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FLUCTUATION THEORY OF AMBIPOLAR DIFFUSION IN A MAGNETO-PLASMA

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We consider a weakly ionized plasma in a strong magnetic field, and include the effects of small fluctuations, called micro-turbulence. The hydrodynamic equations are formulated from the basic equations including the mean motion and the superposed fluctuations. Due to the non-linearity, the statistical effects of the fluctuations enter in the mean motion as correlations. The theory is valid for a plasma where a large scale turbulent motion is active, for example as generated by wind shears and gravity waves in the low ionosphere. It is also valid for a plasma where such a turbulence ceases, and is replaced by micro-disturbances, as observed by incoherent scattering of electromagnetic waves, for example in the high ionosphere beyond the turbopause. Both types of fluctuations will be called turbulence in the broader sense.

The fundamental equations of the mean motion including the effects of fluctuations are as follows:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \underset{\sim}{v}) = \Psi^{\circ} \qquad (1)$$

$$\frac{\partial \underset{\sim}{v}}{\partial t} = -\underset{\sim}{L} - \underset{\sim}{S} - \nu_2 \underset{\sim}{v} + \psi_{nv} T + \psi_{Tv} n \qquad (2)$$

Here

$$\underset{\sim}{L} = -\frac{e}{m}\left(\underset{\sim}{E} + \Omega\, \underset{\sim}{v} \times \underset{\sim}{e}_B\right) + v_{th}^2\left(\frac{\nabla T}{T} + \frac{\nabla n}{n} + \frac{\underset{\sim}{e}_g}{H}\right)$$

$$\underset{\sim}{S} = -\nu_1 (\underset{\sim}{v} - \underset{\sim}{U})$$

$\Omega = eB/m$, and $\underset{\sim}{e}_B$ is a unit vector $// \underset{\sim}{B}$.

Further, H = KT/gm is a scale height, and $v_{th} = (KT/m)^{\frac{1}{2}}$ is a thermal velocity, with the Boltzmann constant K. ν_1 is the molecular collision frequency, and ν_2 is the turbulent collision frequency.

Now,

$$\Psi^{\circ} = \frac{n}{v_{th}}\left(\psi^{\circ}_{nn} z + \psi^{\circ}_{nn} T + \psi_{nT} n\right)$$

has a group of diffusion functions. For future use,

$$\tilde{\Psi} = \psi^{\circ} + \psi_{//}\ , \quad \psi^{\circ} = \psi + \psi_{mol}$$

are "diffusion operators", which operate on a mean quantity to be transferred, through a correlation of two fluctuations denoted by subscripts. This operation yields a diffusion. For example the operation

$$\psi_{nT} n = V_{t\parallel} \nabla \left[\frac{1}{n} D_{nT} \nabla n \right]$$

is a diffusion function, involving the eddy diffusivity

$$D_{nT} = \frac{V_{t\parallel}^2}{nT} \int_0^\infty d\tau \; \overline{n'(0) T'(\tau)} \cos \Omega \tau$$

In an analogous way, ψ_{mol} is a diffusion operator by molecular motions. The quantity with a prime is a fluctuation. A catalogue of other operators and eddy diffusivities are generated in this way, and we shall not elaborate on them here. We note that ψ and $\tilde{\psi}$ represent the turbulent effects, so that without them, the equations (1) and (2) degenerate to a laminar system.

The turbulent transport coefficients are calculated on the basis of a reduced form of (2) for a steady and uniform plasma, the solution of which yields

$$n \underset{\sim}{v}^* = \frac{1}{e} \underset{\approx}{\sigma} \cdot E_m - \underset{\approx}{D} \cdot \nabla N \tag{3}$$

where

$$\nabla N = n \left[\frac{\nabla n}{n} + \frac{\nabla T}{T} + \frac{e g}{H} - \frac{1}{v_{t\parallel}^2} \left(\tilde{\psi}_{mv} T + \tilde{\psi}_{Tv} n \right) \right] \tag{4}$$

and the transport with coefficients are:

$$\underset{\approx}{D} = \underset{\approx}{\sigma} \frac{kT}{e^2 n} \; , \quad \underset{\approx}{\sigma} = \frac{n e^2}{m \Omega} \underset{\approx}{K}$$

$$K_\parallel = \kappa \, , \quad K_P = \frac{\kappa}{1+\kappa^2} \, , \quad K_H = \frac{\kappa^2}{1+\kappa^2} \, , \quad \kappa = \Omega / \nu_0$$

$$\underset{\sim}{E}_m = \underset{\sim}{E} + \underset{\sim}{U}_m \times \underset{\sim}{B} / c$$

$$\nu_0 = \nu_1 + \nu_2 \, , \quad \underset{\sim}{U}_m = \frac{\nu_1}{\nu_0} \underset{\sim}{U} \, , \quad \underset{\sim}{v}^* = \underset{\sim}{v} - \underset{\sim}{U}_m$$

In order to illustrate the effects of turbulence, let us consider the special case $\kappa >> 1$. It is seen that in a laminar plasma, the current parallel to the magnetic field is contributed by the fast electron motion, while the current perpendicular to the magnetic field is contributed by the ions. On the other hand, in a turbulent plasma, the parallel current is reduced and the transverse current is increased by turbulence, by a larger value of σ_p and D_H.

