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A COMPARISON OF NUMERICAL TECHNIQUES FOR DETERMINING THE PARAMETERS OF DISTRIBUTED RC NETWORKS

Prepared under Grant NGL-03-002-136 for the
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Abstract: This report describes the theory and implementation of the use of the matrizant for modeling a distributed RC network. Three different techniques for using the digital computer to determine the matrizant are discussed. A comparison is made between the use of matrizant methods and the use of a lumped equivalent circuit to model the distributed RC network. In general, the lumped equivalent model is shown to be superior.

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A Comparison of Numerical Techniques for Determining the Parameters of Distributed RC Networks

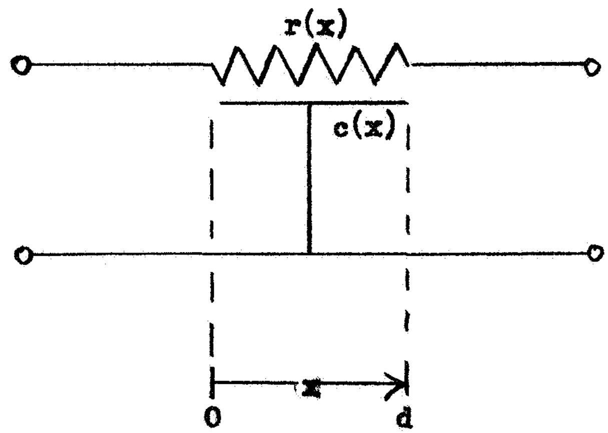
I. Introduction

This is one of a series of reports describing the use of digital computational techniques in the analysis and synthesis of DLA (Distributed-Lumped-Active) networks. This class of networks consists of three distinct types of elements, namely, distributed elements (modeled by partial differential equations), lumped elements (modeled by algebraic relations and ordinary differential equations), and active elements (modeled by algebraic relations). Such a characterization is applicable to a broad class of circuits, especially including those usually referred to as linear integrated circuits, since the fabrication techniques for such circuits readily produce elements which may be modeled as distributed and active, as well as ones which may be considered as lumped.

One of the major problems encountered in the analysis or synthesis of DLA networks is the problem of modeling the distributed elements. Specifically, there are two formidable obstacles. First, immittances associated with distributed RC networks will, in general, involve transcendental irrational functions of the complex frequency variable. This makes the application of standard network analysis techniques difficult, to say the least. Second, closed-form analytical expressions for the network functions describing these distributed elements are available for only a few particular geometries, and even these expressions are completely different for different geometries. For example, the admittance parameters associated with a distributed network with an exponential taper[1] have a form entirely different from the admittance parameters of a distributed network with a linear taper.[2] As a result of these obstacles it has become fairly routine to use analysis techniques which employ the digital computer when dealing with networks which include distributed RC elements.[3] Such techniques require the generation of models which fulfill the dual role of accurately characterizing the distributed elements and also being amenable to digital computational process. In a previous report an evaluation was made of the use of a lumped element model to approximate the transmission parameters of distributed RC networks.[4] Such a model may be analyzed by an iterative matrix multiplication process and has the advantage that it can be applied to networks of a completely arbitrary taper. In this report a different approach to modeling based on the use of the <u>matrizant</u> is presented. The report begins by developing the theory of matrizants and their application to the determination of the transmission parameters of distributed RC networks of arbitrary taper. Several numerical methods for using a digital computer to determine the matrizant are then presented. Finally, a comparison is made between the use of these different methods and the use of the lumped element model previously referred to with respect to the relative computational efficiency and accuracy of the two approaches.

II. The General Theory of the Use of the Matrizant to Determine the Parameters of a Distributed RC Network[5,6]

In this section of the report the general theory of the matrizant method for determining the transmission parameters of a distributed RC network is presented. First of all, consider the network shown below:



The partial differential equations describing this network are

$$\frac{\partial v(x,t)}{\partial x} = - r(x)\ i(x,t)$$

$$\frac{\partial i(x,t)}{\partial x} = - c(x)\ \frac{\partial v(x,t)}{\partial t} \qquad (1)$$

where $v(x,t)$ and $i(x,t)$ are the instantaneous values of the voltage and current along the line, $r(x)$ is the resistance per unit length along the line and $c(x)$ is the capacitance per unit length along the line. Applying the Laplace transformation with respect to time to these equations and assuming that the initial conditions are zero, we obtain

$$\frac{dV(x,p)}{dx} = - r(x)\ I(x,p)$$
$$\frac{dI(x,p)}{dx} = - pc(x)\ V(x,p) \tag{2}$$

where V(x,p) and I(x,p) are the Laplace transforms of v(x,t) and i(x,t) respectively. If we define the matrix K(x,p) as

$$K(x,p) = \begin{bmatrix} 0 & r(x) \\ pc(x) & 0 \end{bmatrix} \tag{3}$$

then the relations of (2) may be written as

$$\frac{d}{dx} \begin{bmatrix} V(x,p) \\ I(x,p) \end{bmatrix} = - K(x,p) \begin{bmatrix} V(x,p) \\ I(x,p) \end{bmatrix} \tag{4}$$

This first-order matrix differential equation may be investigated by first considering a general matrix equation of the form

$$w'(y) = A(y)\ w(y) + R(y) \tag{5}$$

where w(y) and R(y) are vectors, A(y) is a square matrix and the prime (') indicates differentiation with respect to y. To solve such a set of equations let us first assume a product solution for w(y) having the form

$$w(y) = P(y)\ z(y) \tag{6}$$

where P(y) is a square matrix and z(y) is a vector and both these quantities are to be determined. Differentiating (6) and substituting the result in (5) we obtain

$$\left[P'(y) - A(y)\ P(y)\right] z(y) + P(y)\ z'(y) = R(y) \tag{7}$$

Since $P(y)$ is not specified, let us stipulate two requirements on it. These are

$$P'(y) - A(y)\,P(y) = 0 \qquad (8)$$
$$P(0) = I$$

In this case the original relation as given in (7) becomes

$$P(y)\,z'(y) = R(y) \qquad (9)$$

Let us now assume that $P^{-1}(y)$ exists. Then we may write

$$z'(y) = P^{-1}(y)\,R(y) \qquad (10)$$

If we integrate this relation and note that $z(0) = w(0)$ we obtain

$$z(y) = w(0) + \int_0^y P^{-1}(y)\,R(y)\,dy \qquad (11)$$

If we now multiply both sides of this relation by $P(y)$ we obtain

$$w(y) = P(y)\,w(0) + P(y)\int_0^y P^{-1}(y)\,R(y)\,dy \qquad (12)$$

Thus, we have found a general solution for $w(y)$ and, in the reduced set of equations in which $R(y)$ is zero, the solution has the form

$$w(y) = P(y)\,w(0) \qquad (13)$$

Our problem is now one of finding a matrix $P(y)$ such that the relations of (8) are satisfied. Let us begin by integrating both sides of the first relation of (8) and inserting the second relation in the result. Rearranging terms we obtain

$$P(y) = I + \int_0^y A(y)\ P(y)\ dy \tag{14}$$

Since the elements of P(y) are functionally dependent on A(y) and on the integration from zero to y, it is convenient to use the symbol $M_0^y(A)$ to represent P(y). Let us now assume a series solution for P(y) having the form

$$P(y) = P_0(y) + P_1(y) + \ldots = \sum_{k=0}^{\infty} P_k(y) = M_0^y\ (A) \tag{15}$$

