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AERONOMY REPORT NO. 33

FULL-WAVE CALCULATIONS OF REFLECTION COEFFICIENTS FROM D-REGION ELECTRON-DENSITY PROFILES

by
W. A. Viertel
C. F. Sechrist, Jr.

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Supported by
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Aeronomy Laboratory
Department of Electrical Engineering
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ABSTRACT

This report considers the reflection of VLF radio waves from the D-region of the ionosphere. In particular, the calculation of the reflection coefficient matrix below the ionosphere is described. Full-wave theory is employed in a FORTRAN IV computer program for use with the IBM 360 computer. The program is checked by comparing computed reflection coefficients with those obtained by an analytical method for the case of a vertical geomagnetic field and an exponential electron density model, and also with full-wave computations done by other groups for a more general case. The report concludes with suggestions for the application of the full-wave program to the computation of VLF reflection coefficients from D-region electron density profiles measured by ground-based and rocket techniques.

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1. THE D REGION OF THE IONOSPHERE

1.1 Introduction

The D region of the ionosphere is the region between 60 and 90 km, coinciding roughly with the mesosphere and lower thermosphere. A daytime profile showing its place in the entire ionosphere is shown in Figure 1.1. The F region and the topside of the ionosphere are from the observation of the equatorial ionosphere of Farley (1966), who used the incoherent backscatter technique to measure them. The E region portion of the profile was obtained from Nike-Apache rocket measurements at Wallops Island, Virginia, and is from Monro and Bowhill (1969). The D region representation, measured in the same way as the E region, is due to Mechtly and Smith (1968). The entire profile will vary with latitude, longitude, solar zenith angle, season, the solar cycle, and the instantaneous level of solar disturbances.

At sunset with the disappearance of the ionizing solar radiation, the D region electron density drops markedly. At dawn the number of free electrons at any given height gradually returns to its daytime value. The exact shape of the profiles is not universally agreed upon, however, as shown in the comparison made by Deeks (1964). Some show a minimum followed by a maximum at lower altitude in the daytime profile, while others show a more monotonically decreasing electron density with decreasing altitude, apart from some slight fine structure. It may be that when the time and place, as well as the manner of measurement or deduction, are all taken into account and D region theory is better understood, these profiles will be shown to be generally consistent with one another. The difficulty in understanding the D region stems

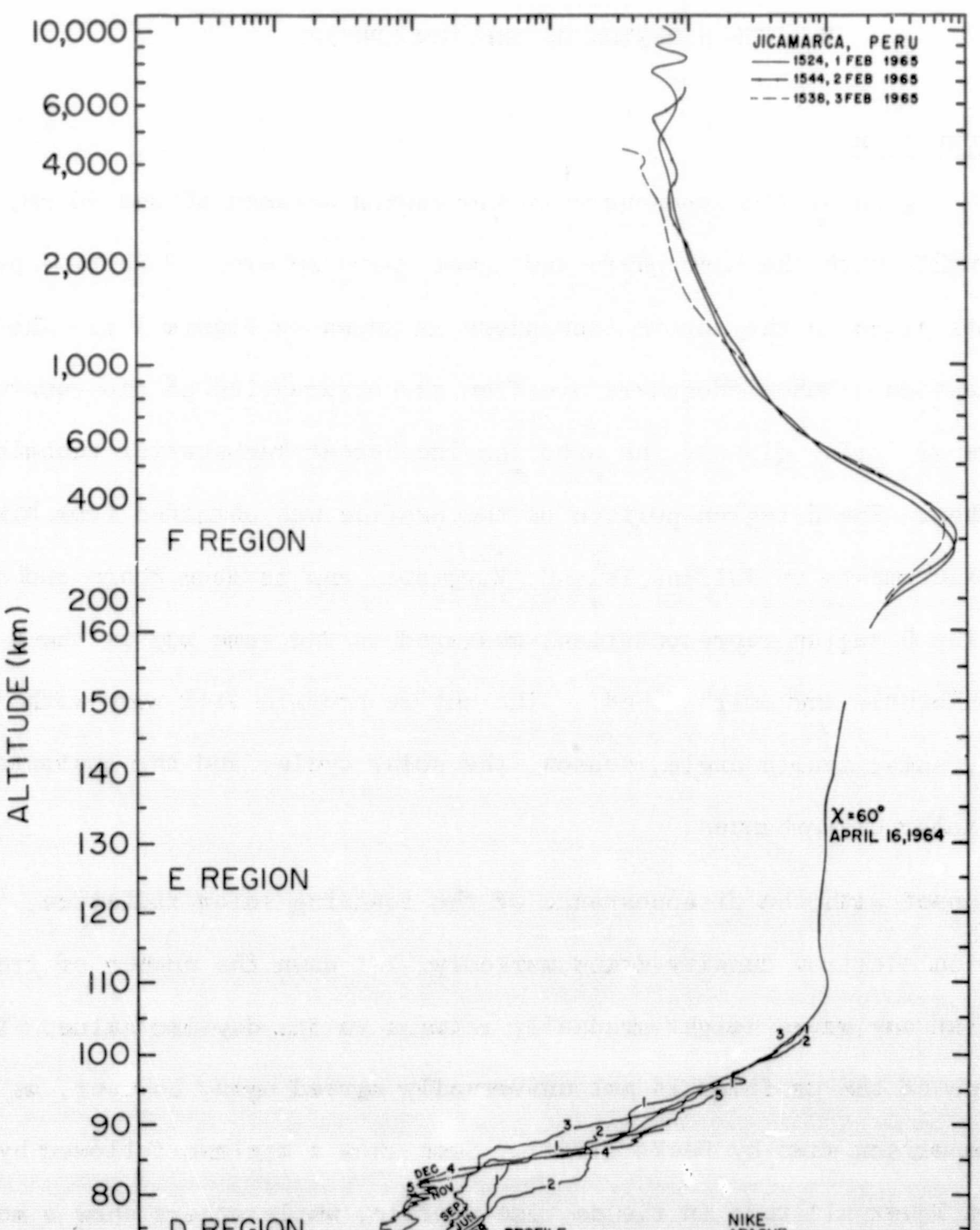


Figure 1.1 The F-region (after Farley, 1966), E-region (after Monro and Bowhill, 1969), and D-region (after Mechtly and Smith, 1968) of the ionosphere.

from its relative inaccessibility to measurements, having an atmospheric concentration too low for aircraft and too high for satellites. Thus rockets and ground-based equipment are the chief tools for studying the D region.

Several types of radiation are thought to be important causes of ionization in the D region. Solar radiations penetrating below 85 km, given by Nicolet and Aikin (1960), are X-rays of $\lambda < 10$ Å, Lyman α ($\lambda = 1215.7$ Å) and radiation in other nearby atmospheric windows, and radiation of $\lambda > 1800$ Å. The X-rays and ultraviolet radiation (Lyman β and the Lyman continuum) are probably important above 85 km during periods of normal solar activity, while an enhanced X-ray flux may affect all constituents, chiefly O_2, N_2, A and O at lower D region heights during solar flares. NO is one of the few species having a sufficiently low ionization potential to be ionized by Lyman α. The importance of the ionization of NO by Lyman α relative to the ionization of other constituents by other radiations depends upon the quantities of NO assumed to be present as is shown by Aikin, <u>et al</u>., (1964). In any case, it seems to be most important in the region around 77 km. Radiations of $\lambda > 1800$ Å require constituents with very low ionization potentials, such as calcium ($\lambda \leq 2028$ Å) and sodium ($\lambda \leq 2413$ Å).

Below the region where ionization of NO is important, galactic and solar cosmic rays are thought to be the principal causes of ionization, although their contribution to the number of free electrons is tempered at these lower D region altitudes by the importance of negative ions due to attachment. The relative importance of contributions to ionization by protons, α-particles, and H-nucleii is treated by Velinov (1968).

Recently metastable $O_2(^1\Delta)$ has been suggested by Hunten and McElroy (1968) as a significant source of ionization in the D region. It can be ionized by the wavelength band 1027-1118 Å, some of which penetrates to the D region through atmospheric windows. In addition, the reaction of this excited O_2 with N may be the most important source of nitric oxide in the D region.

Corpuscular radiation in the form of electrons with energy greater than 40 kev precipitating from the radiation "belts" has also been suggested as an important ionization source at D region altitudes by Tulinov (1967). Further discussion of the chemical and meteorological factors affecting D region free electron concentrations may be found in the record of the Third Aeronomy Conference held at the University of Illinois, September 23-26, 1968.

Another relevant factor is the collisions between electrons and neutral constituents. Electron-electron and electron-ion collisions are not so important because of the far greater number of neutrals present. The altitude dependence of collision frequency given by many workers and compiled by Deeks (1964) is shown for the D region in Figure 1.2. Belrose and Bourne (1966) conclude that the greatest seasonal change in collision frequency occurs at high latitudes. They state that no diurnal changes have been detected, but more measurements are necessary to determine the dependence of collision frequency on the solar cycle and solar disturbances. The relevance of this parameter, along with the electron density, is that it affects radio waves propagating through the ionosphere.

The major objective of this thesis is to devise a computer program for use with the IBM 360 digital computer to calculate the theoretical reflection and conversion coefficients for a VLF radio wave obliquely incident upon the ionosphere. The theory for the ionospheric model and for the model of the wave-ionosphere interaction chosen is discussed in detail.

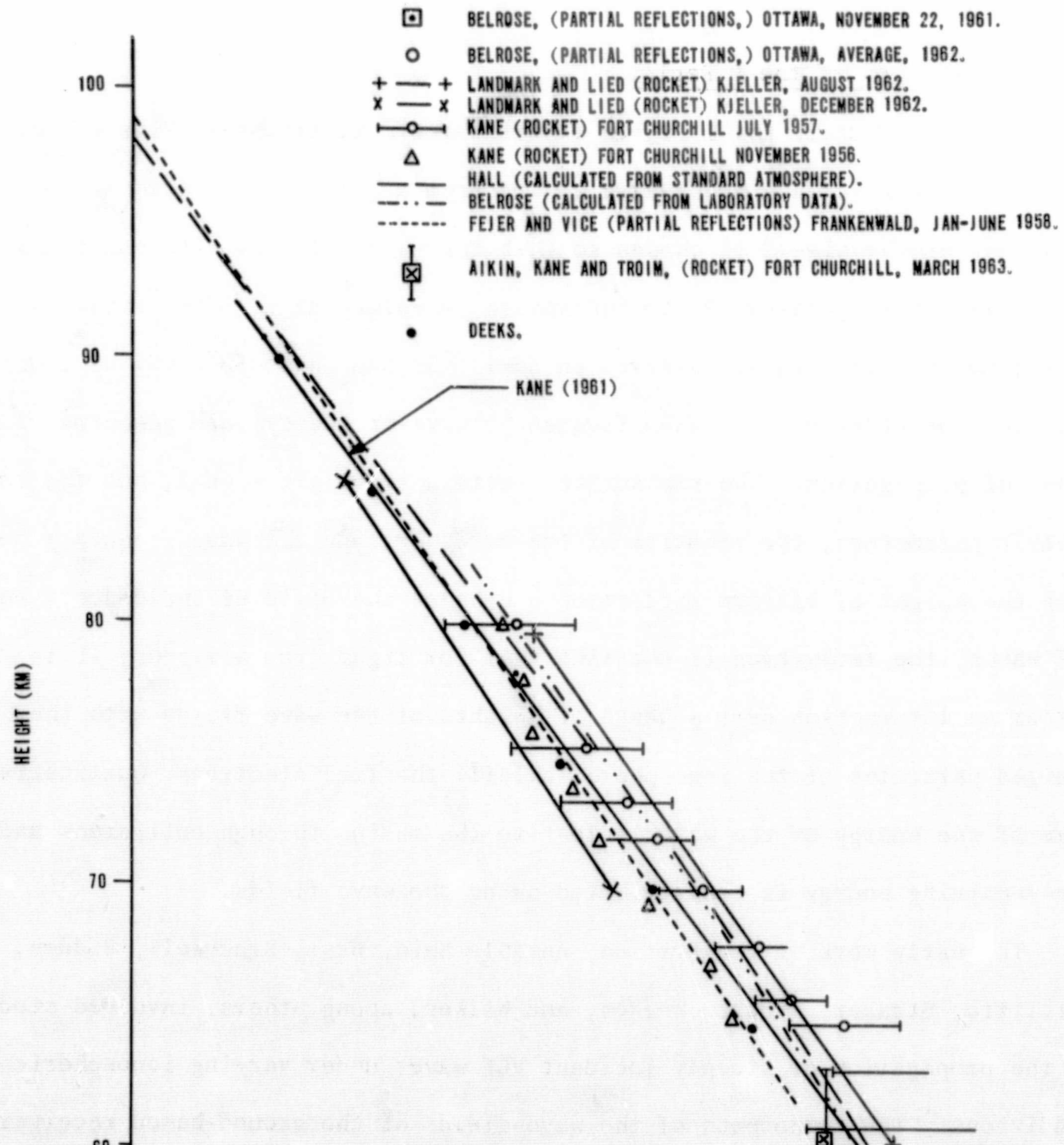


Figure 1.2 Electron collision frequency profiles (after Deeks, 1964).

1.2 VLF Studies of the D Region

One way to study the D region experimentally is to observe its effect on an electromagnetic wave propagated upward from a ground-based transmitter. The frequency of the signal is chosen so that the wave will have its maximum interaction with that portion of the ionosphere in which one is interested. A receiver detects the signal, altered in amplitude and phase in a way dependent upon the electron density, collision frequency, wave-frequency, and geometry of the plane of propagation. The transmitter emits a symmetric signal, but the ionospheric parameters, the location of the receiver, and the wave-frequency determine the height of maximum reflection and hence the angle of incidence. For VLF waves, the reflection is not like that for light from a mirror; it is rather an interaction over a range of heights of the wave fields with the charged particles of the ionosphere, chiefly the free electrons. Qualitatively, some of the energy of the wave is lost to the medium through collisions and the remaining energy is redistributed among the wave fields.

The early work of the British, notably Bain, Best, Bracewell, Budden, Ratcliffe, Straker, Stuart, Weekes, and Wilkes, among others, involved studies of the propagation of steeply incident VLF waves under varying ionospheric conditions. The components of the wave-fields at the ground-based receiver are shown in Figure 1.3 after Bracewell (1952). The groundwave, with subscript 0, is linearly polarized. The downcoming skywave is an elliptically polarized wave arbitrarily split into two linearly polarized components, the normal component, with subscript 1, and the abnormal component with $\bar{H}$ in the plane of propagation, with subscript 2. Also shown is the skywave reflecting from the ground, which is assumed to be a perfect reflector. The various wave-fields making up the total magnetic field at the ground are shown in Figure 1.3B.

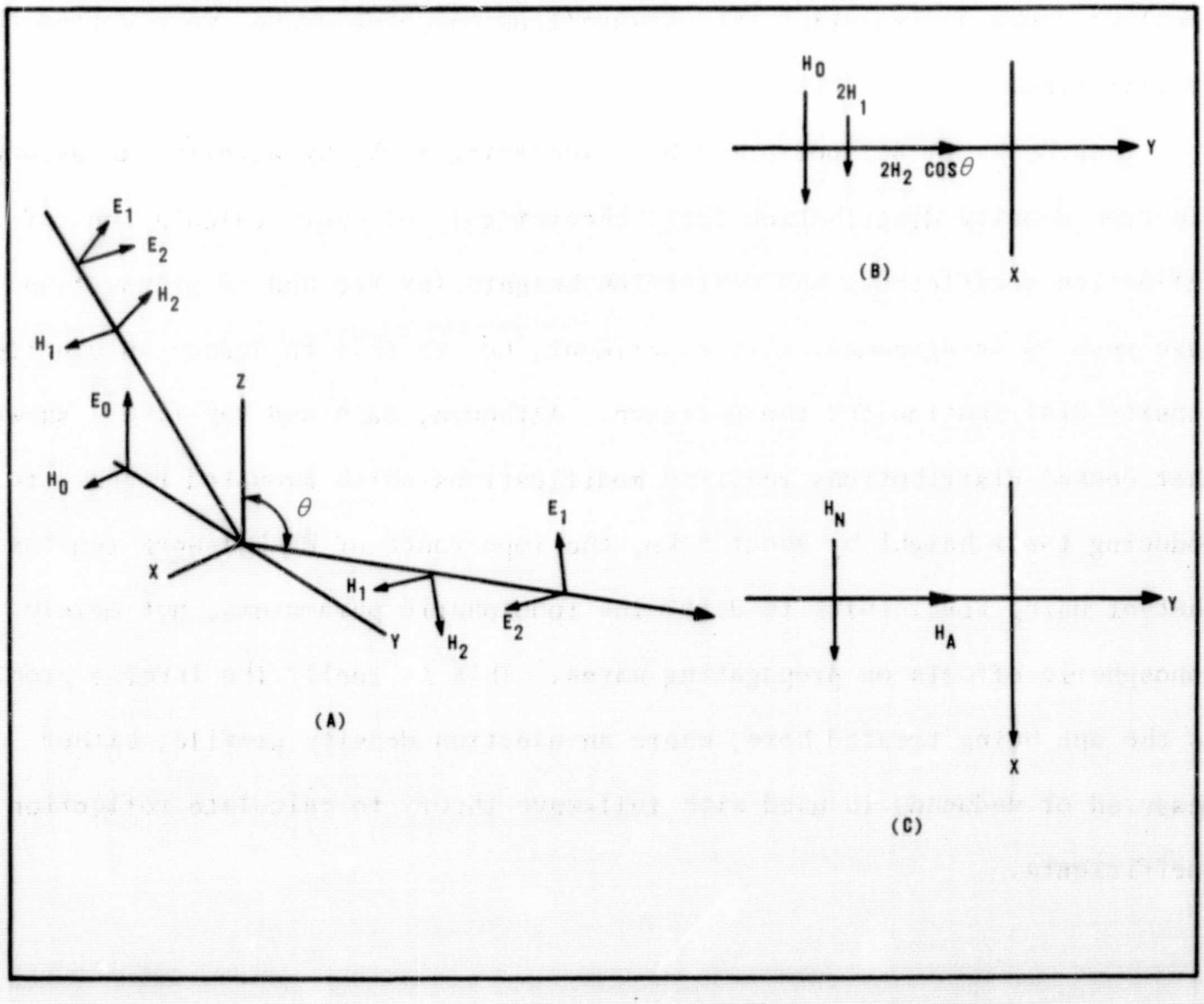


Figure 1.3 Components of the field at the ground (after Bracewell, 1952).

Figure 1.3C depicts the measurable quantities H_N, the total normal component, and H_A, the total abnormal component, the two vectors in general being out of phase. Basically, this work was a cataloguing of the ionosphere's effect on this received signal as a function of time and space. Various experimental problems, such as isolating the skywave from the groundwave, were worked out at this time.

Then Deeks (1966) performed his pioneering work; by altering an assumed electron density distribution until theoretical full-wave calculations of reflection coefficients and reflection heights for VLF and LF propagation gave results in agreement with experiment, he was able to deduce an electron density distribution for the D region. Although, Bain and May (1967) showed that Deeks' distributions required modifications which amounted roughly to reducing their height by about 6 km, the importance of Deeks' work remains, that of using VLF results to determine ionospheric parameters, not merely ionospheric effects on propagating waves. This is really the inverse problem to the one being treated here, where an electron density profile, either measured or deduced, is used with full-wave theory to calculate reflection coefficients.

2. RAY THEORY

2.1 The Theory of Booker

Historically, ray theory preceded full-wave theory. According to the ray theory, a wave incident on the ionosphere from below with an angle of incidence θ measured from the vertical may be thought of as a ray, much as one thinks of light rays in geometrical optics. For simple situations, such as no magnetic field and no electron-neutral collisions, the path of the ray may be traced through the atmosphere by following the phase velocity vector. The orientation of this vector changes along the path through the ionosphere according to Snell's Law

$$\mu \sin \psi = \sin \theta \quad , \qquad (2.1)$$

where μ is the phase refractive index and ψ is the angle the phase velocity makes with the vertical. Below the region of ionization in the atmosphere, $\mu = 1$. In the inhomogeneous, isotropic, collisionless ionosphere above this free space region, μ decreases with increasing altitude. When the level is reached where $\psi = \frac{\pi}{2}$, the wave has achieved its deepest penetration into the ionosphere, and it then begins its descent. The ray path for such a simple siutation is shown in Figure 2.1. Line AB is the boundary between the free space region and the region of ionization.

Booker (1938) pointed out that the inclusion of the earth's magnetic field makes the situation much more complicated. Upon entering the ionosphere, the incident wave may be thought of as being split into two characteristically polarized components, which may propagate independently. The refractive index is now dependent upon ψ, as well as altitude, in a complicated way, and Snell's

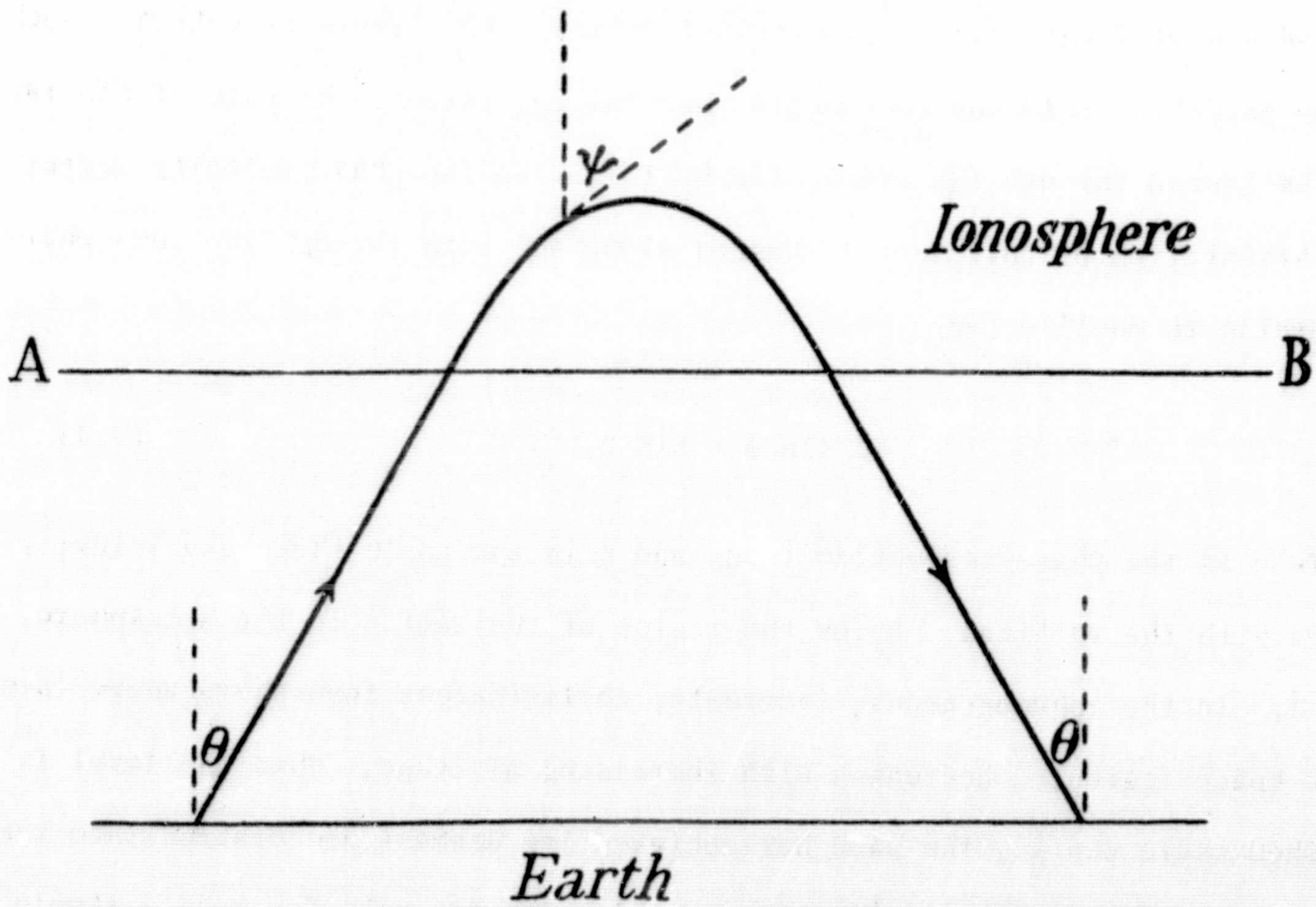


Figure 2.1 A ray path for a wave-packet incident obliquely upon an inhomogeneous, isotropic, collisionless ionosphere (after Booker, 1938).

