ANALYSIS OF POSSIBLE EFFECTS CAUSED BY MAGNETIC PARTICLES IN SPACE

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The assumption that a nutrino may have a scalar magnetic charge of other than zero was first analyzed in [1]. Obviously, the presence of a weak magnetic charge in the nutrino should lead to an entire set of various astrophysical phenomena; several of these will be analyzed in this work. It is sufficient to note, for example, that, as will be shown below, the magnetic nutrinos at velocities $v \ll c$ could form closed trajectories in the field of the terrestrial magnetic dipole. In 1931, Dirac produced the following relationship for the value of a magnetic charge [2]:

$$\mu = \frac{\hbar c}{2e} n, \tag{1}$$

where e is the electrical charge; μ is the magnetic charge; n is an integer; $\hbar$ is Planck's constant; c is the speed of light. It follows from Dirac's theory that a magnetic particle is pseudoscalar. Actually, as we can produce easily from [3], the Maxwell equation with magnetic flux j_i other than zero can be written in the coordinate system x_i (r, ict) in the following form:

$$\frac{\partial F_{ik}^*}{\partial x_k} = \frac{4\pi i}{c} j_i, \tag{2}$$

where $F_{ik}^* = \frac{1}{2} e_{iklm} F_{lm}$; e_{iklm} is the antisymmetrical unit tensor; F_{lm} is the tensor of the electromagnetic field. Obviously, the operation of dualization interchanges the vector components E and H in tensor F_{lm}. Since in Dirac's theory H is an axial vector, the point source of H should be pseudoscalar. Since in relation (1) μ is speudoscalar, n is also pseudoscalar. This can be proven easily if in concluding relationship (1) we select line directions for which the ψ-function of the electrically charged particle is equal to zero, corresponding to the direction of the force line of the magnetic particle field [1]. Thus, the Dirac monopole should be the source of field H, which changes upon space reflection just as does the magnetic field of a polar electric current. Therefore, the affirmation [4] that the monopole may be scalar or pseudoscalar and that these two possibilities are equally probable is doubtful, since it follows directly from Dirac's theory that relationship (1) is correct only for a magnetic pseudoscalar. Many experiments performed with the purpose of detecting a monopole with magnetic charge as defined by (1) have been unsuccessful. It was shown in [5] that the upper limit of the cross section for formation of this monopole is less tnan 10^{-40} cm^2, whereas theoretically a quantity of 10^{-35} cm^2 had been produced. Therefore, it is natural to analyze the question of whether the theory allows the magnetic charge of the particle to be changed. If we assume that the magnetic charge is the source of the potential magnetic field, i.e., field h, which can be defined for a given moment in time t as $h = -\nabla\beta$, where β is a scalar function of coordinates (x,y,x), this magnetic charge will be a scalar. Actually, the sign of the magnetic charge in this case will not depend on the coordinate system, since its magnetic field h is determined by the polar vector and, consequently, the direction of the field is independent of coordinate system. Let us now prove that the value of the scalar magnetic charge is not determined by relationship (1). It is well known that the TCP theorem holds true in quantum electrodynamics, the quantum electrodynamics equations being invariant relative to the transforms P,C,T. Here P,C,T represent respectively the operators for space reflection, charge conjugation and time reflection.

The question as to whether the TCP theorem remains correct if we analyze a field of magnetic particles together with electron-positron and electromagnetic fields has not yet been investigated. However, it is shown in [6] that the equations of classical electrodynamics, in which the flux of magnetic particles is not equal to zero, are invariant to the transforms: C' = CM; P' = PM; T' = TM, where M represents the operation of conjugation of the magnetic charge. In other words, the equations of classical electrodynamics are invariant to space reflection only under conditions such that the electrical charge is not scalar, while the magnetic charge is pseudoscalar and, consequently, is a Dirac monopole with a charge determined by (1). Thus, there is in classical electrodynamics a mutually unambiguous relationship between invariance to space reflection and the fact that the electrical charge is scalar, the magnetic charge pseudoscalar. Consequently, if a magnetic particle is a scalar, the equations of the classical electrodynamics will not be invariant to the operation of space reflection. It is easy to show that a system of equations in which the polar vector of the flux both of electrical and of magnetic particles is not equal to zero will be invariant individually to the operations CT and MP. This conclusion was produced in [7], in which it was shown that disruption of parity during the interaction of a magnetic particle with the field must be analyzed together with disruption of parity during the interaction of an electrical scalar with the field. Therefore, it is natural to expect that it should be possible to produce an estimate of the value of the scalar magnetic charge from analysis of the electromagnetic interaction in which parity is not retained. Work [8] presented an investigation of an electromagnetic interaction in which parity was disrupted, producing the following relationship:

$$\xi\mu_0 \leqslant e \cdot l, \tag{3}$$

where μ_0 is a Bohr magneton; e is the electrical charge; l is the length at which invariance to space reflection begins to be disrupted in quantum electrodynamics; ξ is a number characterizing the degree of weakening of the interaction of the electric charge with the field at which parity is not retained. Using (1), inequality (3) can be rewritten in the following form:

$$\xi \frac{\mu}{n} \frac{r_0}{l} \leqslant e, \tag{4}$$

where μ/n is a scalar; r_0 is the "classical radius" of the electron. Let us assume that $g = \xi(\mu/n)$ determines the value of the scalar magnetic field. Then we produce for n = 1 that $g \leq e(l/r_0)$. Since $l < r_0$, $g < e$. In [8], the value $\xi = 10^{-13}$ is produced; therefore, $g = 6.85 \cdot 10^{-12}$ e, whereas, according to (1), the value of the pseudoscalar magnetic charge μ whose interaction with the electromagnetic field remains invariant to space reflection is 68.5 e. The weakening of the charge is a result of disruption of parity during its interaction with the field. Invariance of the equations of electrodynamics with non-zero polar flux of magnetic particles to MP means that for the interaction of the scalar magnetic charge with a field, the principle of combined inversion of L. D. Landau[1] [9] may be fulfilled. Consequently, it can be assumed that the nutrino has a non-zero scalar magnetic charge, since this is the only particle which has only weak interaction. It will be shown below that the value of the charge $g = 6.8 \cdot 10^{-12}$ e agrees well with the value produced from experimental data on the cross section for ionization of the the nutrino. It is shown in [1] that if the nutrino has a non-zero magnetic

[1] See the end of this article.

charge, it is quite probable that the rest mass of this particle is also other than zero.

