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ON THE ORIGIN AND DISTRIBUTION OF METEOROIDS

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**Bellcomm, Inc.**

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**BELLCOMM, INC.**

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ABSTRACT

The influence of collisional and radiative processes on the population of sporadic and shower meteoroids is examined. It is found that the observed distribution of sporadic meteoroids in the photographic and radio ranges is unstable: collisional processes would rapidly change the distribution of these particles if they are not replenished by a source. It is shown that, in order to maintain the distribution of these sporadic meteoroids in a steady state condition, a source is needed which replaces the sporadic meteoroids destroyed by collisions. The mass distribution of the required source is calculated and found to agree with the distribution of bright radio shower meteors provided that the population index,  $\alpha$ , of sporadic meteors (in the photographic and radio ranges) is taken to be  $\alpha = 13/6$ .

The evolution of the population in a meteor shower under the influence of collisions with sporadic meteoroids is also examined. Since small particles have shorter life times (with respect to collisions) than do larger particles, the population of faint shower meteors is more strongly effected. It is found that the mass distribution of faint radio meteors in the major showers considered here agree with the results of the analysis provided that  $\alpha = 13/6$ , for sporadic meteors. Taking  $\alpha = 13/6$  therefore provides self-consistency for analysis and observation thereby providing new evidence favoring the cometary origin of meteors.

The rate at which the population of sporadic meteoroids is losing mass is calculated and found to compare favorably with Whipple's 1967 estimate as well as his estimate of the rate at which comets are injecting meteoritic mass into the solar system.

The influence of radiation damping on the meteoroid mass distribution is found to be minor; collisional processes dominate the evolution of meteoroids in the faint radio range and brighter.

The influence of radiation pressure may be profound. Using previous work by Harwit, it is shown that the smallest masses present in the major meteoroid showers are in the range of  $10^{-6}$  to  $10^{-10}$  grams. This is precisely the mass range where a leveling off of the distribution of sporadic meteoroids is indicated by satellite penetration measurements. These measurements are therefore consistent with the cometary origin of sporadic meteoroids.

## I. INTRODUCTION

The origin of meteoroids is a subject of considerable interest for students of the solar system (cf. Jacchia, 1963, for a review). Two types of meteor populations can be distinguished, according to their spatial distribution namely shower meteors and sporadic meteors. Shower meteors move in highly correlated orbits so that these meteors are distributed into the volume of an elliptical doughnut with the sun at one of its Foci. Close correlation between the orbit of a meteor shower and that of a known comet has been established for a number of meteor showers (see, e.g., Lovell, 1954) and hence it is believed that meteor showers originate from the partial disruption of comets (Whipple, 1963). Such disruptions occur when the comet is subjected to the thermal forces caused by the sun's radiation (Whipple, 1963).

Sporadic meteoroids, on the other hand, move in random orbits and "fill up" the interplanetary space like ideal gas molecules. It is generally believed that the population of sporadic and shower meteoroids are closely related (c.f. Jacchia, 1963). Planetary perturbations cause shower particles to separate from the "mainstream" and lose themselves among the sporadic meteoroids (Plavec, 1956). This mechanism may account for the origin of the sporadic meteoroids as shower meteoroids that have gone astray under the perturbing influence of planets; it is the purpose of the present paper to supply independent evidence supporting this theory.

Two distinct gravitational dispersal processes can be distinguished for meteor streams. One dispersal effect arises when particles are ejected from the comet near perihelion with a relatively small but limited ejection velocity (Whipple, 1950, 1951). The result is a distribution of orbital elements for the particles around a mean value similar to the orbital elements of the parent comet (see, e.g., Plavec, 1955). This means that the particles assume a spatial distribution of a cloud whose dimension is naturally defined by the spread in the orbital elements of the particles. As time goes on, however, the distribution in the periods of the particles causes a gradual dispersal of the cloud along an orbit similar to that of the parent comet until one has a situation where the shower particles are confined into a doughnut like region of space whose shape is similar to the orbit of the parent comet.

Another perturbational force acting on the meteor shower, independently from the processes we have just discussed, is the influence of planetary encounters. When a particle or group of particles encounter a planet, they will be scattered into orbits that are quite different from their initial orbits.

The magnitude of the effect depends, of course, on the proximity of the encounter. Opik (1966) has shown that the long range statistical effect is that the particles will assume a distribution of orbits having an average inclination to eccentricity ratio (for short period orbits) similar to that of the sporadic meteors reduced by McCrosky and Posen (1961).

In this paper, we examine the mass distribution of sporadic meteors (Section II). Using reasonable assumptions regarding the physics of hypervelocity impact (Section III) we show that the mass distribution of sporadic meteoroids is not stable under the influence of mutual inelastic collisions between individual particles (Section IV). This means that the population of sporadic meteoroids must continuously be replenished by a source function if the former is in a steady state condition. Following the currently favored theory of cometary origin, meteoroid showers are assumed to be such a source function and their mass distribution is estimated.

Theory and observation of the mass distribution of several major showers are compared (Section IV). The present theory is found to be self consistent and in reasonable agreement with observations.

## II. OBSERVATIONAL EVIDENCE

### A. Distribution of Sporadic Meteors

This section is a discussion of the observed distributions of sporadic and stream meteoroids together with the collective properties that the present study must explain.

The distribution of meteors can usually be approximated by the form

$$N = Bm^{-\alpha+1} \quad (1)$$

where  $N$  is the influx rate into the earth's atmosphere (per  $m^2$  sec) of objects having a mass of  $m$  (Kg) or greater.  $B$  and  $\alpha$  are constants, the latter is known as the population index.

Figure 1 is a double logarithmic plot of the cumulative influx rate of sporadic meteors having a mass of  $m$  kilograms or greater. The difference between the various estimates of the flux is an indication of the uncertainty in our knowledge regarding its exact value.

The points labeled "Expl 23" and "Peg" are the influx rates measured by the Explorer and Pegasus satellites. As these satellites measured penetration, it is necessary to identify a certain particle mass with the penetration of a given sensor. Naumann (1968) has calibrated these sensors by equating the masses of laboratory particles fired at meteoric velocities into sensors that were similar to ones actually flown. Since, however, some of these particles were accelerated gas dynamically, it is to be expected that their masses diminished because of ablation during the experiment. The satellite points should therefore be shifted towards the left on Figure 1 by an unknown amount.

The curve labeled "maximum" is based on Whipple's 1967 model (Whipple, 1967). For the sake of mathematical convenience, the  $\alpha$  value of Whipple's model has been adjusted from 2.34 to 7/3 for radio and photographic meteors and from 1.49 to 3/2 for micrometeoroids. The mass value at which these curves meet has been taken here as  $10^{-10}$  Kg; this insures that the model labeled as maximum is a likely upper limit to the flux of microparticles and radio meteors.

The curve labeled "minimum" has an index  $\alpha=2$  and is based on earlier reductions (unpublished) of the Pegasus satellite measurements. The normalization of the microparticle flux has been chosen for this case to represent a probable lower limit.

The model cumulative flux can then be written as:

$$\begin{aligned} N &= B_0 m^{-\alpha_0+1} & \mu_0 \leq m \leq \mu \\ N &= Bm^{-\alpha+1} & \mu \leq m \end{aligned} \quad (2)$$

where the numerical values are listed in Table I.



TABLE I

	<u>Minimum Model</u>	<u>Maximum Model</u>
$\log B_0$ (meters <sup>-2</sup> sec <sup>-1</sup> )	-11.1	-9.99
$\alpha_0 - 1$	1/2	1/2
$\mu_0$ (Kg)	$\sim 10^{-15}$	$\sim 10^{-15}$
$\mu$ (Kg)	$10^{-11}$	$10^{-10}$
$\log B$ (meters <sup>-2</sup> sec <sup>-1</sup> )	-16.6	-18.3
$\alpha - 1$	+1	4/3

Number densities  $f(m)dm$  can readily be obtained from eq-2:

$$f(m)dm = - \frac{4}{\langle v_\infty \rangle} \frac{\partial N}{\partial m} \quad (3)$$

where  $\langle v_\infty \rangle$  is the average meteor earth entry velocity. The factor 4 arises as a result of averaging the velocity distribution over all directions.

We, therefore, have for the number of particles per meter<sup>3</sup> in the mass range  $m$  to  $m+dm$

$$\begin{aligned} f(m)dm &= A_0 m^{-\alpha_0} & \mu_0 \leq m \leq \mu \\ f(m)dm &= Am^{-\alpha} & \mu \leq m \end{aligned} \quad (4)$$

where

$$\begin{aligned} A_0 &= 4(\alpha_0 - 1)B_0 / \langle v_\infty \rangle \\ A &= 4(\alpha - 1)B / \langle v_\infty \rangle \end{aligned} \quad (5)$$

in MKS units.

Because of uncertainties in the mass influx rates based on penetration measurements by satellites, it is difficult to choose between the models labeled minimum and maximum. The maximum model may overestimate the flux of particles for masses smaller than  $10^{-9}$  Kg and the minimum model may underestimate it. In support of the "maximum" model, we may mention that the index  $\alpha$  appears to be greater than 2; Hawkins and Upton, 1958, obtained an index  $\alpha=2.34$  for a sample of the Harvard photographic meteors (cf., however, Erickson, 1968). However, for a larger but less precisely reduced sample of the same meteors (McCrosky and Posen, 1961), Dohnanyi (1966,67) obtained an index of  $\alpha=2$ . Furthermore, radar work indicates that an index of  $\alpha=2$  is appropriate for radio meteors.

Figure 2 is a plot of the index  $\alpha$  for radio meteors versus the radio magnitude  $M_r$  based on the study of Elford (1965). While a precise identification of a given value of  $M_r$  with a meteor mass is uncertain, it may be said that the range in magnitude of  $M_r=0$  to  $M_r=14$ , includes a mass range of the order of about  $10^{-3}$  Kg to about  $10^{-9}$  Kg. It can be seen, from Figure 2, that all of the radio determination of  $\alpha$  yield a value of 2 to 2.5. Because of uncertainties in radio measurements (cg., for example, Greenhow, 1963), however, it is difficult to choose one particular index on the basis of these measurements alone.

We shall, therefore, retain both models under discussion for the purposes of this paper and make appropriate comparisons in the course of the analysis.

#### B. Distribution of Stream Meteors

A meteor stream is a swarm of particles distributed along a well defined orbit. A number of major streams have orbits associated with known comets (see, e.g., Lovell, 1954) and it is well established that the particles originated from these comets. Planets perturb the orbits of these particles to an extent that meteor streams lose many of their members continuously or from time to time. The particles that have thus gone "astray" are believed to constitute the bulk of the population of sporadic meteors. It therefore follows that a close connection between the distribution of stream and sporadic meteoroids exists which is the task of the present paper to explain.

Figure 3 summarizes some of the results of radio work as given by Elford (1965). The figure is a plot of the frequency distribution (of radio magnitudes  $M_r$  and zenithal electron line density  $q_z$ ) of three showers and of sporadic meteors (based on 4 different radio determinations). It is evident that the faint stream meteors have a lower index than do the sporadics giving rise to a "flatter" distribution. There is also an indication that the bright stream meteors have a larger population index (i.e., have a "steeper" distribution) than do the sporadics. As will be seen in Section IV of this paper, the trend indicated by Figure 3 is representative of the mass distribution of meteor streams.

In what follows, an analysis will be developed that relates the distribution of the shower meteors to the distribution of the sporadic background.

### III. IMPACT MECHANICS

In this section, we review the impact mechanics used in the present model when two meteoroids collide. Interplanetary particles frequently collide with each other. Such collisions occur, on the average, at the mean relative velocities between particles. Since the relative particle velocities under discussion are of the order of many kilometers per second, the collisions will be inelastic and fragmentation will result.

Using the results of Gault, Shoemaker and Moore (1963) results based on hypervelocity experiments in basalt, Dohnanyi (1967, 1968) has discussed some of the collective dynamical properties of particle populations undergoing collisions at several km/sec. Spherical particles of uniform material properties will be assumed throughout. It will also be assumed that all collisions occur at an average collisional velocity.

Let two objects, having masses  $M_1$  and  $M_2$ , respectively, collide with each other at a velocity of several km/sec. Let  $M_1 < M_2$  to be specific and one of two situations will result: (1)  $M_1$  will create a crater in  $M_2$ , or (2)  $M_1$  will catastrophically break up  $M_2$ . It will be assumed for the first case that  $M_2$  behaves as a semi-infinite (Dohnanyi, 1967, 1968) target. The mass of debris  $M_e$  cratered out by  $M_1$  is then

$$M_e = \Gamma M_1 \quad (6)$$

where  $\Gamma$  is a nondimensional constant depending only on the particle material properties and on the speed of impact.

$$\Gamma = 5v^2 \quad (7)$$

where  $v$  is the impact velocity in Km/sec.

Equation 7 is a rough fit to experimental data by Gault et al (1963) for basalt targets in an impact energy range of 10 joules to  $10^4$  joules, approximately.

The mass  $M_2$  that can be catastrophically disrupted by  $M_1$  is

$$\Gamma' M_1 \geq M_2 \quad (8)$$

where  $\Gamma'$  is not a function of mass. The value of  $\Gamma'$  is uncertain but for a limited number of experiments on basalt, Moore and Gault (1965) obtained a value of

$$\Gamma' = 50\Gamma \quad (9)$$

The ejecta produced during cratering and catastrophic events will have a certain mass distribution. This distribution may vary from event to event but on the average it will be taken as (Dohnanyi, 1967, 1968)

$$g(m, M_1, M_2) dm = C(M_1, M_2) m^{-\eta} dm \quad (10)$$

where  $g(m, M_1, M_2) dm$  is the number of ejected fragments in the mass range  $m$  to  $m+dm$  produced when an object of mass  $M_1$  impacted  $M_2$ . Ejecta distributions of the form eq-10 have been obtained in the laboratory (Gault et al, 1963); for basalt the value of the index  $\eta$  is

$$\eta = 1.8 \quad (11)$$

We take the mass of the largest fragment  $M_b$  to be proportional to the "projectile" mass  $M_1$

$$M_b = \Lambda M_1 \quad (12)$$

Experimental data (Gault et al, 1963) indicate that

$$\Lambda = \Gamma/10 \quad (13)$$

Using eq-8, 10, and 12 we can define  $C(M_1, M_2)$  by observing that

$$\int_{\mu}^{M_b} g(m, M_1, M_2) m \, dm = M_e \quad (14)$$

where  $\mu$  is the smallest fragment produced and is in the sub-micron range (Gault and Heitovit).

