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DESIGN OF STRAPDOWN GYROSCOPES FOR A DYNAMIC ENVIRONMENT

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ABSTRACT

This report briefly summarizes the work performed during the period 1 December 1968 through 31 May 1969 for the NASA Electronics Research Center under Contract NAS 12-508. A description of the effort planned for the remainder of the contract is also presented.

TABLE OF CONTENTS

1. INTRODUCTION

The purpose of this document is to review briefly the work performed during the first six months of the present contract. Whenever a particular item of research is considered to be completed it will be reported in detail in the form of a technical memorandum. All work will be discussed in depth in the Final Technical Report.

Section 2 discusses work concerning strapdown gyros which employ the time-modulation torquing technique. Comparison is made between the sinusoidal frequency response predicted by describing function analysis and that obtained from simulation. Section 3 deals with an extension of that work and the effort discussed in Ref. 1 to cover random gyro inputs. The fourth section summarizes current investigations related to compensating strapdown sensors by measuring the environment and computing corrective action. Section 5 covers the initial steps in an effort to analyze the dynamics of gyros with gas spin bearings. The final section of the report outlines the direction of future efforts.

2. SINUSOIDAL FREQUENCY RESPONSE FOR THE TIME-MODULATED GYRO LOOP

Previous efforts have centered on the binary and ternary-torqued gyro loops. Another type of pulse rebalanced loop, using the time-modulated torquing scheme, is discussed here. This type of torquing is described in Section 2.3 of Ref. 1. The time-modulated loop is illustrated in Fig. 2.1. Note that the float and torque dynamics are characterized in the same manner as the binary and ternary loops treated earlier.

