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DRC

N69-32375
(ACCESSION NUMBER)
38
(PAGES)
CR-10223
(NASA CR OR TMX OR AD NUMBER)
(THRU)
1
(CODE)
19
(CATEGORY)

FACILITY FORM 602

38 Montvale Avenue
Stoneham, Massachusetts

E-1453U

MATH MODELS FOR NASA/HUNTSVILLE STRAPDOWN SYSTEM TESTS

May 27, 1968

Prepared for

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama

Generated under

Contract No. NAS8-21366

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1. INTRODUCTION

The purpose of this report is to derive math models that characterize the SD-53 Strapdown System during laboratory system tests. The tests are designed to measure primarily the misalignments between the strapdown system optical cube reference axes and the input axes of the three Single-Axis-Platforms (SAP) and three Pendulous Integrating Gyro Accelerometers (PIGAs). The tests contemplated also provide estimates of bias and scale factor error in each SAP and PIGA, and for each of the SAP gyros, the eight g-sensitive drift rates and the two internal misalignment components between gyro IA and platform yoke axis.

Preliminary test considerations are contained in Ref. 5.1, including development of the form of the math models. As mentioned in Ref. 5.1, two series of tests are proposed (with and without rotation of the test table), for a minimum of three orientations of the strapdown system with respect to the test table. This report extends the analysis to include all contemplated test positions, as well as various yoke angles (head positions) of the SAP's whose input axes are nominally perpendicular to the test table axis of rotation. Consideration of required optical measurements and test stand alignment errors is also included. However, specific test sequences, test table and head positions, error analyses, and other test conditions are not included. Subsequent reports will cover those areas.

The coordinate systems, definitions of symbols, etc. defined in Appendix A are compatible with those in Ref. 5.1.

2. GENERAL DEVELOPMENT OF MATH MODELS

2.1 TEST CONFIGURATION AND PHILOSOPHY

The functional relationships of the system test configuration are shown in Fig. 2-1.* The test table base, defined by the test stand elevation axis and the table rotational axis, is oriented with respect to the local geographical coordinate system (East and vertical) by the azimuth and elevation angles, A and E, respectively. It is assumed that errors in this alignment can be made "sufficiently small" to justify the neglecting of second order terms. The test stand is capable of driving the test table at a precise rate $(\dot{\alpha})$ through an angle α.

The strapdown system is mounted to the table and tested in each of three positions, as indicated in Fig. 2-3. Outputs of the system consist of yoke angle readings (β) from each of the three SAPs and three PIGAs. Tests associated with the SAPs are conducted with the test table driven at a precise, constant rate. The table is not rotated during the PIGA tests.

The outputs of the instruments (i. e., SAPs and PIGAs) contain misalignment information only relative to the test stand elevation and rotational axes since the test conditions are changed relative to these axes. However, it is required to determine the misalignments relative to the strapdown system reference axes (as defined by the optical cube). Therefore, the relationship between the test stand coordinate systems and the strapdown system optical cube must be measured accurately for each mounting position of the strapdown system.

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*For reader convenience, Chapter 2 Figures and Tables are located at end of chapter.

Certain misalignments of the test stand coordinate systems contribute to errors in estimating the required calibration terms. These error sources must either be made negligible or else separated from the desired terms by obtaining additional measurements, either as part of the system tests or separately. The analysis in this report is sufficiently flexible to handle any of these alternatives.

The test philosophy is to obtain sufficient test measurements to allow accurate estimation of the required calibration terms. To do this, multiple measurements in various test configurations are necessary.

2.2 COORDINATE TRANSFORMATIONS

The relationships between the various coordinate systems are shown graphically in Figs. 2-1 and 2-2. The mathematical relations derived in this section are used to express the inertial rates and accelerations at the inputs to both the SAP and PIGA instruments, for the test conditions contemplated. Development of the equations for the specific applications, considering the dynamics of each instrument, are contained in Sections 3 and 4. All errors are assumed to be sufficiently small to justify the neglecting of second order terms.

The sequence of coordinate transformations is indicated in Fig. 2-2. Considering the misalignments, (η_s and ν_s) in aligning the test stand elevation axis (y_s) from east and vertical, the relationship between the test stand base and the geographical coordinate system is as follows:

$$\begin{bmatrix} x'_s \\ y'_s \\ z'_s \end{bmatrix} = \begin{bmatrix} 1 & \eta_s & 0 \\ -\eta_s & 1 & \nu_s \\ 0 & -\nu_s & 1 \end{bmatrix} \begin{bmatrix} \cos A & \sin A & 0 \\ -\sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} N \\ E \\ V \end{bmatrix} \qquad (2\text{-}1)$$

The relationship between the test table base and test stand base, including the misalignment (ν_T, ψ_T) of the table axis of rotation relative to the test table base is as follows:

$$\begin{bmatrix} x'_b \\ y'_b \\ z'_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\psi_T \\ 0 & 1 & \nu_T \\ \psi_T & -\nu_T & 1 \end{bmatrix} \begin{bmatrix} \cos E & 0 & -\sin E \\ 0 & 1 & 0 \\ \sin E & 0 & \cos E \end{bmatrix} \begin{bmatrix} x'_s \\ y'_s \\ z'_s \end{bmatrix} \qquad (2\text{-}2)$$

The quantity ψ_T is in essence the misalignment associated with the desired elevation angle E.

Next, a set of rotating axes (x_T, y_T, z_T) fixed to the test stand table is defined by

$$\begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} = \begin{bmatrix} \cos(\alpha + \eta_o) & \sin(\alpha + \eta_o) & 0 \\ -\sin(\alpha + \eta_o) & \cos(\alpha + \eta_o) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'_s \\ y'_s \\ z'_s \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K_T\dot{\alpha} \end{bmatrix}$$

(2-3)

The last term in Eq. (2-3) will be used subsequently when angular rate and acceleration vectors along the coordinate axes are considered. The term η_o represents the inaccuracy in reading out the table angle (α). It will be assumed that the test stand errors are either measured separately from the strapdown system tests and corresponding compensations applied, or that the errors will be estimated as part of the system test program and used to justify the assumption to be made that the errors are sufficiently small to neglect second order terms.

The misalignments (ν_o and ψ_o) between the strapdown system optical cube and the table axis, about the x_T and y_T axes respectively, are used to define the coordinate system of the optical surfaces, as follows:

$$\begin{bmatrix} x'_T \\ y'_T \\ z'_T \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\psi_o \\ 0 & 1 & \nu_o \\ \psi_o & -\nu_o & 1 \end{bmatrix} \begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} \qquad (2-4)$$

To relate the $\{x'_T, y'_T, z'_T\}$ axes to fixed axes on the optical cube, it is necessary to define a resolution matrix (R_1) since three orientations of the strapdown system relative to the test stand table axes $\{x_T, y_T, z_T\}$ are contemplated. Hence,

$$\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} = R_1 \begin{bmatrix} x'_T \\ y'_T \\ z'_T \end{bmatrix} \qquad (2-5)$$

Defining mounting position #1 such that the optical cube axes are identical to the $\{x'_T, y'_T, z'_T\}$ coordinate system, the three mounting positions are defined by matrix R_1 as follows:

Mounting Position #	1	2	3
Matrix R_1	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

(2-6)

Next, the relationships between the bases of the three SAPs and PIGAs are defined in terms of the optical cube by resolution matrix R_2, as follows:

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = R_2 \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} \quad (2\text{-}7)$$

The subscript r is used to designate a particular instrument and mounting position.

Choosing the reference axes of SAP #1 and PIGA #1 to be identical to the optical cube, matrix R_2 is equal to the following:

SAP #	1	2	3
Matrix R_2	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(2-8)

The above relationships are summarized in Fig. 2-3, including the various $R_2 R_1$ matrix products.

Having defined reference axes for each instrument (viz., SAPs and PIGAs), the effect of instrument yoke axis misalignments (ν_r, ψ_r) are defined as follows:

$$\begin{bmatrix} x'_r \\ y'_r \\ z'_r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\psi_r \\ 0 & 1 & \nu_r \\ \psi_r & -\nu_r & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} \qquad (2\text{-}9)$$

The final coordinate transformation is used to define axes on the rotating instrument yoke, to which the inertial reference units (gyros) are referenced. Hence,

$$\begin{bmatrix} x_{Yr} \\ y_{Yr} \\ z_{Yr} \end{bmatrix} = \begin{bmatrix} \cos(\beta_r + \eta_r) & \sin(\beta_r + \eta_r) & 0 \\ -\sin(\beta_r + \eta_r) & \cos(\beta_r + \eta_r) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'_r \\ y'_r \\ z'_r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K_T\dot{\beta}_r \end{bmatrix}$$

(2-10)

The last term in Eq. (2-10) will be used subsequently when angular rate and acceleration vectors along the coordinate axes are considered. The term η_r represents the inaccuracy in reading out the yoke angle (β).

2.3 INSTRUMENT YOKE RATES AND ACCELERATIONS

Equations (2-1) through (2-10) can be combined into three groups. They are the transformations from (a) geographical coordinates $\{N, E, V\}$ to test stand table axes $\{x'_T, y'_T, z'_T\}$, (b) table axes to Instrument Base $\{x_r, y_r, z_r\}$ and (c) Instrument Base to axes on the rotated yoke $\{x_{Yr}, y_{Yr}, z_{Yr}\}$.

