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DELTA-MODULATION FOR THE TRANSMISSION OF IMAGE DATA

by Leonard Ehrman, Herbert Gish

27 August 1969

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National Aeronautics and Space Administration  
Electronics Research Center  
RC/Computer Research Laboratory  
Cambridge, Massachusetts

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## SUMMARY

This final technical report deals with the application of delta-modulation digitization techniques to the efficient transmission of image data. The report reviews the different forms of delta-modulators and compares them on the basis of performance and complexity. On the basis of these comparisons several types are recommended for experimental, high data rate, laboratory use. The report also includes an extensive bibliography on delta-modulation which includes the most recent studies of both United States and foreign origin.

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## I. INTRODUCTION

### 1.1 Purpose of Study

It is common practice today to transmit television pictures by analog means. This is so for both commercial application and for certain government use, such as the transmission of cloud data from weather satellites. However, as technology advances and digital communications techniques become more accepted, television will inevitably be transmitted by digital methods. This report is a study of means of digitizing the analog video signal through the use of the class of systems known as feedback quantizers. In feedback quantizers, the quantizer output is, in some manner, fed back to and compared with the input signal. An error signal is developed which is a function of the error between the (continuous) input signal and the (discrete) quantizer output. The actual quantizer output is some function of this error signal. One of the earliest forms of feedback quantizer is the 1-bit delta modulator, developed in the late 1940's. Since then, many other feedback quantizers, both 1-bit and N-bit, have been developed. While much of the research on these devices has been concerned with speech applications, we will consider them from the point of view of video processors.

The television processing chain is shown in Fig. 1.1. The chain starts with an optical signal which is generated by a picture source, and ends with an optical signal which is generated by a picture display. In order to transmit the optical source signal, it must first be converted into an electrical signal by a video sensor such as a vidicon. The signal is, at this point, analog. It is next encoded into a suitable digital signal, typically a binary data stream, by a digital modulator, and transmitted to the receiver by a RF Encoder-Channel-RF Decoder chain. the RF Decoder output is again a digital (binary) data stream, which is converted back to an analog signal by a digital demodulator. This analog signal, which is hopefully a close replica of the original analog signal from the video sensor, drives the picture display.

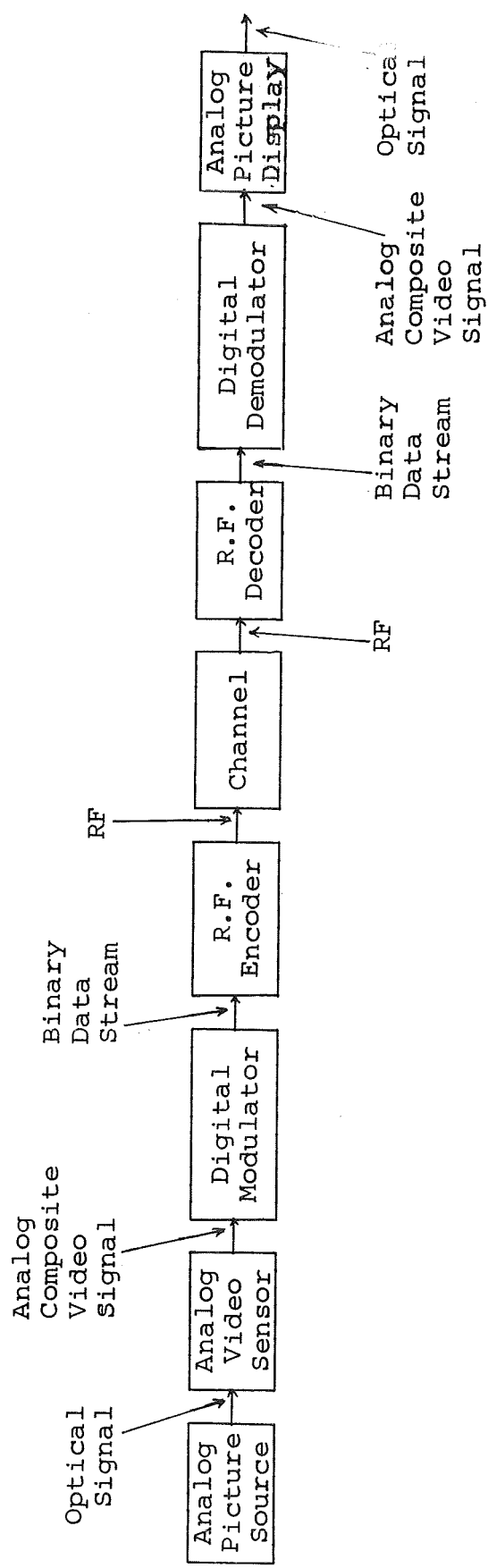


Fig. 1.1.1 Television Processing Chain



The process of digital modulation and demodulation can be performed in many different ways. The most straightforward of these is pulse code modulation, or PCM. Unfortunately, PCM, as will be shown, is quite wasteful of bandwidth. In addition, its implementation is in general complex. Many different means of reducing the bandwidth of digital television have been performed in the past, with the results being of mixed quality and value. Some techniques have been developed which reduce the bandwidth by factors of five to ten, but their implementations are also complex and the resulting pictures of uncertain quality. The purpose of this study is to consider an intermediate class of digital television systems, having a moderate amount of bandwidth compression and still retaining a simple implementation. In our study we define three such systems, each of varying complexity. Depending on the particular system, the bandwidth required ranges from about the same for PCM to approximately 40 percent of that required for PCM, that is, up to a 2.5 bandwidth compression.

## 1.2 Summary of Results

Chapter II provides an introduction to the digital television problem. In this chapter we first present the random process which characterizes the video signal. It is shown that a useful model is a Gaussian random process with a power spectrum given by:

$$S(\omega) = \begin{cases} \frac{1}{\alpha^2 + \omega^2} & \omega \leq \omega_0 \\ 0 & \omega > \omega_0 \end{cases} .$$

The picture content and scanning technique determine  $\alpha$  and  $\omega_0$ . We next introduce the information content of a video picture. From recent measurements of higher order entropies of a picture,

this is estimated to be in the range of 1 bit per picture element. Since a band-limited signal must be sampled at a rate of twice the bandwidth, a commercial television signal, having 4.5 MHz bandwidth would be sampled at a 9 Mbps rate. Therefore the minimum bit rate for transmission of a commercial television signal would also be 9 Mbps. As a point of reference, it is shown that the quantization introduced by PCM encoding is such that 6 bit PCM is required for "good" television reproduction. Good television is defined as a nominal 28 dB RMS video-to-quantizing noise power ratio. Thus PCM requires a bit rate which is six times greater than indicated by information theory.

In Chapter III we define and discuss five types of feedback quantization systems. They are

- 1-bit Delta Modulation
- N-bit Delta Modulation
- 1-bit Direct Feedback Modulation
- N-bit Direct Feedback Modulation
- N-bit Predictive Quantization

Block diagrams, system operation, and signal-to-noise ratios are presented for each system. Both companded and non-companded systems are considered. It is shown that "good" television transmission can be expected at bit rates ranging from factors of 4.5 to 13 times the video bandwidth. In particular, 28 dB RMS video signal-to-noise ratio is expected at the following bit rates for commercial television:

1-bit Delta Modulation	56 x 10 <sup>6</sup> bps
N-bit Delta Modulation	36 x 10 <sup>6</sup> bps
1-bit Direct Feedback -- No companding	36 x 10 <sup>6</sup> bps
1-bit Direct Feedback -- Companding	25 x 10 <sup>6</sup> bps
3-bit Direct Feedback -- No companding	32 x 10 <sup>6</sup> bps
3-bit Direct Feedback -- Companding	21 x 10 <sup>6</sup> bps
N-bit Predictive Quantization: element feedback	24 x 10 <sup>6</sup> bps

Thus the 3-bit direct feedback system with companding has about a factor of two higher bit rate than the theoretical minimum, while 1-bit delta modulation requires about the same bit rate as does PCM. While the bit rates listed above indicate that N-bit systems can be expected to outperform 1-bit systems, the implementation of an N-bit quantizer and the requirement for word-sync indicate that the companded 1-bit direct feedback system would make a useful, low-bit rate television transmission system.

In Chapter IV we summarize the performance of the 1-bit and N-bit systems, rank them in subjective order, and consider the problems involved in multiplexing the various systems. Block diagrams of 1-bit delta and direct feedback modulators and demodulators are given, as is a block diagram of a proposed digital compander. It is estimated that the 1 bit systems can be operated at bit rates as high as 100 Mbps. The problems involved in multiplexing the modulators are discussed, and it is shown that time or frequency multiplexing of the digital outputs of independent digital modulators is the most reasonable way to multiplex different television sources. Consideration is also given to the multiplexing of color television; three ways of transmitting color signals are presented.

The report ends with a bibliography of papers dealing with television bandwidth compression and feedback quantization systems. The time period covered by this bibliography is 1951 to 1969. All reports listed can be found in the open literature, and represent literature from foreign as well as United States research.