Substituting this in the integral equation of (14) we obtain

$$\sum_{k=0}^{\infty} P_k(y) = I + \int_0^y A(y) \sum_{k=0}^{\infty} P_k(y)\ dy \tag{16}$$

It may be shown that the series given in (15) converges absolutely and uniformly in every interval in which the elements A(y) are continuous. Therefore, if we make the following assignments

$$\begin{aligned} P_0 &= I \\ P_1(y) &= \int_0^y A(y)\ P_0\ dy = \int_0^y A(y)\ dy \\ P_2(y) &= \int_0^y A(y)\ P_1(y)\ dy \\ &\ \vdots \\ P_{i+1}(y) &= \int_0^y A(y)\ P_i(y)\ dy \end{aligned} \tag{17}$$

Then the relation given in (16) is satisfied and the quantity $M_0^y(A)$ defines a matrix function of A called the <u>matrizant</u>. This then gives the solutions to the general differential equation given in (5), namely, we obtain

$$w = M_0^y\,(A)\;w(0) + M_0^y(A)\int_0^y \left[M_0^y\,(A)\right]^{-1} R(y)\;dy \tag{18}$$

In the simplified case which we will discuss here where $R(y) = 0$ we obtain

$$w(y) = M_0^y\,(A)\;w(0) \tag{19}$$

To use the matrizant approach to solve the matrix differential equation describing the distributed network given in (4) it is convenient to make a change of variable by substituting d-y for x where x is the original variable for position along the length of the distributed line, y is the new variable, and d is a constant specifying the total length of the distributed line. This in effect reverses the one-dimensional axis shown on page 3 so that y = 0 is the right end of the line and y = d is the left end of the line. With this substitution the matrix differential equation of (4) becomes

$$\frac{d}{dy}\begin{bmatrix} V(d-y,p) \\ I(d-y,p) \end{bmatrix} = K(d-y,p)\begin{bmatrix} V(d-y,p) \\ I(d-y,p) \end{bmatrix} \tag{20}$$

The solution to this equation in terms of the matrizant is thus given as

$$\begin{bmatrix} V(d-y,p) \\ I(d-y,p) \end{bmatrix} = M_0^y\left[K(d-y,p)\right]\begin{bmatrix} V(d,p) \\ I(d,p) \end{bmatrix} \tag{21}$$

If we now evaluate this expression for y = d, i.e., the conditions at the left end of the network shown on page 3, we obtain

$$\begin{bmatrix} V(0,p) \\ I(0,p) \end{bmatrix} = M_0^d \left[K(d-y,p)\right] \begin{bmatrix} V(d,p) \\ I(d,p) \end{bmatrix} \tag{22}$$

The quantity $V(0,p)$, however, is simply the conventional input port voltage $V_1(p)$. Similarly, the quantities $I(0,p)$, $V(d,p)$, and $I(d,p)$ are the conventional port variables $I_1(p)$, $V_2(p)$ and $-I_2(p)$. Thus, the relations of (22) may be written as

$$\begin{bmatrix} V_1(p) \\ I_1(p) \end{bmatrix} = M_0^d \left[K(x,p)\right] \begin{bmatrix} V_2(p) \\ -I_2(0) \end{bmatrix} \tag{23}$$

From this relation we see that the matrizant M_0^y (A) is actually the transmission matrix of a distributed RC line of length d in which the resistance and the capacitance for arbitrary taper are specified by the quantities $r(x)$ and $c(x)$. Obviously, a determination of the matrizant provides a solution to the problem of obtaining a set of parameters or a model for a two-port distributed RC network. In the next section of this report we will investigate some ways of making such a determination.

III. Numerical Methods for Finding the Matrizant

The basic problem in the determination of the matrizant is that of determining a matrix $P(y)$ which satisfies (8). Let us first consider a Taylor's expansion for such a matrix. If we assume that $P(y)$ is known and $y + h$ is a point in the vicinity of y then we may express $P(y + h)$ as follows

$$P(y+h) = P(y) + \frac{hD}{1!} P(y) + \frac{(hD)^2}{2!} P(y) + \dots \tag{24}$$

where D is the derivative operator d/dy. An approximation for such a solution may be made by truncating this expansion after the first derivative term. Thus, we obtain

$$P(y + h) = P(y) + hD\,P(y) \tag{25}$$

Using (8) we may substitute $A(y)\,P(y) = P'(y)$, thus the truncated expansion of (25) may be written in the form

$$P(y + h) = \left[I + hD\right] P(y) \tag{26}$$

Now let us define a matrix E(y) by the expression

$$E(y) = I + h\,A(y) \tag{27}$$

Recalling from (8) that P(0) is equal to the identity matrix, we may write the following expressions for an incremental series of values of $y = 0, h, 2h, \ldots$ Thus we obtain

$$\begin{aligned} P(h) &= E(0)\;P(0) = E(0)\;I = E(0) \\ P(2h) &= E(h)\;P(h) = E(h)\;E(0) \\ P(3h) &= E(2h)\;P(2h) = E(2h)\;E(h)\;E(0) \\ &\vdots \\ P(d) &= P(nh) = E\left[(n-1)h\right]\;E\left[(n-2)h\right]\ldots E(h)\;E(0) = M_0^d\,(A) \end{aligned} \tag{28}$$

The final expression in the above gives us the value of the matrizant, i.e., the transmission parameter matrix. Thus, our determination of the parameters modeling the distributed RC network is simply accomplished by a series of matrix multiplications as indicated in the final expression of (28). The procedure outlined above is usually called the <u>Euler method</u> of determining the matrizant.

The second numerical method which may be used to determine the matrizant is one which includes two derivative terms from the basic Taylor expansion given in (24). Thus, we may write

$$P(y+h) = P(y) + \frac{hD}{1!} P(y) + \frac{(hD)^2}{2!} P(y) \tag{29}$$

Differentiating the expression $P'(y) = A(y)\ P(y)$ of (8), we obtain the result

$$P''(y) = \left[A'(y) + A^2(y)\right] P(y) \tag{30}$$

Thus, the truncated Taylor series which includes the second derivative term may be expressed as

$$\begin{aligned} P(y+h) &= P(y) + hA(y)\ P(y) + \frac{h^2}{2}\left[A'(y) + A^2(y)\right] P(y) \\ &= \left\{I + hA(y) + \frac{h^2}{2}\left[A'(y) + A^2(y)\right]\right\} P(y) \end{aligned} \tag{31}$$

If we define the matrix $E(y)$ as

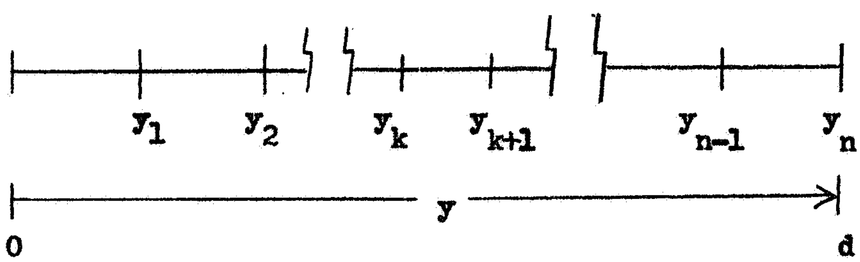
$$E(y) = I + hA(y) + \frac{h^2}{2}\left[A'(y) + A^2(y)\right] \tag{32}$$

then, following the same logic used above, we see that the matrizant is determined by the expression

$$P(d) = E\left[(n-1)h\right]\ E\left[(n-2)h\right]\ \ldots\ E(h)\ E(0) = M_0^d\ (A) \tag{33}$$

The above method of determining the matrizant is frequently called a <u>modified Euler method</u>. Since it basically involves only matrix multiplication it is readily implemented on a digital computer although it has the disadvantage that equations must be supplied for determining the elements of the matrix $A'(y)$ in addition to those determining the matrix $A(y)$.