1. Estimate of Magnetic Charge of Nutrino on the Basis of Ionization Cross Section

Upon moving through matter, a magnetic scalar will lose energy to excitation and ionization of atoms of the medium. Let us analyze the problem of the loss of energy of a particle to ionization upon interaction with the electrons of atoms without considering interaction with the magnetic moments of the atoms. It should be noted that in the external field of the particle, the magnetic moment will undergo the influence of a force couple which tends to orient it either along or strictly against the field. Therefore, the interaction of the field of the magnetic particle with magnetic moments of the electrons cannot produce any significant contribution to the relationship for energy loss to ionization. However, magnetic polarization of the medium is possible, causing some additional energy loss of the particle. Detailed analysis of the effect of polarization is to be performed at a later date. In order to solve the problem, let us use the theory of N. Bohr [10] which basically correctly describes the passage of electrically charged particles through matter. Let us find the value of the electric field created by magnetic scalar g upon movement through matter at velocity β = v/c. In coordinate system K', in which the particle is at rest, its magnetic and electrical fields are, respectively:

$$H' = \frac{gr}{|r'|^3}; \quad E' = 0.$$

Suppose tensor B_{ik} of the electromagnetic field of the magnetic scalar has the following form in arbitrary coordinate system $x_1(r, ict)$:

$$B_{ik} = \begin{vmatrix} 0 & -E_z & E_y & -iH_x \\ E_z & 0 & -E_x & -iH_y \\ -E_y & E_x & 0 & -iH_z \\ iH_x & iH_y & iH_z & 0 \end{vmatrix}; \quad B_{ik} = -B_{ki}.$$

The components of this tensor in system K' are defined by the following relationships:

$$B_{14}' = -\frac{igx'}{r'^3}; \quad B_{24}' = -\frac{igy'}{r'^3}; \quad B_{34}' = -\frac{igz'}{r'^3};$$
$$B'_{12} = B'_{13} = B'_{23} = 0.$$

When we go over to coordinate system K, relative to which system K' moves at velocity β, the components of the tensor B_{ik} will be transformed in the same way as are the corresponding components of the ordinary electromagnetic tensor F_{ik} (see, for example, [11]). We produce:

$$B_{12} = \frac{B'_{12} - i\beta B'_{42}}{\sqrt{1-\beta^2}}; \quad B_{13} = \frac{B'_{13} - i\beta B'_{43}}{\sqrt{1-\beta^2}}; \quad B_{23} = B'_{23} = 0;$$
$$B_{42} = \frac{B'_{42} + i\beta B'_{12}}{\sqrt{1-\beta^2}}; \quad B_{43} = \frac{B'_{43} + i\beta B'_{13}}{\sqrt{1-\beta^2}}; \quad B'_{14} = B_{14} = -iH'_x.$$

Performing substitution and considering that the transition from coordinates (x'y'z't') of system K' to the coordinates (xyzt) of system K is performed using a Lorentz transformation, we obtain the following formulas for field components E and H in system K:

$$H_x = H'_x = \frac{g}{R^3}(x - \beta ct)(1 - \beta^2); \qquad H_y = \frac{H'_y}{\sqrt{1-\beta^2}} = \frac{gy}{R^3}(1-\beta^2);$$
$$H_z = \frac{H'_z}{\sqrt{1-\beta^2}} = \frac{gz}{R^3}(1-\beta^2); \; E_y = \frac{\beta H'_z}{\sqrt{1-\beta^2}} = \frac{g\beta z}{R^3}(1-\beta^2);$$
$$E_z = \frac{\beta H'_y}{\sqrt{1-\beta^2}} = \frac{g\beta y}{R^3}(1-\beta^2); \qquad E_x = E'_x = 0,$$

where

$$R = \sqrt{(1-\beta^2)\rho^2 + (x - \beta ct)^2}, \qquad \rho^2 = y^2 + z^2.$$

Since $E_x = 0$, field E in system K is in a plane perpendicular to the direction of movement of the particle:

$$|E| = \sqrt{E_y^2 + E_z^2}, \quad |E| = \frac{g\beta\rho}{R^3}(1-\beta^2).$$

It is easy to show that vector E is directed at a tangent to the circle of radius ρ with its center on the X axis in system K, i.e.,

$$E = \frac{g\beta\rho}{R^3}(1-\beta^2)\,n, \tag{5}$$

where $|n| = 1$. The direction of vector n corresponds to the direction of a tangent to the circle. Let us now determine the momentum transmitted by the magnetic scalar to the electron at distance ρ from the trajectory of the magnetic particle. It was shown in [1] that the equation of movement of an electron in an external field created by a scalar magnetic charge g in coordinate system $x_i(r,ict)$ is

$$\frac{dP_i}{ds} = \frac{e}{c}(B_{ik} - iB^*_{ik})\,u_k. \tag{6}$$

We will assume that the interaction of the electron with the field of the magnetic particle results primarily from the second term in formula (6). It should be noted that this is a natural assumption, since it is unknown in advance just what the contribution of the first term to the formula for the Lorentz force will be. However, this contribution should be small, since it is difficult to assume that the potential magnetic field could change the energy of the electric charge in the same manner as an ordinary electric field. Therefore, we assume that the first term in (6) is negligibly small in comparison to the second term. Suppose the electron is free and its movement during the time of the collision is slight. If m and M are the masses of the electron and magnetic particle respectively, the condition $m \gg M$ will be fulfilled. Therefore, we can write the equation of movement of the electron (6) in the following form:

$$\frac{dP}{dt} = eE, \tag{7}$$

where P is the momentum transmitted to the electron during the time of the interaction. We note that the expression for E can be represented in the form

$$E = -\left[\frac{v}{c} \times \frac{r}{|r|}\right] \frac{g}{r^2} \frac{1-\beta^2}{(1-\beta^2 \sin^2 \theta)^{3/2}},$$

where r is the vector connecting particles g and e; θ is the angle between r and the X axis. It follows from this relationship that vector E is axial. Therefore, equation (7) is not invariant to the operation of space reflection, but is invariant to operation MP. Suppose during the time of the collision 2T, the value of momentum transmitted ΔP satisfies the condition $P_0 \gg \Delta P$, where P_0 is the momentum of the magnetic particle before the interaction. Therefore, we will assume that the value and direction of the velocity of the magnetic charge remain unchanged with a single collision. We produce:

$$|\Delta P| = \int_{-T}^{+T} e|E|\,dt; \quad \Delta P = \frac{eg}{c\rho}\,\frac{2T}{\sqrt{\frac{\rho^2(1-\beta^2)}{\beta^2c^2} + T^2}}.$$

Where $T \gg \frac{\rho}{\beta c}\sqrt{1-\beta^2}$, the value of $\Delta P = \frac{2eg}{c\rho}$, where $T \sim \frac{\rho}{\beta c}\sqrt{1-\beta^2}$, the quantity $\Delta P = \frac{2eg}{c\rho\sqrt{2}}$. As follows from the theory of N. Bohr, an electron in an atom upon collision with an oncoming particle may be considered free if the time of its "rotation in orbit" τ satisfies the condition $\tau \gg T$. The time of interaction of two electrically charged particles in the theory of N. Bohr is taken as $\rho/\beta c$ for velocities $\beta \ll 1$. From the relationship produced for the momentum transmitted, it is obvious that in this case the time of interaction is $T \sim (\rho/\beta c)\sqrt{1-\beta^2}$. Therefore, the electron can be considered free for velocities of the magnetic particle which satisfy the condition

$$\frac{\rho_{max}}{\beta c}\sqrt{1-\beta^2} \ll \frac{h}{I}, \tag{8}$$

where I is the mean ionization energy of an atom of the material, and ρ_{max} is determined from the condition that the kinetic energy Q transmitted where $\rho = \rho_{max}$ is equal to the ionization energy $\overline{I}$ of the atom in question. The kinetic energy Q transmitted to an electron during the time of the interaction is equal to

$$Q = \frac{\Delta P^2}{2m} = \frac{2e^2g^2}{mc^2\rho^2}. \tag{9}$$

Let us now find the expression for the ionization cross section of the magnetic charge. Suppose the number of magnetic particles passing through 1 cm^2 in one second is equal to n. Then, the cross section for transmission of the energy to the electron from the value of Q to Q + dQ will be