Evaluating eq-14, we obtain for erosive collisions

$$C(M_1, M_2) = (2-\eta)\Gamma\Lambda^{\eta-2}M_1^{\eta-1}, \Gamma M_1 < M_2 \quad (15)$$

and for catastrophic collisions

$$C(M_1, M_2) = (2-\eta)\Lambda^{\eta-2}(M_1+M_2)^{-2}, \Gamma M_1 \geq M_2 \quad (16)$$

When eq-15 and 11 are combined with eq-10, the mathematical description of the comminution process for individual events is complete.

#### IV. ASYMPTOTIC FORM OF METEOR SHOWER DISTRIBUTIONS

It is generally surmised (see, e.g., Plavec, 1956) that meteor showers are dispersed by planetary perturbations into uncorrelated orbits, thereby they give rise to the sporadic meteoroid distribution. In this section, we shall prove that the distribution of sporadic meteoroids is indeed unstable because of mutual inelastic collisions resulting in particle erosion and catastrophic breakups. In other words, we shall show that the present distribution of sporadic meteoroids is either rapidly changing in time or it is fed by a source function replenishing particles over the entire spectrum of masses present in the distribution. Thus, if we assume that the present distribution of sporadic meteoroids is unchanging, i.e., has reached steady state conditions then the source function that would make steady state conditions physically possible can be calculated. Such a source function can then be identified with the gravitational dispersal of shower meteoroids in support of the theory that the meteor streams do indeed feed the sporadic background.

##### A. Instability of the distribution of Sporadic Meteoroids

The population of meteoroids is subjected to frequent interparticle collisions. Since these collisions are inelastic, the target particles may either lose some of their mass (erosive collision) or be completely broken up (catastrophic collisions). The net result is a change in the distribution.

The equation that expresses the dependence of the population on collisions can be written as

$$\begin{aligned} \frac{\partial f(m,t)}{\partial t} dm = & \frac{\partial f(m,t)}{\partial t} dm \Big|_{\text{erosion}} + \frac{\partial f(m,t)}{\partial t} dm \Big|_{\text{catastrophic +}} \\ & \text{collisions} \\ & + \frac{\partial f(m,t)}{\partial t} dm \Big|_{\text{creation by}} \\ & \text{fragmentation} \end{aligned} \quad (17)$$

where  $f(m,t)dm$  is the particle number density function, i.e., the number of particles per unit volume of space in the mass range  $m$  to  $m+dm$ . The number of particles per unit volume (in the mass range  $m$  to  $m+dm$ ) changes because of erosive collisions,

catastrophic collisions and particle creation because the erosive and catastrophic crushing of larger objects give rise to fragments in this mass range. Dohnanyi (1968) discussed some of the properties of this equation (eq-17); it was found that solutions having a population index  $\alpha > 11/6$ , the population is unstable because of a relative overabundance of small particles and relative paucity of larger ones. The larger number of small particles causes a strong increase in the erosive rate ( $\partial f / \partial t$  dm erosion) as well as in the rate of catastrophic collisions ( $\partial f / \partial t$  dm catastrophic collision) relative the particle creation rate ( $\partial f / \partial t$  dm creation); thus many more particles are lost from a given mass range than are replaced by particle creation. The rate of particle creation is therefore seen to decrease with increasing population index (keeping other parameters constant) because the relative number of larger particles (which contribute to this term) decreases with increasing population index. These results were derived using the model crushing law, eq-15 and 16, based on experiment.

In view, however, of the crucial importance of stability considerations in this paper, I shall presently prove a stronger result, which is independent of any particular crushing theorem:

THEOREM-I: Given a population index type distribution, of the type eq-4 which undergoes inelastic collisional processes described by eq-17. The physical parameters  $\Gamma$ ,  $\Gamma'$  and  $\langle v \rangle$  are furthermore taken to be independent of the particle masses. It then follows that if the index  $\alpha > 11/6$ , the population is unstable and will decay in time regardless of what crushing law governs the redistribution of particulate fragments created during erosion and catastrophic collisions.

#### Proof for Catastrophic Collisions

First, we estimate the total amount of mass crushed catastrophically from objects in the arbitrary but finite mass range  $m_1$  to  $m_2$  per unit time and unit volume,  $\dot{M}(m_1, m_2)$ . Since the number of particles in the mass range  $m$  to  $m+dm$  is  $f(m)dm$ , the geometrical collision cross section of two particles with masses  $m$  and  $M$  is  $(3\pi^{1/2}/4\rho)^{2/3} (m^{1/3} + M^{1/3})^2$  where  $\rho$  is the material density; then, clearly,

$$\dot{M}(m_1, m_2) = - \int_{m_1}^{m_2} \left[ dm f(m) \int_{m/\Gamma'}^{M_\infty} dM K f(M) (m^{1/3} + M^{1/3})^2 \right] \quad (18)$$

where the expression in the square brackets is the number of objects destroyed by catastrophic collisions per unit time and unit volume in a mass range of  $m$  to  $m+dm$  (cf. Dohnanyi, 1967, 1968). Multiplying the number of catastrophic collisions per unit time and unit volume by the mass removed during each collision and integrating over appropriate limits gives us the desired expression  $\dot{M}(m_1, m_2)$ .

The constant  $K$  is defined as:

$$K = (3\pi^{1/2}/4\rho)^{2/3} \langle v \rangle. \quad (19)$$

We now substitute a number density function of the type eq-4 into eq-18 and obtain, for  $\alpha \neq 11/6$ .

$$\dot{M}(m_1, m_2) = - \frac{A^2 K (\Gamma')^{\alpha-1}}{\alpha-1} \left[ \frac{m_1^{-2\alpha+11/3} - m_2^{-2\alpha+11/3}}{-2\alpha + 11/3} \right] \quad (20)$$

where grazing collisions have been disregarded, and where  $m_1, m_2 \ll M_\infty$  and use has been made of the fact that  $\alpha > 5/3$  for our present purposes (otherwise terms involving  $M_\infty$  may dominate expression 20).

If  $\alpha = 11/6$ , the expression for  $\dot{M}(m_1, m_2)$  (eq-20) becomes logarithmic in  $m_2/m_1$ , as can easily be shown.

We shall presently estimate the lower limit of the total mass actually lost to the interval  $m_1$  to  $m_2$  (per unit time and unit volume of space) due to catastrophic crushing,  $\dot{L}(m_1, m_2)$ . The quantity  $\dot{M}(m_1, m_2)$  is the total mass being crushed



from objects in the mass range  $m_1$  to  $m_2$  per unit time and volume. Some of the fragments may, however, be redistributed in this same interval, so that  $\dot{M}(m_1, m_2)$  does not necessarily represent the rate of mass loss from the interval. To be specific, suppose that the mass of the largest fragment is  $\epsilon M$  when the mass is disrupted catastrophically by a smaller object,\* and  $\epsilon$  is some number less than one. Then clearly, all the mass catastrophically crushed in the mass range  $m$  to  $m/\epsilon$  will be lost from this interval and hence:

$$|\dot{L}(m_1, m_2)| \geq |\dot{M}(m_1, m_1/\epsilon)|, \quad m_2 \geq m_1/\epsilon \quad (21)$$

We now estimate the total amount of mass crushed per unit time and unit volume from the entire distribution of particles with masses greater than  $m_2$ , i.e., the quantity  $\dot{M}(m_2, M_\infty)$ .

Clearly, the total amount of mass crushed and lost from the interval  $m_2$  to  $M_\infty$  is less than  $\dot{M}(m_2, M_\infty)$ :

$$|\dot{L}(m_2, M_\infty)| \leq |\dot{M}(m_2, M_\infty)| \quad (22)$$

and hence

$$\frac{\dot{L}(m_2, M_\infty)}{\dot{L}(m_1, m_2)} = R < \frac{\dot{M}(m_2, M_\infty)}{\dot{M}(m_1, m/\epsilon)} \quad (23)$$

---

\*It can be shown that, for populations of our present interest ( $\alpha > 11/6$ ), the collisions of a mass  $M$  with masses equal to or greater than  $M$  contribute negligibly to the bracketed expression in eq-18. For example, if  $\alpha = 11/6$ , the mentioned contribution is of the order of  $10^{-2}$  of one per cent.

The right hand side of the above inequality can be easily evaluated using eq-20:

$$R < \frac{m_2^{-2\alpha+11/3}}{m_1^{-2\alpha+11/3}(1-\epsilon^{2\alpha-11/3})} = \left(\frac{m_1}{m_2}\right)^{2\alpha-11/3} (1-\epsilon^{2\alpha-11/3})^{-1} < 1 \quad (24)$$

provided that,

$$\alpha > 11/6$$

$$m_2 \geq m_1/\epsilon \quad (25)$$

$$M_\infty \gg m_2$$

The ratio R in eq-24 is seen to be less than one for all cases subject to the conditions in eq-25. It therefore follows that more mass is lost (because of catastrophic crushing) from any arbitrary interval  $m_1$  to  $m_2$ ,  $m_2 > m_1/\epsilon$  than can possibly be redistributed from the entire population (with masses greater than  $m_2$ ) into this mass range. This completes the proof for THEOREM I for catastrophic processes. We therefore conclude that a particle distribution, having physical parameters  $\Gamma'$ ,  $\epsilon$  which are not functions of the colliding masses and a population index,  $\alpha > 11/6$ , is unstable under the influence of catastrophic collisions. This follows because for such a system of particles more mass is lost from any arbitrary mass range than can be created by the entire distribution of larger masses. The result is furthermore seen to be the result of the conservation of mass only and does in no way depend on any particular communication process. The conclusion is therefore seen to be general (i.e., model independent). THEOREM I can also be proven for erosive processes; this is done in Appendix A.

It therefore follows that, if  $\Gamma$ ,  $\Gamma'$  and  $\epsilon$  are constants, the population having an index  $\alpha > 11/6$  is not stable because mass cannot be conserved. Thus, if the sporadic

meteoroid distribution can be characterized by the approximately mass independent set of parameters,  $\Gamma$ ,  $\Gamma'$  and  $\epsilon \frac{\Delta}{\Gamma'}$ , the distribution is unstable.

We shall assume that we are not experiencing a particular form of a fast decaying transient distribution of sporadic meteors. We shall take the alternate view, that a source function keeps "feeding" the sporadic meteoroid distribution in such a manner that a steady state situation exists and has existed for some time. We now turn our attention to calculating the precise form of the source function which is required to maintain the sporadic meteoroid distribution.

#### B. Asymptotic Form of Initial Stream Distributions

We shall presently estimate the form of the source function which is required to maintain the population of sporadic meteoroids in a steady state. We need to add a source function  $S(m)dm$  to the right hand side of eq-17 and set the left hand side equal to zero. Explicitly expressing the individual terms in the equation will then enable us to solve for the source function.

We write therefore:

$$f(m,t) = f(m) \quad (26)$$

and

$$\frac{\partial f(m)}{\partial t} = 0 \quad (27)$$

Before writing the explicit expression for eq-17 with a source function included, we note some simplifications which are in order. The population index  $\alpha$  for sporadic meteoroids in a mass range of our future interest is in the neighborhood of  $2 \leq \alpha \leq 7/3$ . Dohnanyi (1968) has shown that for  $\alpha > 11/6$ , the influence of the particle creation term due to fragmentation  $\left. \frac{\partial f(m)}{\partial t} \right|_{\text{creation}}$  is small in comparison with the other terms of eq-17. Specifically, as  $\alpha$  approaches 2, this term becomes less than 10% of the other terms and it can be shown that  $\left. \frac{\partial f(m)}{\partial t} \right|_{\text{creation}}$  becomes even smaller in comparison with the other terms for higher values of  $\alpha$ . We therefore disregard  $\left. \frac{\partial f(m)}{\partial t} \right|_{\text{creation}}$  from eq-17, without loss in accuracy, in our present discussion.

We therefore can write for the source function, trivially,

$$S(m)dm = \frac{\partial f(m)}{\partial t} dm \Big|_{\text{erosion}} - \frac{\partial f(m)}{\partial t} dm \Big|_{\substack{\text{catastrophic} \\ \text{collisions}}} \quad (28)$$

Dohnanyi (1967, 1968) derived explicit expressions for the terms on the right hand side of eq-28. The result is:

$$\frac{\partial f(m)}{\partial t} \Big|_{\text{erosion}} = - \frac{\partial}{\partial m} [f(m)\dot{m}] \quad (29)$$

and

$$\frac{\partial f(m)}{\partial t} \Big|_{\substack{\text{catastrophic} \\ \text{collision}}} = - k f(m) \int_{m/\Gamma'}^{M_\infty} f(M) dM (m^{1/3} + M^{1/3})^2 \quad (30)$$

where  $\dot{m}$  is given by eq-25.

It can be seen that eq-29 is reasonable, because  $f(m)\dot{m}$  is the "flux of masses" (in a phase space of  $f(m)$  vs.  $m$ ) past a given value of  $m$ , analogous to the flow of fluids in real space. The net accumulation or loss due to erosion in the number of objects in the mass range  $m$  to  $m+dm$  is then the negative divergence, with respect to mass, of the "flux" term  $f(m)\dot{m}$ , as given in eq-29.

Equation 30 is seen to be the bracketed expression on the right hand side of eq-18 and has been discussed in the text accompanying that equation.

#### Minimum Model

Using a density function  $f(m)$  given by eq-4, we obtain for  $\alpha=2$ ,

$$S(m)dm = k\Gamma A^2 m^{-7/3} \left\{ -\frac{4}{3} \left[ C + \ln \frac{m}{\Gamma' \mu} \right] + 1 \right\} dm + k\Gamma' A^2 m^{-7/3} dm \quad (31)$$

where

$$C = \left[ 1 - (\mu_0/\mu)^{2-\alpha_0} \right] (2-\alpha_0)^{-1} \approx 2 \quad (32)$$

is the contribution to erosion by small particles with masses in the range  $\mu_0$  to  $\mu$ . Eq-31 is valid for masses

$$m \geq \Gamma' \mu \quad (33)$$

i.e., for masses that are equal to or greater than the largest mass catastrophically disrupted by a particle of mass  $\mu$ . For smaller masses, i.e., for  $m < \Gamma' \mu$  a more complicated relation than eq-31 applies, since in this case the distribution of microparticles with masses less than  $\mu$  can produce catastrophic disruptions of our "test particles" with masses  $m$ . It will be shown later, that available information on meteor shower distributions is for masses greater than  $\Gamma' \mu$  and we shall not presently concern ourselves with the dynamics of particles smaller than  $\Gamma' \mu$ .