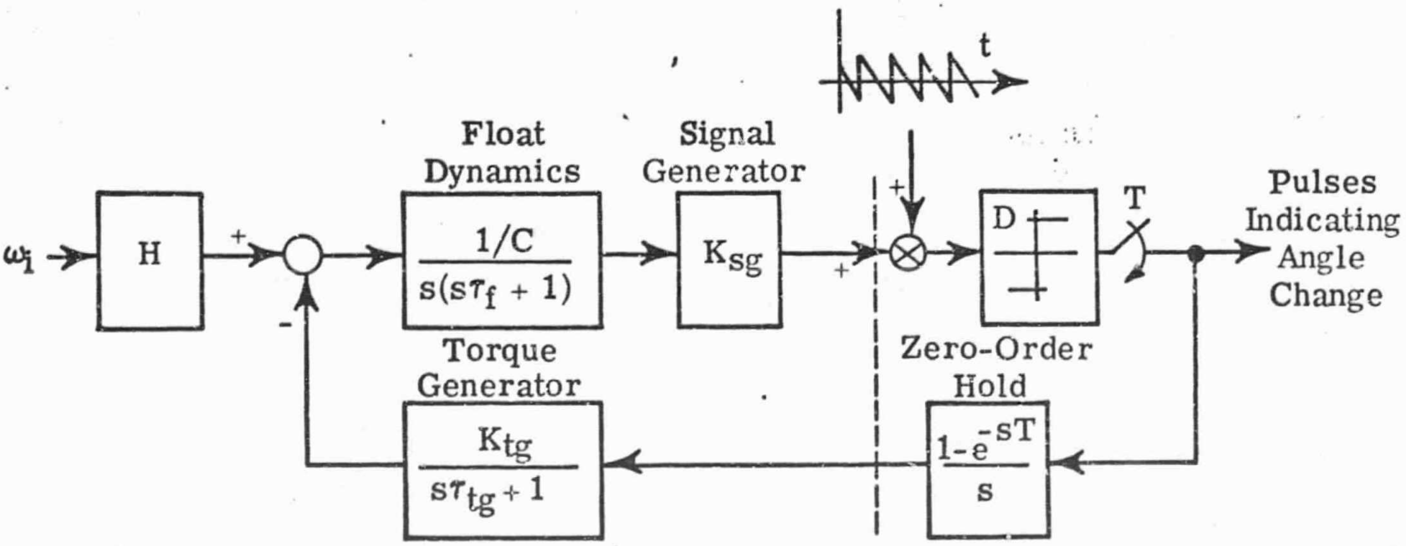

Figure 2.1 The Time-Modulated Torquing Loop

The time-modulated torquing scheme adds the signal representing float angle to a sawtooth or other unbiased periodic waveform (dither) with a frequency considerably higher than gyro inputs of interest. The sum is then passed through a sampled binary nonlinearity. The result is a rebalance torque history which is a square wave composed of positive and negative torque pulses of unequal duration. Float and torquer dynamics sharply attenuate the oscillations at the sawtooth frequency, leaving a float angle which varies slowly with respect to the dither. Consequently, the float angle can be viewed as a slowly varying quantity with some magnitude, B(t), which is essentially constant over each sawtooth period. An equivalent gain of the nonlinearity to the float angle is given in Ref. 2:

$$N_B(B, A_d, D) = \frac{\text{Average Output Over One Sawtooth Period}}{B} \tag{1}$$

where A_d is the amplitude of the sawtooth dither signal shown in Fig. 1. When $B < A_d$ the gain is approximated by

$$N_B(B, A_d, D) \cong \frac{D}{A_d} \tag{2}$$

Note that, as in the case of the limit cycling binary loop, the equivalent gain of the nonlinearity to a low frequency sinusoid is independent of the amplitude and frequency of the sinusoid. For the loop response studies, the nonlinearity was replaced by the gain N_B and the time-modulation gyro loop viewed as a second-order linear system. If the linearization gain is computed using Eq. (2) and sampling effects are neglected, the equivalent damping ratio and natural frequency of the loop are approximately:

$$\zeta = \frac{1}{2}\sqrt{\frac{CA_d}{K_{sg}K_{tg}D\left(\tau_f + \tau_{tg}\right)}} \qquad \omega_n = \sqrt{\frac{K_{sg}\,K_{tg}\,D}{CA_d\left(\tau_f + \tau_{tg}\right)}} \tag{3}$$

Note that the above quantities depend on most of the loop parameters shown in Fig. 2.1, in addition to the dither amplitude.

The linear approximation outlined above was checked using an analog computer simulation of the loop. The set of parameter values chosen to provide a test case is the same as used for the simulation of the binary loop in Ref. 1. The sawtooth amplitude, A_d, was 0.83 mrad of float angle. The results of the study are shown in Fig. 2.2. The analytical response is indicated by the solid line, with simulation data points superimposed. There is generally good agreement between the two methods.