$$d\sigma = \frac{dn}{n} = 2\pi\rho d\rho.$$

Performing elementary conversions, we can find the ionization cross section as the magnetic charge passes through matter:

$$\sigma = \frac{4\pi e^2 g^2}{mc^2}\left(\frac{1}{I} - \frac{1}{Q_{\max}}\right), \tag{10}$$

where Q_{max} is the greatest quantity of energy which can be transmitted to the electron in a single collision. The mean value of energy loss -- $d\mathscr{E}/dx$ -- can be found from the following relationship:

$$-\frac{\overline{d\mathscr{E}}}{dx} = N \int\limits_{Q_{\min}}^{Q_{\max}} Q d\sigma = \frac{4\pi N \cdot e^2 g^2}{mc^2} \ln \frac{Q_{\max}}{Q_{\min}}, \tag{11}$$

where N is the number of electrons per cm^3 of matter. Expressions (10) and (11) do not include the mass of the magnetic particle. The mean energy loss depends only on the value of the scalar magnetic charge. Also, formula (11) does not include the term $1/v^2$ before the logarithm, representing loss to ionization for electrically charged particles at velocities $v \ll c$. This peculiarity, characteristic only for the loss of energy of a magnetic charge upon movement through matter, is also noted in [12].

The greatest quantity of energy transmitted to an electron in a collision is equal to [13]

$$Q_{\max} = Mc^2 \frac{\left(\frac{\mathscr{E}}{Mc^2}\right)^2 - 1}{\frac{M}{2m} + \frac{m}{2M} + \frac{\mathscr{E}}{Mc^2}}, \tag{12}$$

where M and m are the mass of the magnetic scalar and the electron respectively; while $\mathcal{E}$ is the energy of the magnetic scalar in system K. Substituting the relationship for Q_{max} in (10), we produce

$$\sigma = \frac{4\pi e^2 g^2}{mc^2}\left(\frac{1}{\bar{I}} - \frac{\frac{M}{2m} + \frac{m}{2M} + \frac{\mathcal{E}}{Mc^2}}{Mc^2\left[\left(\frac{\mathcal{E}}{Mc^2}\right)^2 - 1\right]}\right). \tag{13}$$

Suppose η = g/e; then,

$$\sigma = 4\pi r_0^2 \eta^2 \frac{e^2}{r_0}\left(\frac{1}{\bar{I}} - \frac{1}{Q_{max}}\right),$$

where $r_0 = e^2/mc^2 = 2.8 \cdot 10^{-13}$ cm is the "classical radius" of the electron. Relationship (12), considering condition $m \gg M$, as well as $\mathcal{E} \gg Mc^2$, can be rewritten in the following form:

$$Q_{max} \simeq \mathcal{E}\frac{1}{1 + \frac{mc^2}{2\mathcal{E}}}. \tag{14}$$

Since for various materials the mean ionization energy does not exceed a few dozen electron volts, inequality $\bar{I} \ll Q_{max}$ will be fulfilled for $\mathcal{E} > mc^2$. Where $\mathcal{E} \sim 1$ Mev, $Mc^2 \leq 250$ ev, we produce the following expression for the ionization cross section as the magnetic particle passes through the material:

$$\sigma \cong 5{,}2 \cdot 10^{-21}\, \eta^2\, cm^2. \tag{15}$$

In works [14-16], upper limits were found for the ionization cross section of a nutrino. These data allow us to estimate the value of the magnetic charge which the nutrino could have:

σ (см²)	10^{-43}	$6\cdot10^{-40}$	10^{-34}
η	$4{,}3\cdot10^{-12}$	$3{,}4\cdot10^{-10}$	$0{,}3\cdot10^{-6}$

We can see that this charge must be extremely small. If we assume that $e^2/mc^2 \sim g^2/Mc^2$ [17], even for $g \sim 10^{-6}$ e, we find $Mc^2 \sim 0.5\cdot10^{-6}$ ev.

2. Movement of Magnetic Particle in Homogeneous Magnetic Fields

Let us now analyze the interaction of a magnetic particle with a homogeneous magnetic field H, created by electrical charges. In analyzing the equations of movement of the magnetic particle in the external electromagnetic field F_{ik}, we can produce the following equation for the Lorentz force:

$$\frac{dP_i}{ds} = \frac{g}{c}\left(F_{ik} - iF^{*}_{ik}\right)u_k. \tag{16}$$

The second term in (16) is produced from analysis quite similar to the analysis performed in [1]. As before, we will assume that the interaction of the magnetic particle with the external field is determined by this term. Thus, the contribution of the first term is assumed small, and it need not be considered in analysis of the equations of movement. Suppose the homogeneous magnetic field H is directed along the X axis, while the initial particle momentum P(0) = 0. Then, it is easy to see that during time t the particle will accumulate energy equal to

$$\mathcal{E} = Mc^2\sqrt{1 + \left(\frac{gHt}{Mc}\right)^2}.$$

Let us assume that the magnetic nutrino has passed through a sector of homogeneous magnetic field H, the size of which is about 1 ps. If $H \sim 10^{-5}$ gs, $\eta = g/e \sim 10^{-12}$, $Mc^2 \leq 250$ ev, after about 10^8 sec its energy will be 9.00 kev.

If we assume that the particle has an initial energy of about 1 Mev and momentum directed along the field, after about 10^8 sec its energy will change insignificantly. We know that the homogeneous field sectors in the galaxy have dimensions on the order of hundreds of parsecs [19]. If the initial momentum is directed along the field, the time required to travel this path is about 10^{10} sec. It is easy to see that with an initial energy of 1 mev, a particle at the end of the path will take on energy about 1.9 Mev. The solar system is in the spiral magnetic field of the galaxy; therefore, if a nutrino has non-zero magnetic charge, it would be expected that the number of high energy nutrinos approaching the earth along this field would be considerably greater than the number of particles moving in the transverse direction.

Let us now analyze the movement in a perpendicular homogeneous magnetic field. Suppose field H is directed along the Z axis, and the particle moves with initial momentum $|P(0)| = P_0$ along the X axis. Let us assume that at moment in time t = 0 the charge enters the external perpendicular field. Then, obviously, the following equations are correct:

$$\begin{array}{llll} P_x(0) = P_0; & \dot{P}_z(t) = gH; & P_x c = \mathcal{E}\beta_x; & \dot{z} = \dfrac{P_z c^2}{\mathcal{E}}; \\ P_z(0) = 0; & P_z(t) = gHt; & P_z c = \mathcal{E}\beta_z; & \\ P_y(t) \equiv 0; & \dot{P}_x(t) = 0; & \dot{x} = \dfrac{P_x c^2}{\mathcal{E}}. & \end{array}$$

From this

$$\mathcal{E}(t) = \sqrt{\mathcal{E}_0^2 + g^2H^2t^2c^2},$$

where

$$\mathcal{E}_0 = \mathcal{E}(0) = \sqrt{(Mc^2)^2 + P_0^2c^2}.$$

Elementary conversions allow us to produce the following equation for the particle trajectory:

$$x(z) = \frac{P_0 c}{gH} \operatorname{ar}\left(\operatorname{ch} \frac{gHz}{\mathscr{E}_0}\right).$$