A third method using numerical techniques to determine the matrizant is the method of <u>mean coefficients</u>. To apply this method we may consider the y axis as being divided into subintervals as shown in the following figure:



Now let us define the quantities P_k and h_{k+1} by the relations

$$P_k = P(y_k)$$

and

$$h_{k+1} = y_{k+1} - y_k \tag{34}$$

Thus, integrating both sides of the first equation of (8) we obtain

$$P_{k+1} = P_k + \int_{y_k}^{y_{k+1}} A(y)\ P(y)\ dy \tag{35}$$

If we now assume that the interval of integration has been chosen small enough so that $P(y)$ is constant, the above equation may be written in the form

$$P_{k+1} = \left[I + \int_{y_k}^{y_{k+1}} A(y)\ dy \right] P_k \tag{36}$$

We may now define a matrix E_k by the relation

$$E_k = I + h_{k+1} A_{k+1} \tag{37}$$

where

$$A_{k+1} = \frac{1}{h_{k+1}} \int_{y_k}^{y_{k+1}} A(y)\ dy \tag{38}$$

We can now compute the matrizant by an iterative process similar to those described previously. The expression for doing this is given in (33). Since numerical integration schemes are readily available, this procedure is readily implemented on the digital computer.

The actual implementation of the three methods of determining the matrizant given above and the results obtained from using the resultant models to represent a distributed RC network element are given in the following section.

IV. A Comparison of Modeling Procedures

In the sections given above we introduced the matrizant and showed how it may be used to define the transmission parameters for a distributed RC network and thus, to provide a model for such a network. In addition, the theory of three numerical methods for determining the matrizant was presented. In this section we shall present some results obtained from programming the three methods for the digital computer and comparing the results obtained from them with the results obtained from the use of the lumped element model described in a previous report.[4] The three methods were programmed in Fortran IV and programs were run in a CDC 6400 digital computer. The distributed RC network configuration tested was a well known one, namely, an exponentially tapered network in which r(x) and c(x) are defined by the relations

$$r(x) = r_0 e^{\alpha x} ; \qquad c(x) = c_0 e^{-\alpha x} \tag{39}$$

The transmission parameters of the network were evaluated under sinusoidal steady state conditions for a range of frequencies from 0.1 to 1,000 rad/sec and for models of 10, 20, and 50 sections. Three different degrees of taper were investigated for values of alpha respectively equal to 1, 2, and 3. For convenience, the quantities r_0 and c_0 given in (39) and the length of the network were set to unity, thus providing a convenient frequency and impedance normalization.

It was found that the Euler method and the method of mean coefficients gave almost identical answers for every case. Thus all the following comments on the Euler method apply equally well to the method of mean coefficients, except for computer time. Table I shows the amount of CDC 6400 computer time required to compute the transmission parameters at a given frequency. This table shows that the method of mean coefficients (using 5 iterations for the trapezoidal integration) is by far the most costly of computer time of the three methods. The relative accuracy of the various methods is summarized in Figures 1 to 10.

Figures 1 and 2 show the magnitude and phase error, respectively, of the 10, 20, and 50 section Euler method and the 10 section modified Euler method solutions for the A parameter of the network with $\alpha = 1$. Figures 3 and 4 show the same information for the C parameter. The other two parameters have characteristics which are quite similar to these. From these figures, one can conclude that the 10 section modified Euler solution is clearly superior to the 10 and 20 section Euler solutions and is valid over a larger frequency range. The 50 section Euler solution is roughly comparable to the 10 section modified Euler solution in accuracy, but it requires almost twice as much computer time.

Figures 5 and 6 show the error of the 10, 20, and 50 section modified Euler method solutions for the A parameter of the network with $\alpha = 1$. The results for the other three parameters are almost identical. These figures show that as the number of sections used to approximate the network is increased, the upper limit of the useful frequency range is increased. The 10 section approximation is good for $\omega < 10$ rad/sec, the 20 section one for $\omega < 30$ rad/sec, and the 50 section one for $\omega < 100$ rad/sec.

Figures 7 and 8 show the error of the 10 section modified Euler method for $\alpha = 1$, $\alpha = 2$, and $\alpha = 3$. The results for this case are typical and show essentially no difference in accuracy for the three different tapers. This result was also found to hold true for the other methods.

To enable a comparison to be made between the matrizant approach for modeling a distributed RC network as characterized by the results described above, and the lumped element model previously investigated, tests similar to those used above were made on the lumped element model. In addition to tests of 10, 20, and 50 sections, the 100 section lumped element model was also tested. The computation times for the results are tabulated in Table I. It is readily verified that for an equivalent number of sections, the running times are all lower than the most efficient of the matrizant methods which were evaluated, namely, the Euler method.

In Figures 9 and 10 the relative magnitude and phase error for the A parameter for 10, 20, 50 and 100 section lumped element models are plotted as functions of frequency. Note that the C curve shown in Figure 9 for the 50 section lumped element model is, in general, superior to the B curve of Figure 5 for the 20 section modified Euler model, despite the fact that (from Table I) the latter required slightly more computational time, namely, 0.082 seconds compared with .080 seconds. Similarly, the D curve of Figure 9 for the 100 section lumped element model is, in general, far superior than the C curve shown in Figure 5 for a 50 section modified Euler model. Again, the running times were considerably in favor of the lumped element model, namely, 0.136 seconds vs 0.158 seconds. When low numbers of sections are used in the lumped element model, however, the results may not be as good as the modified Euler method. For example, the A curve of Figure 9 for the 20 section lumped element model shows results which are, in general, poorer than the A curve of Figure 5 for the 10 section modified Euler model. However, the 20 section lumped element model required less computation time, namely, 0.048 seconds vs 0.056 seconds for the modified Euler method. Conclusions similar to the above with respect to the phase of A parameter may be found by comparing the results shown in Figure 10 for the lumped element model and Figure 6 for the modified Euler model. Studies made of the other transmission parameters, in general, yielded results which are similar to those included with this report. From a study of the data obtained from these various computer runs, we can, in general, conclude that (beyond a certain minimum number of sections) the lumped element model, for a given amount of computer time, will produce more accurate results than any of the numerical methods used to compute the matrizants which have been investigated here.

V. Conclusions

In this report three different numerical methods, namely, the Euler method, the modified Euler method, and the method of mean coefficients, have been applied to generate a matrizant model for a distributed RC network of exponential taper. The best of these results have been compared with the results obtained from the use of a lumped element model for such a network. In comparing the three matrizant methods the results given in section IV show that for the analysis of an exponential taper distributed RC network, the 10 section modified Euler method gives results which are at least as good as models using the Euler method or the method of mean coefficients and using up to 50 sections. In addition, the 10 section modified Euler method model uses much less computer time. Of all the numerical methods used to determine the matrizant investigated in this study, only the 10 section Euler method was faster and its results, in general, were only valid for frequencies less than 2 rad/sec. Even in that range the relative magnitude error of the B and C parameters approached values of 5 per cent. The 10 section modified Euler method model, on the other hand, gives accurate results for values of frequency up to 10 rad/sec, and, using 50 sections, the range may be extended very nearly to 100 rad/sec. Extremeness of taper appears to have very little effect on the accuracy of these methods.

