WEATHER BUREAU
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# Probability of Tropical Cyclone Induced Winds at NASA Manned Spacecraft Center



Technical Memorandum WBTM SOS 4

U.S. DEPARTMENT OF COMMERCE / ENVIRONMENTAL SCIENCE SERVICES ADMINISTRATION

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# PROBABILITY OF TROPICAL CYCLONE INDUCED WINDS AT THE NASA MANNED SPACECRAFT CENTER[1]

Charles J. Neumann

## ABSTRACT

This study presents a statistical analysis of tropical cyclones as they affected the NASA Manned Spacecraft Center (located approximately midway between Houston and Galveston, Tex.) over the period of record from 1886 through 1967. The Poisson distribution is used to estimate the probability of the Center observing at least n occurrences of sustained 35-knot winds over k consecutive years (n = 0,3; k = 1,20). The bivariate normal distribution is used to estimate the probability of an existing tropical storm or hurricane producing sustained 35-knot winds <u>at</u> 24-hourly intervals and <u>within</u> 24-hourly intervals for prognostic periods of up to 4 or 5 days depending on the source region of the storm. Presented also are charts and figures depicting the general Gulf of Mexico tropical cyclone climatology. This study is modeled after a 1968 study which presented similar data for the Kennedy Space Center, Fla.

## 1. INTRODUCTION

This study was undertaken by the Spaceflight Meteorology Group (SMG), Weather Bureau, ESSA, to provide quantitative estimates of the likelihood of the NASA Manned Spacecraft Center (MSC)[2] experiencing certain critical winds associated with tropical storms or hurricanes. Many of the statistical techniques and computer programs utilized in this study were developed earlier by Hope and Neumann (1968). This earlier study provided probability estimates of the Cape Kennedy, Fla., area experiencing critical winds from tropical storms or hurricanes.

Two broad problem areas are considered: (1) What is the climatological probability of critical winds at MSC during various exposure periods within the hurricane season or over consecutive hurricane seasons? (2) What is the probability of an existing tropical storm or hurricane producing critical winds at some future time or within some future time interval?

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1 This study was prepared by the Miami Section of the Spaceflight Meteorology Group, collocated with the National Hurricane Center at the University of Miami, Miami, Fla. Through funds transferred from the NASA Office of Manned Space Flight, the Spaceflight Meteorology Group provides the primary meteorological support for the NASA manned spaceflight programs.

2 The Manned Spacecraft Center is located approximately midway between Houston and Galveston, Tex., near 29.6° N., 95.1° W.

Hope and Neumann (1968) accomplished a further stratification based on the storm's direction of motion. Because of an insufficiency of cases, it was not possible to stratify the MSC storms in this manner. This deficiency was partially offset by other techniques, to be discussed later.

In this report, a critical wind is defined as a 1-minute average wind of 35 knots at the 10-meter level produced by a tropical storm or hurricane. Assuming the gust factors usually associated with winds of this magnitude (1.4 or slightly higher), 35-knot 1-minute winds would be accompanied by gusts approaching 50 knots. The 35-knot threshold was selected not on the basis of any particular operational limit at MSC, but rather on the basis of convenience. A regression equation and other techniques developed for the Cape Kennedy study were based on 35 knots. Also, 35 knots is a convenient figure since it is the lower wind speed limit defining a tropical storm. Furthermore, selection of a threshold wind higher than 35 knots would diminish the number of cases available for statistical analysis to the point that reliable probability estimates could not be obtained.

## 2. DATA SOURCE

As discussed by Hope and Neumann (1968), the basic data source currently available for studies of this type is computer card deck 988, "North Atlantic Tropical Cyclones," available through National Weather Records Center, Environmental Data Service, ESSA, at Asheville, N.C. This card deck was modified, error checked, and brought up to date through the 1967 hurricane season by SMG, Miami, and then transferred to binary tape. This final tape lists all positions (either once or twice a day, depending on whether the storm was pre- or post-1930) of the 660 tropical storms and hurricanes between 1886 and 1967. The storm tracks through 1958 (with some modification) are basically those given by Cry, et al. (1959). Those from 1959 through 1963 are given in Cry (1965). Tracks for the years 1964 through 1967 were obtained from records maintained at the National Hurricane Center, Miami. Also included on the master data tape are the mean maximum wind speed recorded in the storm on each day of the storm's existence and other pertinent storm identification data. All computations were performed on the IBM 7040 or 360 computers available to ESSA through arrangement with the University of Miami Computing Center.

## 3. SELECTION OF CASES

Since detailed wind records are not available from the site of the Manned Spacecraft Center, it was necessary to rely on an indirect method to determine which storms over the 82-year period of record 1886-1967 were likely to have produced at least 35-knot steady-state winds at MSC. Hope and Neumann (1968) found that the regression equation

$$R = 0.6W_{max} + 30 \quad (1)$$

could be used to describe the relationship between the radius of the 35-knot isotach (R) and the maximum wind near the storm center ($W_{max}$). Although this equation was developed for the Kennedy Space Center, it should be reasonably valid for any exposed near-coastal location in the tropics or subtropics.

Details of the storm-selection technique utilizing equation (1) are shown in figures 1 and 2. Figure 1 describes the logic in terms of a standard computer program flow chart, and figure 2 schematically represents a fictitious case. In the actual program described in figure 1, one storm out of the 660-storm data file is read into the computer at a time. A linear interpolation between the recorded 0000 GMT and 1200 GMT storm positions gives 3-hourly storm positions for 0300 GMT, 0600 GMT, and 0900 GMT, whereas interpolations between the 1200 GMT and 0000 GMT recorded positions give intermediate positions for 1500 GMT, 1800 GMT, and 2100 GMT for each day of the storm's existence. The actual decision as to whether the storm ever produced critical winds at the site depends on whether distance Z (DISTZ in figure 2) is equal to or less than the radius of the 35-knot winds as defined by equation (1). The constant 52 in the expression for DISTX is simply the length, in nautical miles, of 1 degree of longitude at the latitude of MSC. The term WIND as used in both figures 1 and 2 refers to the mean maximum wind near the center of the storm on a given day.

It is realized that this method of selecting storms affecting MSC is somewhat arbitrary. In some hurricanes, sustained winds in excess of 35 knots will extend much farther from the storm center than equation (1) indicates. In some minimal tropical storms the radius of 35-knot winds might be smaller. Also, it is generally true that asymmetry in the wind field is observed in most storms which in turn is a function of the intensity of the storm, its direction of movement, and surrounding synoptic features. But it is believed that in dealing with mean conditions where large numbers of storms are considered, the method described in figures 1 and 2 correctly identifies most of the tropical cyclones that have produced sustained 35-knot winds at the Manned Spacecraft Center. In any case, this method of storm selection is believed superior to considering all storms passing within a fixed distance from the site, regardless of storm intensity.

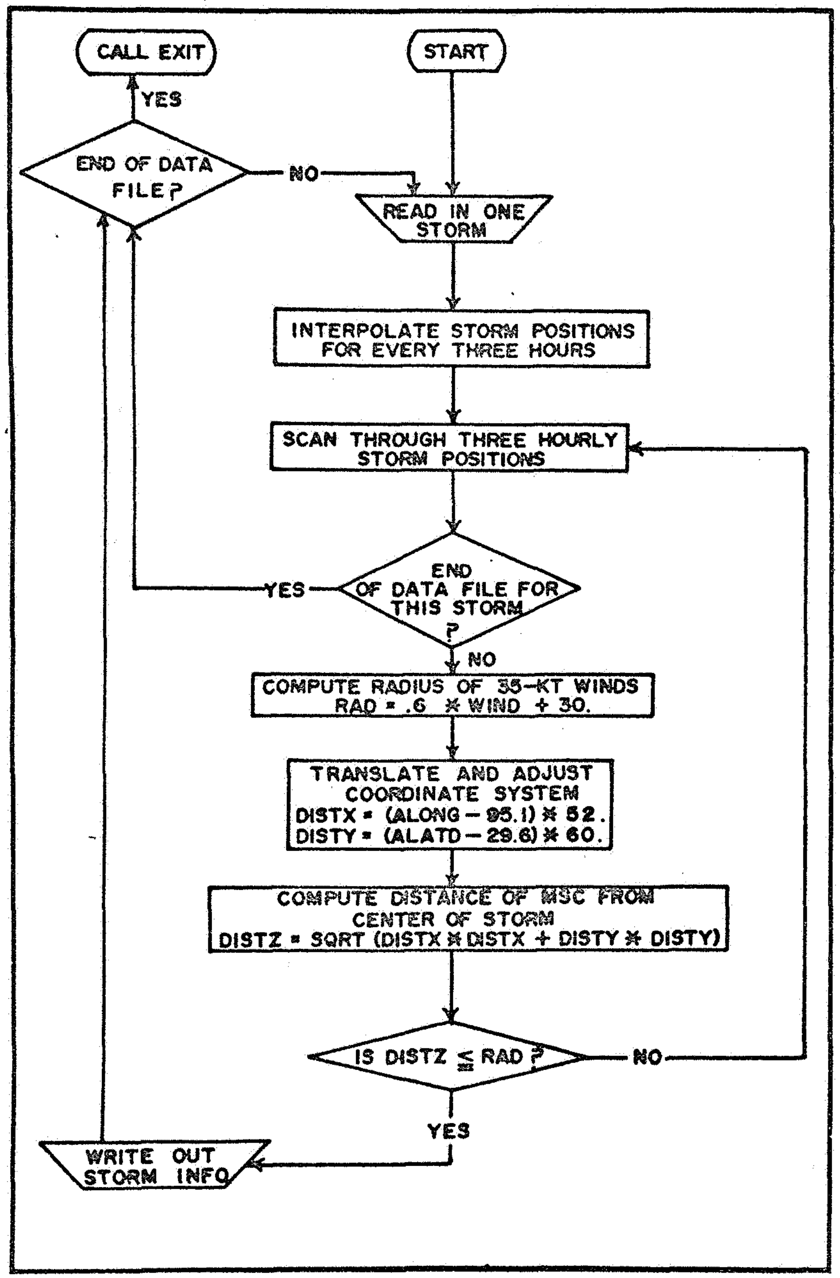



Figure 1. - Computer flow chart showing storm selection technique.

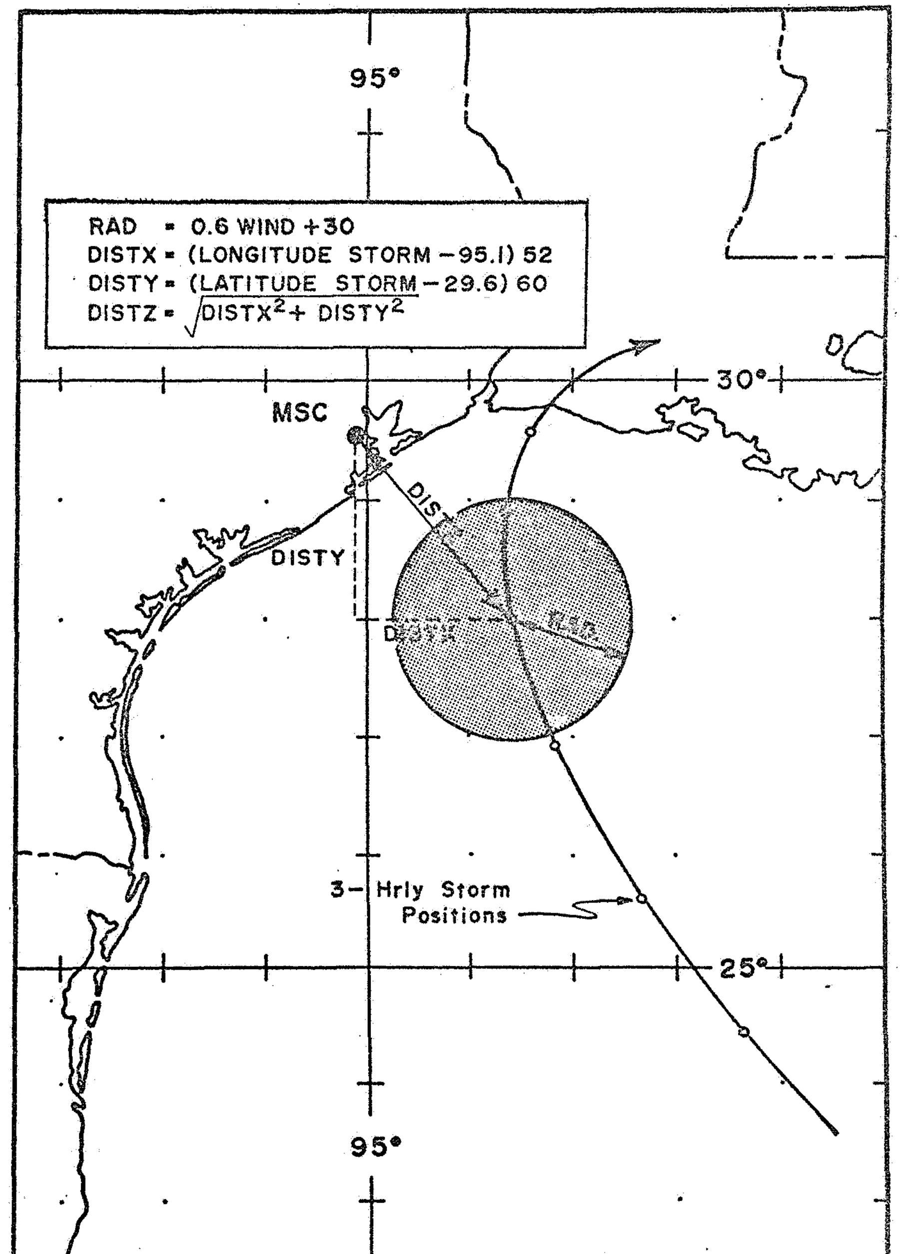



Figure 2. - Schematic illustration of storm selection technique.

The paths of the 25 tropical cyclones 1886-1967 which were calculated to have produced critical winds at MSC according to equation (1) are shown in figure 3. Table 1 lists additional storm data for the same 25 cases. The storm numbers in column 1 of table 1 correspond to those in figure 3. The storm category in column 6 of table 1 refers to the storm source region and will be explained in subsection 4. Both the figure and the table include the tropical depression stages, if any, of the 25 cases.

The program described in figure 1 also has the option of counting the number of tropical cyclones (including depression stage) passing within fixed distances (50, 100, 150, and 200 n.mi.) of MSC, without considering wind. Figure 4 is a plot of these data. The figure shows that there were 76 storms passing within 200 n.mi. of MSC in the 82-year period of record, 52 storms passing within 150 n.mi., etc. It should be noted that, by interpolation, figure 4 indicates that there were 25 storms passing within 78 n.mi. of MSC during the period of record (dashed line). This is the same number of storms that was counted by the use of regression equation (1). This suggests a simpler, alternate procedure for counting the number of cases producing critical winds. But it should be noted that, although this alternate procedure might be expected to count the correct <u>number</u> of cases, some of these would be the wrong <u>individual</u> cases.

## <u>Problem Area 1</u>

As outlined on page 1, an objective of this study is to determine the basic tropical cyclone climatology as it pertains to MSC. What, for example, is the probability of MSC observing critical winds on a particular day, month, year, or over several consecutive years?

A graphical display of the time distribution of the 25 storms composing the data sample is given on figure 5. Twenty-five cases over 82 years of record yield a sample mean of 0.305 storms per year. But in the years 1940 and 1941, there were two occurrences each year, giving a frequency of 23/82 or 28 percent of the years when critical winds occurred <u>at least once</u>. Hope and Neumann (1968) showed that the observed spectrum of storm occurrences in single years closely approximates the probabilities computed by fitting the data to a Poisson exponential distribution, where the sample mean (.305) is used as the Poisson parameter. Similarly in this MSC study, the observed frequencies closely approximate the Poisson probabilities. Table 2 shows the comparison. The Poisson probabilities were computed according to

$$P(x) = e^{-m} m^{x} / x! \qquad (2)$$

where e is the base of natural logarithms, m is the expected (mean) value, and x is the number of presumably independent occurrences over the same time unit as the mean value.

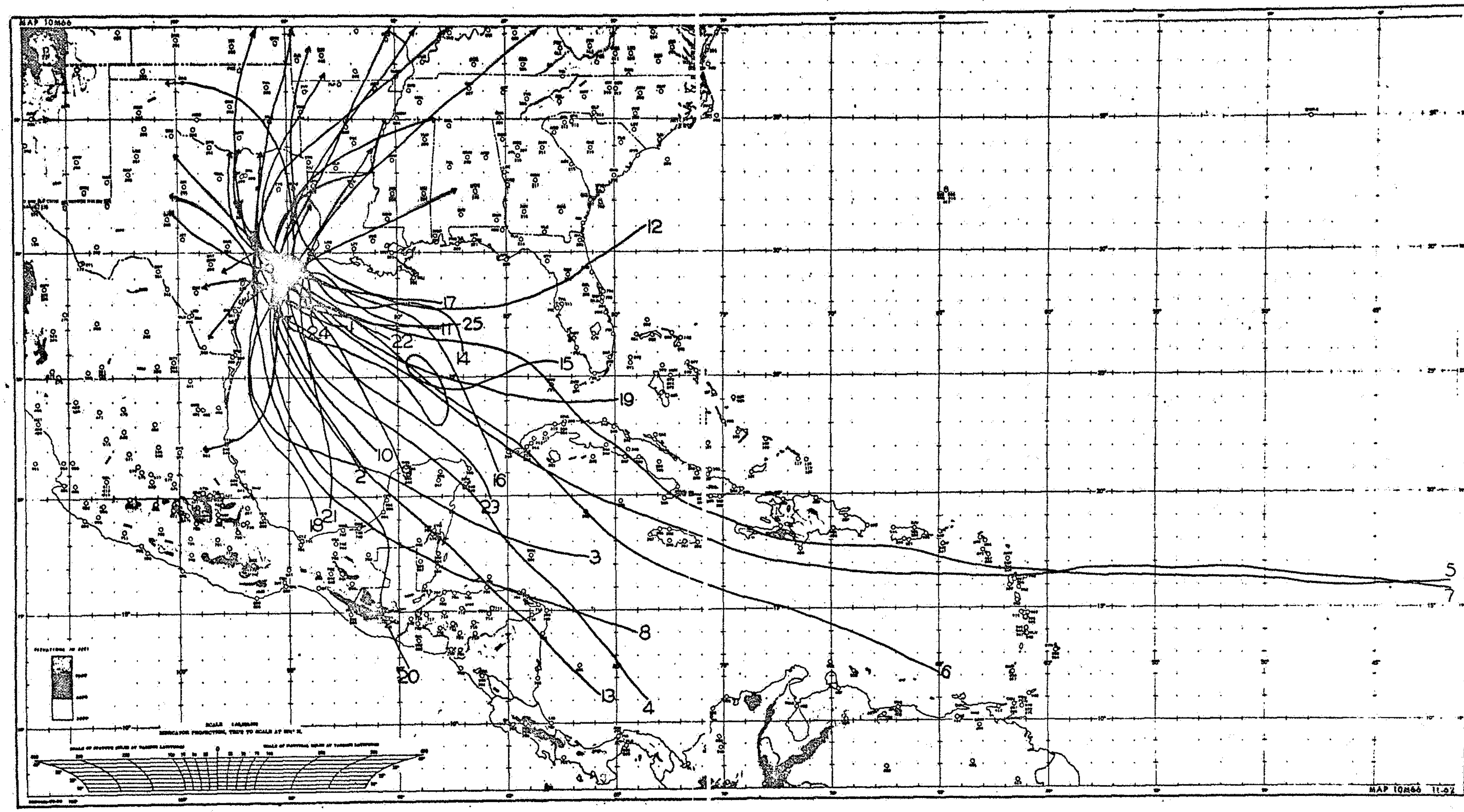



Figure 3. - Tracks of tropical cyclones which produced critical winds at the Manned Spacecraft Center 1886-1967. Numbers are in chronological order and correspond to those given in table 1.

