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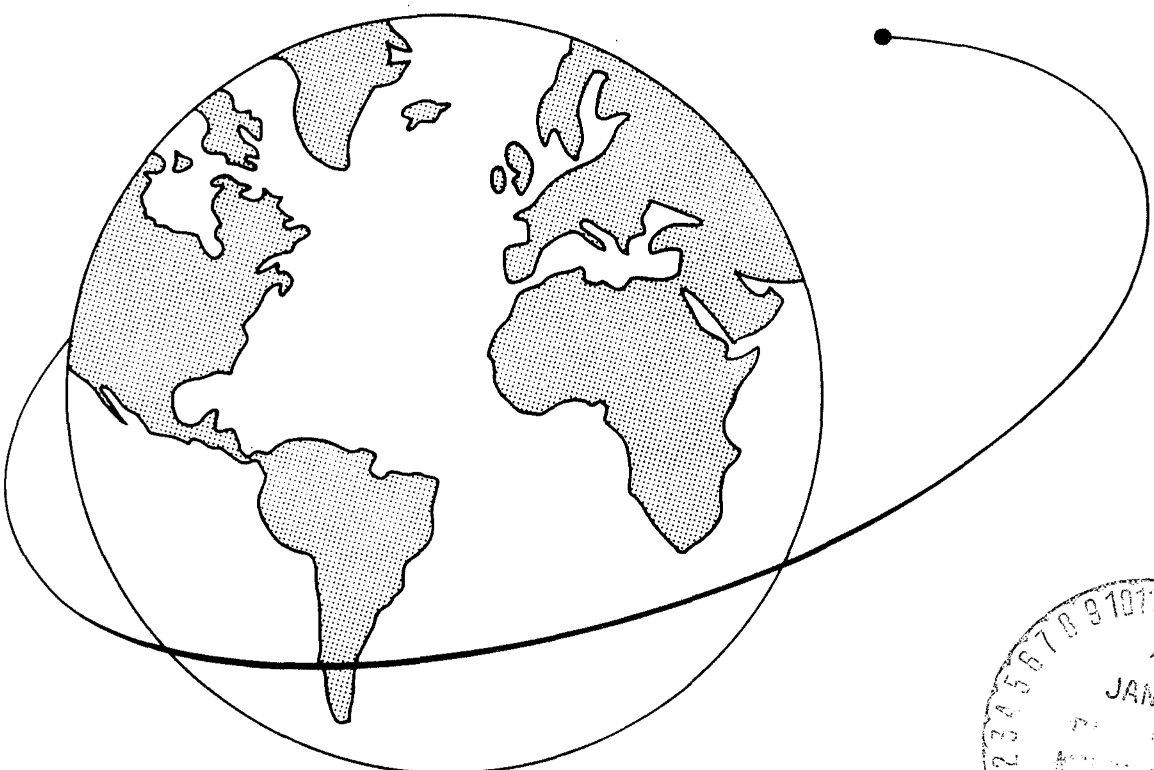
THE MOTION OF A CHARGED SATELLITE IN THE EARTH'S MAGNETIC FIELD

L. SEHNAL



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L. Sehnal

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Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

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ABSTRACT

The perturbations of the orbital elements of a charged artificial earth satellite caused by the earth's magnetic field are studied. A rough estimate of the size of the disturbing effects gives them a very small value. A detailed computation is made in the case of the changes of the inclination of the satellite's orbit.

RÉSUMÉ

On étudie les perturbations causées par le champ magnétique terrestre sur les éléments de l'orbite d'un satellite terrestre artificiel chargé. Une estimation approchée de l'ordre de grandeur des effets perturbateurs indique qu'ils sont de très faible valeur. Une calculation détaillée est faite dans le cas des changements d'inclinaison de l'orbite du satellite.

КОНСПЕКТ

Изучаются возмущения орбитальных элементов заряженного искусственного спутника Земли вызываемые земным магнитным полем. Грубая оценка размеров возбуждающих эффектов указывает на очень малую величину. Сделано детальное вычисление для случая с изменениями наклона орбиты спутника.

THE MOTION OF A CHARGED SATELLITE IN THE EARTH'S MAGNETIC FIELD

L. Sehnal

1. INTRODUCTION

A number of papers discuss the influence of the earth's magnetic field on the motion of a charged artificial satellite; however, most of them deal with the influence of the earth's magnetism on the rotation of the satellite body or with the motion of a satellite in a plasma field, which changes slightly the neutral drag coefficient. The perturbational approach to the problem was probably first studied in a paper of Fain and Greer (1959). Their investigation was further developed by Westerman (1960), who studied the simple case of a circular orbit in the plane of the geomagnetic equator.