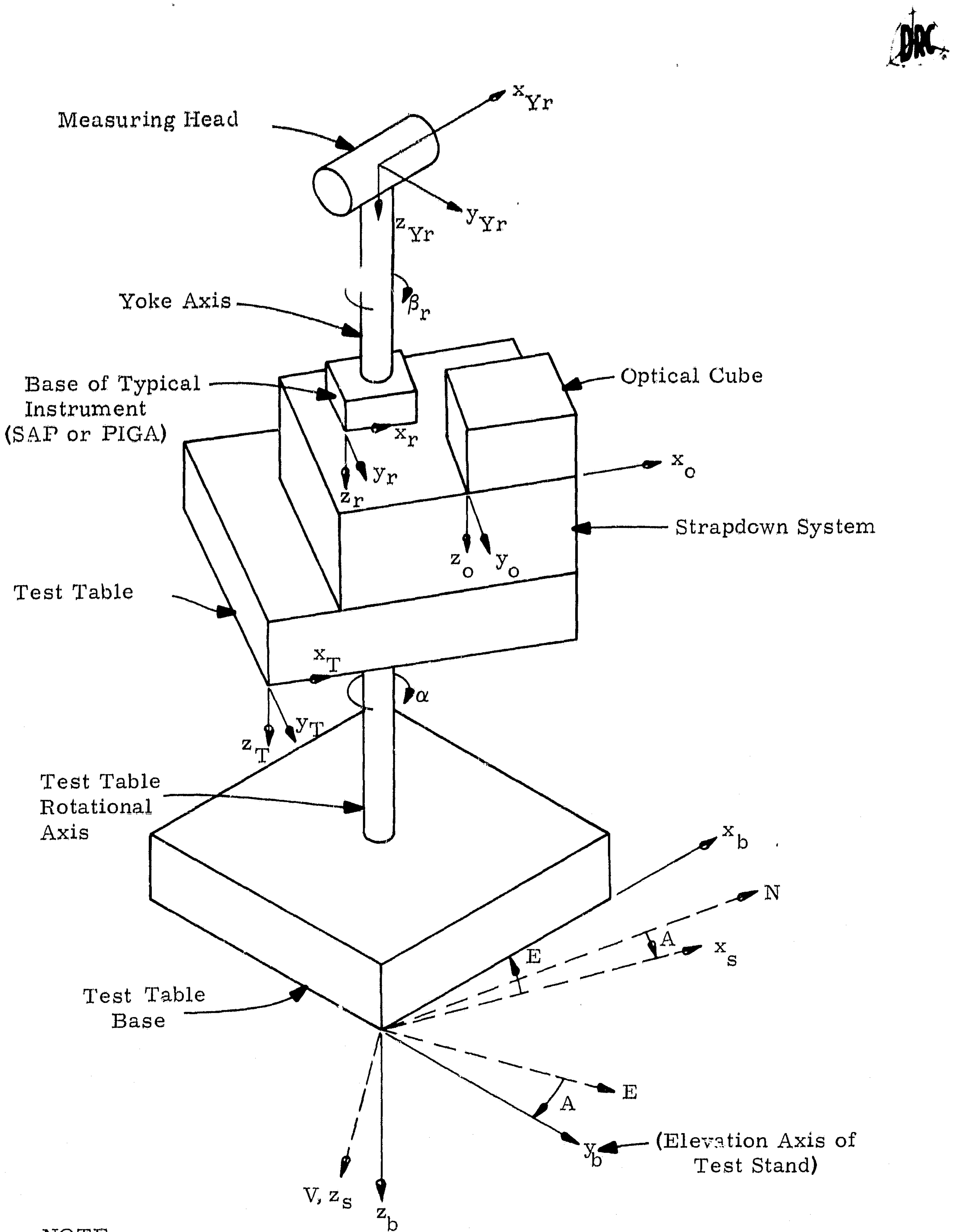
The respective equations are: (a) (2-1) through (2-4), (b) (2-5), (2-7) and (c) (2-9), (2-10).

Since angular rate and linear acceleration vectors acting on the instruments are ultimately desired, the matrices will be combined for rates and accelerations separately, using the following inputs:

$$\begin{bmatrix} N \\ E \\ V \end{bmatrix}_{rates} = \begin{bmatrix} \omega_e \cos L \\ 0 \\ -\omega_e \sin L \end{bmatrix} \text{ for rates} \quad \text{and} \quad \begin{bmatrix} N \\ E \\ V \end{bmatrix}_{accel} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \text{ for accelerations} \qquad (2\text{-}11)$$

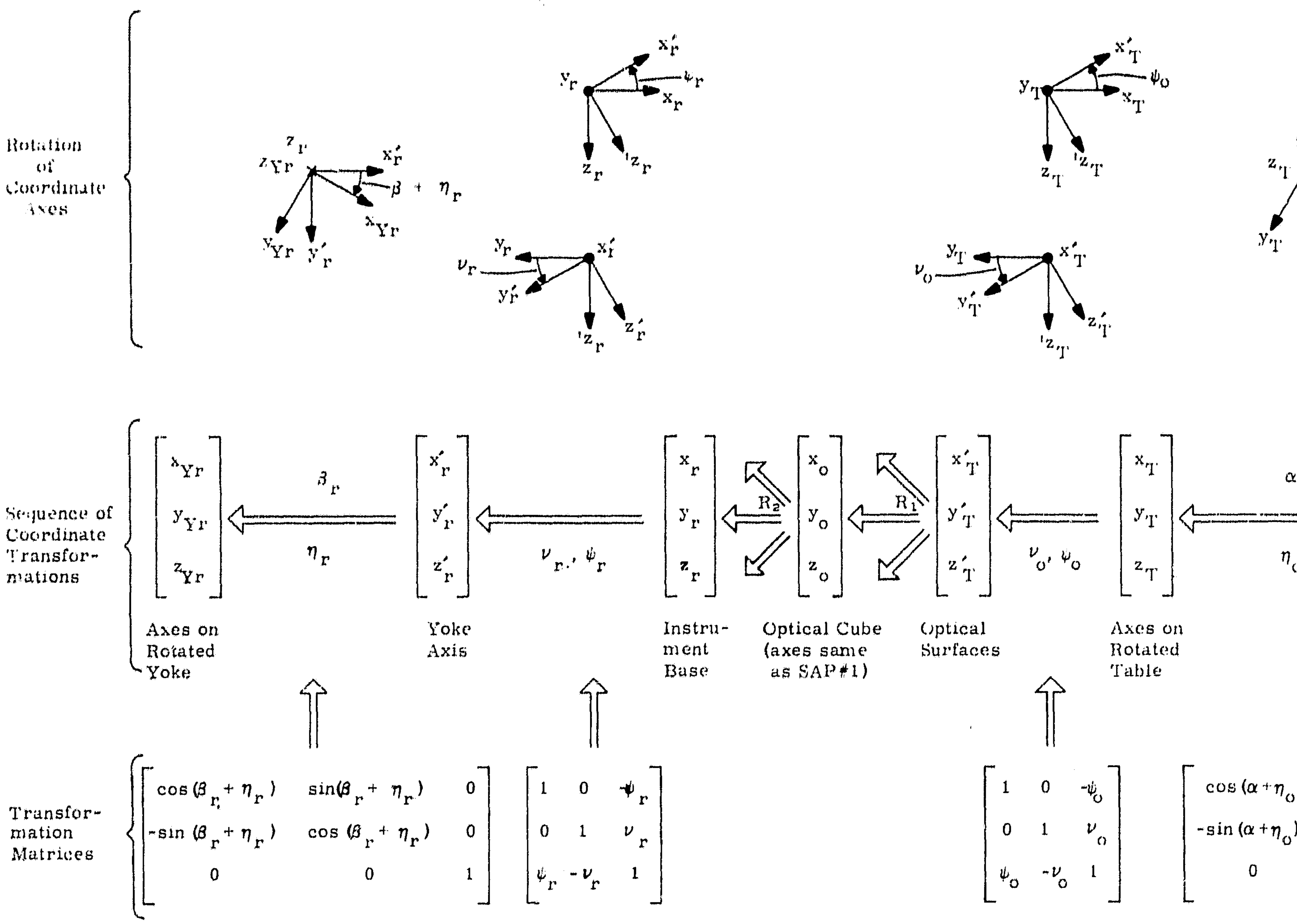
In Eqs. (2-3) and (2-10), the constants K_T and K_Y are set equal to unity when rates are considered and to zero for accelerations.

The results are summarized in Table 2-1. Although second order terms were neglected in combining the various matrices, no terms were eliminated that may ultimately yield second order effects when the specific SAP and PIGA configurations are considered. The test stand azimuth angle (A) is assumed to be equal to zero so that the test table rotational axis may be oriented anywhere in the plane defined by local vertical and the earth's rotational axis.



