returned parameter.)

## Extended Bouadary Intrrcept Calculator and Positioner

CALL INCAL. INCAL calculates the $X$ and $Y$ positions of the intercepts involving the proposed beam vector. Using the extended frame boundaries, INCAL evaluates the position of these intercepts with respect to the frame. The frame boundaries on the vector foot, vector head, intercept second-ccordinates and intercept positions all reside in COMMON.

## Frame-Boundary-Extension Intercept Scanner

CALL INSCN (IX, IY, IF). INSCN scans the list of four frame-boundary-extension intercepts, searching for the first one that is on or within the viewing frame defined by the proposed beam vector.

If an intercept qualifies, the output $f l a g$ is set to 1 and its coordinates are returned in the $X, Y$ arguments passed. if no qualifying intercept is found, the values are returned unchanged.

IX $=$ The $X$-coordinate input value.
$I Y=$ The $Y$-coordinate input value.
IF = Intercept flag, a returned parameter of value 1 or 2.

```
l = Qualifying intercept
2 = Non-qualifying intercept
```


## Reset Beam Position

CALL RESET (IX,IY,IF). RESET checks for in-frame beam reset positions and positions the beam legally if the request is made.

IX = The $X$-coordinate input value.
$I Y=$ The $Y$-coordinate input value.
IF $=$ The input and output flag used in the following manner:

Input flag = 1 - Reset beam if legal reset position.

Input flag $=2$ - Do not reset beam in any case. Output flag = 1 - Transmitted beam position is legal.

```
Output flag = 2 - Illegal beam position trans- mitted.
```


## Double Precision Integer Multiply Function

FUNCTION IMUL (IMULI). IMUL is a double-precision integer function. The multiplicand is in the integer word IAC located in COMMON, and the multiplier is passed as the argument to this function. The most significant left-half of the product is returned in IAC and as the function value, while the least significant right half of the product is left in IMQ. This routine takes advantage of the 11.08 36-bit word length for 32bit accuracy, and must be changed for 516 implementation.

IMULI = Multiplier.

## Double-Precision Integer Divide Function

FUNCTION TDIV (IDIVI). IDIV is a double-precision integer divide function. The dividend is in the integer words IAC-IMQ located in COMMON, and the divisor is passed as the angument to this function. The quotient is returned as the function value and in IAC, while the remainder is left in IMQ. This routine takes advantage of the 1108 36-bit word length for 32-bit accuracy, and must be changed for 516 implementation.
IDIVI = Divisor.

## In Frame Test Function

FUNCTION IFRME (IX,IY). IFRME computes the position of the specified point relative to the viewing frame speciried in COMMON and returns this position as a zone code from 1 to 9.

IX $=$ The $X$-coordinate input value.
IY $=$ The Y -coordinate input value.

## Vector Frame Test Function

FUNCTION IVFT (IX, IY). IVFT computes the position of a specified point relative to the rectangular frame generated by the coordinates of the head and tail of the proposed beam vector (IBPX, IBPY,IXN,IYN). If the point is in or on the frame, the function value is set to 1 . If the point is outside the frame, the function value is set to 2.

IX $=$ The $X$-coordinate input value.
$I Y=$ The $Y$-coordinate input value.