**Table 1.**-Tropical cyclones (including depressions) computed to have produced critical winds at the NASA Manned Spacecraft Center, 1886 through 1967. T-0 (to the closest three hours) refers to the time of initial onset of critical winds. Storm number corresponds to those given in WEABUR Technical Paper Number 55. Storm type refers to classification at T-24 hours. Categories 1 and 2 refer, respectively, to northeastern Gulf of Mexico and Atlantic source region and to southwestern Gulf of Mexico and western Caribbean source region.

| Consecutive Storm Number | Year | Month & Day | Time (CST) | Storm Number | Category | Type | Storm position at T-(hrs.) prior to striking MSC (Lat. N., long. W.). | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | T-0 | T-24 | T-48 | T-72 | T-96 | T-120 | T-144 | T-168 |
| 1 | 1888 | Jun 16 | 2100 | 1 | 2 | H | 28.5 94.6 | 26.3 91.0 | | | | | | |
| 2 | 1891 | Jul 5 | 1200 | 1 | 2 | H | 28.1 95.4 | 24.8 94.8 | 22.5 93.5 | 21.0 92.0 | | | | |
| 3 | 1895 | Oct 6 | 2100 | 4 | 2 | TS | 29.5 94.8 | 23.8 95.8 | 21.6 93.3 | 20.2 89.9 | 18.8 86.7 | 17.5 82.9 | | |
| 4 | 1898 | Sep 27 | 1800 | 6 | 2 | TS | 28.6 94.6 | 26.1 94.3 | 24.1 92.4 | 22.5 90.2 | 20.9 88.0 | 18.6 85.7 | 16.0 83.6 | 12.9 81.2 |
| 5 | 1900 | Sep 8 | 1200 | 1 | 1 | H | 28.6 93.5 | 26.8 89.6 | 26.4 84.6 | 24.2 82.1 | 22.0 80.2 | 20.2 77.3 | 19.3 73.9 | 18.4 69.8 |
| 6 | 1909 | Jul 21 | 0900 | 3 | 1 | H | 28.5 94.2 | 26.6 91.0 | 24.8 88.5 | 22.5 85.4 | 19.8 81.2 | 17.1 77.3 | 15.2 71.7 | 13.3 66.8 |
| 7 | 1915 | Aug 16 | 2100 | 2 | 1 | H | 28.4 94.3 | 25.1 89.9 | 22.5 85.8 | 20.6 81.9 | 18.9 76.9 | 17.7 70.5 | 17.3 63.7 | 16.9 56.0 |
| 8 | 1921 | Jun 22 | 1200 | 1 | 2 | H | 28.7 96.3 | 25.7 96.0 | 23.7 95.5 | 21.6 94.5 | 19.3 91.9 | 17.0 88.2 | 16.0 84.8 | 15.0 81.5 |
| 9 | 1932 | Aug 13 | 1800 | 2 | 2 | H | 28.9 94.7 | 25.4 92.1 | 22.0 90.3 | | | | | |
| 10 | 1933 | Jul 23 | 0300 | 4 | 2 | TS | 29.3 96.0 | 25.8 94.3 | 22.8 91.8 | | | | | |
| 11 | 1934 | Aug 27 | 0600 | 5 | 1 | TS | 29.2 93.8 | 27.2 89.8 | | | | | | |
| 12 | 1940 | Aug 7 | 1800 | 2 | 1 | H | 29.9 93.7 | 28.8 92.0 | 28.0 90.0 | 27.7 87.5 | 27.9 84.9 | | | |
| 13 | 1940 | Sep 23 | 1800 | 6 | 2 | TS | 29.0 94.8 | 25.8 95.0 | 21.7 91.6 | 18.0 88.0 | 14.8 84.8 | 11.2 81.9 | | |
| 14 | 1941 | Sep 15 | 0600 | 1 | 1 | TS | 29.8 94.3 | 29.2 93.2 | 28.1 91.8 | 28.0 88.8 | 27.5 87.7 | | | |
| 15 | 1941 | Sep 23 | 1500 | 2 | 1 | H | 28.7 95.4 | 26.4 93.2 | 24.7 89.8 | 23.1 88.0 | 25.1 88.1 | 26.0 89.1 | 24.8 88.4 | 25.3 84.9 |
| 16 | 1942 | Aug 20 | 2100 | 1 | 1 | H | 28.9 93.9 | 28.1 91.5 | 26.8 88.8 | 22.6 86.3 | | | | |
| 17 | 1943 | Jul 27 | 0000 | 1 | 1 | H | 29.0 93.5 | 28.1 90.2 | 27.9 86.6 | | | | | |
| 18 | 1945 | Aug 27 | 0600 | 5 | 2 | H | 28.2 96.2 | 26.6 96.6 | 24.5 90.4 | 20.9 94.7 | 19.0 93.0 | | | |
| 19 | 1947 | Aug 24 | 0300 | 3 | 1 | H | 28.8 94.1 | 27.6 92.3 | 26.4 90.4 | 25.3 88.3 | 24.4 85.9 | 24.2 82.7 | 24.0 79.5 | |
| 20 | 1949 | Oct 3 | 2100 | 10 | 2 | H | 28.3 95.4 | 23.1 94.8 | 20.3 92.8 | 18.6 90.7 | | | | |
| 21 | 1957 | Jun 27 | 0600 | 2 | 2 | H | 29.3 93.8 | 24.7 93.7 | 22.0 93.4 | 19.9 93.1 | | | | |
| 22 | 1959 | Júl 24 | 0900 | 5 | 2 | TS | 28.4 95.3 | 27.5 93.4 | 26.5 91.4 | | | | | |
| 23 | 1961 | Sep 11 | 0900 | 3 | 2 | H | 28.2 96.2 | 26.7 94.1 | 24.8 91.4 | 23.3 88.6 | 21.3 86.3 | 19.2 85.3 | 16.6 82.8 | 14.5 80.4 |
| 24 | 1963 | Sep 17 | 0300 | 4 | 2 | H | 29.0 94.2 | 26.4 93.6 | | | | | | |
| 25 | 1964 | Aug 7 | 1200 | 3 | 1 | TS | 28.5 95.0 | 27.3 91.9 | 27.0 88.1 | | | | | |

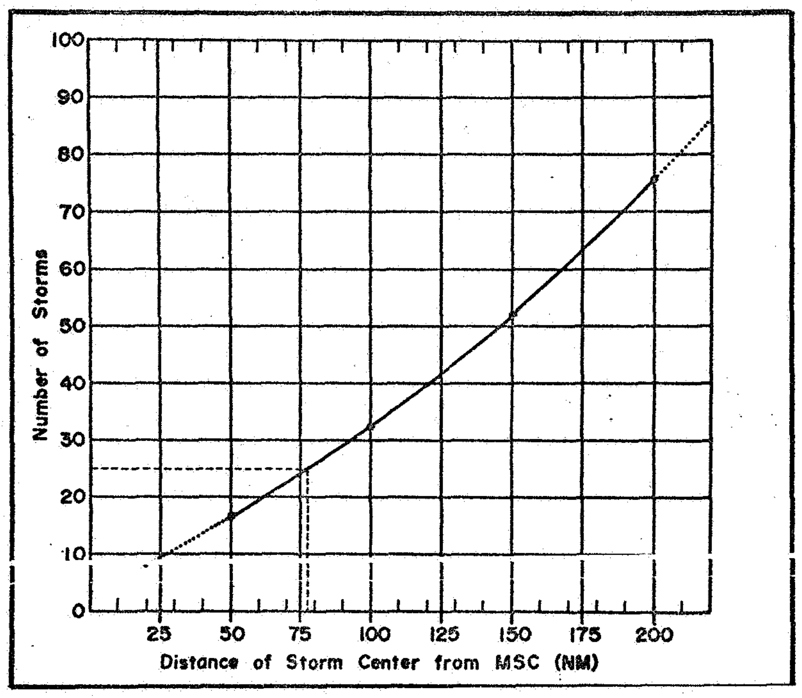



Figure 4. - Number of tropical cyclones passing within fixed distances of the Manned Spacecraft Center, 1886-1967.

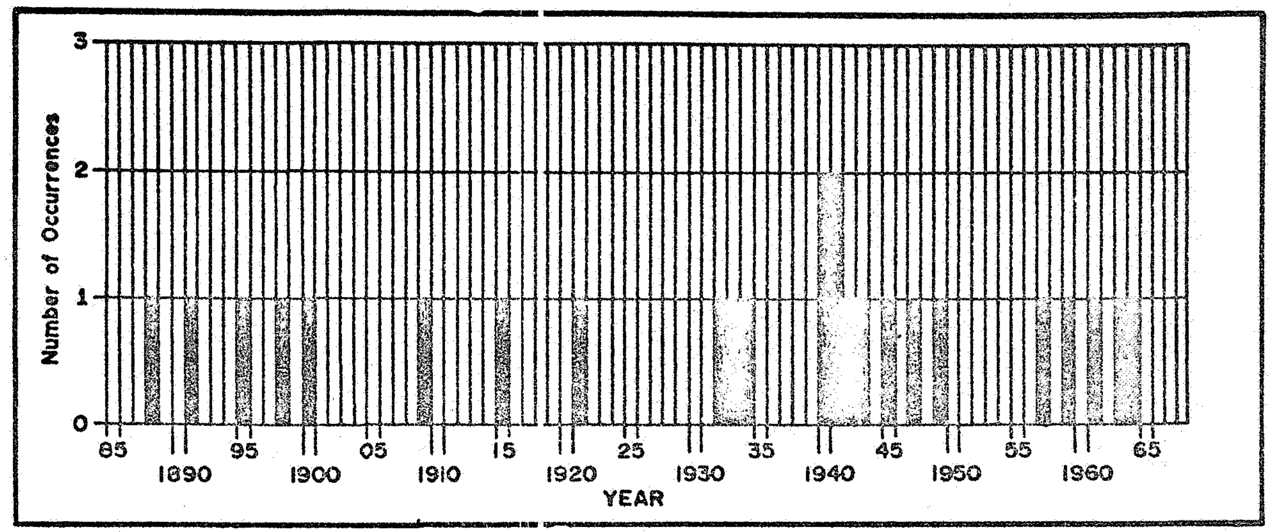



Figure 5. - Occurrence of critical winds at MSC by year, 1886-1967.

Table 2. - Observed frequency and computed probability of yearly critical wind occurrence at MSC

| | Occurrences per year (x) | | | |
|---|---|---|---|---|
| | x = 0 | x = 1 | x = 2 | x = 3 |
| Observed frequency. . . . . . . | 0.719 | 0.256 | 0.025 | 0.000 |
| Poisson probability . . . . . . | 0.737 | 0.225 | 0.034 | 0.004 |

m = 25/82 = 0.305

The probability of at least one occurrence of critical wind at MSC is therefore P(1) + P(2) + P($\geqslant$3) = .263 or 26.3 percent of the years.[3]

The Poisson exponential probability function can also be used to compute the probability of any number of occurrences over any number of consecutive years. In this case, the expected value (m) in equation (2) is computed by multiplying the unit mean (0.305) by the number of consecutive years over which the probability is to be computed. For example, suppose we want the probability of at least one occurrence in 10 consecutive years. The estimated mean occurrence rate in 10 years is computed to be 10 X 0.305 or 3.05. The probability of at least one occurrence is obtained by subtracting the probability of zero occurrences from 1.000. Since zero factorial is 1, equation (2) becomes

$$P(\geqq 1) = 1.000 - \exp(-3.05) = 0.953 \quad (3)$$

This is to say that 95.3 percent of the time, any consecutive 10-year period would be expected to have at least one occurrence of critical winds. In the actual data sample 72 of the 73 or 98 percent of the 10-year overlapping consecutive periods observed at least one occurrence.

Table 3 gives the mean occurrence rates and the computed probabilities of 0, at least 1, at least 2, and at least 3 occurrences for consecutive-year periods of up to 20. The probability of exactly one occurrence is obtained by subtracting the probability of at least two occurrences from the probability of at least one occurrence. The probability of one and only one occurrence in 10 years is therefore 0.953-0.808 or only 0.145. Some of the material of table 3 is displayed graphically in figure 6.

3 This compares with 0.364 or 36.4 percent at Cape Kennedy, Fla.

Table 3.-Poisson probability $P(x) = e^{-m}m^{x}/x!$ of specified occurrence rates of critical winds over extended time periods.

| Number of consecutive years (k) | Mean occurrence rate (m) in k years (0.305k) | Probability (P) of x occurrences | | | |
|---|---|---|---|---|---|
| | | $x=0$ | $x \geq 1$ | $x \geq 2$ | $x \geq 3$ |
| 1 | 0.305 | 0.737 | 0.263 | 0.038 | 0.004 |
| 2 | 0.610 | 0.543 | 0.457 | 0.125 | 0.024 |
| 3 | 0.915 | 0.401 | 0.599 | 0.233 | 0.065 |
| 4 | 1.220 | 0.295 | 0.705 | 0.344 | 0.125 |
| 5 | 1.524 | 0.218 | 0.782 | 0.450 | 0.197 |
| 6 | 1.830 | 0.161 | 0.839 | 0.546 | 0.277 |
| 7 | 2.134 | 0.118 | 0.882 | 0.629 | 0.360 |
| 8 | 2.439 | 0.087 | 0.913 | 0.700 | 0.440 |
| 9 | 2.744 | 0.064 | 0.936 | 0.759 | 0.517 |
| 10 | 3.049 | 0.047 | 0.953 | 0.808 | 0.588 |
| 11 | 3.354 | 0.035 | 0.965 | 0.848 | 0.651 |
| 12 | 3.659 | 0.026 | 0.974 | 0.880 | 0.707 |
| 13 | 3.963 | 0.019 | 0.981 | 0.906 | 0.756 |
| 14 | 4.268 | 0.014 | 0.986 | 0.926 | 0.799 |
| 15 | 4.573 | 0.010 | 0.990 | 0.942 | 0.834 |
| 16 | 4.878 | 0.008 | 0.992 | 0.955 | 0.865 |
| 17 | 5.183 | 0.006 | 0.994 | 0.965 | 0.890 |
| 18 | 5.488 | 0.004 | 0.996 | 0.973 | 0.911 |
| 19 | 5.793 | 0.003 | 0.997 | 0.979 | 0.928 |
| 20 | 6.098 | 0.002 | 0.998 | 0.984 | 0.942 |

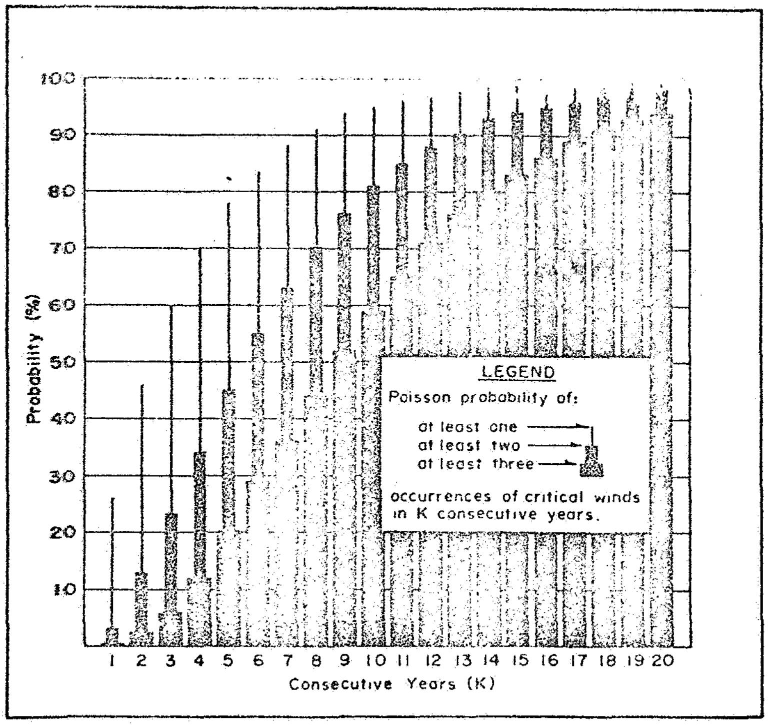



Figure 6. - Probability of critical wind occurrence at MSC over extended time periods.

Figure 7 stratifies the 25 cases into calendar months and suggests that the peak of the tropical cyclone season at MSC can be expected somewhere near the second half of August. Intraseasonal variations are shown in greater detail in figure 8. The frequencies in this figure are based on a 3-week moving average of the daily critical wind occurrence at MSC over the period of record. For example, the percentage frequency read as 0.35 on August 15 indicates that between August 5 and 25, 1886 through 1967 (a total of 1,722 days), critical winds were observed at MSC on six days (.0035 X 1,722 = 6).

