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THE THERMAL NOISE OF CASCADED MISMATCHED PASSIVE TWO-PORTS  
IN THE MICROWAVE RANGE

Das thermische Rauschen von seriengeschalteten, fehlangepassten  
passiven Vierpolen im Mikrowellenbereich

by  
Gernot Kraus

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G. KRAUS: NOISE IN THE MICROWAVE RANGE

THE THERMAL NOISE OF CASCADED MISMATCHED PASSIVE TWO-PORTS  
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by Gernot Kraus\*

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Introduction

Progressively more sensitive, less noisy receivers are being developed in the ultrahigh-frequency range. The great expenditure incurred in measuring low power, however, would not be justified without the exact knowledge of the processes in the transmission system between a generator (e.g., an antenna) and the receiver. In addition to mismatching and loss in the system, the random noise of certain transmitting two-ports must be considered. Their role becomes significant in highly sensitive nuclear resonance measurements, in satellite communication systems, and in radio-astronomical receivers, in which cosmic radio sources (with a radiation flow of several  $10^{-26} \text{ Wm}^{-2} \text{ Hz}^{-1}$ ) produce an antenna noise of only a few degrees Kelvin. Cooled noise standards are used for calibration; i.e., terminal resistances (black body) which produce a specifically known thermal noise of the same order of magnitude. The maximum output  $P_{max}$  that can be transmitted from a noise standard at temperature  $T$  (or from an antenna with an equivalent noise temperature) across an ideal microwave conductor to a connected receiver is, according to Ref. 1, where  $k$  is Boltzmann's constant and  $df$  is the bandwidth of the receiving system. For a system with constant bandwidth,  $P_{max}$  is proportional to  $T$ , so that the power absorbed by the receiver is generally represented by the noise temperature  $T$ .

Normally, additional structural parts such as <sup>(1)</sup> attenuators, insulators, and switches, with loss factors  $L_1, L_2, \dots, L_n$  and temperatures  $T_1, T_2, \dots, T_n$  are connected between a noise output standard  $G$  of

\*Dipl. Phys. G.KRAUS, at the Astronomical Institute of the University, 463 Bochum, P.O. Box 2148.

temperature  $T_0$  and the receiver input. If the matching of all structural elements is ideal, the noise temperature  $T_{Vn}$  absorbed by the receiver is, according to Ref. 2.

$$T_{Vn} = \sum_{i=0}^n T_i (1 - L_i) \prod_{l=i+1}^n L_l \quad \text{mit } L_0 = 0, L_{n+1} = 1. \quad (2)$$

The ideal case of an exact broadband matching of all microwave components of a system of the transmission line cannot be achieved technically. It might be possible to match the individual components to each other with the aid of interconnected tuners. However, this has the disadvantage that, because of the additional coupling flanges and the added loss through the reflectors, the error sources would be increased. Furthermore, tuners are strongly dependent of frequency, so that a consistently good matching over the entire receiver bandwidth, often of several megahertz, is unattainable.

Several attempts have been made already (e.g., in Refs. 3, 4, and 5) to approximate the effect of mismatch of lossy transmission components. Recently, a relation was derived in Ref. 6 by a different means from that used in this work, which is only partly valid in terms of the conditions set forth there: The reflectivity factor of the receiver input must be complex-conjugated to that reflectivity factor which is measured outward from the receiver in the direction of the cascaded circuit. Furthermore, the portion emitted from the receiver, then reflected by the cascaded structural components, and again absorbed by the receiver was not taken into consideration in the calculated noise output. Our goal was, therefore, to find a comprehensive relation for the output transmission by mismatched passive two-ports with their own noise-output contribution. The scattering matrix

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

was used for the description, whose individual elements could be determined by Ref. 7, for example. If  $p_0$  is the reflection coefficient of the generator,  $q_0$  the reflectivity factor of the receiver,  $P_{max}$  again the maximum output capacity of a generator, and if the transmission two-ports between  $G$  and  $E$  are described by matrix  $S$  (Fig. 2), then the output  $P_D$  that the receiver can absorb is calculated, according to Ref. 8, by

$$P_D = \frac{P_{max}(1 - |p_0|^2)(1 - |q_0|^2)|S_{21}|^2}{|1 - p_0 q_s|^2 |1 - S_{22} q_0|^2}, \quad (3a)$$

