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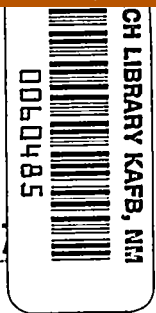
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AN EXPERIMENTAL AND THEORETICAL INVESTIGATION OF STRIATIONS IN A HE-NE LASER

by T. B. Carlson, F. M. Shofner, and C. W. Bray

Prepared by
THE UNIVERSITY OF TENNESSEE SPACE INSTITUTE
Tullahoma, Tenn.
for

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ABSTRACT

This investigation shows that striation oscillations in the He-Ne discharge occur over significant ranges of pressure and current with respect to the optimum values for maximum laser power. This instability, which is of the order of 500 KHz in frequency, effectively modulates the coherent light output of the He-Ne laser as well as the discharge tube voltage and current. A theoretical analysis initiated by Pekarek and extended by Garscadden and Bletzinger has been modified to predict the striation frequency pressure and radius dependences for the single constituent gas. The experimental results for the He-Ne laser show good correlation with the $f \propto p^{-1/2} R^{-3/2}$ functional dependence predicted. One practical design consideration is that effects of striations can be minimized by using small diameter tubes since these have characteristically higher striation frequencies which less effectively modulate the coherent output.

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NOMENCLATURE

A_Z — Initial slope of ionization efficiency as a function of electron energy

a_1 — Reciprocal of the relaxation length for electron temperature

D_a — Ambipolar diffusion coefficient

D_+ — Diffusion coefficient for ions

D_- — Diffusion coefficient for electrons

d — Diameter of tube

dc — Volume element in velocity space

E_o — Axial electric field

e — Electronic charge

f — Frequency of striations

f_v — Velocity distribution function

h — Debye length

I_d — Photodiode current

I_L — Discharge current

k — Boltzmann's constant

m_e — Mass of electron

m_i — Mass of ion

N — Number density in configuration space

N_o — Constant term of number density

N_+ — Number density of ions

N_- — Number density of electrons

n_+ — Small deviation in ion density

n_- Small deviation in electron density

p Pressure of gas

R_i Force per unit mass on a particle in the gas

$S_{coll.}^+$ Inelastic generation rate

T_+ Ion temperature

T_- Electron temperature

t Time

V_I Ionization potential of the gas

V_p Phase velocity of moving striations

v_i Three components of velocity

v_o Phase velocity of striations in free space

Z Ionization rate

Z_o Ionization rate as a function of electron temperature

z Position along the axis of the tube

Γ_i^+ Ion flux density

γ Adiabatic compression coefficient of the electron gas

δ Ratio of electron mass to ion mass

Λ Half-width of initial disturbance

ϵ Small deviation in electric field

λ Wave length

λ_e Mean free path at standard conditions

λ_1 Normalized mean free path for electron-neutral collisions

μ_+	Normalized ion mobility
μ_-	Electron mobility
μ_o^+	Ion mobility at standard conditions
ν_{IN}	Collision frequency of an ion with background gas atoms
ρ	Charge density
σ_p	Dominant wave number
υ	Small deviation in electron temperature
Φ	Any molecular property
ψ	Damping factor
ω	Angular frequency of striation

CHAPTER I

INTRODUCTION

In the direct current glow discharge in the rare gases at pressures of the order of a few Torr, the positive column usually appears homogeneous to the eye. Upon examination by time-resolved techniques it is found that over significant ranges of pressure and current, the positive column is not homogeneous, but consists of alternate bright and dark layers moving along the axis of the tube. These regions have been termed "moving striations". Under certain special discharge conditions, standing striations can exist. Figure 1 shows the characteristics of the development of a striated positive column as seen, for example, by a high-speed camera.

As early as 1874 A. Wullner (1)[1] used a rotating mirror to study striations. He observed uniformly spaced areas of high light emission propagating in the discharge with speeds up to 10^6 cm. per second. Since that time numerous methods such as microwave and image converter measurements, photomultiplier, and Langmuir probe techniques have been used to study striations under widely varying experimental conditions.

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[1]Numbers in parentheses correspond to similarly numbered references listed in the bibliography.

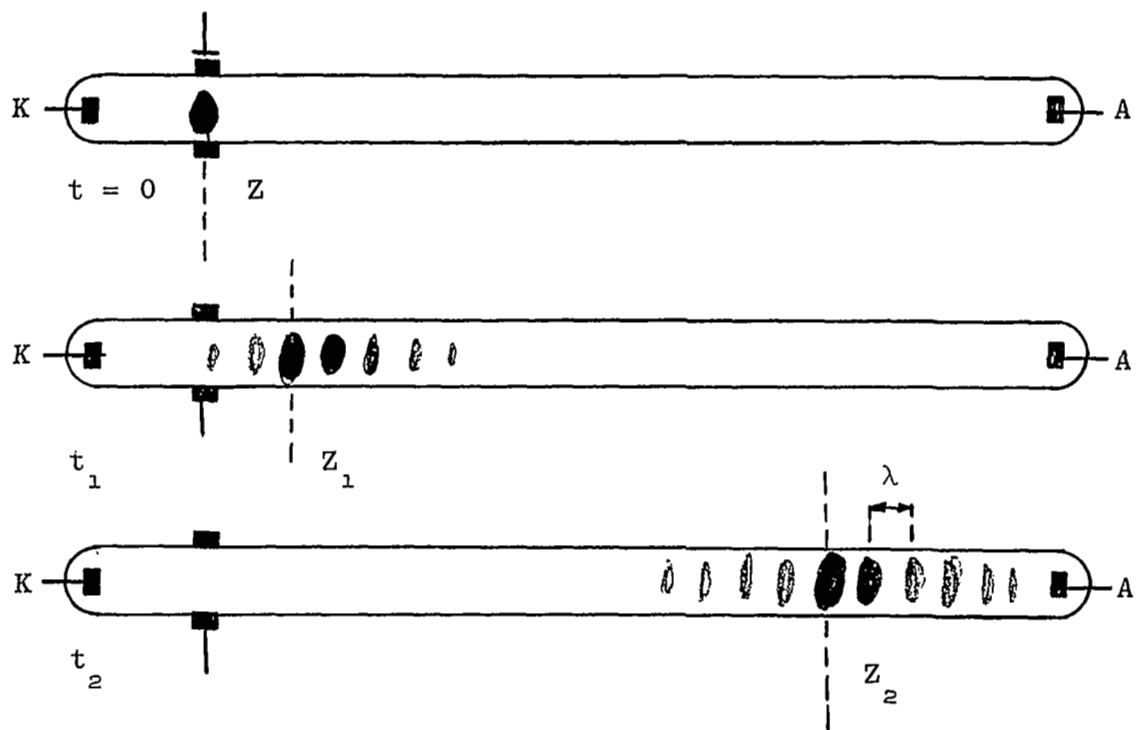K —— t = 0   Z   —— A

t₁   Z₁

t₂   λ   Z₂

Figure 1.   Development of a Striated Positive Column.

Pupp (2) and, recently, Donahue and Dieke (3)
have made the most extensive experimental contributions.
The most significant result of the latter experimenters
is that they showed the functional dependence of frequency
of striations versus discharge current. In the same exper-
iments they showed that as current is increased, the oscil-
lations may become unstable. With a further increase of
current, the oscillations may become stable again but with
a markedly different frequency. Thus, there are several
modes of oscillation. Through his experiments in single-
constituent, rare gas discharges, Pupp showed that there
is a limiting pressure and current at which striations
may exist. These limits are almost independent of tube
radius.

Garscadden and co-workers have published frag-
mentary striation data for the He-Ne laser (4, 5). They
have given specific data points on the pertinent para-
meters of frequency and velocity of the striations in the
He-Ne discharge. There has not been an exhaustive experi-
mental paper describing the continuous variation of the
pertinent parameters.

Theoretical efforts aimed at understanding moving
striations have been divided mainly between attempts to
associate the striations with various plasma waves and
instabilities, and with diffusion-type space-charge, or
ionization waves. Of the latter variety, the most

important contributions have been made by Robertson (6) and by Pekarek (7, 8) who has spent over fifteen years analyzing the problem. His theory, as extended by Bletzinger, predicts qualitatively some of the important properties of the striations. Some attempts have been made to interpret moving striations in terms of ion waves, the most notable of which is the work of Alexeff and Jones (9) who modified the basic ionic-sound wave differential equation and found a damped, spatially-periodic mode of propagation.

It should be emphasized that all the work up to this time on striations is in the rare gases. No theory has been published to even qualitatively explain the phenomenon in multi-constituent rare gases. Pekarek (10) has just recently begun an analysis on the He-Ne discharge. This analysis is much more difficult because of cumulative ionization processes.

The procedures of this study are to present the various theories of striations and to point out the functional relationships that were verified in the Space Institute Laser Laboratory. The experimental results are applicable to and were, in fact, motivated by, studies of fluctuations in the output light intensity of He-Ne lasers. For experimental convenience, data were usually taken on the discharge current since it is known (11) that fluctuations in the discharge current modulate the coherent

output light of the laser. The experimental results of
Chapter III will show that the variation of frequency of
striations versus discharge pressure and tube radius are
in agreement with the theory of Chapter II for a single
constituent plasma. Additional data giving variation of
frequency versus discharge current and other design para-
meters as a function of gas pressure and tube current are
presented. These phenomena are theoretically unexplained
at this time.

CHAPTER II

THEORIES OF STRIATIONS

I. INTRODUCTION

Because of the complexity of the problem, there
is not as yet a theory for striations in a multi-consti-
tuent gas plasma. On the other hand, some rather detailed
studies of striations in single-constituent gas discharges
have appeared in the literature. In this chapter the
three most prominent of those theories will be presented
in varying degrees of detail, and it will be shown later
in this thesis that these theories apply quantitatively
to the He-Ne discharge.

II. ALEXEFF AND JONES SOUND-WAVE THEORY

Alexeff and Jones (12) suggest that moving
striations in direct current glow discharges in the
inert gases may simply be manifestations of ionic-sound
waves propagating in the discharges. By adding an ion-gas
atom collision term to the basic ionic-sound wave differ-
ential equation, the velocity of the running striations
in a single-constituent inert gas is successfully pre-
dicted in the pressure range of 1 micron to 10 Torr. A
serious deficiency in this theory is that a strong spatial

damping is predicted and this damping is contrary to experimental evidence.

The basic equation is

$$\left(\frac{\gamma kT_e}{m_i}\right) \frac{\partial^2 n_+}{\partial z^2} = \frac{\partial^2 n_+}{\partial t^2} + \nu_{In} \frac{\partial n_+}{\partial t} \qquad \text{(II-1)}$$

where,

> γ is the adiabatic compression coefficient of the electron gas,
>
> k is Boltzmann's constant,
>
> m_i is the ion mass,
>
> n_+ is the ion density,
>
> ν_{In} is the collision frequency of an ion, with background gas atoms

and,

> z represents position along the axis.