## II. REQUIREMENTS FOR AN EXPERIMENTAL SYSTEM

In this chapter we will outline the requirements for a delta modulation system to be used for television experiments. The chapter is divided into three parts. They are:

- 2.1 The Video Process
- 2.2 Picture Entropy and Bandwidth Requirements
- 2.3 Signal-to-Noise and Quantization

In 2.1 we introduce the video process and describe its characteristics in the time and frequency domains. This is necessary for an understanding of the modulation and multiplex techniques to be discussed later in the report.

In 2.2 the entropy of the video picture is introduced, and current bandwidth reducing digital techniques described.

In 2.3 we introduce the various signal-to-noise criteria which are used with the video signals, and summarize the subjective effects which quantizers produce on the reproduced signal.

### 2.1 The Television Signal

The television signal is the sum of two signals -- the video and the synchronization signal. The synchronization signal is deterministic and periodic, and has a line spectrum in the frequency domain. The video signal, on the other hand, is random and can only be described by statistical terms. Thus, when describing the video signal, we talk of its autocorrelation function and its power spectrum. When it is necessary to discuss the complete television signal, that is the

combined video and the sync signals, they are denoted as the composite signal.

The video signal is made up of three basic scans.

They are

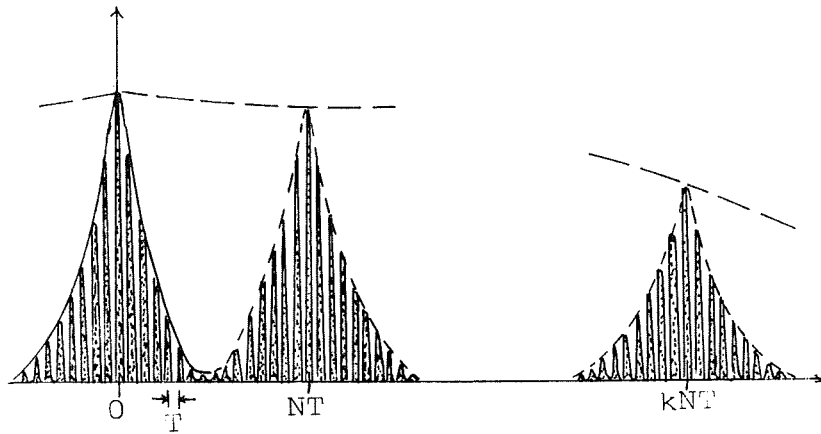
- a) an element scan, i.e., a scan from element to element along a line
- b) a line scan
- c) a frame scan.

There are correlations in all three scanning directions: any picture element has at least one neighbor in the horizontal and vertical directions, as well as in time. Experimental data [Franks, 1966; Kretzmer, 1952; Deriugin, 1957] indicate that, to a first approximation, these correlations can be taken as exponential, and that they can also be considered to be isotropic. The overall correlation function of the video signal is the convolution of the correlation function of the element, line, and frame scans, and the power spectrum of the video signal is the product of their spectra. The "typical" auto-correlation function and power spectrum of the video process are shown in Fig. 2.1. The three structural components of the power spectrum should be noted. The fine structure of the spectrum comes from the frame scan, the intermediate structure comes from the line-scan, and the envelope structure comes from the element-element scan.

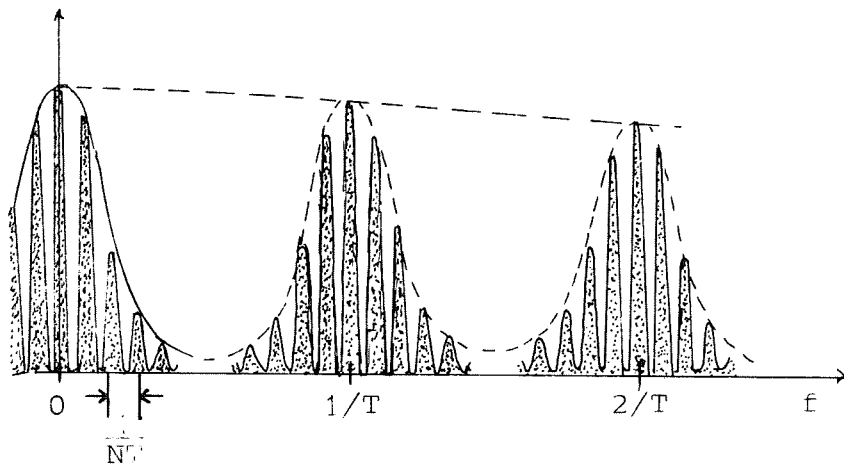
As was pointed out above, the correlation functions can, to a first approximation, be taken as exponential. Therefore the envelope of the power spectrum has the form:

$$\Phi(f) = \frac{1}{(2\pi f)^2 + \alpha^2}$$

This envelope has, however, an infinite bandwidth, while video signals are conventionally truncated at some maximum frequency  $f_0$ . Therefore a useful approximation to the video spectrum is:



AUTO-CORRELATION FUNCTION



POWER SPECTRUM

FIGURE 2.1 Typical Auto-Correlation Function and Power Spectrum of a Video Signal [Franks, 1966]

T = time for one line scan  
 N = number of lines per frame

$$\Phi_v(f) = \begin{cases} \frac{1}{(2\pi f)^2 + \alpha^2} & 0 < f < f_0 \\ 0 & f \geq f_0 \end{cases}$$

For commercial monochrome TV in the United States,  $f_0 = 4.5$  MHz. The composite television spectrum has a corner at: [O'Neal, 1966]

$$\begin{aligned} \alpha &= 0.068 f_0 \\ &= 306 \times 10^3 \text{ radians/sec} \end{aligned}$$

This is a nominal value for  $\alpha$ , the value of which will, in general, vary with picture content.

## 2.2 Picture Entropy and Bandwidth Requirements\*

A measure of the information content, and therefore the required channel capacity, of a television picture is given by its conditional entropy. If the video signal were a Gaussian random process with an exponential correlation function, then it would be a first-order Markov process, and an optimum predictor would be built based on the last sample value. A measure of how closely this is so is given by the higher-order conditional entropies. If all higher-order conditional entropies are equal, then the process is first-order Markov; if the second and higher order conditional entropies are equal, then the process is second-order Markov, etc. Measurements of the conditional entropies of video signals have been made by Schreiber [Schreiber, 1956], and Limb, [Limb, 1968], among others. Schreiber measured the first and second order entropies of "our least complicated picture," using a linear scan and found the following entropies:

---

\* It should be noted that all entropy calculations which have thus far appeared in the literature have used only single-frame pictures.

$H(x)$	$H(y/x)$	$H(z/x,y)$
4.39	1.91	1.49

Measured Picture Entropies: Bits/element  
(from Schreiber)

Limb measured first, second and third order entropies of pictures of varying complexity using both a linear and a rectangular scan. He found the following entropies:

	$H(w)$	$H(x/w)$	$H(y/w,x)$	$H(z/w,x,y)$
Av. Detail	2.87	2.24	1.82	1.46
Av. Detail	2.54	1.99	1.66	1.15
Low Detail	1.33	1.04	0.94	0.90
High Detail	3.77	2.70	2.01	0.87

Measured Picture Entropies: Bits/element  
Rectangular Sampling  
(from Limb)

	$H(w)$	$H(x/w)$	$H(y/w,x)$	$H(z/w,x,y)$
Av. Detail	2.84	2.38	2.10	1.47
Av. Detail	2.49	1.96	1.66	1.28
Low Detail	1.31	0.99	0.97	0.92
High Detail	3.68	3.10	2.23	0.86

Measured Picture Entropies: Bits/element  
Linear Sampling  
(from Limb)

Both Limb's and Schreiber's data indicate that the video process is not a simple first-order Markov process. In addition, Limb's data indicates that if coding using past samples is to be used, the previous element and line prediction is marginally superior to previous element prediction. Limb's data indicates that a reasonable value of conditional picture entropy may be in the order of 1 bit/element; this is in agreement with current conjectures of Schreiber [Schreiber, 1967]. PCM television requires



six to eight bits per element in order to reduce contouring to an acceptable minimum [Roberts, 1962]. Thus, perfect statistical source coding might provide a maximum bandwidth reduction of the order of 6:1 relative to PCM. The more advanced delta modulation techniques which we will discuss in this report provide approximately a 2:1 or 3:1 reduction. Redundancy reduction techniques a general description of which is given by Ehrman, [Ehrman, 1967], have been applied to television signals [Drapkin, 1966; Hochman, et. al., 1967]. The reported results for these algorithmic approaches yield compression ratios in the order of 5:1, although it should be pointed out that these techniques have not been proven on commercial types of television pictures. Roberts [Roberts, 1962] has shown that the addition of pseudo-random noise to a digitized picture before quantization, and removal of the same noise from the picture after reconstruction of the picture breaks up the quantizing noise and produces subjectively acceptable pictures with 3 bit PCM. Thus the noise addition technique results in a 2:1 compression ratio. Bisignani and his associates [Bisignani, et. al., 1966] have developed two coding techniques, the improved gray scale and coarse-fine PCM, both of which result in good pictures with 3 bit PCM. By applying redundancy removal techniques to the coarse-fine PCM, they have achieved good pictures with a typical 5:1 bandwidth reduction [Bisignani, et. al., 1967]. Savings can be achieved by utilizing frame-to-frame redundancies. Manasse, in a technique called directional correlation, [Manasse, 1967], uses 3.77 bits/element to transmit an equivalent 8 bit PCM picture, thus resulting in a 2:1 reduction. Mounts [Mounts, 1969], has shown excellent motion pictures with a 1 bit/element rate in a system which uses frame-to-frame redundancy removal techniques.