More than likely, most of the intraseasonal variations shown on figure 8 can be attributed to chance occurrence owing to the relatively short period of record rather than to actual physical atmospheric processes. Nevertheless, the dip to a minimal value around July 10 is probably real and reflects the overall deficiency in tropical cyclone formation in mid-July. The broad peak in August comes well ahead of the overall September maximum as shown in Cry (1965). This is because most of the September storms which produced the September maximum shown in Cry's data do not penetrate into the Gulf of Mexico but rather recurve northward in the Atlantic. Indeed, as shown in figure 3, only two of the 25 storms were recorded as having originated east of the Antilles. One of these was the devastating 1900 "Galveston" storm.

The earliest seasonal occurrence of critical winds at MSC subsequent to the year 1885 was June 16 (1888) and the latest was October 6 (1895). Most of the October storms which form in

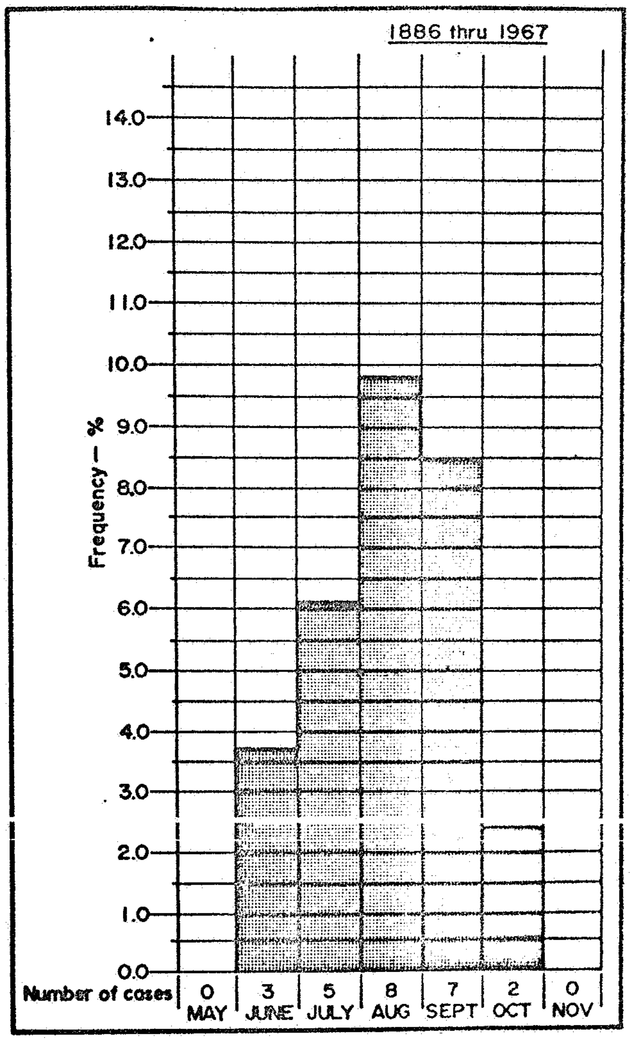



Figure 7. - Percent frequency of critical wind occurrence by month.

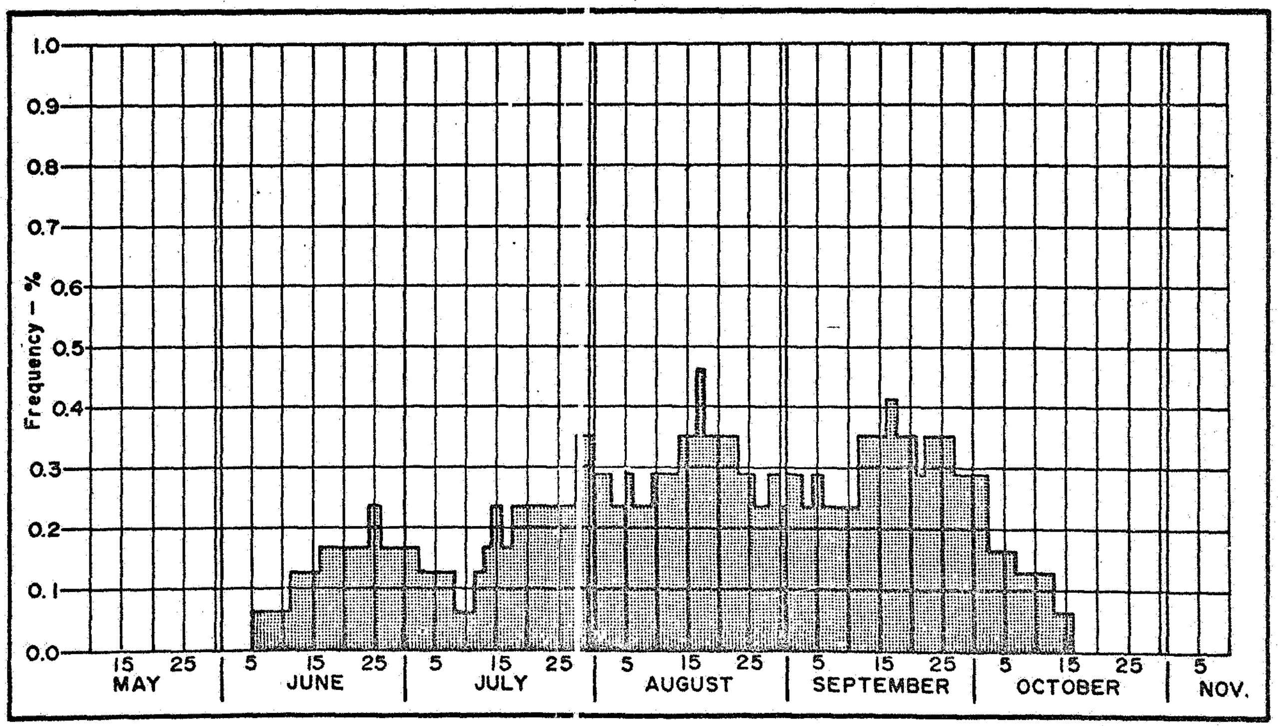



Figure 8. - Percent frequency of critical wind occurrence on any given date, May 15 through November 5, based on 3-week moving average.

the western Caribbean do not penetrate into the Gulf of Mexico as far west as MSC but, as shown in the bottom panel of figure 9, have a strong northerly and northeasterly component after formation. The arrows on figure 9 represent the resultant (mean vector) storm direction of motion (plotted to the nearest whole degree true) for early, mid-, and late season storms for the period of record 1886 through 1968. The isolines on the figure represent "constancy" of storm direction of movement. This value is obtained by dividing the mean translation vector speed of all storms passing through a given box by their mean scalar speed. The numerical value of constancy as defined here is between 0 and 1, zero indicating that storms moved through a box from virtually any direction, and 1 indicating that the direction of motion was always exactly as indicated by the arrow. Since the constancy value is not linear, it can be used in the relative sense only, i.e., a forecast based on a constancy value of 0.8 can be expected to be more reliable than one based on a value of 0.4 but not necessarily "twice" as reliable. The data on figure 9 were derived from Hope and Neumann (1969).

Problem Area 2

In the previous section, probabilities of MSC observing critical winds any given year, years, or portions of years were computed without regard to any current tropical cyclone synoptic situation. These probabilities are nonconditional and intended to be used for longer-range climatological estimates. Once a tropical cyclone is in existence, its location, speed, and direction of motion will alter these climatological probabilities considerably. For example, figure 8 indicates that the probability of MSC experiencing critical winds on any given day is always less than one-half percent. Obviously, if a Gulf of Mexico tropical cyclone is moving directly towards MSC, the probability of observing critical winds approaches 100 percent. The purpose of this section is to estimate the storm strike probabilities from an existing tropical cyclone, given the storm's initial position and point of origin.

In the Cape Kennedy study, 16 out of the 23 (or 70 percent) of the storms that approached Cape Kennedy from the southeast were in existence for 5 or more days before affecting the site. At MSC on the other hand, only nine out of the entire 25 storm data sample, or 36 percent, existed for at least 5 days before producing critical winds. Obviously, this seriously restricts the length of time prior to storm strike for which probabilities can be calculated. As it turns out, depending on the storm's point of origin, the probabilities could be computed out to 5 days (120 hours) or 4 days (96 hours) in advance, depending on the source region. Since actual tropical cyclone forecasts are given for periods up to 72 hours, this means that only 1 or 2 days' advance probabilities can be given over the 72-hour regular forecast. Storm strike probabilities for periods of 72 hours or less can and should be based on the forecasts themselves, and studies are available for this purpose. A list of these studies is given on page 61 of Hope and Neumann (1968).

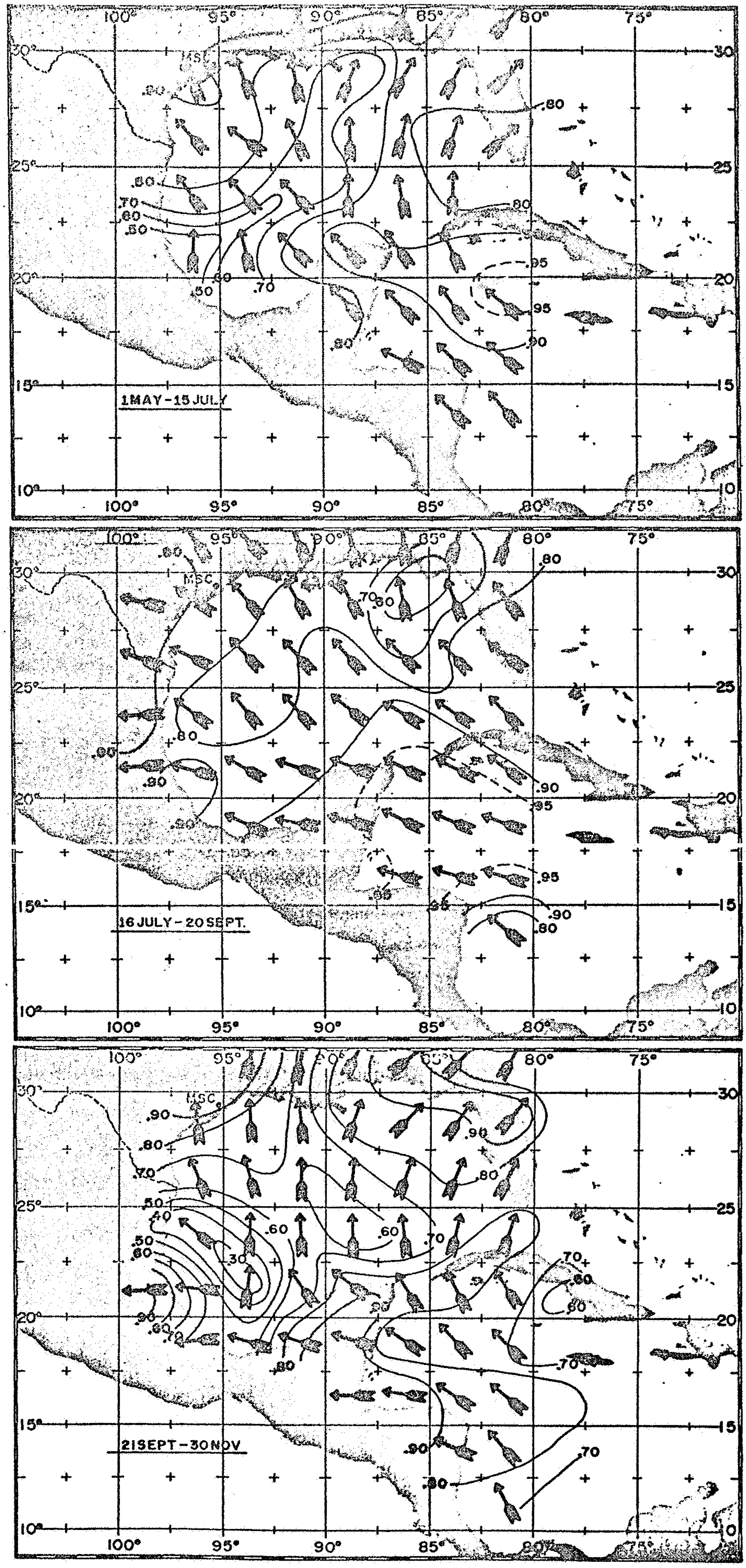



Figure 9. - Resultant storm direction with isolines of direction "constancy" (K) for early, middle and late season storms. $(0 \leqslant K \leqslant 1)$

## 4. SOURCE REGION OF STORMS

Before proceeding further, it is necessary to determine the geographic source regions of the storms which eventually affected MSC. As will be shown later, this determination is necessary in order to insure that an assumption of bivariate normalcy of the two components of storm positions is not violated.

The tracks of the 25 storms which affected MSC were shown on figure 3. For the same period of record, figure 10 shows the overall frequency of tropical storms or hurricanes by 2½-degree latitude-longitude boxes, whether or not they affected MSC. The latter figure shows a well-defined storm track oriented generally SE to NW through the Yucatan Channel (between western Cuba and the Yucatan Peninsula), with the storm frequency diminishing rapidly both northeastward and southwestward of the Channel. Figure 3, on the other hand, does not show a concentration

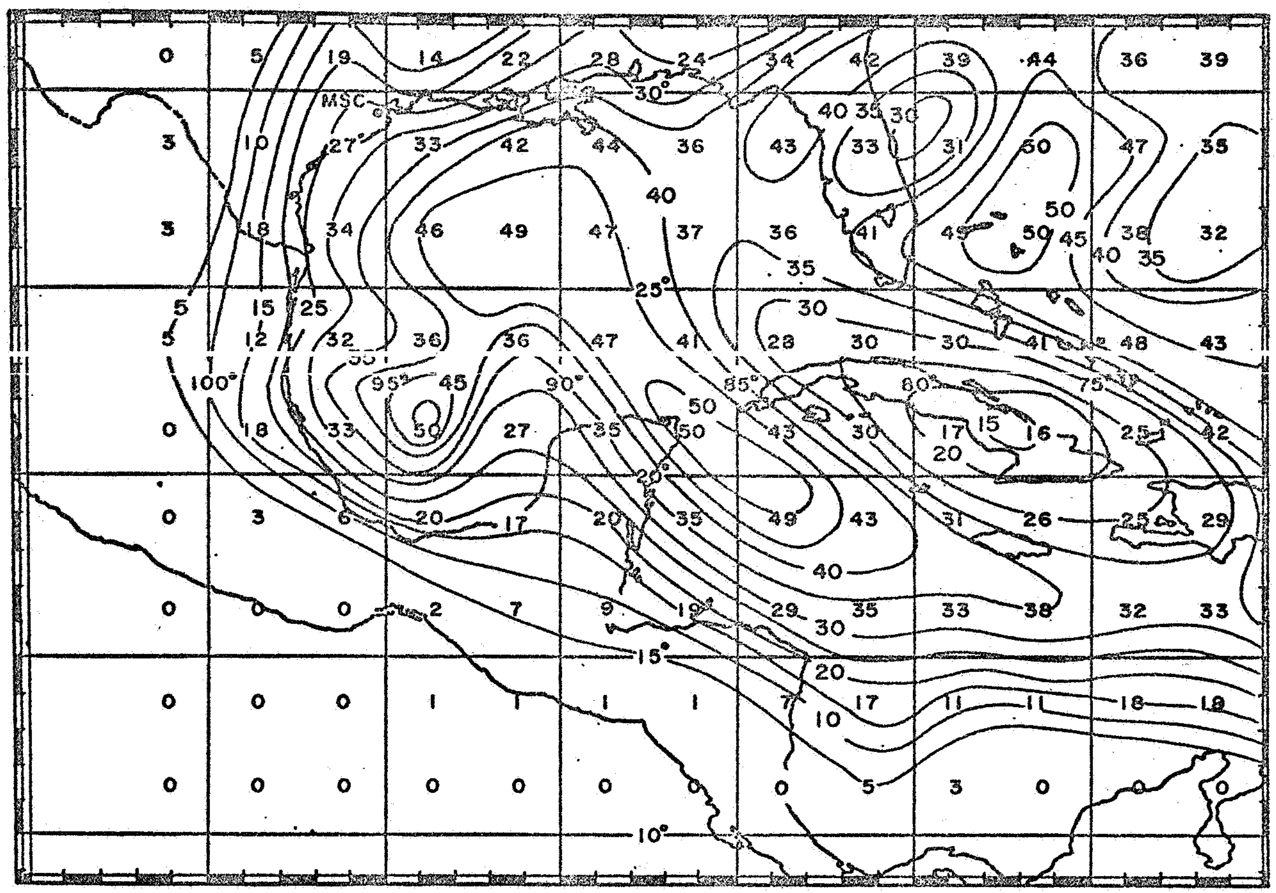



Figure 10. - Number of tropical storms or hurricanes passing through each 2½-degree latitude-longitude box June 15 through October 6, 1886-1967.

of storms passing through the Channel en route to MSC, but rather shows a fairly linear scatter all the way from Florida southwestward to Central America. In the relative sense, then, there is a disproportionate share of storms which eventually affect MSC without first moving through the Yucatan Channel. This suggests two broad storm tracks: one to the northeast and one to the southwest of the main climatological track. One group of storms, hereinafter referred to as category 1 storms, are those which formed in the northeastern Gulf of Mexico or the Atlantic Ocean. The second group of storms, hereinafter referred to as category 2 storms, are those which formed in the western Caribbean or the southwestern Gulf of Mexico. A line extending southeastward from MSC through 0° N., 60° W. was used as the actual dividing line between category 1 and category 2. Even though a storm crossed over the line, its categorical assignment was based on its initially reported position as given in the computer card deck, regardless of storm intensity. Individual storm categorical assignments were given in table 1. Of the 25 tropical cyclones composing the data sample, 11 were category 1 and the remaining 14 were category 2.

The geographical breakdown into category 1 and category 2 storms does not conform to any convenient time breakdown. Most category 1 storms formed in August or early September, but there were a few cases in the earlier months. Category 2 storms formed in each month June through October, without a well-defined modal value.

## 5. ELLIPTICAL DISTRIBUTIONS

The locations of all tropical cyclone centers calculated to have produced critical winds at MSC were plotted at 24-hour intervals, beginning with the time of onset of these winds (T-0) and working backward 6 days (T-144) or to the origin of the storm if it existed less than 6 days. Category 1 storms were plotted separately from category 2 storms.