where

$$q_s = S_{11} + \frac{S_{12} S_{21} q_0}{1 - S_{22} q_0} \quad (4a)$$

is the reflection coefficient measured outward from the generator in the direction of the two-ports with the receiver connected. In the opposite direction, the reflection coefficient  $p_s$  is measured

from the receiver in the direction of the two-ports, with the noise source connected, and can be calculated using

$$p_s = S_{22} + \frac{S_{12} S_{21} p_0}{1 - S_{11} p_0} \quad (4b)$$

from the scattering matrix of the cascaded two-ports and the reflectivity factor of the noise source. If  $q_s$  in Eq. (3a) is replaced by Eq. (4a), a different expression is obtained for Eq. (3a):

$$P_D = \frac{P_{max}(1 - |p_0|^2)(1 - |q_0|^2)|S_{21}|^2}{|1 - p_0 S_{11}|^2 |1 - p_s q_0|^2} \quad (3b)$$

As a special case, with a faulty two-port between  $G$  and  $E$ ,

$$S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(transmission length 0), and the output transmission between the mismatched generator and receiver is obtained from Eq. (3) if  $q_0 = q_s$  and  $p_0 = p_s$  is measured at the same locations between  $G$  and  $E$ , e.g., at the coupling flanges:

$$P_C = \frac{P_{max}(1 - |p_0|^2)(1 - |q_0|^2)}{|1 - p_0 q_0|^2} \quad (5)$$

The required relation for the total noise of cascaded noise resistances can now be derived from Eqs. (3) and (5), if the considerations with which Eq. (2) was derived are carried further: The noise output  $P_1$  of the absorber/twoport combination (Fig. 2), which may at first produce noise together with temperature  $T_1$ , is obtained from Eq. (5) with  $P_{max}$  from Eq. (1):

$$P_{1C} = kT_1 df \frac{(1 - |p_s|^2)(1 - |q_0|^2)}{|1 - p_s q_0|^2} = kT_1 df \cdot C \quad (6)$$

The noise output  $P_{1D}$  that has reached the receiver from the terminal resistance  $G$  of temperature  $T_1$  across the two-port is, with Eq. (3b),

$$P_{1D} = kT_1 df \frac{(1 - |p_0|^2)(1 - |q_0|^2)|S_{21}|^2}{|1 - p_0 S_{11}|^2 |1 - p_s q_0|^2} = kT_1 df \cdot D \quad (7)$$

The contribution of the noisy two-port itself becomes, with abbreviations  $C$  and  $D$  from Eqs. (6) and (7):

$$P_R = P_{1C} - P_{1D} = (C - D)kT_1 df \quad (8)$$

If the terminal resistance now takes on temperature  $T_0$  instead of  $T_1$ , the noise output which the receiver obtains from the absorber becomes  $P_{0D} = kT_0 df \cdot D$ . The total noise output absorbed by the receiver from the noise resistance is thus

$$P_{ED} = P_{0D} + P_R = [DT_0 + (C - D)T_1]kdf \quad (9)$$

In Eq. (2), the circuit is matched to the receiver with two-ports and generator. The total maximum noise output of the receiver input resistance was therefore put into the circuit and consumed there. In the present problem, the portion  $P_{EE} = (1 - C)kT_{EE}df$  emitted from the receiver input, e.g., a circulator (which produces noise with the equivalent temperature  $T_{EE}$  in the direction of the noise standard), reflected by the two-ports, and also registered at the output of the receiver, is added to the output  $P_{ED}$ . Thus, the total reflectivity-dependent output available at the receiver input is obtained:

$$P_{v1} = [DT_0 + (C - D)T_1 + (1 - C)T_{EE}]kdf = \bar{v}_1 kdf \quad (10a)$$

or, expressed in noise temperatures:

$$T_{v1} = DT_0 + (C - D)T_1 + (1 - C)T_{EE}. \quad (10b)$$

For the case in which several mismatched two-ports (with scattering matrices  $S_1, S_2, \dots, S_n$ ) with different temperatures  $T_1, T_2, \dots, T_n$  are connected between the noise standard and the receiver (Fig. 3), a relation corresponding to Eq. (2) must be found. To do so, the considerations with which Eq. (10) was obtained must be executed repeatedly. Thus, the total noise temperature

$$T_{v_n} = D_0 T_0 + (D_1 - D_0) T_1 + (D_2 - D_1) T_2 + \dots + (C - D_{n-1}) T_n + (1 - C) T_{EE} \quad (11a)$$