Law with $\psi = \frac{\pi}{2}$ will no longer give the level of reflection of the wave. The level where the group velocity, not the phase velocity, is horizontal is the true level of reflection. This level was the same for the case of no magnetic field because the group and phase velocity vectors were identical. As a wave-packet propagates through the region of ionization with the group velocity, the individual wave-crests within it are in general moving across the wave-packet with a phase velocity of different magnitude and direction from the group velocity. This results in a cusped ray path for each of the two magneto-ionic components, the ordinary and extraordinary waves. These paths are shown in Figure 2.2 along with the corresponding group-rays. The figure shows only the behavior in the xz plane. Thus the phase-rays do not actually show the path followed by the wave-packet, but only the direction at any altitude of the wave-crests moving across the wave-packet. This is also shown in Figure 2.2 by superimposing phase velocity vectors on the group-ray path.

Because of the dependence of μ on the unknown angle of refraction ψ, it proves convenient to approach the problem of obliquely incident propagation by avoiding the use of μ. The propagation of either of the two magnetoionic components is represented by the wave function

$$\exp\ [ik\ \{ct - \mu(\psi)((\sin\psi)x + (\cos\psi)z)\}] \qquad (2.2)$$

Using Snell's Law and a newly defined variable q, the wave function becomes

$$\exp\ [ik\ \{ct - (\sin\theta)x - qz\}] \qquad (2.3)$$

where q is the only unknown. q contains the electron density, wave-frequency, earth's magnetic field, and angle of incidence as parameters, and the

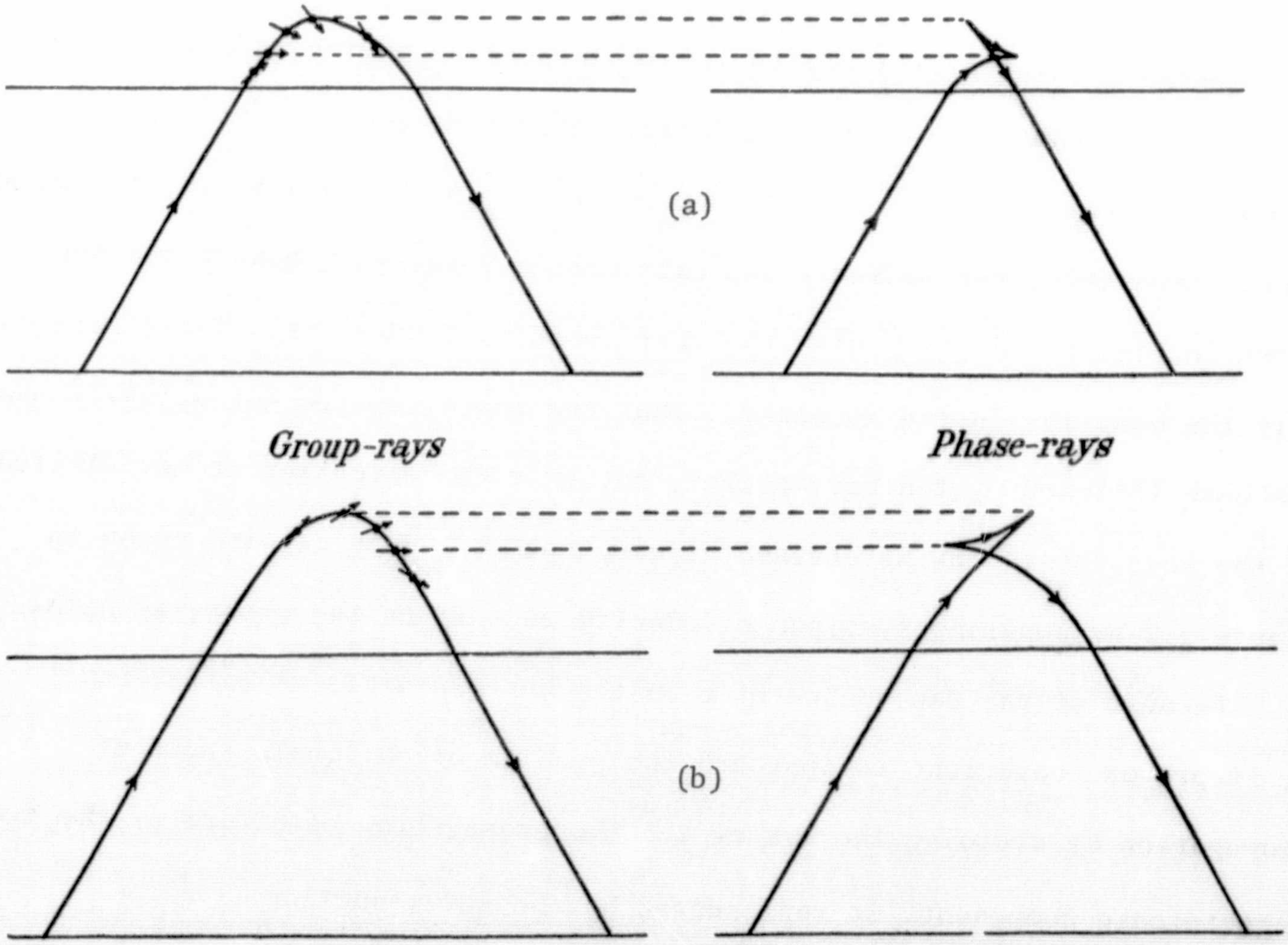


Figure 2.2 Group- and phase-rays of the (a) extraordinary wave, and the (b) ordinary wave, for an inhomogeneous, anisotropic, collisionless ionosphere (after Booker, 1938)

propagation of the wave components through the atmosphere can now be represented by plotting q as a function of the electron density N while keeping the other three parameters constant. Such a plot is shown in Figure 2.3. It would be symmetrical for vertical incidence. Below the ionosphere N = 0 and $q = \cos\theta$. There is a critical electron density (N_A or N_B in Figure 2.3) for each magnetoionic component, above which that component will not penetrate for a given wave-frequency and angle of incidence. Hence the reflection points (A and B in Figure 2.3) are represented by $\frac{\partial q}{\partial N} \to \infty$, not $\psi = \frac{\pi}{2}$ or $q = 0$ (D and E in Figure 2.3) as was the case for the isotropic ionosphere.

Following the example of Appleton, whose wave-function one may retrieve from Booker's by letting $\theta = 0$, the wave function, which was the assumed form of the electromagnetic wave-fields and the polarization, was substituted into the wave equations obtained from Maxwell's equations, resulting in a quartic equation in q, instead of the quadratic obtained by Appleton, represented by

$$F(q) \equiv \alpha q^4 + \beta q^3 + \gamma q^2 + \delta q + \varepsilon = 0 \quad . \tag{2.4}$$

The four Booker quartic roots correspond to the upgoing ordinary, downgoing ordinary, upgoing extraordinary, and downgoing extraordinary waves. The exact nature of the coefficients in the Booker quartic equation depends on the assumptions concerning electron-neutral collisions and the form of the constitutive relation between the polarization $\bar{P}$ and the electric field intensity $\bar{E}$. If the collisional damping due to electron-neutral collisions is included, the coefficients of the Booker quartic equation and the Booker quartic roots become complex quantities.

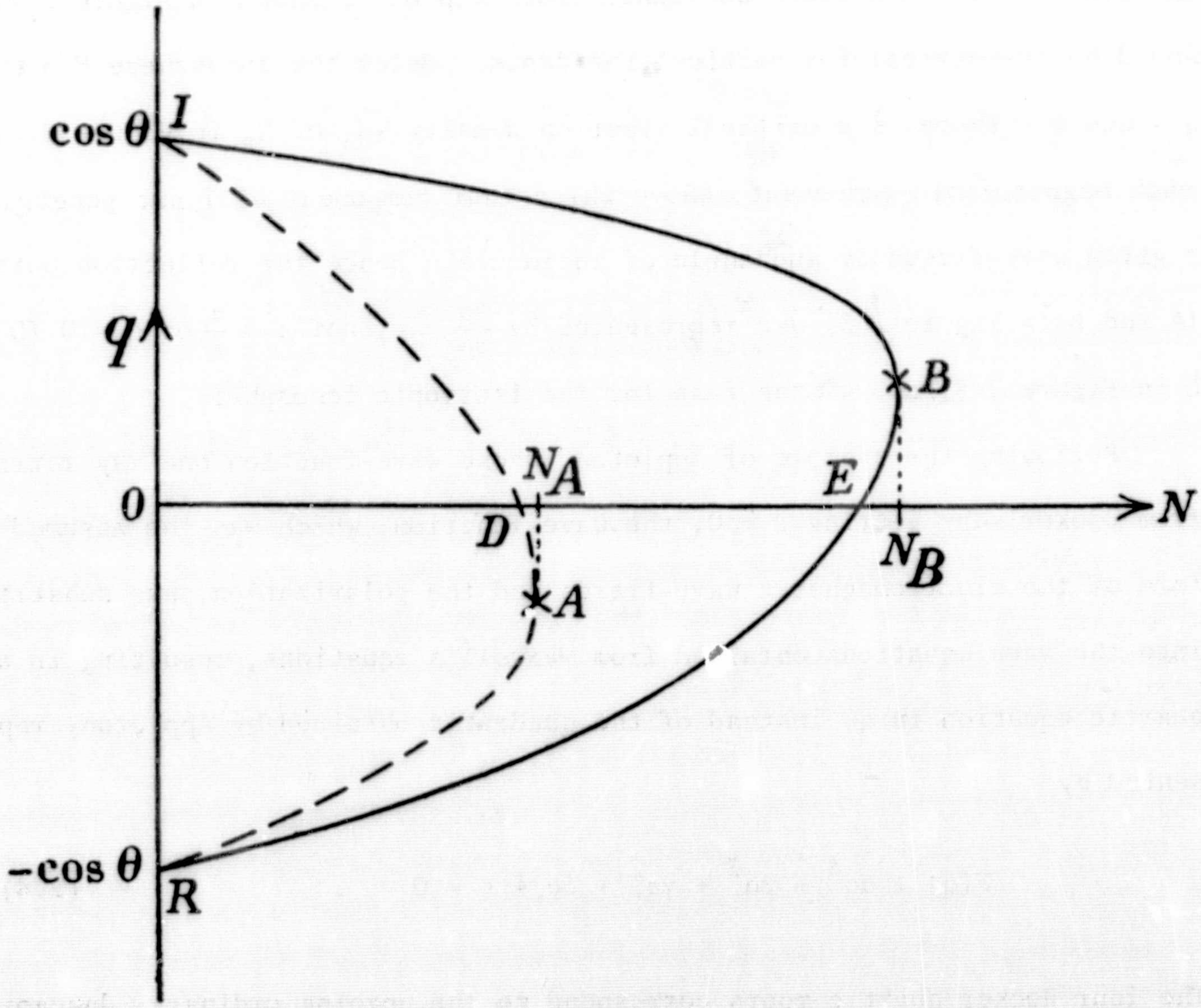


Figure 2.3 The Booker quartic roots q as a function of electron density N, for oblique incidence upon an inhomogeneous, anisotropic ionosphere (after Booker, 1938). The dashed curve represents the extraordinary wave; the solid curve represents the ordinary wave.

The quartic equation may be solved at any altitude, even for complex coefficients, by the method of Burnside and Panton (1904). Figure 2.3 may thus be obtained if the variation of electron density with height, the wave-frequency, the angle of incidence at the ground, and the earth's magnetic field vector, which is assumed to remain constant in magnitude and direction over the region of propagation, are specified.

The wave-fields for the upgoing extraordinary wave, downcoming extraordinary wave, upgoing ordinary wave, and downcoming ordinary wave are proportional at any altitude to

$$\exp \{ik\ (ct - (\sin\theta)y - \int_0^z q\,dz\} \quad , \qquad (2.5)$$

where the root q used is the one appropriate to the magnetoionic component under consideration. Since the form of the relationship between q and z or N can be very complicated, the integration in (2.5) may have to be performed numerically. The real part of q determines the phase change in the wave, whereas the imaginary part arising from collisional damping determines the attenuation as the integration proceeds. The integration for each magnetoionic component might also involve several separate parts due to a situation such as the one shown in Figure 2.4, where a q vs N plot and the corresponding group-ray diagram are depicted. The upgoing ordinary wave is represented by IB and A_2A_3, the downgoing ordinary wave by RB and A_2A_1, the upgoing extraordinary wave by IA_1, and the downgoing extraordinary wave by RA_3.

In any case, the reflection coefficients could in principle be calculated by comparing the wave-fields at the end of the path with those at the start

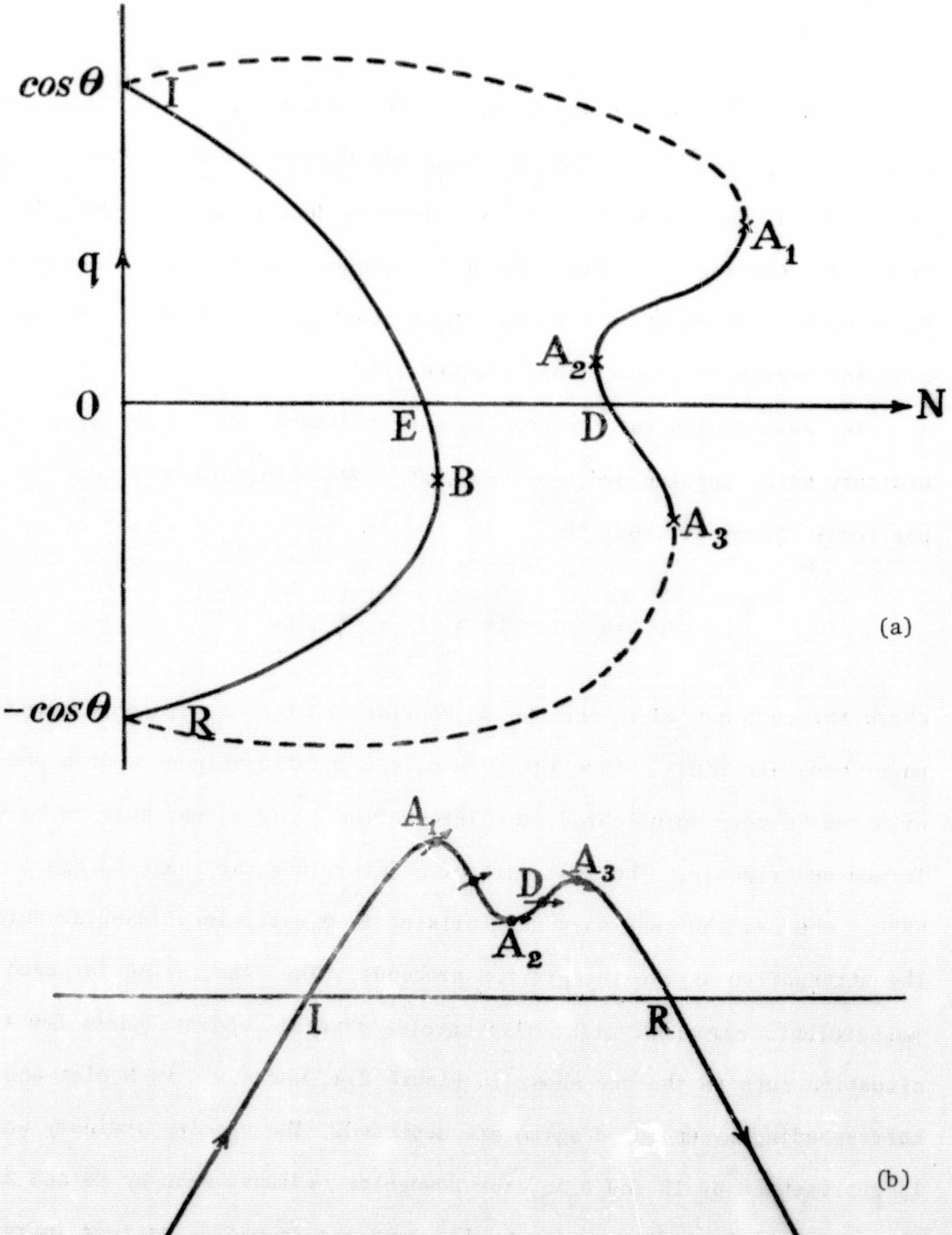


Figure 2.4 Another possible form of (a) q as a function of N for oblique incidence upon an inhomogeneous, anisotropic ionosphere with collisions included, and (b) the corresponding group-ray diagram (after Booker, 1938).

after all the phase change and attenuation, which occur over the whole path, and all the absorption, which occurs only over the portion of the path in the ionosphere, have altered the wave.

2.2 The Necessity for Full Wave-Theory

A wide variety of situations exist for which the ray theory is invalid; it fails to give a good description of reality. Booker (1938) discusses the failure of the theory in the stratum in which reflection takes place, that is, when the quartic roots for the upgoing and downgoing waves of either the ordinary (or extraordinary) component become nearly equal. He also mentions the failure in a region of coupling when the upgoing (or downgoing) waves of the ordinary and extraordinary component become nearly equal, and the two components thus lose their independence. Failure also occurs when the collisional damping is so great that reflection cannot be considered to occur at one height alone, but occurs partially at different levels. The failure of the medium to be slowly varying at the wave-frequency being used is the common factor in all of the inadequacies of ray theory. Usually, below wave-frequencies of 1 MHz, and always below wave-frequencies of 100 KHz, one must resort to a more exact solution of the electromagnetic wave equations. The form of the wave-fields assumed by Booker is no longer valid. The refractive indices of the ionosphere change appreciably within one wavelength. The upgoing energy is converted by the ionosphere to downgoing energy over a range of heights. Since studies of the D region involve frequencies of a few tens of KHz, it is necessary to employ the more exact full-wave theory.

3. FULL-WAVE THEORY

3.1 General Magnetoionic Theory

To develop a mathematical formalism to describe waves obliquely incident upon a horizontally stratified anisotropic, collisional, non-thermal, non-slowly varying, non-permeable ($\mu = \mu_o$), inhomogeneous ionosphere, one must first begin with the equation for conservation of charge, the continuity equation

$$m_\alpha \frac{\partial N_\alpha}{\partial A} + m_\alpha \bar{\nabla} \cdot (N_\alpha \bar{v}_\alpha) = 0 \quad ; \tag{3.1}$$

and the force equation, the magnetohydrodynamic equation

$$m_\alpha \frac{D\bar{v}_\alpha}{Dt} = z_\alpha e\bar{E} + z_\alpha e\bar{v}_\alpha \times \bar{B} + m_\alpha N_\alpha \bar{g} - \frac{\bar{\nabla} p}{N_\alpha} - m_\alpha \nu_\alpha \bar{v}_\alpha \quad , \tag{3.2}$$

where the subscript α indicates a summation over the α charged constituents and the D/Dt is the convective derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{v}_\alpha \cdot \bar{\nabla} \quad , \tag{3.3}$$

which in this case gives the total time rate of change of the αth constituent's velocity, $\bar{v}_\alpha$, on the left hand side of the force equation. Other quantities appearing in Equations (3.1) and (3.2) are the mass of the αth charged constituent, m_α; the number density of the αth charged constituent, N_α; the electric field of the skywave, $\bar{E}$; the total magnetic field, including the earth's magnetic field and the magnetic field of the skywave, $\bar{B}$; the gravitational acceleration field, $\bar{g}$; the scalar fluid pressure of the plasma, p; and the collision frequency, measured in collisions per second, of the

αth charged constituent with ions, electrons, and neutral atoms and molecules, ν_α. The definition of these quantities makes the nature of each of the force terms on the righthand side of Equation (3.2) self-explanatory.

If it was desired to examine the propagation in the plasma in terms of characteristic modes, one would form perturbed equations using

$$N_\alpha = N_{\alpha o} + N_\alpha'$$

$$\bar{v}_\alpha = v_{\alpha o} + \bar{v}_\alpha'$$

$$p_\alpha = p_{\alpha o} + p_\alpha' \tag{3.4}$$

$$\bar{E} = \bar{E}_o + p_\alpha'$$

$$\bar{B} = \bar{B}_o + \bar{B}'$$

where the first quantity on the right hand side is the unperturbed ionospheric parameter, and the second is the perturbation due to the passage of the wave. To simplify matters, $v_{\alpha o}$ and $\bar{E}_o$ might be taken to be negligible. The perturbed counterparts of Equations (3.1) and (3.2) would be solved simultaneously for $\bar{v}_\alpha$ in terms of $\bar{E}$, which would then be substituted into the equation for the electric polarization to obtain the constitutive relations with the susceptibility matrix, and finally the dielectric tensor. It is not relevant here to study the propagation in this manner, however, since the total effect of the ionosphere on the received wave is what is desired. Not only will $v_{\alpha o}$ and $\bar{E}_o$ be neglected, but N_α' as well. Then if non-linear terms in the perturbed quantities, the αth constituent's velocity and the wave-fields, are neglected, the terms

$$m_\alpha(\bar{v}_\alpha \cdot \bar{\nabla})\ \bar{v}_\alpha$$

$$z_\alpha e\bar{v}_\alpha \times \bar{B}_{wave}$$

are dropped. This eliminates the need for Equation (3.1). The Lorentz polarization term has been excluded, on the basis of experimental evidence discussed by Budden (1961).

If only a one component plasma is considered, that is, if only the motion of the electrons is considered and not the motion of the heavier, more sluggish ions, as is proper to do if the number density of ions is not much greater than the number density of electrons, then the summation over α charged constituents is no longer necessary, and z_α becomes equal to unity. The ions simply cannot respond to the passing wave-fields as rapidly as the electrons, and the motion that they do acquire is too small to be greatly affected by the earth's magnetic field for VLF frequencies, as mentioned by Budden (1961).

Finally, neglecting the effects of gravity and the thermal pressure gradient (3.2) reduces to

$$e\bar{E}_{wave} + e\frac{\partial\bar{r}}{\partial t} \times \bar{B}_{earth} = m\frac{\partial^2\bar{r}}{\partial t^2} + m\ \nu\frac{\partial\bar{r}}{\partial t} \quad , \qquad (3.5)$$

where the ionospheric electric fields have also been neglected. The collision frequency, ν, is now that for electrons with ions, electrons, and neutrals, but since the number density of neutrals is so much greater than that of ions, and electrons it may be effectively taken as the collision frequency for electron-neutral collisions. This collision frequency, however, is dependent upon the energy of the electrons, this fact being taken into account by Sen and Wyller (1960) in a generalization of Appleton-Hartree magnetoionic theory.