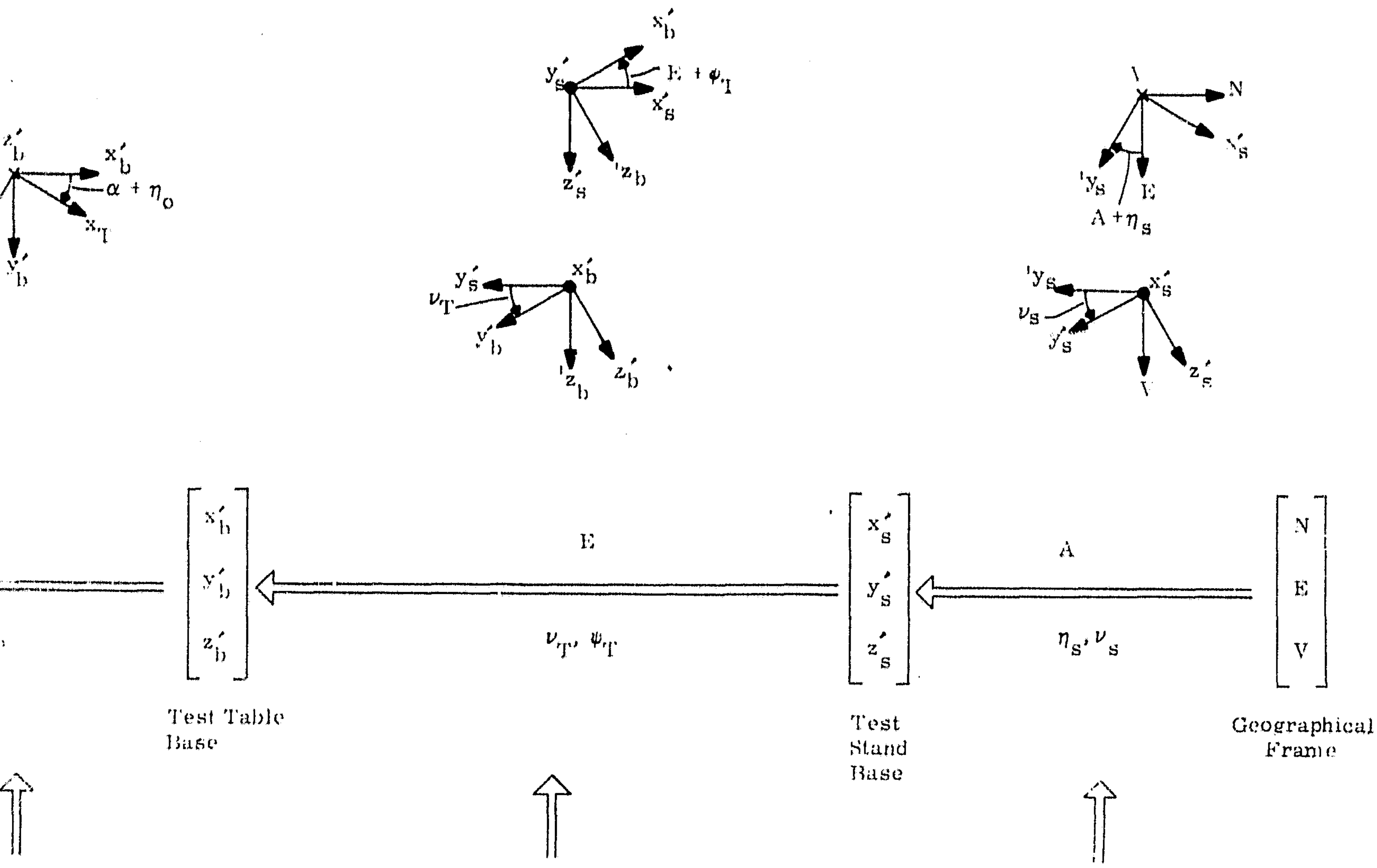
NOTE:
Misalignments of the coordinate systems are not shown in this figure.

Figure 2-1 Functional Relationships of Test Configuration, Showing Major Coordinate Systems



NOTES: (1) The $R_2 R_1$ matrix products are summarized in Fig. 2-3.
(2) A = 0°.

FOLDOUT FRAME 1



$$\begin{matrix} \sin(\alpha+\eta_o) & 0 \\ \cos(\alpha+\eta_o) & 0 \\ 0 & 1 \end{matrix}\Bigg]$$

$$\begin{bmatrix} (\cos E - \psi_T \sin E) & 0 & -(\sin E + \psi_T \cos E) \\ \nu_T \sin E & 1 & \nu_T \cos E \\ (\sin E + \psi_T \cos E) & -\nu_T & (\cos E - \psi_T \sin E) \end{bmatrix}$$

$$\begin{bmatrix} 1 & \eta_s & 0 \\ -\eta_s & 1 & \nu_s \\ 0 & -\nu_s & 1 \end{bmatrix}$$

Figure 2-2

Summary of Coordinate Transformations

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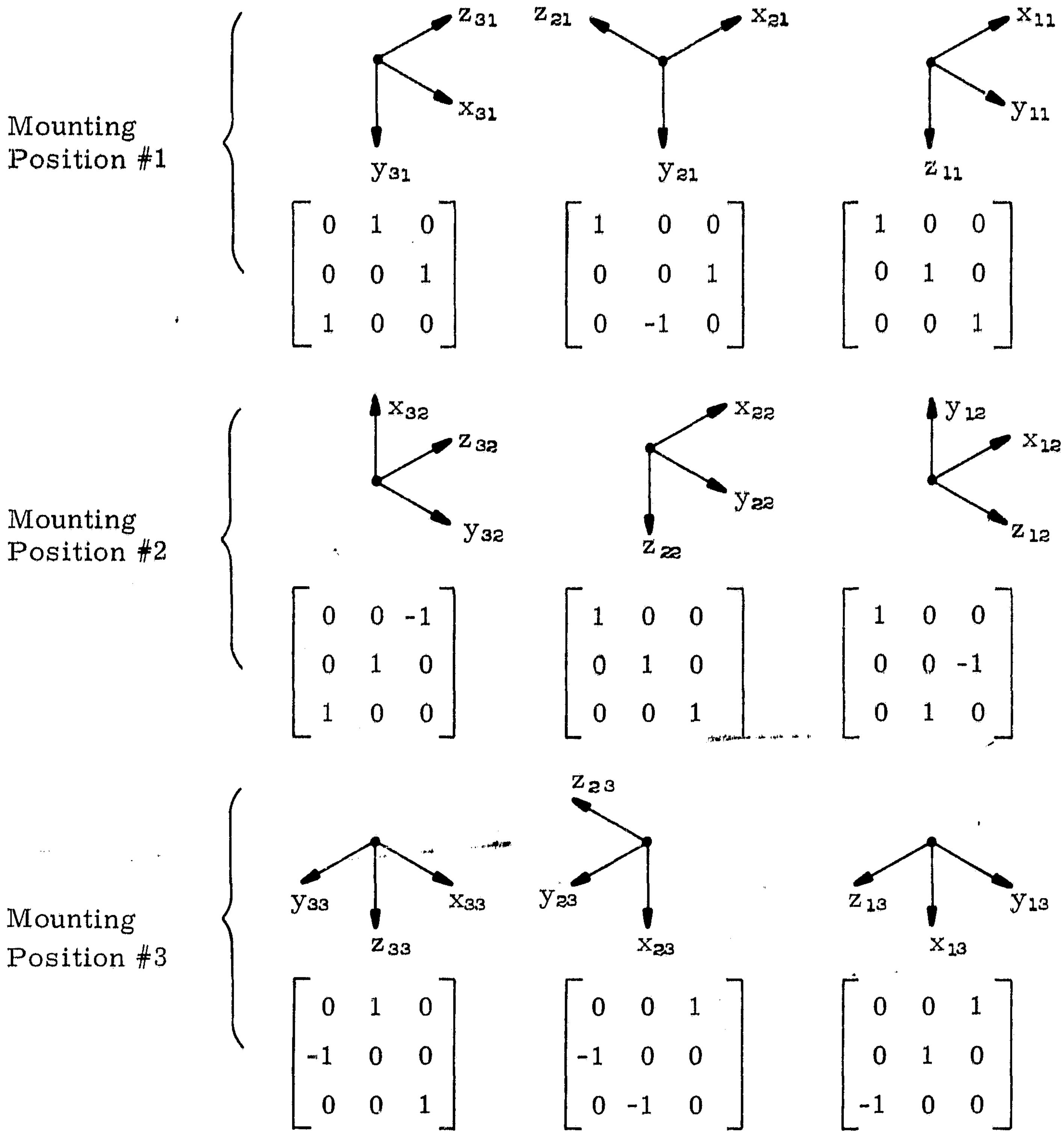

	Instrument #3	Instrument #2	Instrument #1
Mounting Position #1	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Mounting Position #2	$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$
Mounting Position #3	$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

NOTES: (1) Matrices shown are equal to $R_2 R_1$.
(2) Instrument #1 reference axes are chosen to be identical to optical cube axes.
(3) Optical cube axes are nominally same as table axes in position #1.

Figure 2-3 Instrument Reference Axes and Associated Resolution Matrices

Angular Rates

$$\begin{bmatrix} \omega'_{xT} \\ \omega'_{yT} \\ \omega'_{zT} \end{bmatrix} = \omega_e \begin{bmatrix} -\psi_0\left[\frac{\dot{\alpha}}{\omega_e} - \sin(L - E)\right] + (\cos\alpha - \eta_0 \sin\alpha)\cos(L - E) + \delta_{10} \\ +\nu_0\left[\frac{\dot{\alpha}}{\omega_e} - \sin(L - E)\right] - (\sin\alpha + \eta_0 \cos\alpha)\cos(L - E) + \delta_{11} \\ +\left[\frac{\dot{\alpha}}{\omega_e} - \sin(L - E)\right] + (\nu_0 \sin\alpha + \psi_0 \cos\alpha)\cos(L - E) + \delta_{12} \end{bmatrix} \tag{2-12}$$

$$\begin{bmatrix} \omega_{xr} \\ \omega_{yr} \\ \omega_{zr} \end{bmatrix} = R_2 R_1 \begin{bmatrix} \omega'_{xT} \\ \omega'_{yT} \\ \omega'_{zT} \end{bmatrix} \tag{2-13}$$