is obtained or more briefly,

$$T_{v_n} = D_0 T_0 + \sum_{i=1}^{n+1} (D_i - D_{i-1}) T_i = T_{n+1} - \sum_{i=0}^n D_i (T_{i+1} - T_i) \quad (11b)$$

with

$$D_n = C, \quad D_{n+1} = 1, \quad T_{n+1} = T_{EE}.$$

$D_i$  is the transmission factor for the output from the  $i$ th two-port (across two-ports  $i + 1$  to  $n$ ) to the receiver input. (The original noise source is designated as zeroth two-port here.) Thus,

$$D_i = \frac{(1 - |p_i|^2)(1 - |q_{n+1}|^2) |{}^n S_{21}|^2}{|1 - p_i {}^n S_{11}|^2 |1 - p_n q_{n+1}|^2} \quad (12a)$$

with

$$i = 0, 1, \dots, n, \quad {}_{n+1}^n S := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad {}^n S = {}^n S.$$

The reflectivity factors  $p_i, q_i$  and matrices  ${}^n S$  will be calculated in the next section.

For a fixed arrangement, the factor  $(1 - |q_{n+1}|^2)$  is a constant and can also be used in the calculation of the amplification factor of the circuit; it is then automatically taken into account in the calibration.

$$T_{v_n} = D_0 T_0 + \sum_{i=1}^{n+1} (D_i - D_{i-1}) T_i$$

$$T_{v_n} = T_{n+1} - \sum_{i=0}^n D_i (T_{i+1} - T_i)$$

## 2. Calculation of the Reflectivity Factors and the Scattering Matrix for Cascaded Two-Ports

The calculation of the scattering matrix  ${}^n_i S$  of two-ports connected in series (two-port  $i$  to two-port  $n$  inclusive) from the scattering matrix elements of the individual two-ports with the aid of matrix theory is awkward. One would first have to change the individual scattering matrices  ${}^i S$  to  ${}^n S$  into the appropriate "scattering-transfer-matrices," multiply these by one another, and calculate the multiplied matrix back into scattering matrix  ${}^n_i S$ . This conversion for reciprocal two-ports is presented in Ref. 7. The formula can easily be extended to nonreciprocal two-ports.

In Ref. 9, the scattering matrix of an arbitrary passive network is calculated from the scattering matrices of the individual elements with the aid of a signal flow diagram -- similar to the node or loop rule of low-frequency electronics.

The reflectivity factors  $p_i, q_i$  matrix elements  ${}^n_i S_{11}, {}^n_i S_{22}$ , and the amount of  ${}^n_i S_{21}$  required in Eq. (12a), however, can be determined recursively much more easily and quickly from known equations than by using the above procedure:

The complex  ${}^n_i S_{11}$  are calculated from the recursion formula for  $q_i$ , making  $q_{n+1} = 0$ ; thus,

$$({}^n_i S_{11} = q_i)_{i=1, \dots, n+1} \quad \text{für } i = 1, \dots, n+1. \quad (13a)$$

The recursion formula for  $q_i$  follows from Eq. (4a):

$$q_i = {}^i S_{11} + \frac{{}^i S_{12} {}^i S_{21} q_{i+1}}{1 - {}^i S_{22} q_{i+1}} \quad \text{für } i = 0, 1, \dots, n. \quad (13b)$$

All  $q_i$  or  ${}^n_i S_{11}$  can now be written immediately, one after the other, from  $q_{n+1}$  and the individual scattering matrices  ${}^i S$ .

A recursion formula can be given for matrix elements  ${}^n_i S_{21}$ :

$${}^n_i S_{21} = \frac{{}^i S_{21} + {}^n_{i+1} S_{21}}{1 - {}^i S_{22} {}^n_{i+1} S_{11}} \quad (13c)$$

where, again,

$${}^{n+1}_i S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad {}^n_i S = {}^{n+1}_i S.$$

Thus, starting with  ${}^{n-1}_i S_{21}$ , all of the required  ${}^n_i S_{21}$  are calculated one by one, up to  ${}^1_i S_{21}$ , from the individually measured  ${}^i S$ .