To summarize all of the techniques, the current state of delta modulation is that it provides a 2:1 to 3:1 bandwidth reduction relative to PCM with simple implementation. Other techniques, while providing larger compressions, require significantly more sophisticated implementation.

### 2.3 Signal-to-Noise Ratios and Quantization

In speaking of signal-to-noise ratio in regard to television, one must be careful to define the signal under discussion [O'Neal, 1966]. Three signals are used in practice. These are

- . peak-to-peak composite signal
- . RMS composite signal
- . RMS video signal.

The conversions between these are shown below:

<u>Signal Type</u>	<u>Relative Level</u>
peak-to-peak composite signal	+11.6 dB
RMS composite signal	0 dB
peak-to-peak video signal	+2.8 dB
RMS video signal	-5.8 dB

As long as the signal-to-noise ratio of the sync signal is sufficiently high to maintain synchronization, it is meaningful to consider only the RMS video signal-to-noise ratio. Typically "good" pictures require a 35 dB peak-to-peak video signal-to-noise ratios [Schreiber, 1967]. For 6 bit PCM, using the assumption of 4-sigma loading, the RMS video signal-to-quantization noise ratio is approximately 29 dB. The above table shows that this corresponds to approximately a 38 dB peak-to-peak video signal-to-noise ratio\* which can be considered to be a "good" quality picture. Discussions of subjective effects of video noise can be found in Huang, et.al., [Huang, 1967]; Brown [Brown, 1967]; and Brainard [Brainard, 1967], among others.

It is necessary to realize that the eye reacts differently to different types of noise. O'Neal [O'Neal, 1966], points out that in delta modulation quantizing noise subjectively appears

---

\* (peak-to-peak video SNR) - (RMS video SNR) = 2.8 dB - (-5.8 dB)

as grainy noise, slope overload, contouring, and edge busyness. Each appears different to the eye, and by proper design of the delta modulator filters, the subjective effects can be optimized. This is the reason that Roberts is able to utilize 3-bit PCM and still retain good picture quality.

### III. CLASSIFICATION OF DELTA MODULATION SYSTEMS

#### 3.1 Introduction

Delta modulation, as originally developed, was a method of transmitting analog data by means of a one-bit code. It first appears in the Western literature in a 1946 French patent of Deloraine et. al. [Deloraine, 1946]. The first occurrence of the technique in the English language literature, as far as the authors of this report can ascertain, were deJager's 1952 paper in the Phillips Research Report [deJager, 1952] and the paper by Schouten, deJager, and Greefkes in the same year [Schouten et. al., 1952]. According to Menshikov [Menshikov, 1967], in 1948 Korobkov, in the U.S.S.R., independently developed delta modulation. In the intervening years, many forms of delta modulation have evolved. Among these are:

- N-bit delta modulation
- Companded delta modulation
- Direct feedback delta modulation
- Statistical delta modulation

In this chapter we will first introduce the conventional, one-bit delta modulator and describe its operating principles and sources of noise. We will then discuss the systems which have evolved from it, and describe their operating principles and sources of error. We end the chapter with a discussion of the experimental data available in the literature concerning the operation of these various systems with television input signals.

#### 3.2 Conventional Delta Modulation

##### 3.2.1 Description of Conventional Delta Modulation

The block diagram of a conventional delta modulation system is shown in Fig. 3.1. It is seen to consist of two parts, the modulator and the demodulator. The purpose of the modulator is to transform the analog input signal,  $x(t)$ , into the synchronous digital signal  $\{s(t_k)\}$ . It does this in the following manner.

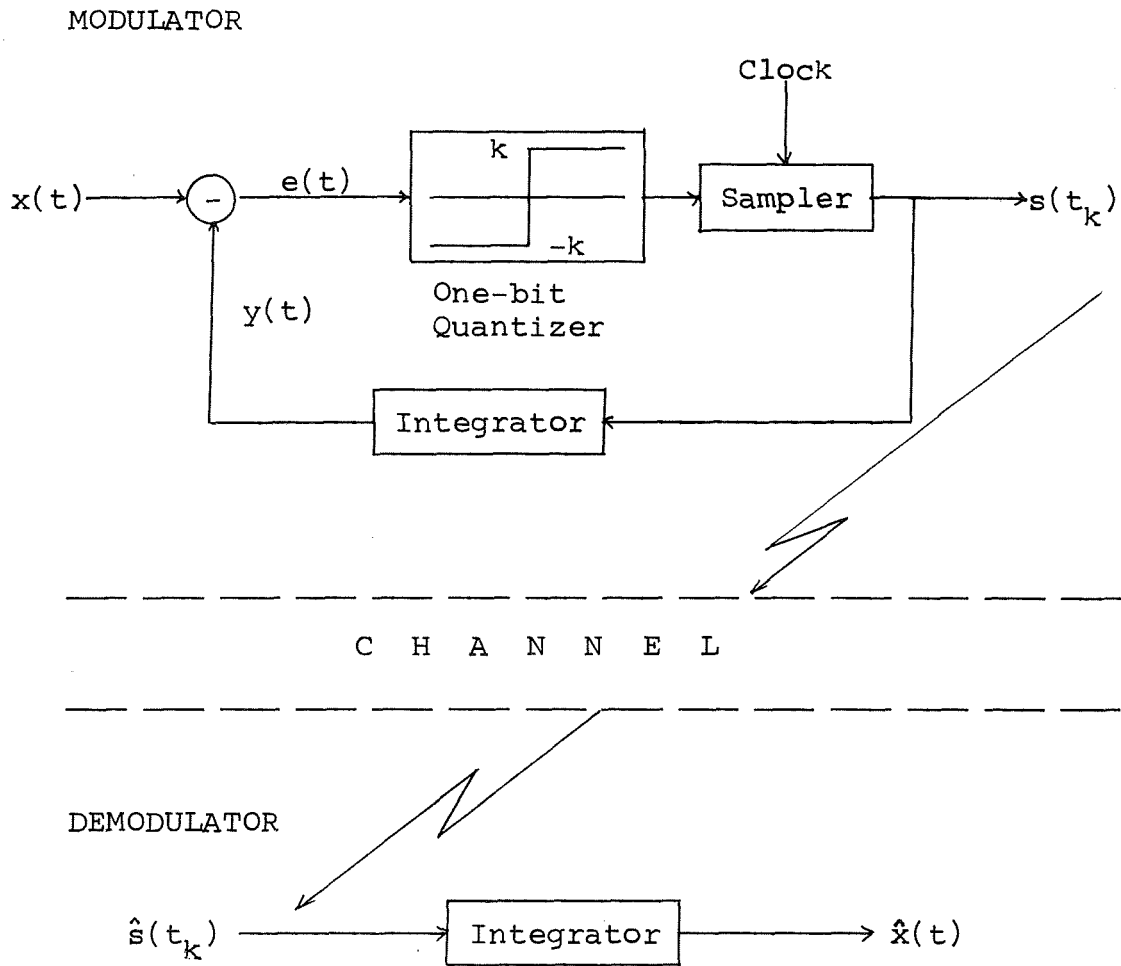


Fig. 3.1 Conventional Delta Modulator

An analog signal  $y(t)$  is developed by integrating the  $\{s(t_k)\}$  bit stream. An error signal,  $e(t)$ , is developed by subtracting  $y(t)$  from  $x(t)$ . This error signal is then used as the input to a one-bit quantizer, the output of which is  $+k$  if  $e(t)$  is greater than zero, and  $-k$  if  $e(t)$  is less than zero. The parameter  $k$  is denoted as the step size of the delta modulator. At time intervals  $\tau$ , corresponding to a sampling frequency  $f_s \equiv 1/\tau$ , the quantizer output is sampled; these samples form the transmitted stream  $\{s(t_k)\}$ .

If we consider the conventional delta modulator as a feedback system, it is intuitively obvious that the loop tries to drive the error signal  $e(t)$  to zero. Thus  $y(t)$ , the integrator output, should be similar to  $x(t)$ , the input signal.

The modulator output,  $\{s(t_k)\}$  is transmitted through a channel and received as  $\{\hat{s}(t_k)\}$ . Passing the received signal through an integrator results in the analog signal  $\hat{y}(t)$ , which, in the absence of channel errors, is identical to  $y(t)$ .

The delta modulator noise is conventionally considered to be made up of two components, granular noise,  $N_g$ , and slope overload noise,  $N_o$ . Granular noise is due to the fact that  $y(t)$  is constrained to be integral multiples of the step size,  $k$ . In order to decrease the granular noise,  $k$  must be decreased. Slope overload noise, on the other hand, is due to the fact that if the sampling interval is  $\tau$ , the maximum slope which the delta modulator can follow is  $k\tau$ . Thus, if the input signal is a sine wave of frequency  $f$  and amplitude  $A$ , slope overload will occur if:

$$k\tau < 2\pi fA \quad .$$

If the input is a zero-mean Gaussian random process of power  $\sigma^2$  and RMS bandwidth  $f_e$ , then the RMS slope of  $y(t)$  is  $2\pi\sigma f_e$ . If a condition known as 4 sigma loading is used, i.e., the maximum slope is taken to be four times the RMS slope, then  $k\tau$  should be chosen so that

$$k\tau = 8\pi\sigma f_e \quad .$$

It is evident that, if the sampling rate is held constant, a too-small value of  $k$  will result in an output dominated by overload noise, while a too-large value of  $k$  will result in an output dominated by granular noise. Thus, for any given input signal there is an optimum value of  $k$  which will result in minimum delta modulator noise. In the next section we shall examine these relationships in detail.