$$\begin{bmatrix} \omega_{x_{Yr}} \\ \omega_{y_{Yr}} \\ \omega_{z_{Yr}} \end{bmatrix} = \begin{bmatrix} \cos\beta_r(\omega_{xr} - \psi_r\omega_{zr} + \eta_r\omega_{yr}) + \sin\beta_r(\omega_{yr} + \nu_r\omega_{zr} - \eta_r\omega_{xr}) \\ -\sin\beta_r(\omega_{xr} - \psi_r\omega_{zr} + \eta_r\omega_{yr}) + \cos\beta_r(\omega_{yr} + \nu_r\omega_{zr} - \eta_r\omega_{xr}) \\ \psi_r\omega_{xr} - \nu_r\omega_{yr} + \omega_{zr} + \dot{\beta}_r \end{bmatrix} \tag{2-14}$$

where

$$\delta_{10} \triangleq -\epsilon_s + [\nu_T \sin(L - E)]\sin\alpha + [\psi_T \sin(L - E)]\cos\alpha \tag{2-15}$$

$$\delta_{11} \triangleq -\epsilon_s + [\nu_T \sin(L - E)]\cos\alpha - [\psi_T \sin(L - E)]\sin\alpha \tag{2-16}$$

$$\delta_{12} \triangleq \psi_T \cos(L - E) \tag{2-17}$$

$$\epsilon_s \triangleq \eta_s \cos L + \nu_s \sin L \tag{2-18}$$

NOTES: (1) The R_2 R_1 matrix produc

(2) Second order terms have b

(3) $A = 0°$

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Linear Accelerations

$$\begin{bmatrix} g'_{xT} \\ g'_{yT} \\ g'_{zT} \end{bmatrix} = g \begin{bmatrix} -\sin E \cos\alpha + [\delta_8 + \dot{\eta}_o \sin E \sin\alpha - \psi_o \cos E] \\ \sin E \sin\alpha + [\delta_9 + \eta_o \sin E \cos\alpha + \nu_o \cos E] \\ \cos E + [\delta_7 - \sin E(\psi_o \cos\alpha + \nu_o \sin\alpha)] \end{bmatrix} \quad (2\text{-}19)$$

$$\begin{bmatrix} g_{xr} \\ g_{yr} \\ g_{zr} \end{bmatrix} = R_2 R_1 \begin{bmatrix} g'_{xT} \\ g'_{yT} \\ g'_{zT} \end{bmatrix} \quad (2\text{-}20)$$

$$\begin{bmatrix} g_{x_{Yr}} \\ g_{y_{Yr}} \\ g_{z_{Yr}} \end{bmatrix} = \begin{bmatrix} \cos\beta_r(g_{xr} - \psi_r g_{zr} + \eta_r g_{zr}) + \sin\beta_r(g_{yr} + \nu_r g_{zr} - \eta_r g_{xr}) \\ -\sin\beta_r(g_{xr} - \psi_r g_{zr} + \eta_r g_{zr}) + \cos\beta_r(g_{yr} + \nu_r g_{zr} - \eta_r g_{xr}) \\ \psi_r g_{xr} - \nu_r g_{yr} + g_{zr} \end{bmatrix} \quad (2\text{-}21)$$

where $\delta_7 \triangleq -\psi_T \sin E$ (2-22)

$\delta_8 \triangleq (\nu_T \sin\alpha - \psi_T \cos\alpha)\cos E + \nu_s \sin\alpha$ (2-23)

$\delta_9 \triangleq (\psi_T \sin\alpha + \nu_T \cos\alpha)\cos E + \nu_s \cos\alpha$ (2-24)

products are summarized in Fig. 2-3.

have been neglected.

Table 2-1

Summary of Rates and Accelerations within Test Configurations

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3. SAP MATH MODELS

3.1 GENERAL FORM OF SAP EQUATIONS OF MOTION

The gyro (measuring head) equations of motion derived in Ref. 5. 2 (Eq. 2-18b) is simplified considerably when the present SAP test conditions are considered. Justification for omitting certain terms is contained in Ref. 5. 2 and additional considerations will be included in the error analysis report on the strapdown system test program. The resulting SAP equation is given by Eq.(1) in Ref. 5. 3 (repeated as Eq. (3-1) below) and is based on the summation of torques on the gyro gimbal/rotor cylinder about its OA being equal to zero. Since the SAP is intended to measure rates relative to inertial space, the gyro torque generator signal (ω_{tg}) is zero and the gyro signal generator output is used to drive the SAP yoke such that the inertial rate of the yoke (ω_z) acting about the input axis of the gyro is nominally zero. Considering internal gyro errors, Eq.(1) of Ref. 5. 3 is as follows:

$$\omega_z + x_1 (g_{x_{Yr}}/g)(g_{y_{Yr}}/g) + x_2 (g_{x_{Yr}}/g)(g_{z_{Yr}}/g) + x_3 (g_{y_{Yr}}/g)^2$$

$$+ x_4 (g_{y_{Yr}}/g)(g_{z_{Yr}}/g) + x_5 (g_{z_{Yr}}/g)^2 + x_6 (g_{x_{Yr}}/g) + x_7 (g_{y_{Yr}}/g)$$

$$+ x_8 (g_{z_{Yr}}/g) + x_9 (\omega_{x_{Yr}}/\omega_e) + x_{10} (\omega_{y_{Yr}}/\omega_e) + x_{11} = 0 \qquad (3\text{-}1)$$

The term ω_z above is equal to $\omega_{z_{Yr}}$ and the internal gyro misalignments θ_x and θ_y contained in x_9 and x_{10} are reinterpreted for the SAP to be referenced to the yoke axis (see Appendix A of Ref. 5. 1 and the definitions of θ_x and θ_y in this report.) Equation (3-1) can be expressed in the

SAP base coordinate system, using the appropriate equations of Table 2-1, as follows:

$$
\begin{aligned}
-\dot{\beta}_r = \; & \omega_{zr} + \psi_r \omega_{xr} - \nu_r \omega_{yr} \\
& + \frac{x_1}{2} \left[-\left(g^2_{xr}/g^2 + g^2_{yr}/g^2\right) \sin 2\beta_r + 2\left(g_{xr} g_{yr}/g^2\right) \cos 2\beta_r \right] \\
& + \left[x_2 \left(g_{zr}/g\right) + x_8 \right] \left[\left(g_{xr}/g\right) \cos \beta_r + \left(g_{yr}/g\right) \sin \beta_r \right] \\
& - \left[x_4 \left(g_{zr}/g\right) + x_7 \right] \left[\left(g_{xr}/g\right) \sin \beta_r - \left(g_{yr}/g\right) \cos \beta_r \right] \\
& + \frac{x_3}{2} \left[\left(g^2_{xr}/g^2 + g^2_{yr}/g^2\right) - \left(g^2_{xr}/g^2 - g^2_{yr}/g^2\right) \cos 2\beta_r \right. \\
& \qquad \left. - 2\left(g_{xr} g_{yr}/g^2\right) \sin 2\beta_r \right] \\
& + x_5 \left(g_{zr}/g\right)^2 + x_3 \left(g_{zr}/g\right) + x_{11} \\
& + x_9 \left[\left(\omega_{xr}/\omega_e\right) \cos \beta_r + \left(\omega_{yr}/\omega_e\right) \sin \beta_r \right] \\
& + x_{10} \left[\left(\omega_{yr}/\omega_e\right) \cos \beta_r - \left(\omega_{xr}/\omega_e\right) \sin \beta_r \right] \qquad (3\text{-}2)
\end{aligned}
$$

Note that the term η_r occurs only as products with other small quantities and therefore has been neglected.

Expressions for the ω and g terms are contained in Table 2-1 (Eqs. 2-12, 2-13, 2-19 and 2-20). Since the g terms are in every case multiplied by a small quantity (the x's), Eq. (2-19) can be simplified:

$$\begin{bmatrix} g'_{xT} \\ g'_{yT} \\ g'_{zT} \end{bmatrix} = g \begin{bmatrix} -\sin E \cos\alpha \\ \sin E \sin\alpha \\ \cos E \end{bmatrix} \qquad (3\text{-}3)$$

Applying Eq. (3-2) to each SAP in each of the three mounting positions, the resulting equations are determined to be functions of the dependent variable $\dot{\beta}$ and trigonometric functions of α, β and $(\alpha+\beta)$. There are two forms of the equations, corresponding to SAP input axes parallel and normal to the test table axis. Each is considered in the following sections in order to solve the transcendental, differential equations in β.

3.2 SAP IA PARALLEL TO TABLE AXIS

Whenever a SAP input axis (z) is nominally parallel to the test table axis, $\dot{\beta}$ is a function of the independent variables, $\dot{\alpha}$, α and $(\alpha+\beta)$. Letting $\dot{\delta}$ represent the rate error equal to the various misalignment and x terms, the resulting equations are of the form:

$$-\dot{\beta} = \dot{\alpha} - \omega_e \sin(L - E) + \dot{\delta}$$
$$+ x_k f[\sin(\alpha+\beta),\ \cos(\alpha+\beta),\ \sin 2(\alpha+\beta),\ \cos 2(\alpha+\beta)] \qquad (3\text{-}4)$$

Integrating Eq. (3-4) from $t = 0$ to $t = t$,

$$\Delta\alpha + \Delta\beta = (\omega_e t)\sin(L - E) - \Delta\delta - x_k \int_0^t f[\ ]dt$$

where Δ represents change in the corresponding quantity.