The inclusion of this energy dependence makes Equation (3.5) invalid, but it can still be used to give good results for VLF studies if the correct collision frequency profile is used, as shown by Deeks (1966a). The use of such an effective collision frequency profile is a mathematical device to equate, as far as possible, the two theories. The profiles of collision frequency for monoenergetic electrons of energy kT used in Sen-Wyller theory and for the effective collision frequency used to make Appleton-Hartree theory give fairly good agreement with the Sen-Wyller results for VLF calculations are shown in Figure 3.1. The dashed curve should be used for all calculations employing the theory of this chapter. The penalty for doing so, according to Deeks (1966a) is slightly increased absorption (smaller reflection coefficient magnitudes) relative to results obtained by using Sen-Wyller theory, but the difference between the results is less than experimental error, especially at VLF frequencies.

Although the wave at the transmitter is spherical, far from the antenna in a localized region it can be approximated by a plane wave of the form

$$e^{i\omega t - ik\bar{r}}$$

Thus $\frac{\partial}{\partial t}$ in (3.5) can be replaced by $i\omega$ to produce an equation in the transform domain. Multiplying this transformed version of (3.5) by $Ne/m\omega^2$, where ω is the angular wave-frequency, and noting that the electric polarization, $\bar{P}$, and geomagnetic field vector, $\bar{Y}$, are given by

$$\bar{P} = Ne\bar{r} \tag{3.6}$$

$$\bar{Y} = \frac{e}{m\omega}\bar{B} \tag{3.7}$$

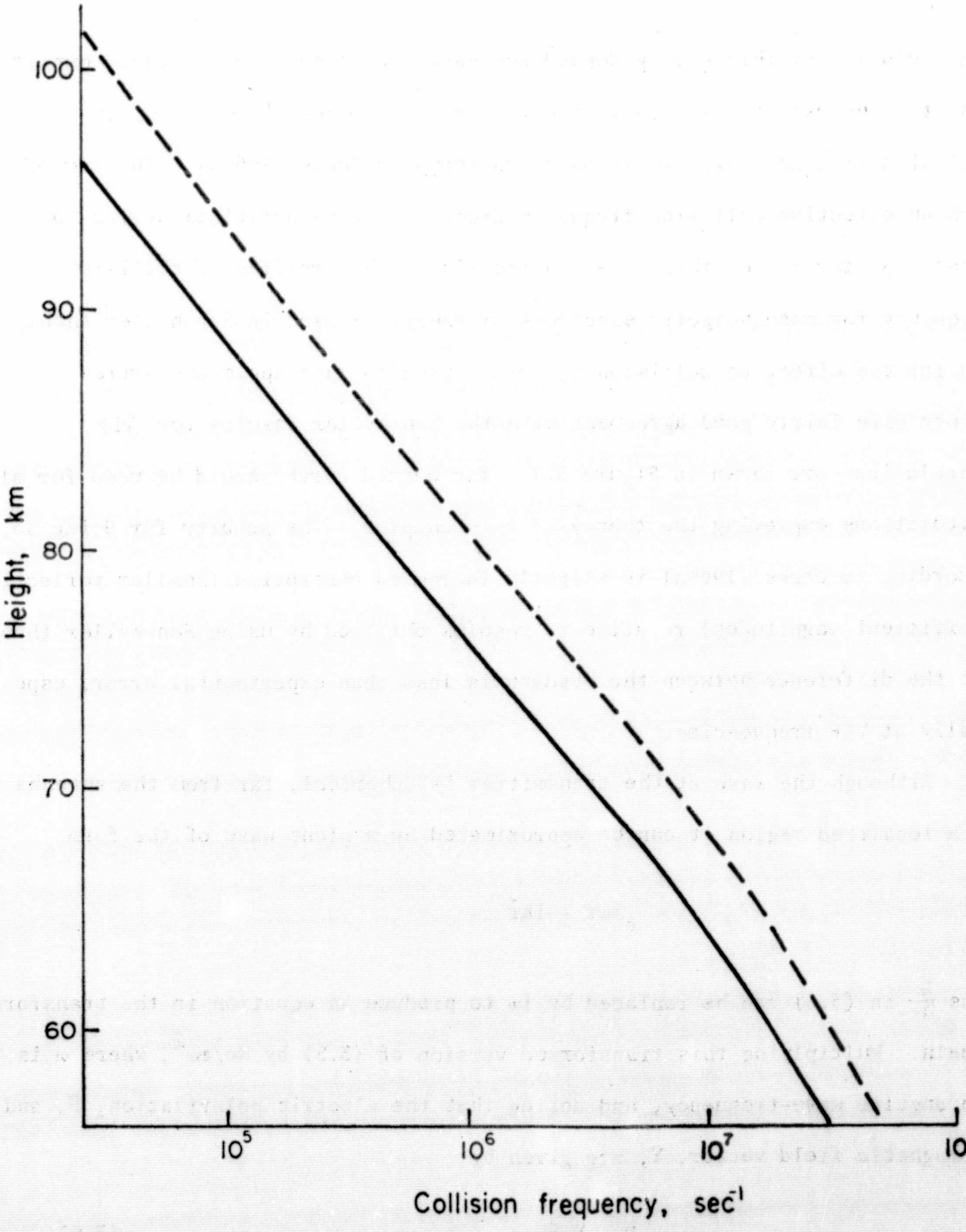


Figure 3.1 Height variation of ν_M, the collision frequency of mono-energetic electrons of energy kT (full curve) and ν_{eff}, the effective classical collision frequency for very long wave calculations (dashed curve), (after Deeks, 1966a).

it is a straightforward matter to obtain the constitutive relations for the ionosphere. These are of the matrix form

$$\bar{P} = \varepsilon_o \tilde{\tilde{M}} \bar{E} \tag{3.8}$$

where $\tilde{\tilde{M}}$ is the susceptibility tensor given by

$$\tilde{\tilde{M}} = \frac{-X}{U(U^2-Y^2)} \begin{pmatrix} U^2-l^2Y^2 & -iUnY-lmY^2 & iUmY-lnY^2 \\ iUnY-lmY^2 & U^2-m^2Y^2 & -iUlY-mnY^2 \\ -iUmY-lnY^2 & iUlY-mnY^2 & U^2-n^2Y^2 \end{pmatrix} \tag{3.9}$$

The quantities of which the susceptibility matrix is composed are

$$X = \frac{Ne^2}{\varepsilon_o m\omega^2} = \frac{\omega_p^2}{\omega^2} \tag{3.10}$$

$$Y = |\bar{Y}| = \left|\frac{e\bar{B}}{m\omega}\right| = \frac{\omega_H}{\omega} \tag{3.11}$$

$$U = 1-iZ = 1 - i\,\frac{\nu}{\omega} \tag{3.12}$$

where ω_H is the gyrofrequency for electrons or the cyclotron frequency, and ω_p is the plasma frequency at the altitude corresponding to $N(z)$. The letters l, m, n represent the direction cosines of the geomagnetic field vector, $\bar{Y}$, with respect to a right-handed coordinate system whose z axis is vertically upward and whose positive x axis is in the direction of the horizontal component of the wave vector, $\bar{k}$. The direction cosines are given by

$$l = - \cos \phi \cos \alpha$$

$$m = - \cos \phi \sin \alpha \qquad (3.13)$$

$$n = \sin \phi$$

where ϕ is the magnetic dip angle, measured down from the horizontal ($0 < \phi \leq 90°$ for Northern Hemisphere), and α is the azimuth east of magnetic north of the x axis, which is in the plane of propagation. The coordinate system is shown in Figure 3.2.

A point which often generates much confusion is that Y can be defined so that the gyrofrequency for electrons is negative

$$Y = \frac{e|\bar{B}|}{m\omega} \quad . \qquad (3.14)$$

If this is done, however, the signs of the direction cosines are changed, which means that they are the direction cosines of $\bar{B}$, which is oppositely directed to $\bar{Y}$. The use of either set of definitions for Y, l, m, and n will yield the same susceptibility matrix elements.

One other point worthy of mention is that the expression for $\bar{P}$ used in Equation (3.6) is the result of taking an average over a volume containing many electrons. The polarization vector and the wave fields used later in Maxwell's equations are assumed fairly constant over distances much less than a wavelength, but large compared with inter-electron distances even for a non-slowly varying medium. That is, changes in the electron density in the vertical direction are not neglected, but small irregularities are smoothed out. As Budden (1961) mentions, there would be no meaning in speaking of the value of $\bar{P}$ at a specific point in the free space between electrons.

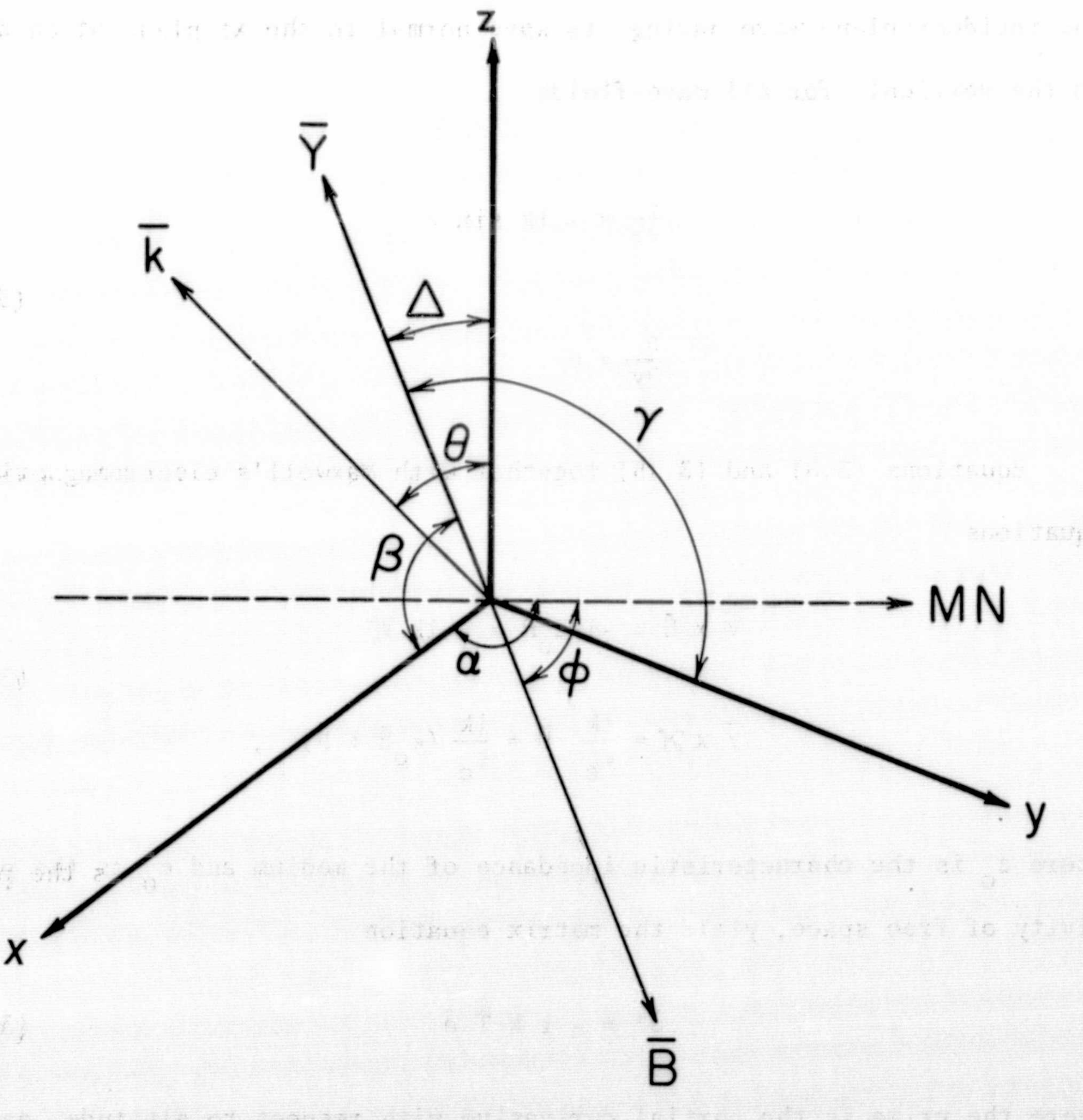


Figure 3.2 The relevant coordinate system at the transmitter. The wave vector $\bar{k}$ is in the xz plane. $\bar{B}$ is in the plane of the magnetic meridian. β, γ, Δ are the arccosines of the direction cosines, l, m, n of the geomagnetic field vector, $\bar{Y}$.

3.2 The Relevant Equations

With the form of the plane wave already assumed below the ionosphere and the incident plane wave having its wave normal in the xz plane at an angle θ to the vertical, for all wave-fields

$$\frac{\partial}{\partial x} = - ik \sin \theta$$
$$\frac{\partial}{\partial y} = 0 \tag{3.15}$$

Equations (3.8) and (3.15) together with Maxwell's electromagnetic equations

$$\bar{\nabla} \times \bar{E} = -ikZ_o \bar{H} = - ik \mathcal{H}$$
$$\bar{\nabla} \times \mathcal{H} = \frac{ik}{\varepsilon_o} \bar{D} = \frac{ik}{\varepsilon_o} (\varepsilon_o \bar{E} + \bar{P}) \quad , \tag{3.16}$$

where z_o is the characteristic impedance of the medium and ε_o is the permittivity of free space, yield the matrix equation

$$\bar{e}' = - i \; k \; \tilde{\tilde{T}} \; \bar{e} \tag{3.17}$$

where the prime is the partial derivative with respect to altitude, and

$$\bar{e} = \begin{pmatrix} Ex \\ -E_y \\ \mathcal{H}x \\ \mathcal{H}y \end{pmatrix} \tag{3.18}$$

and

$$\tilde{\tilde{T}} = \begin{pmatrix} -\frac{SM_{31}}{1+M_{33}} & \frac{SM_{32}}{1+M_{33}} & 0 & \frac{C^2+M_{33}}{1+M_{33}} \\ 0 & 0 & 1 & 0 \\ \frac{M_{23}M_{31}}{1+M_{33}} - M_{21} & C^2+M_{22}-\frac{M_{23}M_{32}}{1+M_{33}} & 0 & \frac{SM_{23}}{1+M_{33}} \\ 1 + M_{11} - \frac{M_{13}M_{31}}{1+M_{33}} & \frac{M_{32}M_{13}}{1+M_{33}} - M_{12} & 0 & \frac{-SM_{13}}{1+M_{33}} \end{pmatrix} \tag{3.19}$$

where S = sin θ, C = cos θ, and the M_{ij} are elements of the susceptibility matrix. X and Z are functions of height, since they depend upon the electron density and collision frequency respectively. Therefore the M_{ij} are functions of height, making the T_{ij} functions of height. The wave-fields and their derivatives with respect to z are also naturally functions of height. If it was desired to solve for the actual wave-fields, Equation (3.17) would be treated. Of interest here, however, are the reflection coefficients, which are ratios of the fields at any altitude desired to the incident fields. The reflection coefficient matrix is defined as

$$\tilde{\tilde{R}} = \begin{pmatrix} {}_{||}R_{||} & {}_{||}R_{\perp} \\ {}_{\perp}R_{||} & {}_{\perp}R_{\perp} \end{pmatrix} = \begin{pmatrix} \frac{E_{||}^{(R)}}{E_{||}^{(I)}} & \frac{E_{\perp}^{(R)}}{E_{||}^{(I)}} \\ \frac{E_{||}^{(R)}}{E_{\perp}^{(I)}} & \frac{E_{\perp}^{(R)}}{E_{\perp}^{(I)}} \end{pmatrix} \tag{13.20}$$

where $E_{||}^{(I)}$ and $E_{\perp}^{(I)}$ are the components of the incident electric field parallel and perpendicular to the plane of propagation respectively, and $E_{||}^{(R)}$ and $E_{\perp}^{(R)}$

are defined similarly for the reflected wave. The dependent variable is then the reflection coefficient matrix and not the wave-field vector. This is the dependent variable used by Budden (1955a). Barron and Budden (1959) resort to another dependent variable, however, termed the admittance matrix $\tilde{\tilde{A}}$. It is related to $\tilde{\tilde{R}}$ by

$$\tilde{\tilde{A}} = \begin{pmatrix} -\left(\dfrac{({}_{\perp}R_{\perp} + 1)}{2\ \mathrm{RPOLY}} + 1\right)\Big/ C & \dfrac{-{}_{\parallel}R_{\perp}}{2\mathrm{RPOLY}} \\[2ex] -\dfrac{{}_{\perp}R_{\parallel}}{2\mathrm{RPOLY}} & -\left(\dfrac{({}_{\parallel}R_{\parallel} - 1)}{2\mathrm{RPOLY}} - 1\right) C \end{pmatrix} \qquad (13.21)$$

where

$$\mathrm{RPOLY} = \left(\frac{{}_{\parallel}R_{\parallel} - 1}{2}\right)\left(\frac{{}_{\perp}R_{\perp} + 1}{2}\right) - \left(\frac{{}_{\parallel}R_{\perp}}{2}\right)\left(\frac{{}_{\perp}R_{\parallel}}{2}\right) . \qquad (13.22)$$

The differential Equations (3.17) become, after this change of variable, in matrix form

$$i\tilde{\tilde{A}} = k\left(\tilde{\tilde{A}}\begin{pmatrix} -T_{11} & T_{12} \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} T_{44} & 0 \\ T_{34} & 0 \end{pmatrix}\tilde{\tilde{A}} + \tilde{\tilde{A}}\begin{pmatrix} -T_{14} & 0 \\ 0 & 1 \end{pmatrix}\tilde{\tilde{A}} + \begin{pmatrix} T_{41} & -T_{42} \\ T_{31} & -T_{32} \end{pmatrix}\right) \qquad (13.23)$$

where this correct version is given by Barron and Budden (1959), the version in Budden (1961) being incorrect. This equation is clearly too complicated to be solved analytically. After numerically integrating it down through the ionosphere from a known starting solution, however, $\tilde{\tilde{R}}$ can be obtained from $\tilde{\tilde{A}}$ by

$$\tilde{R} = 2\begin{pmatrix} -CA_{11} - 1 & A_{12} \\ A_{21} & 1 - A_{22}/C \end{pmatrix}^{-1} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{13.24}$$

The wave admittance approach is used here, although the wave-fields as variables are preferred by others such as Pitteway (1965), and the reflection coefficients as variables by still others such as Sheddy (1968).

4. FULL-WAVE SOLUTIONS

4.1 The Starting Solution

The starting solution necessary to initiate the solution of the relevant differential equations from the full-wave theory of Chapter 3 can be obtained by the method of Sheddy (1968). This method does not depend on a lengthy, and often too slowly converging, preliminary integration, beginning with a crude estimate of the initial solution, down through a fictitious homogeneous ionosphere with X and Z held constant at their values at the height where the preliminary integration is started. This was the method of Budden (1955a). Such a preliminary integration is terminated when $\overset{\approx}{A}'$ is arbitrarily close to zero, and the resultant $\overset{\approx}{A}$ is the admittance matrix corresponding to only up-going waves above the boundary, that is, the matrix for a sharply bounded homogeneous medium. This result may then be used as the starting solution back at the starting level for the preliminary integration, since once $\overset{\approx}{A}$ is found which satisfies $\overset{\approx}{A} = 0$, it will not change over a homogeneous ionosphere. Clearly, this starting level must be well above the height in the real ionosphere where waves of a given frequency are interacting with the ionization. The same method could be used if $\overset{\approx}{R}$ were the dependent variable, but the rate of convergence to $\overset{\approx}{R}' = 0$ would be different.

That the problem is reduced to one of a sharp boundary makes appropriate the boundary matching technique of Crombie (1961), which in turn employs the solutions to the Booker quartic equation with complex coefficients discussed in Chapter 2. The Booker quartic roots may be obtained in closed form using the mathematics of Burnside and Panton (1904). Whereas Crombie's solution was good only for west-east or east-west propagation, Sheddy's method gives the reflection coefficients for a plane of propagation of arbitrary azimuth east of north for a sharply bounded homogeneous ionosphere with a lower boundary at the height where one wishes to commence a full-wave solution.

The flow chart for the initial solution portion of the full-wave program is shown in Figure 4.2, the actual computer printout being given in the Appendix. Flow chart notation follows that of McCracken (1965). This flow chart is preceded by the flow chart in Figure 4.1 for the input portion of the main program, in which all quantities necessary for later calculations are defined or computed.

4.2 The Numerical Integration

Once the initial solution has been found, the numerical integration down through the real ionosphere is carried out using a modified Runge-Kutta process due to Gill (1951). The pertinent constants of nature, input parameters (indicated by asterisks) and integration variables are defined below. The left-hand symbol is the standard notation; the right-hand symbol is the notation used in the computer program, if it differs from the standard notation. MKS units are used in all calculations. All calculations are performed with double precision accuracy.

e	Charge of the electron (it is negative).
m,ELECM	Mass of the electron.
ε_o,PERME	Permittivity of free space.
μ_o,PERMM	Permeability of free space.
c	Speed of light in a vacuum.
B	Magnitude of earth's magnetic field.
θ,THETA(in radians) * ANGLE(in degrees)	Angle of incidence between the vertical and the wave normal of the incident wave below the ionosphere.

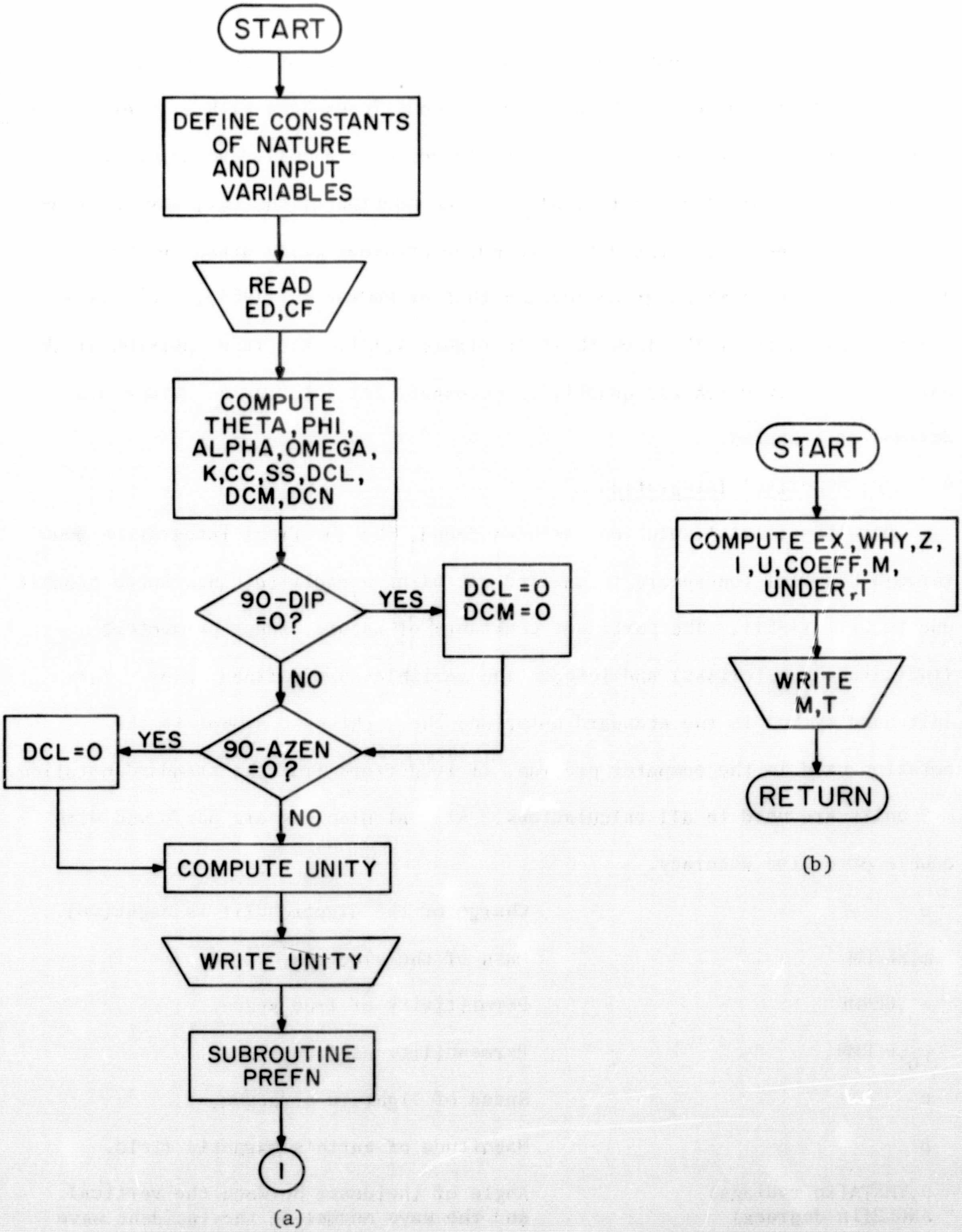


Figure 4.1 Flow chart depicting (a) input portion of main program, including (b) subroutine PREFN.