Let us determine now the radius of curvature of the trajectory of the particle, R(t). Since during its movement the particle remains in plane (XZ), we can use the equation for the radius of curvature of the flat curve in the following form:

$$R(t) = \frac{(\dot{x}^2 + \dot{z}^2)^{3/2}}{\dot{x}\ddot{z} - \dot{z}\ddot{x}}, \qquad (17)$$

where t is the parameter of the curve. The points represent derivatives with respect to this parameter. It is easy to see that the following relationships are correct:

$$\dot{x}^2 + \dot{z}^2 = \left(\frac{c^2}{\mathscr{E}}\right)^2 P^2; \qquad \dot{x}\ddot{z} - \dot{z}\ddot{x} = \frac{c^4}{\mathscr{E}^2}(P_x\dot{P}_z - P_z\dot{P}_x).$$

Here P is the momentum of the particle at moment in time t. Substituting the formulas produced in (17), we can find an expression for R(t):

$$R(t) = \frac{c^2 P^3}{\mathscr{E}(t)(P_x\dot{P}_z - P_z\dot{P}_x)}. \qquad (18)$$

From (18) for moment in time t = 0 we produce

$$R(0) = \frac{\mathscr{E}_0 \beta^2}{gH}. \qquad (19)$$

For field $H \sim 10^{-5}$ gs, $g \sim 10^{-12}$ e, $\mathscr{E}_0 \sim 1$ Mev and $\beta \sim 1$, we find $R(0) \sim 33 \cdot 10^{18}$ cm or $R(0) \sim 11$ ps. The transverse dimension of the galactic spiral field in which the solar system is located is about 400 ps. The magnetic field of the spiral is homogeneous and has a magnitude of 10^{-5} gs [19]. However, heterogeneities may occur within the homogeneous field, the dimensions of these heterogeneities varying from tens to hundreds of parsecs. Consequently, magnetic nutrinos with low energy can be captured by these heterogeneities as they move along the lines of force of the galactic spiral field.

3. Bremstrahlung of a Magnetic Particle

Let us now analyze the Bremstrahlung of a magnetic particle in the external electromagnetic field. In the system of coordinates $x_i(r,ict)$, the equation of movement, considering the force of radiation friction, has the form

$$\frac{dP_i}{ds} = -\frac{ig}{c} F^{*}_{ik} u_k + f_i, \tag{20}$$

where f_i is the four-dimensional force vector of radiation friction. In order to determine f_i, we should note that at particle velocities $v \ll c$, its space components are transformed to components of the radiation friction force vector, which for an individual particle is defined as $f = \frac{2}{3}\frac{g^2}{c^3}\ddot{v}$. Also, components f_i should satisfy the identity $f_i u_i = 0$, correct for any four-dimensional force vector. An expression can be written for the four-dimensional radiation frictionforce vector satisfying both conditions in the following form [11]:

$$f_i = \frac{2}{3}\frac{g^2}{c}\left(\frac{d^2u_i}{ds^2} + u_i u_k \frac{d^2u_k}{ds^2}\right). \tag{21}$$

It is well known that the condition of acceptability of the theory of radiation friction in classical electrodynamics is the fact that in the coordinate system in which the particle is not moving, the Lorentz force should be much greater than the force of radiation friction. In this approximation, we produce

$$\frac{d^2u_k}{ds^2} = -\frac{ig}{Mc^2}\frac{\partial F^*_{kl}}{\partial x_m}u_m u_l - \left(\frac{g}{Mc^2}\right)^2 F^*_{kl}F^*_{lm}u_m. \qquad (22)$$

Substituting (22) into (21), we produce

$$f_i = -\frac{2}{3}\frac{ig^3}{Mc^3}\frac{\partial F^*_{ik}}{\partial x_l}u_l u_k - \frac{2}{3}\frac{ig^4}{M^2c^5}F^*_{ik}A_k - \frac{2}{3}\frac{g^4}{M^2c^5}A_l^2 u_i, \qquad (23)$$

where $A_l = -iF^*_{lk}u_k$. For velocities $\beta \sim 1$, the last term in (23) increases more rapidly, in proportion to $\sim u_i^3$, whereas the two other terms increase as u_i^2. Therefore, we can write

$$f_i \simeq -\frac{2}{3}\frac{g^4}{M^2c^5}A_l^2 u_i. \qquad (24)$$

Let us find $A_l^2 = -(F^*_{ik}u_k)^2$. Substituting the components of the tensor F^*_{lk}, we produce the following relationships for the components of A_l:

$$A_x = \frac{[\mathbf{E}\times\beta]_x + H_x}{\sqrt{1-\beta^2}}; \qquad A_y = \frac{[\mathbf{E}\times\beta]_y + H_y}{\sqrt{1-\beta^2}};$$
$$A_z = \frac{[\mathbf{E}\times\beta]_z + H_z}{\sqrt{1-\beta^2}}; \qquad A_4 = \frac{i\mathbf{H}\beta}{\sqrt{1-\beta^2}}.$$

From this, it is easy to produce an expression for A_l^2:

$$A_l^2 = \frac{1}{1-\beta^2}\{(\mathbf{H} + [\mathbf{E}\times\beta])^2 - (\mathbf{H}\cdot\beta)^2\}. \qquad (25)$$

Substituting (25) in (24), we find:

$$f_l = \frac{2}{3}\frac{g^4}{M^2c^5}\frac{u_l}{\sqrt{1-\beta^2}}\{(\mathrm{H}\cdot\beta)^2 - (\mathrm{H} + [\mathrm{E}\times\beta])^2\}. \tag{26}$$

In order to produce a fomula for the energy loss per unit length of the particle path where $\beta \sim 1$, we must consider that the work of the force of radiation friction f per unit path length is equal to the energy radiated by the particle. Consequently, the following relationship is correct:

$$\frac{d\mathcal{E}}{dt} = \mathrm{f}\cdot\mathrm{v} \quad \text{or} \quad -\frac{d\mathcal{E}}{dx} = |\mathrm{f}|.$$

From (26) we have

$$-\frac{2}{3}\frac{r_g^2}{c}A_l^2\frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{c\sqrt{1-\beta^2}}\mathrm{f},$$

where $r_g = g^2/Mc^2$. Therefore, we produce the following equation:

$$|\mathrm{f}| = -\frac{d\mathcal{E}}{dx} = -\frac{2}{3}r_g^2 A_l^2|\beta|. \tag{27}$$

Substituting relationship (25) for A_l^2 in this last expression, we find the loss of energy to Bremstrahlung under the condition that $\beta \sim 1$:

$$\frac{d\mathcal{E}}{dx} = \frac{2}{3}r_g^2\frac{\beta}{1-\beta^2}\{(\mathrm{H}\cdot\beta)^2 - (\mathrm{H} + [\mathrm{E}\times\beta])^2\}. \tag{28}$$

Let us analyze the losses in a homogeneous magnetic field. Suppose the particle velocity corresponds to the direction of the homogeneous magnetic field H. From (28), we produce

$$-\frac{d\mathcal{E}}{dx} = \frac{2}{3}r_g^2\beta H^2 \tag{29}$$

or

$$-\frac{d\mathcal{E}}{dx}=\frac{2}{3}r_0^2\eta^4\alpha^2H^2\beta,$$

where $r_0 = \frac{e^2}{mc^2}$; $\alpha = m/M$.