Rearranging terms,

$$(\alpha + \beta) = \gamma - \Delta\delta' \qquad (3\text{-}5)$$

where

$$\gamma = \gamma_0 + \omega_e t \sin(L - E)$$

$$\gamma_0 = \alpha_0 + \beta_0$$

$$\Delta\delta' = \Delta\delta + x_k \int_0^t f[\ ]dt$$

and α_0 and β_0 are the initial values of α and β. Since $\Delta\delta'$ is a small quantity, Eq. (3-5) can be substituted back into Eq. (3-4), with the following approximation:

$$\sin(\alpha + \beta) \cong \sin\gamma$$

$$\cos(\alpha + \beta) \cong \cos\gamma$$

$$\sin 2(\alpha + \beta) \cong \sin 2\gamma$$

$$\cos 2(\alpha + \beta) \cong \cos 2\gamma \qquad (3\text{-}6)$$

Note that the $(\Delta\delta')x_k$ products have been neglected. The final form of the SAP math model, for the SAP IA parallel to the table axis, is as follows:

$$\left[\frac{\dot{\beta}_r}{S^{\circ}_{sr}} + P_r\right] = -L_r - B_r \sin\gamma - C_r \cos\gamma - D_r \sin 2\gamma - E_r \cos 2\gamma$$

$$- [Q_{dr} \sin\alpha + Q_{fr} \cos\alpha]\,\omega_e \cos(L - E) \qquad (3\text{-}7)$$

where

$$P_r \triangleq P_{ar}[\dot{\alpha}_r - \omega_e \sin(L - E)] + [P_{br}\sin\alpha + P_{cr}\cos\alpha]\omega_e \cos(L - E)$$

$$-I_r \triangleq -A_r - Q_{ar}[\dot{\alpha}_r - \omega_e \sin(L - E)] - Q_{br}[\omega_e \cos(L - E)]$$

$$\gamma = \gamma_0 + \omega_e t \sin(L - E)$$

The quantities on the left side of the equation are measurable, whereas the constants A_r thru E_r, Q_{ar}, Q_{br}, and Q_{fr} on the right side are to be estimated from inputs of the independent variables $\dot{\alpha}$ and t. Table 3-1 contains a tabulation of the constants as a function of the various terms to be calibrated.

It will be noted that the test stand misalignment errors (η_S, ν_S, ν_T, ψ_T) are included in the Q terms. Although the test stand alignment may be sufficiently precise to make the terms negligible, they were incorporated in order to evaluate the possibility of allowing a less precise alignment without effecting the accuracy of estimating the desired calibration terms. If the misalignments are measured accurately, they can be transferred to the left hand side of Eq. (3-7) and incorporated with the P terms.

3.3 SAP IA NORMAL TO TABLE AXIS

The second form of the β equations occurs when the SAP input axis (z) is nominally normal to the table axis. Representing the rate error equal to the various misalignment and x terms by $\dot{\delta}$, the resulting equations are of the form:

$$-\dot{\beta} = \omega_e \cos(L - E)\sin\alpha + \dot{\delta} + x_k f[\sin\beta,\ \cos\beta,\ \sin 2\beta,\ \cos 2\beta] \quad (3\text{-}8)$$

Integrating Eq. (3-8) from $t = 0$ to $t = t$,

$$\Delta\beta = \frac{-\omega_e \cos(L - E)}{\dot{\alpha}} (1 - \cos\alpha) - \Delta\delta - x_k \int_0^t f[\ ]dt$$

where Δ represents change in the corresponding quantity.

Rearranging terms,

$$\beta = \beta_0 - \Delta\delta' \tag{3-9}$$

where

$$\Delta\delta' = \frac{\omega_e \cos(L - E)}{\dot{\alpha}} (1 - \cos\alpha) + \Delta\delta + x_k \int_0^t f[\ ]dt$$

and β_0 is the initial value of β. The quantity $\Delta\delta'$ will be small providing $\omega_e \cos(L - E)/\dot{\alpha}$ is small. Assuming this to be the case, Eq. (3-9) can be substituted back into Eq. (3-8) with the following approximations:

$$\begin{aligned} \sin\beta &\cong \sin\beta_0 \\ \cos\beta &\cong \cos\beta_0 \\ \sin 2\beta &\cong \sin 2\beta_0 \\ \cos 2\beta &\cong \cos 2\beta_0 \end{aligned} \tag{3-10}$$

Note that the $(\Delta\delta')x_k$ products have been neglected. The final form of the SAP math model, for the SAP IA normal to the table axis, is as follows:

$$\left[\frac{\dot{\beta}_r}{S^{\circ}_{sr}} + P_r \right] = -L_r - M_r \sin\alpha - N_r \cos\alpha - D_r \sin 2\alpha - E_r \cos 2\alpha \tag{3-11}$$

where

$$-M_r \triangleq -B_r - Q_{cr}[\omega_e] - Q_{dr}[\omega_e \cos(L - E)]$$

$$-N_r \triangleq -C_r - Q_{er}[\omega_e] - Q_{fr}[\omega_e \cos(L - E)] \qquad (3\text{-}11)$$

and P_r and L_r are given with Eq. (3-7). The quantities on the left side of the equation are measurable, whereas the constants A_r through E_r and Q_{ar} through Q_{fr} on the right side are to be estimated from inputs of the independent variables $\dot{\alpha}$ and t. Table 3-1 contains a tabulation of the constants as a function of the various terms to be calibrated.

The remarks at the end of Section 3.4 pertaining to the test stand misalignments also applies here.

SAP #	Coeff. X Constants		Values of Constants		
			Position #1	Position #2	Position #3
1	$[\dot{\alpha}_r - \omega_e \sin(L - E)]$	P_a	1	ν_{02}	ψ_{03}
	$\sin\alpha\,[\omega_e \cos(L - E)]$	P_b	ν_{01}	-1	η_{03}
	$\cos\alpha\,[\omega_e \cos(L - E)]$	P_c	ψ_{01}	$-\eta_{02}$	-1
	$[\dot{\alpha}_r - \omega_e \sin(L - E)]$	Q_a	$\delta S_{s1}/S^\circ_{s1}$	ν_1	ψ_1
	$[\omega_e \cos(L - E)]$	Q_b	ψ_{T1}	0	0
	$\sin\alpha\,[\omega_e]$	Q_c	-	$-\psi_{T2} \sin(L - E)$	$-\epsilon_s - \nu_{T3} \sin(L - E)$
	$\sin\alpha\,[\omega_e \cos(L - E)]$	Q_d	ν_1	$-\delta S_{s1}/S^\circ_{s1}$	ν_1
	$\cos\alpha\,[\omega_e]$	Q_e	-	$-\epsilon_s - \nu_{T2} \sin(L - E)$	$-\psi_{T3} \sin(L - E)$
	$\cos\alpha\,[\omega_e \cos(L - E)]$	Q_f	ψ_1	ψ_1	$-\delta S_{s1}/S^\circ_{s1}$
2	$[\dot{\alpha}_r - \omega_e \sin(L - E)]$	P_a	ν_{01}	1	$-\nu_{03}$
	$\sin\alpha\,[\omega_e \cos(L - E)]$	P_b	1	ν_{02}	1
	$\cos\alpha\,[\omega_e \cos(L - E)]$	P_c	η_{01}	ψ_{02}	η_{03}
	$[\dot{\alpha}_r - \omega_e \sin(L - E)]$	Q_a	$-\nu_2$	$\delta S_{s2}/S^\circ_{s2}$	ψ_2
	$[\omega_e \cos(L - E)]$	Q_b	0	ψ_{T2}	0
	$\sin\alpha\,[\omega_e]$	Q_c	$-\psi_{T1} \sin(L - E)$	-	$\psi_{T3} \sin(L - E)$
	$\sin\alpha\,[\omega_e \cos(L - E)]$	Q_d	$\delta S_{s2}/S^\circ_{s2}$	ν_2	$-\delta S_{s2}/S^\circ_{s2}$
	$\cos\alpha\,[\omega_e]$	Q_e	$\epsilon_s + \nu_{T1} \sin(L - E)$	-	$\epsilon_s + \nu_{T3} \sin(L - E)$
	$\cos\alpha\,[\omega_e \cos(L - E)]$	Q_f	ψ_2	ψ_2	ν_2
3	$[\dot{\alpha}_r - \omega_e \sin(L - E)]$	P_a	$-\psi_{01}$	$-\psi_{02}$	1
	$\sin\alpha\,[\omega_e \cos(L - E)]$	P_b	$-\eta_{01}$	$-\eta_{02}$	ν_{03}
	$\cos\alpha\,[\omega_e \cos(L - E)]$	P_c	1	1	ψ_{03}
	$[\dot{\alpha}_r - \omega_e \sin(L - E)]$	Q_a	$-\nu_3$	$-\psi_3$	$\delta S_{s3}/S^\circ_{s3}$
	$[\omega_e \cos(L - E)]$	Q_b	0	0	ψ_{T3}
	$\sin\alpha\,[\omega_e]$	Q_c	$-\epsilon_s - \nu_{T1} \sin(L - E)$	$-\epsilon_s - \nu_{T2} \sin(L - E)$	-
	$\sin\alpha\,[\omega_e \cos(L - E)]$	Q_d	$-\psi_3$	ν_3	ψ_3
	$\cos\alpha\,[\omega_e]$	Q_e	$\psi_{T1} \sin(L - E)$	$\psi_{T2} \sin(L - E)$	-
	$\cos\alpha\,[\omega_e \cos(L - E)]$	Q_f	$-\delta S_{s3}/S^\circ_{s3}$	$\delta S_{s3}/S^\circ_{s3}$	ν_3

For r = 11, 22, 33

$$[\frac{\dot{\beta}_r}{S^\circ_{sr}} + P_r] = -L_r - B_r \sin\gamma - C_r \cos\gamma - D_r \sin 2\gamma - E_r \cos 2\gamma$$
$$- Q_{dr}[\omega_e \cos(L - E)]\sin\alpha - Q_{fr}[\omega_e \cos(L - E)]\cos\alpha \quad (3\text{-}7)$$

For r = 21, 31, 12, 32, 13, 23

$$[\frac{\dot{\beta}_r}{S^\circ_{sr}} + P_r] = -L_r - M_r \sin\alpha - N_r \cos\alpha - D_r \sin 2\alpha - E_r \cos 2\alpha \quad (3\text{-}11)$$

$$P_r \triangleq P_{ar}[\dot{\alpha}_r - \omega_e \sin(L - E)] + [P_{br}\sin\alpha + P_{cr}\cos\alpha]\omega_e \cos(L - E)$$

$$-L_r \triangleq -A_r - Q_{ar}[\dot{\alpha}_r - \omega_e \sin(L - E)] - Q_{br}[\omega_e \cos(L - E)]$$

$$-M_r \triangleq -B_r - Q_{cr}[\omega_e] - Q_{dr}[\omega_e \cos(L - E)]$$

$$-N_r \triangleq -C_r - Q_{er}[\omega_e] - Q_{fr}[\omega_e \cos(L - E)]$$

$$\gamma \triangleq (\alpha_0 + \beta_0) + \omega_e t \sin(L - E)$$

$$\epsilon_s \triangleq \eta_s \cos L + \nu_s \sin L$$

NOTES:

(1) Subscripts on (η_0, ν_0, ψ_0) and (ν_T, ψ_T) refer to mounting position number.