In comparing the matrizant modeling procedure with the lumped element model, in general, the lumped element model was found to be superior. Although more sections were required to achieve accuracies comparable with the modified Euler model, the use of such additional sections was, in general, found to require less computation time. For example, a 100 section lumped equivalent model has an accuracy in the low frequency range which is approximately the same as that of the 50 section modified Euler matrizant model, is valid over a greater frequency range, and uses about 15 per cent less computational time. In addition, the lumped equivalent model has the advantage of being able to accurately model tapers in which the functions $r(x)$ and $c(x)$ are not continuous. Such tapers cannot be treated by the modified Euler approach due to the necessity of providing equations describing the derivative of these functions. In summary, it is felt that although the matrizant approach may be slightly superior to the lumped element model in specific applications, in general, the lumped element model of a distributed RC network has more flexibility and uses less computer time for a given desired accuracy. Although the results given in this report have been derived only with respect to the exponentially tapered distributed RC network, experimental evidence gained in using the various models for networks of different taper has substantiated the conclusions given above for such networks.

Method	Sections	Computer Time (seconds)
Euler	10	0.046
	20	0.062
	50	0.108
Modified Euler	10	0.056
	20	0.082
	50	0.158
Method of Mean Coefficients (5 iterations)	10	0.078
	20	0.124
	50	0.262
Lumped Element	10	0.034
	20	0.048
	50	0.080
	100	0.136

Table I

Time Required to Compute Transmission Parameters at a Single Frequency on CDC 6400.

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Figure 1

Magnitude Error of A Parameter.

ALFA = 1

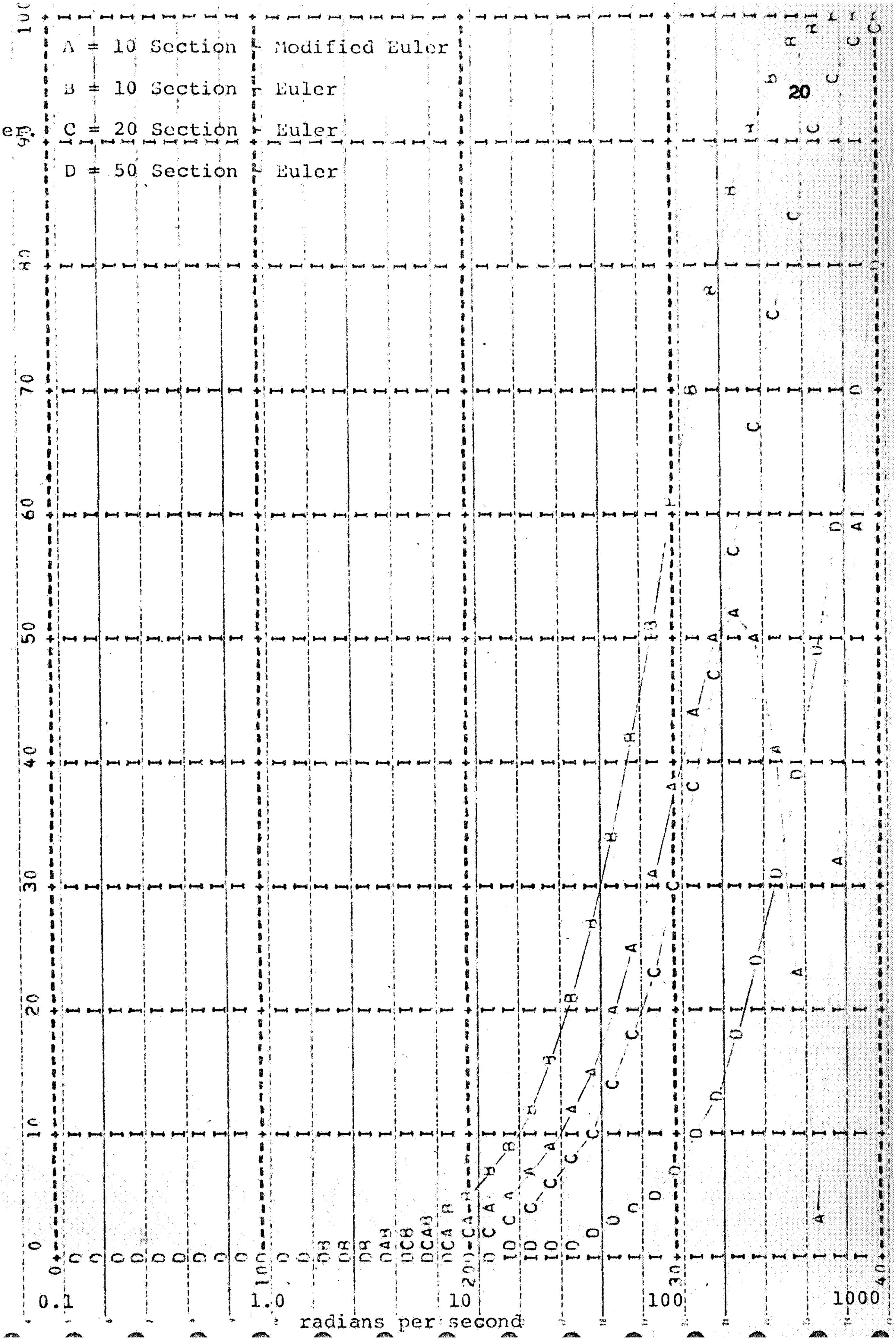


Figure 2

Phase Error of A Parameter.