Where $\beta \sim 1$, $H \sim 10^{-5}$ gs, $\eta \sim 10^{-12}$, $\alpha \geq 2 \cdot 10^3$, we find

$$-\frac{d\mathcal{E}}{dx} \geqslant 13{,}12 \cdot 10^{-18}\eta^4 \text{ ev/cm}$$

Suppose the density of the material is 10^{-24} g/cm^3; then,

$$-\frac{d\mathcal{E}}{d\zeta} \geqslant 13{,}12 \cdot 10^{6}\eta^4 \text{ ev/g/cm}^2,$$

where ζ is the quantity of material in g/cm^2.

If during its movement the particle enters a heterogeneous, moving magnetic field, it is easy to see that the specific energy loss increases sharply. Actually, suppose the magnetic charge at velocity $\beta \sim 1$ enters an area in which there is a magnetic field H moving at velocity u. The moving magnetic field H, in the system of coordinates where its velocity is equal to u and the velocity of the charge is β, creates, if $u \ll c$, an electric field $E = -1/c[u \times H]$. Therefore, expression (28) for the specific energy loss takes on the following form:

$$\frac{d\mathcal{E}}{dx}=\frac{2}{3}r_g^2\beta\left(\frac{\mathcal{E}}{Mc^2}\right)^2\{(H\beta)^2-[H+1/c^2[v\times[u\times H]]]^2\}.$$

Transforming this last expression, we find

$$\frac{d\mathcal{E}}{dx}=\frac{2}{3}r_g^2\beta\left(\frac{\mathcal{E}}{Mc^2}\right)^2\{(H\beta)^2-[H+1/c^2(u(v\cdot H)-H(v\cdot u))]^2\}. \tag{30}$$

Obviously, where $\beta \perp H$, the following equation is correct:

$$-\frac{d\mathscr{E}}{dx}=\frac{2}{3}r_{g}^{2}\beta\left(\frac{\mathscr{E}}{Mc^{2}}\right)^{2}H^{2}\left[1-\frac{1}{c^{2}}(\mathbf{v}\cdot\mathbf{u})\right]^{2}. \tag{31}$$

Where $u \ll c$, we produce

$$-\frac{d\mathscr{E}}{dx}\simeq\frac{2}{3}r_{g}^{2}\beta\left(\frac{\mathscr{E}}{Mc^{2}}\right)^{2}H^{2}.$$

Thus, in this case the specific energy loss depends on the energy of the particle and may become rather large. For example, for $\mathscr{E} \sim 1$ Mev, $H \sim 10^{-5}$ gs, $Mc^2 \leq 250$ ev, we produce

$$-\frac{d\mathscr{E}}{d\zeta}\geqslant 21\cdot 10^{13}\eta^{4}\ \text{ev/g/cm}^{2}$$

Let us now analyze the question of the angular and spectral distribution of Bremstrahlung when a magnetic particle moves in an external electromagnetic field. It is easy to show that in this case the relationships for the magnetic particle will be similar to those which are correct for an electrical charge [11]. For example, the expression for the magnetic field of the wave radiated upon accelerated particle movement has the form

$$\mathbf{H}=-\frac{g}{c^{2}\left(|\mathbf{r}|-\frac{\mathbf{r}\mathbf{v}}{c}\right)^{3}}\left[\mathbf{r}\times\left[\left(\mathbf{r}-\frac{\mathbf{v}}{c}|\mathbf{r}|\right)\times\dot{\mathbf{v}}\right]\right]-$$
$$-g\frac{1-v^{2}/c^{2}}{\left(|\mathbf{r}|-\frac{\mathbf{r}\mathbf{v}}{c}\right)^{3}}\left(\mathbf{r}-\frac{\mathbf{v}}{c}|\mathbf{r}|\right). \tag{32}$$

From this it is easy to determine the intensity of the flux of radiation at solid angle dO at distance r from the particle:

$$\frac{dI}{dt}=\frac{c}{4\pi}H^{2}r^{2}dO. \tag{33}$$

At sufficiently long range from the particle, only the first term remains in (32). Substituting (32) into (33), we produce

$$\frac{dI}{dt} = \frac{g^2 dO}{4\pi c^3}\left[\frac{2(\mathrm{n}\cdot\mathrm{w})(\mathrm{v}\cdot\mathrm{w})}{c\left(1-\frac{\mathrm{vn}}{c}\right)^5} + \frac{w^2}{\left(1-\frac{\mathrm{vn}}{c}\right)^4} - \frac{\left(1-\frac{v^2}{c^2}\right)(\mathrm{n}\cdot\mathrm{w})^2}{\left(1-\frac{\mathrm{v}\cdot\mathrm{n}}{c}\right)^6}\right], \tag{34}$$

where $n = r/|r|$ is a unit vector directed from the particle to the point of observation; $w = \dot{v}$ is the acceleration of the particle. Expression (34) contains in the denominator high powers of the difference $(1 - vn/c)$. Therefore, if $v \sim c$, the intensity will have a sharp maximum in the direction where $v \cdot n \sim c$, i.e., where n almost corresponds in direction with v. Actually,

$$1 - \frac{\mathrm{vn}}{c} = 1 - |\mathrm{v}/c|\cos\theta \sim 1 - |v/c|(1-\theta^2/2) \sim \theta^2/2.$$

If $v \sim c$, we produce for angle θ [11]:

$$\theta \sim \sqrt{1-|v/c|^2}.$$

During movement through a transverse field in the case when the total angle of deviation of the particle is large in comparison to θ, it can be shown that radiation occurs on the sector of the trajectory over which the velocity of the particle is still parallel to the initial direction of movement [11]. In this case, the main portion of the radiation will be concentrated in the following frequency area:

$$\omega \sim \frac{gH}{Mc}\frac{1}{1-v^2/c^2} = \frac{gH}{Mc}\left(\frac{\mathcal{E}}{Mc^2}\right)^2. \tag{35}$$

for $\mathcal{E} \sim 1$ Mev, $\eta \sim 10^{-12}$, $Mc^2 \leq 250$ ev, $H \sim 10^{-5}$ gs, we produce $\omega \geq 5.76$ Hz, $\lambda \lesssim 3.2 \cdot 10^{10}$ cm. Where $\mathcal{E} \sim 10$ bev, we find $\lambda \lesssim 3.26$ m. Thus, the electromagnetic radiation of a nutrino could be concentrated into the radio wavelength range. It is obvious that the Bremstrahlung of a magnetic nutrino will be directed along the trajectory of the particle and, consequently, will correspond to the direction of the lines of force of the magnetic field, in contrast to the magnetic Bremstrahlung of electrons, which has a maximum in the direction perpendicular to the magnetic field vector. In this connection, we must note the following fact. Work [20] presents data on radiation at a wavelength of 3.5 m in the galaxy. According to these data, the intensity of this radiation increases in the direction along the magnetic field force lines of the "arm" of the galaxy in which the solar system is located. In the review of V. L. Ginzburg and S. I. Syrovatskiy [18], it is stated that the increase in intensity of nonthermal radio radiation along the magnetic lines of force of the "arm" of the galaxy is difficult to explain if we assume that the source of this radiation consists entirely of relativistic electrons. However, if the source of this radiation is a magnetic particle, as was shown above, an increase in the intensity of radiation along the lines of force of the magnetic field should indeed be expected. It is easy to show from (35) that 3.5 m corresponds to a nutrino energy of 10.3 bev, if we consider the rest energy $\bar{M}c^2 \sim 250$ ev. Thus, it is possible that a portion of the nonthermal radio radiation of the galaxy results from magnetic nutrinos.