Equation (II-1) contains the assumption that the amplitude of the wave is small, and that the temperature of the electrons is much greater than the temperature of the ions. Solving Equation (II-1) by the standard technique of assuming n_+ of the form of a constant plus a small oscillating term gives the phase velocity of the striations

$$V_p = V_o \left[1/2 + 1/2 \sqrt{1 + \left(\frac{\nu_{In}}{\omega}\right)^2} \right]^{-1/2} \qquad \text{(II-2)}$$

and the damping factor ψ is given as

$$\psi = (-\frac{v_{In}}{2v_o})\left[1/2 + 1/2\sqrt{1 + (\frac{v_{In}}{\omega})^2}\right]^{-1/2} |x| \quad (II-3)$$

where

v_o is $(\frac{\gamma kT}{m})^{1/2}$, the free space velocity.

The effects of the ion-gas atom collisions on the propagating density are two-fold. Not only is the wave exponentially damped, but also the wave propagates at less than the free space velocity v_o. The importance of this theory is that it shows that the velocity of propagation is $\propto \frac{1}{v_{In}^{1/2}}$ and therefore $\propto \frac{1}{p^{1/2}}$; this dependence also occurs in the experiments reported here.

III. ROBERTSON METASTABLE THEORY (13)

The basis of the theory of Robertson is that striations are influenced by the density of some metastable state. Following are the qualitative arguments which he has made supposedly proving that metastables are necessary for an instability to occur in the positive column.

At a constant current density, under the condition that no metastable states are allowable, the total direct ionization rate decreases as the electron density increases. This phenomenon occurs because the ionization

rate per electron increases rapidly with electric field
and the electric field is approximately inversely pro-
portional to electron density. The fact that the product
of electron density and ionization rate per electron is
a monotone decreasing function of the electron density
contributes very strongly to the stability of the positive
column.

When metastables are present in quantity, how-
ever, the ionization rate may, for a time, continue to
increase as the electron population increases. This is
because, in the inert gases for example, it is much easier
to ionize a metastable than an unexcited atom. Also, a
decrease in the local electric field has a less drastic
effect on the rate per electron for ionizing metastable
than on that for ionizing unexcited atoms. Thus, the
presence of metastables can lead to the growth rather
than to the suppression of instabilities. On this basis
it was predicted that moving striations are critically
dependent upon the metastable density.

An obvious experimental check on the necessity
of metastables for the existence of moving striations is
to examine positive column plasmas which have no meta-
stables. Extensive observations on plasmas that have
no metastables have confirmed the fact that no striations
are present in the positive column of such plasmas (14).

Therefore, it may be inferred that the metastables play
an important function in the production of moving
striations.

It has also been established that the lifetime
of the metastables is related to the velocity of the
striations (15). Since the lifetime of metastables is
inversely proportional to their density and since
Landenburg (15) has shown theoretically the functional
dependence of density of metastables on current, it is
clear that a relationship exists between current and
frequency of striations as well as between pressure and
frequency. Both dependences will be shown experimentally
in Chapter III.

III. PEKAREK-GARSCADDEN THEORY

The Pekarek-Garscadden theory (8, 16, 17, 18)
is the most quoted and most rigorous theory of striations.
The development of this theory given here starts with the
Boltzmann equation, assumes small perturbation theory, and
finally arrives at the result of Pekarek (8). From that
point, a Fourier Series solution is assumed and the analy-
sis of Garscadden (17) is followed. The result

$$f \sim \frac{1}{p^{1/2}R^{3/2}} \qquad \text{(II-4)}$$

is obtained.

Basic to this derivation is that it shows a remarkable characteristic property of a plasma - the development of periodicity after an aperiodic disturbance.

The Boltzmann equation (19, 20) has been employed successfully to predict macroscopic transport phenomena. This formulation is valid for a weakly ionized plasma in which the charges interact principally with neutrals by virtue of short range forces, for example, polarization forces. For highly ionized plasmas, the applicability of the Boltzmann equation is questionable. But because of the much greater mathematical complexity involved in more rigorous approaches, the use of the Boltzmann equation is generally accepted for glow discharges.

The Boltzmann equation, which expresses the time, velocity, and position of the velocity distribution function f_v is written in indicial notation as

$$\frac{\partial f_v}{\partial t} + v_i \frac{\partial f_v}{\partial x_i} + R_i \frac{\partial f_v}{\partial v_i} = \left(\frac{\partial f_v}{\partial t}\right)_{coll.} \qquad (II-5)$$

The average value of any molecular property $\Phi(v_i, x_i, t)$ can be shown to be (21)

$$< \Phi > = \frac{\int \Phi f_v dc}{N} \qquad (II-6)$$

where

dc is a differential element in velocity space

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and

N is the number density in configuration space.
The so-called moment equations are formulated as follows:

$$\int \Phi \left(\frac{\partial f_v}{\partial t} \right) + v_i \frac{\partial f_v}{\partial x_i} + R_i \frac{\partial f_v}{\partial v_i} \right) dc$$

$$= \int \Phi \left(\frac{\partial f_v}{\partial t} \right)_{coll.} dc. \qquad (II-7)$$

Using properties of f_v and Φ this reduces to

$$\frac{\partial (N\langle\Phi\rangle)}{\partial t} + \frac{\partial (N\langle v_i \Phi\rangle)}{\partial x_i}$$

$$- N \left[\langle \frac{\partial \Phi}{\partial t} \rangle + \langle v_i \frac{\partial \Phi}{\partial x_i} \rangle + \langle R_i \frac{\partial \Phi}{\partial v_i} \rangle \right]$$

$$= \int (\hat{\Phi}^S - \Phi^S) f^S f^N S(X) dw dc^N dc^S. \qquad (II-8)$$

Letting $\Phi = 1$ in Equation (II-8) gives for the Maxwell
Boltzmann (equilibrium) solution,

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x_i} (N\langle v_i \rangle) = 0. \qquad (II-9)$$

Equation (II-9) is obvious and needs to be modified to
account for inelastic collisions.

Continuity of positive ions is thus described by

$$\frac{\partial N_+}{\partial t} + \frac{\partial}{\partial x_i} (N_+ \langle v_i^+ \rangle) = S^+{}_{coll.} \qquad (II-10)$$

where $S^+{}_{coll.}$ is the inelastic generation rate.

The effects of ionization in the continuity equation are included by defining the ionization rate Z such that the number of ions produced per second is $N_- Z$. After defining the ion flux density as

$$\Gamma_i^+ \triangleq N_+ \langle v_i^+ \rangle, \qquad (II-11)$$

substitution of Equation (II-11) into (II-10) gives

$$\frac{\partial N_+}{\partial t} + \Gamma_{i,i}^+ = N_- Z \qquad (II-12)$$

which is a modified continuity equation.

The momentum equation is obtained by letting $\Phi = mv_i$ in Equation (II-8):

$$\frac{1}{\nu_{In}} \frac{\partial \Gamma_i^+}{\partial t} + \frac{kT_+}{m_+ \nu_{In}} \frac{\partial N_+}{\partial x_i} - \frac{N_+ q_+}{m_+ \nu_{In}} E_i = -\Gamma_i^+. \qquad (II-13)$$

If the characteristic time (21) for density changes (the diffusion time) is long compared with the time between collisions $(1/\nu_{In})$, then the first term in Equation (II-13) can be neglected compared to the last term. The flux of

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particles due to the transport phenomenon of diffusion is expressed as the density gradient multiplied by the diffusion coefficient; thus from Equation (II-13),

$$D_+ \triangleq \frac{kT_+}{m_+ \nu_{In}} .$$

(II-14)

The flux of particles due to the electric field is given by the electric field times the number density of drifting particles, multiplied by the mobility; thus

$$\mu_+ \triangleq \frac{q_+}{m_+ \nu_{In}}$$

(II-15)

Taking the divergence of Equation (II-13) gives

$$\Gamma^+_{i,i} = - \left[D_+ \frac{\partial N_+}{\partial x_i} - N_+ \mu_+ E_i \right]_{,i} .$$

(II-16)

Substitution of Equation (II-16) into Equation (II-12) gives

$$\frac{\partial N_+}{\partial t} = \left[D_+ \frac{\partial N_+}{\partial x_i} - N_+ \mu_+ E_i \right]_{,i} + ZN_- .$$

(II-17)

Transform the three-dimensional Equation (II-17) into a one-dimensional equation for small deviations by the substitutions:

$$N_+ = N_o + n_+$$

(II-18)

$$N_- = N_o + n_-$$

(II-19)

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$$E_z = E_o + \epsilon(z,t) \ . \qquad\qquad\qquad (II-20)$$

Substitution of Equations (II-18), (II-19) and (II-20) into Equation (II-17) gives

$$\frac{\partial n_+}{\partial t} = D_+ \frac{\partial^2 n_+}{\partial z^2} - \frac{\partial n_+}{\partial z} \mu_+ E_o - N_o \mu_+ \frac{\partial \epsilon}{\partial z} + ZN_- \quad (II-21)$$

where terms of products of small deviations have been neglected.

From this point the derivation will essentially follow that of Pekarek (8).

Solve Equation (II-21) together with an initial condition that represents a small perturbation in the ion density. Such an initial condition can be represented as

$$n_+(z, t = 0) = n_o(z) = n_o \frac{1}{\Lambda\sqrt{\pi}} e^{-\frac{z^2}{\Lambda^2}} ,$$

$$- L \leq z \leq L \qquad\qquad\qquad (II-22)$$

where Λ is the half-width of the disturbance. Note that this represents an aperiodic disturbance.