The first term on the right hand side of eq-31 is the rate of change per unit time and unit volume in the number of particles in the mass range  $m$  to  $m+dm$  due to erosion, the second expression is the analogous rate due to catastrophic collisions. It can be seen that the erosion term increases indefinitely as  $\ln(m/\Gamma' \mu)$  for large masses in comparison with the catastrophic collision term. For relatively small masses ( $m \approx \Gamma' \mu$ ), however, the catastrophic collision term is of the order of  $\Gamma'/2\Gamma=25$  times greater than the erosion term. The two terms equal for masses of about  $m \approx 3 \times 10^{12} \Gamma' \mu$ . Our interest will be confined to masses considerably smaller than  $3 \times 10^{12} \Gamma' \mu$  (which is of the order of many tons), and therefore in our mass range of interest catastrophic collisions will dominate for populations with  $\alpha=2$ . Erosion plays a minor but non-negligible role. The logarithmic term in eq-31 is so slowly varying that we can replace it by a constant over any reasonable mass range.

One may therefore write:

$$S(m)dm = \text{constant } m^{-7/3} dm \quad (34)$$

provided that

$$\begin{aligned} \alpha &= 2 \\ m &\geq \Gamma' \mu \end{aligned} \quad (35)$$

Equation 34 tells us the important result that in order to maintain in a steady state a population with index  $\alpha=2$  we must have a source function injecting particles into the population with an index of  $7/3$ .

It therefore follows that meteor streams, if initially created with an index  $7/3$  for masses greater than  $\Gamma'\mu$ , provide an adequate mechanism for maintaining the sporadic backgrounds through the dispersal of stream meteors by planetary perturbations. Conversely, if we knew definitely that the sporadic population has an index  $\alpha=2$ , we would expect meteor streams to have an initial distribution with index  $\alpha=7/3$ .

#### Maximum Model

The expression for the source function can easily be obtained when  $\alpha=7/3$ . Using eq-4 and eq-28, 29 and 30 one obtains:

$$S(m)dm = KA^2 \gamma^{\alpha-1} m^{-2\alpha+5/3} dm/(\alpha-1) \\ + K\Gamma A^2 \mu^{-\alpha+2} (\alpha - \frac{2}{3}) m^{-\alpha-1/3} \left[ C + \frac{1}{\alpha-2} - (1 + \frac{1}{\alpha-2}) (m/\Gamma'\mu)^{-\alpha+2} \right] dm$$

(36)

where

$$m > \Gamma'\mu$$

and where the first term is the contribution of catastrophic collisions and the second term is the contribution of erosion. The constant  $C$  is the contribution to the erosion term of microparticles with masses in the range  $\mu_0$  to  $\mu$ .

It can be seen that the last expression in the erosion term (inside the square bracket) becomes negligible for large masses. This is to be expected since for large enough masses the erosion rate of populations with index  $\alpha>2$  should be dominated by collisions with small particles. The mass erosion rate of an individual object,  $m$  (in eq-25) is then directly proportional to  $m^{2/3}$  giving rise to a so-called "linear erosion rate" where the rate of change of the particle radius is constant, i.e.,  $d/dt(m^{1/3}) = \text{constant}$ .

A comparison between the catastrophic collision term and erosion term on the right hand side of eq-36 indicates that, for  $\alpha=7/3$ , these two terms are equal when the mass  $m \approx 150\Gamma'\mu$  which is of the order of milligrams. For smaller particles, catastrophic collisions are more important than erosion; when  $m=\Gamma'\mu$ , the erosion term is only about 14% of the catastrophic collision term. For masses  $m \gtrsim 150\Gamma'\mu$  the erosion term becomes greater than the catastrophic collision term and eventually dominates.

Thus, if we force  $S(m)dm$  to be a population index type function, with

$$S(m)dm \approx S(1) m^{-\sigma} dm \quad (37)$$

we have, for  $\alpha=7/3$ ,

$$\sigma \approx 2\alpha - 5/3 = 3 \text{ for small masses, } m \ll 150\Gamma'\mu \quad (38)$$

and

$$\sigma \approx \alpha + 1/3 = 8/3 \text{ for larger masses, } m \gg 150\Gamma'\mu \quad (39)$$

For masses of the order of  $150\Gamma'\mu$  we have:

$$8/3 < \sigma < 3, m \approx 150\Gamma'\mu \quad (40)$$

It therefore follows that an initial stream meteoroid distribution of the type eq-38, 39 and 40 is required to maintain a sporadic distribution with an index of 7/3.

### C. Asymptotic Form Of Steady State Stream Distributions

When a meteor stream is created, it has a certain initial distribution. Portions of this distribution will experience planetary perturbations and be randomly scattered to join the sporadic environment. The distribution of particles remaining in the

given stream  $h(m,t)dm$  will evolve in time because of collisional processes. Collisions between the stream particles themselves will be disregarded because of the following considerations; their relative velocities will be of the order of the ejection velocities from their parent comets, believed to be less than a km/sec (Whipple, 1951). The initial number density of particles in a stream is not known at the present time but we shall use observations made from the Giacobinid shower to estimate their number density relative to the sporadic environment. Since the Giacobinid shower has not apparently been spread out over its entire orbit (Lovell, 1954), it is probably similar to its initial distribution. Particles in this shower have an average geocentric velocity of about 23 km/sec which is about 1.5 times higher than is the average geocentric velocity of sporadic meteors ( $17 \frac{\text{km}}{\text{sec}}$ ). Maximum recorded rates of this shower are of the order of 4,000 to 6,000 visual meteors per hour (Lovell, 1954) compared with an average sporadic rate of about 12 meteors per hour. The Giacobinid particles therefore have a number density of about  $\frac{5000}{12} \frac{1}{1.5} \approx 300$  times the number density of sporadic meteors (in the visual range). Their relative velocities are, however, of the order of .5 km/sec. Therefore, the flux of Giacobinid particles relative each other is about 34 times lower than is the flux of sporadic meteors because of the different relative velocities (i.e., if the sporadics had the same number density). The influence of collisions, however, is even greater since the parameters  $\Gamma$ ,  $\Gamma'$  scale with kinetic energy, i.e., with the square of the encounter velocity. Hence, values of  $\Gamma$ ,  $\Gamma'$  should be scaled down by a factor of  $(\frac{1}{34})^2 = 10^{-3}$  from similar values for sporadic meteors. Combining these two factors, we obtain the result that collisional rates for the Giacobinid particles between each other should be about  $10^{-3}/34 \approx 3 \times 10^{-5}$  times slower than would be the case with sporadic meteors at the same number densities. Since the collisional rates are also proportional to the number densities, the self destruction rate of a "stream cloud" like the Giacobinids will be  $300 \times 3 \times 10^{-5} \approx 10^{-2}$  times the influence of the sporadic environment. This order of magnitude argument may be inaccurate inasmuch as much higher particle densities may initially exist in new meteor streams than we have considered. The various life times we shall obtain will therefore be upper limits.



Thus, considering the interaction of stream particles with sporadic meteoroids, we have a situation where stream meteors will undergo collisional processes described by eq-17. A final steady state will be reached when fragments of the stream particles in a given mass range will be created at the same rate as particles are removed by collisional processes from the same mass range.

Thus, if  $h(m,T)dm$  is the number of stream particles per unit volume at a time  $T$  in a range  $m$  to  $m+dm$ , and if  $T$  is much longer than a certain life time, we have:

$$\begin{aligned} \frac{\partial h(m,T)}{\partial t} = 0 = & \frac{\partial h(m,T)}{\partial t} \Big|_{\text{erosion}} + \frac{\partial h(m,T)}{\partial t} \Big|_{\text{catastrophic}} \\ & \text{collisions} \\ & + \frac{\partial h(m,T)}{\partial t} \Big|_{\text{creation by}} \\ & \text{fragmentation} \end{aligned} \quad (41)$$

Anticipating our results, we can disregard the erosion term in eq-41 since shower mass distributions for small particles tend to have a lower index than the sporadic flux, specifically if they tend to have an index of about 1.8.

Taking

$$h(m,T)dm = H m^{-\sigma} dm, \quad (42)$$

one obtains the following set of relations

$$\frac{\partial h(m,T)}{\partial t} \Big|_{\text{erosion}} = - \frac{\partial}{\partial m} [h(m,T)\dot{m}] \quad (43)$$

where  $\dot{m}$  is given by equation A-1 in the Appendix,

$$\frac{\partial h(m,T)}{\partial t} \Big|_{\text{catastrophic}} = - h(m,T)KA \Gamma^{\alpha-1} \frac{m^{-\alpha+5/3}}{\alpha-1} \quad (44)$$

collisions

and

$$\frac{\partial h(m, T)}{\partial t} \Big|_{\text{creation}} = m^{-\eta} K \int_{m/\Lambda}^{M_\infty/\Gamma'} dM_1 \int_M^{M_\infty} dM_2$$

$$\times C(M_1, M_2) f(M_1) h(M_2, T) (M_1^{1/3} + M_2^{1/3})^2 \quad (45)$$

Equation 45 is the result of the model crushing law assumed in the text (eq-10, 15 and 16); and can be seen to be reasonable because  $f(M_1) h(M_2, T) K (M_1^{1/3} + M_2^{1/3})^2 dM_1, dM_2$  is the "influx" of sporadic particles in the mass range  $M_1$  to  $M_1 + dM_1$  into stream particles of masses in the range  $M_2$  to  $M_2 + dM_2$  per unit time and unit volume. Since  $C(M_1, M_2) m^{-\eta} dm$  is the number of fragments produced during each such collision in the mass range into  $m + dm$ , the relation eq-45 follows.

Equation 41 can be simplified somewhat; it can be shown with some algebra, that the catastrophic term eq-44 is greater than the erosion term eq-43 in the ratio of approximately

$$\left( \frac{\Gamma'}{\Gamma} \right) \left( \frac{\Gamma' \mu}{m} \right)^{\alpha-2} \quad \text{for } \alpha > 2 .$$

Since for  $\alpha = \frac{7}{3}$ , this ratio is about unity for objects with masses in the Kg range only, and since this ratio increases with decreasing  $\alpha$ , we may disregard the erosion term from eq-41 altogether. This gives us a reasonable description of particles in our size range of interest which is in the range of milligrams to grams.

A further approximation can be introduced because it can be shown that the contribution of catastrophic collisions to the particle creation term eq-45 is greater than the

contribution of erosive collisions in the ratio of about  $\Gamma'/\Gamma \approx 50$ . Hence we shall disregard the contribution of erosive collisions to eq-45.

Using eq-4, 41, 42, 44 and 45, we obtain, after applying the simplifications discussed,

$$\begin{aligned} & \text{AHK}(\Gamma')^{\alpha-1} (\alpha-1)^{-1} m^{-\sigma-\alpha+5/3} dm \\ & = \text{AHK}(\Gamma')^{-\sigma+8/3} \Lambda^{\alpha+\sigma-11/3} (8/3-\sigma)^{-1} (\alpha-\eta+\sigma-5/3)^{-1} m^{-\alpha-\sigma+5/3} dm \end{aligned} \quad (46)$$

where  $\alpha+\sigma-\eta-5/3 > 0$  and where grazing collisions have not been included. The left hand side of eq-46 is the particle removal rate in the mass range  $m$  to  $m+dm$  because of catastrophic collisions and the right hand side is the particle creation rate because of catastrophic fragmentation in the same mass range.

Equation 46 expresses the relationship that must exist between the various parameters if a steady state condition exists. It is satisfied, identically if

$$\alpha-1 = -\sigma+8/3 \quad (47)$$

in which case eq-46 reduces to  $1=1$ . The only restrictions on the parameters used are:

- (i)  $\Gamma' \gg \Gamma$
- (ii)  $\alpha+\sigma-\eta-5/3 > 0$ , i.e.,  $\eta < 2$  (48)
- (iii)  $\alpha > 5/3$ ,  $\alpha \neq 8/3$

The first of these relations expresses the experimentally observed fact that the largest (rock) object disrupted by a projectile is much greater than is the mass cratered out by a similar projectile impacting a semi-infinite target. Condition (ii) is the only restriction on the index  $n$  of the comminution process. It can be shown that (cf. eq-10) for  $n < 2$ , most of the crushed mass is contained in the larger fragments but for  $n > 2$ , practically all of the crushed mass is contained in rock flour. It is obvious that if the crushed mass consists of rock flour, blown away by radiation pressure, no steady state population of the type under discussion can exist.

We therefore have the significant result that for arbitrary  $n < 2$  stream particles will come to a steady state distribution with an index  $\sigma$  given by eq-47. Thus,

$$\begin{aligned} \text{for } \alpha = 7/3, \quad \sigma = 4/3 \\ \text{for } \alpha = 2, \quad \sigma = 5/3. \end{aligned} \tag{49}$$

These relations were derived from the approximate relation eq-46 where the contribution of erosion eq-43 has been disregarded. The values for  $\sigma$  in eq-49 are therefore also to be regarded as approximate.

#### V. APPLICATION TO PERMANENT SHOWERS

In this section we discuss the observed mass index  $\sigma$  of six permanent showers. More specifically, we compare the observed values of the shower mass index  $\sigma_{\infty}$  for very long and  $\sigma_0$  very short shower ages for given values of the index  $\alpha$  of the sporadic flux.

The observational material we consider consists in shower mass indexes obtained from radio work and some obtained from photographic work. The photographic data have been obtained from meteor data published by McCrosky and Posen (1961) by printing out from the tape the respective shower meteors. The photographic data are somewhat inhomogeneous insofar as their bulk consists in the approximately reduced meteors (graphic method, McCrosky, 1957) while a significant portion

of the shower meteors have been precisely reduced by Whipple and Jacchia (1957). While individual meteors reduced by the graphic method may be sufficiently in error to be useless for precise study (McCrosky and Posen, 1961), they should be adequate for our present statistical purposes. The mass index values we obtained are a result of fitting the shower mass distribution with a method which gives the maximum likelihood estimate of the mass index (Dohnanyi and Marcus, unpublished).

The photographic data for the Geminid and Perseid showers had to be discarded because the type of sampling technique employed by McCrosky and Posen for these two showers is not suitable for the purposes of the present analysis. The investigators were mainly interested in sporadic meteoroids and hence over a hundred Geminids have not been treated photometrically leaving a treated sample of 70 meteors; a strong systematic error may therefore be present in the distribution of these meteors (McCrosky, private communication). The Perseids comprise a sample of 43 photometrically treated meteors and 5 meteors have been omitted. In view, however, of the unusual shape of the mass distribution of these meteors, an addition of 5 bright meteors may significantly change the distribution.