### 3.2.2 Performance of Conventional Delta Modulator

#### 3.2.2.1 Granular Noise

The early studies of delta modulation quantization noise considered only sinusoidal input signals. In deJager's original work [deJager, 1952], it was shown that if the baseband is band limited to a frequency  $f_o$ , and the transmitted signal is a sinusoidal signal of frequency  $f$ , then at slope overload the signal-to-noise granular noise power in the reconstructed signal is approximately given by:

$$\left(\frac{S}{N}\right)_g \cong 0.04 \frac{f_s^3}{f^2 f_o}$$

More exact analysis of the granular noise, based on the input signal being a zero-mean Gaussian random process, have been performed by VandeWeg [VandeWeg, 1953] and Goodman [Goodman, 1969], among others. Goodman's results reduce to those of VandeWeg for  $R$ , the delta modulator bandwidth expansion ratio, greater than 4.  $R$  is the ratio of the sampling rate to the Nyquist rate, and is given by:

$$R \cong f_s / (2f_o)$$

In terms of  $R$ , deJager's signal-to-quantizing ratio becomes:

$$\left(\frac{S}{N}\right)_g \cong 0.31 (f_o/f)^2 R^3$$

It is necessary, in talking about the random process, to define the further parameters:

$S(f)$  = power spectrum of  $x(t)$

$$\sigma^2 = \int_{-\infty}^{\infty} S_{xx}(f)df = \text{mean-square power of } x(t)$$

$$f_e = \left[ \int_{-\infty}^{\infty} f^2 S_{xx}(f)df / \sigma^2 \right]^{1/2} = \text{RMS bandwidth of } x(t)$$

$\beta = k/\sigma = \text{normalized step size}$

$\sigma^2 \rho(\cdot) = \text{auto covariance function of } x(t),$

$F = f_s/f_e = \text{normalized sampling rate}$

Under the assumption of  $4\sigma$  slope loading, i.e..

$$k = 8\pi\sigma f_e / f_s$$

then the granular noise is equal to: [Goodman, 1969]

$$N_g = \frac{1}{R} \left[ Q_0 + 2 \sum_{n=1}^{\infty} Q_n \frac{\sin\left(\frac{\pi n}{R}\right)}{\left(\frac{\pi n}{R}\right)} \right],$$

where

$$Q_0 = \frac{64\pi^2\sigma^2}{F^2} \left\{ \frac{1}{3} + \sum_{k=1}^{\infty} \frac{1}{\pi^2 k^2} \exp\left[-\frac{F^2 k^2}{32}\right] \right\}$$

and

$$Q_n = \frac{128\sigma^2}{F^2} \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \frac{g(m,k,n)}{mk} \exp\left[-\frac{F^2(k^2+m^2)}{128}\right] \sinh\left[\frac{mk\rho_n}{64}\right]$$

where

$$g(m,k,n) = \begin{cases} 1+(-1)^{m+k} & ; \text{ n even} \\ 2 & ; \text{ n odd} \end{cases}$$



This formulation of granular noise is quite complex. Goodman has computed  $N_g$  for  $x(t)$  being white, band-limited, Gaussian noise, and presented the results in graphical form. [Goodman, 1969]. From it the following asymptotic signal-to-granular noise relations can be derived:

$$\left(\frac{S}{N}\right)_g \cong \begin{cases} 9 + 10 \log_{10} (R/4)^4 \text{ dB} & R < 4 \\ 9 + 10 \log_{10} (R/4)^3 \text{ dB} & R > 4 \end{cases}$$

It should be remembered that these results are for 4 sigma loading.

In an independent analysis, Abate [Abate, 1967] has derived the asymptotic slopes for the granular noise:

$$N_g = \begin{cases} \frac{k^2}{6R} & S < 8 \\ \frac{\sqrt{3}k^3}{48\pi} & S > 8 \end{cases},$$

where  $S$  is a slope loading factor defined by:

$$S = \frac{k f_s}{\sqrt{\frac{1}{\pi f_m} \int_0^{f_m} S_{xx}(\omega) d\omega}}$$

For a flat, band limited Gaussian process,  $S$  is equal to  $\sqrt{3} k R/\pi$ . Thus the signal-to-granular noise in dB is, for the flat Gaussian process:

$$\left(\frac{S}{N}\right)_g = \begin{cases} 7.8 + 10 \log_{10} R - 20 \log_{10} k \text{ dB} & kR < 14.5 \\ 19.4 - 30 \log_{10} k \text{ dB} & kR > 14.5 \end{cases}$$

For 4 sigma loading,  $k$  is equal to  $8\pi f_e/f_s$ . Therefore:

$$S = \frac{\sqrt{3} 8f_e R}{f_s} \quad ; \quad 4 \text{ sigma loading}$$

But  $R = f_s / (2f_o)$ , and for a flat band-limited process,  $f_e = f_o / \sqrt{3}$ . Therefore:

$$S = \sqrt{3} \frac{8f_o}{\sqrt{3} f_s} \frac{f_s}{2f_o} = 4 \quad ; \quad 4 \text{ sigma loading}$$

Therefore, for a Gaussian process with a flat, band-limited spectrum, and 4 sigma loading, Abate's result is:

$$N_g = \frac{16\pi^2}{18R^3} \quad ,$$

which results in a signal-to-granular noise ratio of:

$$\left(\frac{S}{N}\right)_g = -9.7 + 30 \log_{10} R \text{ dB},$$

which is essentially the same as Goodman's result for  $R > 4$ . It should be noted that Abate's asymptotes provide a good fit to experimental data of O'Neal's [O'Neal 1966].

### 3.2.2.2 Slope Overload Noise

Slope overload noise, as previously noted, occurs from the inability of the delta modulator to track rapid changes in input signal amplitude. Zetterberg [Zetterberg, 1955], performed the first analysis of slope overload noise of a delta modulator with a random process input signal. An approximate analysis of slope overload noise, based on a third moment expansion of the random process, was performed by O'Neal and Rice in 1966. [O'Neal, 1966]. Their results, however, did not agree with overload noise found by simulation in the limit as the step size became very small. Protonotarios [Protonotarios, 1967] re-examined Zetterberg's work and determined a conceptual error in it. Using Zetterberg's technique, but with the error corrected, he arrived at a new

formulation for slope overload noise which provides an extremely good fit to the experimental data of O'Neal. His result is that  $N_o$ , the slope overload noise, is given by:

$$N_o = \frac{1}{4\sqrt{2\pi}} \left( \frac{b_1^2}{kf_s} \right) \left( \frac{3b_1^{1/2}}{kf_s} \right)^5 \exp \left( - \frac{b_2 f_s}{2b_1} \right) A(x)$$

where

$$x = \frac{\sqrt{2}}{3} \left( \frac{2b_2}{b_1} \right)^{1/8} \frac{kf_s}{\sqrt{b_1}}$$

$$A(x) = 1 + [-\exp(-x^2/2)P(x) + \phi(x)Q(x)]/\sqrt{2\pi}$$

$$P(x) = 2 \left( \frac{16}{15} x^5 + \frac{1}{3} x^3 + x \right)$$

$$Q(x) = 2 \left( \frac{16}{15} x^6 + x^4 - 1 \right)$$

$$\phi(x) = \int_x^\infty e^{-z^2/2} dz$$

$$b_n = \int_0^\infty (2\pi f)^n S_{xx}(f) df$$

This expression is, again, quite complicated. Abate [Abate, 1967] has arrived at a much simpler expression for the slope overload noise which again provides a good fit to O'Neal's data. This is:

$$N_o = \frac{8}{27} \left( \frac{kR}{S} \right)^2 e^{-3S} (1 + 3S)$$

For a band-limited white Gaussian process,  $N_o$  is equal to:

$$N_o = \frac{8\pi^2}{81} e^{-\frac{3\sqrt{3}}{\pi} kR} \left( 1 + \frac{3\sqrt{3}}{\pi} kR \right)$$

Thus the signal-to-overload noise is, for a flat Gaussian process:

$$\left(\frac{S}{N}\right)_o = 0.1 + 7.15kR \cdot 10 \log_{10} \left(1 + \frac{3\sqrt{3}}{\pi} kR\right)$$

For  $kR > 1$ , this is approximately:

$$\left(\frac{S}{N}\right)_o \approx 7.15kR - 2.1 - 10 \log_{10} kR \text{ dB.}$$

### 3.2.2.3 Conventional Delta Modulation Signal-to-Noise Ratio

Figure 3.2 shows the RMS signal-to-noise ratio for a conventional delta modulation system, using the asymptotic equations of Abate for a flat, band-limited Gaussian process. For a television video signal, the same curves apply if 11 dB is added to the signal-to-noise ratio scale and the abscissa values are multiplied by 7.\* Since "good" television requires about a 28 dB RMS signal-to-noise ratio, a delta modulator must be run at a minimum of  $R = 8$  for television transmission. The typical delta modulation characteristics are shown as follows. The abscissa is the parameter  $kR$ , while the family of curves are for  $R$  equal a constant. Thus, for a given value of  $kR$ , an increase in  $R$  requires an inverse decrease in  $k$ . Therefore, as  $R$  increases and  $kR$  is held constant, the system will eventually be in the region of slope overload. The locus of these points is given by the left-hand curve marked "slope overload region." If the system is operated to the right of this locus, granular noise becomes the limiting factor.