If the initial condition is chosen such that the width of the initial disturbance is much greater than the Debye Length for electrons, the electron density deviation is then very nearly equal to the deviation in ion density and can be expressed as (8)

$$n_- \cong n_+ + \frac{h^2}{D_-} \mu_- E_o \frac{\partial n_+}{\partial z} + h^2 \frac{\partial^2 n_+}{\partial z^2} + \dots \qquad (II\text{-}23)$$

where, in Gaussian units, the Debye Length is

$$h = \left(\frac{kT_-}{4\pi N_o e^2}\right)^{1/2} . \qquad (II\text{-}24)$$

The space charge term can then be written as

$$\rho = e(n_+ - n_-) = - eh^2 \left[\frac{\mu E_o}{D_-} \frac{\partial n_+}{\partial z} + \frac{\partial^2 n_+}{\partial z^2}\right] \qquad (II\text{-}25)$$

and ρ is found from Poisson's Equation

$$\frac{\partial \epsilon}{\partial z} = 4\pi\rho . \qquad (II\text{-}26)$$

Performing a simple integration gives

$$\epsilon = - e\, 4\pi\, h^2 \left[\frac{\mu_- E_o}{D_-} n_+ + \frac{\partial n_+}{\partial z}\right] . \qquad (II\text{-}27)$$

Consider the third term of Equation (II-21). Substitution of Equation (II-27) into Equation (II-21) gives

$$- N_o \mu_+ \frac{\partial \epsilon}{\partial z} = N_o \mu_+ 4\pi e h^2 \left[\frac{\mu_- E_o}{D_-} \frac{\partial n_+}{\partial z} + \frac{\partial^2 n_+}{\partial z^2} \right]$$

$$= \mu_+ E_o \frac{\partial n_+}{\partial z} + \frac{kT_-}{e} \mu_+ \frac{\partial^2 n_+}{\partial z^2} \quad . \quad \text{(II-28)}$$

For the glow discharge (22)

$$D_a \cong \frac{kT_- \mu_+}{e} \quad . \quad \text{(II-29)}$$

Substituting Equation (II-29) and Equation (II-28) into Equation (II-21) gives, realizing that the second term in Equation (II-21) cancels in the expansion of the third term,

$$\frac{\partial n_+}{\partial t} = D_a \frac{\partial^2 n_+}{\partial z^2} + ZN_- \quad . \quad \text{(II-30)}$$

It is intuitively acceptable that $Z = Z(kT)$. The concern here is the spatial variation of Z as providing the causal effect of striations and not on net average generation of charges. Therefore, following Pekarek, one writes

$$Z = Z_\upsilon \Big|_{T_o} + \frac{\partial Z_\upsilon}{\partial (kT)} \Big|_{T_o} k(T_- - T_o) + \ldots$$

$$\triangleq Z_\upsilon \Big|_{T_o} + Z'_\upsilon \upsilon \quad \text{(II-31)}$$

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where T_O is the equilibrium electron temperature. The
constant term will be hereafter disregarded.

The temperature deviation has been expressed by
Granovski (8) in terms of n_+ as

$$\frac{\partial \upsilon}{\partial z} - a_1 \upsilon = - b_1 \epsilon(z) \tag{II-32}$$

where

$$b_1 \cong \frac{3}{2} e \tag{II-33}$$

and

$$a_1 = \frac{32}{3\pi} \frac{p^2 k}{E_o \lambda_1^2} \quad . \tag{II-34}$$

The solution of Equation (II-32) is

$$\upsilon e^{-\int_0^z a_1 dz} = - b_1 \int_0^z \epsilon(z') e^{-\int^{z'} a_1 dz'} dz' + c \tag{II-35}$$

$$= - b_1 \int_0^z \epsilon(z') e^{-a_1 z'} dz' + c \quad . \tag{II-36}$$

Evaluating c at $z = \infty$ gives

$$\upsilon = - b_1 \int_\infty^z \epsilon(z') e^{(z-z')a_1} dz' \quad . \tag{II-37}$$

After substitution of Equation (II-27) into (II-37)

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and defining

$$\alpha = 4\pi e h^2 \tag{II-38}$$

and

$$\beta = \frac{\mu_- E_o}{D_-} \quad . \tag{II-39}$$

Equation (II-37) becomes

$$\upsilon(z) = b_1 \int_\infty^z \alpha(\beta n_+ + \frac{\partial n_+}{\partial z'}) \, e^{(z-z')a_1} \, dz' \quad . \tag{II-40}$$

Performing the integration by parts gives

$$\upsilon(z) = b_1 \int_\infty^z \alpha\beta n_+ e^{(z-z')a_1} dz'$$

$$+ b_1 \alpha n_+ e^{(z-z')a} \Big|_\infty^z - b_1 \alpha \int_\infty^z n_+ e^{(z-z')a_1}(-a_1) dz'$$

$$= b_1 \alpha n_+ - \int_z^\infty b_1 \alpha(n_+\beta + n_+ a_1) e^{(z-z')a_1} dz'.$$

Finally,

$$\upsilon(z) = b_1 4\pi e h^2 n_+ - b_1 4\pi e h^2 [\frac{\mu_- E_o}{D_-} + a_1] \times$$

$$\int_z^\infty n_+(z') e^{a_1(z-z')} dz' \quad . \tag{II-41}$$

Substitution of Equation (II-41) into (II-30) gives

$$\frac{\partial n_+}{\partial t} = D_a \frac{\partial^2 n_+}{\partial z^2} + z'_v b_1 \frac{kT_-}{e} n_+ - z'_v b_1 \left(\frac{kT_-}{e} a_1\right.$$

$$\left. + E_o\right) \int_z^\infty e^{a_1(z-z')} n_+(z')dz' \quad . \tag{II-42}$$

This is the equation originally derived by Pekarek (8) and solved on an electronic computer. After substitution for appropriate constants and the initial condition of Equation (II-22), he found that the solution is oscillatory on the left hand side of the initial disturbance. The importance of this result is that it is shown that an aperiodic initial disturbance in the positive column of a glow discharge can have an oscillatory response.

In order to find a useful form of the solution of Equation (II-42), the procedures of Bletzinger et al. (17, 18) will be followed to obtain a relationship between the frequency of striations and pressure, radius of discharge tube, and temperature. Equation (II-42) can be written as

$$\frac{\partial n_+}{\partial t} = D_a \frac{\partial^2 n_+}{\partial z^2} + c_1 n_+$$

$$- c_2 \int_z^\infty \exp[-a_1(z'-z)]n_+(z',t)dz' \tag{II-43}$$

where

$$c_1 = z'_{\upsilon} b_1 \frac{kT_-}{e} \qquad (II\text{-}44)$$

$$c_2 = z'_{\upsilon} b_1 [(\frac{kT_-}{e})a_1 + E_o] \quad . \qquad (II\text{-}45)$$

Assuming a Fourier Series solution of Equation (II-43)

$$n_+(z,t) = Re[\sum_{m=0}^{\infty} C_m(t)\exp(i\sigma_m z)] \qquad (II\text{-}46)$$

where $\sigma_m = \frac{m\pi}{L}$, $m = 0, 1, 2 \ldots$ \qquad (II-47)

and the initial condition of Equation (II-22) is imposed.
Substitution of Equation (II-46) into Equation (II-43)
gives, term by term, omitting the operation Re until
later in the analysis,

$$\frac{dC_m(t)}{dt} \exp(i\sigma_m a) = D_a C_m(t)i^2 \exp(i\sigma_m z)$$

$$+ c_1 C_m(t)\exp(i\sigma_m z)$$

$$- c_2 \int_z^{\infty} [\exp -a_1(z-z')]C_m(t)\exp(i\sigma_m z')dz'.$$
$$\qquad (II\text{-}48)$$

Consider the last term:

$$- C_m(t) c_2 e^{a_1 z} \int_z^\infty \exp[(-a_1 + i\sigma_m)z']dz'$$

$$= - c_2 C_m(t) \exp(i\sigma_m z) \frac{a_1 + i\sigma_m}{a_1^2 + \sigma_m^2} \quad . \quad \text{(II-49)}$$

Now Equation (II-48) can be written as

$$\frac{dC_m(t)}{dt} = - [D_a \sigma_m^2 - c_1 + c_2 \frac{(a_1 + i\sigma_m)}{a_1^2 + \sigma_m^2}]C_m(t) \quad \text{(II-50)}$$

whose solution is

$$C_m(t) = C_m(0)\exp[- D_a \sigma_m^2 - c_1$$

$$+ c_2 \frac{(a_1 + i\sigma_m)}{a_1^2 + \sigma_m^2}]t \quad . \quad \text{(II-51)}$$

$C_m(0)$ is the Fourier coefficient

$$C_m(0) = \frac{1}{L} \int_{-L}^{L} n_0(z)\exp[- i\sigma_m z]dz \quad \text{(II-52)}$$

which was obtained from Equations (II-46) and (II-22).
For the initial condition given previously, the solution
for the ion density is

$$n_+(z,t) = \sum_{m=0}^{\infty} C_m(0)\exp[- (D_a\sigma_m^2 - c_1$$

$$+ \frac{c_2 a_1}{a_1^2 + \sigma_m^2})t] \cos(\frac{\sigma_m c_2 t}{a_1^2 + \sigma_m^2} + \sigma_m z) \qquad \text{(II-53)}$$

which is seen to have the form of a damped, propagating wave.

The attenuation factor will have a minimum when

$$0 = 2D_a\sigma_m - \frac{c_2 a_1 2\sigma_m}{(a_1^2 + \sigma_m^2)^2} \qquad \text{(II-54)}$$

or when

$$\sigma_m = \sqrt{(\frac{a_1 c_2}{D_a})^{1/2} - a_1^2} \triangleq \sigma_P \qquad . \qquad \text{(II-55)}$$

Thus, as time increases, the solution is more closely given by the term in which $\sigma_m = \sigma_P$, the others being more strongly damped. Then σ_P could be called the "dominant wave number" of the discharge and this is the mode most likely to propagate.

Associate an angular frequency with Equation (II-53) of the form

$$\omega(\sigma_P) = \frac{c_2\sigma_P}{a_1^2 + \sigma_P^2} \qquad . \qquad \text{(II-56)}$$

23

By definition, the phase velocity is

$$V_p(\sigma_p) \triangleq \frac{\omega(\sigma_p)}{\sigma_p} = \frac{c_2}{a_1^2 + \sigma_p^2} = (\frac{c_2 D_a}{a_1})^{1/2} \qquad \text{(II-57)}$$

where Equation (II-55) was used in the last equality.

Several semi-empirical expressions relating pressure p and tube radius R to the significant parameters of glow discharges are given below. These relations permit prediction of the p and R dependence of V_p, and are taken from von Engle's book on ionized gases (23). The electron temperature can be calculated from the transcendental relation

$$\exp(\frac{V_I}{\theta}) = 1.2 \times 10^7 (cpR)^2 (\frac{V_I}{\theta})^{1/2} \qquad \text{(II-58)}$$

where V_I is the ionization potential of the gas

$$\theta = \frac{kT}{e} \quad , \qquad \text{(II-59)}$$

and

$$c = (\frac{A_z V_I}{\mu_o^+})^{1/2} \quad . \qquad \text{(II-60)}$$

A_z is the initial slope of ionization efficiency as a function of electron energy and μ^+ is the normalized ion mobility defined by

24

$$\mu^+ \triangleq \frac{\mu_o^+}{p} \quad . \tag{II-61}$$

The rate of change of the ionization frequency Z with respect to θ can be found from the relation

$$Z_\upsilon = 9 \times 10^7 \, A_Z p V_I \theta^{1/2} [\exp(-\frac{V_I}{\theta})] \tag{II-62}$$

for a Maxwellian velocity distribution.