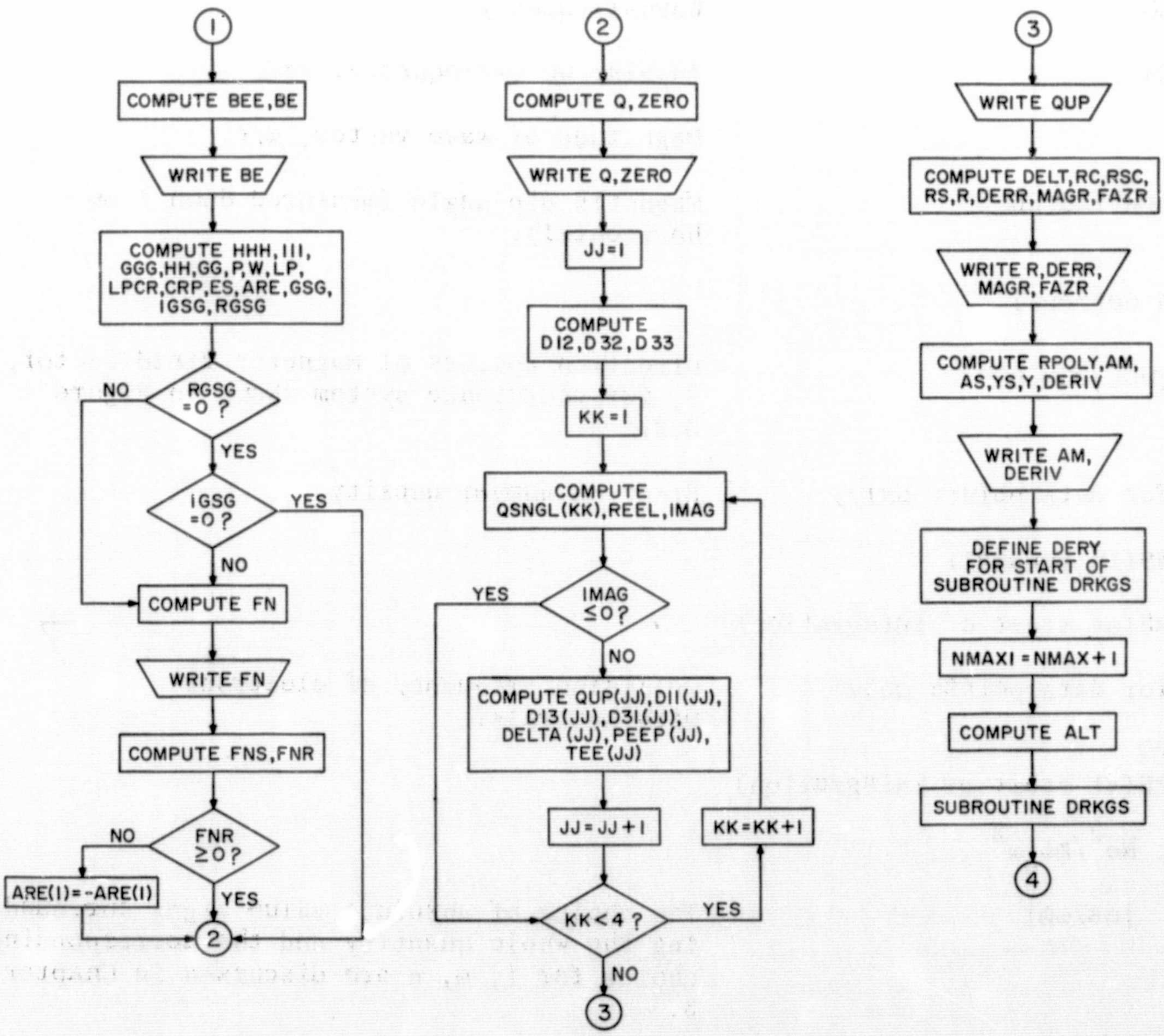


Figure 4.2 Flow chart depicting initial solution (after Sheddy, 1968) portion of main program.

α,ALPHA(in radians)	Azimuth east of magnetic north of positive x axis in the plane of propagation.
* AZEN(in degrees)	
* f, FREQ	Wave-frequency.
ω,OMEGA	Angular wave-frequency, $2\pi f$.
k	Magnitude of wave vector, ω/c.
ϕ,PHI(in radians)	Magnetic dip angle (measured down from horizontal).
* DIP(in degrees)	
l,m,n;DCL,DCM,DCN	Direction cosines of magnetic field vector, $\bar{Y}$, for coordinate system shown in Figure 3.2.
* N,ED(For data points only)	Electron number density.
EDENS(in general)	
EDPRE(at start of integration)	
* ν,CF(for data points only)	Collision frequency of electrons with neutrals.
CFREQ	
CFPRE(at start of integration)	
X,EX $Ne^2/m\varepsilon_o\omega^2$	
Y,WHY $\lvert eB/\omega m\rvert$	The choice of absolute value signs surrounding the whole quantity and the corresponding choice for l, m, n are discussed in Chapter 3.
Z	ν/ω
i	$\sqrt{-1}$
$\underset{\approx}{M}$,M=MR+iMI	Susceptibility matrix.
$\underset{\approx}{T}$,T=TR+iTI	T matrix.
q	Root of Booker quartic equation.
$R_{11} = {}_{\parallel}R_{\parallel}$,RC(1)=R(1)+iR(5)	Elements of reflection coefficient matrix, $\underset{\approx}{R}$

$R_{12} = {}_{\parallel}R_{\perp}$, RC(2)-R(2)+iR(6)	
$R_{21} = {}_{\perp}R_{\parallel}$, RC(3)=R(3)+iR(7)	
$R_{22} = {}_{\perp}R_{\perp}$, RC(4)=R(4)+iR(8)	
$\lvert\tilde{R}\rvert$, MAGR	Matrix of reflection coefficient magnitudes or amplitudes
$\arg\tilde{R}$, FAZR	Matrix of reflection coefficient phases or arguments
$\tilde{R}'$, DERR	Matrix of reflection coefficient derivatives with respect to altitude
A_{11}, AM(1)=Y(1)+iY(5)	Elements of wave admittance matrix, $\tilde{A}$
A_{12}, AM(2)=Y(2)+iY(6)	
A_{21}, AM(3)=Y(3)+iY(7)	
A_{22}, AM(4)=Y(4)+iY(8)	
$\tilde{A}'$, DERIV DERIV(I)=DERY(I)+iDERY(I+4)	Matrix of wave admittance derivatives with respect to altitude
* NMAX	The number of data points comprising the digitized form of the electron density profile (same for collision frequency profile).
* PRMT(1)	Height at start of integration.
* PRMT(2)	Height at end of integration.
* PRMT(3)	Integration interval.
* PRMT(4)	Accuracy maintained per step in the integration.
* ZDATA	Height interval between data points.
ALT	Altitudes corresponding to data points representing profiles of N and ν.
IHLF	Number of times integration interval is halved in order to ensure accuracy supplied in PRMT(4).
C,CC	$\cos\theta$
S,SS	$\sin\theta$

The electron density and collision frequency profiles are specified by data points given every (-ZDATA) meters between PRMT(1) and (PRMT(2) + ZDATA) inclusive. If PRMT(3) < ZDATA, the values in between data points are computed by linear interpolation; no sophisticated fitting of polynomial curves to points is used. PRMT(1) is chosen so that EX $\approx$ 100(1+WHY) and PRMT(2) so that EX < 1 for the frequency being used. Although these limits are vestiges of ray theory, and should not be considered to be related to precise levels of reflection for VLF waves, they give reasonable boundaries for the region of significant wave-ionosphere interactions.

The angle of incidence is computed by geometry, knowing the distance between the transmitter and receiver, and assuming an average height of reflection equal to the altitude where EX = (1+WHY). This height decreases during the sunrise period, causing θ to increase.

PRMT(3) is chosen small enough so that IHLF does not become too large, yet large enough so that an unnecessary number of calculations over gradual electron density gradients are avoided. PRMT(4) is chosen large enough so that IHLF does not become too large over steep electron density gradients, yet small enough so that the final results are meaningful.

Figure 4.3 shows the flow chart for the DRKGS IBM SSP Library subroutine used in the numerical integration. Figure 4.4 shows the flow chart for the subroutines FCT and OUTP called from within DRKGS. Use of the OUTP subroutine can provide a continuous print-out of the reflection coefficients down through the entire ionosphere, producing diagrams analogous to those of Pitteway (1964), who used the penetrating and non-penetrating wave-fields as dependent variables. The locations for this use of the OUTP subroutine are shown as dashed boxes in Figure 4.3. If this is not desired, the OUTP subroutine is not necessary. The

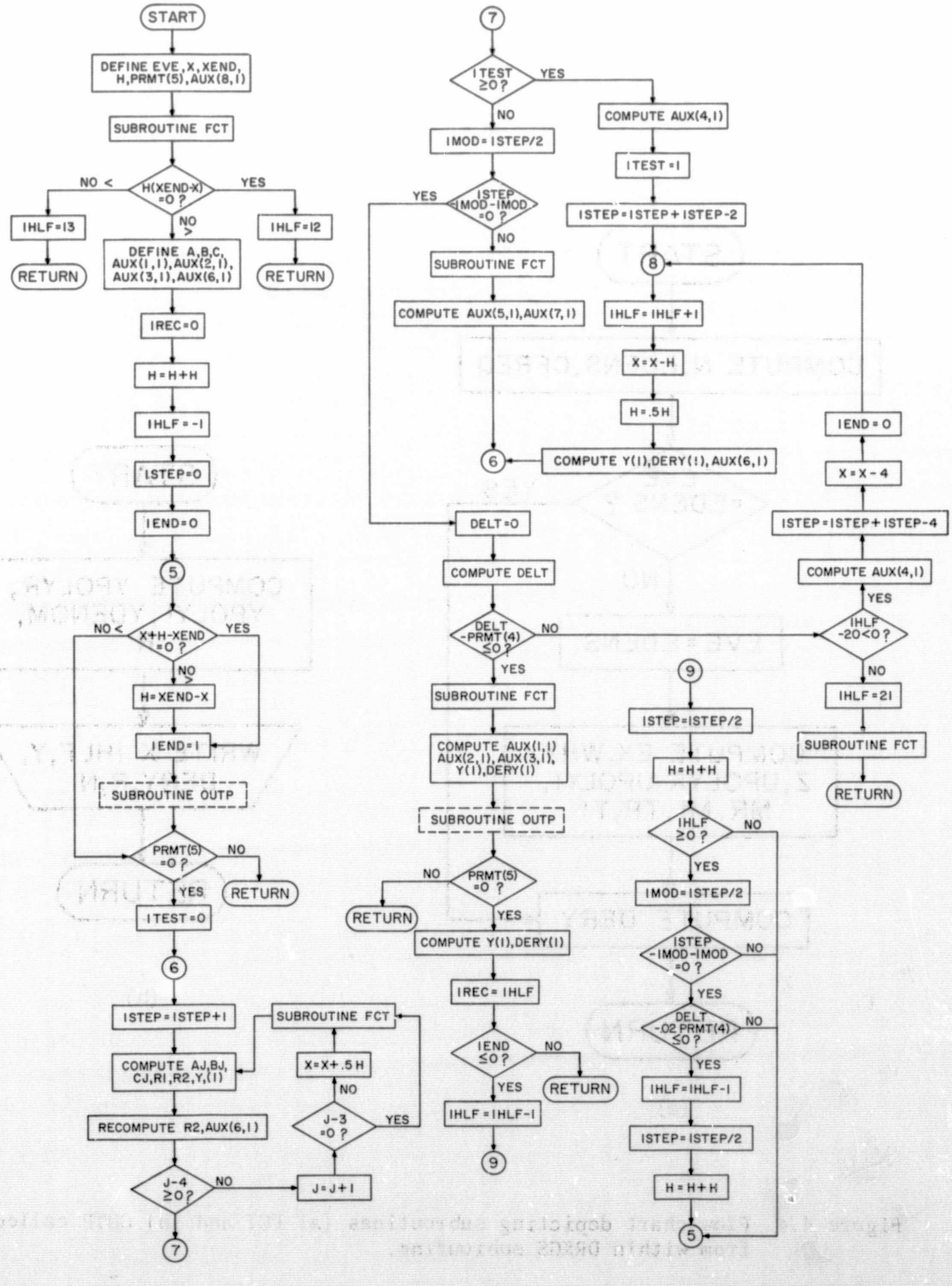
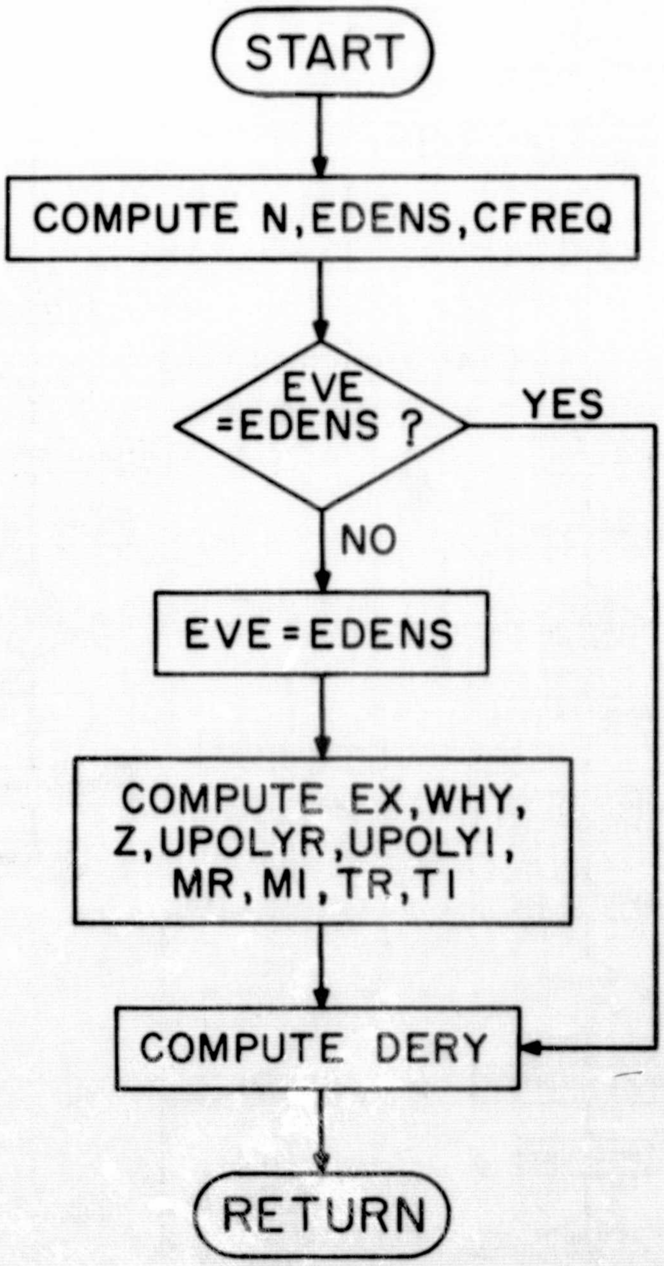
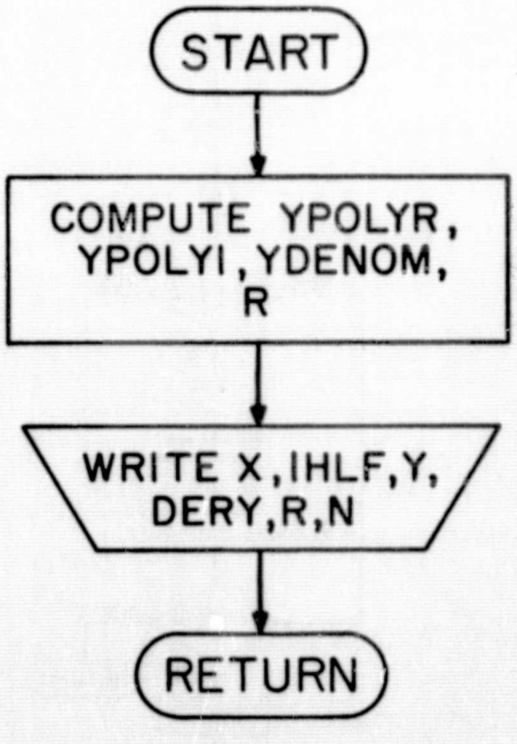


Figure 4.3 Flow chart depicting numerical integration subroutine DRKGS.



(a)



(b)

Figure 4.4 Flow chart depicting subroutines (a) FCT and (b) OUTP called from within DRKGS subroutine.

FCT subroutine computes the derivatives of the wave admittances as they steadily change down through the ionosphere because of the changing electron density and collision frequency. Figure 4.5 shows the output portion of the program, which gives the final reflection coefficient values below the ionosphere.

No provision was made to prevent numerical swamping, since with the dependent variables being of the same order of magnitude and the computer having great accuracy when operating in the double precision mode it was felt unnecessary.

4.3 Checks of the Full-Wave Program

A complicated computer program such as the one used in full-wave solutions cannot be used with any confidence until after its results have been thoroughly checked against known results.

The correctness of the starting solutions was verified in several ways. First, the Booker quartic roots were substituted into the Booker quartic equation to see if they satisfied it. The value of the Booker quartic equation, which should strictly be zero, was always at least 10^2 times smaller than any of the roots themselves, and often 10^5 times smaller.

The starting reflection coefficients and admittance matrix elements were checked by substituting them into the differential equations. The values of the derivatives, which again for an exact solution should be zero, were at least 10^4 times smaller than any of the matrix elements themselves, and some of the derivatives were as much as 10^{17} times smaller. In other words, with matrix element magnitudes on the order of unity, the derivatives had magnitudes of 10^{-4} and smaller, or as close to the value zero as one might reasonably expect from a digital computer following all the steps of Sheddy's method while carrying 17 digits in its double precision mode of operation.

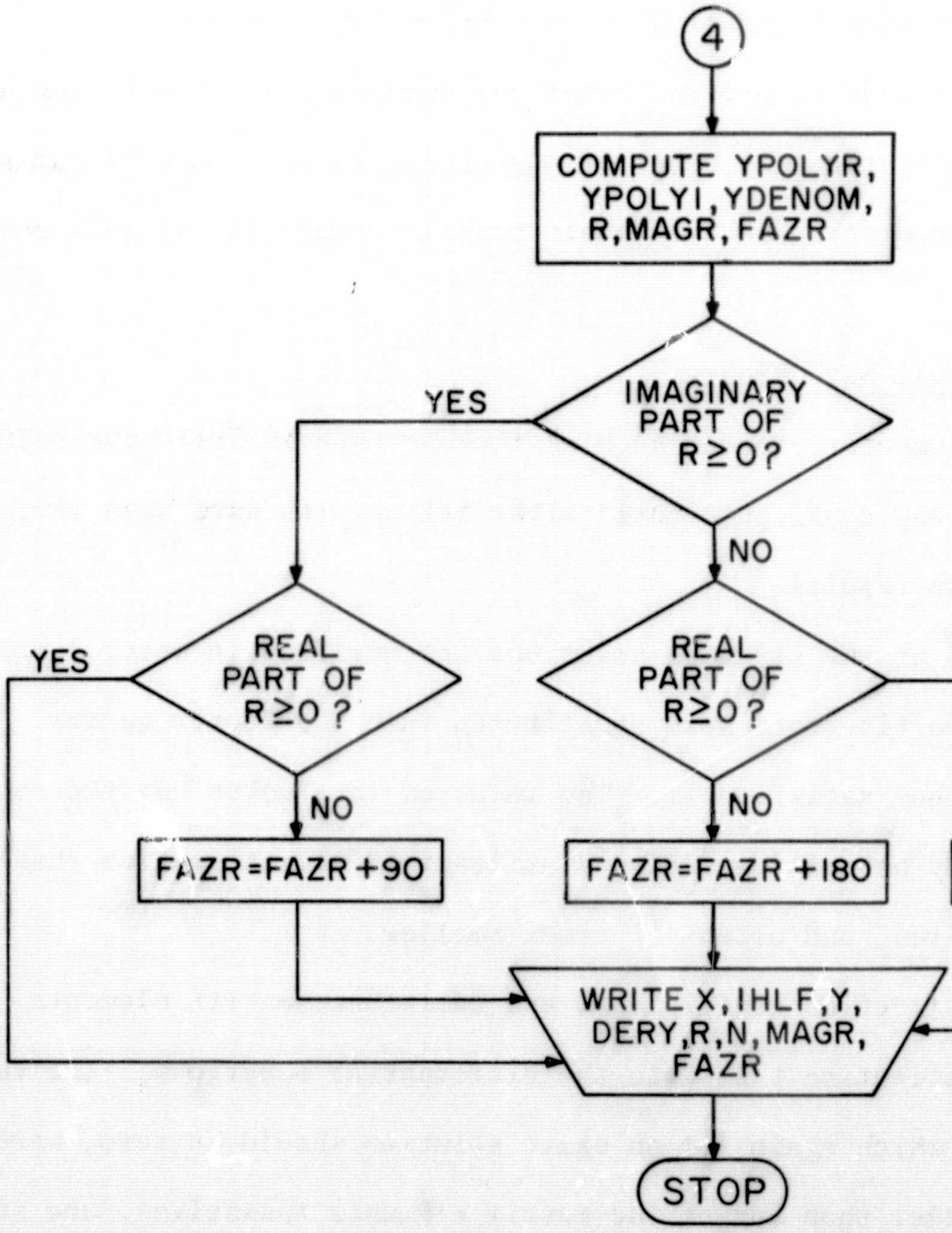


Figure 4.5 Flow chart depicting final output portion of main program.

In addition, a comparison was made with results of initial solutions numerically calculated by Fedor, et al., (1964) for the special case of west-east propagation. Table 4.1 shows the results of the comparison, with all numbers rounded off to 2 places to the right of the decimal for brevity.