After plotting these positions, equiprobability ellipses were computed from the distribution of the storm center locations for each day prior to affecting MSC for each of the two groups of storms, assuming a bivariate normal distribution of the latitude and longitude components. Hope and Neumann (1968) established that this assumption was reasonable providing storms fitted to an ellipse were moving along a reasonably common track. If the storm tracks are not treated separately, fitting the data to an ellipse will artificially create a storm track where one does not exist. In the case we are dealing with here, this would have created the main storm track through the Yucatan Channel where, as discussed in the previous subsection, one does not exist. These storm locations and the computed ellipses at T-0 for both category 1 and category 2 storms are shown in figure 11. The storm locations and computed ellipses for category 1 storms at 24-hourly intervals starting at T-24 are shown in figures 12 and 13, and those for

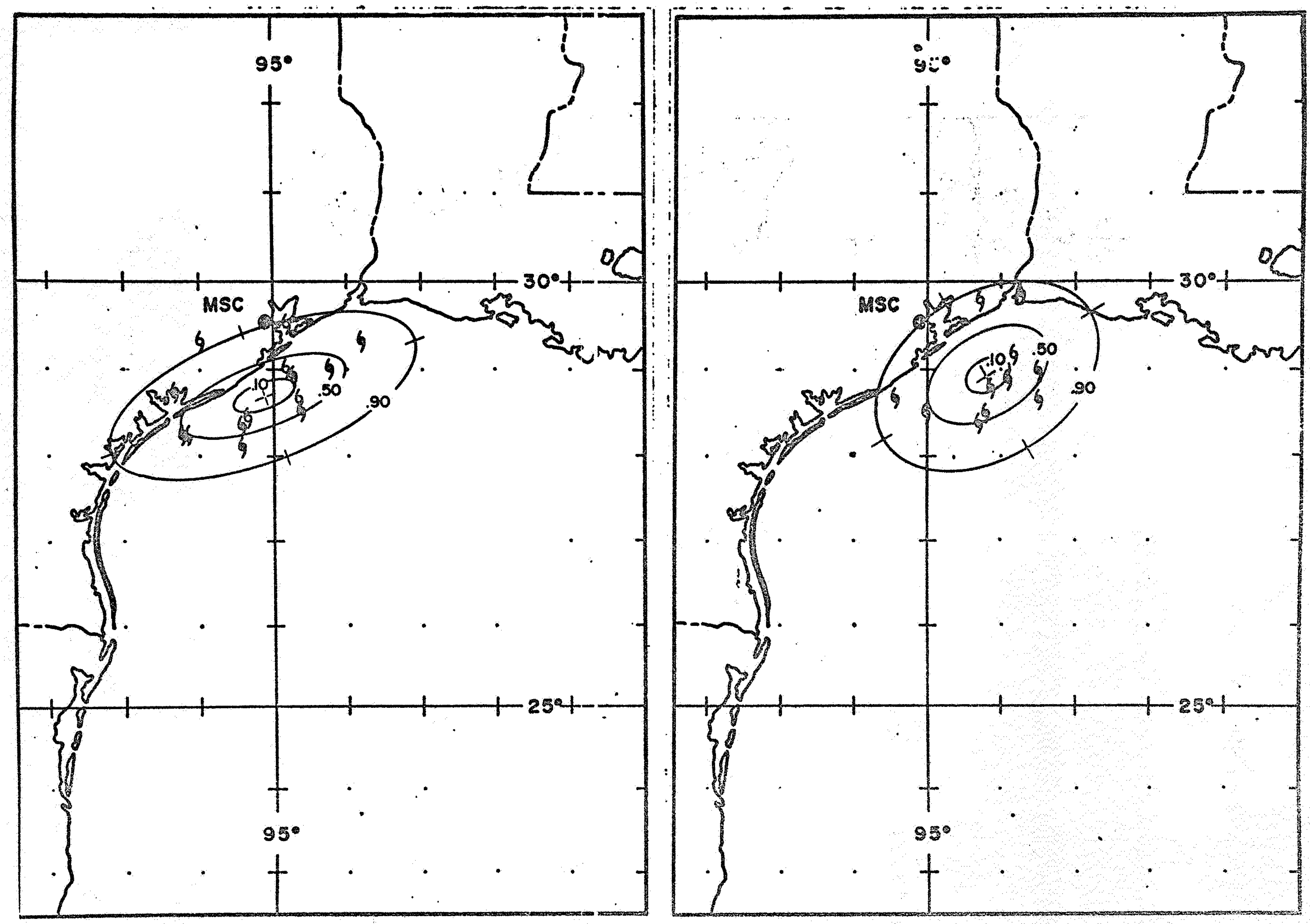



Figure 11. - Probability ellipses of the distribution of tropical storms or hurricanes, 1886-1967, when they first produced critical winds at MSC: Left, category 2 storms; right, category 1 storms.

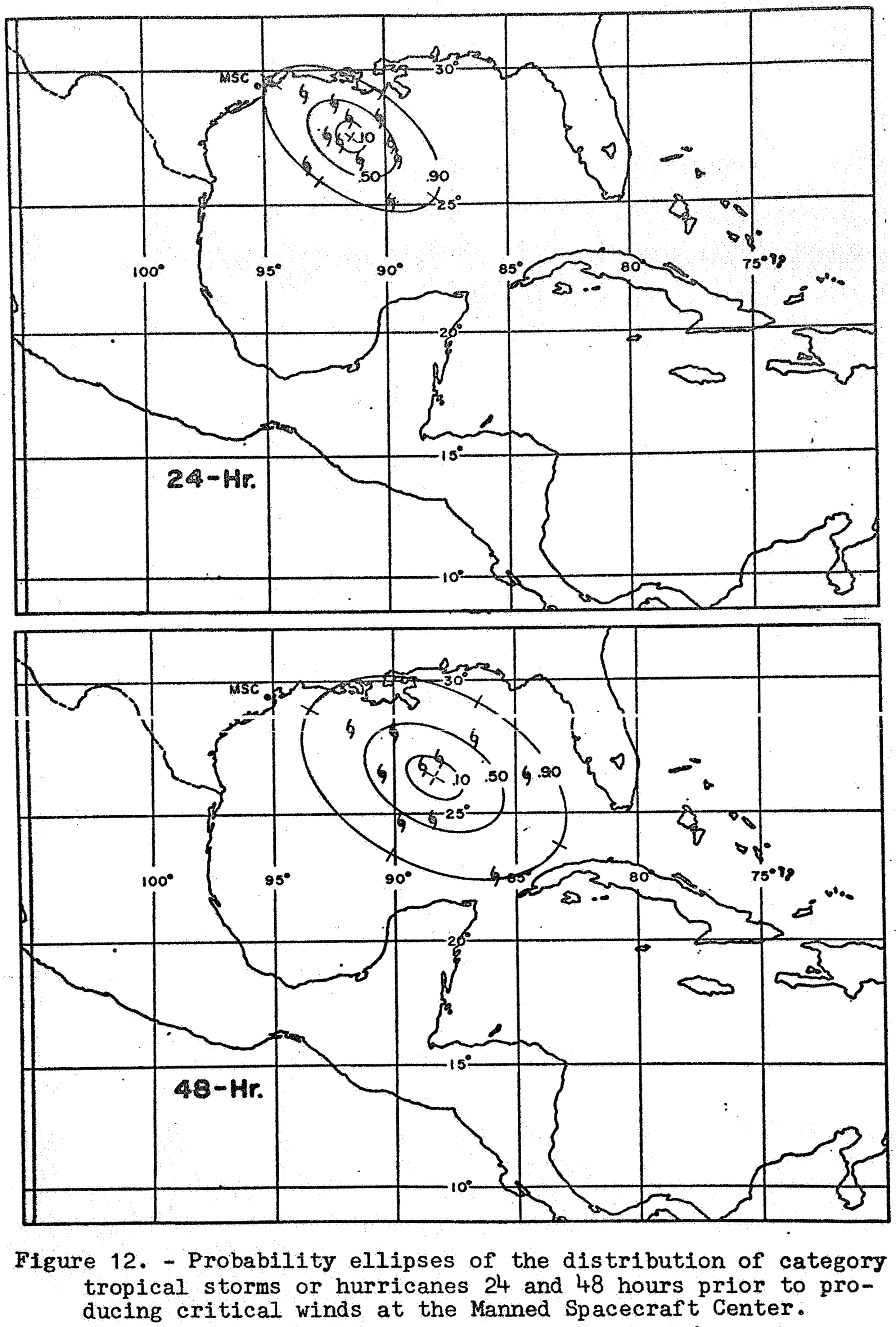
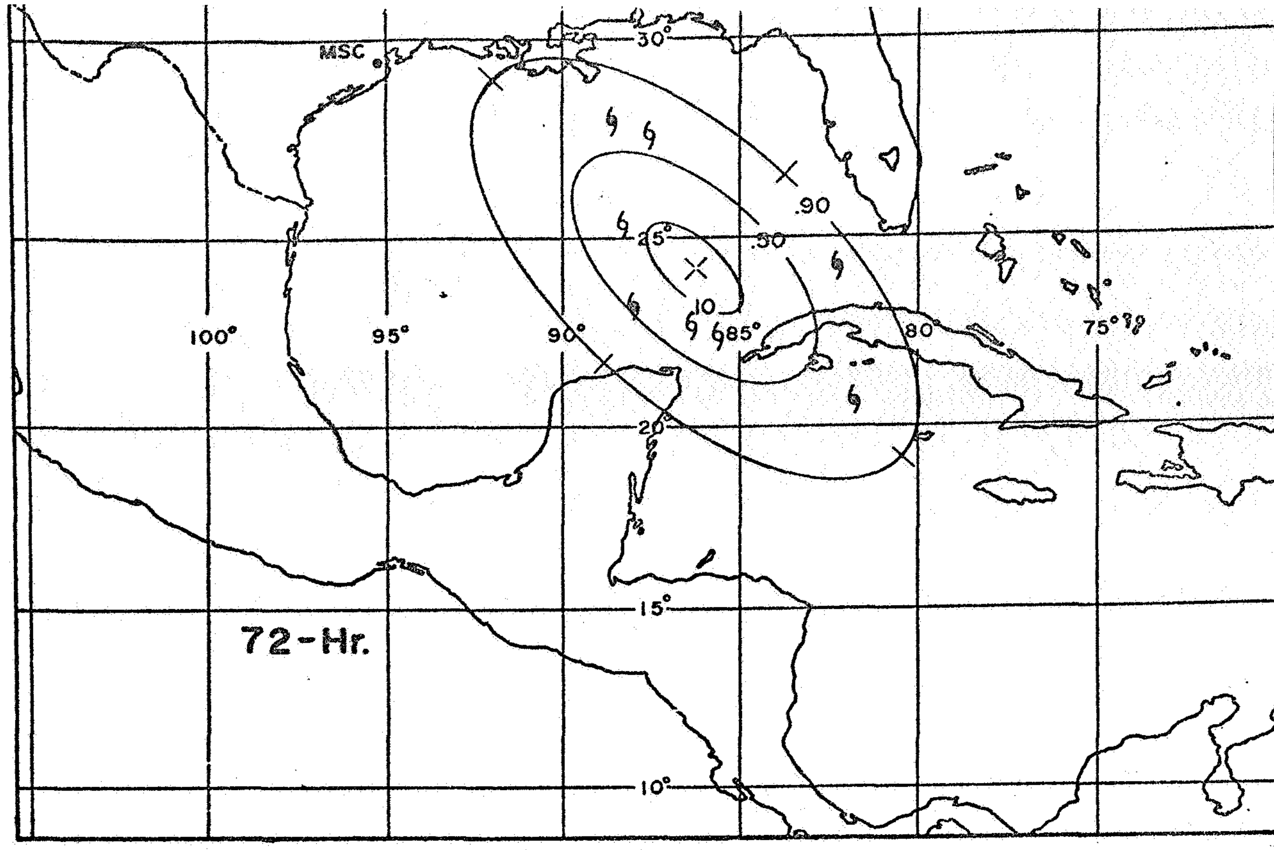



Figure 12. - Probability ellipses of the distribution of category 1 tropical storms or hurricanes 24 and 48 hours prior to producing critical winds at the Manned Spacecraft Center.



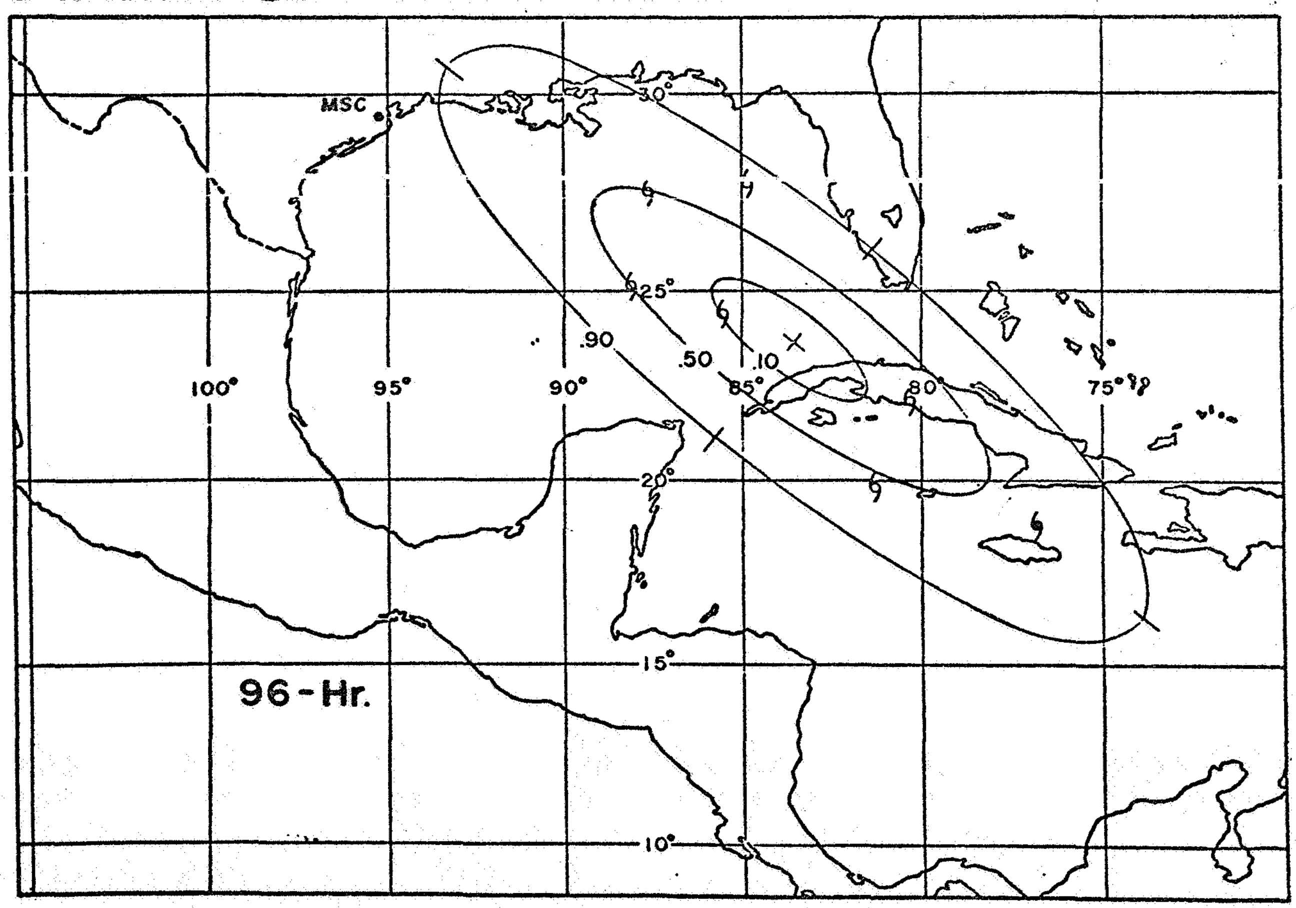




Figure 13. - Probability ellipses of the distribution of category 1 tropical storms or hurricanes 72 and 96 hours prior to producing critical winds at the Manned Spacecraft Center.

category 2 storms are shown in figures 14, 15, and 16. It was not possible to construct a 120-hour ellipse for the category 1 group because the size of the ellipse became unmanageably and probably unrealistically large. In the figures, storm symbols with open circles are either tropical storms or depressions and those with darkened centers are hurricanes.

These ellipses depict the theoretical distribution of storms that would be initially producing critical winds at MSC in exactly the number of hours indicated. Elliptical probability rings have been drawn for probabilities of 0.10, 0.50, and 0.90. Over a long period of record, 90 percent of the storm centers should lie within the 0.90 ring, 40 percent between the 0.50 and 0.90 ring, etc. The most recent comprehensive publication that deals with this subject that has come to the attention of the author is by Groenewould, et al. (1967).

A rather thorough treatment of the mathematical logic behind these ellipses was given by Hope and Neumann (1968) and need not be repeated here. For a given set of storm positions, each envelope of ellipses is uniquely defined by five derived parameters. These are (1) the mean latitude of the storms, (2) the mean longitude, (3) the standard deviation of the latitudes from the mean latitude, (4) the standard deviation of the longitudes from the mean longitude, (5) the linear correlation coefficient between the latitude and the longitude components. If the standard deviations are equal and the correlation coefficient is zero, then the ellipse reduces to a circle.

The centroids (mean latitude and longitude components) of the two groups of storms are plotted at 24-hour intervals (fig. 17) as they approached the Manned Spacecraft Center. Category 1 storms were plotted out to T-144 hours and category 2 storms to T-120 hours. The restriction here was the number of cases diminishing to less than five. The line connecting the centroids can be considered "the most critical tracks" insofar as MSC is concerned. This figure is particularly useful for estimating the time a storm will be initially producing critical winds at the site. For example, if a storm located at 19° N., 86° W. were to produce critical winds at MSC, it would be expected to do so in about 110 hours.

This estimate is obtained by constructing a line perpendicular to the appropriate centroid track through the storm position and reading the interpolated time.