In this paper, we check the magnitude of the perturbations arising from the influence of the earth's magnetism, in a case approximating reality more closely than previous papers.

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2. BASIC EQUATIONS

Let us consider the magnetic field of the earth to be given in the form of a series of spherical harmonics; furthermore, we shall suppose the earth's magnetic axis to be identical with the earth's axis of rotation. We shall seek changes of the orbital elements of an electrically charged satellite caused by that magnetic field. We shall start with the Lagrangian equations in Gaussian form. We can write them generally in the form

$$\frac{d\sigma}{dt} = K_\sigma f(S, T, W) \quad , \tag{1}$$

where σ denotes an arbitrary element, and S, T, and W are the components of disturbing accelerations in the direction of, and perpendicular to, the radius vector and binormal to the orbit. The constant K_σ is a simple combination of the orbital elements appearing in the Lagrangian equation; it is different for the change of each element.

The components of the disturbing force we are looking for will be the components of a vector

$$\vec{F} = Q\,(\vec{v} \times \vec{B}) \quad , \tag{2}$$

where $\vec{v}$ is the velocity of a satellite in its orbit, $\vec{B}$ is the vector of the magnetic intensity, and Q is the satellite's electrical charge. The components of the vector $\vec{F}$, from equation (2), in the directions S, T, and W will then be

$$(\vec{v} \times \vec{B})_S = v_T \, B_W - v_W \, B_T \quad , \qquad \text{etc.} \tag{3}$$

The components of the satellite's velocity vector are

$$v_S = \dot{r} \quad , \qquad v_T = r \, \dot{w} \quad , \qquad v_W = 0 \quad , \tag{4}$$

where w denotes the true anomaly. We can obtain the components of $\vec{B}$ from the expression for the earth's magnetic field V, which can be written

$$V = P_1^{(0)} (\cos \theta) \, g_1^0 \left(\frac{R}{r}\right)^2 R + P_1^{(1)} (\cos \theta) \left(g_1^1 \cos \lambda + h_1^1 \sin \lambda\right) \left(\frac{R}{r}\right)^2 R + \ldots \quad , \tag{5}$$

where g_1^0, g_1^1, and h_1^1 are the numerical constants that describe the earth's magnetic field, θ is the angular distance from zenith, $P_1^{(0)}$, $P_1^{(1)}$ are the associated Legendre functions, and R is the radius of the earth.

The second term in this expression depends on λ, the geocentric longitude, so that it does not give rise to secular perturbations. Thus, we shall consider only the first term in equation (5).

The components of the magnetic force X (horizontal, northward), Y (horizontal, eastward), and Z (vertical, downward) can be obtained from equation (5) by partial differentiation. Considering the first term only, we have

$$X = \frac{1}{r} \frac{\partial V}{\partial \theta} = - \, g_1^0 \sin \theta \left(\frac{R}{r}\right)^3 ,$$

$$Y = - \frac{1}{r} \frac{1}{\sin \theta} \frac{\partial V}{\partial \lambda} = 0 \quad ,$$

$$Z = \frac{\partial V}{\partial r} = - 2 \ g_1^0 \ \cos \ \theta \left(\frac{R}{r}\right)^3 \tag{6}$$

These components will now be transformed into the directions S, T, and W merely by rotation around the radius vector, since

$$Z = - \ B_S \tag{7}$$

For the transformation, we shall make use of the relations that appear in Figure 1.

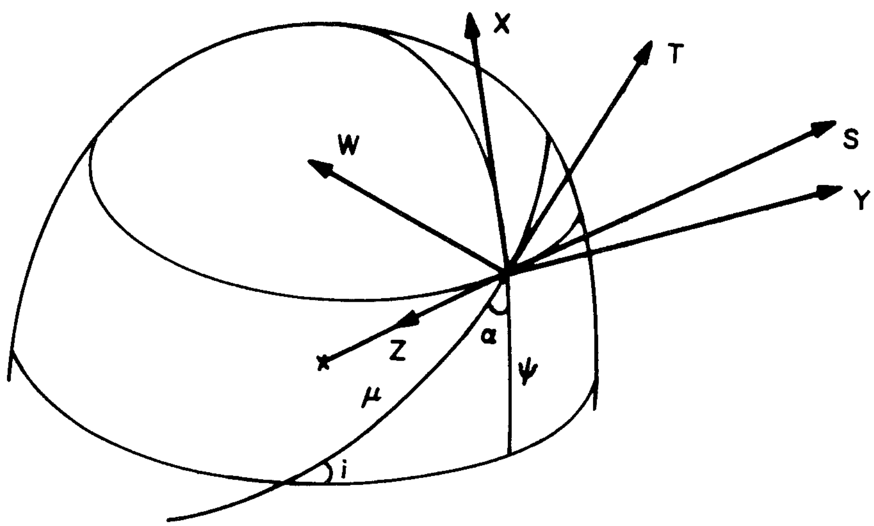

Figure 1. Geometry of X, Y, Z and S, T, W.