The axial electric field E_o, gas pressure p, and the average fraction δ of energy lost by an electron in a collision are related by the following

$$\frac{E_o}{p} = \frac{\theta(2\delta)^{1/2}}{\lambda_1} \tag{II-63}$$

where

$$\delta = \frac{2m_e}{m_i} \tag{II-64}$$

and λ_e is the normalized mean free path for electron-neutral collisions defined through the relation

$$\lambda_e \triangleq \frac{\lambda_1}{p} \quad . \tag{II-65}$$

It is convenient to simplify the relations for a_1 and E_o by writing them as

$$a_1 \triangleq c_3 p: \quad c_3 \triangleq \frac{32}{3\pi} \frac{(\delta^{1/2} 2^{1/2})^{-1} k}{\lambda_1} \qquad (II-66)$$

$$E_o \triangleq c_4 p\theta: \quad c_4 \triangleq \frac{(2\delta)^{1/2}}{\lambda_1} . \qquad (II-67)$$

Substitution of Equation (II-58) into (II-62) after differentiation gives

$$e \frac{\partial Z_\upsilon}{\partial (kT)} = \frac{\partial Z_\upsilon}{\partial \theta} = 9 \times 10^7 \ A_Z p V_I [\frac{1}{2}$$

$$+ \frac{V_I}{\theta}] \ \frac{1}{\theta^{1/2}} \ exp[-\frac{V_I}{\theta}]$$

$$= \frac{1}{pR^2} [\frac{7.5\mu_o^+}{V_I^{1/2}} (\frac{1}{2} + \frac{V_I}{\theta})] . \qquad (II-68)$$

Thus, Equation (II-45) becomes

$$c_2 = \frac{c_5}{R^2} \qquad (II-69)$$

where

$$c_5 \triangleq \frac{3}{2} \times 7.5 \ V_I^{-1/2} (\frac{\theta}{2} + V_I)(c_3 + c_4). \qquad (II-70)$$

Finally,

$$V_p = (\frac{c_5 \theta \mu^+}{R^2 c_3 p})^{1/2} = c_6 \frac{\theta^{1/2}}{pR} \qquad (II-71)$$

26

where

$$c_6 \triangleq (\frac{c_5}{c_3} \mu_o^+)^{1/2} \quad .$$ (II-72)

This is the final result of Bletzinger, et al. (17).
Equation (II-55) reduces to

$$\sigma_p = \sqrt{\frac{c_7 p}{\theta^{1/2} R} - c_3^2 p^2}$$ (II-73)

where

$$c_7 \triangleq (\frac{c_3 c_5}{\mu_o^+})^{1/2} \quad .$$ (II-74)

Equation (II-57) can be rewritten as

$$f = \frac{\sigma_p V_p}{2\pi} \quad \frac{1}{2\pi} \frac{c_6 \theta^{1/2}}{pR} \left[\frac{c_7 p}{\theta^{1/2} R} - c_3^2 p^2 \right]^{1/2}$$

$$= \frac{1}{2\pi} \frac{c_6 (c_7)^{1/2} \theta^{1/4}}{p^{1/2} R^{3/2}} (1 - \frac{c_3^2 \theta^{1/4} Rp}{c_7})^{1/2} \quad .$$ (II-75)

The final relationship for the frequency is

$$f \triangleq \frac{c_8 \theta^{1/4}}{p^{1/2} R^{3/2}} (1 - \frac{c_3^2 \theta^{1/4} Rp}{c_7})^{1/2}$$

$$\cong \frac{c_8 \theta^{1/4}}{p^{1/2} R^{3/2}}$$ (II-76)

where

$$c_8 \triangleq \frac{c_6 c_7^{1/2}}{2\pi} \; .$$ (II-77)

Substitution of appropriate values shows that the last term in the brackets of Equation (II-75) is small compared to unity. Thus, the functional relationship is

$$f = \frac{c_8 \; \theta^{1/4}}{p^{1/2} R^{3/2}} \; .$$ (II-78)

It should be noted that this same result is obtained by using Equations (II-71) and (II-57) and substituting the relation for wavelength as given by Cobine (24). This functional relationship has been derived from basic plasma equations. In Chapter III the general validity of Equation (II-78) will be demonstrated. It should be pointed out that θ is not a function of current, that is, this equation gives only the dependence of frequency of striations on pressure and radius of discharge tube.

CHAPTER III

EXPERIMENTAL RESULTS

I. CONSTRUCTION AND INSTRUMENTATION

The He-Ne lasers that were used in the following
experiments were constructed in our laboratory of Pyrex
glass. Coherent output power obtainable is comparable to
that achieved in commercial He-Ne lasers. The laser tubes
have physical dimensions of forty to fifty centimeters
active length and 2.5 to 5.0 millimeters in diameter.
Electrodes for this cold cathode design tube were purchased
from the Tubelite Company, Moonachie, New Jersey, and from
the E. G. L. Company, Newark, New Jersey, and are standard
neon sign electrodes. The external mirror configuration
required the use of Brewster-angle windows. Purchased
from Edmund Scientific, Barrington, New Jersey, these
surplus windows were attached with Torr-Seal epoxy. Mirrors
of surface quality $\lambda/4$ and of various transmissivities
were acquired from Spectra-Physics, Mountain View,
California. The highest reflectivity mirror was usually
used to widen the range over which the laser would oscillate.

Standard glass-blowing techniques were used in
the fabrication of the lasers, and care was exercised to
keep the glass-blower's breath from depositing on the
windows. For successful operation of the lasers, it is

essential to keep the Brewster-angle windows free of dirt
and film. This chapter will not elucidate further on
construction techniques since that area has already been
sufficiently investigated (25). Figure 2 illustrates
the general shape and dimensions of a typical laboratory
laser plasma tube.

Equipment used in the experiments is of high
quality and results are repeatable. The vacuum system
used in the experiments was built around a Vac-Ion Model
911-5014 pump which eliminates oil contamination problems.
A base pressure of approximately 10^{-7} Torr is easily attain-
able with this system. This ultra-high vacuum system was
found necessary in order to eliminate contaminants from
seriously affecting experimental results. Tube pressure p
was measured with a Hastings thermocouple gauge Model SP-1S.
Coherent output power measurements were made with an E.G.& G.
Model 560 "Lite-Mike" in conjunction with a Wratten #29
filter which eliminated spurious radiation from the dis-
charge. Frequency measurements were made with a Tektronix
1L5 Spectrum Analyzer, and laser current levels I_L were
measured with a Hewlett-Packard Model 412-A Voltmeter
connected across a 100 ohm resistor in series with the
discharge. A schematic diagram of the system is shown in
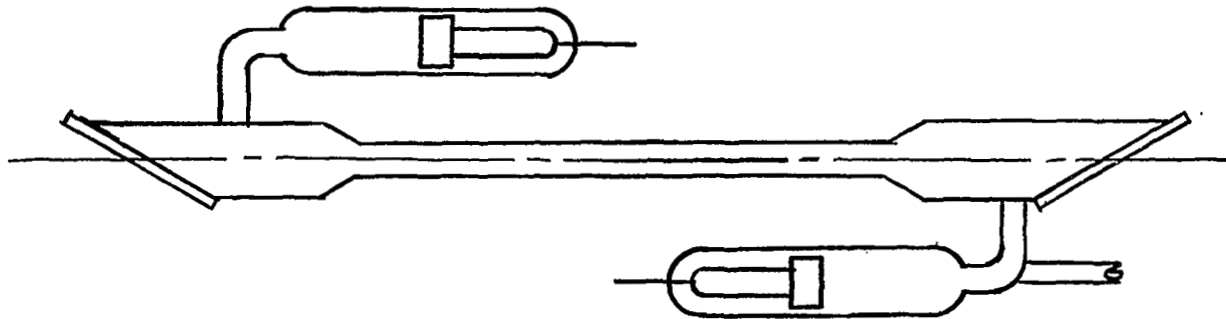Figure 3.
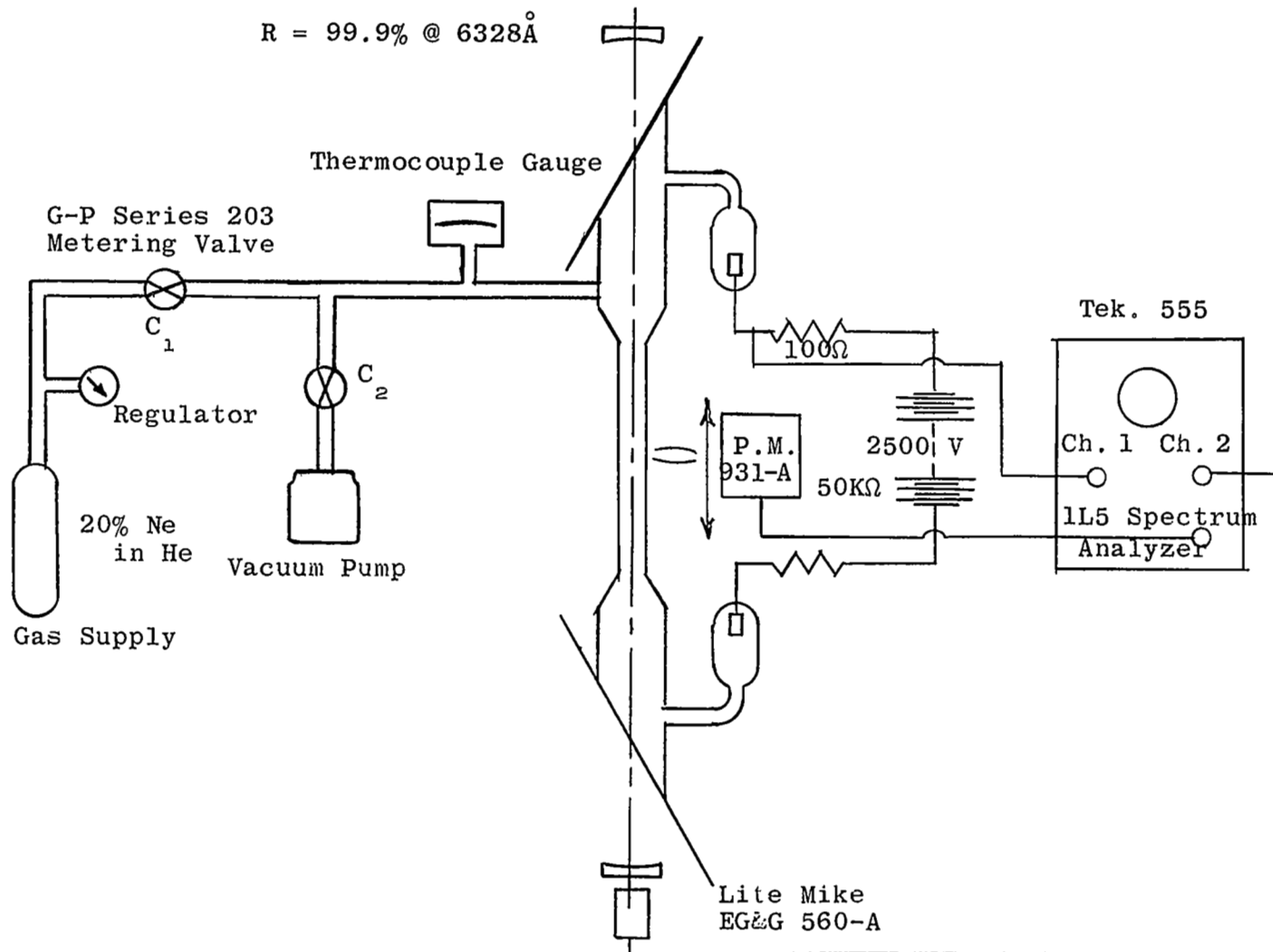
Figure 2. Laboratory Plasma Laser Tube.