(2) Subscripts on (ν, ψ, and $\delta S_s / S^\circ_s$) refer to SAP numbers.

Table 3-1

Tabulation of SAP Coefficients

SAP #	Constant	Position #1					Position #2			
		$\sin\beta_0$	$\cos\beta_0$	$\sin 2\beta_0$	$\cos 2\beta_0$	Unity	$\sin\beta_0$	$\cos\beta_0$	$\sin 2\beta_0$	(
1	A	-	-	-	-	$+\frac{x_3}{2}\sin^2 E + x_5\cos^2 E$ $+ x_8 \cos E + x_{11}$	$- x_6 \cos E$ $- x_9 R_\omega$	$- x_7 \cos E$ $- x_{10} R_\omega$	$+\frac{x_1}{8}(1 + 3\cos 2E)$	$+\frac{x_3}{8}($
	B	-	-	-	-	$+\frac{x_4}{2}\sin 2E + x_7 \sin E$	$- x_2 \sin 2E$	$- x_4 \sin 2E$	-	
	C	-	-	-	-	$-\frac{x_2}{2}\sin 2E - x_8 \sin E$	$+ x_7 \sin E$	$- x_6 \sin E$	$-\frac{x_3}{2}\sin 2E$	$+\frac{x_1}{2}$
	D	-	-	-	-	$-\frac{x_1}{2}\sin^2 E$	$+\frac{x_4}{2}\sin^2 E$	$-\frac{x_2}{2}\sin^2 E$	-	
	E	-	-	-	-	$-\frac{x_3}{2}\sin^2 E$	-	-	$-\frac{x_1}{4}\sin^2 E$	$-\frac{x_3}{4}$
2	A	$+ x_6 \cos E$ $+ x_9 R_\omega$	$+ x_7 \cos E$ $+ x_{10} R_\omega$	$+\frac{x_1}{8}(1 + 3\cos 2E)$	$+\frac{x_3}{8}(1 + 3\cos 2E)$	$+\frac{x_3}{8}(3 + \cos 2E)$ $+\frac{x_5}{2}\sin^2 E + x_{11}$	-	-	-	
	B	$- x_2 \sin 2E$	$-\frac{x_4}{2}\sin 2E$	-	-	$- x_8 \sin E$	-	-	-	
	C	$x_7 \sin E$	$- x_6 \sin E$	$+\frac{x_3}{2}\sin 2E$	$-\frac{x_1}{2}\sin 2E$		-	-	-	
	D	$-\frac{x_4}{2}\sin^2 E$	$\frac{x_2}{2}\sin^2 E$	-	-		-	-	-	
	E	-	-	$-\frac{x_1}{4}\sin^2 E$	$-\frac{x_3}{4}\sin^2 E$	$(\frac{x_3}{2} - x_5)\frac{\sin^2 E}{2}$	-	-	-	
3	A	$+ x_6 \cos E$ $+ x_9 R_\omega$	$+ x_7 \cos E$ $+ x_{10} R_\omega$	$+\frac{x_1}{8}(1 + 3\cos 2E)$	$+\frac{x_3}{8}(1 + 3\cos 2E)$	$+\frac{x_3}{8}(3 + \cos 2E)$ $+\frac{x_5}{2}\sin^2 E + x_{11}$	$+ x_7 \cos E$ $+ x_{10} R_\omega$	$- x_6 \cos E$ $- x_9 R_\omega$	$-\frac{x_1}{8}(3 + \cos 2E)$	$-\frac{x_3}{8}($
	B	$- x_7 \sin E$	$+ x_6 \sin E$	$-\frac{x_3}{2}\sin 2E$	$+ x_1 \sin 2E$	-	$+ x_6 \sin E$	$+ x_7 \sin E$	$+\frac{x_3}{2}\sin 2E$	$-\frac{x_1}{2}$
	C	$-\frac{x_2}{2}\sin 2E$	$-\frac{x_4}{2}\sin 2E$	-	-	$- x_8 \sin E$	$-\frac{x_4}{2}\sin 2E$	$+\frac{x_2}{2}\sin 2E$	-	
	D	$+\frac{x_4}{2}\sin^2 E$	$-\frac{x_2}{2}\sin^2 E$	-	-	-	$-\frac{x_2}{2}\sin^2 E$	$-\frac{x_4}{2}\sin 2E$	-	
	E	-	-	$\frac{x_1}{4}\sin^2 E$	$+\frac{x_3}{4}\sin^2 E$	$-(\frac{x_3}{2} - x_5)\frac{\sin^2 E}{2}$	-	-	$-\frac{x_1}{1}\sin^2 E$	$-\frac{x_3}{4}$

NOTES: (1) All table entries to be multiplied by corresponding column headings.
(2) Constants of second column equal sum of terms in corresponding row for each position.
(3) β_0 is the initial value of measurement head yoke angle.
(4) The term R_ω equals $[\frac{\dot{\alpha}}{\omega_e}$ - si

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n #2		Position #3				
$\cos 2\beta_0$	Unity	$\sin\beta_0$	$\cos\beta_0$	$\sin 2\beta_0$	$\cos 2\beta_0$	Unity
$+\frac{x_3}{8}(1+3\cos 2E)$	$+\frac{x_3}{8}(3+\cos 2E)$ $+\frac{x_5}{2}\sin^2 E + x_{11}$	$-x_7\cos E$ $-x_{10}R_\omega$	$+x_8\cos E$ $+x_9 R_\omega$	$-\frac{x_1}{8}(1+3\cos 2E)$	$-\frac{x_3}{8}(1+3\cos 2E)$	$+\frac{x_3}{8}(3+\cos 2E)$ $+\frac{x_5}{2}\sin^2 E + x_{11}$
-	$+x_8\sin E$	$+x_6\sin E$	$+x_7\sin E$	$-\frac{x_3}{2}\sin 2E$	$+\frac{x_1}{2}\sin 2E$	-
$+\frac{x_1}{2}\sin 2E$	-	$-\frac{x_4}{2}\sin 2E$	$+\frac{x_2}{2}\sin 2E$	-	-	$+x_8\sin E$
-	-	$+\frac{x_2}{2}\sin^2 E$	$+\frac{x_4}{2}\sin^2 E$	-	-	-
$-\frac{x_3}{4}\sin^2 E$	$(\frac{x_3}{2}-x_5)\frac{\sin^2 E}{2}$	-	-	$-\frac{x_1}{4}\sin^2 E$	$-\frac{x_3}{4}\sin^2 E$	$-(\frac{x_3}{2}-x_5)\frac{\sin^2 E}{2}$
-	$+\frac{x_3}{2}\sin^2 E + x_5\cos^2 E$ $+x_8\cos E + x_{11}$	$-x_7\cos E$ $-x_{10}R_\omega$	$+x_8\cos E$ $+x_9 R_\omega$	$-\frac{x_1}{8}(1+3\cos 2E)$	$-\frac{x_3}{8}(1+3\cos 2E)$	$+\frac{x_3}{8}(3+\cos 2E)$ $+\frac{x_5}{2}\sin^2 E + x_{11}$
-	$+\frac{x_4}{2}\sin 2E + x_7\sin E$	$+\frac{x_4}{2}\sin 2E$	$-\frac{x_2}{2}\sin 2E$	-	-	$-x_8\sin E$
-	$-\frac{x_2}{2}\sin 2E - x_6\sin E$	$+x_6\sin E$	$+x_7\sin E$	$-\frac{x_3}{2}\sin 2E$	$+\frac{x_1}{2}\sin 2E$	-
-	$-\frac{x_1}{2}\sin^2 E$	$-\frac{x_2}{2}\sin^2 E$	$-\frac{x_4}{2}\sin^2 E$	-	-	-
-	$-\frac{x_3}{2}\sin^2 E$	-	-	$+\frac{x_1}{4}\sin^2 E$	$+\frac{x_3}{4}\sin^2 E$	$+(\frac{x_3}{2}-x_5)\frac{\sin^2 E}{2}$
$-\frac{x_3}{8}(1+3\cos 2E)$	$+\frac{x_3}{8}(3+\cos 2E)$ $+\frac{x_5}{2}\sin^2 E + x_{11}$	-	-	-	-	$+\frac{x_3}{2}\sin^2 E + x_5\cos^2 E$ $+x_4\cos E + x_{11}$
$-\frac{x_1}{2}\sin 2E$	-	-	-	-	-	$+\frac{x_2}{2}\sin 2E + x_6\sin E$
-	$-x_8\sin E$	-	-	-	-	$+\frac{x_4}{2}\sin 2E + x_7\sin E$
-	-	-	-	-	-	$+\frac{x_1}{2}\sin^3 E$
$-\frac{x_3}{4}\sin^2 E$	$-(\frac{x_3}{2}-x_5)\frac{\sin^2 E}{2}$	-	-	-	-	$+\frac{x_3}{2}\sin^2 E$