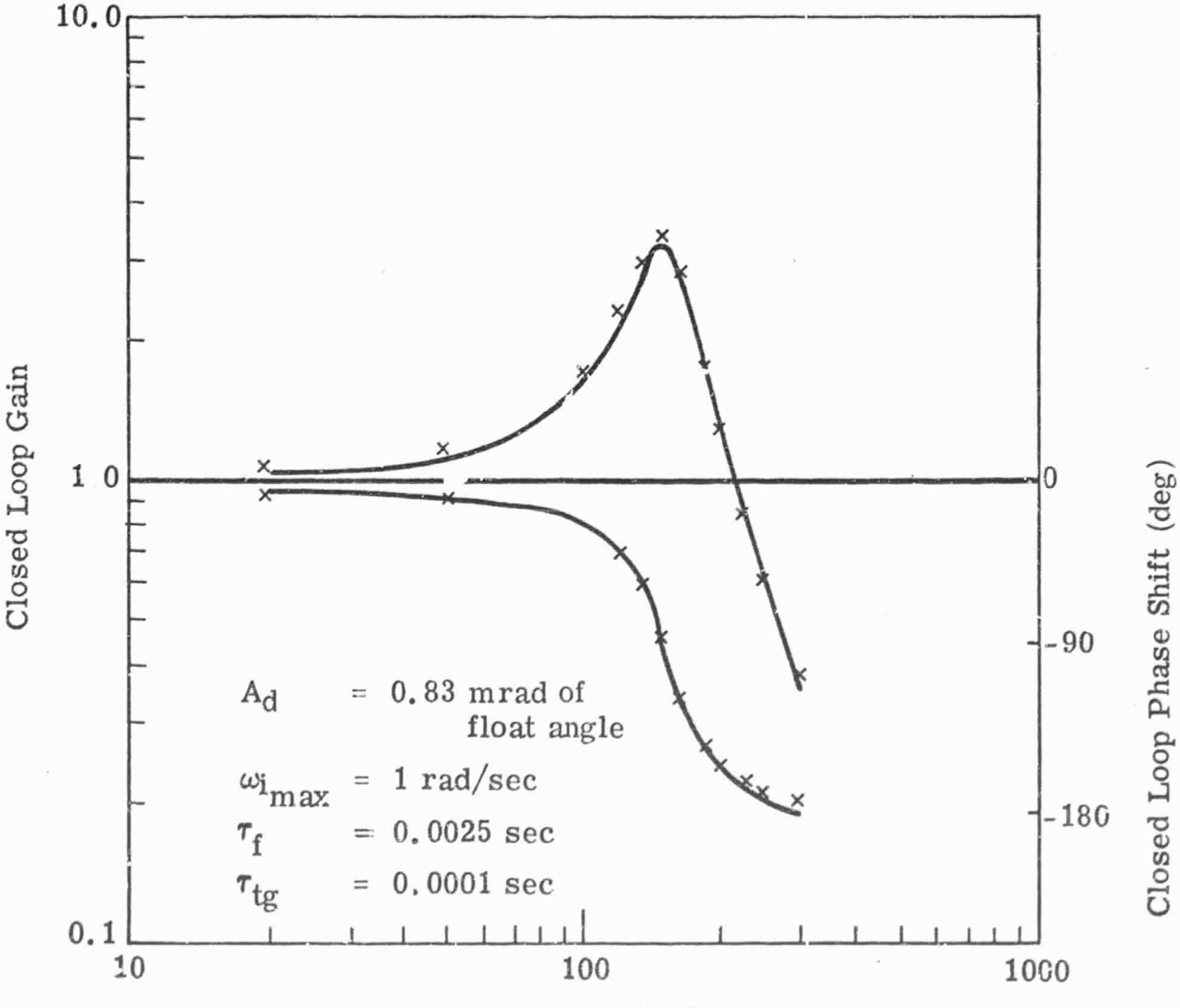

Figure 2.2 Closed Loop Gain and Phase Shift for the Time-Modulated Gyro

The uncompensated gyro loop response illustrated in Fig. 2.2 is not generally representative of the performance sought in strapdown gyros. However linear compensation acting on the output of the signal generator can be used to shape the response of a gyro using time-modulation torquing. Furthermore, synthesis of the compensation network is easier to perform for this gyro rebalance technique than for the binary or ternary torqued instruments.

The research discussed above illustrates that a simple analytical technique can provide accurate descriptions of the response of a nonlinear system to sinusoidal excitations. As in the case of the binary and ternary loops, simulation results support the validity of the describing function approach for analyzing the time-modulated loop. Furthermore, there is sound theoretical basis for expecting this same analytical technique to enable computation of the gyro response to random and other input signal forms.

3. MEASURING LOOP RESPONSES TO RANDOM INPUTS

Reference 1 contains describing function treatments of the binary and ternary loops for single sinusoid inputs. It is shown that the assumption of small signal inputs to the binary loop permits a closed loop sinusoidal transfer function to be written which is equivalent to that of a linear second-order system. The ternary loop requires a more complex treatment and generally retains its nonlinear character. Reference 2 indicates that the "linearizing" effect of the binary loop limit cycle on the nonlinearity will cause the describing function representation of the closed loop to be independent of the shape of the input signal waveform. Approximation of the binary loop as a linear system should also be valid for random inputs. The activity presently underway seeks to verify this statement by analog simulation, and more generally, to investigate the responses of the binary, ternary and time-modulation loops to random inputs when a variety of input spectra are assumed.