The results are indicated in Figures 4 through 8. Plotted in these figures are the observed shower mass indices (ordinates) plotted against radio and photographic magnitudes (abscissae). Radio observations are labelled with the letter R followed by the respective reference number while photographic observations are labelled P and the shaded area corresponds to the variance from the mean value. Probable errors for radio measurements are not indicated since they have not been published. The calibration of radio magnitude  $M_R$  in terms of photographic magnitudes  $M_p$  obtained by Verniani and Hawkins (1964) has been used in the figures.

The horizontal lines labelled  $T \sim 0$  corresponding to sporadic flux population indices  $\alpha=2$ ,  $13/6$  and  $7/3$ , respectively are the initial shower mass indices required to maintain the sporadic flux at the particular value of  $\alpha$ . By the initial distribution of a shower we mean the distribution at the time when the rate of dispersal of its mass into the sporadic environment is the greatest. It is also assumed that the particles are sufficiently small and catastrophic collisions dominate.

The horizontal lines in Figure 4 through 8, labeled  $T \gg 0$  corresponding to the sporadic meteoroid indices  $\alpha=2$ ,  $13/6$  and  $7/3$ , respectively, are the shower indices that will be reached after a very long time due to the destructive action of the sporadic flux on the shower population. As has been

discussed in Section IV of this paper, the stream particles will reach a steady state distribution under the influence of particle removal by collisions with the sporadic particles and particle creation by the fragmentation of other stream particles.

The datum line for the daytime Arietid shower in the magnitude range  $.5 \leq M_R \leq 2$  is a lower limit because experimental difficulties precluded a more precise reduction of the observations (Browne et al, 1956).

Inspection of the Figures 4 through 8 reveal a systematic trend of the shower mass indices,  $\sigma$ : for faint meteors  $\sigma$  tends toward smaller values for increasing  $M_R$  and vice versa,  $\sigma$  tends to increase with decreasing magnitude.

The shower index  $\sigma$  tends toward the value  $1.5 \pm .1$  for faint meteors in every shower considered save the Quadrantids which have a low value of  $\sigma \approx 1.7$  for faint meteors. The value of  $\sigma \approx 1.5$  corresponds to the steady state distribution of shower particles when the sporadic flux has an index of about  $13/6$ . Conversely, if the sporadic meteoroid population has an index of  $13/6$  (which is suggested by the evidence in Figure 1), we may assume that the five out of the six showers considered are sufficiently old that their small particles have reached a steady state distribution under the collisional environment of the sporadic flux.

For bright meteors, the minimum value of  $\sigma$  reached by bright meteors is about 2.7 for the Arietids, and Aquarids and Geminids. A value of about  $2 \pm .1$  is reached by bright Aquarids, Perseids and Quadrantids. It therefore follows that, if the sporadic flux has an index of  $13/6$  and if the Arietids, and Aquarids and Geminids are representative, they are sufficiently young that their bright meteors still have a distribution similar to the showers' initial distribution. By "representative", we mean representative of the mass distribution of the source function necessary for the maintenance of the sporadic meteoroid environment.

Assuming that our interpretation of the observations summarized in Figure 4 through 8 is correct, we can establish the following inference regarding the mass index  $\alpha$  of the sporadic meteoroids. Bright meteors from the Arietid ( $M_R < \sim 6$ ), Aquarid ( $M_R < \sim 1$ ) and Geminid ( $M_R < \sim 1$ ) streams have the right

form to maintain a sporadic environment with index  $\alpha=13/6$  when the bright stream meteors are dispersed by gravitational perturbations into the sporadic background. We therefore have demonstrated that at mass values corresponding to the magnitude range under discussion source functions do exist which have the right form required to maintain the sporadic population. The mass range under discussion can approximately be estimated from our tentative identification of radio and photographic magnitudes and the corresponding mass ranges obtained in the McCrosky and Posen (1961) meteors. The latter will be multiplied, however, by a factor of 6.46 (Dohnanyi, 1966) to bring them into agreement with the more recent Harvard mass scale (Veriani, 1965). The result is then that  $\delta$  Aquarid meteors are a satisfactory source function for sporadic meteors (with index  $\alpha=13/6$ ) for masses of about  $5 \times 10^{-2}$  grams, and the Geminids would do the same for sporadics having masses in the neighborhood of about .1 gm. To estimate the approximate masses of Arietids with  $M_R \approx 6$ , we take an average velocity of 39 Km/sec (Kashcheyev, et al, 1967) and assume that  $M_R$  is proportional to  $-2.5 \log_{10}$  (meteor mass). Since the Arietids have a comparable velocity to the Aquarids ( $\approx 42$  Km/sec) I shall assume that the particle masses for a given magnitude of these two showers are comparable. We then have a value of the order of  $5 \times 10^{-4}$  grams for the mass of  $6M_R$  Arietids. It must be emphasized, however, that the masses of radio meteors are not well known. Radio masses are uncertain by perhaps an order of magnitude or more. Thus, we have established that a source function of the right form exists that helps to maintain the population of sporadic meteors in the mass range of the order of  $5 \times 10^{-4}$  grams.

Presently I estimate the order of magnitude of the mass range of sporadic meteors for which an index  $\alpha \approx 13/6$  is implied by the distribution of faint shower meteors. The masses of sporadic meteors that most strongly influence the population of shower particles are about  $\Gamma'$  times smaller than the masses of the shower particles affected by catastrophic collisions with the sporadic meteors. Since  $\Gamma'$  is of the order of  $10^4$ , the mass index  $\alpha$  of sporadic meteors suggested by faint shower meteors in steady state distribution refers therefore to the sporadic meteor masses about  $10^{-4}$  times the respective shower particle masses.

In the previous paragraph we estimated that the  $6^{M_R}$  Arietid meteor has a mass of the order of  $5 \times 10^{-4}$  grams. Arietids appear to have a steady state distribution for  $M_R > 9$  or 10 and hence, this would correspond approximately to masses of the order of or less than  $10^{-5}$  grams. The masses of sporadic meteors for which an  $\alpha = 13/6$  is indicated would then be about  $10^{-9}$  grams or smaller. It is interesting to note that for these sporadic masses the satellite data indicate a general "levelling off" of the sporadic distribution. No such effect is, however, indicated in the distribution of faint Arietid particles. If the sporadic population were levelling off (i.e., the index  $\alpha$  of sporadic meteors would suddenly diminish), the number of sporadic meteors capable of producing catastrophic collisions would sharply decrease. The result would be a relative increase in the number of small shower particles, i.e., the mass index of the shower particles would begin to increase. In view, however, of the considerable uncertainty in the masses of radio meteors, it is possible that I have over estimated the masses of the radio meteors and therefore the apparent discrepancy does not exist.

For a  $0^{M_R}$   $\delta$  Aquarid meteor I estimated a mass of about  $5 \times 10^{-2}$  grams. The magnitude at which steady state appears to be reached is  $M_R > 6$ . This would correspond to a  $\delta$  Aquarid meteor of about  $5 \times 10^{-4}$  grams or smaller. Thus, the sporadic meteors of about  $10^{-4}$  times their value, i.e., with masses at about  $5 \times 10^{-8}$  grams or smaller one would estimate a mass index of  $\alpha = 13/6$ . Since the satellite measurements imply a definite change in  $\alpha$  at about  $10^{-8}$  grams, the inference drawn from the Arietids is still consistent with current estimates of the sporadic population. We have estimated that Geminids in the magnitude range of  $0 < M_R < 1$ , approximately, have a mass of about .1 gm. Geminids appear to have reached steady state in the range  $\sim 7 < M_R < 11$ . This would correspond to a mass of about  $10^{-4}$  grams and hence implies a sporadic mass index of  $13/6$  for sporadic particles with masses of about  $10^{-8}$  grams.

Using the data of McCrosky and Posen (corrected by a factor of 6.46, as discussed earlier) we obtain a mass of about  $10^{-2}$  grams for a  $0^{M_P}$  Perseid meteor. Perseids seem to



have reached steady state at magnitudes  $M_R > 6$  or 7. This would correspond to a mass value of about  $10^{-4}$  grams and imply an  $\alpha \approx 13/6$  for sporadic particles with masses of the order of  $10^{-8}$  grams.

In the foregoing discussion, it was assumed that the shower mass indices are correct as published in the literature. A possible source of uncertainty persists, however, inasmuch as the radio results plotted in Figure 4 through 8 assume a direct proportionality between meteor masses and maximum electron line densities,  $q_M$ . In the case of fragmenting meteors, however, strong "flares" may exist, undermining this relationship. Furthermore, the relationship between meteor mass and  $q_M$  may vary from shower to shower depending on the latter's physical composition. It is not known, at the present time, how to take these effects into account.

The present model predicts that, given sufficient time, the fainter meteors in a shower of fairly arbitrary population index type distribution will reach a value of  $\sigma_{\infty} = 11/3 - \alpha$  with  $\alpha$  the population index of the sporadic environment. If the population index of the initial shower distribution is smaller than  $\sigma_{\infty}$ , than the fragments created during catastrophic and erosive collisions of larger shower particles will increase the relative number of the smaller particles (c.f., Dohnanyi, 1968) and hence "steepen" the distribution until the index  $\sigma_{\infty}$  is reached and vice versa, if the shower had an initial distribution with an index greater than  $\sigma_{\infty}$  (which is the assumed situation here) then the analysis in Section IV indicates that an equilibrium index of  $\sigma_{\infty}$  will be reached. The agreement between this result with the observations of faint shower meteors would imply that the masses of faint shower meteors are approximately proportional to the maximum zenithal electron line densities they produce. Otherwise it is unlikely that such a close agreement would exist between the present theory and experiment.

The case of bright meteors is somewhat more tenuous; our model requires only that the average mass in all showers should be distributed with a population index  $\sigma_0 = 2\alpha - 5/3$  if

the collisional processes in the sporadic environment are dominated by catastrophic collisions. If the collisional processes are dominated by erosion, we have

$$\sigma = \alpha + 1/3$$

In practice, for meteors in the range of our interest, catastrophic collisions will dominate, but for bright meteors, an intermediate value for  $\sigma$  may be appropriate. This argument applies to the distribution of masses averaged over all showers; since more recent showers presumably contain more mass than do the old ones, the argument applies only to very recent showers and to the bright meteor portion of older showers. The argument does not imply that every shower is initially distributed according to the formula under discussion; it merely requires that the average over the showers comprising most of the mass is distributed accordingly. It therefore follows that even if our tentative identification of the bright Arietids,  $\delta$  Aquarids and Geminids as representing their initial distribution is in error because the population indices of these showers should be adjusted downwards because of fragmentation, (as would be the case for many bright photographic meteors, Jacchia et al, 1965), our model would still be applicable.

## VI. INFLUENCE OF RADIATION EFFECTS

### A. Radiation Damping

In this paper we have analyzed some aspects of the evolution of meteor streams without having considered the influence of the Poynting Robertson effect (Robertson, 1936). We shall presently consider the relative times required for collision processes and for the separation of small particles from meteor streams.

According to Robertson (1936) radiation from the sun's radiation field causes particles to spiral in toward the sun. The rate at which the semi-major axis and eccentricity of the particles diminishes is proportional to the product of the particle radius times it's material density. This means that small particles spiral in toward the sun faster than do larger ones. One would, therefore, expect that a given meteor stream loses all of its particles with masses smaller than a given amount

(the "cutoff" mass would actually be a function of time). This has not been observed at the present time (Wyatt and Whipple, 1950).

The situation is actually more complicated, as has been pointed out by Wyatt and Whipple (1950). The mechanism responsible for the dispersal of meteor streams, namely gravitational perturbations by planets, constantly acts on the meteor stream particles. This process does not depend on the particle masses and hence retards the action of radiation damping in the sense that a very small particle in the earth's atmosphere is slowed down in its gravitational free fall by Brownian motion. This analogy is correct if it can be shown that the time scale of meteor dispersal by gravitational perturbations is much shorter than is the corresponding time scale of radiation damping. Specifically one needs to know the characteristic time for the dispersal by gravitational perturbations of a swarm of particles moving in highly correlated orbits around the sun with semi-major axis in the range  $a$  to  $a+\Delta a$  and eccentricities in the range  $e$  to  $e+\Delta e$ . This, to the knowledge of the writer, is still an unsolved problem. We can only say then that gravitational perturbations lengthen the characteristic Poynting Robertson decay times for shower particles (and sporadic meteoroids as well).

Another radiation force (for a review, see Jacchia, 1963) acting on meteoroids is the so-called Yarkovski-Radzievski effect (Radzievski, 1952, et al, Opik, 1951). This effect consists in the fact that more photons are radiated from a hot surface than from a cold one. Hence, if a particle is spinning in the same sense as it is rotating around its orbit, its "evening side" will be hotter than its morning side and will therefore experience an acceleration; if it is spinning in an opposite sense to its orbital motion, a damping will occur. Since we do not know anything about the period or sense of rotation of meteoroids, the magnitude of the effect cannot be estimated. It is, however, important to point out that the Yarkovski-Radzievski effect may partially obscure the significance of the Poynting Robertson effect.

In the light of the foregoing, estimates of particle lifetimes due to the Poynting Robertson effect are likely to be lower limits. Even then, mass distributions of meteor showers due to collisional processes between shower particles and sporadic particles will be unaffected unless radiation damping completely removes a large fraction of the particles from the shower. If radiation damping only gives rise to a relatively small separation between large and small particles (during a certain length of time), the model discussed in this paper still

applies since we are not concerned with the "self interaction" of stream particles but in their interaction with the sporadic flux which is independent of the relative spatial distribution of particle sizes in a stream (as long as they are not completely removed).

We shall presently estimate the relative importance of radiation damping compared with collisional processes. Following Wyatt and Whipple (1950) we compute the time required for a shower particle to change the true anomaly of the intersection of its orbit with earth's orbit by  $5^\circ$ . This corresponds to a travel time of 5 days for the earth. Our method of calculation differs from that of Wyatt and Whipple (1950) inasmuch as I used a short cut for the estimation of the change in the orbital eccentricity  $\Delta e$ . Instead of graphically calculating  $\Delta e$ , I calculated  $\Delta e$  from the total derivative  $d\theta$  of the true anomaly  $\theta$ ; the result is a much faster but somewhat less accurate calculation.

The results for the time of separation from the original orbit are listed in Table II for the showers under our present consideration. Two values of material densities have been used; one for basalt with  $\rho = 3.5 \times 10^3 \text{ Kg/meter}^3$  and one for pumice with  $\rho = .6 \times 10^3 \text{ Kg/m}^3$ .