4. Movement of a Magnetic Particle in the Field of the Terrestrial Magnetic Dipole

As before, we will assume that the equation of movement of the magnetic particle with charge g in the magnetic field H has the form

$$\frac{dP}{dt} = gH.$$

Suppose at the origin of the spherical system of coordinates (r,θ,ϕ) we locate a magnetic moment directed along the Z axis. Usually, the direction of the dipole moment is selected from the negative fictitious magnetic charge image to the positive. Since the north magnetic pole is located in the southern geographic hemisphere of the earth, usually taken as positive, the direction of the magnetic moment k corresponds to the geographic direction of south. Let us for clarity analyze movement of a negative particle $g < 0$ in the field of the magnetic dipole. Since the external field in which the movement of the charge being analyzed is constant, its Lagrange function does not depend explicitly on time. As we know, in this case the energy is retained, corresponding with the Hamilton function. Consequently, the following equation is correct:

$$\mathcal{E} = \varkappa = T + U,$$

where $\mathcal{E}$ is the total energy upon movement of the particle in the external field; T,U are its kinetic and potential energies respectively; κ is the Hamilton function of the particle. We know that the magnetostatic potential of the dipole field in the coordinate system selected has the form

$$U = \frac{\mathrm{kr}}{r^3}.$$

Therefore, let us find the following expression for the Hamiltonian of the particle:

$$\varkappa = \mathcal{E} = \sqrt{M^2c^4 + c^2P^2} + \frac{gk\cos\theta}{r^2}, \tag{36}$$

where P is the momentum of the particle during movement through the dipole field. In order to solve the problem, let us use the Hamilton-Jacoby method. We note first of all that, since the azimuthal component of the field intensity in the dipole field $H_\phi \equiv 0$, obviously, $\dot{P}_\phi = 0$, i.e., P_ϕ is the integral of movement and, therefore, ϕ is a cyclical coordinate of the particle. The Hamilton-Jacoby equation has the following form [21]:

$$\varkappa\left(r,\ \theta,\ \varphi, \frac{\partial S}{\partial \theta},\ \frac{\partial S}{\partial r},\ \frac{\partial S}{\partial \varphi}\right) + \frac{\partial S}{\partial t} = 0,$$

where S, as usual, represents the action function. Since the system is conservative, the following equality is correct:

$$\varkappa = \mathscr{E} = -\frac{\partial S}{\partial t}.$$

We produce from (36)

$$\mathscr{E} = -\frac{\partial S}{\partial t} = \sqrt{M^2c^4 + c^2(\nabla S)^2} + \frac{gk\cos\theta}{r^2}, \qquad (37)$$

where

$$|P| = |\nabla S|^2 = P_r^2 + \frac{1}{r^2}P_\theta^2 + \frac{1}{r^2\sin^2\theta}P_\varphi^2.$$

Since ϕ is a cyclical coordinate of the particle, it is easy to show that equation (37) should be sought in the following form [21]:

$$S = -\mathscr{E}t + P_\varphi\varphi + S_0(r, \theta). \qquad (38)$$

From this we produce the equation for S_0:

$$\mathscr{E} - \frac{gk\cos\theta}{r^2} = \sqrt{(Mc^2)^2 + c^2\left[\left(\frac{\partial S_0}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S_0}{\partial \theta}\right)^2 + \frac{P_\varphi^2}{r^2\sin^2\theta}\right]}. \qquad (39)$$

If we square this relationship, it is not possible to perform separation of variables r, θ in general form. However, since we must find the solution to the problem in the case when $g \sim 10^{-12}$ e, as we square the expression we need not consider the term $\sim g^2 k^2 \cos^2 \theta / r^4$, and therefore separation of variables r, θ becomes possible. The solution of the problem in this case is similar to the problem of the movement of a charge g in a dipole field at nonrelativistic velocities, i.e., when the Hamiltonian has the form:

$$\varkappa = \frac{P^2}{2M} + \frac{gk\cos\theta}{r^2}.$$

For $v \ll c$, the problem is analyzed in [22], where it is solved as an electron moves through the field of an electric dipole. Thus, performing the transforms indicated, we find

$$\left[\frac{\mathcal{E}^2 - (Mc^2)^2}{c^2} - \left(\frac{\partial S_0}{\partial r}\right)^2\right] r^2 = \frac{2\mathcal{E}gk\cos\theta}{c^2} + \frac{P_\varphi^2}{\sin^2\theta} + \left(\frac{\partial S_0}{\partial \theta}\right)^2 = \alpha_2. \qquad (40)$$

Let us introduce the following symbols: $\mathcal{E} = \alpha_1$, $P_\phi = \alpha_3$, and seek the solution for S_0 in the form $S_0(r_1\theta) = S_1(\theta) + S_2(r)$. We produce

$$P_\theta = \frac{dS_1}{d\theta} = \sqrt{\alpha_2 - \frac{2\alpha_1 kg\cos\theta}{c^2} - \frac{\alpha_3^2}{\sin^2\theta}},$$

$$P_r = \frac{dS_2}{dr} = \sqrt{\frac{\alpha_1^2 - (Mc^2)^2}{c^2} - \frac{\alpha_2}{r^2}}.$$

Consequently, the solution for function S has the form

$$S(r, \theta, \varphi, t) = -\alpha_1 t + \alpha_3\varphi + \int\sqrt{\alpha_2 - \frac{2\alpha_1 gk\cos\theta}{c^2} - \frac{\alpha_3^2}{\sin^2\theta}}\, d\theta + \int\sqrt{\frac{\alpha_1^2 - (Mc^2)^2}{c^2} - \frac{\alpha_2}{r^2}}\, dr + \alpha_4. \qquad (41)$$

Let us differentiate (41) with respect to constants $\alpha_1, \alpha_2, \alpha_3$ and set the expressions thus produced equal to the new constants $\beta_1, \beta_2, \beta_3$. It is well known that this operation corresponds to the canonical transform to new momentums $\alpha_1, \alpha_2, \alpha_3$ and coordinates $\beta_1, \beta_2, \beta_3$, the Hamilton function being equal to zero in the new variables [21]. The relationships produced by differentiation allow the coordinates r, θ, ϕ to be expressed through time and the constants $\alpha_1, \alpha_3, \alpha_2, \beta_1, \beta_2, \beta_3$. This allows us to find the trajectory equations for the particle. Performing these operations, we find the following relationship:

$$\beta_1 = -t - \frac{gk}{c^2} \int \frac{\cos\theta \, d\theta}{\sqrt{\alpha_2 - \frac{2\alpha_1 gk \cos\theta}{c^2} - \frac{\alpha_3^2}{\sin^2\theta}}} + \frac{\alpha_1}{c^2} \int \frac{dr}{\sqrt{\frac{\alpha_1^2 - (Mc^2)^2}{c^2} - \frac{\alpha_2}{r^2}}}; \tag{42}$$

$$\beta_2 = \frac{1}{2} \int \frac{d\theta}{\sqrt{\alpha_2 - \frac{2\alpha_1 gk \cos\theta}{c^2} - \frac{\alpha_3^2}{\sin^2\theta}}} - \frac{1}{2} \int \frac{dr}{r^2 \sqrt{\frac{\alpha_1^2 - (Mc^2)^2}{c^2} - \frac{\alpha_2}{r^2}}}; \tag{43}$$

$$\beta_3 = \varphi - \alpha_3 \int \frac{d\theta}{\sin^2\theta \sqrt{\alpha_2 - \frac{2\alpha_1 gk \cos\theta}{c^2} - \frac{\alpha_3^2}{\sin^2\theta}}}. \tag{44}$$