ALFA = 1

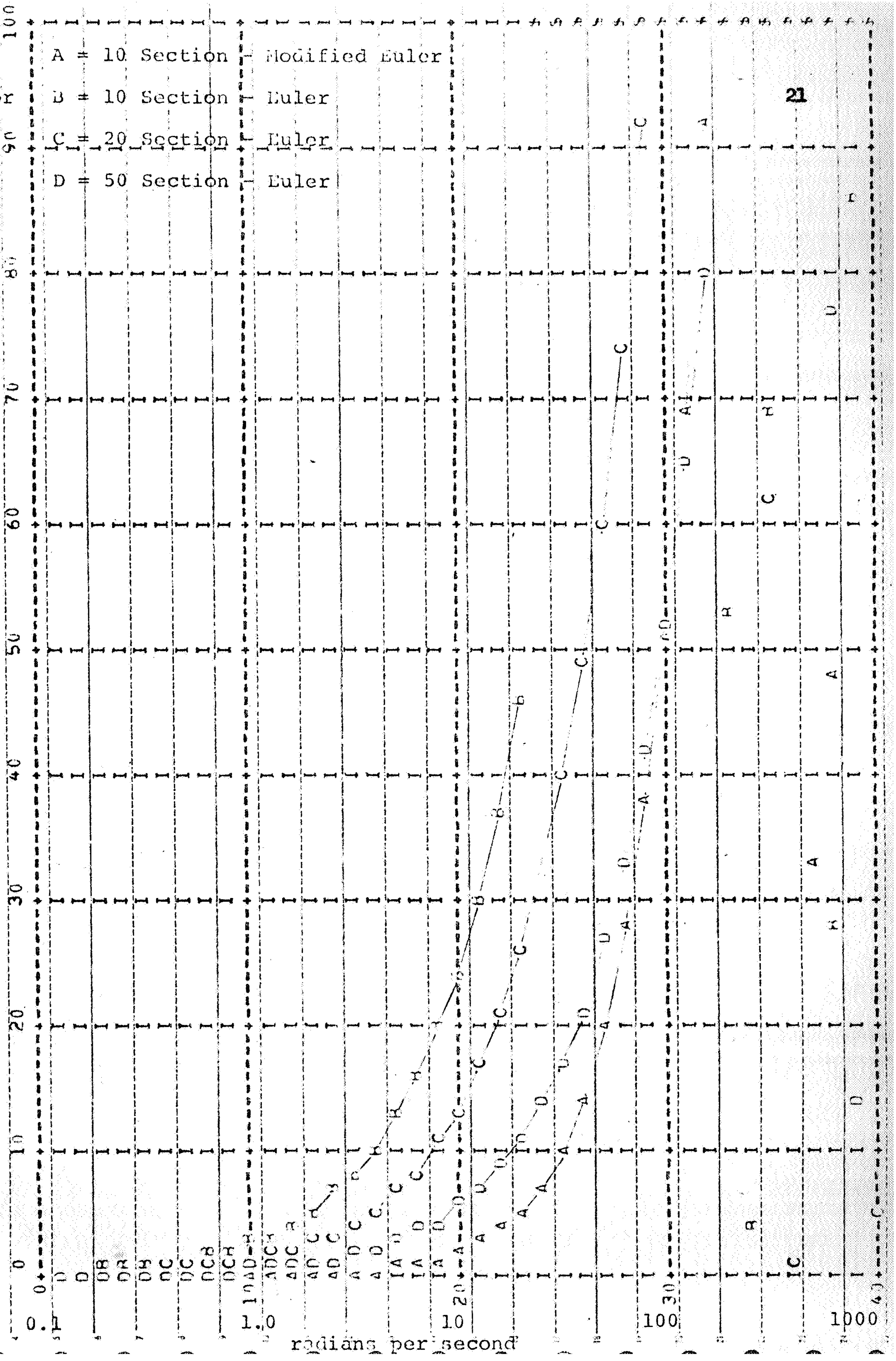
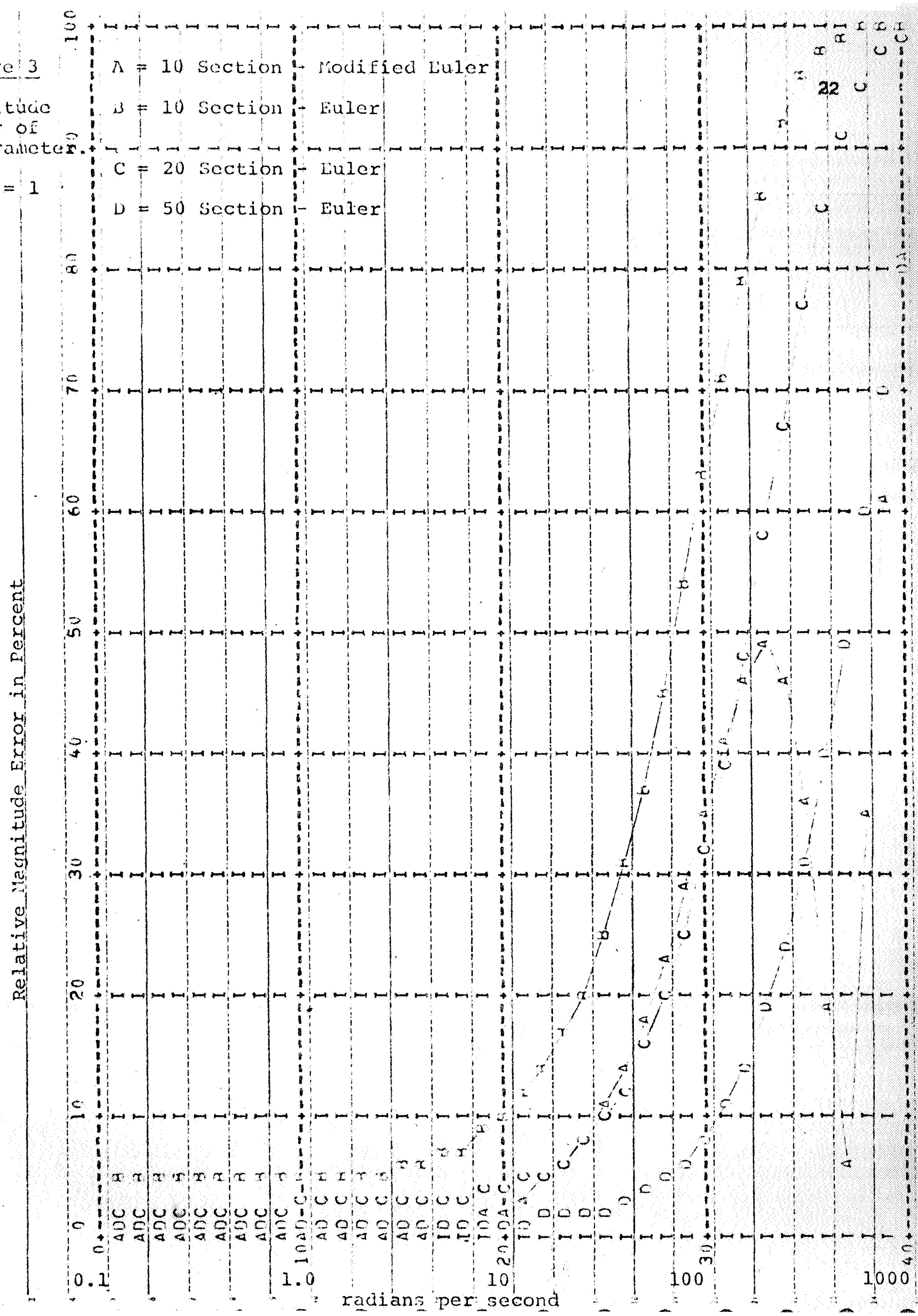


Figure 3

Magnitude Error of C Parameter.