We also list in Table II the life time  $\tau_{cc}$  with respect to catastrophic collisions of shower particles. We use the "minimum model" for the sporadic flux in order to obtain a conservative estimate (for the "maximum model"  $\tau_{cc}$  is shorter). Using eq-30, one obtains,

$$\tau_{cc} = \left[ AK \Gamma' m^{-1/3} \right]^{-1} \quad (50)$$

where the symbols have been defined earlier. The quantity  $\tau_{cc}$  is the time required for the number of particles in a mass range  $m+dm$  to decay to  $1/3$  of its value because of catastrophic collisions provided that no other processes operate. For showers with a population index  $\sigma = 11/3$  - a particle creation by fragmentation is negligible (Dohnanyi, 1968) and hence the only collisional processes operating are catastrophic and erosive collisions. The quantity  $\tau_{cc}$  is therefore an upper limit to the particle lifetime since we will not include the contribution of erosion to the effective particle lifetime.

In what follows, two types of material composition will be considered: basalt having a density of 3.5 gm/cc which may be representative of moderately dense particles during hypervelocity impact processes, and pumice with a density of about .6 gm/cc which may be representative of fluffy particles. For basalt,  $\Gamma'$  can be taken as (Dohnanyi, 1967):

$$\Gamma' = 250 v^2$$

where  $v$  is the impact velocity in Km/sec. For pumice, I take a value for  $\Gamma'$  about .1 times its value for basalt (Gault, 1968, private communication, also cf. Moore and Robertson, 1966).

In order to estimate the collisional environment of the meteor stream particles, I assume that the collisional environment they experience over their entire orbit is similar to the environment they experience near Earth. This is not accurate, inasmuch as shower particles experience a considerably more severe environment at distances less than 1 AU from the sun and a considerably "milder" environment toward aphelion (where the particles spend a greater part of their time).

From eq-50, it can be seen that  $\tau_{cc}$  is inversely proportional to the constant  $K$  and the parameter  $\Gamma'$  which is the largest mass completely disrupted by a projectile with unit mass.  $K$  is proportional to the mean relative velocity of the colliding particles;  $\Gamma'$  is proportional to the projectile kinetic energy with respect to the target particles and hence to the square of the mean relative velocity. Collisional rates are therefore proportional to the cube of the mean velocity. Since the heliocentric velocity of particles increases as  $R^{-1/2}$  where  $R$  is their distance to the sun, we see that collisional rates are greater than at 1 AU from the sun by about a factor of 4 at Mercury's orbit ( $R \sim .31$  AU) and by a factor of about 10 at  $R = .2$  AU due to the velocity factor. In turn, collisional rates decrease (because of the decrease in velocity) by about an order of magnitude near Jupiter.

We also note, from eq-50, that  $\tau_{cc}$  is inversely proportional to the number density normalization constant (i.e., number density of unit mass)  $A$  of sporadic meteors. The spatial variation of  $A$  in the solar system is difficult to determine.

According to a recent study by Southworth (1967)  $A$  is proportional, approximately to  $R^{-3/2}$ . This would decrease the collisional rates by an order of magnitude near Jupiter and increase the rates by an order of magnitude at  $R \sim .2$  AU.

Taking the spatial variation of the mean encounter velocity and the sporadic number density into account, I estimate that collisional rates increase by about 2 orders of magnitude near  $R \sim .2$  AU and decrease by about the same factor near Jupiter. Whence, if a group of shower particles spend one percent of their time near  $R = .2$  AU and the rest of the time they experience no collisions at all, they would have average collisional rates correctly given by their collisional environment at  $R = 1$  AU. This forms the basis for an order of magnitude estimate for  $\tau_{cc}$ , which I take on the average to equal its value at  $R = 1$  AU.

For the normalization constant,  $A$ , I use the "minimum model" value given by eq-4 and Table I.1 estimate the approximate value of mean collisional velocities  $\bar{V}$  (shower) of shower meteors by combining the geocentric velocities  $V_g$  (shower) with the mean geocentric velocities  $V_g$  (sporadic) of sporadic meteors:

$$\bar{V}(\text{shower})^2 = V_g(\text{shower})^2 + V_g(\text{sporadic})^2 \quad (51)$$

We are now ready to discuss the significance of Table II. It can be seen, from the table, that for a given material composition  $\tau_{cc}$  is shorter than  $\tau_{PR}$  for every shower considered.  $\tau_{cc}$  is shorter than  $\tau_{PR}$  by half an order of magnitude to two and a half orders of magnitude, depending on the particular shower. It should also be pointed out that the Poynting Robertson separation times  $\tau_{PR}$ , under discussion are the times of separation that can be traveled by earth in about 5 days. If such a separation actually occurs, the small particles are still recognizable members of the "main shower" and hence the mass distribution of the shower particles averaged over their entire radiant is still determined by collisional processes. The separation times  $\tau_{PR}$  are likely to be underestimated because of the other factors we have discussed earlier,

and hence the influence of radiation damping can be disregarded in comparison with collisional processes for purposes of estimating the mass distribution of shower meteors. The mass range for the shower particles under discussion for which the argument applies includes masses equal to or greater than  $\Gamma'\mu$  where  $\Gamma'\mu$  is the largest mass that can be completely disrupted by a particle with mass  $\mu$ .  $\mu$  is the mass at which the distribution of sporadic meteors changes (cf Figure 1) from a distribution with index  $\alpha$  to a distribution with a smaller index (for smaller meteors). Since in  $\tau_{cc}$  we used the minimum model ( $\alpha=2$ ), it follows that use of a model sporadic environment with  $\alpha>2$  normalized to agree with the distribution of photographic meteors will give even shorter values for  $\tau_{cc}$  and hence  $\tau_{PR}$  becomes even less significant in determining shower mass distribution.

An interesting feature of the values of  $\tau_{cc}$  in Table II is the relatively small difference between  $\tau_{cc}$  for pumice and  $\tau_{cc}$  for basalt for any given shower. This is due to the fact that even though less mass is cratered out during an impact into pumice than would be the case with basalt, the pumice particle has a greater geometrical cross sectional area because of its lower density than does the basalt particle. This has the effect of increasing the collisional rates for the under dense particles. The net effect is that the values of  $\tau_{cc}$  are only moderately effected by material density.

#### B. Influence of Radiation Pressure

When the sun's radiation is incident on a particle in the solar system, a certain amount of pressure will be exerted on the particle. This classical effect has been treated in the literature (see, e.g., Robertson, 1937, Van De Hulst, 1962) with the result that the resultant forces, gravitational and electromagnetic, binding the particle to the sun decreases with the particle's radius. For particle radii smaller than a certain critical value, the electromagnetic force exceeds the gravitational force and the particle is blown out of the solar system. The critical radius at which this occurs is a function of the shape of the particle, its material density and its optical properties.

An interesting effect occurs, as a result of radiation pressure, if we consider the origin of cometary particles. Comets are believed to emit most of their debris near perihelion. Since

we are dealing with a central force problem the particles are given initial orbits whose perihelia are identical to that of the comet but the other orbital elements of the particles will be modified by radiation pressure. This problem has been discussed by Harwit (1963); the result is that a particle's semi-major axis  $a'$  is given by

$$a' = \frac{av(1-e)}{2v-e-1} \quad (52)$$

where  $a$  and  $e$  are the semi-major axis and eccentricity, respectively, of the parent comet. The quantity  $v$  is, for spherical particles (Harwit, 1963):

$$v = 1 - \frac{5.8 \times 10^{-5} \beta}{\rho \pi} \quad (53)$$

where  $\rho$  and  $\pi$  are the material density and radius, respectively, of the particle in c.g.s. units.  $\beta$  is the fraction of light, incident on the particle, which transfers momentum to it.

From eq-52, one can see that dust particles are sent into hyperbolic orbits if the eccentricity of the parent comet

$$e > 2v - 1 \quad (54)$$

Figure 9 is a plot of eq-54. Plotted in the Figure is the threshold mass of meteoroids at which they will be blown away by radiation pressure vs.  $1-e$  of the parent comet. Meteor densities of 1/2, 1, 2, and 3.5 gm/cc have been considered. The  $1-e$  values of the major showers are also indicated. One may assume that each comet which presumably gave rise to a major shower, had at the time it created the shower, an eccentricity similar to that of the shower. The intersection of each vertical line (representing the appropriate  $1-e$  value of the shower) with any line labelled by a density gives the smallest particle mass having the given density that can be



present in the shower. On that basis, one finds that none of the major showers contain particles smaller than about  $10^{-6}$  to  $10^{-9}$  grams having a density of 1/2 gm/cc. The radiation pressure cut-off on the shower masses occurs in the range of  $10^{-8}$  to  $10^{-10}$  grams for meteoroids having a density similar to basalt (3.5 gm/cc).

If the well known major showers are representative of meteor showers in the solar system, then one would expect a drop in the population of sporadic meteoroids with masses less than  $10^{-6}$  to  $10^{-8}$  grams, depending on their density. This would follow if the present model is correct and meteor showers do in fact sustain the sporadic background. Inspection of Figure 1 indicates that a drop in the sporadic meteor population does occur for masses less than about  $10^{-7}$  grams, and observations appears to support the present result.

It must be pointed out that the major showers may still contain a modest number of particles which are smaller than the cut-off values under consideration. Collisional fragments having masses less than the cut-off value under discussion but greater than the radiation pressure limit for bound particles will populate the mass distribution after a sufficiently long time.

## VII. TOTAL RATE OF METEORITIC MASS LOSS

In this section, the rate of losing mass by the population of sporadic meteoroids is estimated. The result is compared with existing sources of mass input. More specifically, we shall compute the total mass of particles that are created per second by fragmentation into a particle mass range equal to or less than  $\mu \approx 10^{-10.5}$  Kg; this is the mass loss rate, due to crushing, from particles with masses greater than  $\mu$ . Since, however, no attempt will be made to account for the spatial variation inside the solar system of the velocity and number distribution of meteoroids the results will be order of magnitude approximations only.

The number of collisional fragments created per unit volume and unit time in the mass range  $m$  to  $m+dm$  in the meteoritic cloud is given by an expression analogous to eq-45.

$$\left. \frac{\partial f(m)}{\partial A} \right|_{\text{creation}} = km^{-n} \int_{m/\Lambda}^{M_\infty/\Gamma'} dM_1 \int_M^{M_\infty} dM_2 C(M_1, M_2) f(M_1) f(M_2) (M_1^{1/3} + M_2^{1/3})^2 \quad (55)$$

where the symbols have been defined earlier. For  $f(m)$  we use eq-4 with  $\alpha=13/6$ ,  $\alpha_0=3/2$  and  $\mu=10^{-10.5}$  Kg.

The rate at which the population of particles having masses greater than  $\mu$  is losing mass (per unit time and unit volume) can now be obtained when eq-55 is multiplied by  $m$  and integrated over appropriate limits.

The result is

$$\dot{M}/\text{volume} = \Lambda^{n-2} K_\mu^{-1/3} \Gamma'^{1/2} M_T^2 [5(\Lambda^2 - 1) + (8/3 - n)^{-1}] / 18 \quad (56)$$

where  $\dot{M}/\text{volume}$  is the rate (per unit volume and time) at which the population of meteoroids with masses greater than  $\mu$  is losing mass because of fragmentation.  $M_T$  is the total mass per unit volume of meteoroids having masses equal to or greater than  $\mu$ .

The expression on the right hand side of eq-56 is the contribution of catastrophic collisions; erosion plays an insignificant role. The first expression inside the square brackets is the contribution of projectile particles with masses smaller than  $\mu$  breaking up target particles with masses greater than  $\mu$ . The second expression inside the square bracket is the contribution of catastrophic collisions when the projectile as well as the target masses are greater than  $\mu$ .

Whipple (1967) estimated that the total mass of the meteoritic cloud is about  $2.5 \times 10^{16}$  Kg distributed over a volume of 3.5 AU radius about the sun within an inclination  $i < 20^\circ$  from the ecliptic ( $2.25 \times 10^{35}$  meter<sup>3</sup>). The ratio of the total mass of particles with masses greater than  $\mu$  to the total mass of particles with masses less than  $\mu$  is) for a distribution of the type eq-4)  $(2 - \alpha_0) / (\alpha - 2)$ , where  $\alpha_0 < 2$  and  $\alpha > 2$ . Taking  $\alpha_0 = 3/2$  and  $\alpha = 13/6$ , this ratio is 3. Hence, we can take for  $M_T$ , 75% of Whipple's (1967) estimate.

Using appropriate values for the collision parameters we then obtain, for basalt particles,

$$\dot{M} = 2.8 \times 10^4 \text{ Kg/sec} \quad (57)$$

and for pumice particles

$$\dot{M} = 2.5 \times 10^4 \text{ Kg/sec} \quad (58)$$

These figures compare favorable with Whipple's (1967) estimate of about 1 or 2 x 10<sup>4</sup> Kg/sec.

It can be seen, from eq-57 and 58 that  $\dot{M}$  is not very sensitive to the material composition of the particles. Stones fragment more extensively than do "fluffy" particles but stones have smaller physical dimensions (viz collision crosssections) and therefore encounter less collisions than do "fluffy" particles. The net result is a compensation giving rise to similar values for  $\dot{M}$  in each case.

Whipple (1967) has discussed the contribution to the meteoritic cloud by comets. He estimated that Halley's comet contributes about 5 x 10<sup>3</sup> Kg/sec and Comet Encke contributes about 3.5 x 10<sup>3</sup> Kg/sec. Other comets do not appear to contribute significantly to the meteoritic cloud. While the results of eq-57 and 58 are about three times higher than comets apparently can supply, it must be born in mind that our result for  $\dot{M}$  is approximate and this order of magnitude agreement should be considered encouraging.

#### VIII. DISCUSSION AND CONCLUSIONS

The currently favored theory of the origin of sporadic meteoroids is that they are shower particles dispersed by gravitational perturbations. The present study was undertaken in order to check the validity of this theory and to gain further insight into the physical significance of the known meteoroid distribution. Inelastic (two body) collisions destroy some particles and create a spectrum of fragments. The result is an evolution of the mass distribution of the meteoric population which, together with the influence of solar radiation on the particle population is described by a statistical model. Precise observations of sporadic and shower meteors are examined and their respective mass distributions are found consistent with the results of the analysis. Since meteor showers are believed to originate from comets, the results of this study constitute new evidence favoring the cometary origin of meteors.