From Fig. 3.2 it is seen that one of the principal weaknesses of delta modulation is the relatively narrow maximum in the signal-to-noise ratio. In order to flatten out the maximum region, various types of companding have been developed for delta modulation systems.

We next discuss them.

### 3.3 Variable Step Delta Modulation

In order to increase the operating range of delta modulators, companding, in the form of a variation of the basic transmitted signal, has been tried by many different researchers. Among the

\* See equations in Sections 3.2.2.1 and 3.2.2.2, also Figs. 6-11 in [Abate, 1967].

# Delta Modulation

S/N in dB

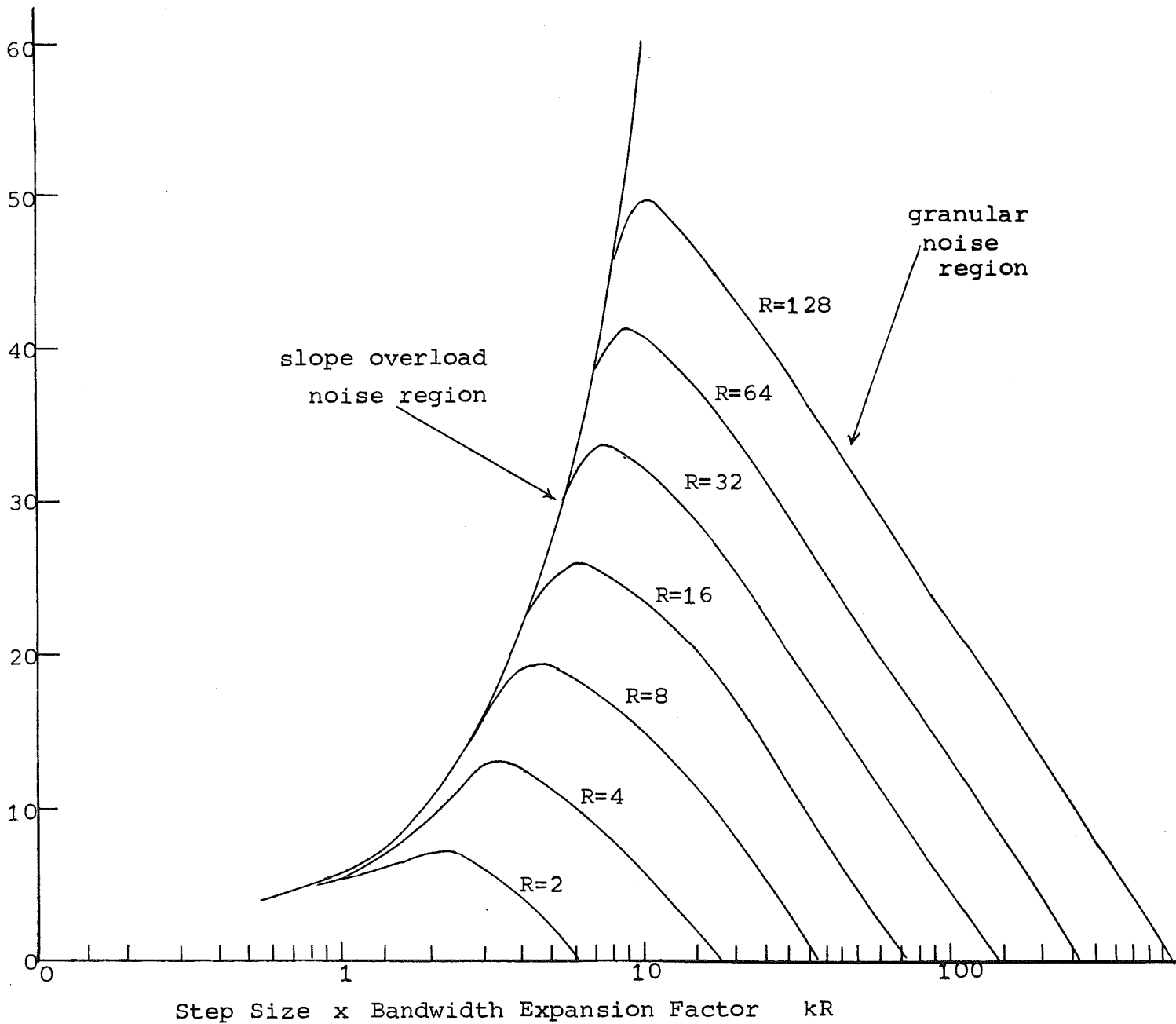


Fig. 3.2 Delta Modulation RMS Signal-to-Noise Ratio as a Function of Step Size and Sampling Rate. Uniform, Band-Limited Gaussian Input Signal. (For television video signal, add 11 dB to ordinate and multiply abscissa by 7.) [Ref. Abate, 1967].

techniques which have been tried are varying the step amplitude continuously, varying the step amplitude discretely, and varying the duty cycle between 1's and -1's as well as varying the step size.

In 1963 deJager and Greefkes, [deJager, 1963] demonstrated a speech system which they called "continuous delta modulation," in which a DC bias, derived from the transmitted bit stream, was used to develop a DC bias which was a function of the input signal amplitude. This bias was then used to both vary the amplitude of the step in the feedback loop, and change the ratio of 1's to -1's in the transmitted signal. Another version of this system was reported by Greefkes and deJager in 1968 [Greefkes, 1968], in which they claimed that 20 dB variations in the speech input signal level could easily be tolerated with their system.

Tomozawa and Kaneko [Tomozawa, 1968], have described a delta modulation system, also designed for speech, in which the step amplitude is continually changed but the 1:-1 ratio is kept constant at unity. They claim a dynamic range of 45 dB for a 25 dB signal-to-noise ratio, and 35 dB for a 30 dB signal-to-noise ratio, at 56 KHz sampling rate and a 0.8 KHz tone. Brolin and Brown [Brolin, 1968], have published results on a speech system in which an auxiliary bit stream is used to carry the companding information; their system, which also has a continuously variable step size, results in a 26 dB companding range.

Winkler [Winkler, 1963, and Winkler 1965] has developed a system which he named "high information delta modulation," or HIDM, in which the step size is changed in a discrete fashion using an algorithm which is based on the transmitted bit stream. In HIDM, the basic pulse amplitude is taken as 1. If two sequential pulses of the same polarity are transmitted the next feedback pulse amplitude is 2. If three pulses of the same polarity are transmitted, the next feedback pulse amplitude is  $2^2$ . In general,

if  $N$  pulses of the same polarity are transmitted, the next feedback pulse amplitude is  $2^{N-1}$ . Finally, when the transmitted pulse reverses polarity, the next feedback pulse is reduced in magnitude by a factor of 2. Thus, in principle, HIDM has a small step size to reduce granular noise, but can still realize a low slope overload noise by means of the variable step size. Abate [Abate, 1967] has investigated delta modulation systems in which the step sizes change both linearly and exponentially [the exponential being similar to HIDM], and shown that good performance results when there are four possible step sizes, with linear increments outperforming exponential increments. As expected, if  $K_n$  is the ratio of the maximum to the minimum step size, the signal range is increased by  $20 \log_{10} K_n$ ; this factor is termed the companding range. Abate also shows that if the minimum signal power is  $S_1$  and the maximum signal power is  $S_2$ , then