To date, random input responses have been measured for the binary and ternary loops with broad bandwidth inputs employed. Some difficulty has been encountered in accurately measuring the power density spectra involved. For the present, a method which gives approximate results has been chosen. Two sixth-order Butterworth filters are tuned at different break frequencies and the random signal to be measured is applied to each filter input. An electronic squarer and low-pass filter are used to measure the average power output of each filter. The difference is approximately equal to the power contained in the band between the two filter break frequencies. This method is limited in accuracy because the filters involved do not have ideal cut-off characteristics. Response measurements have been obtained for the binary loop which shows general agreement with what would be expected from the previously derived sinusoidal response characteristics.

Future random input studies are to be implemented using a commercially-manufactured spectrum analyzer. Arrangements are currently being made at NASA-ERC for the selection and purchase of such an instrument.

4. THREE-AXIS SELF-COMPENSATION

One area of current investigation is the removal of motion-induced errors from strapdown inertial navigators. When gimballed single-degree-of-freedom gyros are used, many errors cannot be significantly reduced through parameter selection. It was proposed in Ref. 3 that the angular rates measured by a gyro triad be used to generate compensation torques for nulling error torques on the individual gyros. The accuracy and stability of such a compensation scheme is difficult to assess analytically because of the complexity of the interconnected information loops and the presence of time-dependent gains. Thus, a simulation study has been undertaken on an analog computer to investigate these properties and draw conclusions about the value of such a scheme.

Initial efforts have centered on compensation of the cross-coupling error of a single gyro. Two gyros are simulated, with the instrument sensitive axes assumed to be at right angles to each other. One gyro (A) experiences angular rates about its spin and input axes which generate a cross-coupling error. The other gyro (B) measures the angular rate about the spin axis of the first in order to provide information for compensation. The cross-coupling error torque appears as the product of the spin-axis rate and the float angle of A, as shown in Fig. 4.1. The compensation scheme takes the output pulse train of gyro B, multiplies it by the float angle of gyro A and applies a correction torque pulse to A.

Ignoring the dynamics of the three torque generators, the compensation torque, M_c, is given by

