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STATISTICAL INVESTIGATION OF LOCAL VARIATIONS OF THE HORIZONTAL COMPONENT OF A GEOMAGNETIC VECTOR IN QUIET DAYS (SUMMER SEASON)

Each daily (24 hour) record of components of a geomagnetic vector should be considered as a single event in a process which contains true as well as random components. The true components of daily variations have been analyzed with a sufficient carefulness at various macro-conditions in a series of articles by different investigators (1,2,3).

These components were usually segregated from initial data by averaging or on the basis of assembling homogeneous - in a certain sence - events or by one event which could be traced back for a sufficiently long time, and in posing conditions silently accepted, or assumptions announced beforehand on the stationary character and ergodicity of the process. The statistical properties of the random components have, apparently, as yet, not been studied. However, in order to clarify the correctness of applying the corresponding probability methods when studying geomagnetic variations, it is absolutely necessary to investigate the random components in each given macro-condition (space coordinates, time, degree of perturbation, etc.), from the point of view of their satisfying the stationary principle, the ergodicity and normalcy (gauss). Without the knowledge of the behavior of the random component it is impossible, for instance, to evaluate with any degree of certitude the precision with which any of the mean characteristics of the process has been achieved; it is equally impossible to foretell to what extent and limits and with what probabilities individual events in the characteristics of geomagnetic processes may be deflected from the average event characteristics , etc.

The present article reports on a statistical investigation of daily variations of a horizontal component of a geomagnetic vector according to data of the Irkutsk (Zuy) station. The first stage-working on the methodology of a statistical investigation - considered the comparatively little-variable and, therefore, easily studied data for the quiet 24 hours of the "summer" season of 1952-53 (called for abbreviation purposes further on the variant I) and the "summer"season of 1963-64 (variant II).

The years of a minimum of sun activity were selected because they contain a larger volume of observations, relating to quiet days.

As a result of visual analysis of magnetograms, 53 were selected for a further study; these 53 were representative of the most quiet days in the I variant and 58 analogous days of variant II.

The random component was segregated from the initial mean hourly data by eliminating the average in the assembly of characteristical events in daily variations (Sq-variations) and removing from the data for each day the corresponding mean daily values (of the secular trend-motion). As a result of these operations only random deflections remained in the initial data; these random deflections will be designated below as

$$x_i^j, \; i = \overline{1,24}, \; j = \overline{1,N},$$

where N = 52 or 58.

The very method of separating out the random component points out that

$$\bar{x}_i = \frac{1}{N} \sum_{i=1}^{N} x_i^j = 0, \qquad (1)$$

$$\bar{x}^j = \frac{1}{24} \sum_{i=1}^{24} x_i^j = 0. \qquad (2)$$

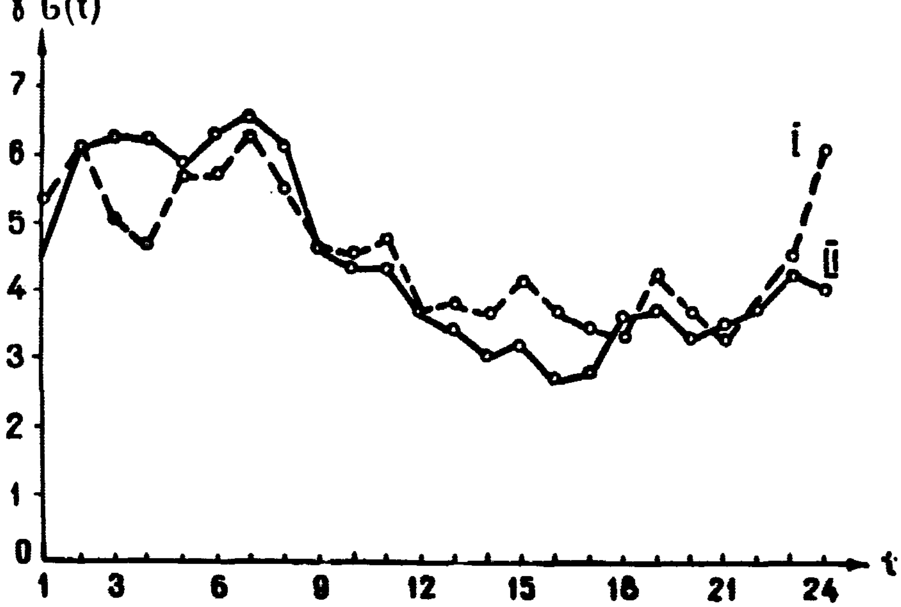

Fig. 1. Daily trend of mean square deflection of a random component at the universal time.