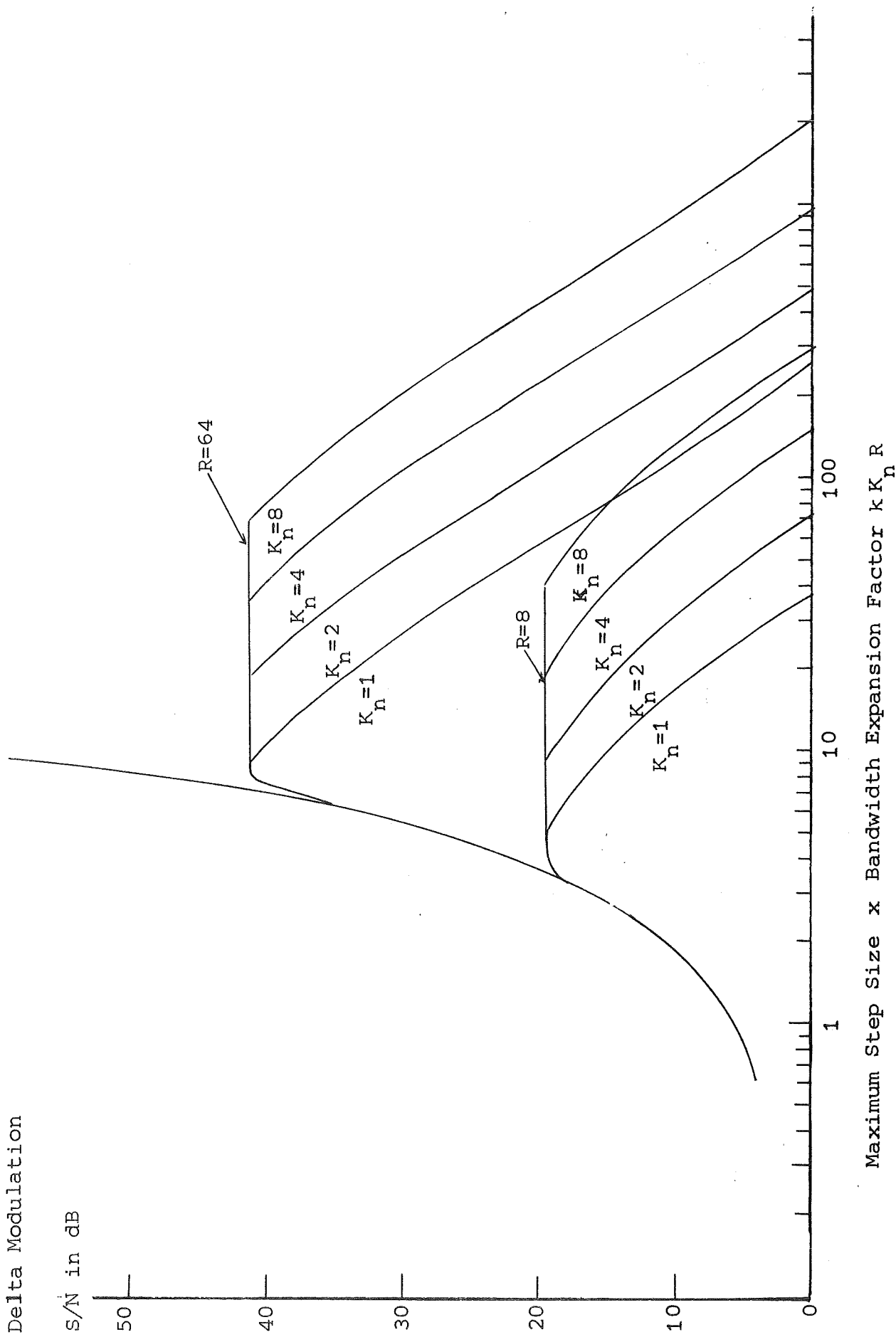
$$K_n = \sqrt{S_2/S_1}$$

and the minimum step size,  $k$ , should be chosen as:

$$k = \sqrt{S_1} \frac{2\pi f_e}{f_s} \ln 2R,$$

where, as before,  $f_e$  is the RMS signal bandwidth.

Figure 3.3 shows the RMS signal-to-noise ratio bounds of a delta modulator for various values of  $K_n$ . The input signal is, as in Fig. 3.2, a band-limited white Gaussian process. The figure is for four values of companding, i.e.,  $K_n = 1, 2, 4,$  and  $8$ , and two sampling rates, i.e.,  $R = 8$  and  $64$ . The value  $K_n = 1$  corresponds to conventional delta modulation. The action of the companding is seen to increase the signal range over which the signal-to-noise ratio is a maximum; it does not, however, increase the maximum value over that of a conventional delta modulator with step size  $k$ . Again, for a television video signal, add 11 dB to the RMS SNR scale and multiply the abscissa by 7. (See footnote on page 22.)



Maximum Step Size x Bandwidth Expansion Factor  $kK_n R$

Fig. 3.3 Variable Step Size Delta Modulation Signal-to-Noise Ratio as a Function of Step Size, Sampling Rate and Companding-Uniform, Band-Limited Gaussian Input Signal. [ref. Abate, 1967].

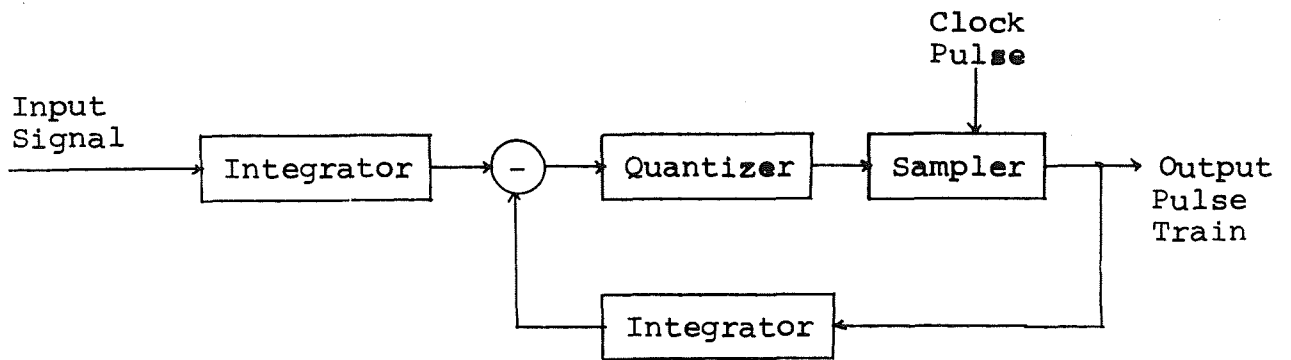


One of the most fundamental, and general techniques for varying step size, which includes the previously described systems as special cases, is in the statistical delta modulation (SDM) system developed by Fine [Fine, 1964, Fine 1968, U.S. Patent No. 3,393,364]. In this approach the output of the delta modulator feedback which is sent to comparator is determined by the pattern of digital data which has previously been transmitted. The feedback levels associated with particular patterns are determined by the statistics of the data being processed. At present, this system has not been built, however a simulation study has been performed [Bello, Lincoln, Gish, 1967] which indicates the possibility of a substantial improvement in performance over conventional delta modulation.

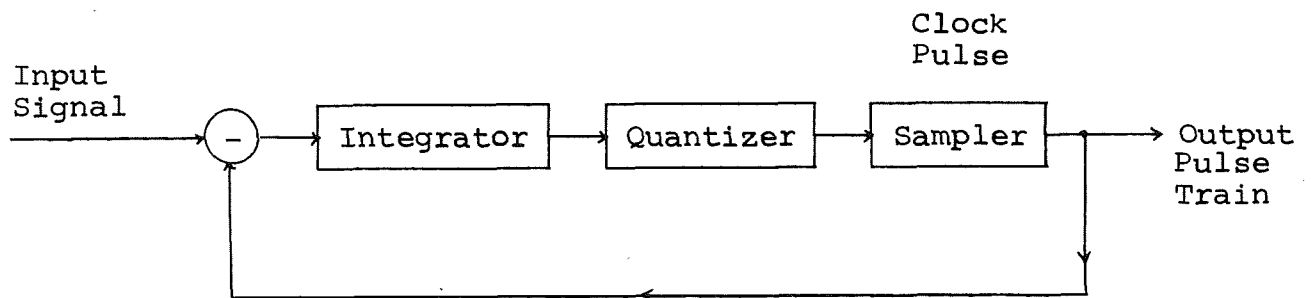
It is possible to make further improvements in the performance of delta modulation systems by changing the feedback network and introducing pre-emphasis. Techniques such as these will be discussed next.

#### 3.4 Direct Feedback Delta Modulation

The delta modulators discussed up to this point all have a filter network in the feedback loop. It was realized early in the development of delta modulation that other configurations might result in improved performance. In 1961, Inose, Tagaki and Murakami, [Inose, 1961] developed what they called delta-sigma ( $\Delta$ - $\Sigma$ ) modulation. This technique has since been extensively studied [Inose 1962, Inose 1963, Johnson, 1968]. Inose, working under the erroneous idea that delta modulation could not be used to transmit a DC level, decided that the input signal should be integrated before being applied to the delta modulator. The resulting modulator is shown in Fig. 3.4a. However, the combination of the integrator before the modulator, and the integrator in the feedback loop is equivalent to having only an integrator in the forward loop before the quantizer. This leads to the realization of Fig. 3.4b, which is the standard  $\Delta$ - $\Sigma$  modulator. Brainard, in 1967 [Brainard, 1967] suggested a variation of