The numerical integration and the resulting final solutions were checked by comparing results with those of Budden (1955b), who in turn compared his results with the analytic solutions of Heading and Whipple (1952). The comparison was made for the case of vertical incidence, an exponential electron density profile, a constant collision frequency, a vertically downward magnetic field, and west-east propagation. Figure 4.6a shows Budden's comparison with Heading and Whipple for $X = \exp(.295z)$ and $Z = 8$ where z is in km. Points marked X were calculated by the author. A similar comparison is shown in Figure 4.6b, but the electron density gradient is sharper, $X = \exp(2.36z)$, and $Z = 2$.

A further comparison was made with the computing facilities at NELC and the Radio Research Station, Slough, England. For this check only, the collision frequency profile of Fejer and Vice (1959) was used, along with the nighttime electron density profile of Deeks (1964). The angle of incidence was taken to be 40°, the dip angle 70°, the azimuth of the propagation path 152°, and the wave-frequency 21.4 KHz. The numerical integration was started at 108.00 km and stopped at 63.55 km. Table 4.2 compares the results.

All of the comparisons made indicate that full-wave calculations with the program described can be used with reasonable confidence to find the variation of the reflection coefficient matrix under different circumstances, even for an ionosphere with steep electron density gradients.

TABLE 4.1

A comparison of the author's numerically calculated Starting solutions with those of Fedor, et al., (1964) of NELC.

	${}_{\parallel}R_{\parallel}$	${}_{\parallel}R_{\perp}$	${}_{\perp}R_{\parallel}$	${}_{\perp}R_{\perp}$
NELC	$9.66\times10^{-1}-i3.43\times10^{-2}$	$3.31\times10^{-2}+i3.05\times10^{-2}$	$3.31\times10^{-2}+i3.05\times10^{-2}$	$-9.68\times10^{-1}+i3.24\times10^{-2}$
VIERTEL	$9.48\times10^{-1}-i5.28\times10^{-2}$	$5.08\times10^{-2}+i4.51\times10^{-2}$	$5.08\times10^{-2}+i4.51\times10^{-2}$	$-9.51\times10^{-1}+i4.97\times10^{-2}$

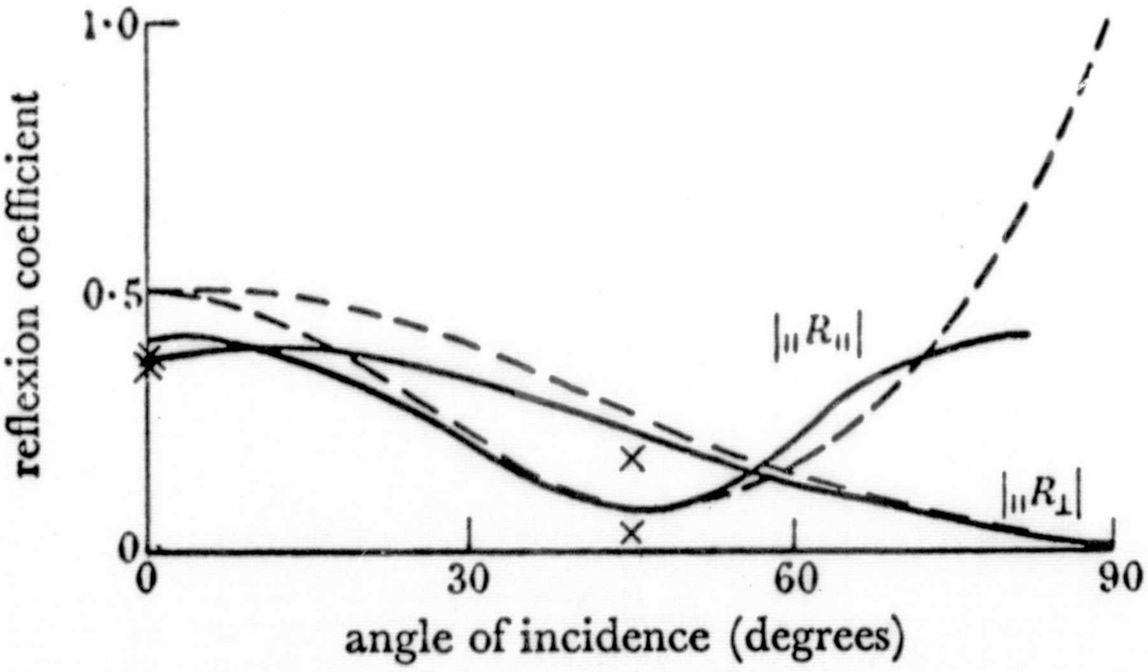
	A_{11}	A_{12}	A_{21}	A_{22}
NELC	15.26-i15.11	14.96-i15.25	14.96-i15.25	-15.39+i15.11
VIERTEL	9.91-i 9.82	9.75-i 9.97	9.74-i 9.97	-10.05+i 9.81

	A'_{11}	A'_{12}	A'_{21}	A'_{22}
NELC	$-1.48\times10^{-5}-i3.01\times10^{-4}$	$3.02\times10^{-4}+i1.48\times10^{-5}$	$3.02\times10^{-4}+i1.48\times10^{-5}$	$1.49\times10^{-5}+i3.03\times10^{-4}$
VIERTEL	$0.00\ -i3.20\times10^{-17}$	$6.97\times10^{-6}-i8.19\times10^{-6}$	$-2.28\times10^{-17}+i2.74\times10^{-17}$	$-5.97\times10^{-8}-i3.52\times10^{-9}$

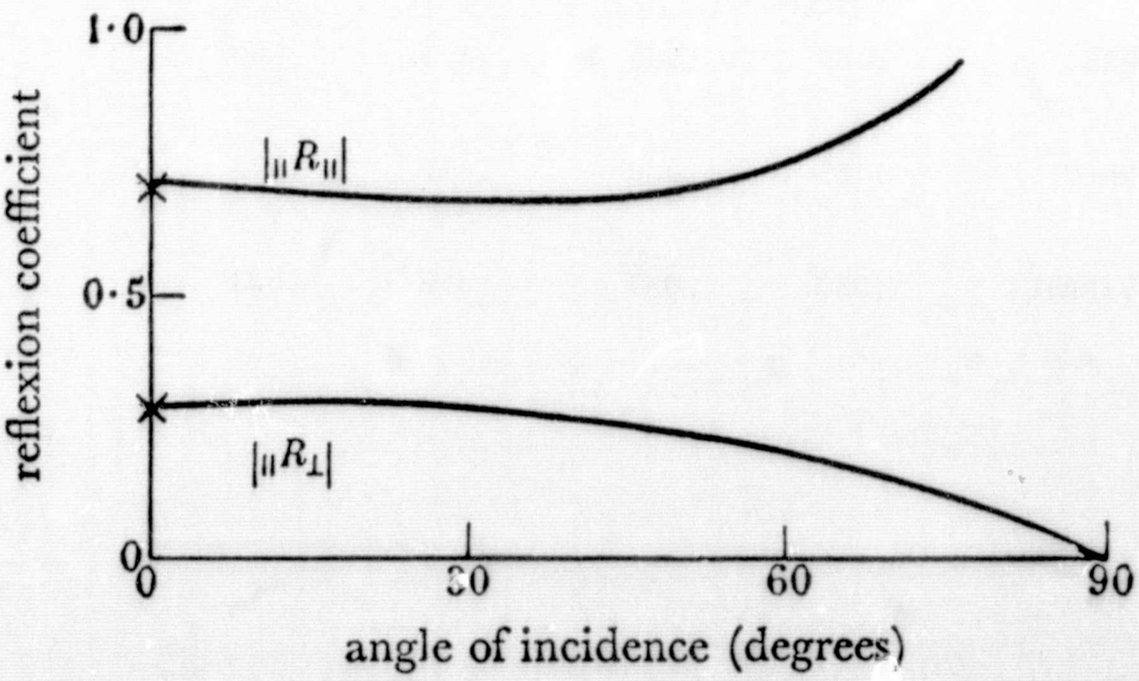
TABLE 4.2

A comparison of full-wave solutions for the nighttime electron density profile of Deeks (1964) and the collision frequency profile of Fejer and Vice (1959).

	$\lvert {}_{\parallel}R_{\parallel} \rvert$	$\lvert {}_{\parallel}R_{\perp} \rvert$	$\lvert {}_{\perp}R_{\parallel} \rvert$	$\lvert {}_{\perp}R_{\perp} \rvert$
RRS	.088	.089	.408	.509
NELC	.085	.088	.436	.573
VIERTEL	.057	.087	.458	.621



(a)



(b)

Figure 4.6 A comparison of the full-wave results, reflection coefficient magnitude vs angle of incidence, for ν constant, $\phi = 90°$, and $\alpha = 90°$, of (a) Budden (1955b) (solid line), Budden using the formulas of Heading and Whipple (1952) (dashed line), and the author (points marked x) for $X = \exp(.295z)$, $Z = 8$ and (b) Budden (1955b) (solid line) and the author (points marked x) for $X = \exp(2.36z)$, $Z = 2$.

4.4 Suggestions for Applications of the Full-Wave Program

One important application of the full-wave program would be to test the consistency of theoretical and experimental methods of studying the D region by comparing the temporal behavior of the reflection coefficient matrix over the sunrise period obtained by full-wave solutions with the results from ground-based VLF studies. For the electron density profiles used in the full-wave program, the results of Smith et al. (1966), shown in Figure 4.7, might be used, with suitable lower E region profiles added to the upper portions of the curves to extend them to sufficiently high levels where the integration would be started. For experimental VLF results, those given by Sechrist (1968) might be employed. The results of both Smith and Sechrist were obtained during a period of minimum solar activity, but their measurements were made at mid-latitudes in different hemispheres. Also, Smith's profiles were not all determined in the same season. Clearly, the optimum comparison would involve data obtained at the same place and time, but such information is regrettably scarce.

Another valuable comparison would involve replacing Smith's electron density profiles above, which were deduced from ground-based experiments, with profiles obtained from rocket measurements, such as those of Mechtly and Smith (1968) shown in Figure 4.8. The degree of agreement of the theoretical results, obtained using both sets of sunrise electron density profiles, with the experimental VLF results might solve the controversy over which measurement technique is better for measuring electron densities.

If the degree of agreement between theory and experiment proved to be good, more confidence could be displayed in studying not only the sunrise period, but the similar sunset and eclipse conditions by the theoretical and

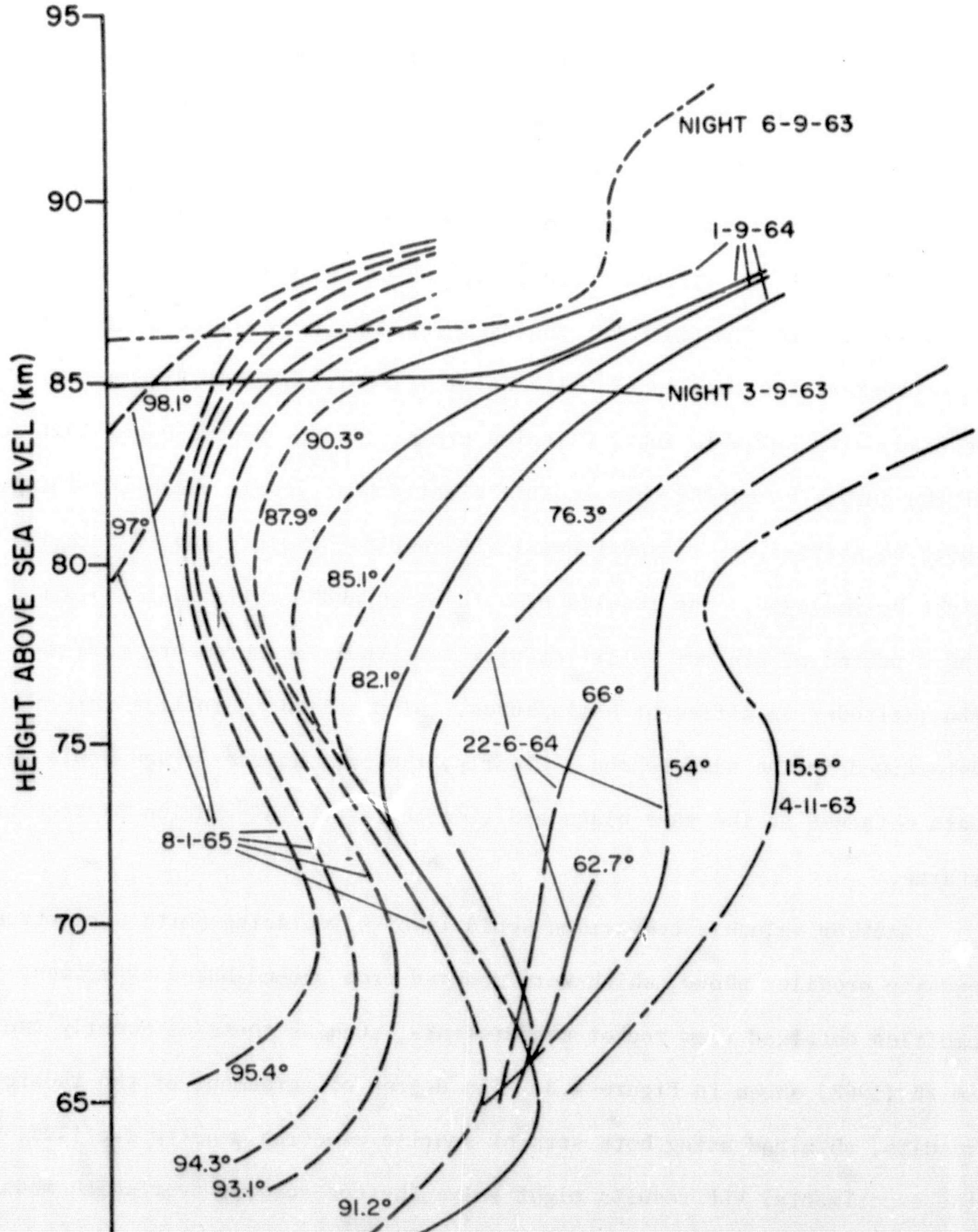


Figure 4.7 D-region electron density profiles for night, dawn, and daytime periods (after Smith et al., 1966) deduced from ground-based measurements.

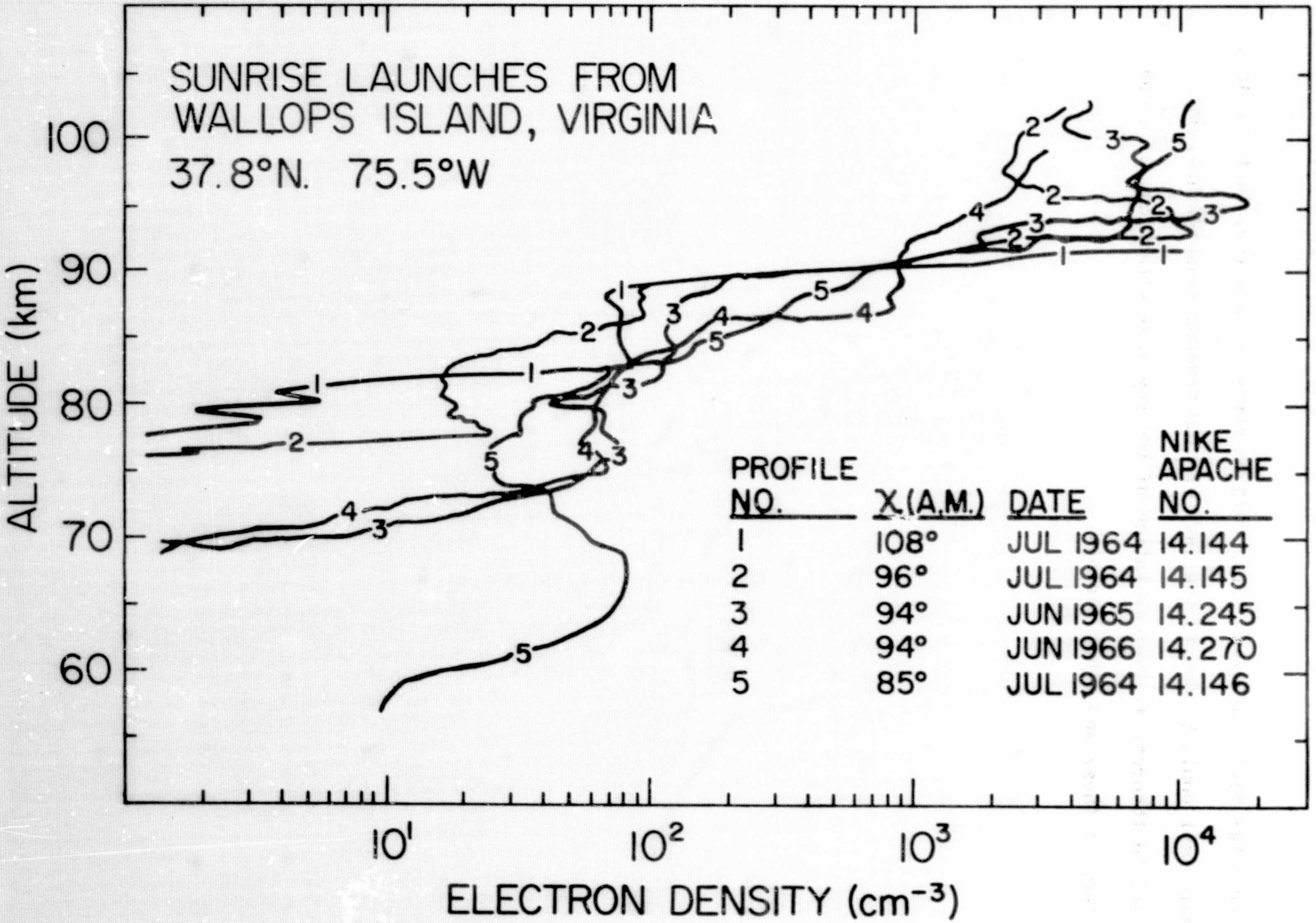


Figure 4.8 D-region electron density profiles over the sunrise period measured with rockets (after Mechtly and Smith, 1968).

experimental means now used. It should be stressed, however, that the lack of agreement could be due to the lack of simultaneous measurements, an incomplete theoretical representation of the physical situation, or a combination of these and other factors.