## IV. ALGORITHM

## A. General

The algorithm for rotation, scaling and three-space to two-space projection is centered around an input transform block of 17 words. The block consists of the following format:

```
IFLAGS = Flag used to signal a new transform blonk.
IDELTX = X-base value for transformation, data coordin-
    ates (signed integer).
IDELTY =: Y-base value for transformation, data coordin-
    ates (signed integer).
IANGLE = Angle of rotation, expressed in multiples of
        .0001 radian; that is, +32767 = +3.2767 radians.
ISCALE = Multiplying or dividing factor as indicated by
        positive or negative numbers. For example:
        +200 is 200X; -3042 is }\frac{1}{3042}\textrm{X}
            3042
IVXO = View origin X-coordinate.
IVYO = View origin Y-coordinate (VXO and VYO) specify
    the point in data coordinates which is at (511, 5ll)
        in scope coordinates.
IVXL = View left-edge X-coordinate (minimum 0).
IVXR = View right-edge X-coordinate (maximum 1023).
IVYB = View bottom-edge Y-coordinate (minimum 0).
IVYT = View top-edge Y-coordinate (maximum 1023). These
        four scope coordinates specify the area within
        which the transformed data is to appear. Scissor-
        ing to view occurs for points, lines, centers of
        circles and character origins.
```

```
ITX3D = View origin X'-coordinate.
ITY3D = View origin Y'-coordinate.
ITZ3D = View origin Z'-coordinate (ITX3D, ITY3D, and
        ITZ3D specify that point in user data coordin-
        ates relative to the original X, Y, Z viewing
        coordinates).
IGAMA = Primary angle rotation about X-axis, expressed
        in multiples of .0001 radian.
IBETA = Secondary angle, rotation abcut Y-axis.
IALPH = Tertiary angle, rotation about Z-axis. (The 3-D
        user coordinates are effected by Euler angle
        rotation and Z' made equal to zero for projection
        to 2-D view, X2D = X', Y2D = Y'.)
```

Transforming operations are performed in the following order:

1. Three-space to two-space projection.
2. Rotation about $(0,0)$.
3. Translation by $\Delta X, \Delta Y$.
4. Scaling relative to (IVXO, IVYO), the resultant coordinate +511 giving scope coordinates.

## B. Three Dimensional Algorithm

Translation of Coordinate Areas. Let $X, Y, Z$ be the coordinates of any point $P$ with respect to the right-handed rectangular cartesian reference system. Let $X^{\prime}, Y^{\prime}, Z^{\prime}$ be the coordinates of the same point $P$ with respect to a second right-handed
rectangular cartesian reference system. If these axes have the same directions as the corresponding axes of the $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ system, and the origin has the coordinates $X=X_{0}, Y=Y_{0}$, $Z=Z_{0}$ in the $X, Y, Z$ system, and also if equal scales are used to measure the coordinates in both systems, the coordinates $\mathrm{X}^{\prime}$, $Y^{\prime}, Z^{\prime}$ are related to the coordinace; $X, Y, Z$ by the transformation equations:

$$
\begin{aligned}
& X=X^{\prime}+X_{0} \\
& Y=Y^{\prime}+Y_{0} \\
& Z=Z^{\prime}+Z_{0}
\end{aligned}
$$



Rctation of Coordinate Axes. Given any point $P \equiv(X, Y, Z)$, let $X^{\prime}, Y^{\prime}, Z^{\prime}$ be the coordinates of the same point with respect to a second right-handed rectangular cartesian reference system having the same origin $\mathscr{P}$ as the $X, Y, Z$ system and oriented with respect to the latter so that:

The X -axis has the direction cosines $\mathrm{IT}_{11}, \mathrm{IT}_{12}, \mathrm{IT}_{13}$.
The $Y$-axis has the direction cosines $I T_{21}, I T_{22}, I T_{23}$.
The Z -axis has the direction cosines $\mathrm{IT}_{31}, \mathrm{IT}_{32}, \mathrm{IT}_{33}$.

If equal scales are used to measure $X, Y, Z, X^{\prime}, Y^{\prime}$, and $Z^{\prime}$, the transformation equations relating the coordinates $X^{\prime}, Y^{\prime}, Z^{\prime}$, to the coordinates $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ are:

$$
\begin{aligned}
& X=I T 11^{\prime}+I T T_{12} Y^{\prime}+I T_{13} Z^{\prime} \\
& Y=I T_{21} X^{\prime}+I T_{22^{\prime}+I T_{23} Z^{\prime}} \\
& Z=I T_{31} X^{\prime}+I T_{32} Y^{\prime}+I T_{33^{\prime}} Z^{\prime}
\end{aligned}
$$

It should be noted at this point that the transformations above are orthogonal projections.