(a) Prototype  $\Delta$ - $\Sigma$  Modulator



(b) Practical  $\Delta$ - $\Sigma$  Modulator

Fig. 3.4  $\Delta$ - $\Sigma$  Modulators

Fig. 3.4a, designed for television transmission, in which the integrators were replaced by two different networks. Further work on this device was reported in 1966 by Brainard and Candy [Brainard, 1969]. This system, called a direct feedback coder, is shown in Fig. 3.5. In Fig. 3.5 the coder may be an N bit quantizer; the decoders in the modulator feedback loop and at the input to the demodulator serve as digital-to-analog converters. The switches shown in the figure are used to isolate different portions of the circuit during the transient intervals when quantization is being performed in the coder, and digital-to-analog conversion is being performed in the decoder. The block labeled "accumulating amplifier" is effectively an integration circuit. The filter  $H_1$ , is a pre-emphasis network, with transfer function:

$$H_1(j\omega) = (\alpha + j\omega)h,$$

where  $h$  is an arbitrary gain factor. The recommended value for  $\alpha$  is

$$\alpha = 3(2\pi f_L) \text{ seconds}^{-1},$$

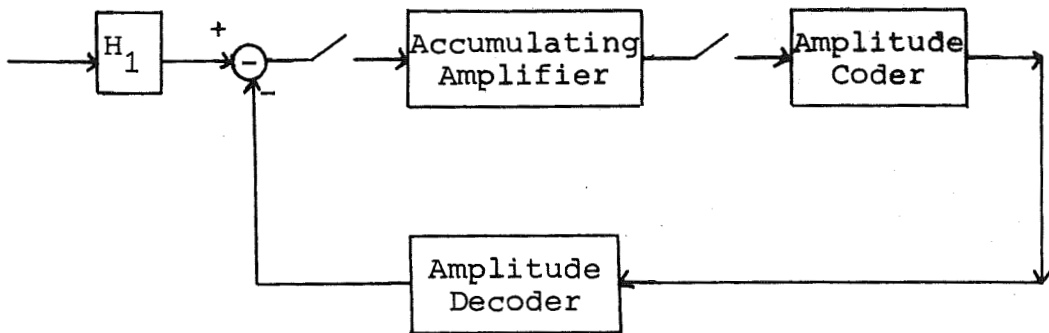
where  $f_L$  is the line frequency of the television picture. The receiver filter,  $H_2$ , is  $H_1^{-1}(j\omega)$ . The step size of the coder,  $k$ , can be optimized in accordance with the picture content. However, it has been experimentally determined that if the voltage increment between the black and white voltage levels is  $U$  volts, then a reasonable choice for the step size  $k$  is:

$$k \cong \frac{hUf_0}{2} \text{ volts}.$$

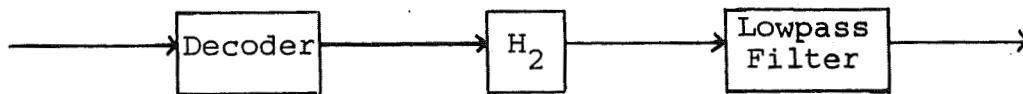
In video systems, the black-white difference is usually\* in the order of 0.6V [Fink, 1957, p 11-52]. Since  $f_0$  is 4.5 MHz,  $k$  would be approximately  $1.35 h \times 10$  volts. Thus the filter gain factor,  $h$ , will be in the order of  $0.75 \times 10^{-6}$ , so that  $k$  can be in the order of one volt. This results in

$$H_1 \cong 2.25(1 + j\omega/3 \times 10^5)$$

\*National Television Systems Committee Standard.



(a) Direct Feedback Modulator



(b) Direct Feedback Demodulator

Fig. 3.5 Direct Feedback Coder

The signal-to-granular noise ratio of a direct feedback coder is given by [Brainard, 1969]

$$(S/N)_g = \frac{\int_0^{\omega_0} |V(\omega)|^2 d\omega}{\int_0^{\omega_0} |N_o(\omega)|^2 d\omega},$$

where  $V(\omega)$  is the voltage spectrum of the input signal and  $N_o(\omega)$  is the voltage spectrum of the quantization noise. This can be shown to be approximately equal to:

$$(S/N)_g \cong \frac{\int_0^{\omega_0} |V(\omega)|^2 d\omega}{\left[ \sqrt{\frac{1}{3\pi f_s}} \int_0^{\omega_0} V(\omega) 2 \sin(\omega/2f_s) d\omega \right]^2}.$$

For a flat signal spectrum:

$$V(\omega) = \begin{cases} \frac{1}{\omega_0} & 0 < \omega < \omega_0 \\ 0 & \omega > \omega_0 \end{cases},$$

and

$$\begin{aligned} \left(\frac{S}{N}\right)_g &\cong \frac{\frac{1}{\omega_0}}{\frac{4}{3\pi f_s \omega_0^2} \left[ 2f_s \left[ \cos \frac{\omega_0}{2f_s} - 1 \right] \right]^2} \\ &= \frac{3\pi^2}{64R} \frac{1}{\sin^4(\pi/4R)} \end{aligned}$$

Therefore, for  $R > 1$ :

$$(S/N)_g \cong 12/\pi^2 R^3.$$

$$\cong 0.8 + 30 \log_{10} R .$$

This is approximately 10 dB greater than the signal-to-granular noise ratio of a conventional delta modulator, corresponding to a factor of 2.2 sampling rate reduction.

Similarly, for the commercial TV spectrum, the composite spectrum is given by (see discussion in page 7):

$$V(\omega) = \begin{cases} \frac{1}{\omega_c + j\omega} & 0 < \omega < \omega_o \\ 0 & \omega > \omega_o \end{cases} ,$$

and

$$(S/N)_g \cong \frac{\int_0^{\omega_o} \frac{d\omega'}{\omega_c^2 + \omega'^2}}{\frac{4}{3\pi f_s} \left[ \int_0^{\omega_o} \sin \frac{\omega}{2f_s} \frac{1}{\omega_c^2 + \omega^2} d\omega \right]^2}$$

Since  $\sin \frac{\omega}{2f_s} < \frac{\omega}{2f_s}$ ,

$$\begin{aligned} (S/N)_g &\geq \frac{\int_0^{\omega_o} \frac{d\omega}{\omega_c^2 + \omega^2}}{\frac{1}{3\pi f_s^3} \left[ \int_0^{\omega_o} \frac{\omega^2}{\omega_c^2 + \omega^2} d\omega \right]^2} \\ &\cong \frac{3\omega_o}{2\omega_c \pi} R^3 . \end{aligned}$$

For commercial TV,  $\omega_o/\omega_c \cong 30\pi$ , resulting in

$$(S/N)_g \cong 16 + 30 \log_{10} R \text{ dB. (RMS composite signal)}$$

Since the RMS video signal is about 6 dB lower than the RMS composite signal,

$$(S/N)_g \cong 10 + 30 \log_{10} R \text{ dB} \quad (\text{RMS video signal})$$

Thus "good" television, i.e.  $(S/N)_g \geq 27 \text{ dB}$ , can be expected at a rate of about 4.

In Fig. 11 of [Brainard and Candy, 1969], a comparison is shown between an original picture and two processed versions of it, one being processed by a direct feedback modulator and the other by a conventional delta modulator. The rate,  $R$ , was 2.5 in both cases. There was a significant amount of granular noise in the delta modulator picture, while the direct modulator picture was significantly better, with the granular noise being barely perceptible.

A further improvement in the direct feedback coder was reported by Bosworth and Candy [Bosworth, 1969], who applied a variable step size technique to a direct feedback coder. In their system, the relative weights were 1,1,2,3,5 .... 5, that is, weights were changed only after two successive transmitted bits of the same polarity. When a string of similar bits ended, the weight was returned to unity. Subjective measurements on a television picture at a rate  $R = 3.15$  showed a companding gain of about 10 dB and a signal-to-noise improvement of 5 dB relative to a fixed step size system. Since the signal-to-noise ratio goes as  $R^3$ , a 5 dB increase in signal-to-noise ratio implies that the sampling rate can be decreased by a factor of 1.45 for the same quality picture. The peak signal-to-RMS noise ratio at  $R = 3.15$  was in the order of 45 to 50 dB; this corresponds to an RMS signal-to-noise ratio of the order of 34 to 39 dB. Since, from Fig. 3.2, a conventional delta modulator would have to operate at a rate of about 16 to provide the same signal-to-noise ratio, the 1 bit variable step size direct feedback coder provides about a factor of 4 reduction in bit rate relative to a 1 bit delta modulator for this quality.

In the next section we will discuss the family of  $N$  bit feedback quantizer systems.

### 3.5 N-Bit Feedback Quantizer Systems

N-bit feedback quantizer systems can be considered to be delta modulation systems which utilize N-bit instead of 1 bit quantizers. As such, the different classes of 1 bit systems which have been discussed can be fit into equivalent N-bit systems. The principal advantage of N bit systems over 1 bit systems is that the quantizer can have, simultaneously, a small quantization step for maintaining a low granular noise level, and a large quantization step for maintaining a small slope overload noise level. Remm [Remm, 1966], for example, investigated a two-bit system, in which the small step was  $\pm 4$  and the large step was  $\pm 15$ , relative to a 6 bit PCM which has a  $\pm 32$  range. In order to achieve good performance, a sampling rate R equal to 2 was experimentally found to be required; as each sample contains two bits this is a higher bit rate than is required by the direct feedback systems discussed previously, although it is superior to 1 bit delta modulation.