Equation (42) allows us to find the position of the particle on the trajectory at moment in time t. The two latter relationships (43), (44) determine the form of the particle trajectory. Let us assume now that P = 0, so that it is easy to produce from (37) the following condition for points (r_1, θ_1) where the velocity of the particle becomes zero:

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$r \gg r_0$. Let us now analyze the possible forms of trajectories. For this, we introduce the following symbols:

$$I_1 = \int \frac{\cos\theta\, d\theta}{\sqrt{\alpha_2 - \frac{2\alpha_1 gk}{c^2}\cos\theta - \frac{\alpha_3^2}{\sin^2\theta}}},$$
$$I_2 = \int \frac{d\theta}{\sin^2\theta \sqrt{\alpha_2 - \frac{2\alpha_1 gk}{c^2}\cos\theta - \frac{\alpha_3^2}{\sin^2\theta}}}.$$

Suppose $\xi = \cos\theta$, $d\xi = -\sin\theta\, d\theta$:

$$I_1(\xi) = -\int \frac{\xi\, d\xi}{\sqrt{(1-\xi^2)\left(\alpha_2 - \frac{2\alpha_1 gk}{c^2}\xi - \frac{\alpha_3^2}{1-\xi^2}\right)}},$$
$$I_1(\xi) = -\int \frac{\xi\, d\xi}{\sqrt{\left(\alpha_2 - \frac{2\alpha_1 gk}{c^2}\xi\right)(1-\xi^2) - \alpha_3^2}},$$
$$I_2(\xi) = -\int \frac{d\xi}{(1-\xi^2)\sqrt{\left(\alpha_2 - \frac{2\alpha_1 gk}{c^2}\xi\right)(1-\xi^2) - \alpha_3^2}}. \qquad (46)$$

Equation (44) determines the change in angles θ and ϕ as the particle moves. It can be looked upon as a projection of the trajectory of the particle on a sphere of unit radius. As was first noted in [22], this expression is identical to the equation for a spherical pendulum, the center of which is located at the coordinate origin, with the force of gravity directed along the axis of the dipole. This equation is well known [22, 23]. Therefore, let us analyze the main properties of (44) briefly. Suppose

$$\psi(\xi) = \left(\alpha_2 - \frac{2\alpha_1 gk}{c^2}\xi\right)(1-\xi^2) - \alpha_3^2.$$

Obviously, the integral in (46) has a real value if $\psi > 0$. Let us analyze the function $\psi(\xi)$. From equation $d\psi/d\xi = 0$, we can find ξ_1 and ξ_2, for which the function $\psi(\xi)$ has extreme values. We produce

$$\xi_{1,2} = \frac{\alpha_2 c^2}{6\alpha_1 gk} \pm \sqrt{\left(\frac{\alpha_2 c^2}{6\alpha_1 gk}\right)^2 + \frac{1}{3}}.$$

Let us investigate here the two limiting cases. We will assume $\alpha_2 \geq 0$, $\alpha_1 < 0$, $gk < 0$. Let us assume at first that $\alpha_2 c^2 \gg 6\alpha_1 gk$. Then $\xi_1 = \alpha_2 c^2/3\alpha_1 gk \gg 1$, $\xi_2 \sim 0$,

$$\begin{aligned}\psi(\xi_1 \gg 1) &= -\left[\frac{\alpha_2}{3}\left(\frac{\alpha_2 c^2}{3\alpha_1 gk}\right)^2 + \alpha_3^2\right] \leqslant 0,\\ \psi(0) &= \alpha_2 - \alpha_3^2.\end{aligned} \tag{47}$$

Let us assume that $\psi(0) \geq 0$. Let us now analyze how $\psi(\xi)$ changes if ξ changes from $-\infty$ to $+\infty$.

ξ . . .	$-\infty$	-1	$\xi_2 \sim 0$	$+1$	ξ_1	$+\infty$
$\psi(\xi)$ . .	$-\infty$	$-\alpha_3^2$	$\alpha_2 - \alpha_3^2 \geqslant 0$	$-\alpha_3^2$	$\leqslant 0$	$+\infty$

Thus, it is easy to see that $\psi(\xi)$ has three roots, which can be represented in increasing order as $\xi_3 < \xi_4 < \xi_5$. For the task at hand, the solution can be produced in the area where $|\xi| \leq 1$, while $\psi(\xi) \geq 0$, i.e., can change only from ξ_3 to ξ_4. Obviously, the problem has a solution if condition $\psi(0) \geq 0$ is fulfilled, since if $\psi(0) < 0$, the integrals in (46) have no real values for $|\xi| \leq 1$. Therefore, angle θ changes between arc cos ξ_3 to arc cos ξ_4. Since $\alpha_3 = P_\phi$, $P_\phi = (Mr^2 \sin^2 \theta/\sqrt{1-\beta^2})\dot{\phi}$, where $\alpha_3 > 0$, $\dot{\phi} > 0$ or vice versa. Consequently, angle ϕ either decreases alone or increases alone as the particle moves. Thus, the projection of the trajectory on the sphere of unit radius will be contained between two parallel lines arc cos ξ_3 and arc cos ξ_4, which it will contact alternately. Let us analyze the other limit case when $\alpha_2 c^2 \ll 6\alpha_1 gk$. Then $\xi_{1,2} = \pm 1/\sqrt{3}$. It is easy to show that in this case the range of possible values of ψ moves toward negative ξ. Detailed analysis of the change in angle θ and ϕ for the problem of the spherical pendulum can be found in [22, 23]. Here we will note only the two main results of these investigations. The projection of the trajectory on a sphere of unit radius is symmetrical relative to the meridians, passed through the points of contact of the trajectory with the parallel lines arc cos ξ_3 and arc cos ξ_4. The azimuthal angle of rotation ϕ of the projection of the trajectory resulting from movement from one contact point to another cannot be less than $\pi/2$. Let us now analyze the change in r as the particle moves. As before, we will assume for definition

$$\alpha_2 \geqslant 0, \quad \alpha_1 < 0, \quad gk < 0. \tag{48}$$

Equations (42) and (43) can be rewritten in the following form:

$$t + \beta_1 = -\frac{gk}{c^2} I_1(\theta) + \frac{R_0}{\sqrt{\alpha_2}} \frac{\alpha_1}{c^2} \int \frac{r\,dr}{\sqrt{r^2 - R_0^2}},$$
$$\beta_2 = \frac{1}{2} I_1(\theta) + \frac{1}{2\sqrt{\alpha_2}} \int \frac{d(1/r)}{\sqrt{\frac{1}{R_0^2} - \frac{1}{r^2}}},$$

where $R_0 = \sqrt{\frac{\alpha_2 c^2}{\alpha_1^2 - (Mc^2)^2}}$.