APPENDIX

The full-wave computer program printout is shown for a sample application, that of finding the reflection coefficients for a sunrise electron density profile. The flow charts for this program are given in Figures 4.1-4.5.

```
      $JOB              KP=29,LINES=3000,TIME=900,PAGES=100
  1          DIMENSION PRMT(5),DERY(8),AUX(8,8),ED(119) ,CF(119) ,ALT(120) ,     VARIABLE
            CTR(4,4),TI(4,4),MR(3,3),MI(3,3),Y(8),R(8),MAGR(4),FAZR(4),T(4,4),
            CDERIV(4),  QSNGL(4),RSC(4), M(3,3),BEE(5),BE(4),W(3),ES(3),ARE(3),
            CQ(4),QUP(2),D11(2),D13(2),D31(2),DELTA(2),PEEP(2),TEE(2),RC(4),
            CAM(4),ZERO(4),AMS(4),RS(8),YS(8),DERR(4)
  2          REAL*4 FNR,SNGL,REAL,AIMAG,REEL,IMAG,RGSG,IGSG,RS,YS
  3          REAL*8 PRMT,Y,DERY,PERME,PERMM,E,EX,WHY,Z,C,B,OMEGA,K,FREQ,DCL,
            CDCM,DCN,ELECM,PHI,     THETA,CC,SS,X,ED,CF,PI,UNITY,MAGR,FAZR,R,
            CDCOS,DSIN,DABS,TR,TI,MR,MI,UR,UI,EDPRE,CFPRE,ALPHA,  AUX,UPOLYR,
            CUPOLYI,YPOLYR,YPOLYI,UDENOM,DSQRT, DBLE,ALT,  DATAN,MDENOM,YDENOM
            C,ANGLE,DIP,AZEN,ZDATA
  4          COMPLEX*8 CSQRT,CMPLX,CLOG,CEXP,GSG,FNS,QSNGL,RSC,AMS
  5          COMPLEX*16 M,HH ,GG ,BEE,BE,HHH,III,GGG,ARE,Q,QUP,ZERO,D11,D12,
            CD13,D31,D32,D33,DELTA,PEEP,TEE,DELT,RC,AM,RPOLY,P,LP,LPCR,CRP,ES,
            CFN,CDLOG,CDEXP,CDSQRT,DCMPLX,W,DCONJG,U,I,COEFF
            C,T,DERIV,DERR,CYU,ZIP
  6          EXTERNAL FCT,OUTP
      C
      C      THE FOLLOWING ARE CONSTANTS OF NATURE
      C          E IS THE ELECTRON CHARGE
      C          ELECM IS THE ELECTRON MASS
      C          PERME IS THE ELECTRIC PERMITTIVITY OF FREE SPACE
      C          PERMM IS THE MAGNETIC PERMEABILITY OF FREE SPACE
      C          C IS THE SPEED OF LIGHT IN A VACUUM
      C          B IS THE EARTH'S MAGNETIC FIELD
      C      THE FOLLOWING ARE INPUT VARIABLES
      C          THETA IS THE ANGLE OF INCIDENCE
      C          PHI IS THE MAGNETIC DIP ANGLE, MEASURED FROM THE HORIZONTAL.
      C         IF B FIELD DIPS DOWN AS IN NORTHERN HEMISPHERE, PHI IS POSITIVE
      C         ANC 0<PHI<90.  IF B FIELD DIPS UP AS IN SOUTHERN HEMISPHERE,
      C         PHI IS NEGATIVE AND -90<PHI<0.
      C          ALPHA IS THE AZIMUTH OF PROPAGATION, MEASURED EAST OF MAGNETIC
      C         NORTH, THAT IS, IT IS THE ANGLE BETWEEN MAGNETIC NORTH AND THE
      C         POSITIVE X AXIS, WHERE THE XZ PLANE IS THE PLANE OF PROPAGATION.
      C         POSITIVE Z IS UPWARDS, POSITIVE X IS IN THE DIRECTION OF THE
      C         HORIZONTAL COMPCNENT OF K, THE WAVEVECTOR.  THE COORDINATE
      C         SYSTEM IS RIGHT-HANDED.
      C          FREQ IS THE FREQUENCY OF THE INCIDENT PLANE WAVE
      C          ED IS THE ELECTRON DENSITY, WHICH IS A FUNCTION OF CHI, THE
      C         SOLAR ZENITH ANGLE, AS WELL AS ALTITUDE (ED PROFILE IS ONDATA
      C         CARCS)
      C          CF IS NU, THE COLLISION FREQUENCY(CF PROFILE IS ON DATA CARDS)
      C      CF IS THE NUMBER OF COLLISIONS PER SEC EACH ELECTRON MAKES, NOT THE
      C      NUMBER OF ELECTRONS COLLIDING PER SEC
      C          X = ALTITUDE
      C          DCL, DCM, DCN ARE THE DIRECTION COSINES OF-B WITH RESPECT TO
      C      A COORDINATE SYSTEM WHOSE XZ PLANE IS THE PLANE OF INCIDENCE AND
      C      WHOSE Z AXIS VERTICAL.
      C      ALL VALUES ARE IN MKS UNITS
  7          E      =-1.6021D-19
  8          ELECM = 9.1091D-31
  9          PERME = 8.8542D-12
 10          PERMM = 1.25664D-6
 11          C  ·  = 2.997925D8
 12          PI    = 3.141593D0
 13          B     = 0.560D-4                                                VARIABLE
      C      CHI = 95.4                                                       VARIABLE
 14          ANGLE = 37.8D0                                                   VARIABLE
 15          DIP   = 70.0D0                                                   VARIABLE
 16          AZEN  =152.0D0                                                   VARIABLE
 17          FREQ  = 21.40D3                                                  VARIABLE
 18          NMAX = 119                                                       VARIABLE
 19          EDPRE = 3.70D10                                                  VARIABLE
 20          CFPRE = 9.40D04                                                  VARIABLE
 21          PRMT(1) =   94000.0D0                                            VARIABLE
 22          PRMT(2) =   64750.0D0                                            VARIABLE
 23          PRMT(3) =-25.CD0                                                 VARIABLE
 24          PRMT(4) =       .001CCD0                                         VARIABLE
 25          ZDATA =-250.0D0                                                  VARIABLE
 26          NDIM  = 8
```

```
      READ(5,25) ED,CF
   25 FORMAT(11(1P10D8.2/),1P9D8.2/(1P1D8.2))
C     THE FOLLOWING VARIABLES ARE FUNCTIONS OF THE ABOVE INPUT VARIABLES
      THETA =ANGLE*PI/180.0D0
      PHI   =   DIP * PI / 180.0D0
      ALPHA = AZEN*PI/180.0D0
      OMEGA =2.0D0* PI * FREQ
      K = OMEGA / C
      CC = DCOS(THETA)
      SS = DSIN(THETA)
      DCL =-DCOS(PHI) * DCOS(ALPHA)
      DCM =-DCOS(PHI) * DSIN(ALPHA)
      DCN =+DSIN(PHI)
C     THE FOLLOWING CARDS ENSURE THE CORRECT VALUES OF DCL AND DCM.
C     THEY ARE NECESSARY BECAUSE OF THE FINITE ACCURACY OF THE COMPUTER.
      IF(90.0D0 - DIP)92,91,92
   91 DCL = 0.0D0
      DCM = 0.0D0
   92 IF(90.0D0 - AZEN)94,93,94
   93 DCL = 0.0D0
   94 UNITY = DCL*DCL + DCM*DCM + DCN*DCN                              CHECK
      WRITE(6,1) UNITY                                                 CHECK
    1 FORMAT(25X,5HUNITY/20X,1P1D19.10//)                              CHECK
C     BEGIN CALCULATION OF INITIAL SOLUTIONS USING METHOD OF SHEDDY.
             CALL PREFN(M,T   ,SS,CC,DCL,DCM,DCN,EDPRE,CFPRE,OMEGA,ELECM,
     CPERME,E,K,B)
      BEE(4) =1.0D0+ M(3,3)
      BEE(3) =   SS * (M(1,3) + M(3,1))
      BEE(2) = -(( CC* CC) + M(3,3)) *(1.0D0+ M(1,1)) + (M(1,3)*M(3,1))
     C+(M(2,3)*M(3,2))-(1.0D0+ M(3,3))* (( CC* CC) + M(2,2))
      BEE(1) =   SS*(M(1,2)*M(2,3) + M(2,1)*M(3,2) -(( CC* CC) + M(2,2))
     C*(M(1,3) + M(3,1)))
      BEE(5) =(1.0D0+ M(1,1))*(( CC* CC) + M(2,2))*(( CC* CC) + M(3,3))
     C+M(1,2)*M(2,3)*M(3,1) + M(1,3)*M(2,1)*M(3,2) -M(1,3)*(( CC* CC)+
     CM(2,2))*M(3,1)-(1.0D0+ M(1,1))*M(2,3)*M(3,2) -M(1,2)*M(2,1)*((
     C CC* CC) + M(3,3))
      BE(3) = BEE(3)/(4.0D0*BEE(4))
      BE(2) = BEE(2)/(6.0D0*BEE(4))
      BE(1) = BEE(1)/(4.0D0*BEE(4))
      BE(4) = BEE(5)/BEE(4)
      WRITE(6,3)  BE(3),BE(2),BE(1),BE(4)
    3 FORMAT(25X,2HBE/4(20X,1P2D19.10/)//)
      HHH = BE(2) - (BE(3)*BE(3))
      III = BE(4) -(4.0D0* BE(3) * BE(1)) +(3.0D0* BE(2) * BE(2))
      GGG = BE(1)-(3.0D0* BE(3) * BE(2)) +(2.0D0* BE(3) * BE(3) * BE(3))
      HH = - III/12.0D0
      GG = -(GGG *GGG/4.0D0)-(HHH * ((HHH*HHH) +(3.0D0* HH )))
      P = (-GG +CDSQRT((GG*GG) +(4.0D0* HH*HH*HH)))/2.0D0
      W(1) =(1.0D0,0.0D0)
      W(2) = DCMPLX(DCOS(2.0D0*PI/3.0D0),DSIN(2.0D0*PI/3.0D0))
      W(3) = DCMPLX(DCOS(4.0D0*PI/3.0D0),DSIN(4.0D0*PI/3.0D0))
      LP =CDLOG(P)
      LPCR = LP/3.0D0
      CRP =CDEXP(LPCR)
      DO  4 L=1,3
      ES(L) = W(L) * CRP
      ARE(L) =CDSQRT(ES(L) - (HH/ES(L)) - HHH)
    4 CONTINUE
C     TEST(IS -2.0*ARE(1)*ARE(2)*ARE(3)/GGG = 1.0?)
      GSG = GGG
      IGSG = AIMAG(GSG)
      RGSG = REAL(GSG)
      IF(RGSG)6,5,6
    5 IF(IGSG)6,9,6
    6 FN = -2.0*ARE(1)*ARE(2)*ARE(3)/GGG
      WRITE (6,7)  FN
    7 FORMAT(25X,2HFN/20X,1P2D19.10//)
      FNS = FN
      FNR = REAL(FNS)
      IF(FNR)8,9,9
    8 ARE(1) = -ARE(1)
    9 CONTINUE
```

```
C       SOLUTIONS TO THE BCOKER QUARTIC
        Q(1) = ARE(1)+ARE(2)+ARE(3) -BE(3)
        Q(2) = ARE(1)-ARE(2)-ARE(3) -BE(3)
        Q(3) =-ARE(1)+ARE(2)-ARE(3) -BE(3)
        Q(4) =-ARE(1)-ARE(2)+ARE(3) -BE(3)
        WRITE(6,10) Q(1),Q(2),Q(3),Q(4)
     10 FORMAT(25X,1HQ/4(2CX,1P2D19.10/)///)
        DO  11 LLL=1,4                                                  CHECK
        ZERO(LLL) = Q(LLL)*Q(LLL)*Q(LLL)*Q(LLL)+4.0D0*BE(3)*Q(LLL)*Q(LLL)*CHECK
       CQ(LLL)+6.0D0*BE(2)*Q(LLL)*Q(LLL)+4.0D0*BE(1)*Q(LLL) + BE(4)       CHECK
     11 CONTINUE                                                        CHECK
        WRITE(6,12)ZERO(1),ZERO(2),ZERO(3),ZERO(4)                      CHECK
     12 FORMAT(25X,4HZERO/4(20X,1P2D19.10/)//)                          CHECK
        I = (0.0DC,1.CD0)
C       D SIGNIFIES A DISPERSICN MATRIX ELEMENT
C          WE CHOOSE THE SCLUTIONS TC THE QUARTIC EQUATION WHICH CORRESPOND
C       TO UPGOING WAVES, THOSE WITH POSITIVE REAL AND NEGATIVE IMAGINARY PARTS
        JJ = 1
        D12 = M(1,2)
        D32 = M(3,2)
        D33 = ( CC* CC) + M(3,3)
        DO 13 KK=1,4
        QSNGL(KK) = Q(KK)
        REEL = REAL( QSNGL(KK))
        IMAG = -AIMAG( QSNGL(KK))
        IF(.NOT. (                         IMAG .GT. 0.0))GO TO 13
        QUP(JJ) = Q(KK)
        D11(JJ) =1.0D0+ M(1,1) -(QUP(JJ)*QUP(JJ))
        D13(JJ) = M(1,3) + (QUP(JJ)* SS)
        D31(JJ) = M(3,1) + (QUP(JJ)* SS)
        DELTA(JJ) = D11(JJ)*D33 - D13(JJ)*D31(JJ)
        PEEP(JJ) = (-D12*D33 + D13(JJ)*D32)/DELTA(JJ)
        TEE(JJ) =  QUP(JJ)*PEEP(JJ) - (SS*(-D11(JJ)*D32 + D12*D31(JJ)) /
       CDELTA(JJ))
        JJ = JJ + 1
     13 CONTINUE
        WRITE(6,14) QUP(1),QUP(2)
     14 FORMAT(25X,3HQUP/2(20X,1P2D19.10/)///)
        DELT = (TEE(1)* CC + PEEP(1))*( CC+QUP(2)) - (TEE(2)* CC + PEEP(2)
       C)*( CC+QUP(1))
        RC(1)  = ((TEE(1)* CC - PEEP(1))*( CC + QUP(2)) - (TEE(2)* CC -
       CPEEP(2))*( CC + QUP(1)))/DELT
        RC(2)  =-2.0D0*CC*(QUP(1) - QUP(2))/DELT
        RC(3)  =-2.0D0*CC*(TEE(1)*PEEP(2) - TEE(2)*PEEP(1))/DELT
        RC(4)  = ((TEE(1)* CC + PEEP(1))*( CC - QUP(2)) - (TEE(2)* CC +
       CPEEP(2))*( CC - QUP(1)))/DELT
        DO  15 NN=1,4
        II=NN+4
        RSC(NN) = RC(NN)
        RS(NN)=       REAL(RSC(NN))
        R(NN) = DBLE(RS(NN))
        RS(II)=       AIMAG(RSC(NN))
        R(II) = DBLE(RS(II))
     15 CONTINUE
C       R(1)=REAL (PARA R PARA)          R(5) = IMAG(PARA R PARA)
C       R(2)=REAL (PARA R PERP)          R(6) = IMAG(PARA R PERP)
C       R(3)=REAL (PERP R PARA)          R(7) = IMAG(PERP R PARA)
C       R(4)=REAL (PERP R PERP)          R(8) = IMAG(PERP R PERP)
        WRITE(6,40) RC
     40 FORMAT(28X,18HINITIAL VALUE OF R/4(20X,1P2D18.10/))              CHECK
        DERR(1) = -(I/2.0DC)*(-T(1,1)+T(4,4)-(T(1,4)/CC)+CC*T(4,1)+RC(1)*
       C(T(1,1)+T(4,4)-(T(1,4)/CC)-CC*T(4,1))+RC(3)*((T(1,2)/CC)-T(4,2))
       C-(T(1,1)+T(4,4)+(T(1,4)/CC)+CC*T(4,1))*RC(1)-(-T(3,1)-(T(3,4)/CC))
       C*RC(2)-RC(1)*RC(1)*(-T(1,1)+T(4,4)+(T(1,4)/CC)-CC*T(4,1))-RC(1)*
       CRC(2)*(T(3,1)-(T(3,4)/CC))-RC(1)*RC(3)*(-(T(1,2)/CC)-T(4,2))-RC(2)
       C*RC(3)*(-CC+(T(3,2)/CC)))*K
        DERR(2) =-(I/2.0D0)*((T(1,2)/CC)-T(4,2)+(T(1,1)+T(4,4)-(T(1,4)/CC)
       C-CC*T(4,1))*RC(2)+((T(1,2)/CC)-T(4,2))*RC(4)-(-(T(1,2)/CC)-T(4,2))
       C*RC(1)-(CC+(T(3,2)/CC))*RC(2)-RC(1)*RC(2)*(-T(1,1)+T(4,4)+(T(1,4)/
       CCC)-CC*T(4,1))-RC(2)*RC(2)*(T(3,1)-(T(3,4)/CC))-RC(1)*RC(4)*(-(T(1
       C,2)/CC)-T(4,2))-RC(2)*RC(4)*(-CC+(T(3,2)/CC)))*K
```

```
            DERR(3) = -(I/2.0D0)*(T(3,1)+(T(3,4)/CC)+(-T(3,1)+(T(3,4)/CC))*RC
           C(1) +(-CC-(T(3,2)/CC))*RC(3) -RC(3)*(T(1,1)+T(4,4)+(T(1,4)/CC)+
           CCC*T(4,1)) -RC(4)*(-T(3,1)-(T(3,4)/CC)) -RC(1)*RC(3)*(-T(1,1)+
           CT(4,4)+(T(1,4)/CC)-CC*T(4,1)) -RC(1)*RC(4)*(T(3,1)-(T(3,4)/CC))
           C-RC(3)*RC(3)*(-(T(1,2)/CC)-T(4,2)) -RC(3)*RC(4)*(-CC+(T(3,2)/CC)))
           C *K
            DERR(4) = -(I/2.0D0)*(CC-(T(3,2)/CC)+RC(2)*(-T(3,1)+(T(3,4)/CC))
           C+RC(4)*(-CC-(T(3,2)/CC)) -RC(3)*(-(T(1,2)/CC)-T(4,2)) -RC(4)*(CC+
           C(T(3,2)/CC)) -RC(2)*RC(3)*(-T(1,1)+T(4,4)+(T(1,4)/CC)-CC*T(4,1))
           C-RC(2)*RC(4)*(T(3,1) -(T(3,4)/CC)) -RC(3)*RC(4)*(-(T(1,2)/CC)-T(4,
           C2)) -RC(4)*RC(4)*(-CC+(T(3,2)/CC)))*K
            WRITE(6,33) DERR
         33 FORMAT(28X,21HINITIAL VALUE OF DERR/4(20X,1P2D18.10/))
            MAGR(1) = DSQRT((R(1)**2) + (R(5)**2))
            MAGR(2) = DSQRT((R(2)**2) + (R(6)**2))
            MAGR(3) = DSQRT((R(3)**2) + (R(7)**2))
            MAGR(4) = DSQRT((R(4)**2) + (R(8)**2))
            FAZR(1) = DATAN(R(5)/R(1))*180.0/PI
            FAZR(2) = DATAN(R(6)/R(2))*180.0/PI
            FAZR(3) = DATAN(R(7)/R(3))*180.0/PI
            FAZR(4) = DATAN(R(8)/R(4))*180.0/PI
            WRITE(6,16)
         16 FORMAT(///25X,32HSTARTING REFLECTION COEFFICIENTS//27X,4HMAGR,14X,
           C4HFAZR//)
            WRITE(6,17)MAGR(1),FAZR(1),MAGR(2),FAZR(2),MAGR(3),FAZR(3),
           CMAGR(4),FAZR(4)
         17 FORMAT(4(20X,1P2D19.10/))
            RPOLY = ((( RC(1) - 1.0D0)/2.0D0)*(( RC(4) + 1.0D0)/2.0D0) -
           C(RC(3)/2.0D0)*( RC(2)/2.0D0))
            AM(1) = -((( RC(4) + 1.0D0)/(2.0D0 * RPOLY)) + 1.0D0)/CC
            AM(2) = - RC(2)/(2.0D0*RPOLY)
            AM(3) = - RC(3)/(2.0D0*RPOLY)
            AM(4) = -((( RC(1) - 1.0D0)/(2.0D0 * RPOLY)) - 1.0D0) * CC
            DO  18 MM=1,4
            LL=MM + 4
            AMS(MM) = AM(MM)
            YS(MM) = REAL(AMS(MM))
            Y(MM) = DBLE(YS(MM))
            YS(LL) = AIMAG(AMS(MM))
            Y(LL) = DBLE(YS(LL))
         18 CONTINUE
      C         Y(1) =REAL (A11)            Y(5) = IMAG (A11)
      C         Y(2) =REAL (A12)            Y(6) = IMAG (A12)
      C         Y(3) =REAL (A21)            Y(7) = IMAG (A21)
      C         Y(4) =REAL (A22)            Y(8) = IMAG (A22)
            WRITE(6,19)
         19 FORMAT(///30X,18HINITIAL VALUE OF Y/27X,5HREALY,14X,5HIMAGY/)
            WRITE(6,20) AM
         20 FORMAT(4(20X,1P2D19.10/))
            DERIV(1) =(-I*(T(4,4)*AM(1) -T(1,1)*AM(1) +T(4,1) +AM(2)*AM(3)    CHECK
           C -T(1,4)*AM(1)*AM(1)))*K                                           CHECK
            DERIV(2) =(-I*(T(4,4)*AM(2) +T(1,2)*AM(2) -T(4,2) +AM(2)*AM(4)    CHECK
           C -T(1,4)*AM(1)*AM(2)))*K                                           CHECK
            DERIV(3) =(-I*(-T(1,1)*AM(3) +T(3,4)*AM(1) +T(3,1) +AM(3)*AM(4)   CHECK
           C -T(1,4)*AM(1)*AM(3)))*K                                           CHECK
            DERIV(4) =(-I*(T(1,2)*AM(3) +T(3,4)*AM(2) -T(3,2) +AM(4)*AM(4)    CHECK
           C -T(1,4)*AM(2)*AM(3)))*K                                           CHECK
            WRITE(6,22)                                                        CHECK
         22 FORMAT(///28X,21HINITIAL VALUE OF DERY/24X,8HREALDERY,14X,8HIMAGDECHECK
           CRY/)                                                               CHECK
            WRITE(6,23) DERIV
         23 FORMAT(20X,1P2D19.10/20X,1P2D19.10/20X,1P2D19.10/20X,1P2D19.10//) CHECK
      C     END CALCULATION OF INITIAL SOLUTIONS.
         24 DERY(1) = .1250D0
            DERY(2) = .1250D0
            DERY(3) = .1250D0
            DERY(4) = .1250D0
            DERY(5) = .1250D0
            DERY(6) = .1250D0
            DERY(7) = .1250D0
            DERY(8) = .1250D0
      C
```

```
C       ASSOCIATE ALTITUDES WITH ED AND CF DATA POINTS.
        NMAXI = NMAX + 1
        DO 32 J=1,NMAXI
        ALT(J) = PRMT(1) + (PRMT(3)*(J-1))
     32 CONTINUE
C       IT MUST BE REMEMBERED THAT THE INDEPENDENT VARIABLE X IN THIS
C           PROGRAM REPRESENTS Z    OR H   , THE ALTITUDE   , NOT THE X
C           DIRECTION, THAT THE DEPENDENT VARIABLE Y REPRESENTS THE
C           ADMITTANCE MATRIX A, NOT THE Y DIRECTION, AND THAT Z IS
C           CF(N)          / OMEGA, NOT THE Z(OR H OR VERTICAL) DIRECTION.
C           THIS IS CONFUSING NOTATION BUT ELIMINATES THE NECESSITY TO
C           CHANGE VARIABLE NAMES IN THE DRKGS SUBROUTINE.
C       THE INTEGRATION IS STARTED ARBITRARILY WHERE EX= 100 * ( 1 + WHY)
C       ,I.E.,WELL ABOVE THE HEIGHT OF REFLECTIONS, AND CONTINUES DOWN TO
C       LEVELS WHERE N < 1 ELECTRON/CM**3
C        BEGIN CALCULATIONS OF FINAL SOLUTIONS.
        CALL DRKGS (PRMT,Y, DERY,NDIM,IHLF,FCT,OUTP,AUX,ALT,  X,  TR,TI,
       CMR,MI,R,CC,DCL,DCM,DCN,WHY,SS,ED,CF,OMEGA,ELECM,PERME,E,K,ZDATA)
        YPOLYR=-1.0D0 -(CC*Y(1)) +(Y(4)/CC)+Y(1)*Y(4) -Y(5)*Y(8) -Y(2)*Y(3
       C) +Y(6)*Y(7)
        YPOLYI= -CC*Y(5) +(Y(8)/CC) +Y(1)*Y(8) +Y(4)*Y(5) -Y(2)*Y(7)
       C-Y(3)*Y(6)
        YDENOM = (YPOLYR**2) + (YPOLYI**2)
        R(1) =(2.0D0 * (YPOLYR*(1.0D0-(Y(4)/CC))-(Y(8)*YPOLYI/CC))/YDENOM
       C)+ 1.0D0
        R(2)= -2.0D0*(Y(2)*YPOLYR +Y(6)*YPOLYI)/YDENOM
        R(3)= -2.0D0*(Y(3)*YPOLYR +Y(7)*YPOLYI)/YDENOM
        R(4) = (-2.0D0*(YPOLYR*(CC*Y(1) + 1.0D0) + YPOLYI * CC*Y(5))/
       CYDENOM) - 1.0D0
        R(5) = (-2.0D0*(YPOLYI*(1.0D0 - (Y(4)/CC)) + YPOLYR*(Y(8)/CC))/
       CYDENOM)
        R(6)=-2.0D0*(-Y(2)*YPOLYI +Y(6)*YPOLYR)/YDENOM
        R(7)=-2.0D0*(-Y(3)*YPOLYI +Y(7)*YPOLYR)/YDENOM
        R(8) = 2.0D0*(-YPOLYR*CC*Y(5) + YPOLYI*(CC*Y(1)+1.0D0))/YDENOM
        DO 46 LA=1,4
        LO = LA + 4
        MAGR(LA) = DSQRT((R(LA)*R(LA)) + (R(LO)*R(LO)))
        FAZR(LA) = (DATAN(DABS(R(LO)/R(LA))))*180.0/PI
        IF(R(LO))41,43,43
     41 IF(R(LA))42,45,45
     42 FAZR(LA) = FAZR(LA) + 180.0
        GOTO 46
     43 IF(R(LA))44,46,46
     44 FAZR(LA) = FAZR(LA) + 90.0
        GOTO 46
     45 FAZR(LA) = FAZR(LA) + 270.0
     46 CONTINUE
        WRITE(6,26)
     26 FORMAT(///7X,1HX,8X,4HIHLF,7X,5HREALY,14X,5HIMAGY,12X,8HREALDERY,
       C11X,8HIMAGDERY,13X,5HREALR,14X,5HIMAGR/19X,1HN/)
        WRITE(6,27)X,IHLF,Y(1),Y(5),DERY(1),DERY(5),R(1),R(5),Y(2),Y(6),
       CDERY(2),DERY(6),R(2),R(6),Y(3),Y(7),DERY(3),DERY(7),R(3),R(7),
       CY(4),Y(8),DERY(4),DERY(8),R(4),R(8),N
     27 FORMAT(1X,1P1D16.10,I3,1P6D18.10/20X,1P6D18.10/20X,1P6D18.10/
       C20X,1P6D18.10/I20//)
        WRITE(6,28)MAGR(1),FAZR(1),MAGR(2),FAZR(2),MAGR(3),FAZR(3),
       CMAGR(4),FAZR(4)
     28 FORMAT(/////30X,35HFINAL REFLECTION COEFFICIENT VALUES//36X,
       C4HMAGR,18X,4HFAZR//15X,5H11R11,5X,1P2D22.10/15X,5H11R +,5X,
       C1P2D22.10/15X,5H+ R11,5X,1P2D22.10/15X,5H+ R +,5X,1P2D22.10)
     29 STOP
        END
```

```
      SUBROUTINE PREFN(M,T   ,SS,CC,DCL,DCM,DCN,EDPRE,CFPRE,OMEGA,ELECM,
     CPERME,E,K,B)
      DIMENSION M(3,3)
     C,T(4,4)                                                          CHECK
      REAL*8       SS,CC,DCL,DCM,DCN,EDPRE,CFPRE,OMEGA,ELECM,PERME,E,K,
     CB,EX,WHY,Z
      COMPLEX*16 M,U,COEFF,I
     C,T,UNDER                                                         CHECK
C     CALCULATIONS ARE DONE IN COMPLEX NUMBERS.
      EX = EDPRE*E*E/(PERME*ELECM*OMEGA*OMEGA)
      WHY =-E*B/(OMEGA*ELECM)
      Z = CFPRE/OMEGA
C     U = UR + I * UI
      I = (0.0D0,1.0D0)
      U = 1.0D0 - I * Z
      COEFF = -EX/(U*(U*U-WHY*WHY))
      M(1,1) = COEFF*(U*U -DCL*DCL*WHY*WHY)
      M(1,2) = COEFF*(-I*U*DCN*WHY -DCL*DCM*WHY*WHY)
      M(1,3) = COEFF*(I*U*DCM*WHY -DCL*DCN*WHY*WHY)
      M(2,1) = COEFF*(I*U*DCN*WHY -DCL*DCM*WHY*WHY)
      M(2,2) = COEFF*(U*U - DCM*DCM*WHY*WHY)
      M(2,3) = COEFF*(-I*U*DCL*WHY -DCM*DCN*WHY*WHY)
      M(3,1) = COEFF*(-I*U*DCM*WHY -DCL*DCN*WHY*WHY)
      M(3,2) = COEFF*(I*U*DCL*WHY -DCM*DCN*WHY*WHY)
      M(3,3) = COEFF*(U*U - DCN*DCN*WHY*WHY)
      UNDER = 1.0D0 + M(3,3)                                           CHECK
      T(1,1) = -SS*M(3,1)/UNDER                                        CHECK
      T(1,2) = SS*M(3,2)/UNDER                                         CHECK
      T(1,3) = (0.0D0,0.0D0)                                           CHECK
      T(1,4) = ((CC*CC) + M(3,3))/UNDER                                CHECK
      T(2,1) = (0.0D0,0.0D0)                                           CHECK
      T(2,2) = (0.0D0,0.0D0)                                           CHECK
      T(2,3) = (1.0D0,0.0D0)                                           CHECK
      T(2,4) = (0.0D0,0.0D0)                                           CHECK
      T(3,1) = (M(2,3)*M(3,1)/UNDER) - M(2,1)                          CHECK
      T(3,2) = (CC*CC) + M(2,2) - (M(2,3)*M(3,2)/UNDER)                CHECK
      T(3,3) = (0.0D0,0.0D0)                                           CHECK
      T(3,4) = SS*M(2,3)/UNDER                                         CHECK
      T(4,1) = 1.0D0 + M(1,1) - (M(1,3)*M(3,1)/UNDER)                  CHECK
      T(4,2) = (M(3,2)*M(1,3)/UNDER) - M(1,2)                          CHECK
      T(4,3) = (0.0D0,0.0D0)                                           CHECK
      T(4,4) = -SS*M(1,3)/UNDER                                        CHECK
      WRITE(6,41)M(1,1),M(1,2),M(1,3),M(2,1),M(2,2),M(2,3),M(3,1),     CHECK
     CM(3,2),M(3,3)                                                    CHECK
   41 FORMAT(40X,16HM(COMPLEX CALC.)/3(2X,1P2D18.10,2X,1P2D18.10,2X,   CHECK
     C1P2D18.10/))                                                     CHECK
      WRITE(6,42)T(1,1),T(1,2),T(1,3),T(1,4),T(2,1),T(2,2),T(2,3),     CHECK
     CT(2,4),T(3,1),T(3,2),T(3,3),T(3,4),T(4,1),T(4,2),T(4,3),T(4,4)   CHECK
   42 FORMAT(40X,16HT(COMPLEX CALC.)/4(2X,1P2D13.5,2X,1P2D13.5,2X,     CHECK
     C1P2D13.5,2X,1P2D13.5/))                                          CHECK
      RETURN
      END
```

```
      SUBROUTINE FCT(X,Y,DERY,TR,TI,MR,MI,SS,CC,DCL,DCM,DCN,ED,CF,OMEGA,
     CELECM,PERME,E,K,ALT,PRMT,N,EVE,ZDATA)
      DIMENSION Y(8),DERY(8),TR(4,4),TI(4,4),MR(3,3),MI(3,3),PRMT(5),
     CED(119) ,CF(119) ,ALT(120)                                  VARIABLE
      REAL*8 X,Y,DERY,TR,TI,MR,MI,SS,CC,DCL,DCM,DCN,ED,CF,OMEGA,ELECM,
     CPERME,E,K,B,EX,WHY,Z,UR,UI,UPOLYR,UPOLYI,UDENOM,MDENOM,DABS
     C,ALT,EDENS,PRMT,EVE,ZDATA,CFREQ
C     CALCULATIONS ARE DONE IN REAL NUMBERS- COMPLEX NUMBERS SEPARATED
C     INTO REAL AND IMAGINARY PARTS.
C BETWEEN ALTITUDES FOR WHICH ED AND CF HAVE BEEN SPECIFIED ON DATA CARDS
CED AND CF ARE LINEARLY INTERPOLATED.
      N = -((PRMT(1)-X)/ ZDATA ) + 1
      L = N + 1
      EDENS= -((ALT(N)-X)*ED(L)/ ZDATA ) +(ALT(L)-X)*ED(N)/ZDATA
      CFREQ= -((ALT(N)-X)*CF(L)/ ZDATA ) +(ALT(L)-X)*CF(N)/ZDATA
      IF(EVE-EDENS)47,46,47
   47 EVE = EDENS
      B     = 0.560D-4
C      EX IS DIMENSIONLESS AND IS THE RATIO OF THE SQUARE OF THE PLASMA
C         FREQUENCY TO THE SQUARE OF THE WAVE FREQUENCY
C     PERMM IS NOT INCLUDED IN WHY IN MKS UNITS. WHY IS DIMENSIONLESS
C         AND IS THE RATIO OF THE GYROFREQUENCY TO THE WAVE FREQUENCY
C     Z IS DIMENSIONLESS AND IS THE RATIO OF THE COLLISION FREQUENCY
C         TO THE WAVE ANGULAR VELOCITY(OR 2*PI*WAVE FREQUENCY)
      EX = EDENS*E*E/(PERME*ELECM*OMEGA*OMEGA)
      WHY = DABS(E*B/(OMEGA*ELECM))
      Z = CFREQ/OMEGA
C     U = UR + I * UI
      UR = 1.0
      UI = -Z
C     UPOLY = UPOLYR + I * UPOLYI
      UPOLYR = (UR**3) - (3.0D0*UR*(UI**2)) - (UR * (WHY**2))
      UPOLYI =-(UI**3) + (3.0D0*UI*(UR**2)) - (UI * (WHY**2))
      UDENOM = UPOLYR**2  +  UPOLYI**2
C     M = MR + I * MI
      MR(1,1)=-EX*((UPOLYR*((UR**2)-(UI**2)-((DCL**2)*(WHY**2))))+(
     CUPOLYI*2.0D0*UI*UR))/UDENOM
      MI(1,1)=-EX*((UPOLYR*2.0D0*UI*UR)-(UPOLYI*((UR**2)-(UI**2)-((DCL**
     C2)*(WHY**2)))))/UDENOM
      MR(1,2)=EX*((UPOLYR*((DCL*DCM*(WHY**2))-(UI*DCN*WHY)))+(UPOLYI*UR*
     CDCN*WHY))/UDENOM
      MI(1,2)=EX*((UPOLYI*((UI*DCN*WHY)-(DCL*DCM*(WHY**2))))+(UPOLYR*UR*
     CDCN*WHY))/UDENOM
      MR(1,3)=EX*((UPOLYR*((DCL*DCN*(WHY**2))+(UI*DCM*WHY)))-(UPOLYI*UR*
     CDCM*WHY))/UDENOM
      MI(1,3)=EX*((UPOLYI*((UI*DCM*WHY)+(DCL*DCN*(WHY**2))))+(UPOLYR*UR*
     CDCM*WHY))/(-UDENOM)
      MR(2,1)=EX*((UPOLYR*((DCL*DCM*(WHY**2))+(UI*DCN*WHY)))-(UPOLYI*UR*
     CDCN*WHY))/UDENOM
      MI(2,1)=EX*((UPOLYI*((UI*DCN*WHY)+(DCL*DCM*(WHY**2))))+(UPOLYR*UR*
     CDCN*WHY))/(-UDENOM)
      MR(2,2)=-EX*((UPOLYR*((UR**2)-(UI**2)-((DCM**2)*(WHY**2))))+(
     CUPOLYI*2.0D0*UI*UR))/UDENOM
      MI(2,2)=-EX*((UPOLYR*2.0D0*UI*UR)-(UPOLYI*((UR**2)-(UI**2)-((DCM**
     C2)*(WHY**2)))))/UDENOM
      MR(2,3)=EX*((UPOLYR*((DCN*DCM*(WHY**2))-(UI*DCL*WHY)))+(UPOLYI*UR*
     CDCL*WHY))/UDENOM
      MI(2,3)=EX*((UPOLYI*((UI*DCL*WHY)-(DCN*DCM*(WHY**2))))+(UPOLYR*UR*
     CDCL*WHY))/UDENOM
      MR(3,1)=EX*((UPOLYR*((DCL*DCN*(WHY**2))-(UI*DCM*WHY)))+(UPOLYI*UR*
     CDCM*WHY))/UDENOM
      MI(3,1)=EX*((UPOLYI*((UI*DCM*WHY)-(DCL*DCN*(WHY**2))))+(UPOLYR*UR*
     CDCM*WHY))/UDENOM
      MR(3,2)=EX*((UPOLYR*((DCM*DCN*(WHY**2))+(UI*DCL*WHY)))-(UPOLYI*UR*
     CDCL*WHY))/UDENOM
      MI(3,2)=EX*((UPOLYI*((UI*DCL*WHY)+(DCM*DCN*(WHY**2))))+(UPOLYR*UR*
     CDCL*WHY))/(-UDENOM)
      MR(3,3)=-EX*((UPOLYR*((UR**2)-(UI**2)-((DCN**2)*(WHY**2))))+(
     CUPOLYI*2.0D0*UI*UR))/UDENOM
      MI(3,3)=-EX*((UPOLYR*2.0D0*UI*UR)-(UPOLYI*((UR**2)-(UI**2)-((DCN**
     C2)*(WHY**2)))))/UDENOM
      MDENOM = ((1.0D0 + MR(3,3))**2) + (MI(3,3)**2)
C     T = TR + I * TI
      TR(1,1)=-(SS*((MR(3,1)*(1.0D0+MR(3,3)))+(MI(3,1)*MI(3,3))))/MDENOM
      TI(1,1)=-(SS*((-MR(3,1)*MI(3,3))+(MI(3,1)*(1.0D0+MR(3,3)))))/MDENO
     CM
```

```
      TR(1,2)=+(SS*((MR(3,2)*(1.0D0+MR(3,3)))+(MI(3,2)*MI(3,3))))/MDENOM
      TI(1,2)=+(SS*((-MR(3,2)*MI(3,3))+(MI(3,2)*(1.0D0+MR(3,3)))))/MDENO
     CM
      TR(1,3)=0.0D0
      TI(1,3)=0.0D0
      TR(1,4)=(((CC**2)+MR(3,3))*(1.0D0+MR(3,3))+(MI(3,3)**2))/MDENOM
      TI(1,4)=(MI(3,3)*(1.0D0+MR(3,3))-MI(3,3)*((CC**2)+MR(3,3)))/MDENOM
      TR(2,1)=0.0D0
      TI(2,1)=0.0D0
      TR(2,2)=0.0D0
      TI(2,2)=0.0D0
      TR(2,3)=1.0D0
      TI(2,3)=0.0D0
      TR(2,4)=0.0D0
      TI(2,4)=0.0D0
      TR(3,1)=((MR(2,3)*MR(3,1)*(1.0D0+MR(3,3))-MI(2,3)*MI(3,1)*(1.0D0+
     CMR(3,3))+MR(2,3)*MI(3,1)*MI(3,3)+MR(3,1)*MI(2,3)*MI(3,3))/MDENOM)
     C-MR(2,1)
      TI(3,1)=((MI(2,3)*MI(3,1)*MI(3,3)-MR(2,3)*MR(3,1)*MI(3,3)+MR(2,3)*
     C(1.0D0+MR(3,3))*MI(3,1)+MI(2,3)*MR(3,1)*(1.0D0+MR(3,3)))/MDENOM)
     C-MI(2,1)
      TR(3,2)=((MR(3,2)*MR(2,3)*(1.0D0+MR(3,3))-MI(3,2)*MI(2,3)*(1.0D0+
     CMR(3,3))+MR(3,2)*MI(2,3)*MI(3,3)+MR(2,3)*MI(3,2)*MI(3,3))/(-MDENOM
     C))+(CC**2)+MR(2,2)
      TI(3,2)=((MI(3,2)*MI(2,3)*MI(3,3)-MR(3,2)*MR(2,3)*MI(3,3)+MR(3,2)*
     C(1.0D0+MR(3,3))*MI(2,3)+MI(3,2)*MR(2,3)*(1.0D0+MR(3,3)))/(-MDENOM)
     C)+MI(2,2)
      TR(3,3)=0.0D0
      TI(3,3)=0.0D0
      TR(3,4)=+(SS*((MR(2,3)*(1.0D0+MR(3,3)))+(MI(2,3)*MI(3,3))))/MDENOM
      TI(3,4)=+(SS*((-MR(2,3)*MI(3,3))+(MI(2,3)*(1.0D0+MR(3,3)))))/MDENO
     CM
      TR(4,1)=((MR(3,1)*MR(1,3)*(1.0D0+MR(3,3))-MI(3,1)*MI(1,3)*(1.0D0+
     CMR(3,3))+MR(3,1)*MI(1,3)*MI(3,3)+MR(1,3)*MI(3,1)*MI(3,3))/(-MDENOM
     C))+1.0D0+MR(1,1)
      TI(4,1)=((MI(3,1)*MI(1,3)*MI(3,3)-MR(3,1)*MR(1,3)*MI(3,3)+MR(3,1)*
     C(1.0D0+MR(3,3))*MI(1,3)+MI(3,1)*MR(1,3)*(1.0D0+MR(3,3)))/(-MDENOM)
     C)+MI(1,1)
      TR(4,2)=((MR(3,2)*MR(1,3)*(1.0D0+MR(3,3))-MI(3,2)*MI(1,3)*(1.0D0+
     CMR(3,3))+MR(3,2)*MI(1,3)*MI(3,3)+MR(1,3)*MI(3,2)*MI(3,3))/MDENOM)
     C-MR(1,2)
      TI(4,2)=((MI(3,2)*MI(1,3)*MI(3,3)-MR(3,2)*MR(1,3)*MI(3,3)+MR(3,2)*
     C(1.0D0+MR(3,3))*MI(1,3)+MI(3,2)*MR(1,3)*(1.0D0+MR(3,3)))/MDENOM)
     C-MI(1,2)
      TR(4,3)=0.0D0
      TI(4,3)=0.0D0
      TR(4,4)=-(SS*((MR(1,3)*(1.0D0+MR(3,3)))+(MI(1,3)*MI(3,3))))/MDENOM
      TI(4,4)=-(SS*((-MR(1,3)*MI(3,3))+(MI(1,3)*(1.0D0+MR(3,3)))))/MDENO
     CM
   46 DERY(1)=(-TR(1,1)*Y(5)-TI(1,1)*Y(1) +TR(4,4)*Y(5) +TI(4,4)*Y(1)
     C-2.0D0*Y(1)*Y(5)*TR(1,4) -(Y(1)**2)*TI(1,4) +(Y(5)**2)*TI(1,4)
     C+Y(2)*Y(7) +Y(6)*Y(3) +TI(4,1))*K
      DERY(2)=(TI(1,2)*Y(1)+TR(1,2)*Y(5) +TI(4,4)*Y(2) +TR(4,4)*Y(6)
     C-Y(1)*Y(2)*TI(1,4) -Y(1)*Y(6)*TR(1,4) -Y(2)*Y(5)*TR(1,4)