With respect to equations (6) we have

$$B_T = X \cos \alpha \quad ,$$

$$B_W = X \sin \alpha \quad ;$$

according to equations (6) and (7), after substitution of $\phi = 90° - \theta$ we have immediately

$$B_S = 2\, g_1^0 \left(\frac{R}{r}\right)^3 \sin\phi \ ,$$

$$B_T = -\, g_1^0 \left(\frac{R}{r}\right)^3 \cos\phi \, \cos\alpha \ ,$$

$$B_W = -\, g_1^0 \left(\frac{R}{r}\right)^3 \cos\phi \, \sin\alpha \tag{8}$$

Again from Figure 1 we have the relations

$$\sin\phi = \sin i \sin u \ ,$$

$$\cos\phi \sin\alpha = \cos i \ ,$$

$$\cos\phi \cos\alpha = \sin i \cos u \ , \tag{9}$$

where the argument of the latitude $u = \omega + w$. After substituting equations (9) into equations (6) and using equations (3), we have for the components of the disturbing force the expressions

$$S = -\, Q\, g_1^0 \left(\frac{R}{r}\right)^3 \cos i \; r\dot{w} \ ,$$

$$T = +\, Q\, g_1^0 \left(\frac{R}{r}\right)^3 \cos i \; \dot{r} \ ,$$

$$W = -\, Q\, g_1^0 \left(\frac{R}{r}\right)^3 \sin i \left[\cos(\omega + w)\, \dot{r} + 2 \sin(\omega + w)\, r\, \dot{w}\right] \tag{10}$$

3. ESTIMATE OF THE NUMERICAL VALUES OF THE PERTURBATIONS

Substituting equations (10) into the Lagrangian equations in expression (1), we obtain the relations for the changes of the orbital elements. The magnitudes of the components of the disturbing accelerations (according to equations (10)) can be determined by the product

$$Q\, g_1^0 \frac{1}{m} \quad .$$

The value of this product will not be too high; thus, we can expect only small changes in the orbital elements. In cgs units, we can accept for g_1^0 the approximate value of -0.3 Gauss. Let us transform the charge of the satellite Q into volts. Accepting the value of the dielectrical constant to be 1 in our case, we have

$$Q = \frac{1}{9}\, 10^{-12}\, \delta F \quad ,$$

where δ is the radius of the satellite's body in centimeters, and F is the voltage of the satellite. Then the factor $Q\, g_1^0\, m^{-1}$ will be approximately

$$-\frac{1}{3}\, 10^{-13}\, \frac{\delta}{m}\, F \quad .$$

If we suppose the ratio δ/m to be of the order 10^{-2} or 10^{-3}, we shall have, by computing the changes of the elements, the coefficient 10^{-15} F, where time is measured in seconds and the length in centimeters. The remaining expressions from equation (1), like the constants K and the results of the computation of the changes of the elements, give some combinations of orbital elements only, which will not substantially affect the final numerical results. Thus, we find that the changes of angular elements (Ω, ω) will not exceed 10^{-5} deg day^{-1}, and the change of the radius vector of the orbit during one revolution will undergo short-periodic perturbations measurable in fractions of centimeters.

4. CHANGE OF THE INCLINATION

There is one element — the inclination — whose change is so small that it may be comparable to our above results. The secular decrease of the inclination, which is caused mainly by the rotation of the atmosphere, was observed to be greater than the expected theoretical values (see King-Hele, 1967; Nigam, 1963). Since the discrepancy between theory and observations is of the order 10^{-6} to 10^{-5} deg day^{-1}, let us see if the effect of the earth's magnetic field does not contribute somehow to this value.