ALFA = 1



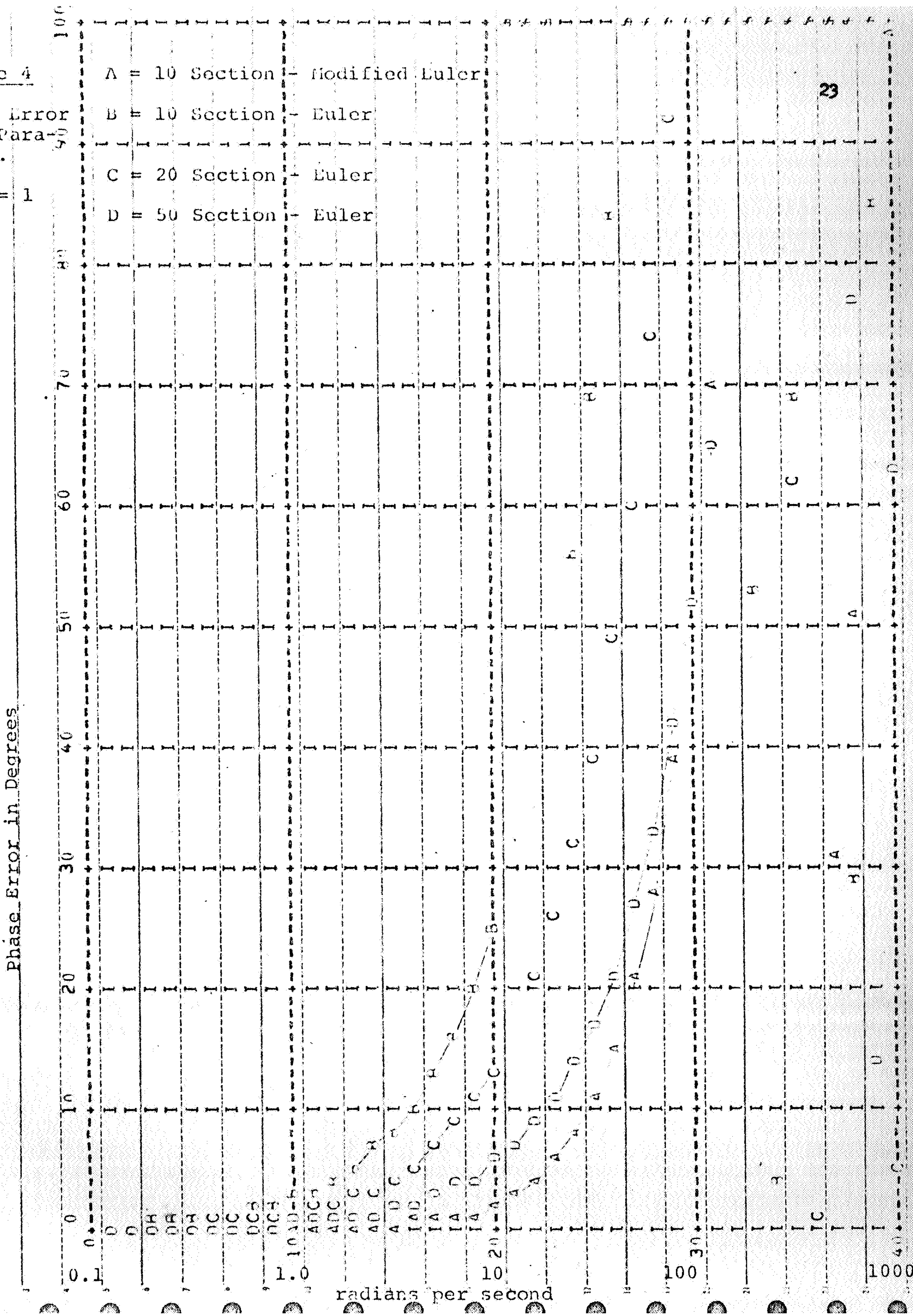


Figure 4

Phase Error of C Parameter.

ALFA = 1

Figure 5

Magnitude Error of A Parameter

ALFA = 1

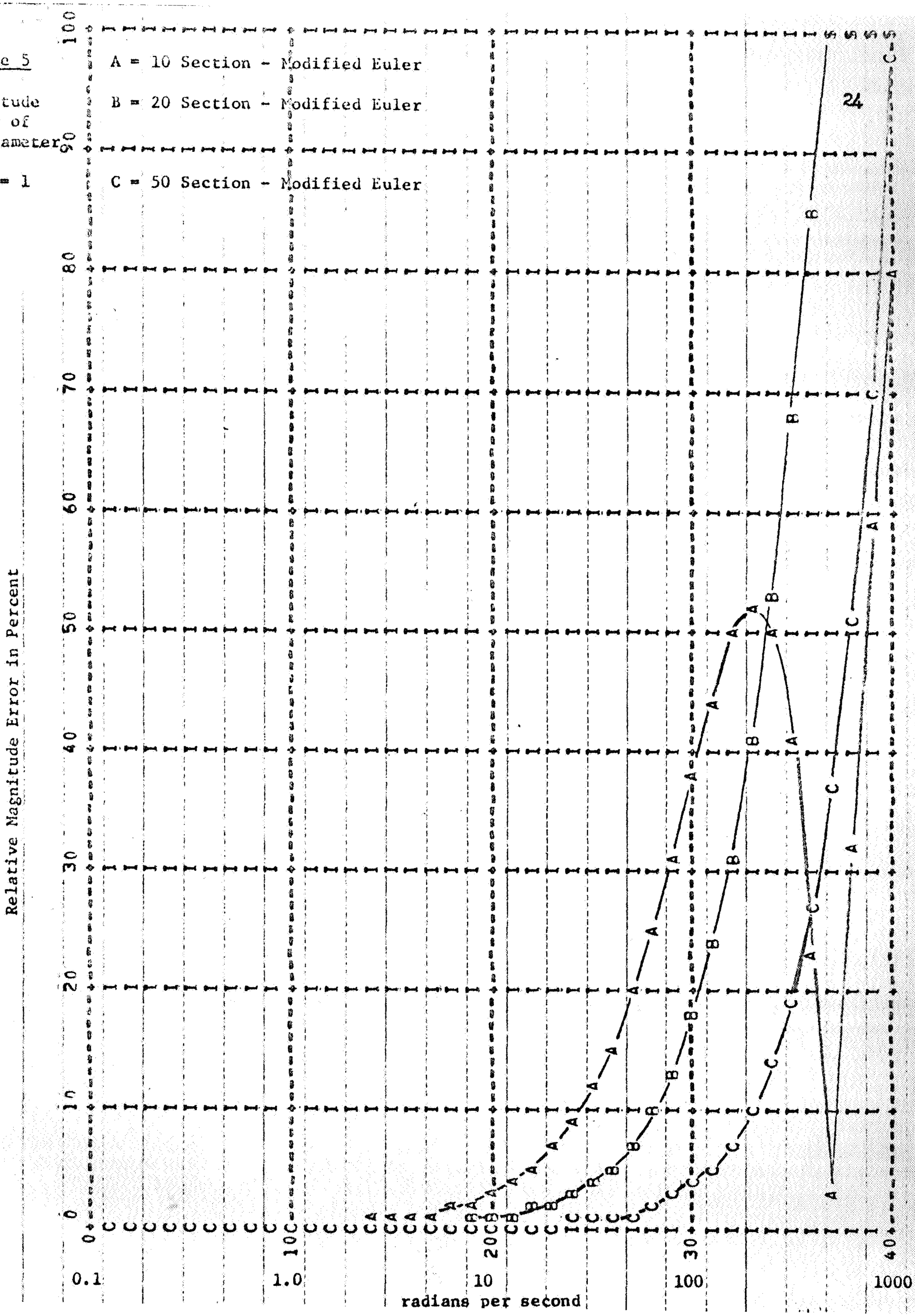


Figure 6

Phase Error of A Parameter.