     C+Y(2)*Y(8) +Y(4)*Y(6) +Y(5)*Y(6)*TI(1,4) -TI(4,2))*K
      DERY(3)=(-TR(1,1)*Y(7)-TI(1,1)*Y(3) +TR(3,4)*Y(5) +TI(3,4)*Y(1)
     C-Y(1)*Y(3)*TI(1,4) -Y(5)*Y(3)*TR(1,4) -Y(7)*Y(1)*TR(1,4) +TI(3,1)
     C+Y(5)*Y(7)*TI(1,4) +Y(3)*Y(8) +Y(7)*Y(4))*K
      DERY(4)=(TR(1,2)*Y(7)+TI(1,2)*Y(3) +TR(3,4)*Y(6) +TI(3,4)*Y(2)
     C-Y(2)*Y(3)*TI(1,4)-Y(6)*Y(3)*TR(1,4) -Y(2)*Y(7)*TR(1,4) -TI(3,2)
     C+Y(6)*Y(7)*TI(1,4) +2.0D0*Y(4)*Y(8))*K
      DERY(5)=(TR(1,1)*Y(1)-TI(1,1)*Y(5) -TR(4,4)*Y(1) +TI(4,4)*Y(5)
     C+(Y(1)**2)*TR(1,4) -(Y(5)**2)*TR(1,4) -2.0D0*Y(1)*Y(5)*TI(1,4)
     C-Y(2)*Y(3) +Y(6)*Y(7) -TR(4,1))*K
      DERY(6)=(-TR(1,2)*Y(1)+TI(1,2)*Y(5) -TR(4,4)*Y(2) +TI(4,4)*Y(6)
     C+Y(1)*Y(2)*TR(1,4) -Y(5)*Y(6)*TR(1,4) -TI(1,4)*Y(6)*Y(1) +TR(4,2)
     C-TI(1,4)*Y(5)*Y(2) -Y(2)*Y(4) +Y(6)*Y(8))*K
      DERY(7)=(TR(1,1)*Y(3)-TI(1,1)*Y(7) -TR(3,4)*Y(1) +TI(3,4)*Y(5)
     C+Y(1)*Y(3)*TR(1,4) -Y(5)*Y(7)*TR(1,4) -Y(1)*Y(7)*TI(1,4) -TR(3,1)
     C-Y(3)*Y(5)*TI(1,4) -Y(3)*Y(4) +Y(7)*Y(8))*K
      DERY(8)=(-TR(1,2)*Y(3)+TI(1,2)*Y(7) -TR(3,4)*Y(2) +TI(3,4)*Y(6)
     C+Y(2)*Y(3)*TR(1,4) -Y(6)*Y(7)*TR(1,4) -Y(2)*Y(7)*TI(1,4) +TR(3,2)
     C-Y(6)*Y(3)*TI(1,4) -(Y(4)**2) +(Y(8)**2))*K
      RETURN
      END
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      SUBROUTINE OUTP (X, Y, DERY, IHLF, NDIM, PRMT,N,R,CC)
      DIMENSION Y(8), DERY(8), PRMT(5), R(8)
      REAL*8 X,PRMT,Y,DERY,R,YPOLYR,YPOLYI,YDENOM,DABS,CC
C     YPOLY = YPOLYR + I * YPOLYI
      YPOLYR=-1.0D0 -(CC*Y(1)) +(Y(4)/CC)+Y(1)*Y(4) -Y(5)*Y(8) -Y(2)*Y(3
     C) +Y(6)*Y(7)
      YPOLYI= -CC*Y(5) +(Y(8)/CC) +Y(1)*Y(8) +Y(4)*Y(5) -Y(2)*Y(7)
     C-Y(3)*Y(6)
      YDENOM = (YPOLYR**2) + (YPOLYI**2)
      R(1) =(2.0D0 * (YPOLYR*(1.0D0-(Y(4)/CC))-(Y(8)*YPOLYI/CC))/YDENOM
     C)+ 1.0D0
      R(2)= -2.0D0*(Y(2)*YPOLYR +Y(6)*YPOLYI)/YDENOM
      R(3)= -2.0D0*(Y(3)*YPOLYR +Y(7)*YPOLYI)/YDENOM
      R(4) = (-2.0D0*(YPOLYR*(CC*Y(1) + 1.0D0) + YPOLYI * CC*Y(5))/
     CYDENOM) - 1.0D0
      R(5) = (-2.0D0*(YPOLYI*(1.0D0 - (Y(4)/CC)) + YPOLYR*(Y(8)/CC))/
     CYDENOM)
      R(6)=-2.0D0*(-Y(2)*YPOLYI +Y(6)*YPOLYR)/YDENOM
      R(7)=-2.0D0*(-Y(3)*YPOLYI +Y(7)*YPOLYR)/YDENOM
      R(8) = 2.0D0*(-YPOLYR*CC*Y(5) + YPOLYI*(CC*Y(1)+1.0D0))/YDENOM
    8 WRITE(6,9)X,IHLF,Y(1),Y(5),DERY(1),DERY(5),R(1),R(5),Y(2),Y(6),   CHECK
     CDERY(2),DERY(6),R(2),R(6),Y(3),Y(7),DERY(3),DERY(7),R(3),R(7),    CHECK
     CY(4),Y(8),DERY(4),DERY(8),R(4),R(8),N                             CHECK
    9 FORMAT(1X,1P1D16.10,I3,1P6D18.10/20X,1P6D18.10/20X,1P6D18.10/     CHECK
     C20X,1P6D18.10/I20//)                                              CHECK
   12 RETURN
      END
C                                                                       DRKGS001
C     ..................................................................DRKGS002
C                                                                       DRKGS003
C        SUBROUTINE DRKGS                                               DRKGS004
C                                                                       DRKGS005
C        PURPOSE                                                        DRKGS006
C           TO SOLVE A SYSTEM OF FIRST ORDER ORDINARY DIFFERENTIAL      DRKGS007
C           EQUATIONS WITH GIVEN INITIAL VALUES.                        DRKGS008
C                                                                       DRKGS009
C        USAGE                                                          DRKGS010
C           CALL DRKGS (PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)             DRKGS011
C           PARAMETERS FCT AND OUTP REQUIRE AN EXTERNAL STATEMENT.      DRKGS012
C                                                                       DRKGS013
C        DESCRIPTION OF PARAMETERS                                      DRKGS014
C           PRMT   - DOUBLE PRECISION INPUT AND OUTPUT VECTOR WITH      DRKGS015
C                    DIMENSION GREATER THAN OR EQUAL TO 5, WHICH        DRKGS016
C                    SPECIFIES THE PARAMETERS OF THE INTERVAL AND OF    DRKGS017
C                    ACCURACY AND WHICH SERVES FOR COMMUNICATION BETWEENDRKGS018
C                    OUTPUT SUBROUTINE (FURNISHED BY THE USER) AND      DRKGS019
C                    SUBROUTINE DRKGS. EXCEPT PRMT(5) THE COMPONENTS    DRKGS020
C                    ARE NOT DESTROYED BY SUBROUTINE DRKGS AND THEY ARE DRKGS021
C           PRMT(1)- LOWER BOUND OF THE INTERVAL (INPUT),               DRKGS022
C           PRMT(2)- UPPER BOUND OF THE INTERVAL (INPUT),               DRKGS023
C           PRMT(3)- INITIAL INCREMENT OF THE INDEPENDENT VARIABLE      DRKGS024
C                    (INPUT),                                           DRKGS025
C           PRMT(4)- UPPER ERROR BOUND (INPUT). IF ABSOLUTE ERROR IS    DRKGS026
C                    GREATER THAN PRMT(4), INCREMENT GETS HALVED.       DRKGS027
C                    IF INCREMENT IS LESS THAN PRMT(3) AND ABSOLUTE     DRKGS028
C                    ERROR LESS THAN PRMT(4)/50, INCREMENT GETS DOUBLED.DRKGS029
C                    THE USER MAY CHANGE PRMT(4) BY MEANS OF HIS        DRKGS030
C                    OUTPUT SUBROUTINE.                                 DRKGS031
C           PRMT(5)- NO INPUT PARAMETER. SUBROUTINE DRKGS INITIALIZES   DRKGS032
C                    PRMT(5)=0. IF THE USER WANTS TO TERMINATE          DRKGS033
C                    SUBROUTINE DRKGS AT ANY OUTPUT POINT, HE HAS TO    DRKGS034
C                    CHANGE PRMT(5) TO NON-ZERO BY MEANS OF SUBROUTINE  DRKGS035
C                    OUTP. FURTHER COMPONENTS OF VECTOR PRMT ARE        DRKGS036
C                    FEASIBLE IF ITS DIMENSION IS DEFINED GREATER       DRKGS037
C                    THAN 5. HOWEVER SUBROUTINE DRKGS DOES NOT REQUIRE  DRKGS038
C                    AND CHANGE THEM. NEVERTHELESS THEY MAY BE USEFUL   DRKGS039
C                    FOR HANDING RESULT VALUES TO THE MAIN PROGRAM      DRKGS040
C                    (CALLING DRKGS) WHICH ARE OBTAINED BY SPECIAL      DRKGS041
C                    MANIPULATIONS WITH OUTPUT DATA IN SUBROUTINE OUTP. DRKGS042
C           Y      - DOUBLE PRECISION INPUT VECTOR OF INITIAL VALUES    DRKGS043
C                    (DESTROYED). LATERON Y IS THE RESULTING VECTOR OF  DRKGS044
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      C                   DEPENDENT VARIABLES COMPUTED AT INTERMEDIATE        DRKGS045
      C                   POINTS X.                                           DRKGS046
      C        DERY     - DOUBLE PRECISION INPUT VECTOR OF ERROR WEIGHTS      DRKGS047
      C                   (DESTROYED). THE SUM OF ITS COMPONENTS MUST BE      DRKGS048
      C                   EQUAL TO 1. LATERON DERY IS THE VECTOR OF           DRKGS049
      C                   DERIVATIVES, WHICH BELONG TO FUNCTION VALUES Y AT   DRKGS050
      C                   INTERMEDIATE POINTS X.                              DRKGS051
      C        NDIM     - AN INPUT VALUE, WHICH SPECIFIES THE NUMBER OF       DRKGS052
      C                   EQUATIONS IN THE SYSTEM.                            DRKGS053
      C        IHLF     - AN OUTPUT VALUE, WHICH SPECIFIES THE NUMBER OF      DRKGS054
      C                   BISECTIONS OF THE INITIAL INCREMENT. IF IHLF GETS   DRKGS055
      C                   GREATER THAN 10, SUBROUTINE DRKGS RETURNS WITH      DRKGS056
      C                   ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM. ERROR      DRKGS057
      C                   MESSAGE IHLF=12 OR IHLF=13 APPEARS IN CASE          DRKGS058
      C                   PRMT(3)=0 OR IN CASE SIGN(PRMT(3)).NE.SIGN(PRMT(2)-DRKGS059
      C                   PRMT(1)) RESPECTIVELY.                              DRKGS060
      C        FCT      - THE NAME OF AN EXTERNAL SUBROUTINE USED. THIS       DRKGS061
      C                   SUBROUTINE COMPUTES THE RIGHT HAND SIDES DERY OF    DRKGS062
      C                   THE SYSTEM TO GIVEN VALUES X AND Y. ITS PARAMETER   DRKGS063
      C                   LIST MUST BE X,Y,DERY. SUBROUTINE FCT SHOULD        DRKGS064
      C                   NOT DESTROY X AND Y.                                DRKGS065
      C        OUTP     - THE NAME OF AN EXTERNAL OUTPUT SUBROUTINE USED.     DRKGS066
      C                   ITS PARAMETER LIST MUST BE X,Y,DERY,IHLF,NDIM,PRMT.DRKGS067
      C                   NONE OF THESE PARAMETERS (EXCEPT, IF NECESSARY,     DRKGS068
      C                   PRMT(4),PRMT(5),...) SHOULD BE CHANGED BY           DRKGS069
      C                   SUBROUTINE OUTP. IF PRMT(5) IS CHANGED TO NON-ZERO,DRKGS070
      C                   SUBROUTINE DRKGS IS TERMINATED.                     DRKGS071
      C        AUX      - DOUBLE PRECISION AUXILIARY STORAGE ARRAY WITH 8     DRKGS072
      C                   ROWS AND NDIM COLUMNS.                              DRKGS073
      C                                                                       DRKGS074
      C     REMARKS                                                           DRKGS075
      C        THE PROCEDURE TERMINATES AND RETURNS TO CALLING PROGRAM, IF    DRKGS076
      C        (1) MORE THAN 10 BISECTIONS OF THE INITIAL INCREMENT ARE       DRKGS077
      C            NECESSARY TO GET SATISFACTORY ACCURACY (ERROR MESSAGE      DRKGS078
      C            IHLF=11),                                                  DRKGS079
      C        (2) INITIAL INCREMENT IS EQUAL TO 0 OR HAS WRONG SIGN          DRKGS080
      C            (ERROR MESSAGES IHLF=12 OR IHLF=13),                       DRKGS081
      C        (3) THE WHOLE INTEGRATION INTERVAL IS WORKED THROUGH,          DRKGS082
      C        (4) SUBROUTINE OUTP HAS CHANGED PRMT(5) TO NON-ZERO.           DRKGS083
      C                                                                       DRKGS084
      C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                     DRKGS085
      C        THE EXTERNAL SUBROUTINES FCT(X,Y,DERY) AND                     DRKGS086
      C        OUTP(X,Y,DERY,IHLF,NDIM,PRMT) MUST BE FURNISHED BY THE USER.DRKGS087
      C                                                                       DRKGS088
      C     METHOD                                                            DRKGS089
      C        EVALUATION IS DONE BY MEANS OF FOURTH ORDER RUNGE-KUTTA        DRKGS090
      C        FORMULAE IN THE MODIFICATION DUE TO GILL. ACCURACY IS          DRKGS091
      C        TESTED COMPARING THE RESULTS OF THE PROCEDURE WITH SINGLE      DRKGS092
      C        AND DOUBLE INCREMENT.                                          DRKGS093
      C        SUBROUTINE DRKGS AUTOMATICALLY ADJUSTS THE INCREMENT DURING    DRKGS094
      C        THE WHOLE COMPUTATION BY HALVING OR DOUBLING. IF MORE THAN     DRKGS095
      C        10 BISECTIONS OF THE INCREMENT ARE NECESSARY TO GET            DRKGS096
      C        SATISFACTORY ACCURACY, THE SUBROUTINE RETURNS WITH             DRKGS097
      C        ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM.                       DRKGS098
      C        TO GET FULL FLEXIBILITY IN OUTPUT, AN OUTPUT SUBROUTINE        DRKGS099
      C        MUST BE FURNISHED BY THE USER.                                 DRKGS100
      C        FOR REFERENCE, SEE                                             DRKGS101
      C        RALSTON/WILF, MATHEMATICAL METHODS FOR DIGITAL COMPUTERS,      DRKGS102
      C        WILEY, NEW YORK/LONDON, 1960, PP.110-120.                      DRKGS103
      C                                                                       DRKGS104
      C     ..................................................................DRKGS105
      C                                                                       DRKGS106
362         SUBROUTINE DRKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX,ALT,  X,
           CTR,TI,MR,MI,R,CC,DCL,DCM,DCN,WHY,SS,ED,CF,OMEGA,ELECM,PERME,E,K,
           C ZDATA)
      C                                                                       DRKGS108
      C                                                                       DRKGS109
363         DIMENSION Y(8),DERY(8),AUX(8,8),A(8),B(8),C(8),PRMT(5),TR(4,4),   DRKGS110
           C TI(4,4),MR(3,3),MI(3,3),ED(119) ,CF(119) ,ALT(120) ,R(8)         DRKGS111
364         REAL*8 PRMT, Y, DERY,AUX,   A, B, C, X, XEND, H, AJ, BJ, CJ,     DRKGS112
           C           DELT, DABS, DCL, DCM, DCN, WHY, ED, CF, SS, CC,
           COMEGA, PERME, E, ELECM,TR,TI,R1,R2,MR,MI,R,K,ALT,EVE,ZDATA
365         EVE = 0.0D0
```

```
      DO 1 I=1,NDIM                                                     DRKGS113
    1 AUX(8,I)=.06666666666666667D0 *DERY(I)                            DRKGS114
      X=PRMT(1)                                                         DRKGS115
      XEND=PRMT(2)                                                      DRKGS116
      H=PRMT(3)                                                         DRKGS117
      PRMT(5)=0.D0                                                      DRKGS118
            CALL FCT(X,Y,DERY,TR,TI,MR,MI,SS,CC,DCL,DCM,DCN,ED,CF,OMEGA,DRKGS119
     CELECM,PERME,E,K,ALT,PRMT,N,EVE,ZDATA)
C                                                                       DRKGS120
C     ERROR TEST                                                        DRKGS121
      IF(H*(XEND-X))38,37,2                                             DRKGS122
C                                                                       DRKGS123
C     PREPARATIONS FOR RUNGE-KUTTA METHOD                               DRKGS124
    2 A(1)=.5D0                                                         DRKGS125
      A(2)=.2928932188134525D0                                          DRKGS126
      A(3)=1.707106781186548D0                                          DRKGS127
      A(4)=.1666666666666667D0                                          DRKGS128
      B(1)=2.D0                                                         DRKGS129
      B(2)=1.D0                                                         DRKGS130
      B(3)=1.D0                                                         DRKGS131
      B(4)=2.D0                                                         DRKGS132
      C(1)=.5D0                                                         DRKGS133
      C(2)=.2928932188134525D0                                          DRKGS134
      C(3)=1.707106781186548D0                                          DRKGS135
      C(4)=.5D0                                                         DRKGS136
C                                                                       DRKGS137
C     PREPARATIONS OF FIRST RUNGE-KUTTA STEP                            DRKGS138
      DO 3 I=1,NDIM                                                     DRKGS139
      AUX(1,I)=Y(I)                                                     DRKGS140
      AUX(2,I)=DERY(I)                                                  DRKGS141
      AUX(3,I)=0.D0                                                     DRKGS142
    3 AUX(6,I)=0.D0                                                     DRKGS143
      IREC=0                                                            DRKGS144
      H=H+H                                                             DRKGS145
      IHLF=-1                                                           DRKGS146
      ISTEP=0                                                           DRKGS147
      IEND=0                                                            DRKGS148
C                                                                       DRKGS149
C                                                                       DRKGS150
C     START OF A RUNGE-KUTTA STEP                                       DRKGS151
    4 IF((X+H-XEND)*H)7,6,5                                             DRKGS152
    5 H=XEND-X                                                          DRKGS153
    6 IEND=1                                                            DRKGS154
C                                                                       DRKGS155
C     RECORDING OF INITIAL VALUES OF THIS STEP                          DRKGS156
    7 IF(PRMT(5))40,8,40                                                DRKGS158
    8 ITEST=0                                                           DRKGS159
    9 ISTEP=ISTEP+1                                                     DRKGS160
C                                                                       DRKGS161
C                                                                       DRKGS162
C     START OF INNERMOST RUNGE-KUTTA LOOP                               DRKGS163
      J=1                                                               DRKGS164
   10 AJ=A(J)                                                           DRKGS165
      BJ=B(J)                                                           DRKGS166
      CJ=C(J)                                                           DRKGS167
      DO 11 I=1,NDIM                                                    DRKGS168
      R1=H*DERY(I)                                                      DRKGS169
      R2=AJ*(R1-BJ*AUX(6,I))                                            DRKGS170
      Y(I)=Y(I)+R2                                                      DRKGS171
      R2=R2+R2+R2                                                       DRKGS172
   11 AUX(6,I)=AUX(6,I)+R2-CJ*R1                                        DRKGS173
      IF(J-4)12,15,15                                                   DRKGS174
   12 J=J+1                                                             DRKGS175
      IF(J-3)13,14,13                                                   DRKGS176
   13 X=X+.5D0*H                                                        DRKGS177
   14       CALL FCT(X,Y,DERY,TR,TI,MR,MI,SS,CC,DCL,DCM,DCN,ED,CF,OMEGA,DRKGS178
     CELECM,PERME,E,K,ALT,PRMT,N,EVE,ZDATA)
      GOTO 10                                                           DRKGS179
C     END OF INNERMOST RUNGE-KUTTA LOOP                                 DRKGS180
C                                                                       DRKGS181
C                                                                       DRKGS182
C     TEST OF ACCURACY                                                  DRKGS183
   15 IF(ITEST)16,16,20                                                 DRKGS184
C                                                                       DRKGS185
```

```
C        IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY   DRKGS186
      16 DO 17 I=1,NDIM                                                     DRKGS187
      17 AUX(4,I)=Y(I)                                                      DRKGS188
         ITEST=1                                                            DRKGS189
         ISTEP=ISTEP+ISTEP-2                                                DRKGS190
      18 IHLF=IHLF+1                                                        DRKGS191
         X=X-H                                                              DRKGS192
         H=.5D0*H                                                           DRKGS193
         DO 19 I=1,NDIM                                                     DRKGS194
         Y(I)=AUX(1,I)                                                      DRKGS195
         DERY(I)=AUX(2,I)                                                   DRKGS196
      19 AUX(6,I)=AUX(3,I)                                                  DRKGS197
         GOTO 9                                                             DRKGS198
C                                                                           DRKGS199
C        IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE                    DRKGS200
      20 IMOD=ISTEP/2                                                       DRKGS201
         IF(ISTEP-IMOD-IMOD)21,23,21                                        DRKGS202
      21       CALL FCT(X,Y,DERY,TR,TI,MR,MI,SS,CC,DCL,DCM,DCN,ED,CF,OMEGA, DRKGS203
        CELECM,PERME,E,K,ALT,PRMT,N,EVE,ZDATA)
         DO 22 I=1,NDIM                                                     DRKGS204
         AUX(5,I)=Y(I)                                                      DRKGS205
      22 AUX(7,I)=DERY(I)                                                   DRKGS206
         GOTO 9                                                             DRKGS207
C                                                                           DRKGS208
C        COMPUTATION OF TEST VALUE DELT                                     DRKGS209
      23 DELT=0.D0                                                          DRKGS210
         DO 24 I=1,NDIM                                                     DRKGS211
      24 DELT=DELT+AUX(8,I)*DABS(AUX(4,I)-Y(I))                             DRKGS212
         IF(DELT-PRMT(4))28,28,25                                           DRKGS213
C                                                                           DRKGS214
C        ERROR IS TOO GREAT                                                 DRKGS215
      25 IF(IHLF-20)26,36,36                                                DRKGS216
      26 DO 27 I=1,NDIM                                                     DRKGS217
      27 AUX(4,I)=AUX(5,I)                                                  DRKGS218
         ISTEP=ISTEP+ISTEP-4                                                DRKGS219
         X=X-H                                                              DRKGS220
         IEND=0                                                             DRKGS221
         GOTO 18                                                            DRKGS222
C                                                                           DRKGS223
C        RESULT VALUES ARE GOOD                                             DRKGS224
      28       CALL FCT(X,Y,DERY,TR,TI,MR,MI,SS,CC,DCL,DCM,DCN,ED,CF,OMEGA, DRKGS225
        CELECM,PERME,E,K,ALT,PRMT,N,EVE,ZDATA)
         DO 29 I=1,NDIM                                                     DRKGS226
         AUX(1,I)=Y(I)                                                      DRKGS227
         AUX(2,I)=DERY(I)                                                   DRKGS228
         AUX(3,I)=AUX(6,I)                                                  DRKGS229
         Y(I)=AUX(5,I)                                                      DRKGS230
      29 DERY(I)=AUX(7,I)                                                   DRKGS231
         IF(PRMT(5))40,30,40                                                DRKGS233
      30 DO 31 I=1,NDIM                                                     DRKGS234
         Y(I)=AUX(1,I)                                                      DRKGS235
      31 DERY(I)=AUX(2,I)                                                   DRKGS236
         IREC=IHLF                                                          DRKGS237
         IF(IEND)32,32,39                                                   DRKGS238
C                                                                           DRKGS239
C        INCREMENT GETS DOUBLED                                             DRKGS240
      32 IHLF=IHLF-1                                                        DRKGS241
         ISTEP=ISTEP/2                                                      DRKGS242
         H=H+H                                                              DRKGS243
         IF(IHLF)4,33,33                                                    DRKGS244
      33 IMOD=ISTEP/2                                                       DRKGS245
         IF(ISTEP-IMOD-IMOD)4,34,4                                          DRKGS246
      34 IF(DELT-.02D0*PRMT(4))35,35,4                                      DRKGS247
      35 IHLF=IHLF-1                                                        DRKGS248
         ISTEP=ISTEP/2                                                      DRKGS249
         H=H+H                                                              DRKGS250
         GOTO 4                                                             DRKGS251
C                                                                           DRKGS252
C                                                                           DRKGS253
C        RETURNS TO CALLING PROGRAM                                         DRKGS254
      36 IHLF=21                                                            DRKGS255
               CALL FCT(X,Y,DERY,TR,TI,MR,MI,SS,CC,DCL,DCM,DCN,ED,CF,OMEGA, DRKGS256
        CELECM,PERME,E,K,ALT,PRMT,N,EVE,ZDATA)
         GOTO 39                                                            DRKGS257
      37 IHLF=12                                                            DRKGS258
         GOTO 39                                                            DRKGS259
      38 IHLF=13                                                            DRKGS260
      39 CALL OUTP(X,Y,DERY,IHLF,NDIM,PRMT,N,R,CC)                          DRKGS261
      40 RETURN                                                             DRKGS262
         END                                                                DRKGS263
```

```
$ENTRY
                          UNITY
                       1.0000000000D 00

                                         M(COMPLEX CALC.)
-3.9785399124D 02 -2.7967909108D 02     2.1216613366D 02  6.4698608386D 01     -1.2417872526D 03 -8.8191673325D 02
 2.1220968545D 02  2.3181439932D 02    -1.1160852567D 02 -7.9678289626D 01      6.6026055190D 02  4.3447874893D 02
-1.2417798108D 03 -8.5336107163D 02     6.6027454799D 02  4.8818417536D 02     -3.8628583396D 03 -2.7006908802D 03

                                         T(COMPLEX CALC.)
-1.95959D-01  1.60360D-03    -1.06761D-01 -2.81786D-03    -0.00000D-01 -0.00000D-01     1.00007D 00 -4.56837D-05
-0.00000D-01 -0.00000D-01    -0.00000D-01 -0.00000D-01     1.00000D 00  0.00000D-01     0.00000D-01  0.00000D-01
 2.56529D-02 -9.46303D 01     2.02804D 00 -9.61541D-01    -0.00000D-01 -0.00000D-01    -1.02757D-01  2.90518D-03
 2.47780D 00 -9.61571D-01     8.42041D-02  9.46303D 01     0.00000D-01  0.00000D-01    -1.98088D-01 -1.43939D-03

                          BE
                       9.8511668580D-02  -4.1052340573D-05
                      -7.4452278704D-01   3.2054206645D-01
                      -6.1505255221D-02   2.5630818368D-05
                      -8.9513429810D 03   1.5692306167D 00


                          FN
                      -1.0000000616D 00   1.1298477043D-08


                          Q
                       9.7435241374D 00  -4.9398453145D-02
                      -9.9439380440D 00   5.1482942608D-02
                      -1.4521257849D-01   9.6101521659D 00
                      -4.8420189219D-02  -9.6120724460D 00


                          ZERO
                       1.0905157833D-03  -3.7385745080D-03
                       1.0896860822D-03  -3.7416783549D-03
                       1.1511382600D-03   3.3972853300D-03
                       1.1492692111D-03   3.3994246232D-03


                          QUP
                       9.7435241374D 00  -4.9398453145D-02
                      -4.8420189219D-02  -9.6120724460D 00
```