Details of the analysis indicate that the present population of sporadic meteors is unstable: the number of particles destroyed by collisions in a given mass range per unit time is greater than the rate at which fragments, in this same mass range, are created by the disruption of larger particles. If the observed distribution of sporadic meteor masses is to be maintained in a steady state, a source function must exist "feeding" the population of sporadic meteoroids. The mathematical form of the required source function is calculated and compared with the observed mass distribution of several showers. It is found that the showers considered qualify for such a source function provided that the population index,  $\alpha$ , of the sporadic meteoroids is taken to be  $\alpha = 13/6$ .

The evolution of the mass distribution of shower particles is then considered. It is shown that the action of sporadic meteoroids colliding with the shower particles will cause the latter to approach, asymptotically, a definite distribution after a sufficiently long time. Small particles have shorter life times than do larger particles and therefore the asymptotic distribution occurs first at the small mass "region" of shower meteor mass distributions. This result is found to be consistent with the observed mass distribution of the meteor showers considered provided that, again, the population index  $\alpha$  of the sporadic distribution is taken to be  $\alpha = 13/6$ .

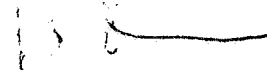
The rate at which the meteoritic population is losing mass by fragmentation is also considered. It is found that the population of particles with masses greater than  $10^{-10.5}$  Kg is losing mass at the rate of  $2.8 \times 10^4$  Kg/sec or  $2.5 \times 10^4$  Kg/sec depending on whether they are assumed to consist of basalt or pumice, respectively. These numbers are comparable to Whipple's (1967) estimates of the same mass loss rate as well as his estimate of the creation rate of new material by comets.

The Poynting Roberston effect is examined; it is shown that, in the range of radio and photographic meteors, collisional processes dominate the evolution of distribution of meteor masses.

Radiation pressure provides a small particle cut-off in the mass distribution of meteoroid masses. Appropriate consideration of the cometary origin of shower meteoroids leads to the inference that the major showers initially contained no particles having masses smaller than about  $10^{-11}$  Kg and hence

sporadic meteoroids having smaller masses  $10^{-11}$  Kg are mostly collisional fragments. In the absence of an effective source function for meteor masses smaller than about  $10^{-11}$  Kg, the number of sporadic meteoroids (in this mass range) is then expected to show a relative decrease. This supersedes my earlier suggestion that collisional processes along may explain the break in the mass distribution of sporadic meteoroids in this mass range (Dohnanyi, 1967).

To sum up, the following conclusions may be advanced: (1) sporadic meteoroids are very likely shower particles that have gone astray, (2) the best theoretical estimate of the population index,  $\alpha$ , of sporadic meteoroids in the radio and photographic range is  $\alpha = 13/6$ , (3) the break in the mass distribution of sporadic meteoroids in the range of satellite measurements may be due to radiation effects suffered by meteor shower particles shortly after their separation from the parent comet. (4) Radiation damping plays a minor role, compared with collisional processes, in determining the distribution of photographic and radio meteors.



J.S. Dohnanyi

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Attachment  
Appendix A  
References  
Figure 1 - 9

APPENDIX - A

A. PROOF OF THEOREM-1 for erosive processes. Since this proof is similar to the proof for catastrophic processes, we shall only sketch the highlights.

We first estimate the amount of mass  $\dot{M}(m_1, m_2)$  crushed by erosive collisions from objects in the mass range  $m_1$  to  $m_2$ . Disregarding grazing collisions, the rate  $\dot{M}$  at which an object of mass  $m$  is losing mass, due to erosion is (Dohnanyi 1967, 1968):

$$(A-1) \quad \dot{M} = -\Gamma K m^{2/3} \int_{\mu}^{m/\Gamma'} M f(M) dM$$

where  $\Gamma M$  is the mass of the crater dug by a projectile having a mass  $M$  (cf. Section II) and  $M/\Gamma'$  is the mass of the largest eroding projectile, i.e., any projectile with a mass greater than  $M/\Gamma'$  would completely disrupt  $m$ .

The mass crushed erosively per unit time and volume from a mass range  $m_1$  to  $m_2$ ,

$$(A-2) \quad \dot{M}(m_1, m_2) = \int_{m_1}^{m_2} f(m) \dot{M} dm$$

can then be calculated, using eq. 4 and A-2 with  $\alpha > 11/6$  and  $m_1 > \Gamma' \mu$ . The result is:

$$(A-3) \quad \dot{M}(m_1, m_2) = -a \left( \frac{b+1}{\alpha-5/3} \left[ \left( \frac{m_1}{\Gamma' \mu} \right)^{-\alpha+5/3} - \left( \frac{m_2}{\Gamma' \mu} \right)^{-\alpha+5/3} \right] - \frac{\left( \frac{m_1}{\Gamma' \mu} \right)^{-2\alpha+11/3} - \left( \frac{m_2}{\Gamma' \mu} \right)^{-2\alpha+11/3}}{2\alpha - 11/3} \right)$$

where

$$(A-4) \quad a = K \Gamma A^2 \Gamma'^{\alpha+5/3} \mu^{-2\alpha+11/3} (\alpha-2)^{-1}$$

and where

$$(A-5) \quad b = \frac{\alpha-2}{2-\alpha_0} \quad (1 - [\mu_0/\mu]^{2-\alpha_0})$$

represents the residual influence on the erosion process of small particles with masses in the range  $\mu_0 \leq m \leq \mu$ .

The mass of the largest particle created will certainly be smaller than is the mass of the crater produced by the largest erosive particle. We therefore take  $\epsilon'M$  as the mass of the largest particle created during an erosive collision with  $m$ , where

$$(A-6) \quad \epsilon' < \Gamma/\Gamma' \approx 1/50 = 2 \times 10^{-2}$$

The analogous expression for  $R$  in eq. 22 is:

$$(A-7) \quad R < \frac{1}{1 - (\Gamma'/\Gamma)^{-2\alpha+11/e}} (m_1/m_2)^{2\alpha-11/3} \quad 11/6 < \alpha < 2$$

and

$$(A-8) \quad R < \frac{1}{1 - (\Gamma'/\Gamma)^{-\alpha+5/3}} (m_1/m_2)^{\alpha-5/3}, \quad 2 < \alpha.$$

where

$$m_2 \geq (\Gamma/\Gamma')m_1.$$

When  $\alpha=2$ , the expression for  $R$  includes a slowly varying logarithmic factor in  $m_1$  and  $m_2$ .

It can be seen, from eq. A-7 and 8, that in both of these expressions,  $R$  is less than one, and hence more mass is lost in an arbitrary range  $m_1$  to  $m_2$ ,  $m_2 > (\Gamma'/\Gamma)m_1$  than can be created by erosive crushing from the entire distribution of masses greater than  $m_2$ .

TABLE II

Shower	$\tau_{PR}$ =Separation* Time For Pumice (yr)	$\tau_{PR}$ =Separation* Time For Basalt (yr)	$\tau_{cc}$ (yr)* Pumice	$\tau_{cc}$ (yr)* Basalt	$\langle \frac{1}{R} \rangle$	$\bar{R} = \left[ \langle \frac{1}{R} \rangle \right]^{-\frac{1}{3}}$
$\delta$ Aquarids	$1.8 \times 10^6$	$6.0 \times 10^6$	$1.6 \times 10^5$	$5.2 \times 10^4$	12	.44
Arietids	$8.5 \times 10^6$	$2.8 \times 10^7$	$1.8 \times 10^5$	$6.0 \times 10^4$	1.8	.82
Geminids	$2.5 \times 10^6$	$8.1 \times 10^6$	$2.4 \times 10^5$	$8.0 \times 10^4$	6.2	.54
Perseids	$1.9 \times 10^8$	$6.2 \times 10^8$	$5.5 \times 10^4$	$1.8 \times 10^4$	.080	2.3
Quadrantids	$3.0 \times 10^7$	$9.7 \times 10^7$	$1.6 \times 10^5$	$5.2 \times 10^4$	.26	1.6
$\eta$ Aquarids	$5.2 \times 10^7$	$1.7 \times 10^8$	$4.8 \times 10^4$	$1.5 \times 10^4$	.86	1.05

\*In units of  $m^{1/3}$  where m is the particle mass in Kg.



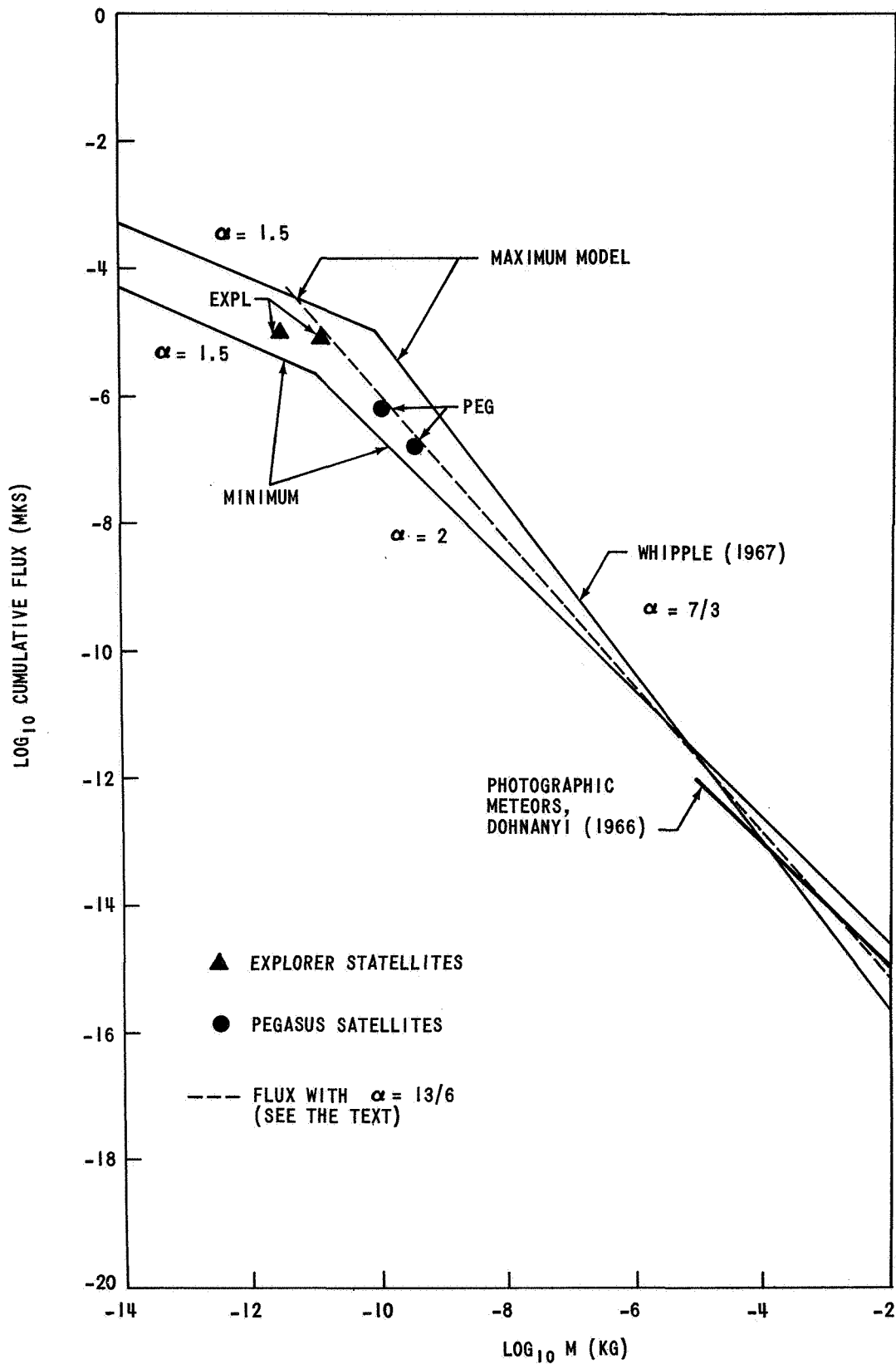


FIGURE-1 - CUMULATIVE INFLUX INTO EARTH'S ATMOSPHERE (IN METER-2 SEC-1) OF METEORS HAVING A MASS OF M (KG) OR GREATER;  $\alpha$  IS THE POPULATION INDEX OF THE CORRESPONDING NUMBER DENSITIES

- (1) WEISS (1961) RADIO
- (2) KAISER (1961a) RADIO
- (3) WEISS (1961) RADIO
- (4) MCKINLEY (1961) RADIO
- (5) ELFORD (1964) RADIO
- (6) SIMEK AND McINTOSH (1968)

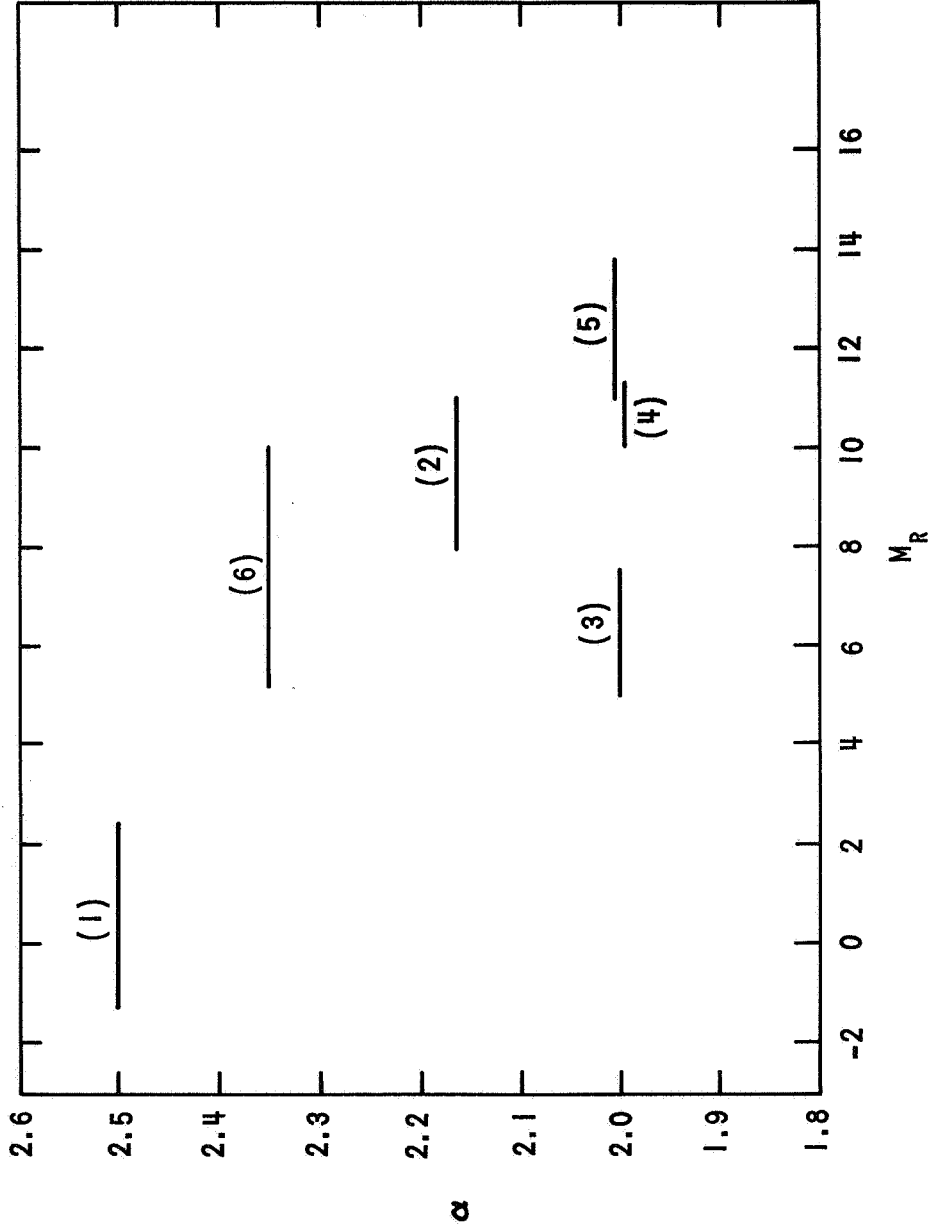


FIGURE 2 - VALUES OF THE POPULATION INDEX  $\alpha$  OF SPORADIC METEORS, DETERMINED BY RADAR METHODS. THE LENGTH OF THE LINE INDICATES THE RANGE OF RADIO MAGNITUDES APPROPRIATE TO EACH DETERMINATION