The system of equations (1) and (2) is nonlinear, we shall linearize them, by substituting $n \underset{\sim}{v}$ from (3) into (1). We rewrite such an equation for $\partial n/\partial t$ twice, for ion and for electron, enabling us to eliminate the self-consistent field. After some simplifications, we obtain:

$$(\underset{\approx}{\sigma}_i + \underset{\approx}{\sigma}_e)\cdot\nabla^2\frac{\partial n}{\partial t} + (\underset{\approx}{\sigma}_i\cdot\nabla_1^2 \underset{\sim}{V}_e\cdot\nabla + \underset{\approx}{\sigma}_e\cdot\nabla_1^2 \underset{\sim}{V}_i\cdot\nabla)n$$

$$= (\underset{\approx}{\sigma}_i\cdot\nabla_1^2)\eta_e + (\underset{\approx}{\sigma}_e\cdot\nabla_1^2)\eta_i \tag{5}$$

$$\underset{\approx}{\sigma}\cdot\nabla_1^2 = \sigma_\perp\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) + \sigma_\parallel\frac{\partial^2}{\partial x_3^2}$$

where

$$\eta = \underset{\approx}{D}_i\cdot\nabla_1^2 N + \Psi^0 \tag{6}$$

Since ∇N involves n according to (4), we find the turbulent ambipolar transport coefficients as follows:

(1) ambipolar drift

$$\underset{\sim}{V}_a = \frac{\sigma_{oi}\underset{\sim}{V}_e + \sigma_{oe}\underset{\sim}{V}_i}{\sigma_{oi} + \sigma_{oe}}; \quad \underset{\sim}{V} = \frac{1}{en}\underset{\approx}{\sigma}\cdot E_{eff} + \underset{\sim}{V}_n \tag{7a}$$

(2) ambipolar diffusion

$$D_a^* = \frac{\sigma_{oe}D_{oi}^* + \sigma_{oi}D_{oe}^*}{\sigma_{oi} + \sigma_{oe}}; \quad \sigma_o = \sigma_\perp \sin^2\beta + \sigma_\parallel\cos^2\beta$$

$$D_o^* = D_\perp^*\sin^2\beta + D_\parallel^*\cos^2\beta; \quad \underset{\approx}{D}^* = \underset{\approx}{D} + \underset{\approx}{D}_{nT}$$

$$\underset{\sim}{E}_{eff} = \underset{\sim}{E}_o + \frac{\underset{\sim}{V}_m\times\underset{\sim}{B}}{c} + \frac{m\underset{\sim}{g}}{e}, \quad \cos\beta = \underset{\sim}{r}\cdot\underset{\sim}{e}_B/r \equiv \mu$$

where $\underset{\sim}{E}_o$ is an external electric field.

The turbulent diffusion equation (5) involves diffusion processes other than that along the density flux, as seen from (4), (5), and (6). However, if we restrict ourself to the density flux, then (7) degenerates to the relation

$$\frac{D_a^*}{D_{i\parallel}^*} = \left[1 + \frac{D_{e\parallel}^*}{\sigma_{e\parallel}}\left(\frac{D_{i\parallel}^*}{\sigma_{i\parallel}}\right)^{-1}\right]\frac{1+\kappa_i^2\mu^2}{1+\kappa_i^2}\;\frac{1+\kappa_e^2\mu^2}{1+\kappa_e^2} \tag{8}$$

The term between the brackets is $1 + T_e/T_i$ for a laminar plasma according to the Einstein relation, but the turbulent diffusion coefficients do not obey such a relation. The ambipolar diffusion is more dispersive in a turbulent plasma than in a laminar plasma, since (8) is more sensitive to μ.

We have applied equation (5) to study the spreading and diffusion of a plasma inhomogeneity in ionosphere, and found that the turbulent process accelerates the dispersion and spreading.

Let us investigate also the linear instability of the density diffusion, as governed by (5). For constant $\underset{\sim}{V}_a$ and D_a in a laminar plasma, the dispersion relation is found to be

$$\gamma = \underset{\sim}{k_o} \cdot \underset{\sim}{V_a} - k^2 D_a \qquad (9)$$

where γ is the growth rate, F_a and D_a are the laminar values of (, $\underset{\sim}{V_a}$ is in the direction of ∇n_o, and $\underset{\sim}{k_o} = -(\nabla n_o)/n_o$ represents the scale of density. The inhomogeneity is unstable if $\underset{\sim}{k_o} \cdot \underset{\sim}{V_a} > k^2 D_a$ requiring at least a plasma of decreasing density in the direction of the drift, a phenomenon similar to the Rayleigh-Taylor instability. **Striations parallel to the magnetic field are formed, and have a size**

$$k = (\underset{\sim}{k_o} \cdot \underset{\sim}{V_a} / D_a)^{\frac{1}{2}}$$

For a plasma inhomogeneity of diameter $4 \cdot k_o^{-1}$ = 30 km, with a drift V_a = 100 m/sec and a diffusion coefficient D_a = $10^3 m^2$/sec the striations have a wavelength of 1.5 km, and an onset time of 75 sec. For a plasma with $K_e \gg K_i \gg 1$, it can be shown that $k_{//} = 0$, giving striations parallel to the magnetic field.

For the low ionosphere (<200 km), where the collision of the charge particles with the neutral particles dominate, $\underset{\sim}{V_a}$ and D_a are dispersive, so that an inhomogeneity will drift apart. For the higher ionosphere (>250 km) where the collisions between the electrons and ions are dominant, $\underset{\sim}{V_a}$ and D_a are not very dispersive, so that an inhomogeneity will not easily split, but striations are governed by **the same relation (9), with other transport coefficients which are not dispersive.**