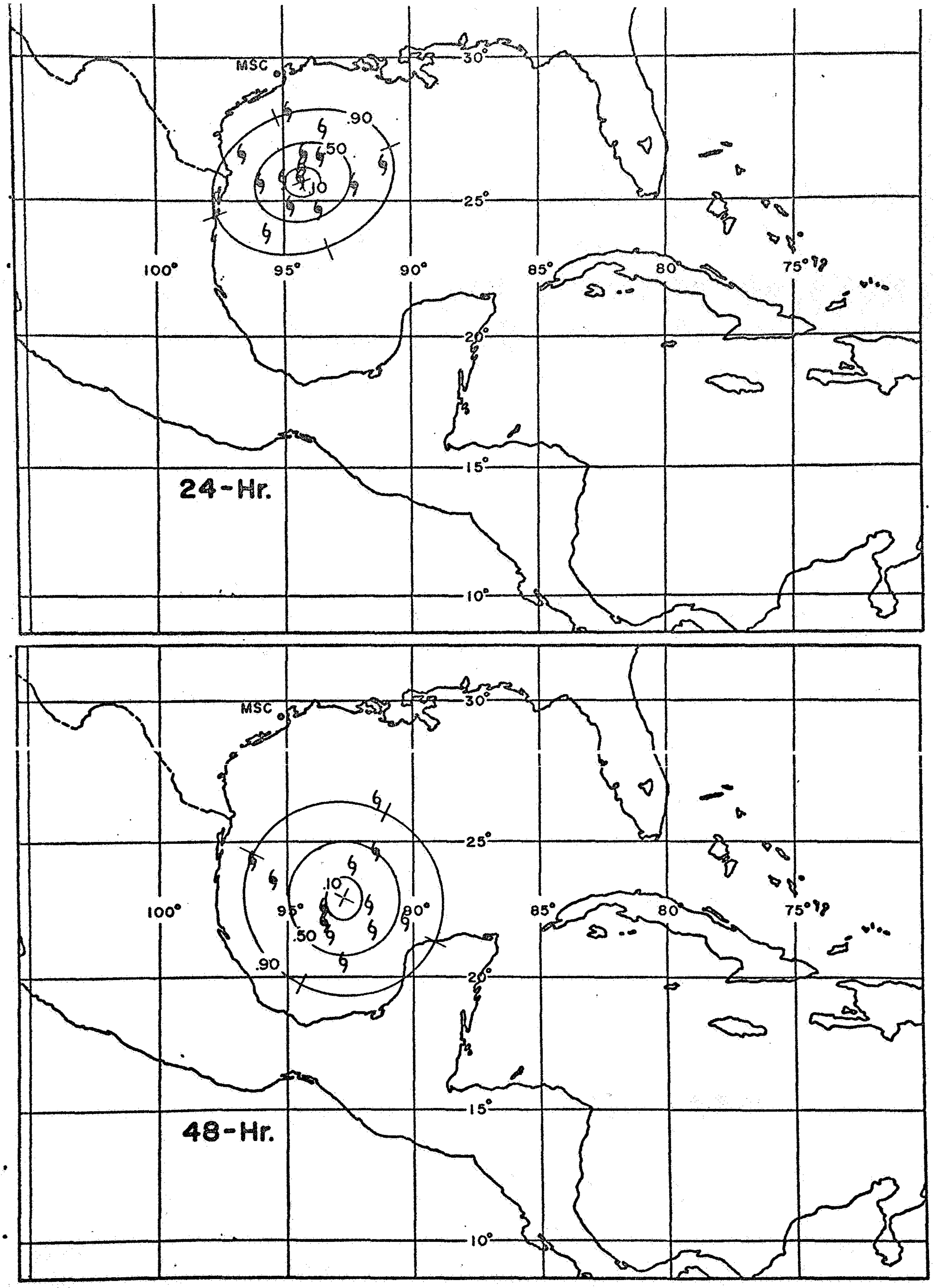



Figure 14. - Probability ellipses of the distribution of category 2 tropical storms or hurricanes 24 and 48 hours prior to producing critical winds at the Manned Spacecraft Center.

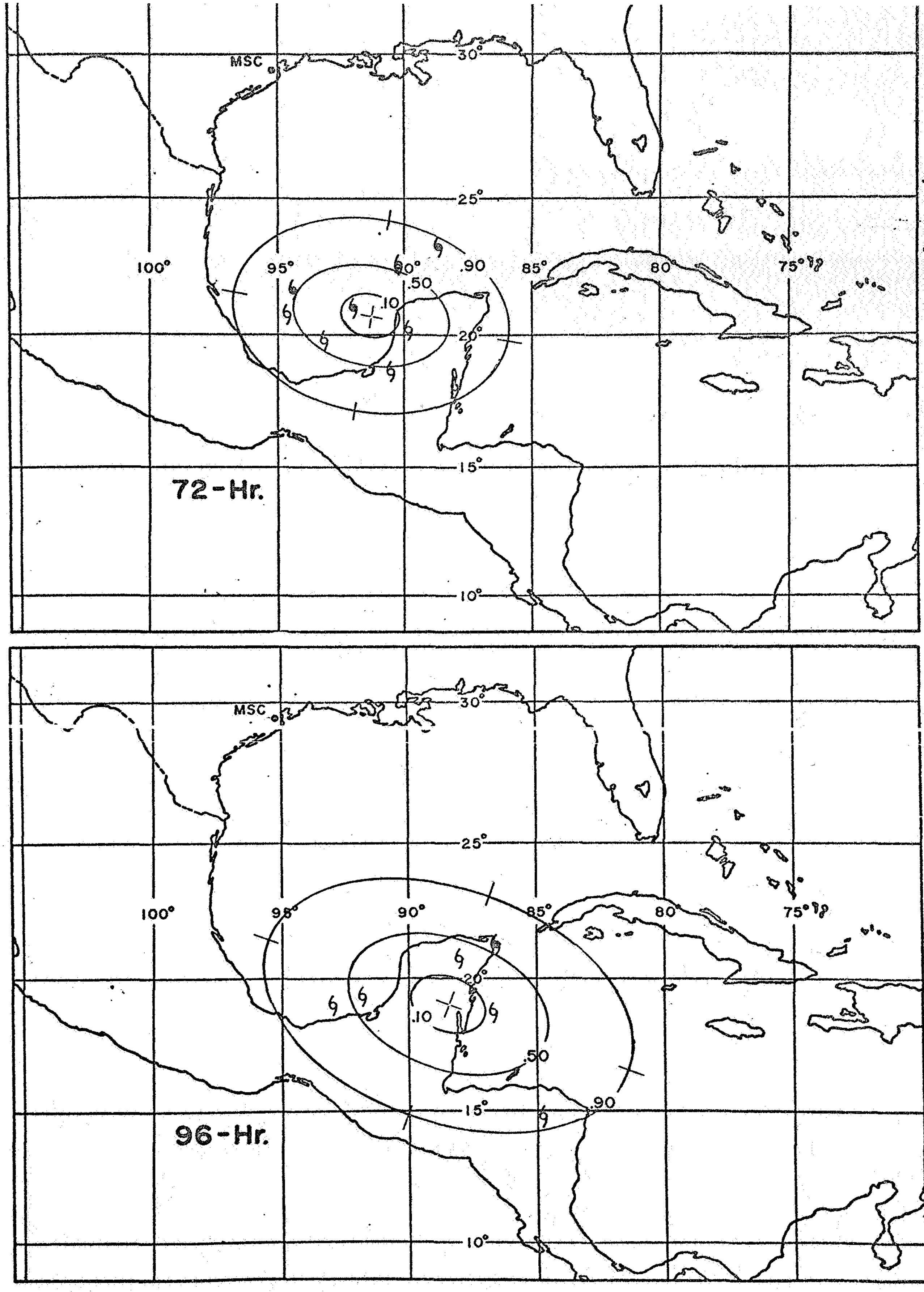



Figure 15. - Probability ellipses of the distribution of category 2 tropical storms or hurricanes 72 and 96 hours prior to producing critical winds at the Manned Spacecraft Center.

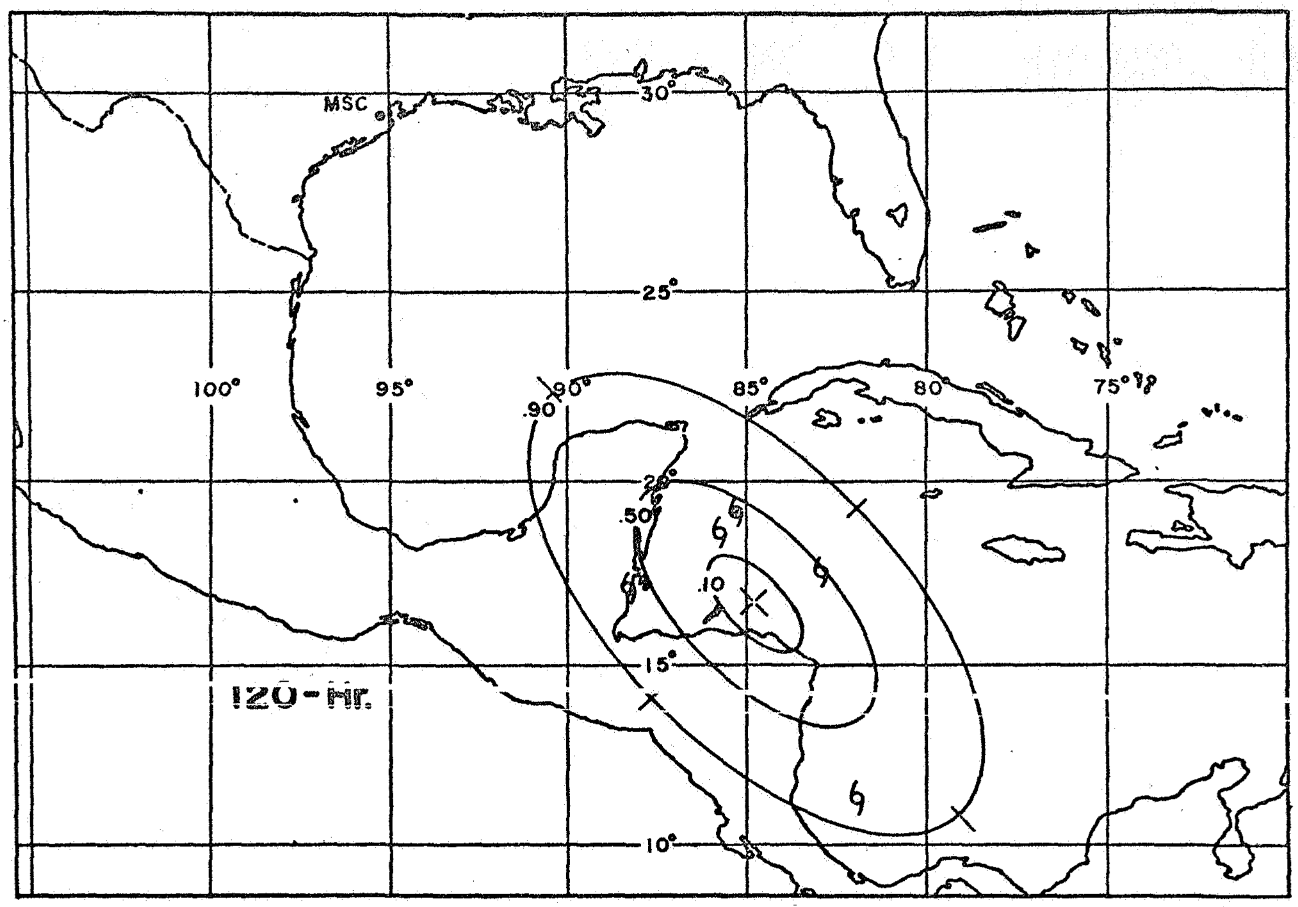



Figure 16. - Probability ellipses of the distribution of category 2 tropical storms or hurricanes 120 hours prior to producing critical winds at the Manned Spacecraft Center.

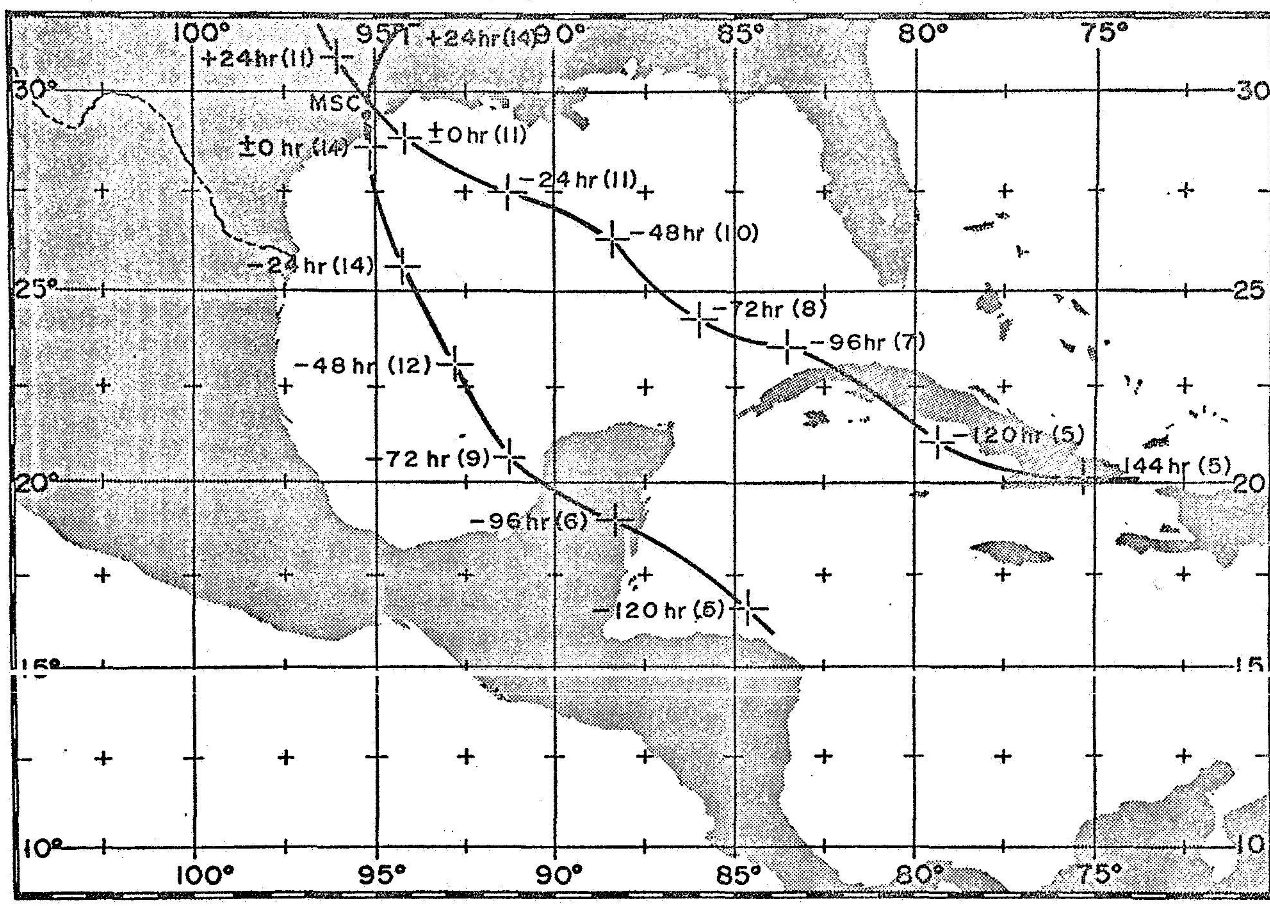



Figure 17. - Location of storm center distribution centroids at specified 24-hour time intervals before and after producing critical winds at the Manned Spacecraft Center. Numbers in parentheses are number of cases from which ellipses were computed.

## 6. PROBABILITY COMPUTATIONS AT FIXED TIMES

The frequency of an event, F(E), is given by

$$F(E) = C/N_t \qquad 0 \leq F(E) \leq 1 \qquad (4)$$

where C represents the number of occurrences of the event, and $N_t$ represents the number of possible occurrences. If $N_t$ becomes sufficiently large, then F(E) becomes a good estimate of the probability of E.

The ellipses given in figures 11 through 16 merely specify the theoretical distribution of storms that would be affecting MSC in exactly the number of hours indicated. In order to arrive at a definite probability, C in equation (4) must represent the number of storms passing over some geometrical area within the total elliptical area and then affecting MSC, while $N_t$ in equation (4) must be replaced by the total number of storms moving over the same geometrical area during the same period of record, whether or not they affected MSC. As discussed by Hope and Neumann (1969), a convenient area to work with is a 2½-degree latitude-longitude box. Five-degree boxes are too large, because significant nonlinear variations can occur within the box. Any box size less than 2½ degrees is too small because the number of cases is seriously curtailed. Figure 10 presented an analysis of the total storm count by 2½-degree boxes. The data presented in figure 10 cannot be used as $N_t$ in equation (4), however, since the source regions of the storms were not considered. Figures 18 and 19, on the other hand, give the storm count by category 1 and 2 respectively. Figure 18 includes only those storms which initially formed to the right of the dashed line extending almost exactly southeastward from MSC. These are category 1 storms. Storms on figure 18 that appear on the left side of the line initially formed to the right of the line but moved across the line. Similarly, figure 19 includes only initial category 2 storms. As was the case with figure 10, both figures 18 and 19 exclude the tropical depression stages of storms. The sums of any given box total from figures 18 and 19 will, of course, equal the box total given on figure 10.