The integrals with respect to r can be easily calculated. We produce

$$t + \beta_1 = -\frac{gk}{c^2} I_1(\theta) + \frac{\alpha_1}{c\sqrt{\alpha_1^2 - (Mc^2)^2}} \sqrt{r^2 - R_0^2},$$
$$\beta_2 = \frac{1}{2} I_1(\theta) + \frac{R_0^2}{2\sqrt{\alpha_2}} \arcsin(R_0/r).$$

Obviously, the solution produced is real if $r \geq R_0$, and also if condition (48) is fulfilled. We will assume that $|\alpha| \gg Mc^2$. Then $R_0 \sim (c/\alpha_1)\sqrt{\alpha_2}$. Thus, the trajectory of a particle as it approaches the earth will be contained between the two angles arc cos ξ_3 and arc cos ξ_4. The distance r to the dipole will vary from infinity to R_0. At distance R_0, the particle will be reflected and once more recede to infinity. Obviously, the particle will reach the earth if $R_e \geq R_0$. If $\alpha_3 = P_\phi = 0$, the trajectory will be located in the meridian plane. If, however, $P_\phi \neq 0$, even rotation about axis Z will be added to the movement in the meridian plane. Let us now analyze the movement with particle velocity $v \ll c$. In this case, the Hamilton-Jacoby equation for function $S_0(r,\theta)$ has the form

$$\frac{1}{2M}\left[\left(\frac{\partial S_0}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S_0}{\partial \theta}\right)^2 + \frac{\alpha_3^2}{r^2 \sin^2\theta}\right] = \alpha_1 - \frac{gk\cos\theta}{r^2}. \quad (49)$$

The solution of this equation should be sought in the form:

$$S_0(r,\theta) = S_1(r) + S_2(\theta).$$

The exact solution of (49) was produced in [22]. Let us apply the results of [22] to the problem of the movement of a magnetic nutrino in the dipole field of the earth. It can be shown that in the case $\alpha_3 = 0$, $\alpha_1 < 0$, $\alpha_2 < 0$, the particle forms closed trajectories in the dipole field. The trajectory in the meridian plane is included between the two lines $\pm$arc cos $(\alpha_2/2Mgk)$, and the distance of greatest separation from the dipole moment satisfies the following equation:

$$d = \sqrt{\frac{\alpha_2}{2M\alpha_1}}. \quad (50)$$

The greatest value of r is achieved at $t = -\beta_1$. The entire trajectory is covered in time $\sqrt{-\alpha_2/\alpha_1^2}$.

Let us analyze one more form of circular closed trajectory, first found in [22]. We know that the magnetic field intensity of the dipole is determined by the expression

$$H = \frac{3(kr)r - kr^2}{r^5}.$$

The projection of this vector H_z on the axis of the dipole is obviously equal to

$$\frac{k}{r^3}(3\cos^2\theta - 1) = H_z. \tag{51}$$

If $H_z = 0$, movement is possible in a plane perpendicular to the axis of the dipole. These trajectories will be circular [22]. It is obvious from (51) that the half-angle at the peak of cone on which the trajectory is located is determined from condition $\cos\theta = 1/\sqrt{3}$. This movement is possible only if the force component acting on the particle in the plane perpendicular to the Z axis is balanced by centrifugal force. From this condition we can produce an equation correct for the circular trajectory:

$$\mathcal{E}\left[1 - \left(\frac{Mc^2}{\mathcal{E}}\right)^2\right] = -\frac{2gk}{\sqrt{3}r^2}, \tag{52}$$

where $\mathcal{E} = Mc^2/\sqrt{1-\beta^2}$ is the total energy of the particle. In relationship (52), $g < 0$. For $g > 0$, circular trajectories can arise in the lower hemisphere where $|\theta| > \pi/2$. If $\rightarrow Mc^2$, in order for condition (52) to continue to be observed, $r \rightarrow \infty$. Consequently, if the particle energy decreases, the condition of existence of a circular orbit (52) will be fulfilled with high values of r. Solving (53) for $\mathcal{E}$, we produce

$$\mathcal{E}_{1,2} = -\frac{gk}{\sqrt{3}r^2} \pm \sqrt{\frac{g^2k^2}{3r^4} + (Mc^2)^2}. \quad (53)$$

Let us analyze first those r for which $Mc^2 \gg \frac{|g|k}{\sqrt{3}r^2}$.

Then $\mathcal{E} = Mc^2 + \frac{|g|k}{\sqrt{3}r^2}$. Thus, where $r \sim R_e$, $g \sim 10^{-12}$ e, $k = 8 \cdot 10^{25}$ gs$\cdot$cm^3, $(|g|k/\sqrt{3}R_e^2) \sim 0.08$ ev. If $Mc^2 \sim 250$ ev, then where $r \sim R_e$ the particle energy on the circular orbit differs very little from Mc^2. Thus, for particle energies $\mathcal{E} \gg Mc^2$, circular orbits are located deep within the earth. If $(|g|k/\sqrt{3}r^2) \gg Mc^2$, $\mathcal{E} = (2|g|k/\sqrt{3}r^2)$. Let us define the r beginning with which this condition is fulfilled. Suppose $(|g|k/\sqrt{3}r^2) \sim 10\ Mc^2$. Then $r \sim \sqrt{2|g|k/10\sqrt{3}Mc^2}$. Where $Mc^2 \leq 250$ ev, we find $r \gg 23.8$ km. Consequently, circular orbits can be filled with high energy particles at distances amounting to a few dozen kilometers from the position where the magnetic dipole k is located. Let us now show that these trajectories will not be stable. Actually, when moving within the earth a particle will lose energy to Bremstrahlung and ionization. When the kinetic energy is decreased, condition (52) will be disrupted and the orbital radius ρ will decrease, decreasing the centrifugal force acting on the particle. When this occurs, force $g\Delta H_z$ arises, perpendicular to the plane of the orbit. Let us take the complete differential expression (51). Substituting into this expression $\cos\theta \sim 1/\sqrt{3}$ and considering that $\partial H_z/\partial r = 0$ where $\cos\theta \simeq 1/\sqrt{3}$, we find the following condition:

$$g\Delta H_z = -\frac{2\sqrt{2}gk}{r^3}\Delta\theta.$$

Since $\Delta\theta < 0$, $g < 0$, $g\Delta H_z < 0$. Consequently, the force arising upon reduction of energy $g\Delta H_z$ will tend to return the particle to the circular orbit, where $\cos\theta = 1/\sqrt{3}$. However, at this point stable motion is impossible, since with lower values of the radius the kinetic energy required to maintain condition (52) is greater than the initial energy.

If the radius of the stable orbit increases due to external excitation, $g\Delta H_z > 0$, since $\Delta\theta > 0$. In this case, the particle can return to a circular orbit under the condition that expression (52) remains correct for the greater value of radius. Let us note in conclusion the following fact. Since circular orbits for high energy particles are located where $r \ll R_e$, the process of transmission of energy to the material in the earth from magnetic nutrinos formed within the earth during β-decay, and as a result of nuclear reactions, is possible.

As follows from [1, 8], the theory of the magnetic nutrino is not T-invariant. However, this does not contradict the theory of the weak interaction, since after the discovery of nonretention of CP, the problem of the invariance of the weak interaction in relation to T-reflection remains open. Furthermore, since this theory is separately invariant in relation to C' = CM(PT), this fact could explain the experimentally detected failure of CP-parity.

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