- - sin(L - E)]

Table 3-1

Tabulation of SAP Coefficients
(cont'd)

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4. PIGA MATH MODELS

4.1 GENERAL FORM OF PIGA EQUATIONS OF MOTION

The PIGA equation of motion derived in Ref. 5.4 (Eq. 14) can be simplified when the present PIGA test conditions are considered. Justification for omitting certain terms will be included in the error analysis report on the strapdown system test program. The resulting PIGA equation is given by Eq. (23) in Ref. 5.4 (repeated as Eq. (4-1) below) and is based on the summation of torques on the gyro gimbal/rotor cylinder about its OA being equal to zero. Therefore,

$$m\ell \ddot{z} - M_u = \omega_z H \qquad (4\text{-}1)$$

where $\ddot{z}$ and ω_z are along the input axis of the gyro itself (not the PIGA yoke). Considering internal gyro alignment errors ν_P and ψ_P, Eq. (4-1) can be expressed in the PIGA yoke axis coordinate system, as indicated by Eqs. (21) and (22) in Ref. 5.4. The resulting equation is:

$$\dot{\beta}_r = A_r \cos\beta_r + B_r \sin\beta_r + C_r \qquad (4\text{-}2)$$

where

$$A_r \triangleq (S'_{pr} g'_{xr} - S''_{pr} \omega'_{xr})\nu_P - (S'_{pr} g'_{yr} - S''_{pr} \omega'_{yr})\psi_P$$

$$B_r \triangleq (S'_{pr} g'_{xr} - S''_{pr} \omega'_{xr})\psi_P + (S'_{pr} g'_{yr} - S''_{pr} \omega'_{yr})\nu_P$$

$$C_r \triangleq S'_{pr} g'_{zr} - S''_{pr} \omega'_{zr} - B_{pr}$$

and S'_{pr} and S''_{pr} are the PIGA acceleration and angular velocity scale factors, respectively. The PIGA velocity scale factor (S_V) equals $2\pi/S'_p$ where indicated velocity = $S_V \Delta\beta$, with $\Delta\beta$ in revolutions. Note that the term η_r occurs only as products with ν_P and ψ_P and therefore has been neglected. Eq. (4-2) can be expressed in the PIGA base coordinate system, using the appropriate equations of Table 2-1, as follows:

$$\dot{\beta}_r = A_r \cos\beta_r + B_r \sin\beta_r + C_r \qquad (4\text{-}3)$$

where

$$A_r = (S'_{pr} g_{xr} - S''_{pr} \omega_{xr}) \nu_P - (S'_{pr} g_{yr} - S''_{pr} \omega_{yr}) \psi_P$$

$$B_r = (S'_{pr} g_{xr} - S''_{pr} \omega_{xr}) \psi_P + (S'_{pr} g_{yr} - S''_{pr} \omega_{yr}) \nu_P$$

$$C_r = S'_{pr} [\psi_{pr} g_{xr} - \nu_{pr} g_{yr} + g_{zr}] + S''_{pr} [-\psi_{pr} \omega_{xr} + \nu_{pr} \omega_{yr} - \omega_{zr}] - B_{pr}$$

and products of small quantities have been neglected. Expressions for the ω and g terms are contained in Table 2-1 (Eqs. 2-12, 2-13, 2-19 and 2-20).

4.2 PIGA EQUATIONS FOR TEST PROGRAM

A solution of the transcendental, differential equation (4-3) is given by Eq. (35) of Ref. 5.4, which can be approximated by the following equation:

$$\dot{\beta}_r = C_r + \sqrt{A_r^2 + B_r^2} \sin(C_r t + \phi) \qquad (4\text{-}4)$$

The solution is based on A_r, B_r, and C_r being equal to constants, which requires $\dot{\alpha} = 0$, and that second degree terms of $X = \sqrt{(A^2 + B^2)/C^2}$ are negligible. Further simplification of the test program is possible if full revolution tests are specified. In this case, the integral of the sine term in Eq. (4-4) is zero and

$$\Delta\beta_r = C_r \Delta T \qquad (4\text{-}5)$$

where

$$\Delta\beta_r = 2\pi \text{ radians}$$

ΔT = time to complete one revolution of the PIGA yoke

and C_r is given with Eq. (4-3). The quantity C_r, which is a function of the calibration terms to be estimated, is simply determined from Eq. (4-5) by measuring the time required to complete one revolution of the PIGA yoke.

Upon substituting expressions for the ω and g terms from Table 2-1 into the equation for C_r and allowing for errors in S'_{pr} and S''_{pr}, the quantity C_r can be represented as follows:

$$C_r + R_r = S_r \tag{4-6}$$

or

$$C_r + [R_{ar} + R_{br}\sin\alpha_r + R_{cr}\cos\alpha_r] = [S_{ar} + S_{br}\sin\alpha_r + S_{cr}\cos\alpha_r]$$

The term α_r is the constant angle of the test table and the R terms are functions of the table elevation angle and misalignments of the strapdown system optical cube relative to the test stand coordinates. Therefore, the quantities on the left side of Eq. (4-6) are measurable and can be used to estimate the S constants, which are functions of the calibration terms to be estimated. Table 4-1 contains a tabulation of the R and S terms for each PIGA in each mounting position of the strapdown system on the test table.

The remarks at the end of Section 3.4 pertaining to the test stand misalignments also applies to the PIGA tests.

Position *	PIGA *	Constant	R_{ar}, S_{ar} Unity	R_{br}, S_{br} $\sin\alpha$	
1	1	R_r	$-A_z$	$\nu_{01} A_x$	
		S_r	$\delta A_{z1} - \psi_{T1} A_x - B_{p1}$	$-\nu_{p1} A_x$	
	2	R_r	$\nu_{01} A_z$	A_x	
		S_r	$-\nu_{p2} A_z - B_{p2}$	$-\delta A_{x2} - \psi_{T1} A_z$	$-\psi$
	3	R_r	$\psi_{01} A_z$	$-\eta_{01} A_x$	
		S_r	$-\nu_{p3} A_z - B_{p3}$	$\psi_{p3} A_x + \nu_{T1} A_z + A_y$	$-\delta$
2	1	R_r	$-\nu_{02} A_z$	$-A_x$	
		S_r	$\nu_{p1} A_z - B_{p1}$	$\delta A_{x1} + \psi_{T2} A_z$	$-\psi$
	2	R_r	$-A_z$	$\nu_{02} A_x$	
		S_r	$\delta A_{z2} - \psi_{T2} A_x - B_{p2}$	$-\nu_{p2} A_x$	
	3	R_r	$\psi_{02} A_z$	$-\eta_{02} A_x$	
		S_r	$-\psi_{p3} A_z - B_{p3}$	$-\nu_{p3} A_x + \nu_{T2} A_z + A_y$	$-\delta$
3	1	R_r	$-\psi_{03} A_z$	$\eta_{03} A_x$	
		S_r	$\psi_{p1} A_z - B_{p1}$	$-\nu_{p1} A_x - \nu_{T3} A_z - A_y$	δA
	2	R_r	$\nu_{03} A_z$	A_x	
		S_r	$\psi_{p2} A_z - B_{p2}$	$-\delta A_{x2} - \psi_{T3} A_z$	$-\nu$
	3	R_r	$-A_z$	$\nu_{03} A_x$	
		S_r	$\delta A_{z3} - \psi_{T3} A_x - B_{p3}$	$\psi_{p3} A_x$	

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	R_{cr}, S_{cr}
	$\cos\alpha$
	$\psi_{01}A_x$ $-\psi_{p1}A_x$
	$\eta_{01}A_x$ $-\psi_{p2}A_x - \nu_{T1}A_z - A_y$
$+ A_y$	A_x $-\delta A_{x3} - \psi_{T1}A_z$
	$-\eta_{02}A_x$ $-\psi_{p1}A_x + \nu_{T2}A_z + A_y$
	$\psi_{02}A_x$ $-\psi_{p2}A_x$
$+ A_y$	A_x $-\delta A_{x3} - \psi_{T2}A_z$
$- A_y$	$-A_x$ $\delta A_{x1} + \psi_{T3}A_z$
	$\eta_{03}A_x$ $-\nu_{p2}A_x - \nu_{T3}A_z - A_y$
	$\psi_{03}A_x$ $-\nu_{p3}A_x$

$$C_r = \Delta\beta_r/\Delta T_r, \quad \text{where } \Delta\beta_r = 2\pi \qquad (4\text{-}5)$$

$$C_r + R_r = S_r \qquad (4\text{-}6)$$

$$[C_r + R_{ar} + R_{br}\sin\alpha + R_{cr}\cos\alpha] = S_{ar} + S_{br}\sin\alpha + S_{cr}\cos\alpha$$

$$A_x \triangleq (S_p'g)\sin E + (S_p''\omega_e)\cos(L - E)$$

$$A_z \triangleq (S_p'g)\cos E + (S_p''\omega_e)\sin(L - E)$$

$$\delta A_{xr} \triangleq \left(\frac{\delta S_p'}{S_p'}\right)_r (S_p'g)\sin E + \left(\frac{\delta S_p''}{S_p''}\right)_r (S_p''\omega_e)\cos(L - E)$$

$$\delta A_{zr} \triangleq \left(\frac{\delta S_p'}{S_p''}\right)_r (S_p'g)\cos E + \left(\frac{\delta S_p''}{S_p''}\right)_r (S_p''\omega_e)\sin(L - E)$$

$$A_y \triangleq \nu_s(S_p'g) + \epsilon_s(S_p''\omega_e)$$

$$\epsilon_s \triangleq \eta_s\cos L + \nu_s\sin L$$

NOTES:

(1) All table entries to be multiplied by corresponding column headings.