R = 99.9% @ 6328$\overset{\circ}{\text{A}}$

Thermocouple Gauge

G-P Series 203
Metering Valve

C_1

Regulator

20% Ne
in He

Vacuum Pump

Gas Supply

100Ω

P.M.
931-A

2500 V

50KΩ

Tek. 555

Ch. 1   Ch. 2

1L5 Spectrum
Analyzer

Lite Mike
EG&G 560-A

C_2

Figure 3.   Schematic Diagram of System.

II. MEASUREMENTS

The onset of noise and striations in the He-Ne discharge is shown in the three photographs of Figure 4. Only low frequency noise of small amplitude is apparent at low values of current (Figure 4-a). As the current is increased, the noise level begins to increase slightly until the point at which the striations appear is reached. Suddenly, the low frequency noise disappears, and the major fluctuation in the current is the striative oscillation (Figure 4-b). Increasing the current still further will decrease the striation frequency. Finally a point is reached at which the low frequency noise is the dominant perturbation (Figure 4-c). The plasma is thus usually in one of two different modes:

1. The low frequency noise is present without the striations, or

2. The striations predominate with the excess low frequency noise considerably reduced in magnitude.

No theoretical explanation for this mode phenomenon appears in the literature. The experimental evidence is presented here so that the experimenter may realize that these conditions exist with potential detrimental results to some practical uses of the laser.
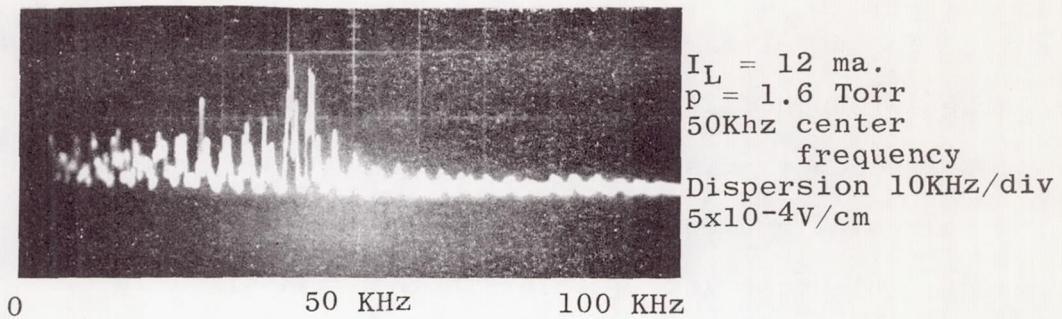
I_L = 12 ma.
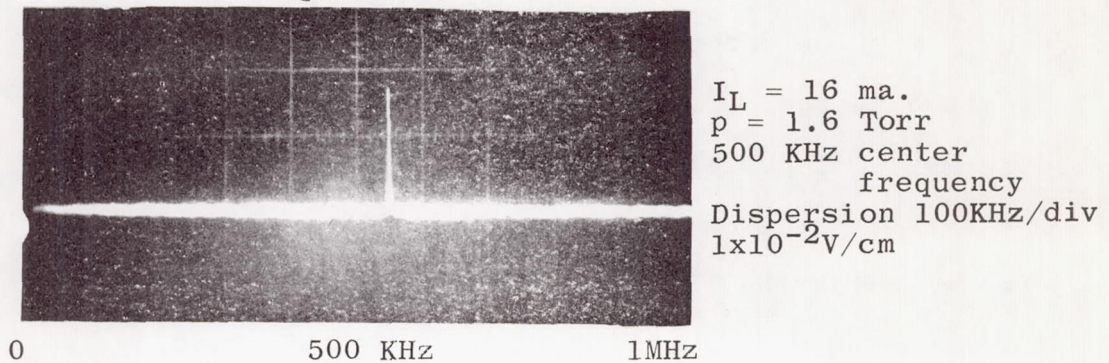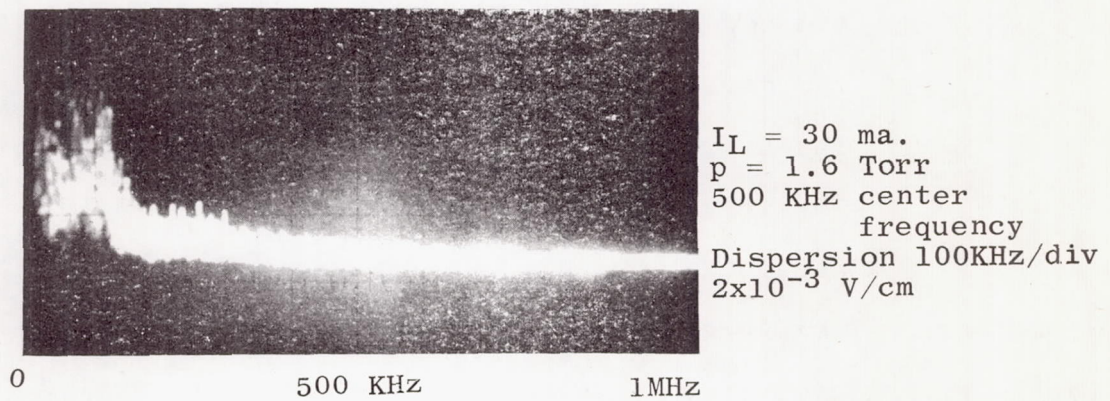p = 1.6 Torr
50Khz center
       frequency
Dispersion 10KHz/div
5×10^{-4} V/cm

0   50 KHz   100 KHz

Figure 4-a

I_L = 16 ma.
p = 1.6 Torr
500 KHz center
        frequency
Dispersion 100KHz/div
1×10^{-2} V/cm

0   500 KHz   1MHz

Figure 4-b

I_L = 30 ma.
p = 1.6 Torr
500 KHz center
        frequency
Dispersion 100KHz/div
2×10^{-3} V/cm

0   500 KHz   1MHz

Figure 4-c

Figure 4.   General Discharge Characteristics

That these oscillations are moving striations
in the plasma and not, for example, relaxation oscillations
was confirmed by the following experiment. The quasi-
sinusoidal fluctuations in the tube current were used to
trigger the sweep of the oscilloscope. The phase of the
side-light output was then observed with a photodetector
to depend on the point in the tube from which the measure-
ment was taken.

Most of the data reported here were observations
of fluctuations in the discharge current rather than of
fluctuations in the coherent light output. This was done
for experimental convenience, and the extent to which the
discharge data is applicable to the coherent light and
output is shown by the modulation experiments of Figures
5 and 6. Up to a frequency of several hundred Khz the
modulation percentage is such that discharge fluctuation
phenomena would show up in the light output at 6328 $\overset{o}{A}$.
It should be pointed out that the modulation of the infra-
red line at 1.153 microns is effective to only about
100 Khz (26).

Kubo, Kawabe, and Inuishi (26) have theoretically
investigated the modulation effects and have determined
that a close relationship exists between the lifetime of
the He metastable atom and the cutoff frequency. Their
theoretical results are in quantitative agreement with
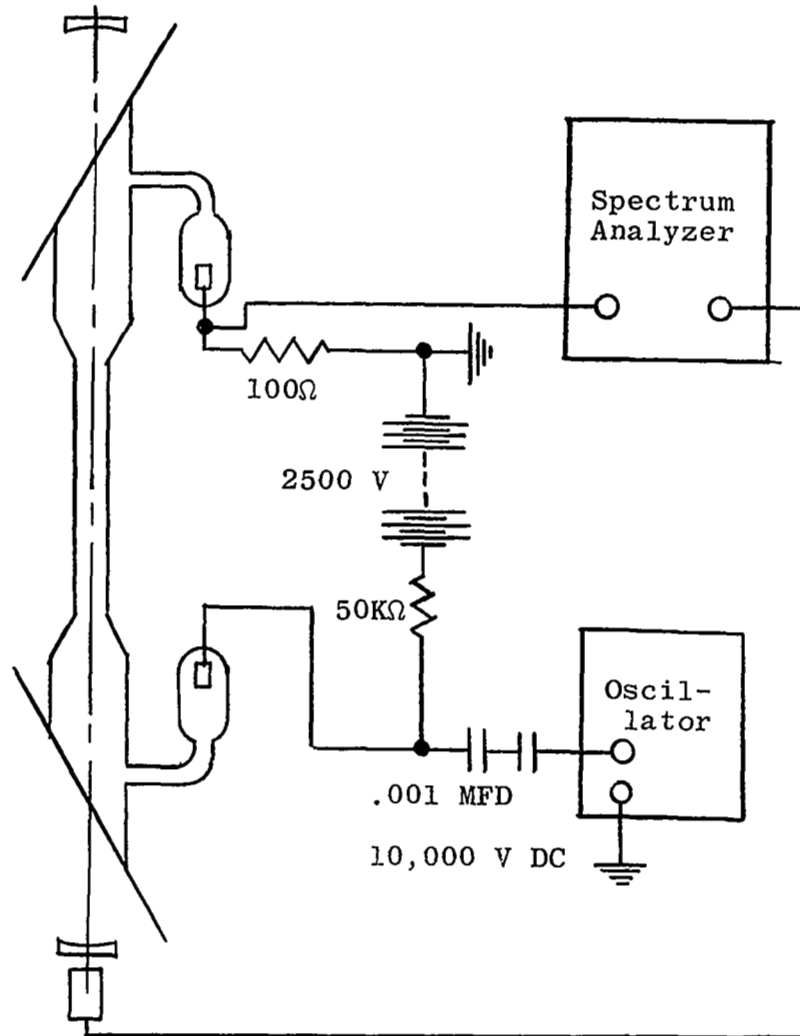their experiment.