$$M_c(t) = K_{tg}\,\alpha_o(t)\,x(t) \tag{4-1}$$

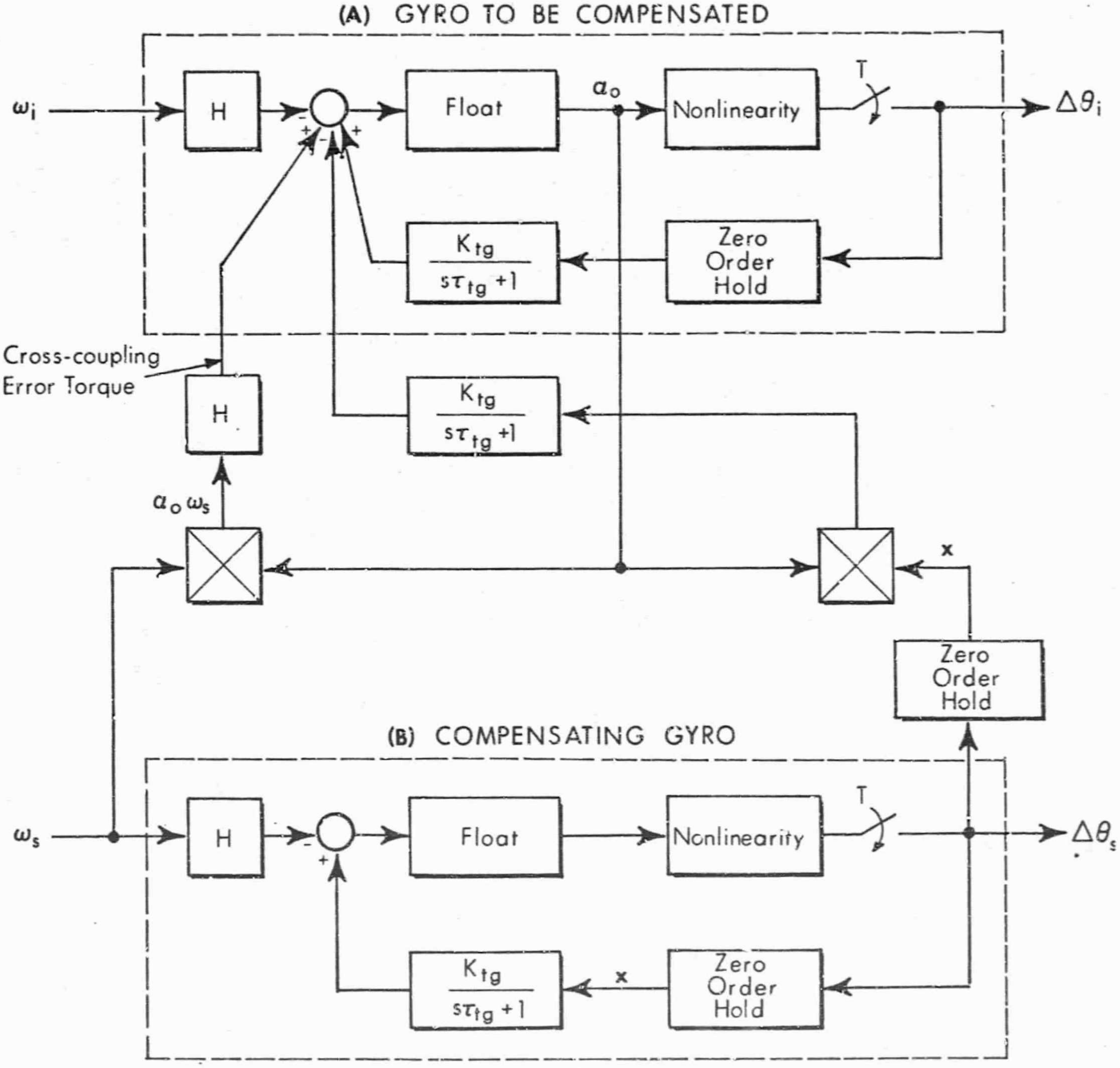

Figure 4.1 Block Diagram of Cross-Coupling Error Compensation of One Gyro

and the crosscoupling error torque, M_e, is

$$M_e(t) = H\,\alpha_o(t)\,\omega_s(t) \tag{4-2}$$

These two torques oppose each other in the compensated gyro giving a net torque

$$M_c(t) - M_e(t) = \alpha_o(t)\,(K_{tg}\,x(t) - H\omega_s(t)) \tag{4-3}$$

In Ref. 1 it was demonstrated that, at least when the input ω_s is a sinusoid in a certain range of frequencies, the fundamental component of the torquer current $x_f(t)$ is related to the input to gyro B by

$$x_f(t) = \frac{H}{K_{tg}}\,\omega_s(t) \tag{4-4}$$

(The current work reported in Section 3 seeks to extend the validity of Eq. (4-4) to the case where random inputs are present.) As a consequence of Eq. (4-4)

$$M_c(t) - M_e(t) = \alpha_o(t)\,K_{tg}\,x_h(t) \tag{4-5}$$

where $x_h(t)$ represents harmonics of the torquer current. If $x_h(t)$ and $\alpha_o(t)$ are uncorrelated and one of them is an unbiased variable the average value of the expression in Eq. (4-5) will be zero.

Simulation studies of this compensation have assumed that the input and spin axis rates are in-phase sinusoids. Thus far, the compensation has proven successful in removing constant cross-coupling errors from the compensated gyro. With sinusoidal inputs of amplitude 0.1 rad/sec on both axes, the compensation is effective at frequencies up to at least 100 Hz.