The change of the inclination is given (Brouwer and Clemence, 1961) by

$$\frac{di}{dt} = \frac{1}{na} \frac{1}{\sqrt{1 - e^2}} \frac{r}{a} \cos (\omega + w)\, W \quad .$$

Substituting the expression for W, which now stands for the disturbing acceleration, from equation (10), we have

$$\frac{di}{dt} = - \frac{1}{na} \frac{1}{\sqrt{1 - e^2}} \frac{Q}{m} g_1^0 \left(\frac{R}{a}\right)^3 \sin i \left(\frac{a}{r}\right)^2 \cos (\omega + w)[\cos (\omega + w) \dot{r} + 2 \sin (\omega + w)\, r \dot{w}\,] \, . \tag{11}$$

Although the charge Q of the satellite can be considered constant, it is more probable that it changes with height, according to the change from the ionized to the neutral portions of the atmosphere. Let us suppose that the potential changes with some power n of the ratio of the height of the satellite in its orbit to the height of the perigee, h_p:

$$Q = Q' \left(\frac{h}{h_p}\right)^n \quad , \tag{12}$$

where Q' is the charge of the satellite in the perigee height. Since we have the simple relations

$$h = a(1 - e \cos E) - R \ ,$$

$$h_p = a(1 - e) - R \ ,$$

where R is the radius of the earth, we shall develop the expression (12) according to the powers of $\cos^i E$, where E is the eccentric anomaly. Thus, we obtain

$$Q = Q' \left(\frac{a - R}{r_p - R} \right)^n \sum_{\ell = 0}^{L} k_\ell \cos^\ell E \ .$$

The coefficients k_ℓ are given as

$$k_\ell = \frac{n(n-1)(n-2) \dots (n - \ell + 1)}{\ell !} \left(\frac{a - r_p}{R - a} \right)^\ell \tag{13}$$

Writing

$$R' = Q' \left(\frac{a - R}{r_p - R} \right)^n \ ,$$

we have finally

$$Q = R' \sum_{\ell = 0}^{L} k_\ell \cos^\ell E \tag{14}$$

Considering expression (13) to be of the order of the eccentricity e, we shall put the limit L = 4, the fourth power of the eccentricity.

We shall make use of the formulas

$$r \dot{w} = a \sqrt{1 - e^2}\, \dot{E}$$

$$\dot{r} = a\ e\ \sin E\ \dot{E}$$

$$\sin w = \sqrt{1 - e^2}\, \frac{a}{r}\ \sin E$$

$$\cos w = \frac{a}{r}\ (\cos E - e)$$

$$\frac{a}{r} = (1 - e \cos E)^{-1}$$

by transformation of equation (11) into an expression for the eccentric anomaly. Now we can develop the expression $(a/r)^4$ into a series,

$$\left(\frac{a}{r}\right)^4 = \sum_{j=0}^{4} f_j \cos^j E \quad ,$$

and we can write the rest of the right-hand side of equation (11) as

$$\cos (\omega + w) \left[\cos (\omega + w)\, \dot{r} + 2 \sin (\omega + w)\, r \dot{w} \right]$$
$$= \left(\sum_{m=0}^{3} a_m \cos^m E + \sum_{n=0}^{2} b_n \cos^n E \sin E \right) \dot{E}$$

After substituting the above two equations and equation (14) into equation (11), we multiply the series and obtain the expression

$$\frac{di}{dt} = -\frac{1}{n}\frac{1}{\sqrt{1-e^2}}\frac{R'}{m}g_1^0\left(\frac{R}{a}\right)^3 \sin i$$

$$\times\left(\sum_{s=0}^{11} A_s \cos^s E + \sum_{t=0}^{10} B_t \cos^t E \sin E\right)\dot{E} \quad .$$

The powers of the trigonometric functions of the eccentric anomaly can readily be transformed into the trigonometic functions of the multiples of the argument (see, e.g., Sehnal and Mills, 1966, equation (28)), so that we obtain

$$\frac{di}{dt} = -\frac{1}{n}\frac{1}{\sqrt{1-e^2}}\frac{R'}{m}g_1^0\left(\frac{R}{a}\right)^3 \sin i$$

$$\times \sum_{i=0}^{11} \cos i\,E\, Z_i + \sum_{i=1}^{11} \sin i\,E\, W_i \quad \dot{E}$$