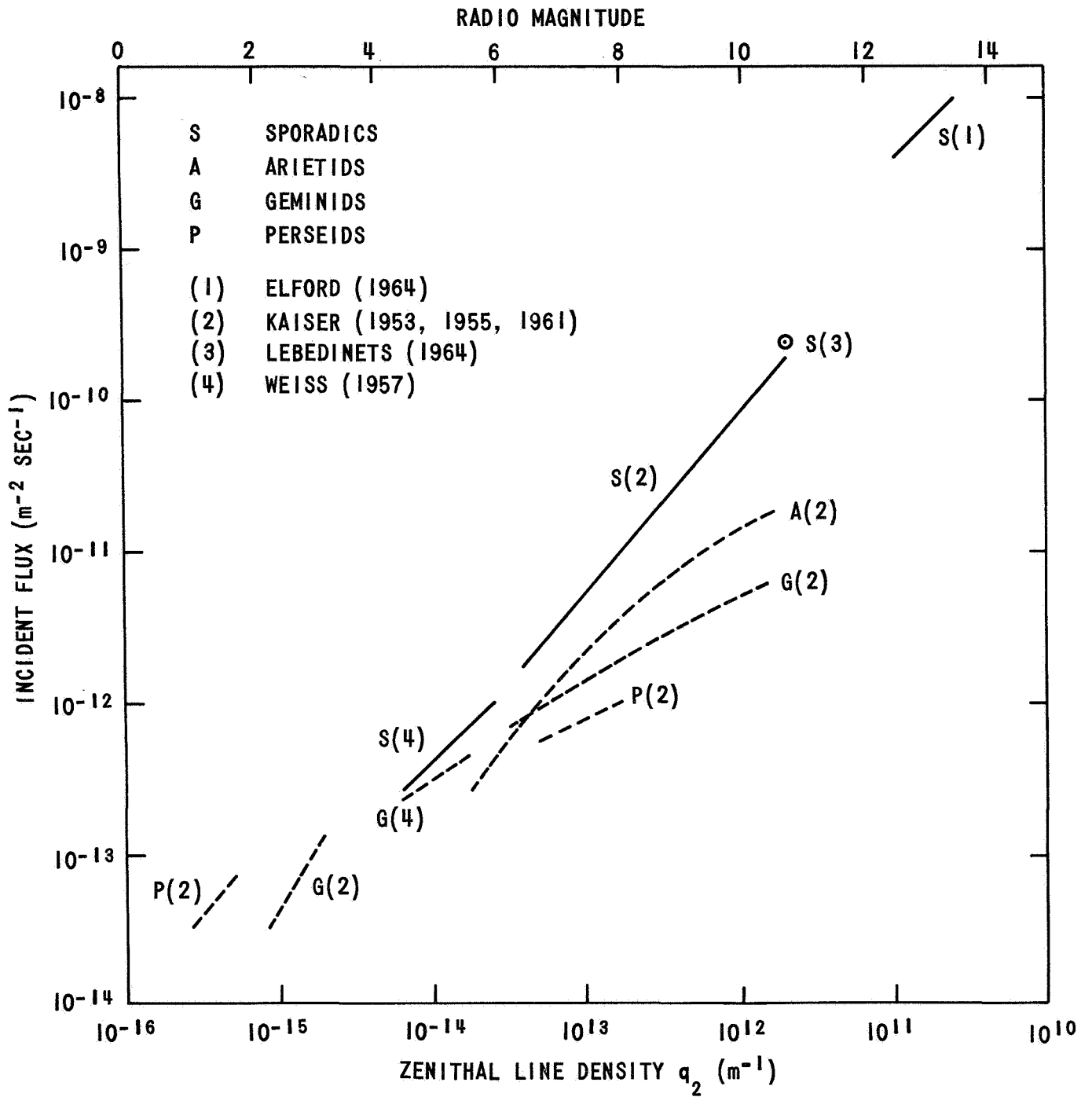


FIGURE 3 - MEAN INCIDENT FLUX OVER THE WHOLE EARTH OF METEORIDS CAPABLE OF PRODUCING METEORS WITH ZENITHAL ELECTRON LINE DENSITY GREATER THAN  $q_2$ . SOLID LINES, SPORADIC FLUX; BROKEN LINES, SHOWER FLUX