The "BESM-2M" computer calculated for both variants: the single-dimensional selected function of density probability, the daily trend of the mean square deflection and the auto-correlation function, the dependence of the latter upon the time interval $\tau = |t_i - t_k|$ between the time moments t_i and t_k compared and other statistical characteristics of the random process investigated.

From the comparison of results obtained we have uncovered the following:

1. The mean square deflection σ_x (Fig.1), and thus therefore, the dispersion $D_x = \sigma^2_x$, has a clearly expressed daily trend which reminds a simple wave with an accuracy up to random deflections which coincide in the case of both variants.

Fig. 1 and further on the dotted curve relates the variant I, the solid curve presents variant II. The maximum of the wave (σ_x is about $6,4\gamma$) occurs at 7 o'clock and the minimum ($\sigma_x \approx 3\gamma$) at about 17 o'clock of the universal time (14 and 24 o'clock respectively, local time).

The auto-correlation function

$$z(t_i,\tau)=\frac{\sum_{j=1}^{N} x^j(t_i)\cdot x^j(t_i+\tau)}{(N-1)\sigma(t_i)\cdot\sigma(t_i+\tau)}, \quad i=\overline{1,24} \qquad (3)$$

for $\tau = 1$ hour is presented in Fig. 2. The figure clearly demonstrates that in a daily trend of the argument t_i $(i=\overline{1,24})$ function (3) does not reveal any tendency for regular changes and is subject to fluctuations - approximately only at the mean level $r \approx 0,7$ random fluctuations, non-coincident for the two variants; these fluctuations have an amplitude R_r if we neglect to consider the outflow, in the first variant which occurs at 3 o'clock UT and is approximately equal to 0.25.

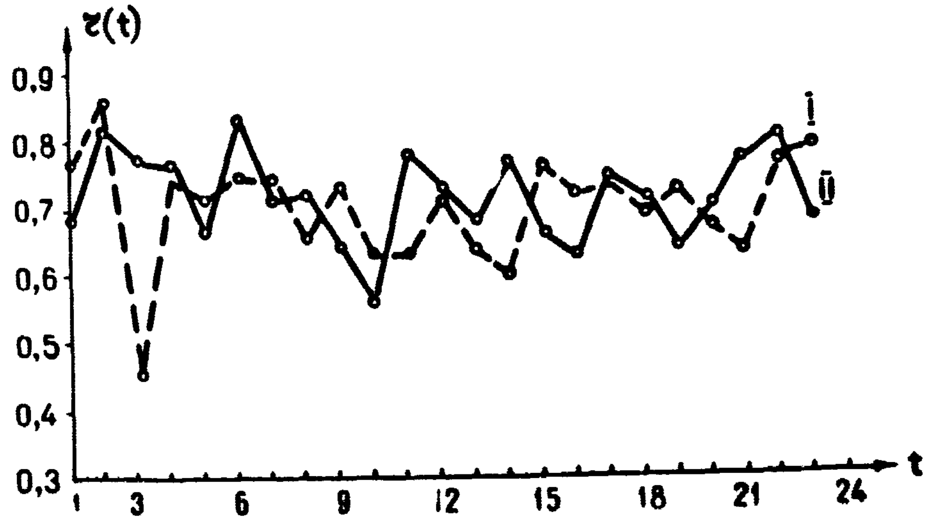

Fig. 2. Behavior of the autocorrelation function of random components at τ = 1 hour depending upon universal time.

According to the well known formula $R_r = \alpha(n)\sigma_r$, where $\alpha(n)$ depending upon the utilized volume of selection is taken from tables, obtaining σ_r equalling about 0.08. The approximated formula

$$\sigma_\tau \approx \frac{1-\tau^2}{\sqrt{n-1}}, \qquad (4)$$

taken from (4) gives us σ_r equalling about 0.07 or, about the same value.

We have considered, as well, the behavior of $r(t, \tau)$ at $\tau = 2$ and $\tau = 3$ hours. In these cases at t describing the daily trend no regular, intentional deflections $r(t, \tau)$ from certain fixed levels were observed. Random deflections from this level were larger but their amplitude was in a satisfactory agreement with the one computed according to formula (4).

Thus, at least, for a τ order not exceeding 3 hours a normalized auto-correlational function of the studied process $r(t, \tau)$ does not, practically depend upon the t. It follows from this (5) that a random component in the process may be approximately introduced as

$$X(t) \approx \sigma(t) \cdot X_0(t), \qquad (5)$$

where $X_0(t)$ is a normalized and centered stationary random function $\sigma(t)$ is a known non-random function, the selected model of which is shown in Fig. 1.

2. Based on the stationary characteristics of the random function, we may consider that different (in time) sections of the given function contain random magnitudes subject to the same distribution law. In order to establish this law and, naturally trying to be supported by a large volume of material, we may unite into one combination the ordinates of the events characteristics

of the random function $X_0(t)$ out of all 24 time sections which are at our disposal within the limits of one day.