## 3. Approximation Equation for Use of One Isolator

Thus far, any two-ports permanently connected in series to the receiver input have been included without further consideration in the calculations for the receiver. It was not necessary

to know the values of these two-ports, since they were automatically taken into account in the receiver calibration. Naturally, this simplification is possible only if the output transmission across these fixed two-ports is not degraded by the cascaded variable two-ports. With the amplification equation for receivers

$$U^2 = g(T_E + T_V), \quad (14)$$

where  $U$  is the voltage at the output of the receiver (recorder),  $g$  the amplification factor of the receiver,  $T_E$  the receiver background noise, and  $T_V$  the noise temperature of those two-ports, this means: The portion of the noise temperature of those two-ports that are included with the receiver must be separated from  $T_V$  and added to  $T_E$ . This yields new expressions  $T'_V$  and  $T'_E$ ; the amplification factor  $g$  may also change ( $g'$ ).

If these transformations are to be made, the following conditions must be fulfilled:

1. Since the registration at the receiver output does not change when a two-port is no longer counted with the noise source but with the receiver, the following must be true for the same measuring arrangement:

$$(T'_V + T'_E)g' = (T_V + T_E)g = U^2.$$

2. The noise temperature  $T'_V$  absorbed by the expanded receiver  $E'$  must not be dependent on the parameters of this receiver except for its input reflectivity factor.
3.  $T'_E$  must be independent of the parameters of the two-ports of the new receiver input.
4. The new amplification factor  $g'$  must not be dependent on the parameters connected to the receiver  $E'$ , except on the available noise temperature.

These requirements can be fulfilled for a matched measuring arrangement. This can easily be seen if one substitutes  $T_{Vn}$  from Eq. (2) for  $T_V$  in Eq. (14) and converts it. If, for example, the  $n$ th two-port is added to the receiver, then

$$T'_E = \frac{T_E + (1 - L_n)T_n}{L_n} \quad \text{and} \quad g' = gL_n.$$

Now the only thing that remains is to calculate  $T'_V$  from the magnitudes of the generator and of two-ports 1 to  $n - 1$ :

$$T'_V = T_{V, n-1}.$$

To examine the problem for mismatched networks, Eq. (11b) with (12a) is entered into Eq. (14). It is necessary for the two-ports  $i = t + 1$  ( $t$  whole number  $< n$ ) to  $i = n$  to be counted with the receiver, and the question arises when all four conditions are fulfilled simultaneously: According to Eq. (12a), magnitude  $D_i$  in Eq. (11b) is dependent on the factor  $|1 - p_{ii+1}^n S_{11}|$ , which, for  $i = 0, 1, 2, \dots, t - 1$ , cannot be counted unequivocally either with  $T'_V$  or  $g'$  and, by assumption, not with  $T'_E$  either ( $i+1$ th  $S_{11}$  for the



given  $i$  is dependent on the two-ports in front of as well as within receiver  $E'$ ;  $p_i$ , on the other hand, only on the two-ports in front of  $E'$ ): the requirements cited can therefore not be satisfied. All of these conditions can be fulfilled only if two-port  $t + 1$  is an ideal insulator or circulator. These prevent the "reciprocal amplification (penetration)" of the reflectivity factors from one side of the two-port to the other. With the input to two-port  $t + 1$  as the input to receiver  $E'$ , the new factors  $D'_i$  become somewhat simpler than the transmission factors  $D_i$ :

$$D'_i = \frac{(1 - |p_i|^2) |q_{t+1}|^2}{|1 - p_i q_{t+1}|^2} \quad \text{with} \quad i = 0, 1, \dots, t, \quad (12b)$$

where

$${}^i S'_{21} = {}^i S_{21} (1 - {}^i S'_{21} q_{t+1}) \quad \text{and} \quad {}^{t+1} S'_{21} = 1.$$

Now  $q_{t+1}$  is the reflectivity factor at the input to receiver  $E'$ . The changed amplification factor and the changed noise temperature of the amplifier need not be calculated, since they are automatically taken into account in the calibration.

If two-port  $t + 1$  is an ideal decoupler, additional two-ports  $t$ ,  $t - 1$  can be counted with receiver  $E'$  only if they have the same temperature ( $T_{t+1}$ ) as the insulator.

If a single two-port with the same temperature is connected in series with the isolator, with temperature  $T_1$ , Eq. (10b) is simplified with  $D = D'_0$  and the noise temperature  $T'_{EE} = T_2 = T_1$  of the new receiver input:

$$T'_{v1} = T_1 - D'_0(T_1 - T_0).$$

where  $D'_0$  is calculated from Eq. (12b), with  $t = 1$ .