ALFA = 1

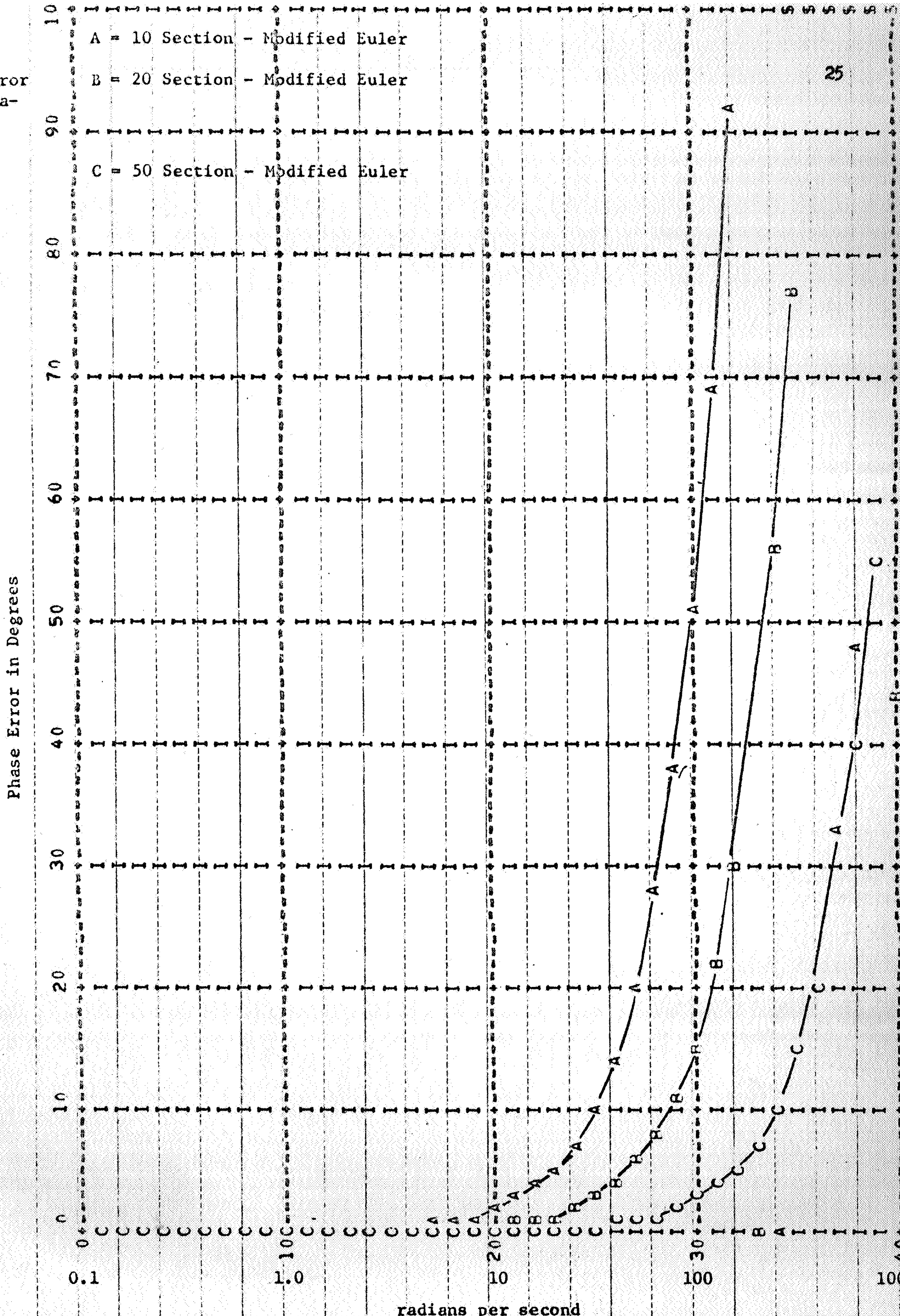


Figure 7

Magnitude Error of A Parameter

10 Section Modified Euler

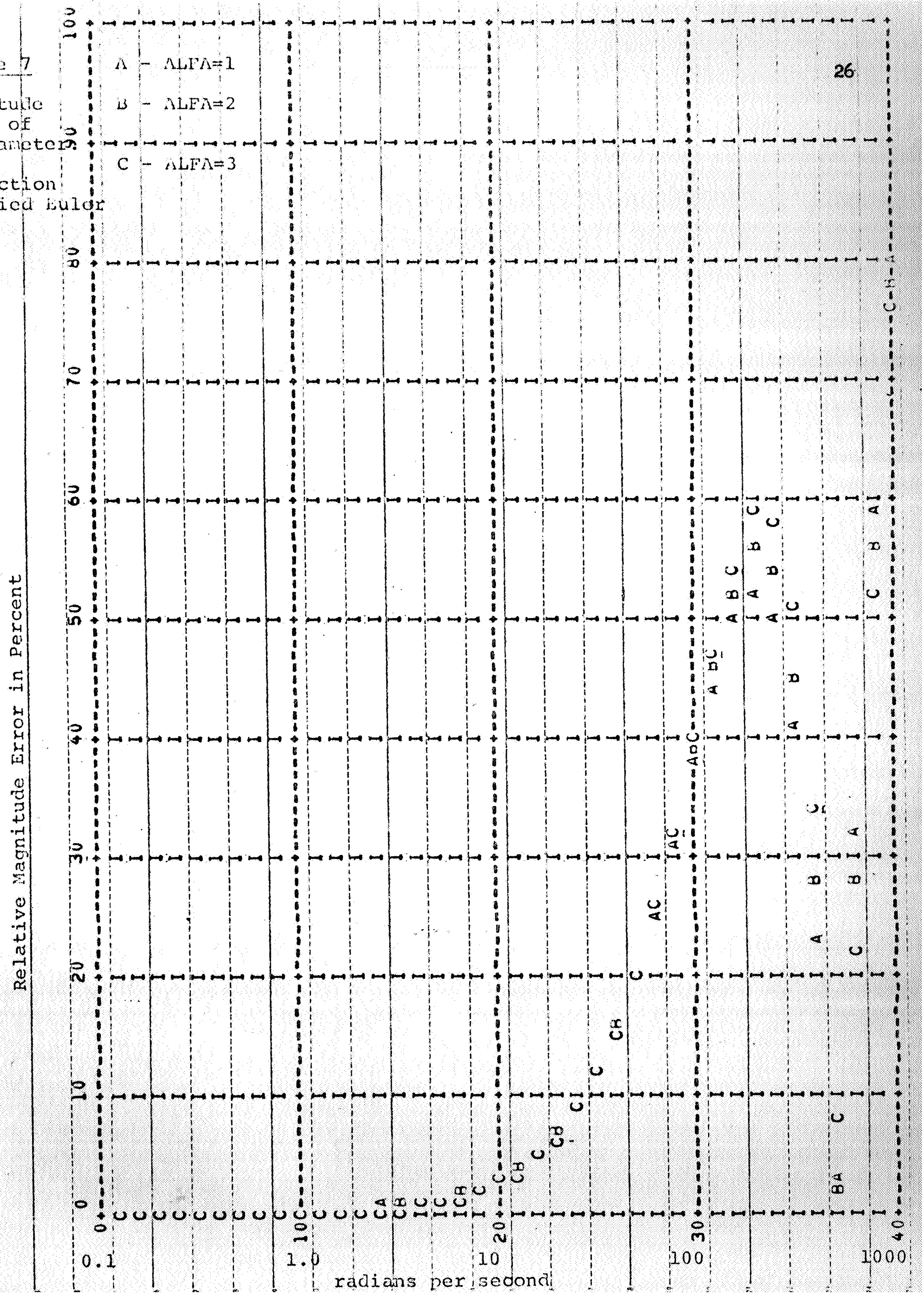


Figure 8

Phase Error of A Parameter.