Fig. 3 brings the histograms of distributions: a) for variant I ($n = 52 \times 24 = 1248$ ordinates); b) for variant II ($n = 58 \times 24 = 1392$ ordinates). Solid lines describe the theoretical curves of normal distrubtions, which posses the same parameters - a mathematical expectation and dispersion.

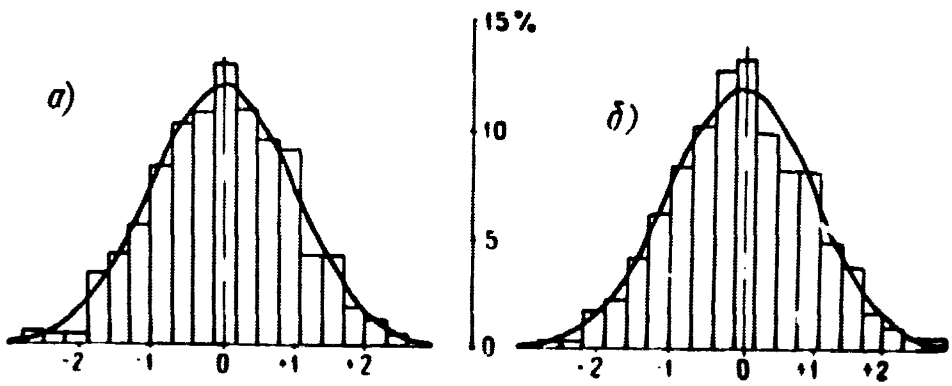

Fig. 3. Histograms of the random component distribution. a) Summer 1952-53, b) Summer 1963-64.

It is apparent, that the selected one-dimensional distributions are easily described by the gauss law. The quantitative indices of deflections from the gauss properties, the assymmetry A and the excess E are as follows: for variant I they are: $A_1 = -0{,}07$, $E_1 = 0{,}06$; for variant II they are: $A_2 = 0{,}16$, $E_2 = 0{,}18$.

Inasmuch as practical computations of a statistical character are usually achieved only within the frame of the so-called correlation theory, when we consider moments to the second order inclusively, there remains to analyze the two-dimensional distribution - a joint distribution of ordinates of the $X_0(t)$ process in two different time sections.

Table I shows the joint distribution of coordinates x_i and x_{i+1} $(i = 1,23)$ according to 1963-64 data (variant II). And here, just as we did before, taking into consideration the stationary characteristics of the process we reduce into one association the neighboring (differing from each other by $\tau = 1$ hour) pairs of sections within the limits of full 24 hours.

The table shows clearly enough the ellipse of scattering, characteristic for a two-dimensional normal distribution. Distinct single-dimensional distribution in the sections (along the main diagonals, the mean vertical and along the mean horizontal of the table) result in histograms which are in complete agreement with the gaussian curves - with a precision up to the unavoidable fluctuations.

Thus, at $\tau=1$ hour the joint two-dimensional distribution is close to the gaussian distribution. At $\tau>1$ hour joint two-dimensional distributions will be the more close to the gaussian ones, as the deflections from the gauss law established for the single-dimension distributions may be caused only by internal interconnections of the process and these weaken as the τ value increases.

3. It is apparent from (1) and (2) that the random function $X_0(t)$ has ergodicity in relatiion to the mathematical expectation. Due to normalization it also displays ergodicity in relation to dispersion. The combination of the gaussian character of the process with ergodicity in relation to mathematical expectation provides a basis for statement (6) that $X_0(t)$ possesses ergodicity also in rleation to the auto-correlation function.

4. Fig. 4 presents a correlogram which expresses the dependence (on the average, for all hours of the day) of the auto-correlation function upon the τ interval which varies within the 48 hour limits. (The calculation was made on selected data containing 32 uninterrupted 48-hours observation of characteristical events in 1952-53).

$$\overline{z(\tau)} = \frac{1}{48-\tau}\sum_{k=1}^{48-\tau} z_k(\tau), \quad \tau=\overline{0,47} \ . \qquad (6)$$

Table 1

X_{i+1} \ X_i	−2,7	−2,1	−1,5	−0,9	−0,3	+0,3	+0,9	+1,5	+2,1		+2,7
+ 2,7							1	2	3	2	3
+ 2,1				1	1			2	8	4	3
+ 1,5						6	15	25	15	6	4
+ 0,9				1	4	34	41	37	19	2	
+ 0,3			1	6	15	74	93	37	20	1	1
− 0,3			5	27	77	129	67	30	10	1	
− 0,9		3	15	46	107	68	18	6	2		
− 1,5		6	16	60	36	25	9	4			
− 2,1	1	7	16	15	15	4		1			
− 2,7		1	3	6	5	1					
	2			2							

Remark:

The joint distribution of values of the random component in two time sections of the process which are removed from each other at a τ equal 1 hour interval.