Summarizing the above, the probability of an existing storm located at the center of a 2½-degree latitude-longitude box affecting the MSC at a fixed time is given by

$$P' = BN/N_t \qquad BN \leq N_t > 0 \qquad (5)$$

where P' = Probability that an existing hurricane or tropical storm will affect MSC at a specific time

N = Actual number of hurricanes or tropical storms of a particular category affecting MSC from which the ellipse was constructed (excluding tropical depressions)

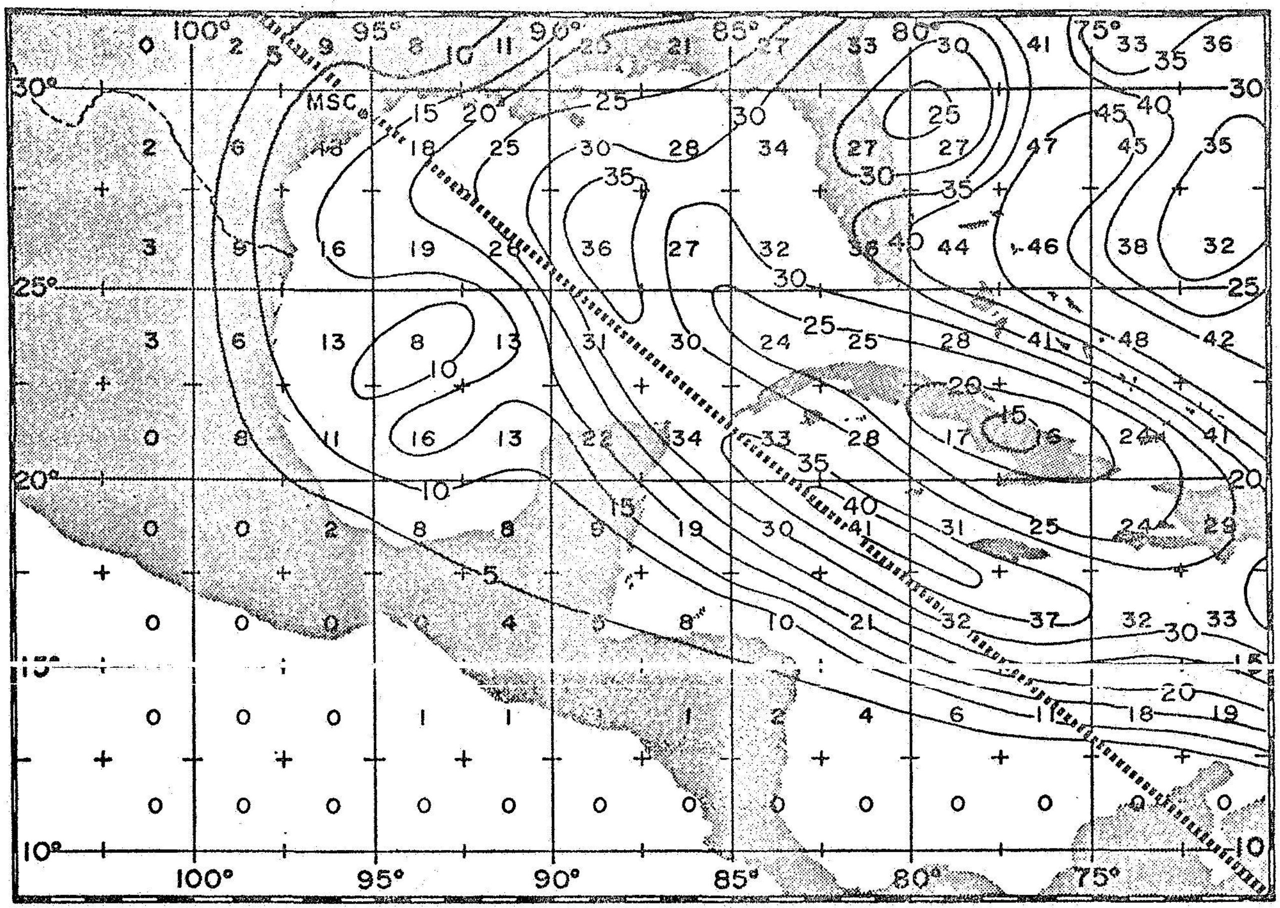



Figure 18. - Number of tropical storms or hurricanes passing through each 2½-degree latitude-longitude box June 15 through October 6, 1886-1967 and having originated to the right of dashed line (category 1 storms).

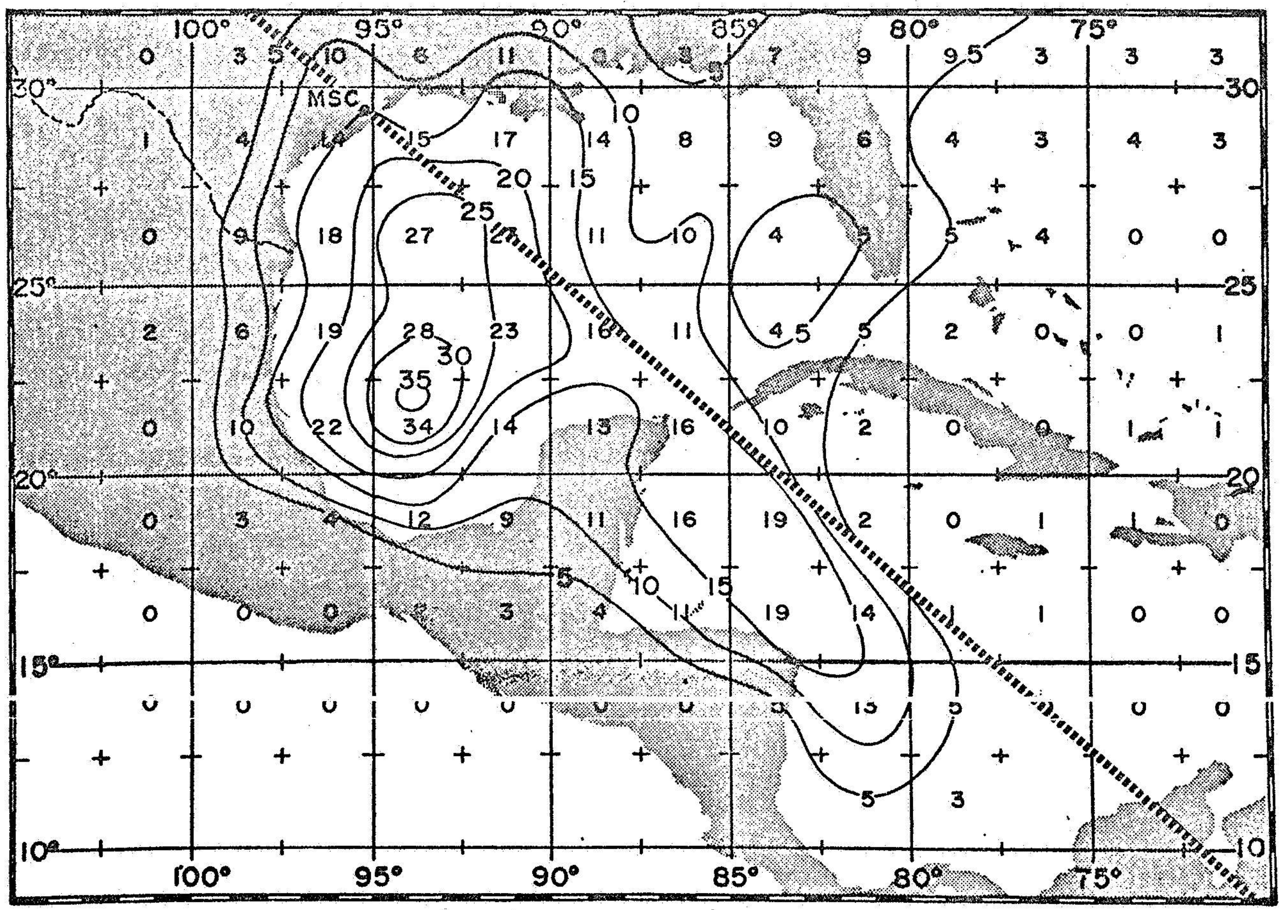



Figure 19. - Number of tropical storms or hurricanes passing through each $2\frac{1}{2}$-degree latitude-longitude box June 15 through October 6, 1886-1967 and having originated to the left of dashed line (category 2 storms).

B = Contribution of the $2\frac{1}{2}$-degree latitude-longitude box to a 99 percent ellipse

$N_t$ = Total number of hurricanes or tropical storms of this given category passing through the $2\frac{1}{2}$-degree latitude-longitude box over the period of record

A sample computation of B is given in the appendix. As an example of a particular computation of P', consider a category 1 storm located at 28.8° N., 91.3° W. What is the probability of this storm producing critical winds at MSC in exactly 48 hours?

N = 9 (Note from figure 17 that there were 10 storms in this group; however, one of these was a depression)

B = .070 (See Appendix)

$N_t$ = 25 (from figure 18)

P' = .070 X 9/25 = .025 or 2.5%

The results of these computations for category 1 storms are given in figures 20 and 21 and for category 2 storms in figures 22 through 24. The analysis on the figures is based on computed values for the center of each $2\frac{1}{2}$-degree latitude-longitude box. Some subjective smoothing of the data was required where the ellipses intersect land masses (mainly over Central America). This was a greater problem in this study than in the Cape Kennedy study where most of the ellipses were over open water. Actually, the assumption of bivariate normalcy does not hold over portions of the ellipses that extend large distances over land masses. The net effect is to force more storms over this land area than actually exist in nature. In equation (5), therefore, $N_t$ approaches zero so that the specified bounding conditions are no longer valid and P' becomes fictitiously large. Hence, some subjective smoothing was required in these areas.

Figures 20 through 24 should be used to estimate the probabilities of MSC observing critical winds in exactly the time period indicated. A category 2 storm located at 25° N., 95° W., for example, has a 16.0 percent chance of initially affecting MSC in exactly 24 hours, a 3.5 percent chance in exactly 48 hours, and less than 1.0 percent in exactly 72 hours. These data were extracted directly from figures 22 and 23. According to figure 17, the maximum probability would be expected in about 27 hours.

## 7. PROBABILITIES OVER EXTENDED TIME INTERVALS

So far, all of the probabilities computed refer only to specific times prior to the onset of critical winds at MSC. Of greater importance operationally is the total probability that an existing storm will affect MSC <u>within</u> a specified period of time rather than <u>at</u> a specified time. In the previous subsection, it

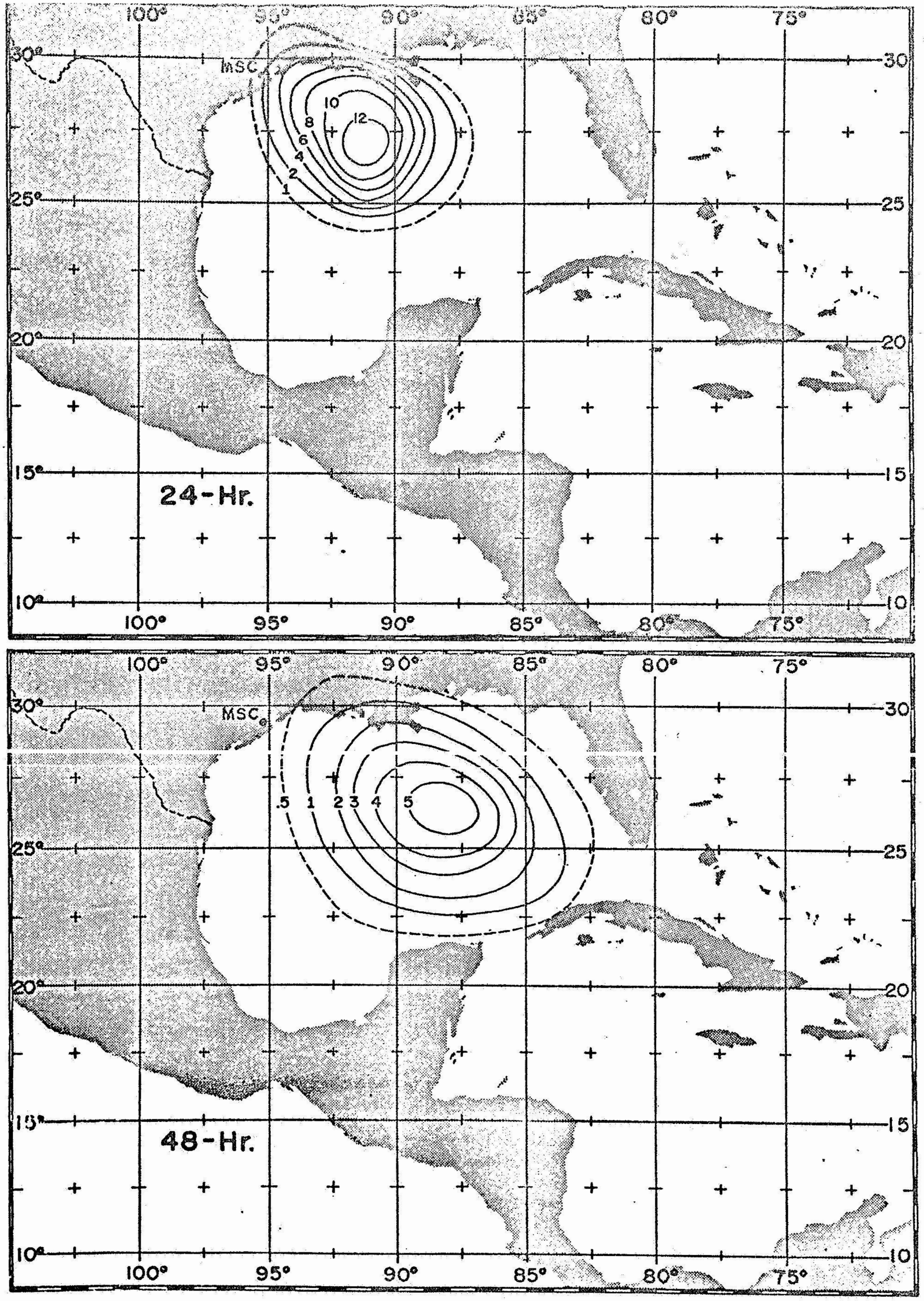



Figure 20. - Percent probability of category 1 tropical storms or hurricanes producing critical winds at the Manned Spacecraft center in exactly 24 and 48 hours.

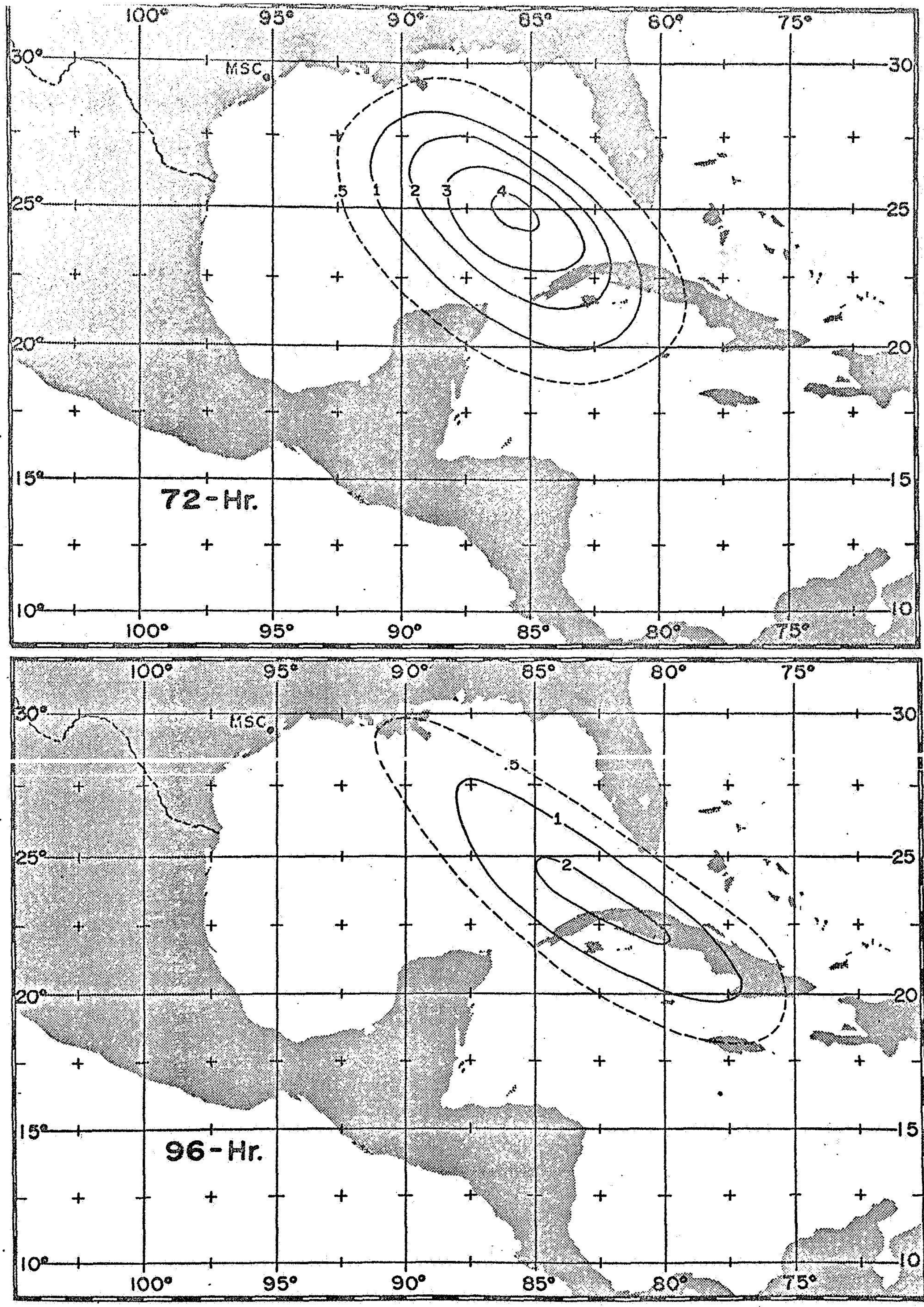



Figure 21. - Percent probability of category 1 tropical storms or hurricanes producing critical winds at the Manned Spacecraft Center in exactly 72 and 96 hours.