5. GAS BEARING DYNAMICS

The presence of rotor bearing compliance in a gyroscope can greatly effect the performance of a sensor loop containing the instrument. Reference 1 contains an analysis based on a damped mass-spring model of the compliance, an assumption appropriate to ball or roller bearing rotor supports. However, if a gas bearing is used, generally nonlinear and frequency dependent compliance terms appear and the analysis becomes more complex. The paragraphs below describe research directed towards modelling the gas rotor bearing in such a way as to permit analysis of the gyro's sinusoidal response characteristics.

The technique employed in analyzing the sinusoidal response of the gas bearing gyro makes use of work reported in Ref. 4. There it is shown that the stiffness parameters can be determined and used to compute a compliance matrix describing the rotor displacement in response to a sinusoidal force of constant amplitude and direction. The matrix is written in phasor notation with a particular whirl ratio and whirl sense assumed. This matrix can reportedly be used to compute by superposition the rotor motion in response to any number of any such forces of the same frequency or different frequencies.* The only necessary assumption is that the net rotor displacement is small compared to the nominal rotor-bearing clearance. Making use of this superposition property, any relative motion

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*Telephone conversation of May 28, 1969 with Dr. C.H.T. Pan of Mechanical Technology, Inc., Latham, New York.

between rotor and gimbal can be calculated as a function of the gyro input and output axis torques. If sinusoidal excitation of a particular frequency is assumed, the gain and phase shift between the gyro float angle and the net torque applied about the output axis can be computed. Consequently, the describing function techniques detailed in Ref. 1 can again be used to obtain closed loop frequency responses for the gas bearing gyro, the major difference being a frequency dependent rotor bearing stiffness.

Implementation of the above analysis depends on the availability of data for computing the entries in the compliance matrix. The radial and tangential stiffnesses can be generated via a digital computer program which is currently available.* Alternatively, the compliance matrix may be evaluated directly from vibration tests data of the gas bearing gyro of interest. In the latter case a particular gyro, the Hamilton Standard RI 1170, is currently being studied. It is expected that test data for this instrument will be made available by November, 1969. In the meantime a study will be performed using the stiffness coefficients presented in Ref. 4.

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*From Mechanical Technology, Inc., Latham, New York.

6. CONTINUATION OF EFFORT

During the remainder of the present contract the investigation of gyro loop responses to random inputs will be concluded. The result will be a series of comparisons between the spectral densities observed at the output of a simulated gyro with those predicted from some simplified (quasi-linear) description of its transmission characteristics. Initially the gains to sinusoidal inputs described in Ref. 1 and Section 2 of this report will be used to project random input transmission characteristics. It is anticipated that modifications to these analytic techniques will be made as further experience dictates. Once good agreement between analyses and simulations is obtained, linear compensation networks will be designed to produce more satisfactory closed loop gyro responses.

The simulation study of self-compensation of gyro dynamic errors will continue, with anisoinertia and output axis inertia effects given particular attention. Some simulation work will also be performed regarding compensation using additional sensors or by applying corrections to the gyro outputs.

Analysis of the closed loop response characteristics of gas-bearing gyros will continue. This effort will increase when the Hamilton Standard bearing test data becomes available. A brief study of the laser gyro will be conducted to determine what kinds of strapdown system errors may be generated when these instruments are used to measure angular motion.

A set of equations will be written, describing the constant errors generated in single-degree-of-freedom gyros and pendulous accelerometers by a vibration environment. These equations will be incorporated in a computer program to generate inertial sensor errors, given vibration density spectra. Two system-generated errors, pseudo-coning and pseudo-skulling, will also be treated. The vibration-induced errors will then be converted into position, velocity and attitude errors in a strapdown navigator fixed to the earth.

In addition to the deterministic errors computed in the above manner, sensitivity coefficients will be found to permit calculation of the covariance of the navigator errors arising from random coefficients of dynamic errors. The study of vibration-induced errors will be enhanced by comparison of predicted errors with real data gathered during helicopter ground tests. At the end of the present contract a considerable body of knowledge regarding accurate prediction of vibration-induced strapdown system errors will have been generated.

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