#### EXAMPLE:

If we assume the case in which the isolator has a stop-band attenuation of 20 dB, then it will decouple the amounts of the (voltage) reflectivity factor by about a factor of 10. With a measuring arrangement such as that commonly used for radio-astronomical measurements, the maximum errors of the most unfavorable phase position of the two-ports should be given. A noise source with  $|p_0| = 0.02$  and a receiver with  $|q_4| = 0.05$ , between which two reciprocal two-ports with reflectivity elements in the amounts of  $|{}^1 S_{11}| = |{}^1 S_{22}| = |{}^2 S_{11}| = |{}^2 S_{22}| = 0.02$  and an isolator as the third two-port, with  $|{}^3 S_{11}| = |{}^3 S_{22}| = 0.05$ , are connected shall serve as an example. Two-ports 1 and 2 and the reciprocal amplification direction of the isolator may not produce any attenuation losses but only reflectivity losses. The kinetic temperatures shall be  $T_0 = 80^\circ\text{K}$  for the noise source,  $T_3 = 290^\circ\text{K}$  for the insulator. Since the loss-free two-ports 1 and 2 cannot make their own noise contribution, it is expected that the measured noise output is independent of their temperatures  $T_1$  and  $T_2$ . From Eq. (11), it therefore follows that  $D_0 = D_1 = D_2$  must be for any reflectivities. This cannot readily be recognized from Eq. (12a) or (12b), but it could generally be demonstrated.

If the mismatching that occurs when an isolator with infinite stop-band attenuation is used is not taken into consideration at all, then the error in the absolute calibration of the measuring arrangement in the above example (according to Eq. 11) is at most  $3^{\circ}\text{K}$  with an unfavorable phase position. For an isolator with infinite stop-band attenuation at 20 dB, the noise temperature can deviate at most  $0.2^{\circ}\text{K}$  from that calculated assuming an ideal isolator and taking into consideration the given mismatch. Since the measuring accuracy for the circuit losses produces an error of similar magnitude, the above-mentioned simplification of the absolute calibration of receivers can be used with good isolators.

The noise of cascaded one- and two-ports was calculated in Eq. (11). In addition to this, the total noise of a circuit of different multi-ports is of interest. Such arrangements are found, for example, in circulators of parametric amplifiers. This problem will be investigated in a later study.

#### NOTE

During the editing of this contribution, the rough manuscript of a study was received which is related to the above topic: "The Effect of Mismatched Components on Microwave Noise-Temperature Calibrations," by T.Y. Otoshi, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California.

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FIGURES

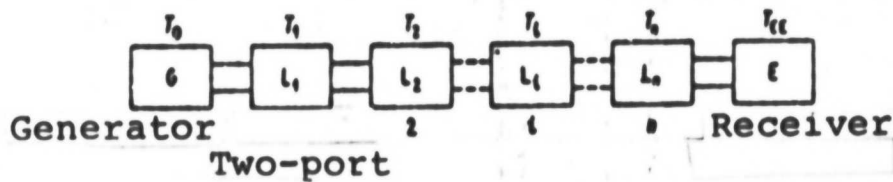


Fig. 1. Several serially connected, mismatched attenuation elements with different temperatures in a mismatched measuring arrangement.

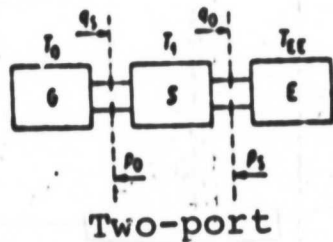


Fig. 2. Any two-port with temperature  $T_1$  in a mismatched measuring arrangement.

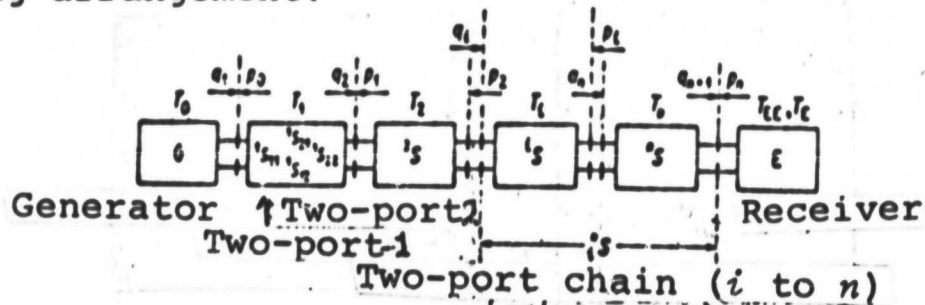


Fig. 3. Serially connected noise two-ports with different noise temperatures in a mismatched measuring arrangement.