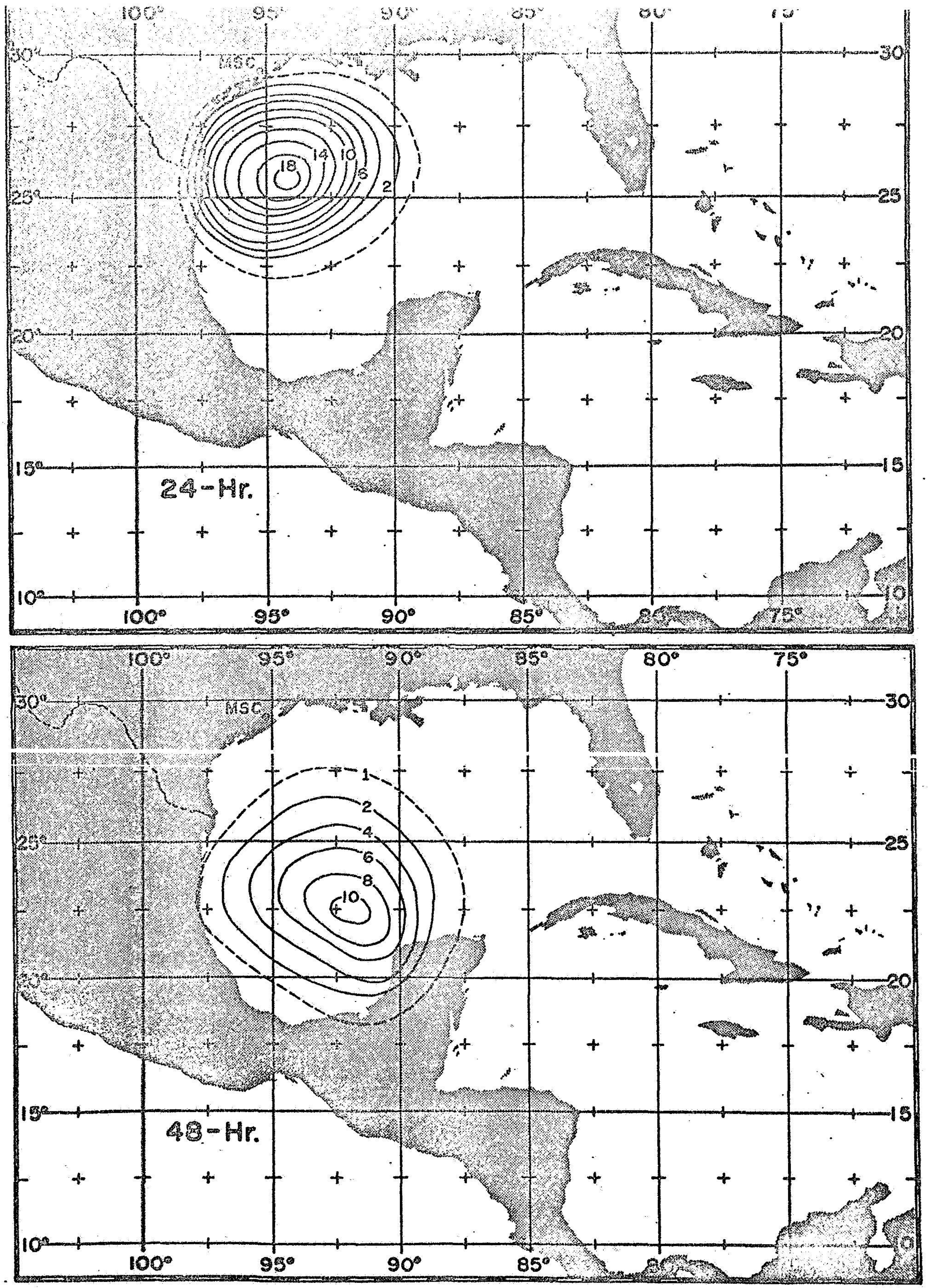



Figure 22. - Percent probability of category 2 tropical storms or hurricanes producing critical winds at the Manned Spacecraft Center in exactly 24 and 48 hours.

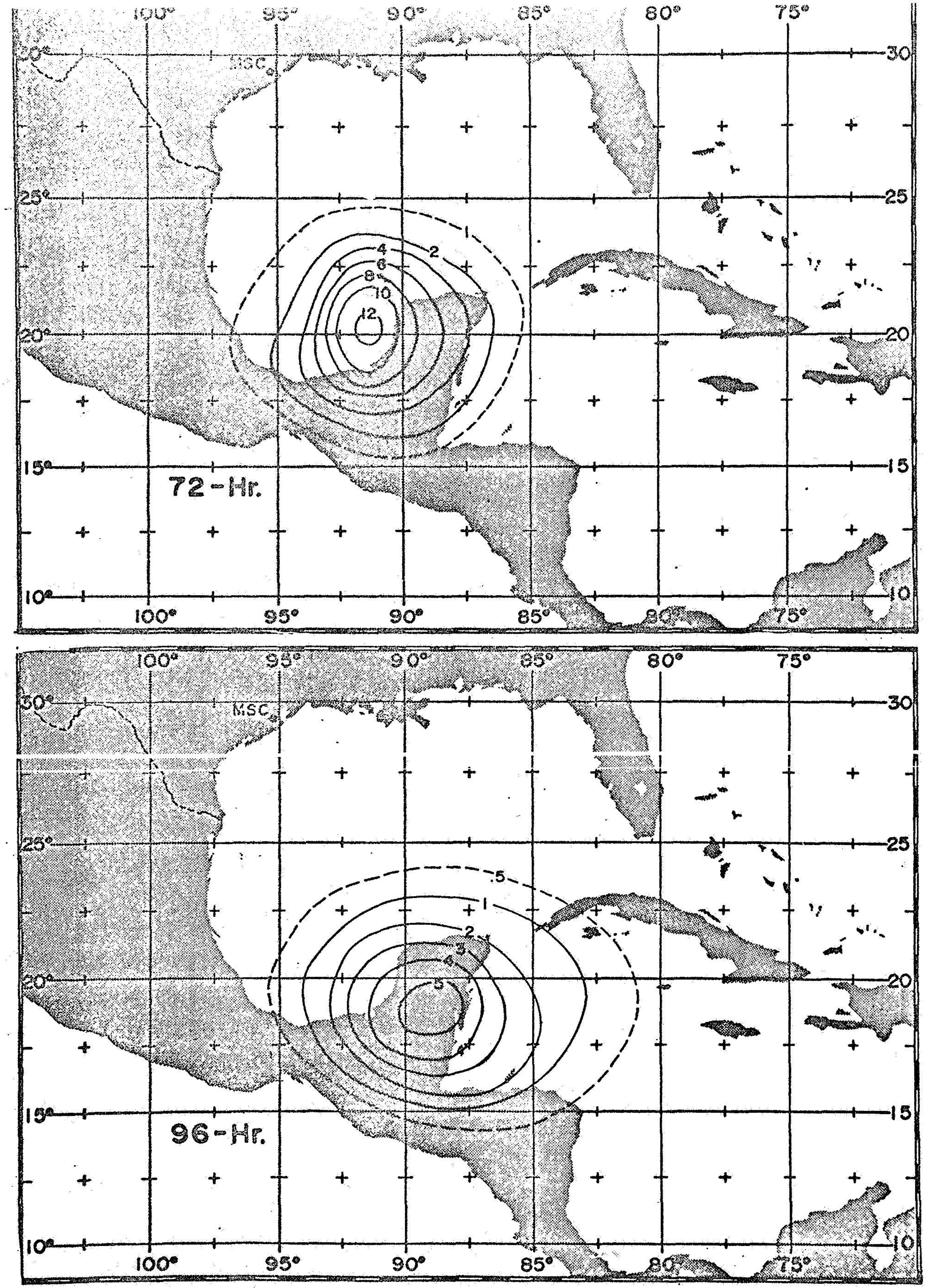



Figure 23. - Percent probability of category 2 tropical storms or hurricanes producing critical winds at the Manned Spacecraft Center in exactly 72 and 96 hours.

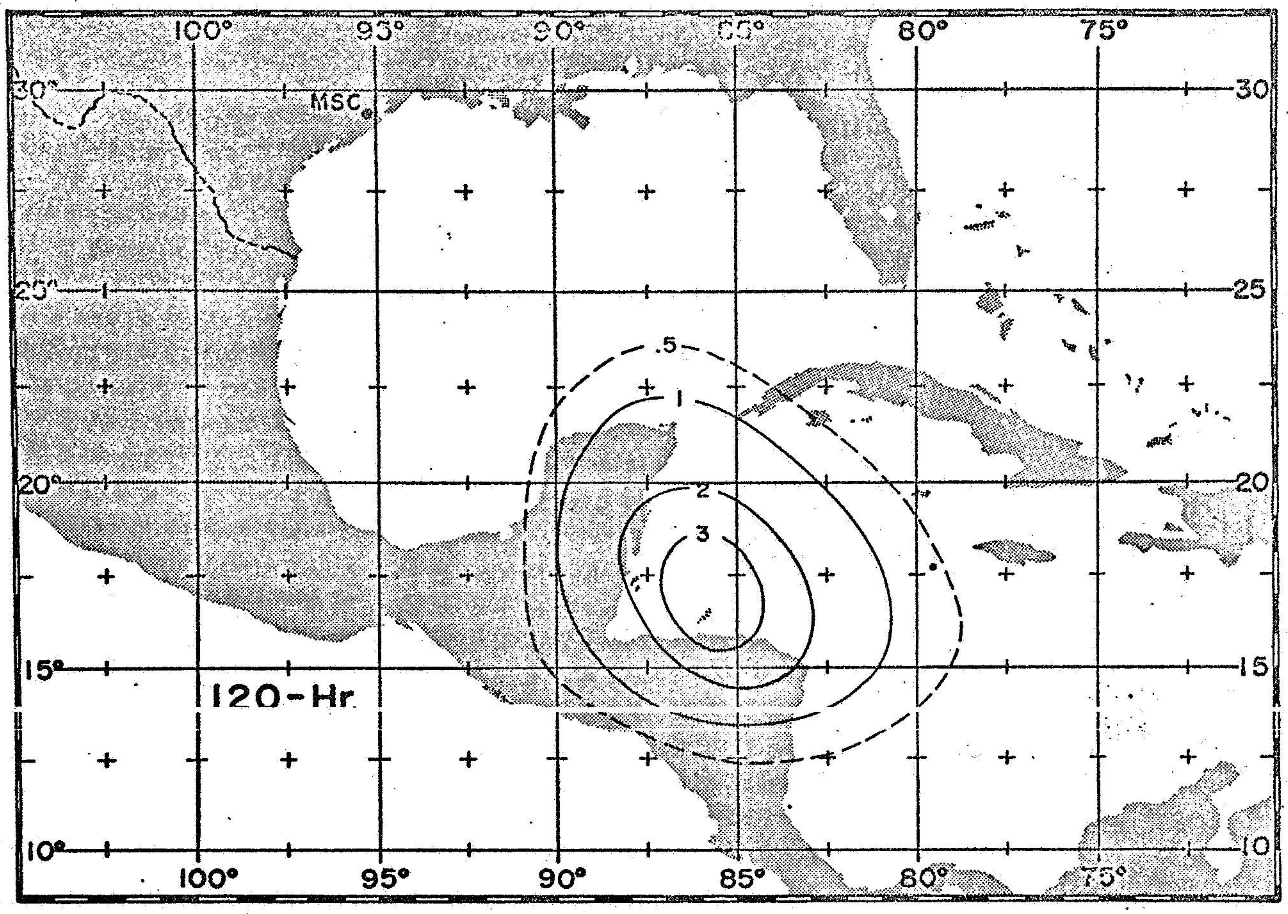



Figure 24. - Percent probability of category 2 tropical storms or hurricane producing critical winds at the Manned Spacecraft Center in exactly 120 hours.

was pointed out that a tropical cyclone located at 25° N., 95° W. has maximum probability of slightly over 16 percent of affecting MSC in exactly 24 hours. The probability at the other 24 intervals was considerably less. The total probability of this storm affecting MSC would, of course, be much greater than 16 percent. Figure 3, for example, indicates that seven storms (numbers 2, 4, 8, 10, 13, and 20) which passed through a 2½-degree latitude-longitude box centered at 25° N., 95° N. went on to produce critical winds at MSC at some future time. Figure 19 indicates that there would be 23 category 2 storms moving through this same 2½-degree box whether or not they eventually affected MSC. The observed frequency of "hits" from this box is therefore 7/23 or about 30 percent, considerably higher than the maximum fixed time computed probability of 16 percent.

The individual 24-hourly probabilities cannot simply be added to obtain the total probability. The sum would still be less than the total probability. In the cumulative process, it is necessary to consider the number of boxes in which the storms affecting MSC were located during each 24-hour period. When these are counted and the sum divided by the number of storms involved, a factor, M, is obtained which can be applied to the fixed-time box probabilities computed from a particular ellipse. Effectively, this adds storms to each box because the number of storms in each box during a given time span is considered, not just the storms in the box at a specified time. This factor M varied between 1.8 and 2.6 depending on the storm category and the probability forecast period.

In summary, the total or cumulative probability within a specified time period is obtained by adding the sum of the probabilities obtained from equation (5) plus a quantity which depends on the number of additional boxes through which a certain group of storms have passed during each applicable 24-hour period. The cumulative probability for a particular 24-hour period starting at T-48 and ending at T-24 can be expressed

$$P_{(48 \rightarrow 24)} = P'_{48} + \left(M_{(48 \rightarrow 24)} - 1\right)\left(\frac{\Delta T P'_{48}}{24}\right) \qquad (6)$$

where $P_{48}$ is the probability at precisely 48 hours, $M(48 \rightarrow 24)$ is the mean number of boxes in which the storms have been during the 24-hour period, and $\Delta T$ is the time in hours between ellipses. If the ellipses are spaced at 24-hour intervals (as they are here), equation (6) becomes

$$P_{(48 \rightarrow 24)} = P'_{48} M_{(48 \rightarrow 24)} \qquad (7)$$

The total or cumulative probability within a specified period of time and for a given box is then

$$P = \left( \sum_{i=1}^{j} P_i M_{(i \rightarrow i-1)} \right) + P_0 \qquad (8)$$

where j is the number of days within which the cumulative probabilities are computed, i refers to a particular day, and $P_0$ refers to the zero-hour probabilities. (Note: Values for $P_0$ are not included in this study but can be calculated from equation 5). The cumulative probabilities of category 1 storms producing critical winds at MSC within 96 hours and computed according to equation (8) are shown on figure 25, and the probabilities of category 2 storms producing critical winds within 96 and 120 hours are shown on figures 26 and 27 respectively. The data presented on these charts were smoothed by averaging the box probabilities at their common intersection, boxes with no entry being counted as zero.

## 8. DISCUSSION

The probabilities computed in this study are most useful for time scales beyond those for which forecasts are issued. Normally, military and NASA installations will have forecasts available for periods extending out to 72 hours. Studies have been made on the accuracy of these forecasts, and some use the forecasts themselves to compute storm strike probabilities. Among these are Tracy (1966), Appleman (1962), U.S. Navy Weather Research Facility (1963), and Veigas, et al. (1959). Although climatological probabilities for periods less than 72 hours are presented in this paper, they are not intended to replace those based on the latest available forecast.

The Cape Kennedy study achieved a further stratification of the storm sample, based on a storm's direction of motion. Obviously, a storm which is moving through a particular 2½-degree latitude-longitude box but heading away from MSC has less probability of affecting MSC than one heading directly up the centroid track. No attempt was made to perform these additional calculations because of an insufficiency of cases. A minimum of 20 storms per category would be required. This deficiency was partially offset by dividing the 25-case storm sample into the two categories. Eliminating category 1 storms when dealing strictly with category 2 storms will eliminate from consideration many of the cases of Atlantic or Caribbean storms moving westward or westnorthwestward across the Gulf of Mexico at almost right angles to the category 2 centroid track. Similarly, when dealing with the category 1 group, those storms moving across the track from the southwest would be largely eliminated. Nevertheless, some subjective

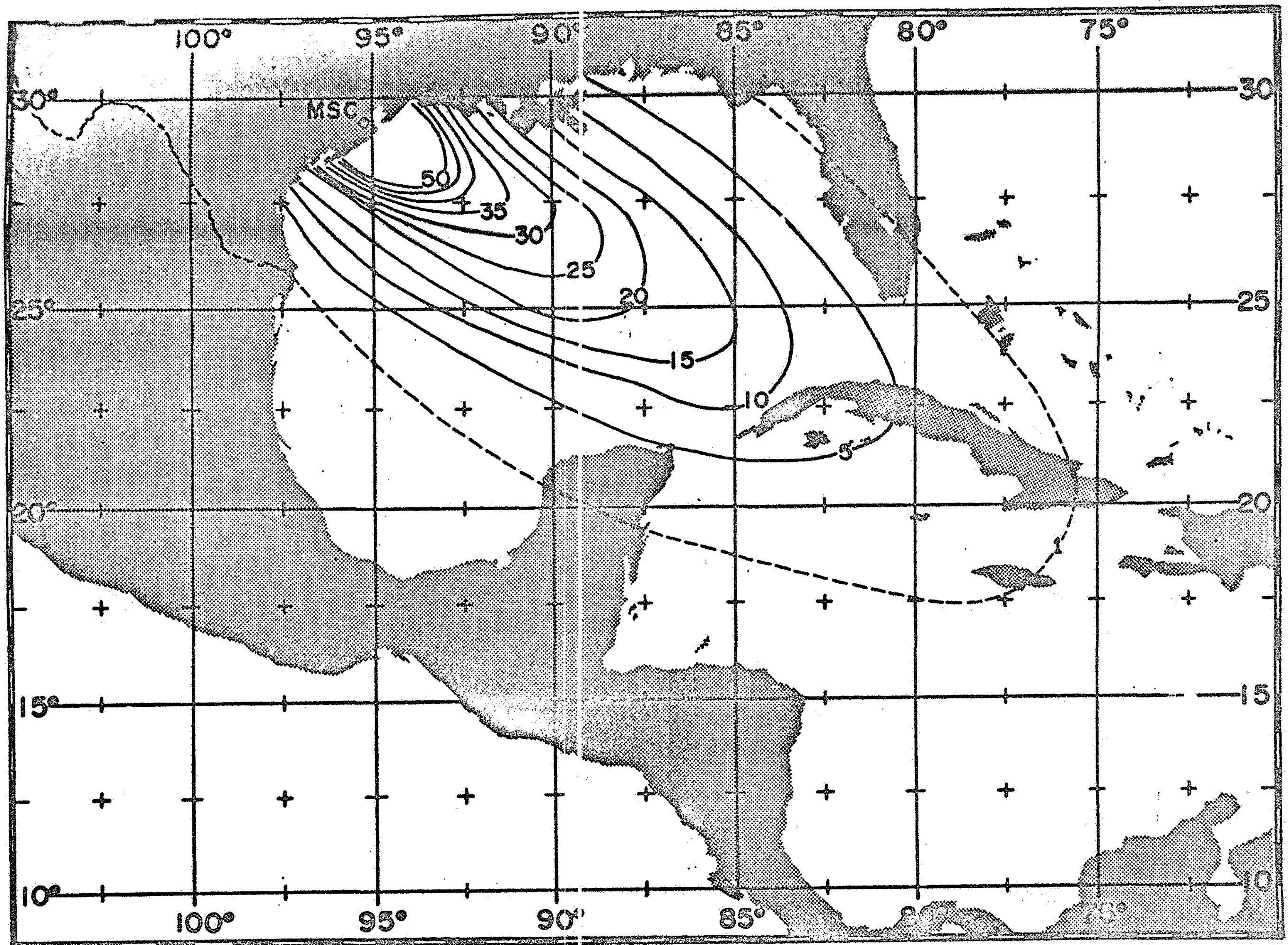



Figure 25. - Cumulative percent probability of an existing category 1 tropical storm or hurricane producing critical winds at the Manned Spacecraft Center within 96 hours.