Figure 5. Schematic Diagram of Modulation Experiments.
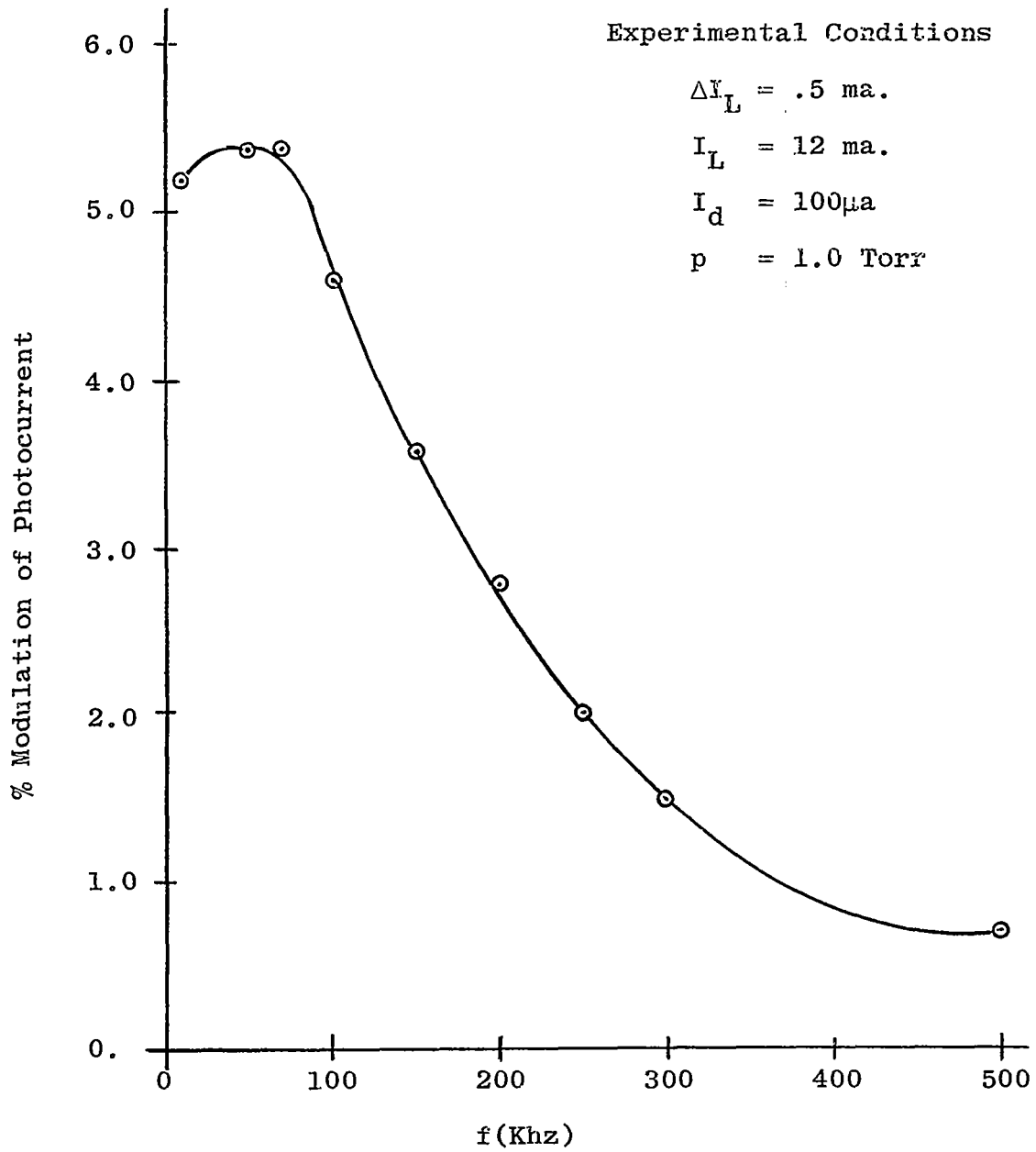
Figure 6. Modulation Characteristics for Tube 1.

An interesting phenomenon that shows the effect
of lasing action on the discharge current was observed.
By spoiling the cavity and simultaneously monitoring the
striation frequency, it was observed that a frequency shift
occurs whose magnitude is proportional to the coherent out-
put power. This decrease in striation frequency was ob-
served to be about 1 percent of the undisturbed frequency.
It has been hypothesized (4) that the change in striation
frequency is related to the population density of the neon
metastables. Lasing action causes approximately a 1 per-
cent increase in the neon metastable level.

Concommitant with the change in striation fre-
quency, the discharge current increases by a fraction of a
milliampere when the cavity is spoiled. A phenomenolog-
ical explanation (27) is that since a population level
near the ionization potential is depopulated by the laser
action, the number of ionizations will be decreased as
compared to the case when laser action is absent. There-
fore, when the cavity is spoiled, the conductivity of the
positive column increases because the electron density
increases. This permits an increase of current.

All the tubes tested showed approximate agreement
with the theoretical analysis of Chapter II. That is, the
functional dependence of frequency of striations versus
pressure is quantitatively verified in Figures 7 and 8.
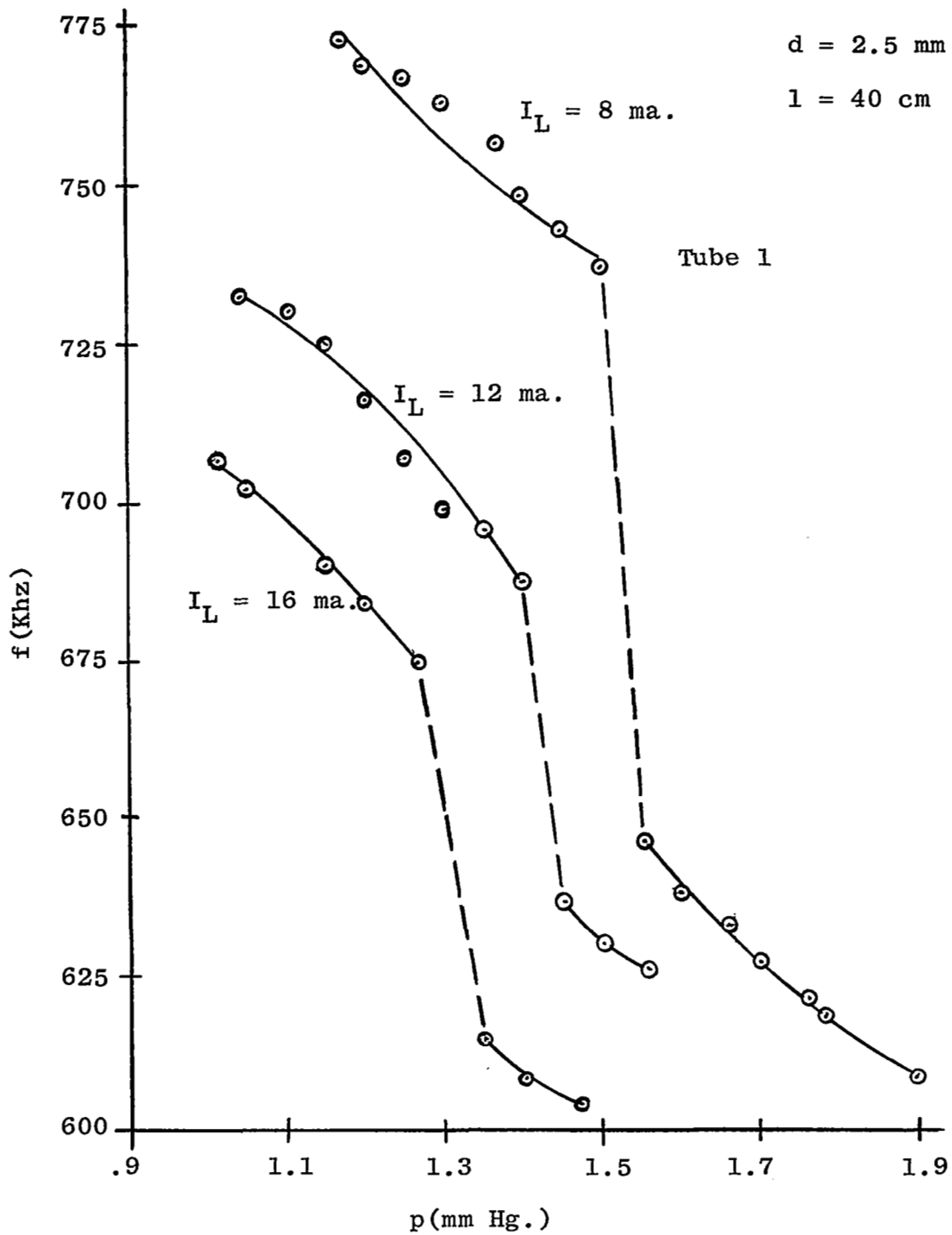One aspect of data that is not fully understood is the

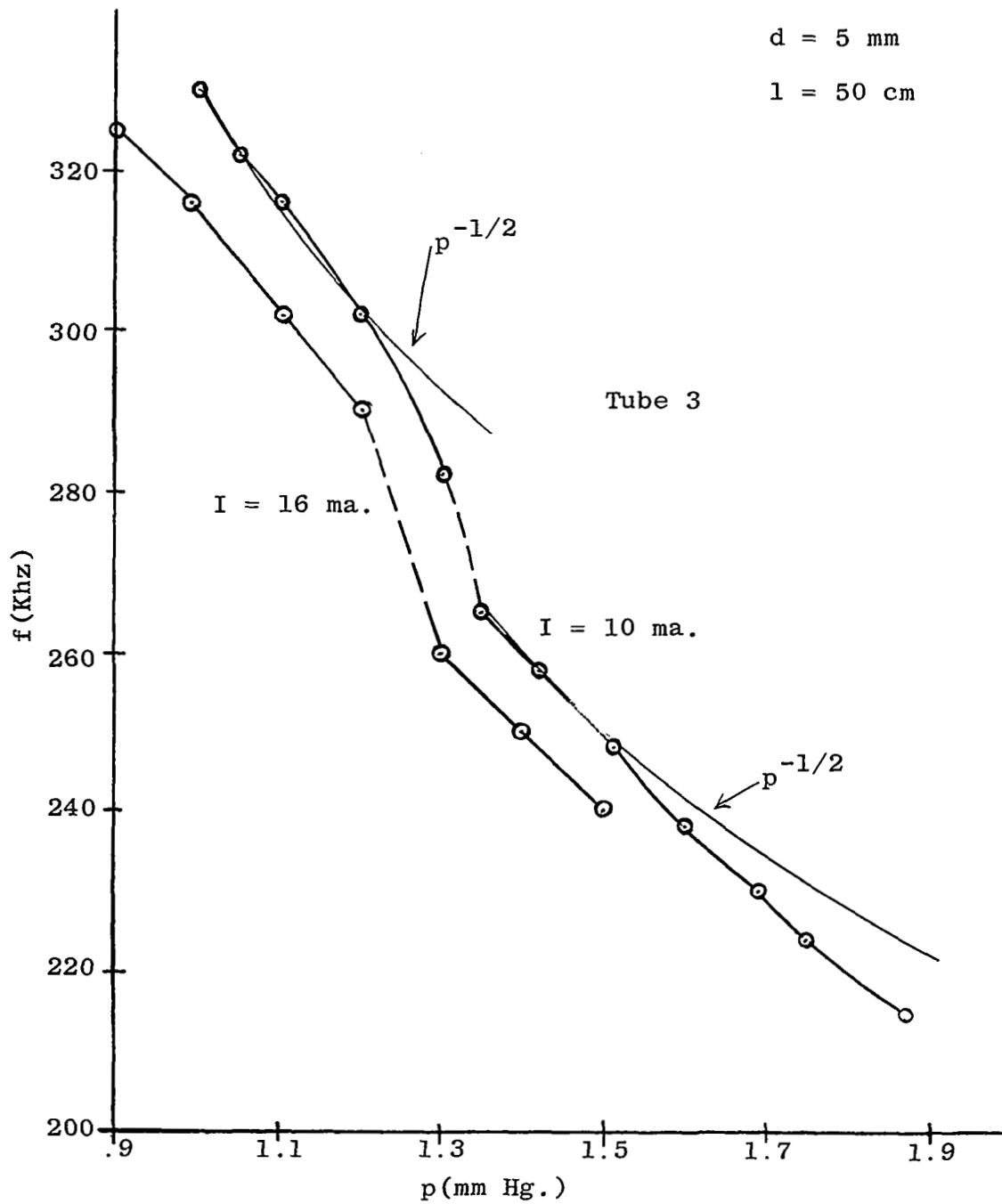Figure 7.   Frequency of Striations Versus Pressure. for Tube 1.