Simultaneous Translation and Rotation. If the origin of the $X^{\prime}, Y^{\prime}, Z^{\prime}$ system is not that of the $X, Y, Z$ system but has the coordinates $X=X_{0}, Y=Y_{0}, Z=Z_{0}$ in the $X, Y, Z$ system, the transformation equations become:

$$
\begin{aligned}
& Y=I T_{21} X^{\prime}+I T_{22^{\prime}}{ }^{\prime}+I T_{23^{\prime}} Z^{\prime}+Y_{0} \\
& Z=I T_{31} X^{\prime}+I T_{32} Y^{\prime}+I T_{33^{\prime}} Z^{\prime}+Z_{0}
\end{aligned}
$$



Euler Angles. Given a matrix $A \equiv\left[{ }_{\mathrm{ik}}\right.$ ] representing a proper rotation in three-dimensional Euclidean space, it can be variously expressed as a product of three matrices as follows:

$$
\begin{aligned}
& I T_{11}=\cos \alpha \cos \beta \cos \gamma-\sin \alpha \sin \gamma \\
& I T_{12}=-(\sin \alpha \cos \beta \cos \gamma+\cos \alpha \sin \gamma) \\
& I T_{13}=\sin \beta \cos \gamma \\
& I T_{21}=\cos \alpha \cos \beta \sin \gamma+\sin \alpha \cos \gamma \\
& I T_{22}=-\sin \alpha \cos \beta \sin \gamma+\cos \alpha \cos \gamma \\
& I T_{23}=\sin \beta \sin \gamma
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{IT}_{31}=-\cos \alpha \sin \beta \\
& \mathrm{IT}_{32}=\sin \alpha \sin \beta \\
& \mathrm{IT}_{33}=\cos \beta
\end{aligned}
$$

Geometical Interpretation of the Euler Angles. A set of cartesian $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ axes, initially aligned with $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ can be rotated into alignment with $\mathrm{X}^{\prime}, \mathrm{Y}^{\prime}, \mathrm{Z}^{\prime}$ by three successive rotations:

1. Rotate about Z-axis through the Euler angle $\alpha$.
2. Rotate about Y-axis through the Euler angle B.
3. Rotate about $Z$-axis through the Euler angle $\gamma$.

## C. Two-Dimensional Algorithm

Rotation of the Coordinate Axes. Given $X, Y$ are the coordinates of any point $P$ with respect to a right-handed rectangular cartesian reference system. Let $\mathrm{X}^{\prime}, \mathrm{Y}^{\prime}$ be the coordinates of the same point $P$ with respect to a second right-handed rectangular cartesian reference system having the same origin and rotated with respect to the $X, Y$ system. If equal scales are used to measure all four coordinates $X, Y, X^{\prime}, Y^{\prime}$, the coordinates $X^{\prime}, Y^{\prime}$ are related to the coordinates $X, Y$ by the transformation equations:

$$
\begin{aligned}
& X^{\prime}=X \cos \varnothing+Y \sin \varnothing \\
& Y^{\prime}=-X \sin \varnothing+Y \cos \varnothing
\end{aligned}
$$

## Simultaneous Translation and Rotation of Coordinate Area.

If the origin of the $X^{\prime}, Y^{\prime}$ system is not the same as the origin of the $X, Y$ system but has the coordinates $X=X_{0}$ and $Y=Y_{0}$ in the $X, Y$ system, the transformation equations becone:

$$
\begin{aligned}
& X=X^{\prime} \cos \varnothing-Y^{\prime} \sin \varnothing+X_{0} \\
& Y=X^{\prime} \sin \varnothing+Y^{\prime} \cos \varnothing+Y_{0}
\end{aligned}
$$

## D. Reference

Korn, G. A., and Korn, T. M.: "Mathematical Handbook for Scientist and Engineers." McGraw-Hill Book Co., Inc., 1961.
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Appendix A

MAINLINE ROUTINE FLOWCHARTS

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## Appendix C

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