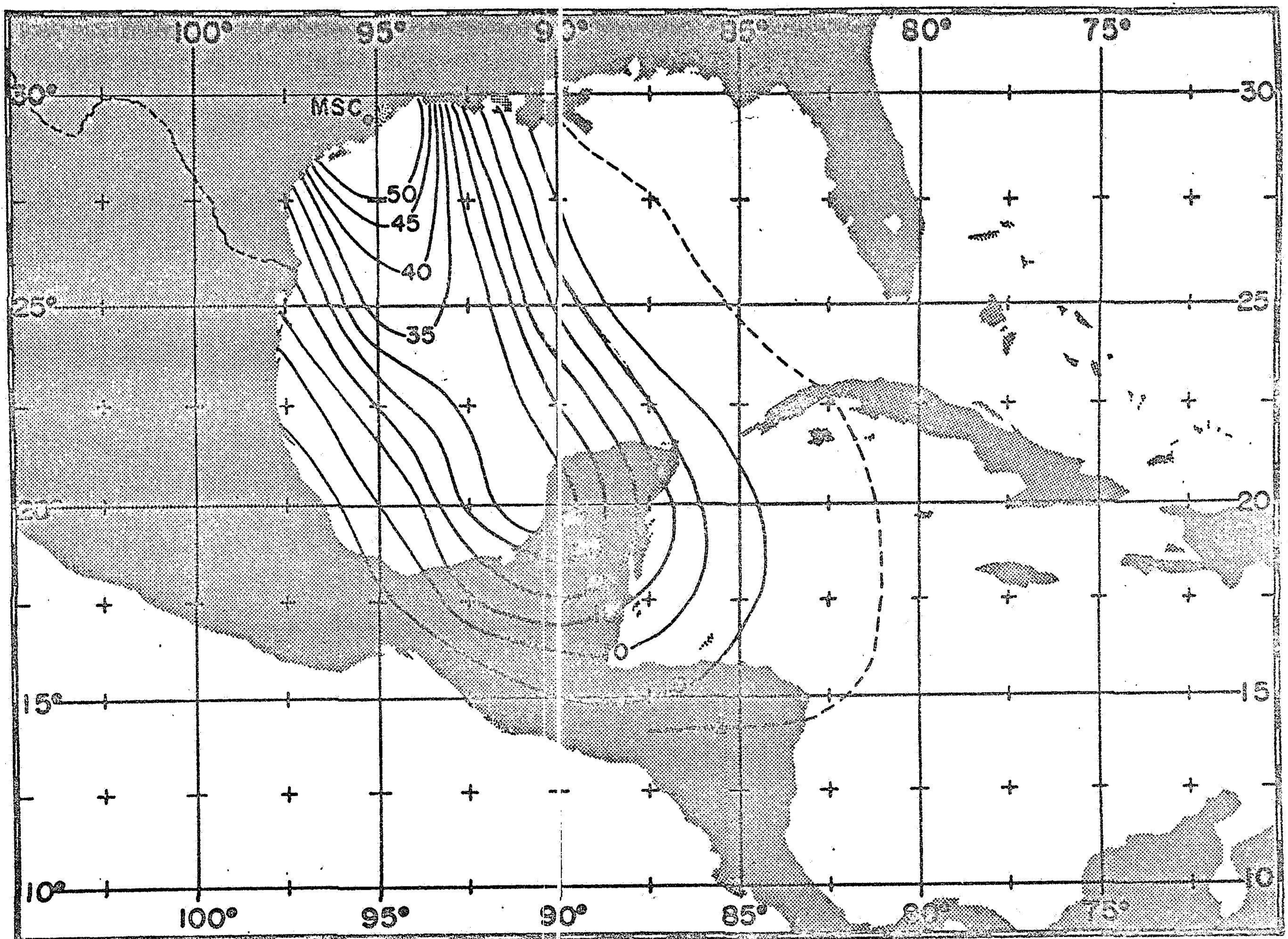



Figure 26. - Cumulative percent probability of an existing category 2 tropical storm or hurricane producing critical winds at the Manned Spacecraft Center within 96 hours.

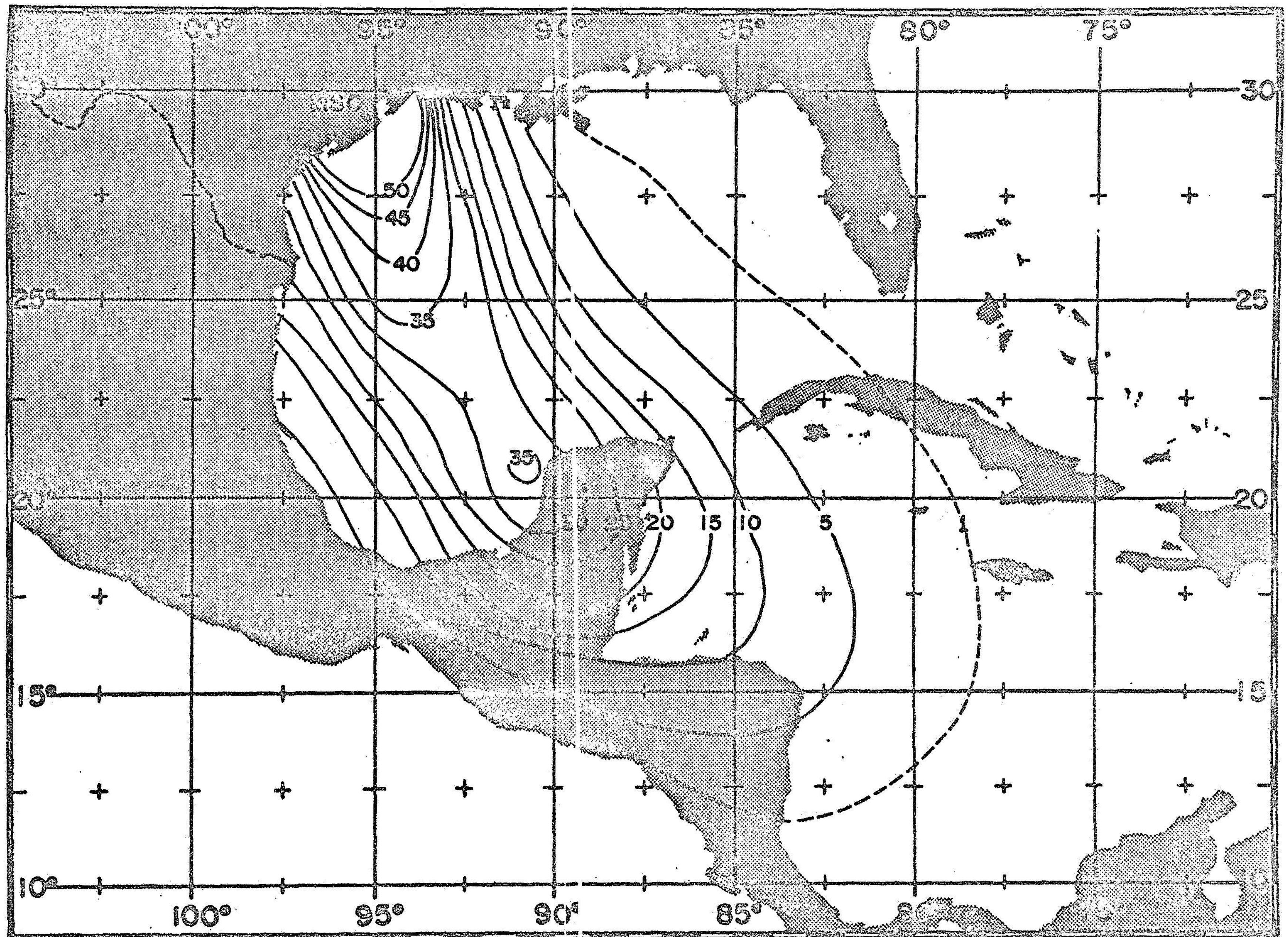



Figure 27. - Cumulative percent probability of an existing category 2 tropical storm or hurricane producing critical winds at the Manned Spacecraft Center within 120 hours.

modification of the probabilities derived from figures 25, 26, and 27 would be in order and the following procedure is recommended. If the storm is moving within 10 degrees of the appropriate centroid track given in figure 17, the probabilities should be increased by about 15 percent of the indicated value (that is, a probability read from figure 25, 26, or 27 as 30.0 percent should be increased to 34.5 percent. For storms moving between 10 and 20 degrees of the centroid track, no correction is needed. For storms moving at even greater angles to the centroid track, 15 percent of the indicated value should be subtracted for each multiple of 10 degrees the observed storm track is farther than 20 degrees off the centroid track. A storm, therefore, which is moving at right angles to the track will have a probability of near zero of affecting MSC. Negative "corrected probabilities" should be considered as having a value near zero. These subjective corrections were suggested by reference to the directional probabilities vs. nondirectional probability charts given in the Cape Kennedy study.

Somewhat more confidence can be placed in the category 2 probabilities than the category 1, since the former were derived from a larger sample of cases. Furthermore, the category 2 storm tracks were somewhat more uniform than those of category 1. Indeed, the 120-hour category 1 ellipse became so large it was impossible to compute category 1 probabilities beyond 96 hours.

This study was patterned as closely as possible after the Cape Kennedy study (Hope and Neumann, 1968), which should be read before attempting to use the probabilities derived in this later MSC study. The original study includes a rather thorough mathematical treatment of the bivariate normal distribution. Also, there are many examples of the use of the various charts and figures.

ACKNOWLEDGMENTS

The author wishes to thank Mr. John Hope of the National Hurricane Center for his helpful advice during the course of this study; Mr. Dale Martin of the Miami Section, SMG, for his excellent drafting work on the many figures; and Mr. Peter Chase of the National Hurricane Research Laboratory for his assistance in deriving the equal-area ellipse formula in the appendix.

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APPENDIX

PART I. COMPUTATION OF B IN EQUATION (5)

$$P' = BN/N_t \qquad BN \leqslant N_t > 0$$

This technique of estimating the probability contribution of a given box to a 99 percent ellipse is a refinement of the technique given in Hope and Neumann (1968).

(1) The 99 percent ellipse is subdivided into 11 elliptical rings, each ring incremented by .09 or 9 percent. See figure A.

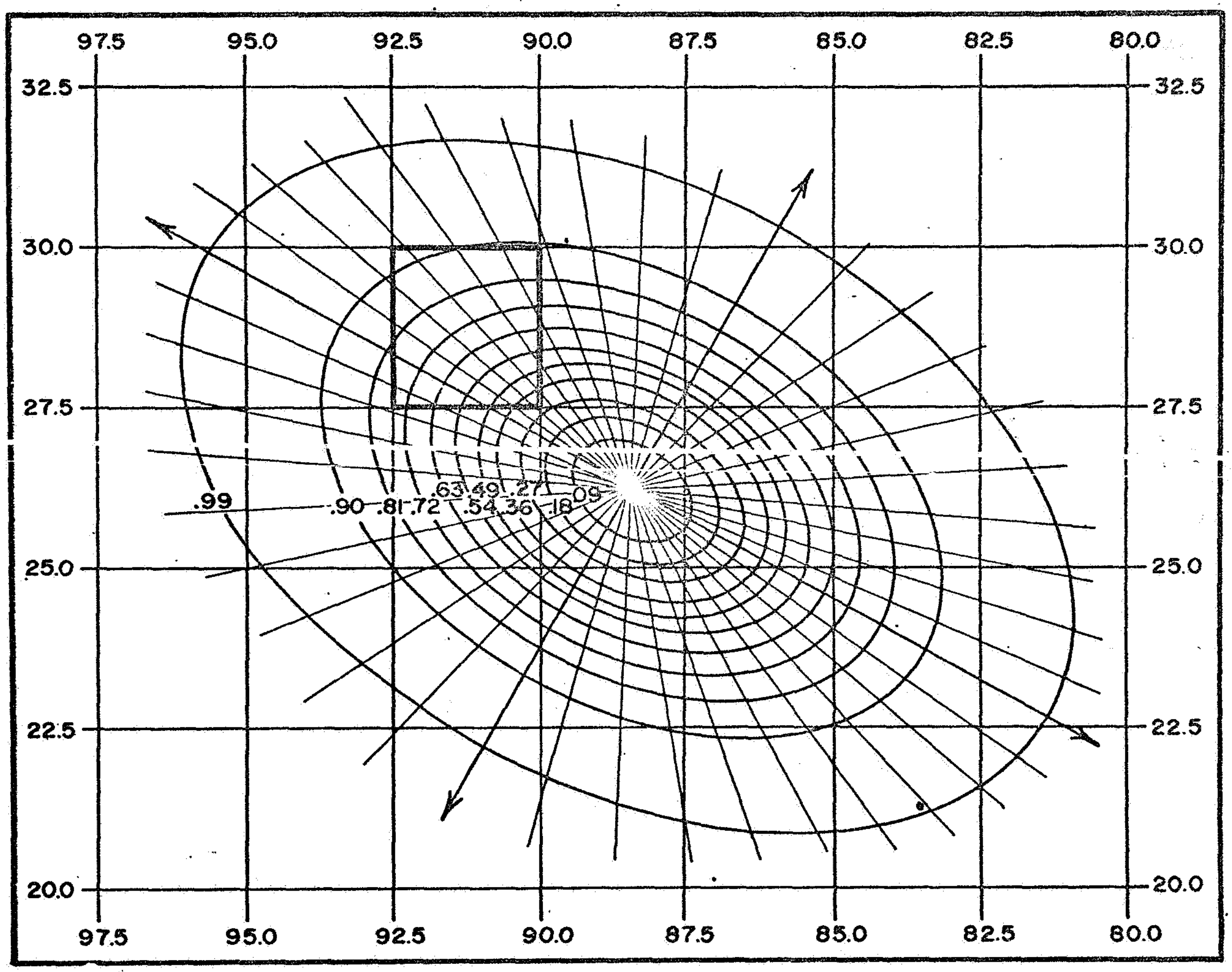


Figure A. - Ellipse used to illustrate the computation of B in equation (5), $P' = BN/N_t$

(2) The ellipse is further subdivided into 40 equal-area sectors. For the derivation of the equal-area sector angle formula, see Appendix, Part II.

(3) The ellipse has been subdivided into 11 x 40 or 440 approximately equal-probability areas (EPA). Each EPA is worth 99 X 440, or 0.225 percent. That is to say, if there were 440 storms contained within the 99 percent ellipse, the density distribution would be such that each EPA would include exactly one storm.

(4) Now consider the $2\frac{1}{2}$-degree latitude-longitude box bounded by $27.5^\circ$ N. and $30.0^\circ$ N., $90.0^\circ$ W. and $92.5^\circ$ W. (Outlined in figure A).

(5) By actual count, there are 31.9 EPA's contained within the box. In terms of probability, the $2\frac{1}{2}$-degree box is worth 31.9 X 0.225 or 7.0 percent. This is the value of B for this particular box. That is, 7.0 percent of the storms within the ellipse would have been in this box, assuming a bivariate normal storm distribution. Since there were 10 storms from which this 48-hour ellipse was computed, the total in the box should be

$$BN = .070 \times 10 = 0.70 \text{ storms}.$$

There are more precise methods available for integrating the bivariate normal elliptical density function over an offset circle. Extensive tables of values so obtained have been published; the latest and most complete is probably that by Groenewould, et al. (1967). This method would require that the storm count shown on figures 10, 18, and 19 be made on the basis of $2\frac{1}{2}$-degree overlapping circles rather than the nonoverlapping boxes. The resultant degree of precision is probably unnecessary, since the EPA method when tested against the circle method gave a maximum probability error of 0.4 percent, assuming the offset circle method to be the standard.

## PART II: DERIVATION OF ELLIPSE EQUAL-AREA SECTOR FORMULA

The equation of an ellipse in a rectangular x,y coordinate system where a is the semilength of the major axis, measured along the x-axis, and b is the semilength of the minor axis, measured along the y-axis, is given by

$$x^2/a^2 + y^2/b^2 = 1 \qquad (1)$$

Converting to a polar coordinate system and rearranging terms

$$r^2 = \frac{1}{(\cos^2\theta)a^2 + (\sin^2\theta)/b^2} \qquad (2)$$

where r is the radius vector and θ is the angle between the x-axis and the radius vector. The general formula for the area A swept by the radius vector is

$$A = \tfrac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta \tag{3}$$

Substituting (2) in (3) gives

$$A = \tfrac{1}{2}\int_{\theta_1}^{\theta_2} \frac{d\theta}{(\cos^2\theta)/a^2 + (\sin^2\theta)/b^2} \tag{4}$$

but, using the identity

$$\cos^2\theta = 1-\sin^2\theta,$$

$$A = \tfrac{1}{2}\int_{\theta_1}^{\theta_2} \frac{d\theta}{1/a^2 + (1/b^2-1/a^2)\sin^2\theta} .$$

According to standard mathematical tables,

$$\tfrac{1}{2}\int_{\theta_1}^{\theta_2} d\theta/(c+d\sin^2\theta) = \tfrac{1}{2}\left[1/\sqrt{(c^2+cd)}\right] \text{Arctan} \left[(\sqrt{c^2+cd}\ \text{Tan}\theta)/c\right]\Big|_{\theta_1}^{\theta_2}$$

where c and d are constants. In this case, $C = 1/a^2$ and $d = (1/b^2-1/a^2)$; therefore,

$$A = (ab)/2\ \text{Arctan}\ \left[(a/b)\ \text{Tan}\theta\right]\Big|_{\theta_1}^{\theta_2} \tag{5}$$

The area of an ellipse is given by $\pi ab$. The area of Q/40th of an ellipse is given by

$$A' = Q\pi ab/40 \tag{6}$$

If A = A', substituting (6) in (5) and rearranging gives

$$Q\pi/20 = \Big[\text{Arctan } (a/b)\text{Tan}\theta\Big]\Bigg|_{\theta_1}^{\theta_2}$$

If $\theta_1$ is taken as the x-axis, then $\text{Tan}\theta_1 = 0$ and

$$\theta_2 = \text{Arctan } \Big[(b/a)\text{Tan}(\pi Q/20)\Big] \qquad (7)$$

where $\theta_2$ is the angle of the sector measured from the x-axis which will subdivide the ellipse in Q/40th of its area. Taking Q from 1 to 9 will give all the required angles, since those in the other three quadrants have equivalent or complementary values. If the ellipse is rotated as in figure A, the angular measurements are made from the major axis rather than from the x-axis.