Figure 8. Frequency of Striations Versus Pressure
for Tube 3

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"mode jumping" phenomenon. The critical pressure at which this instability occurs is in the range of 1.2 to 1.4 Torr and the critical pressure seems to be related inversely to the DC current value.

The following procedures were used to obtain the data of Figures 7 and 8. The discharge was initiated and the current adjusted to the desired value. The pressure was slowly increased, holding the current fixed, until the discharge comes into a striative mode. Usually this initial striative mode was unstable and it "died out" in a short period (approximately ten seconds) due to some transient thermal process. A slight increase of current will restore the striative process to a steady-state level. The pressure was slowly increased in 0.05 Torr increments, readjusting the current after each pressure increment. The Granville Phillips Model GP-203 metering valve was particularly useful in these experiments. A fixed time interval of two minutes was allowed before the frequency data were recorded from the spectrum analyzer.

Data shown in Figure 9, are typical of the frequency versus current characteristics. These curves have not been previously published for the He-Ne discharge, and are shown here so that the prospective user may be aware of this phenomenon. In these experiments, as can be seen by the frequent mode jumping, it is essential to use small increments in the current. A power supply and
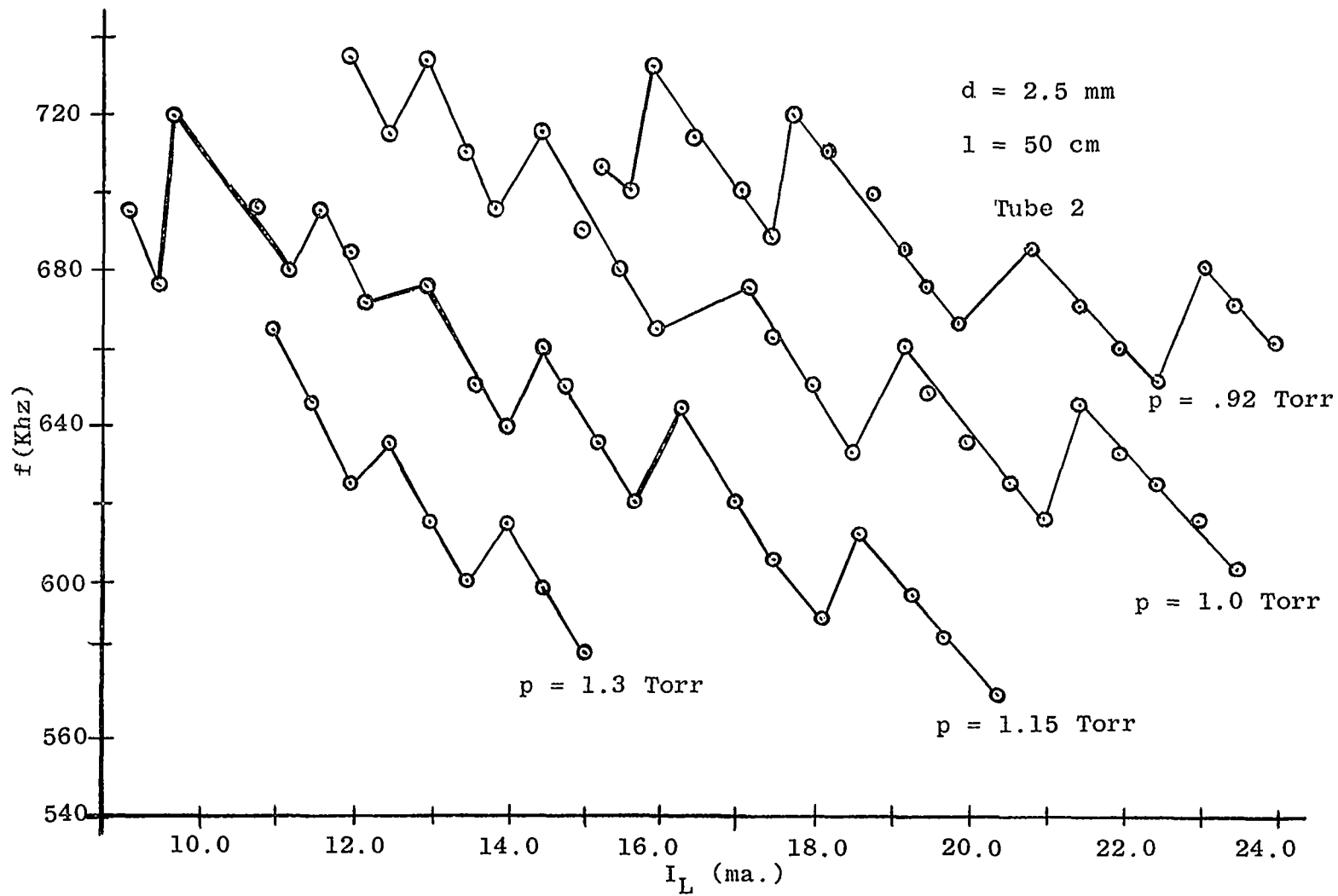
Figure 9.   Frequency of Striations Versus Current for Tube 2.

instrumentation system were constructed so that increments of 0.5 ma. in the discharge current were easily facilitated. The schematic diagram of the system as shown in Figure 3, page 32, clarified the experimental procedure.

Figure 10 shows the linear decrease of striation frequency versus current for a Spectra-Physics Model 130-c plasma tube. This plasma tube exhibited the same behavior as tubes built in this laboratory. Pressure data were not available for this tube.

Pupp (28) found that striations occur only when the current and gas pressure are less than certain limiting values. He found that in pure rare gases, the limiting current and pressure are almost independent of the tube radius and connected by

$$pI_L = C \qquad\qquad\qquad\qquad \text{(III-1)}$$

where C is a constant determined for each gas.

The data of Figure 11, correspond quantitatively with this empirical relationship even though the data are for a two-gas system. Combined with the data at onset of striations as a function of pressure and current, Figure 11 gives design parameters for use in a discharge tube that does not exhibit striations.