After integration and substitution of E = nt + e sin E, we have the expression for the change of the inclination:

$$\Delta i = -\frac{1}{n}\frac{1}{\sqrt{1-e^2}}\frac{R'}{m}g_1^0\left(\frac{R}{a}\right)^3 \sin i$$

$$\times\left[Z_0' \, nt + \sum_{i=1}^{11}\frac{Z_i'}{i}\sin i\,E + \sum_{i=1}^{11}\frac{W_i}{i}(1-\cos i\,E)\right] \quad , \tag{15}$$

where

$$Z_0' = Z_1 + e\,Z_0 \quad ,$$

$$Z_i' = Z_i \quad , \qquad \text{if } i \neq 1$$

The term most interesting to us well be the one that is not periodic with the eccentric anomaly E. From previous analysis, we can derive the expression for the term Z_0 in the form

$$Z_0 = \sum_{t=0}^{11} k_t \sum_{s=0}^{11} d_0^{(1)} \sum_{j=0}^{4} f_j \times a_{s-t-j}$$

The coefficients $d_0^{(s)}$ arise from the transformation

$$\cos^s E = \sum_{i=0}^{s} d_i^{(s)} \cos i E \quad ,$$

and the coefficients a_k are found to be

$$a_0 = -(1-e^2)\sqrt{1-e^2} \sin 2\omega \quad ,$$

$$a_1 = -3 e\sqrt{1-e^2} \sin 2\omega \quad ,$$

$$a_2 = +2(1-e^2)\sqrt{1-e^2} \sin 2\omega \quad ,$$

$$a_3 = +e\sqrt{1-e^2} \sin 2\omega \quad .$$

Finally, the expression for Z_0 is

$$Z_0 = \frac{1}{2}\sqrt{1-e^2} \sin 2\omega \left[-\frac{1}{4} e \left(1+e^2\right) n \frac{a-r_p}{R-a} + \frac{1}{2}\left(1-\frac{5}{16}e^4\right)\frac{n(n-1)}{2!}\left(\frac{a-r_p}{R-a}\right)^2 + \frac{3}{8} e \left(1+\frac{1}{4}e^2\right)\frac{n(n-1)(n-2)}{3!}\left(\frac{a-r_p}{R-a}\right)^3 + \ldots \right] \quad (16)$$

Now we see that, owing to the presence of sin 2ω, this term is not really constant, but rather long periodic.

For integer values of n, there will be a finite number of terms in the series (16). We see immediately that for $n = 0$, that is, for the constant value of the satellite's electrical charge, $Z_0 = 0$. Consequently (viz. equation (15)), in this case we have the short-periodic perturbations of the inclination only.

If we suppose that the charge is growing in linear dependence on the height of the satellite, $n = 1$, we have for the change of the inclination

$$\Delta i = - \frac{1}{8} e \left(1 + e^2\right) \frac{Q'}{m} g_1^{(0)} \sin i \sin 2\omega \left(\frac{R}{a}\right)^3 \frac{a - r_p}{h_p}$$

This expression vanishes in the case of a circular orbit, which is, in our case, identical with the condition of a constant charge, so that we have $\Delta i = 0$ if $e = 0$.

Introducing voltage instead of charge Q and using the transformation of the units into the cgs system, we have finally the equation

$$\Delta i = - 4 \times 10^{-15} \frac{\delta}{m} Fe \left(1 + e^2\right) \sin i \sin 2\omega \left(\frac{R}{a}\right)^3 \frac{a - r_p}{h_p} \quad .$$

To compare the numerical value of this change with the observed one, let us neglect the dependence of this expression on sin 2ω, putting $\sin 2\omega = 1$. If we imagine an orbit of a satellite with a perigee height $h_p = 300$ km, $e = 0.3$, $a = 9540$ km, and $\sin i = 0.9$, we then have, for a change of the inclination in degrees per day, the equation

$$\Delta i = - 2 \times 10^{-8} \frac{\delta}{m} F \quad .$$

This means that if we want to be in agreement with the observed values, we should have

$$\frac{\delta}{m} F \sim 10^{+2} \text{ to } 10^{+3} \quad ,$$

which would require the voltage of a satellite to be at least 10^3 volts. Since no such high potential of a satellite body was observed and cannot even be supposed theoretically, we do not obtain any observable effects of the earth's magnetic field on the change of the inclination of the satellite's orbit. Nevertheless, some measurements of the charge of a satellite body should provide more precise knowledge of the phenomenon; at the present time, information available does not allow us to make any precise evaluation of all the effects of the earth's magnetic field on the orbit of a charged satellite.

5. ACKNOWLEDGMENTS

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BIOGRAPHICAL NOTE

LADISLAV SEHNAL received the M.Sc. degree from Charles University in Prague, Czechoslovakia, in 1954 and the Ph.D. degree from the Astronomical Institute in Prague in 1959.

Dr. Sehnal is a research worker at the Astronomical Institute in Prague. In 1966 he was a celestial mechanician at SAO, a position he resumed in 1969.

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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