1. The daily periodicity, clearly defined during the first

24 hours and somewhat deformed during the second 24 hours can be traced. When formula (6) is used a deformation at the end of the chart should be expected, as the number of components in the analyzed sum decreases with the increase of the τ and the importance of random deflections grows in relation to the regular process traced.

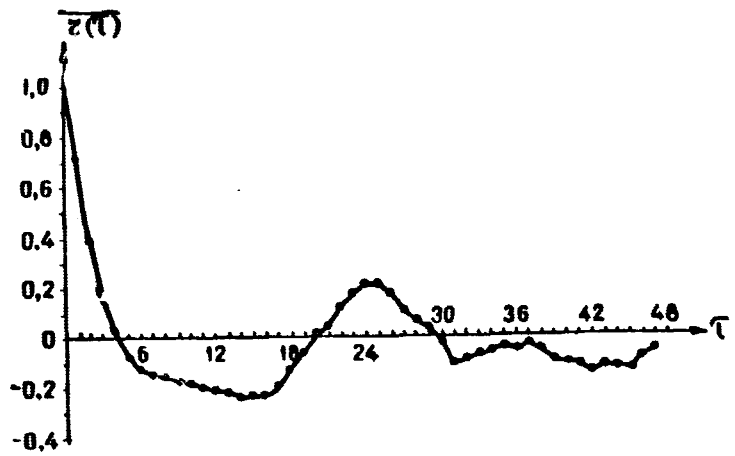

Fig. 4. The dependence of the auto-correlation function of a random component upon the τ interval, expressed in hours.

2. The correlation-gram appears as a tapering out curve.

The values pertaining to function $r(\tau)$ reveal the degree of interconnection between the process sections and may be considered as a sum of common cumulative causes which create the process in these sections. The periodicity of the auto-correlation function should be analyzed from the same point of view.

The tapering out character of the correlation-gram provides a basis for the building of the instantaneous values of the random function $X_0(t)$ into auto-regression series:

$$x_{t+2}=0{,}85x_{t+1}-0{,}19x_t+\varepsilon_{t+2}\,. \qquad (7)$$

The coefficients, in this expression, are calculated according to the rules reported in (7), ε_{t+2} is the normally distributed residual term with a mathematical expectation equalling zero and a mean square scattering $\sigma_\varepsilon = 3{,}1\gamma$.

The instantaneous value of the random process x_{t+2} is, and for which $\sigma_x = 4{,}54\gamma$ is therefore, linearly described over each two preceeding value (one interval= one hour) with a precision up to the random component ε_{t+2} which has only one-half of the dispersion of the x_{t+2}.

If the subsequent instantaneous value is to be described linearly over the six preceding ones (with an interval of one hour each) we find that the corresponding random residual term will have a rather small scattering $\sigma_\varepsilon = 0{,}9\gamma$.

Thus, we have studied the random $X_0(t)$ process which consists of normalized and centered fluctuations of the horizontal component of the geomagnetic vector at the Irkutsk (Zuy)station during the quiet days of the "summer" season in the years of a minimum sun activity.

As a result of this study, the following conclusions may be drawn:

1. The random component of X(t) geomagnetic variations described by formula (5) may be considered as cyclic random process, which is described linearly by means of the stationary process. This means that X(t) can be subjected to spectral analysis and the distribution of this process along the frequency spectrum can be described statistically.

2. The gaussian character of the random stationary process considerably simplifies statistical computations connected with the investigation. The gaussian character has been checked up to the second order (to two-dimensional distributions, inclusive), therefore, it may serve as a basis in all aspects within the frame of the correlation theory.

3. The ergodicity of $X_0(t)$ in relation to the auto-correlation function makes possible the computation of this function along one, characteristic event in the process, which can be traced back far enough in time, instead of calculations involving the entire assembly of characteristic events.

4. Expression (7) and others, similar to it, which contain a large number of components are the basis for linear prognostication. There is a possibility to evaluate future, not yet achieved, events on the basis of preceding instantaneous values of the process with a definite degree of precision and certainty.

5. On the basis of the 24 hour periodicity found in the au-- to correlation function $r(\tau)$ we may assume that, among the causes responsible for the formation of the random process $X_0(t)$ in a fixed point of the Earth surface there are certain factors which exist longer than 24 hours making periodic contributions and, on days sufficiently removed from each other are found in different phases (otherwise, their contributions would make a part of Sq-variations.)

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