(2) R_r and S_r constants equal sum of terms in corresponding row.

(3) Subscripts on (η_0, ν_0, ψ_0) refer to mounting position number.

(4) Subscripts on $(\nu_p, \psi_p, \delta A_{xr}, B_p, \delta A_{zr})$ refer to PIGA number.

Table 4-1

Tabulation of PIGA Coefficients

FOLDOUT FRAME 2

5. REFERENCES

5.1 Nelson, R.H., Jr., "Preliminary Considerations in Testing the NASA/Huntsville Strapdown System", DRC Report E-1259U, Dynamics Research Corporation, Stoneham, Mass., October 26, 1967.

5.2 Quagliata, L., "Equations of Motion of the NASA Air Bearing Gyro in an Inertially Stabilized Configuration and in a Strapdown Configuration", DRC Report E-940U, Dynamics Research Corporation, Stoneham, Mass., September 6, 1966.

5.3 Merz, A.W., "Gyro Test Methods", DRC Report E-1170U, Dynamics Research Corporation, Stoneham, Mass., July 17, 1967,

5.4 Nelson, R.H., Jr., "Equations of Motion for a Pendulous Integrating Gyro Accelerometer", DRC Report E-477, Dynamics Research Corporation, Stoneham, Mass., November 14, 1964.

APPENDIX A

LIST OF SYMBOLS

A, A_o	azimuth angle of test stand, defined as desired angle from true east to stand stand elevation axis, measured about local vertical. Subscript zero indicates initial value.
A_r thru E_r	functions of the SAP calibration terms to be estimated and certain constant test conditions (see Table 3-1). A_r, B_r, C_r also used in PIGA equations (see Eq. (4-2)).
A_x, A_y, A_z	angular rate of PIGA yoke due to components of g and ω_e relative to test table base coordinates (x_b, y_b, z_b)
B_p	PIGA bias ($= M_u/H$)
E	elevation angle of test stand, defined as desired angle from local horizontal to normal to table axis of rotation, measured about test stand elevation axis (y_s)
f	function of
g	gravity
g_{xc}, g_{yc}, g_{zc}	x, y and z components of gravity in "coordinate system c"
H	angular momentum of gyro wheel
K_{ab}	gyro compliance coefficients (displacement along axis "a" due to force along axis "b")
K_T, K_Y	constants used when angular rate and acceleration vectors along coordinate axes are considered. $K_T = K_Y = 1$ for rates and zero for accelerations.
L	local geodetic latitude
L_r, M_r, N_r, P_r	coefficients used in final form of SAP math model

M_u	PIG uncertainty torque
m	mass of gyro gimbal/rotor cylinder. Also mass unbalance of PIG
N, E, V	North, East, Vertical orthogonal coordinate system
P_a, P_b, P_c	measurable constants used in SAP math model (functions of η_o, ν_o, ψ_o)
Q_a thru Q_f	functions of constant SAP calibration terms (includes test stand misalignments)
R_1	resolution matrix relating strapdown system optical cube axes to test stand table axes
R_2	resolution matrix relating SAP and PIGA bases coordinate system to strapdown system optical cube axes
R_r, S_r	coefficients used in final form of PIGA math model
R_a, R_b, R_c	measurable constants used in PIGA math model (functions of $\eta_o, \nu_o, \psi_o, A_x, A_y$)
R_ω	constant equal to $[\dot{\alpha} - \omega_e \sin(L - E)]/\omega_e$
S_a, S_b, S_c	functions of PIGA calibration terms and certain test conditions (includes test stand misalignments)
S'_p	desired PIGA acceleration scale factor ($= m\ell/H$)
S''_p	desired PIGA angular velocity scale factor
S_v	desired PIGA velocity scale factor ($= 2\pi/S'_p$)
S_s, S_s^o	SAP scale factor. Superscript zero denotes desired value.
T	time measured from start of test table rotation during SAP tests
x_1	coefficient relating $g_x g_y/g^2$ to SAP gyro drift rate $= -(mg)^2 K_{zx}/H$
x_2	coefficient relating $g_x g_z/g^2$ to SAP gyro drift rate $= +(mg)^2 K_{yx}/H$
x_3	coefficient relating g_y^2/g^2 to SAP gyro drift rate $= +(mg)^2 K_{zy}/H$

x_4	coefficient relating $g_y g_z / g^2$ to SAP gyro drift rate = $+(mg)^2 (K_{yy} - K_{zz})/H$
x_5	coefficient relating g_z^2/g^2 to SAP gyro drift rate = $+(mg)^2 K_{yz}/H$
x_6	coefficient relating g_x/g to SAP gyro drift rate
x_7	coefficient r lating g_y/g to SAP gyro drift rate $= + mg\delta_z$
x_8	coefficient relating g_z/g to SAP gyro drift rate $= - mg\delta_y$
x_9	SAP gyro drift rate $= \theta_y \omega_e$
x_{10}	SAP gyro drift rate $= -\theta_x \omega_e$
x_{11}	SAP gyro drift rate proportional to gyro bias error
x_k	general term representing x_1 thru x_{11}
x, y, z	gyro output, spin, and input axes, respectively
x_b, y_b, z_b	test table base coordinate axes
x_o, y_o, z_o	strapdown system optical cube coordinate system
x_r, y_r, z_r	reference a[illegible] of Instrument (SAP and PIGA) bases
x_s, y_s, z_s	test stand base coordinate axes
x_T, y_T, z_T	test table coordinate axes
x_Y, y_Y, z_Y	instrument (SAP and PIGA) yoke coordinate axes
$\ddot{z}$	acceleration along input axis of gyro (measurement head)
α, α_o	test stand table angle, defined as desired angle between strapdown system optical cube "y surface" (y'_T) and test stand elevation axis (y_s), measured about table rotational axis. Subscript zero indicates initial value.
$\dot{\alpha}$	rate of change of α with respect to test stand base
β, β_o	instrument (SAP or PIGA) yoke angle, defined as desired angle between inertial reference unit (gyro) spin axis and the strapdown system optical cube "y surface" (y_r), measured about the yoke axis. Subscript zero indicates initial value.

$\dot{\beta}$	rate of change of β with respect to strapdown systems optical cube coordinate system
γ, γ_o	angle equal to integral of earth rate component along test table axis (nominally equal to $\alpha + \beta$). Subscript zero indicates initial value ($= \alpha_o + \beta_o$)
$\delta_y m, \delta_z m$	gyro pendulosity due to mass unbalance along spin and input axes, respectively (see x_7 and x_3)
$\delta S'_p$	error in PIGA acceleration scale factor. True scale factor $= S'_p + \delta S'_p$
$\delta S''_p$	error in PIGA angular velocity scale factor. True scale factor $= S''_p + \delta S''_p$
δS_v	error in PIGA velocity scale factor ($= S_v \delta S'_p / S'_p$)
$\Delta\beta$	change in β during system testing
ΔT	time during PIGA tests for one complete revolution ($\Delta\beta = 2\pi$)
ϵ_s	tilt of test stand elevation axis (y_s) about axis in earth's equatorial plane and normal to y_s ($= \eta_s \cos L + \nu_s \sin L$)
θ (θ_x, θ_y)	inertial reference unit (gyro) internal misalignment, defined as angle between gyro input axis (z) and yoke input axis (z'). θ_x is angle measured about output (x) axis and θ_y is angle measured about spin (y) axis.
η, ν, ψ	misalignments measured about z, x and y axes, respectively. Positive angles are measured about positive coordinate axes in accordance with the right hand rule. These terms represent quantities to be added to desired values to obtain actual values.
η_o, η_r, η_s	these terms are the misalignments associated with α, β, and A, respectively.
ν_o, ψ_o	misalignments between optical cube z'_T surface and test table z_T axis, measured about x_T and y_T, respectively.
ν_P, ψ_P	PIG/PIGA misalignments between PIG input axis and yoke input axis, about the PIG spin (y) and output (x) axes, respectively
ν_r, ψ_r	misalignment between SAP yoke axis (z'_r) and optical cube z_r surface, measured about x_r and y_r , respectively

ν_s	misalignment between test stand elevation axis (y_s') and local horizontal, measured about x_s'.
ν_T	nonorthogonality of test table axis of rotation to test stand elevation axis, measured about x_b'.
ψ_T	misalignment associated with desired elevation angle E, such that $(E + \psi_T)$ is the actual angle from local horizontal to normal to table axis of rotation, measured about test stand elevation axis (y_s)
ω_e	earth rate
$\omega_{xc}, \omega_{yc}, \omega_{zc}$	x, y and z components of inertial rates in "coordinate system c"
ω_z	angular velocity about input axis of gyro (measuring head).
ω_{IA}	angular velocity of PIGA yoke with respect to PIGA case

Miscellaneous Notes

(1) References to optical surfaces are understood to mean the normals to the surfaces.

(2) Primes added to axes designations generally denotes small perturbations from the basic coordinate system.

(3) The following subscript designators are defined:

i - instrument (SAP or PIGA) number, i = 1, 2, 3

j - test configuration = $f(E, \alpha_o, \beta_o, \dot{\alpha})$, to be defined

m - mounting position of strapdown system relative to test stand table, m = 1, 2, 3

r - combination of instrument number and mounting position (r = im)