It was found that a relation similar to Equation (III-1) described the lower limits of striations.
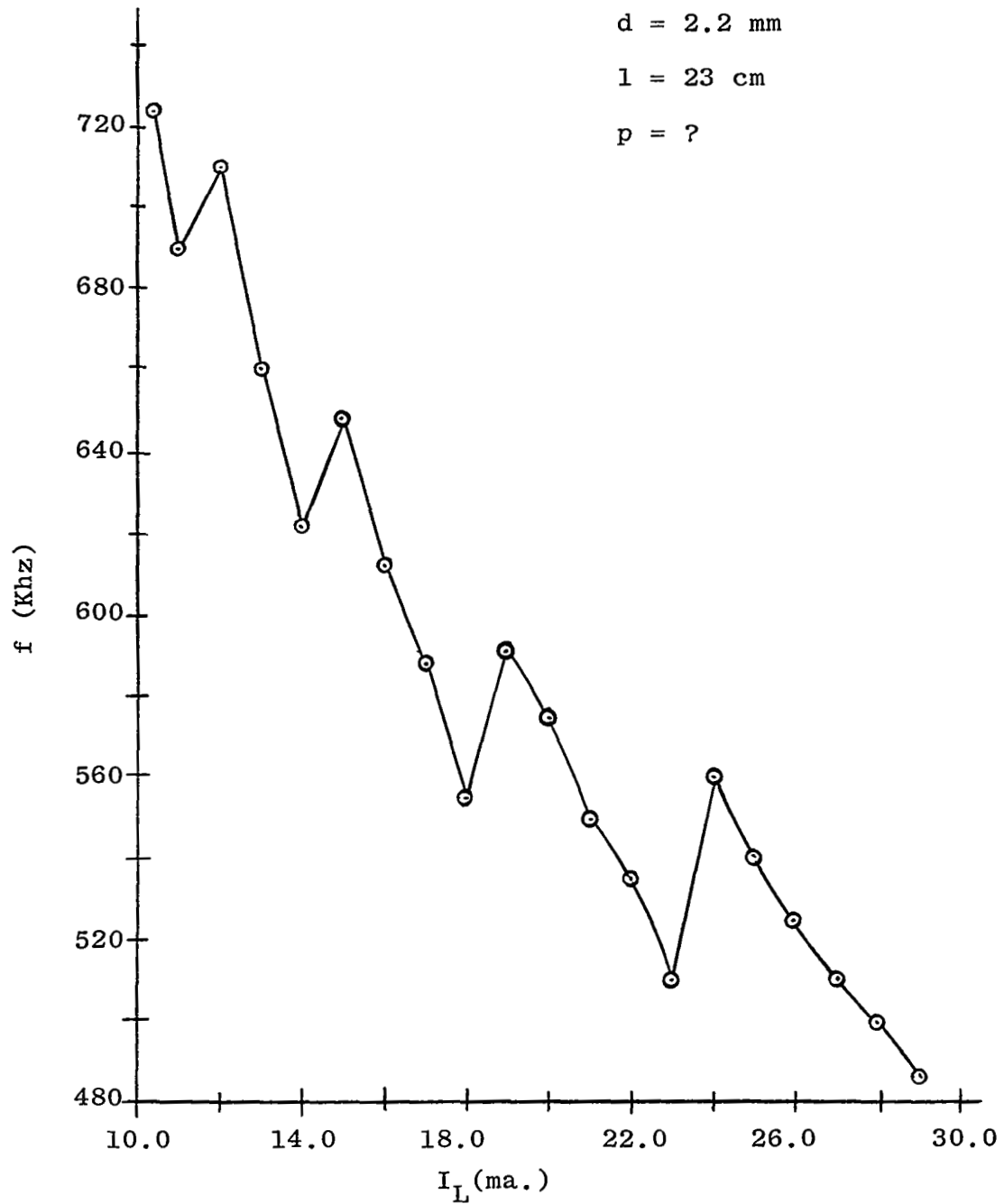
Figure 10. Frequency of Striations Versus
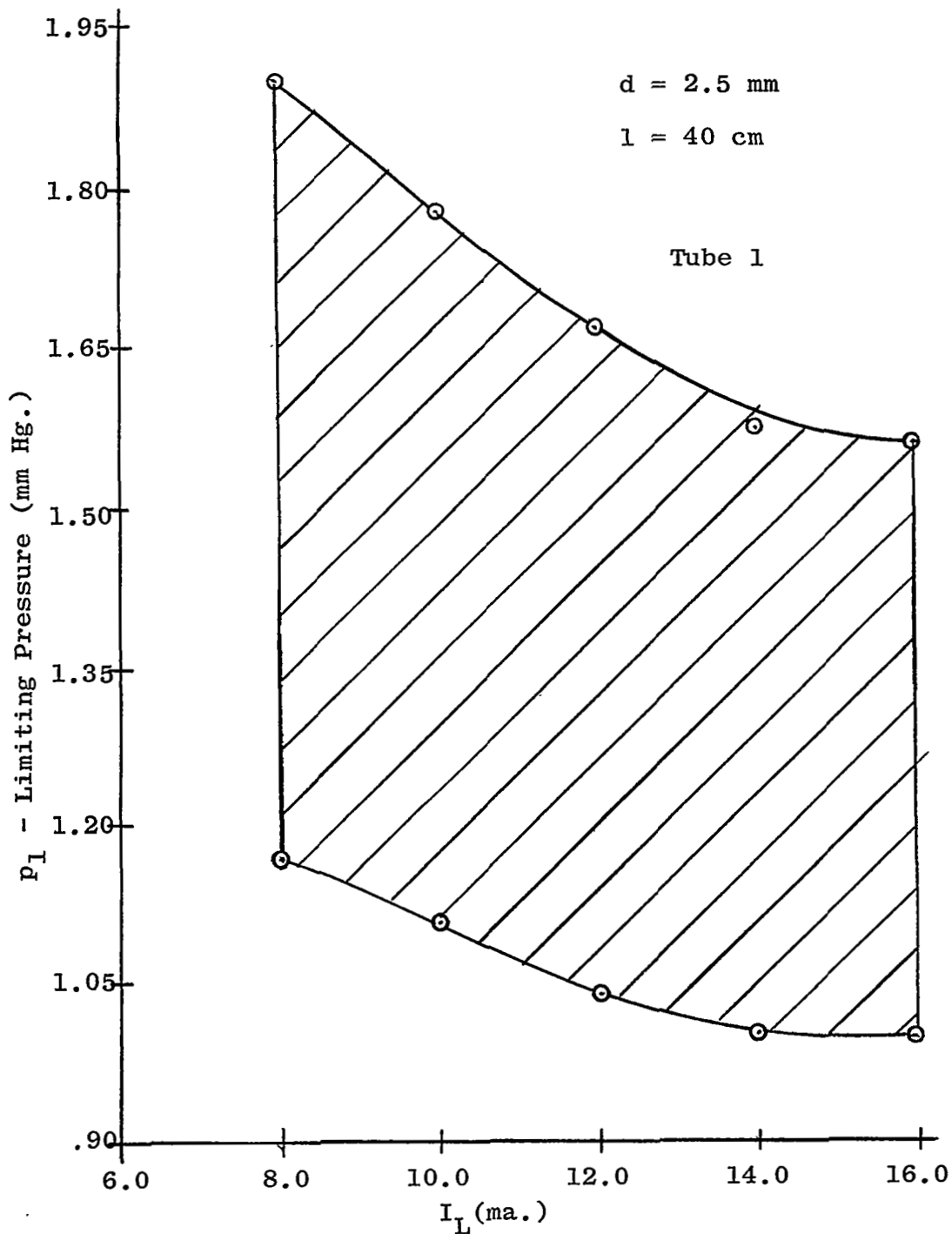Current for Spectra-Physics Discharge Tube.

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Figure 11.   Design Parameters for Stable
Discharge for Tube 1.

The plasma tube radius dependence is given in Figure 12. The two 2.5 mm tubes were nominally of the same diameter but exhibited a slightly different frequency for identical discharge conditions. An error in the measurement at the actual diameter could explain the discrepancy.

Similar data to these have not been reported, but one unpublished report (29) shows an erratic dependence of striation phase velocity versus radius. The cause for the erratic data was probably an insufficient (dirty) vacuum system.
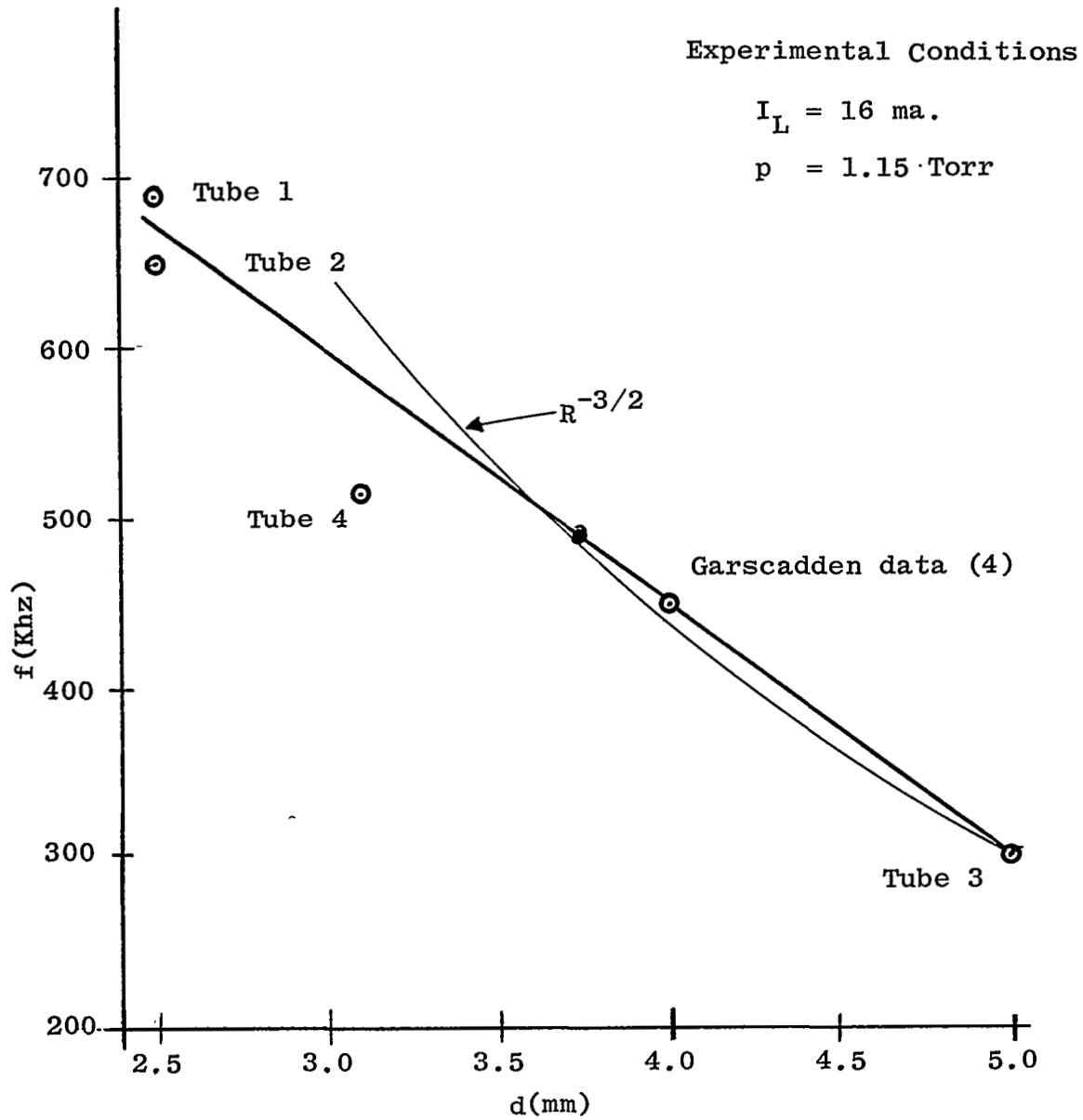
Figure 12. Frequency of Striations Versus
Diameter of Discharge Tube.

CHAPTER IV

CONCLUSION AND DISCUSSION

The frequency dependence of striations in He-
Ne lasers on plasma parameters was developed. The experi-
mental results in Chapter III show qualitative agreement
for the He-Ne plasma laser.

The data of Figure 7, page 39, and Figure 8,
page 40, show that the pressure dependence of striations
can be reasonably predicted in accordance with $p^{-\frac{1}{2}}$
relationship of Equation (II-2), page 7, and Equation
(II-78), page 8. The agreement of the data of Figure 12,
page 47, with the $R^{-3/2}$ dependence of the theoretical
analysis is considered to be reasonable.

Experimentally, this research has also shown
data relating frequency of striations to discharge current.
No theoretical analysis is available to explain this
phenomenon.

The three principle variables that affect
striation frequency - pressure, radius, and current -
have been investigated to the point where it is now
possible to experimentally and/or theoretically predict
their dependences. A thorough theoretical investigation
of the current dependence would be an instructive and
useful next step since, finally, it is desirable to

predict all three dependences for the He-Ne discharge. Further investigation of the role of the metastables would be a reasonable starting point.

Examination of Figure 12, page 47, and Figure 6, page 37, reveals information that tubes with diameters less than 2 mm would effectively eliminate the problem of striations in He-Ne lasers. Small tubes exhibit higher striation frequencies which less effectively modulate light output 6328Å than lower frequencies.

A further consequence of this research is that it is now possible to construct a diagram such as Figure 11, page 45, which gives design conditions for a "quiet" tube by properly adjusting pressure and current.

Any theory dealing with random fluctuations or noise in a He-Ne laser must take into consideration the theory and data given in this research. Figure 4, page 34, and other data compiled during the course of this research reveal that the magnitude of the striations is of the same order as the wide-band noise, moreover, under most discharge conditions, the striation is the stronger perturbation.

It is suggested that future experimental investigations should be conducted with a multielement tube. Not only would this facilitate collection data, but it would also ensure greater reliability between tubes since contaminants would be present to the same degree in each tube.

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