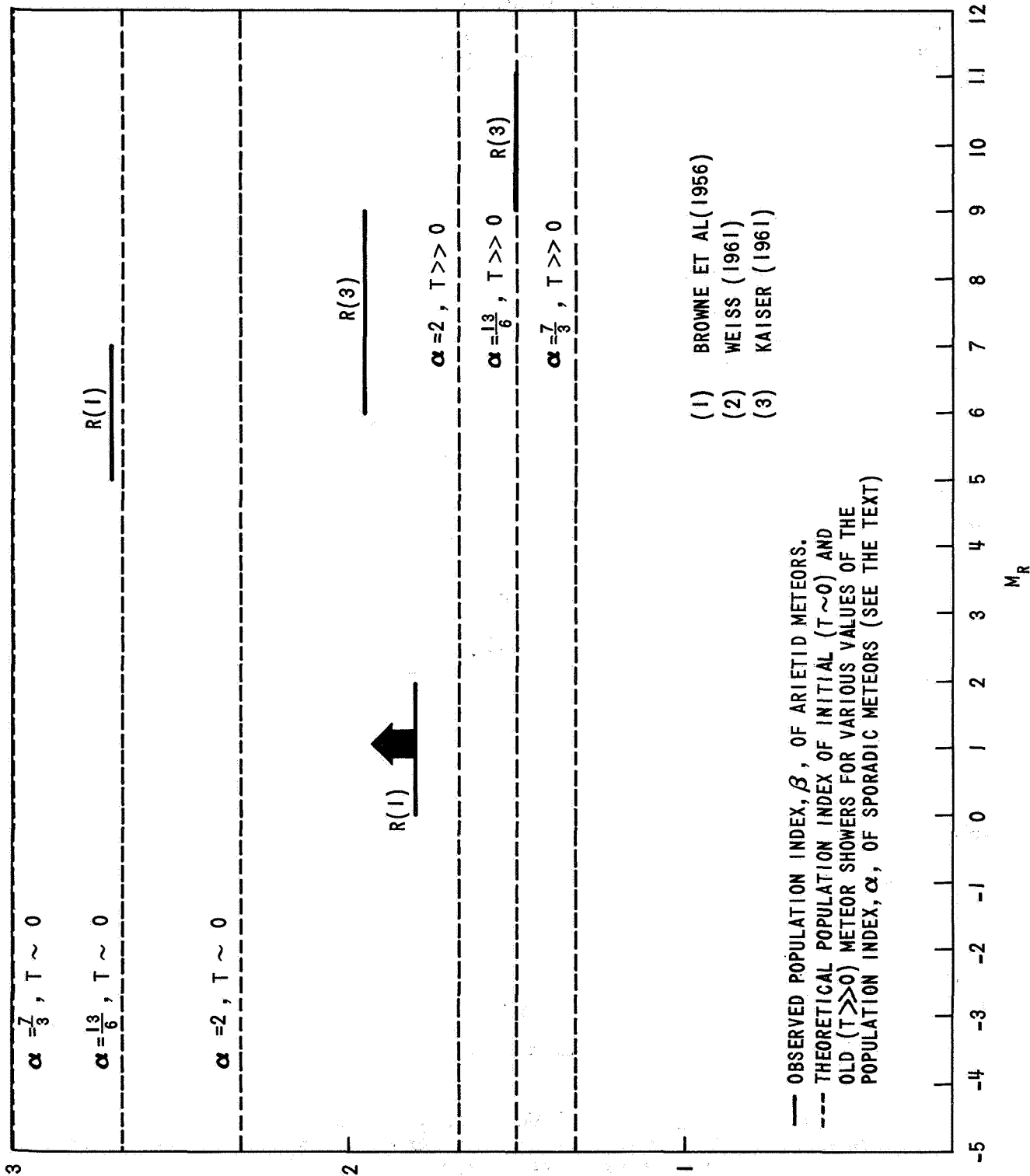


FIGURE 4 ARIETIDS

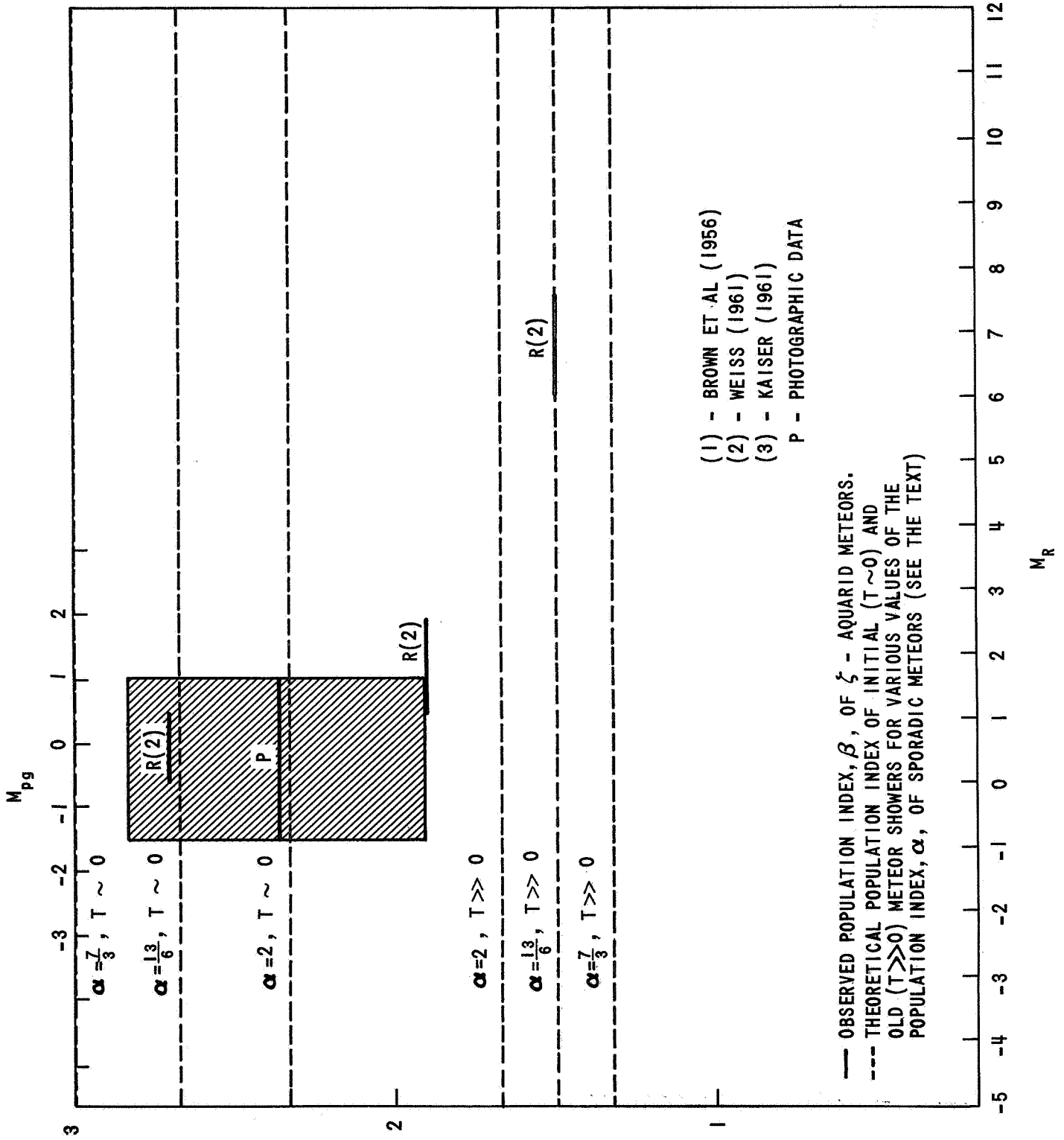


FIGURE-5  $\zeta$  - AQUARIDS

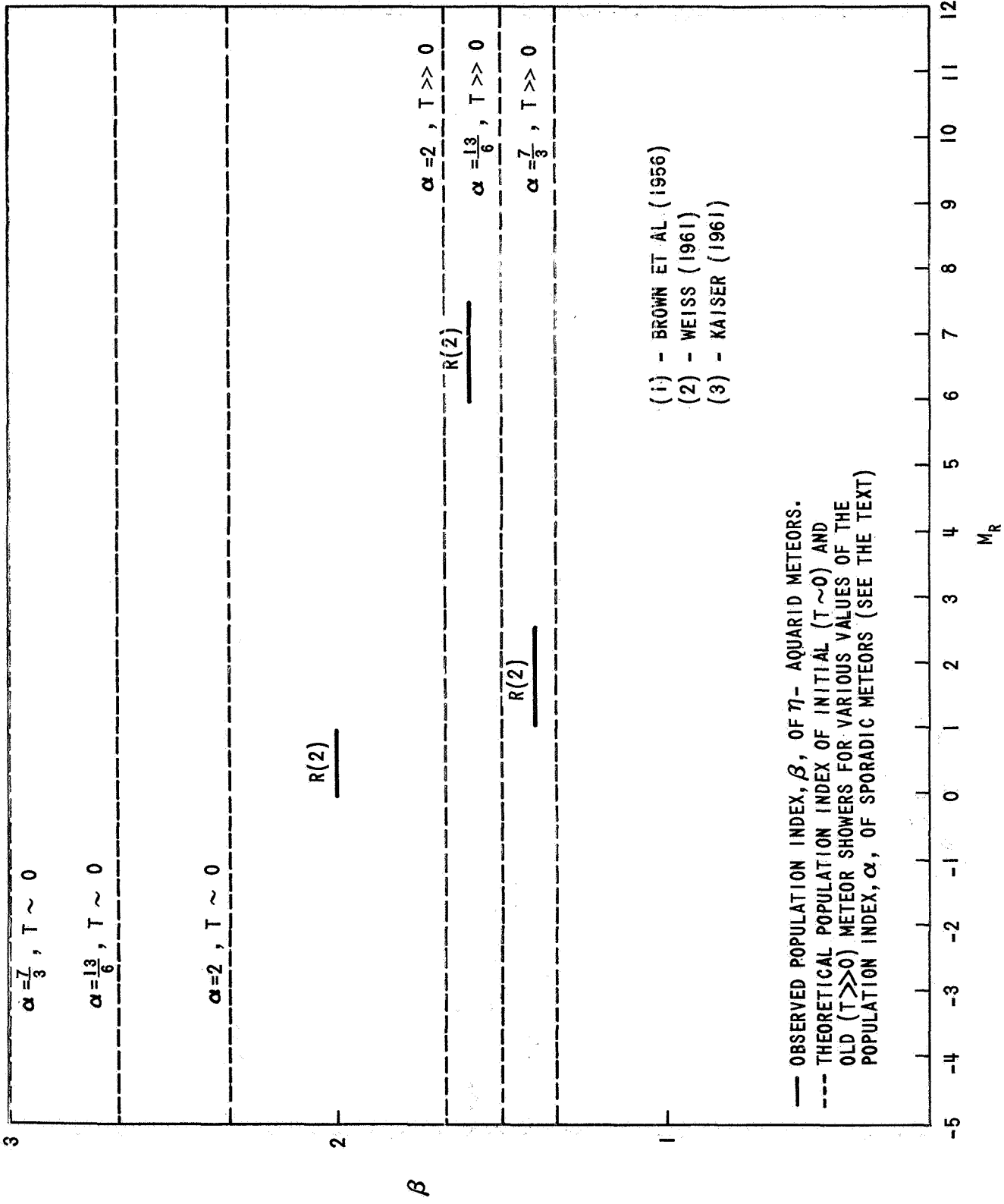


FIGURE-6  $\eta$  -AQUARIDS

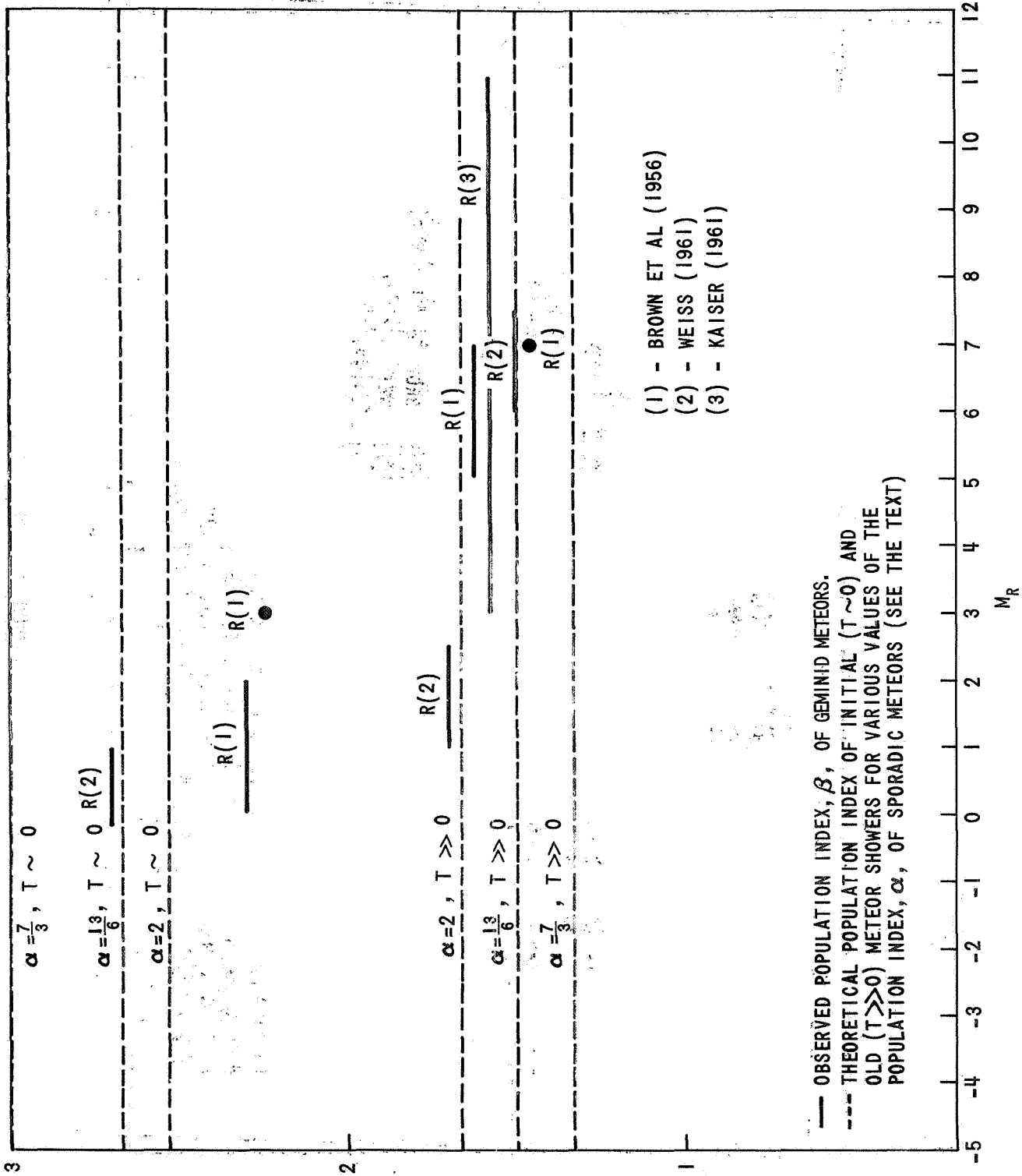


FIGURE-7 GEMINIDS

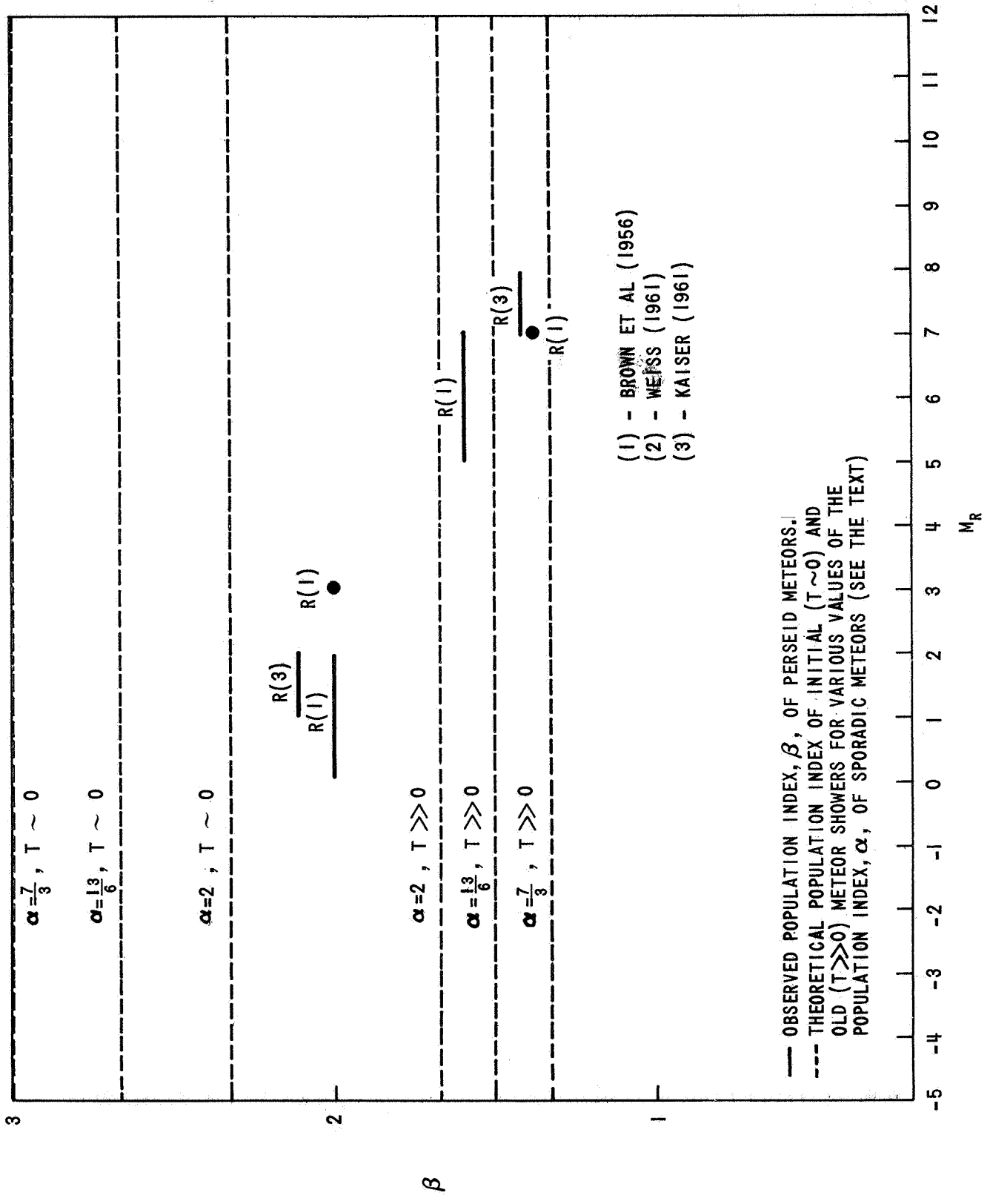


FIGURE-8 PERSEIDS



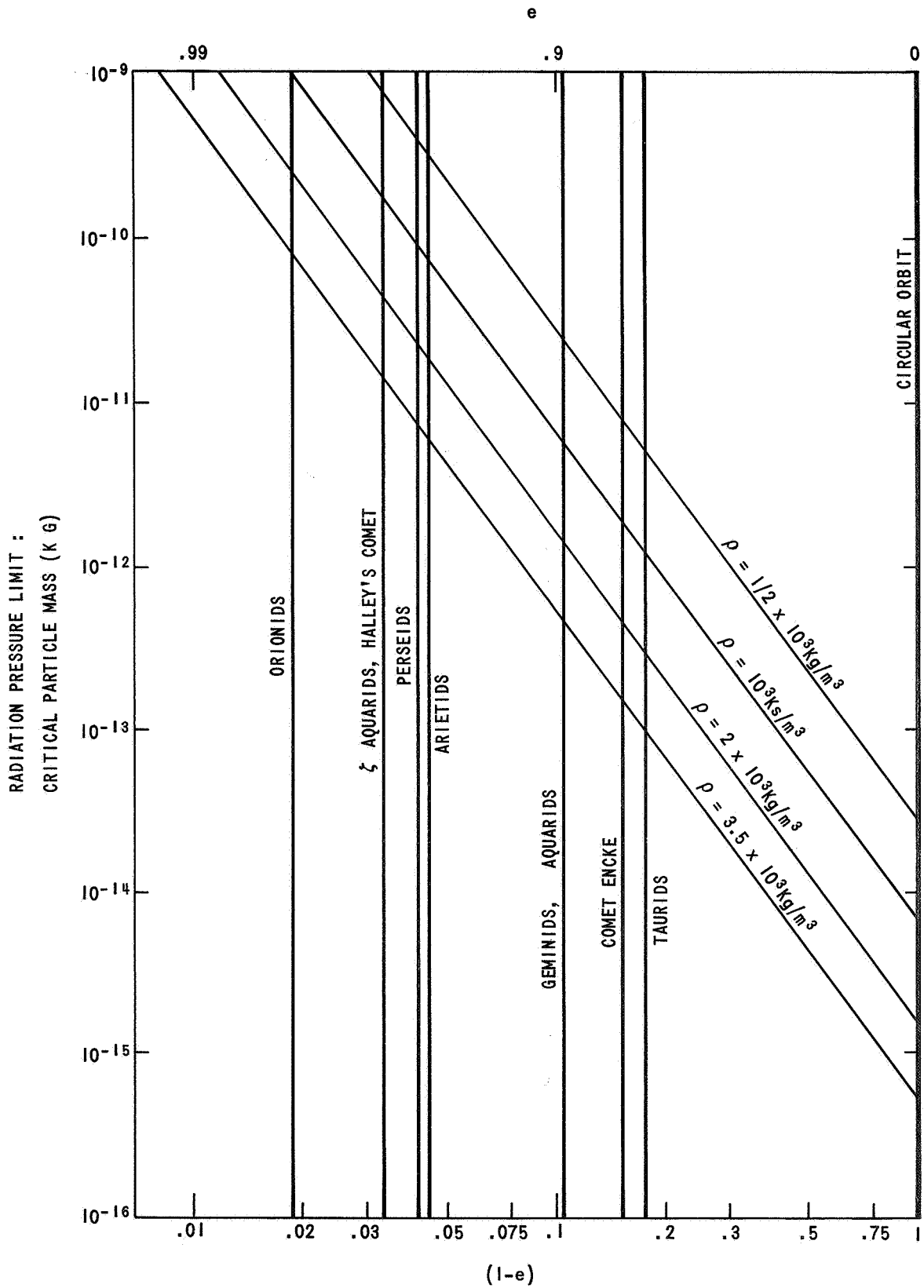


FIGURE-9 - RADIATION PRESSURE LIMIT FOR SEVERAL STREAMS AND COMETS AS A FUNCTION OF PARTICLE DENSITIES AND ECCENTRICITIES

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