10 Section Modified Euler.

A - ALFA=1

B - ALFA=2

C - ALFA=3

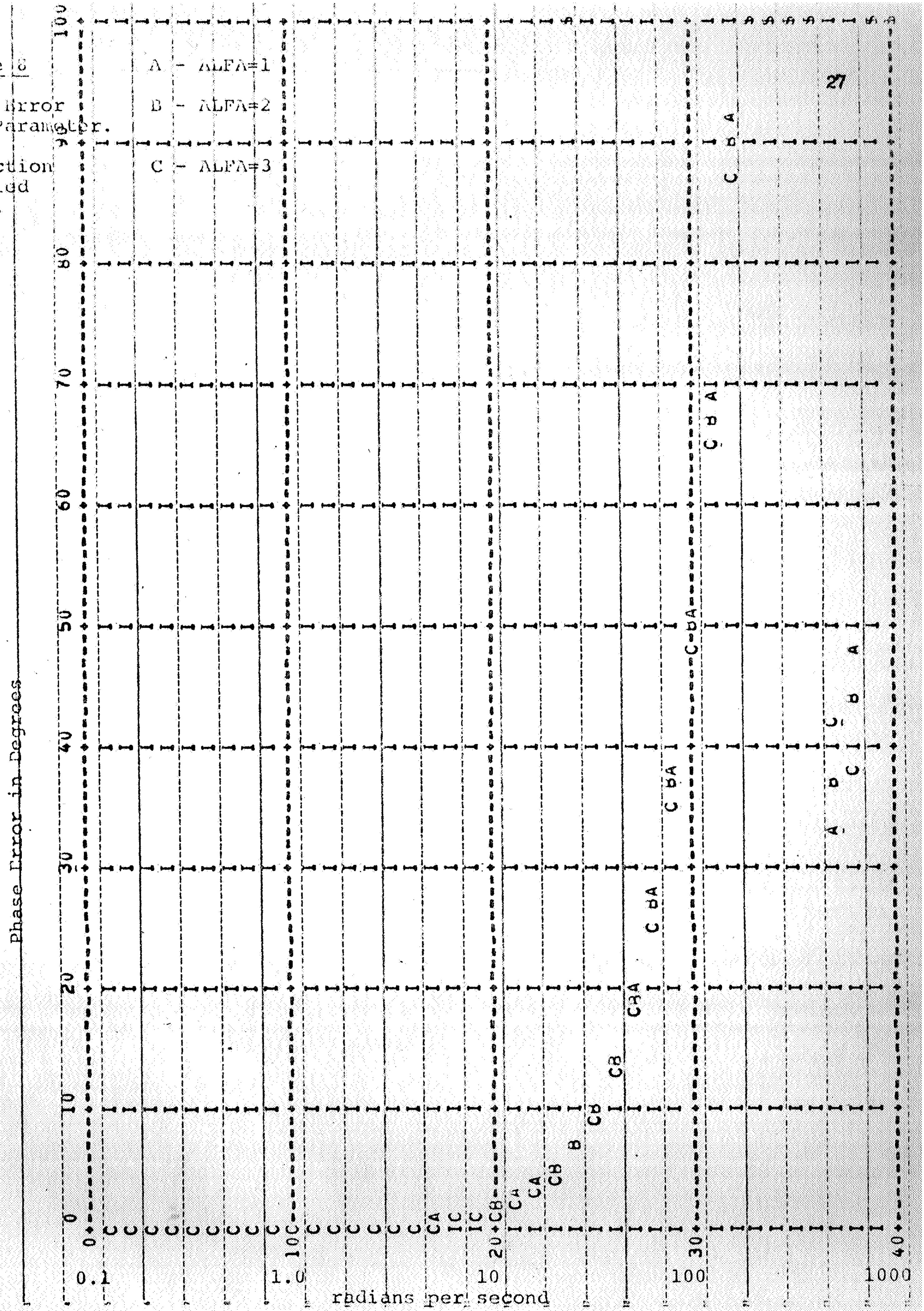


Figure 9

Magnitude Error of A Parameter

ALFA = 1

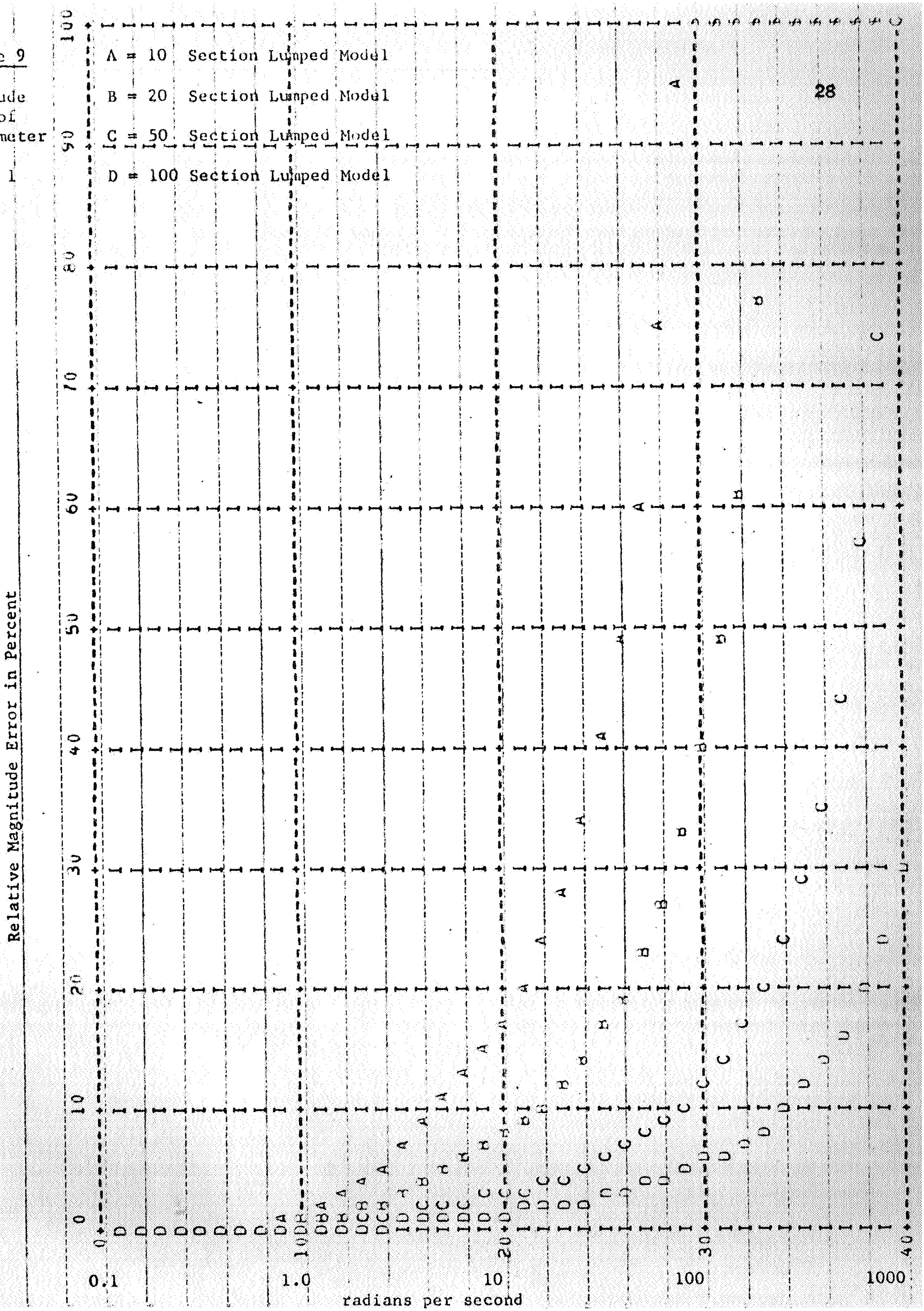


Figure 10

Phase Error of A Parameter.

ALFA = 1