INITIAL VALUE OF R

8.6910140192D-01	-1.2709250980D-01
1.0255621493D-01	8.2323240644D-02
1.0221290519D-01	7.8483021349D-02
-9.1803711576D-01	8.5317074115D-02

INITIAL VALUE OF DERR

2.3378398674D-08	2.1103077460D-08
-1.5900082246D-05	1.8188395264D-05
1.5200690695D-05	-1.8139230186D-05
-8.5807963210D-09	-6.6649396075D-09

STARTING REFLECTION COEFFICIENTS

MAGR	FAZR
8.7834483402D-01	-8.3196415161D 00
1.3150998442D-01	3.8754469178D 01
1.2886832410D-01	3.7518447210D 01
9.2199296692D-01	-5.3094848225D 00

INITIAL VALUE OF Y

REALY	IMAGY
4.9049952455D 00	-4.8188981424D 00
4.8299810777D 00	-4.8948591669D 00
4.6283928767D 00	-4.8975146836D 00
-4.9859671542D 00	4.8436374827D 00

INITIAL VALUE OF DERY

REALDERY	IMAGDERY
-1.7527735830D-17	6.3936400016D-17
8.6526744146D-04	9.2021673507D-04
-8.6198437341D-04	-8.8808565503D-04
5.0543953345D-07	-3.5497083304D-07

```
6.4750000000D 04   0 -1.6482993321D-01  1.3666819054D-01  6.3875952753D-05 -2.1614137969D-03  4.4963581403D-01 -7.2691793566D-01
                        2.0819946549D 00 -7.8650709482D-02 -1.5410148799D-03 -4.6413627779D-04  8.4528582356D-01 -4.3174822275D-01
                        1.8325055974D 00  1.1771692887D-01 -1.3093142998D-03 -5.4515346748D-04  7.7984217674D-01 -3.0275843491D-01
                        4.5497244701D-01 -1.5454688800D 00 -6.7376783577D-04  2.3292883283D-03 -6.1987083388D-01 -1.2216063004D-01
                 118

        X        IHLF         REALY              IMAGY              REALDERY            IMAGDERY             REALR              IMAGR
                   N

6.4750000000D 04   0 -1.6482993321D-01  1.3666819054D-01  6.3875952753D-05 -2.1614137969D-03  4.4963581403D-01 -7.2691793566D-01
                        2.0819946549D 00 -7.8650709482D-02 -1.5410148799D-03 -4.6413627779D-04  8.4528582356D-01 -4.3174822275D-01
                        1.8325055974D 00  1.1771692887D-01 -1.3093142998D-03 -5.4515346748D-04  7.7984217674D-01 -3.0275843491D-01
                        4.5497244701D-01 -1.5454688800D 00 -6.7376783577D-04  2.3292883283D-03 -6.1987083388D-01 -1.2216063004D-01

                         FINAL REFLECTION COEFFICIENT VALUES

                              MAGR                   FAZR

                11R11         8.5474092591D-01       3.2826108915D 02
                11R +         9.4916523923D-01       2.9705666408D 02
                + R11         8.3655035146D-01       2.9121772322D 02
                + R +         6.3179353449D-01       1.9114865971D 02

COMPILE TIME=      3.14 SEC,EXECUTION TIME=    145.85 SEC,OBJECT CODE=    51120 BYTES,ARRAY AREA=     5728 BYTES,UNUSED=  243152 BYTES

        $STOP
```

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