The theoretical performance of N-bit feedback quantizer systems has been studied by O'Neal [O'Neal, 1966, 1967], and McDonald [McDonald, 1966], around others. They have called these systems differential pulse code modulation systems, or DPCM, and have shown that the signal-to-granular noise ratio of an N bit system is approximately:

$$(S/N)_g \cong -4.35 + 6N + 10 \log_{10} \sigma_g^2 / \sigma_e^2 ,$$

where:

$$\sigma_g^2 = \text{input process power}$$

$$\sigma_e^2 = \text{quantizer input process power.}$$

If an integrator is used as the feedback filter, then

$$\sigma_e^2 \cong 2\sigma^2(1-\rho_1) ,$$



where  $\rho_1$  is the correlation coefficient between successive samples of the input process. This results in:

$$(S/N)_g \cong -7.35 + 6N - 10\log_{10}(1-\rho_1) .$$

$\rho_1$  is typically 0.9 to 0.97. This should be compared with 4 $\sigma$  loaded PCM, for which [O'Neal, 1966]:

$$(S/N)_{PCM} = 6N - 7.35 \text{ dB}$$

Thus the effect of signal correlation is to increase the signal-to-noise ratio of a DPCM system, or, alternatively, to allow a lower sampling rate for the same signal-to-noise ratio.

The above equations for DPCM performance are approximations which are good for  $N \geq 3$ . For  $N$  equal to 1, DPCM and conventional delta-modulation are equivalent. For  $N = 2$ , experimental results indicate that DPCM provides superior television pictures than does fixed step-size delta modulation.

Brown [Brown, 1969] has built a 3 bit direct-feedback coder which has effectively 7 bit resolution through combining a coarse-fine technique reminiscent of the work of Bisignani [Bisignani, 1966] with a direct feedback coder. This is, in effect, a companded 3 bit system. At a rate of 1, i.e., 3 bits per Nyquist interval pictures with 47 dB to 54 dB peak signal-to-noise-RMS noise were produced. This is perhaps 5 dB better than the companded 1 bit direct feedback coder of Bosworth and Candy, which, ran at the rate of 3.15 bits per Nyquist interval. The implementation would seem, however, to be more complex.

With this section as background, in the next section we discuss linear predictive feedback quantization systems. These are the most general class of delta modulation systems, and can be considered to contain all the previous systems as subsets.

### 3.6 Predictive Feedback Quantization Systems

Predictive feedback quantization systems are perhaps the most general of all of the delta modulation systems. Fine, in

1964 [Fine, 1964] [Fine, 1968, U.S. Patent No. 3,393,364] analyzed the optimum analog-to-digital-to-analog conversion system, and showed that it could be modeled as a delta modulator with a non-linear predictor in the feedback loop. His formulation of the problem allowed for N-bit quantization. A brief description of this system, often referred to as a statistical delta modulator is given in Section 3.3. Fine's work was carried further by Bello, et al [Bello, 1967], who examined the problem of generating the non-linear predictor, and presented the results of system simulation on certain random process. Gish [Gish, 1967], feeling that the structural problems involved in the general non-linear problem were sufficiently complex to make a practical realization of the system unlikely, studied the problem of optimum linear quantized feedback systems. Further work, especially with regard to television, has been reported by other researchers, among which we should point out O'Neal [O'Neal, 1966].

The linear predictive feedback systems are characterized by their feedback networks. Specifically, if the input signal sequence  $\{S_n\}$ , then the output of the linear predictive network is

$$Y_n = \sum_{k=1}^K a_k S_{n-k} .$$

A typical implementation of a predictive quantization modulator is shown in Fig. 3.6. [O'Neal, 1966] In this implementation the quantizer input is the difference between the input signal sample  $S_n$  and the quantized feedback estimate,  $\hat{S}_n$ . Therefore  $S_n$  can be estimated by adding  $\hat{S}_n$  to the quantizer output, as is shown in the figure. The  $D_i$  are analog delay lines, one of which is required for each coefficient in the predictor. It should be noted that in principle the delays can have any value. For example, a three-delay system can be postulated, in which  $D_1$  corresponds to a delay of one element,  $D_2$  to a delay of one line, and  $D_3$  to a delay of one frame. It should further be

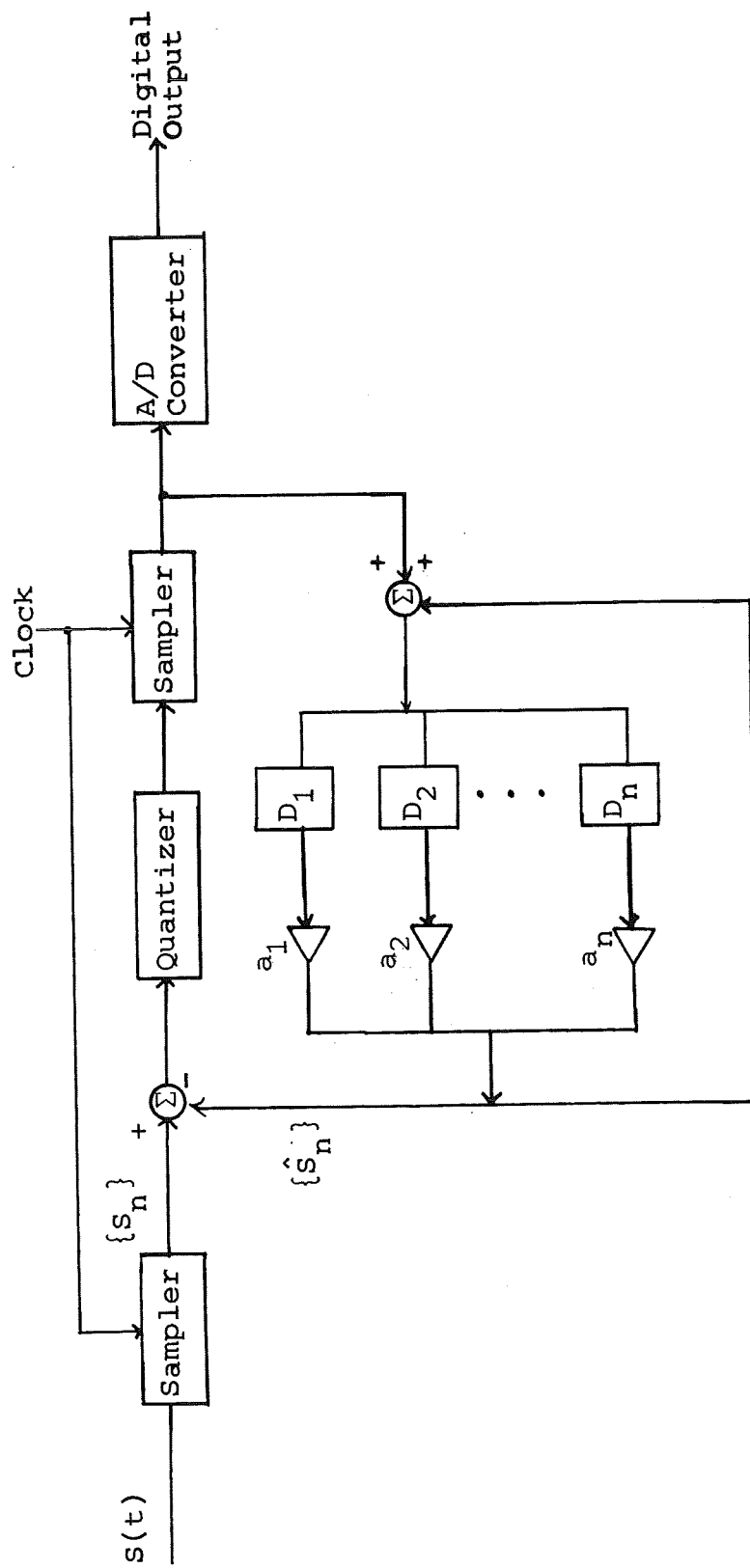


Fig. 3.6 Linear Predictive Quantization Modulator

noted that if only one delay, corresponding to the previous element, is used, and if  $A_1$  is set equal to unity, then the system reduces to a delta modulator with an ideal integrator in the feedback loop. Returning to the general case, the prediction is based on the previous  $K$  samples, and the predictor is characterized by the  $K$  coefficients  $\{a_k\}$ ;  $k = 1, 2 \dots K$ . Gish [Gish, 1967] has shown how to derive the optimum coefficient values for a minimum mean-square error system taking into account the quantizer non-linearity, and has thus the most general results. O'Neal [O'Neal, 1966], using a linearized quantizer model, applied linear prediction theory to show that the approximately optimum feedback network is given by the solution to the  $K$  simultaneous equations:

$$R_{oi} = A_1 R_{1i} + a_2 R_{2i} \dots + a_K R_{Ki}; k=1, 2 \dots K,$$

where  $R_{ij}$  is the covariance of the input signal at time interval  $(i-j)/f_s$ . The  $\{a_k\}$  are then found by solving the set of equations. For the special case of  $S(t)$  being a first-order Markov process, the solution will be  $a_1 = R_{01}/R_{00} \cong \rho_1$ ,  $a_2 = a_3 \dots a_K = 0$ . Thus, for this special case, the optimum linear predictor will simply be based on the last input signal sample:

$$Y_n = \rho_1 S_{n-1}.$$

In general, however, the input signal is not first-order Markov. Limb [Limb, 1966] found by simulation of television systems that previous-sample predictive systems were improved only about 2 or 3 dB by including a previous line prediction into the feedback system; based on measurements made by Kretzman [Kretzman, 1952] and Deriugin [Deriugin, 1957], even a smaller improvement may be achieved by using previous frame feedback. In O'Neal's simulations, it was found that the autocovariance of 1 bit delta modulation quantizing noise required two lags to be essentially uncorrelated; the noise frame higher order quantizers became uncorrelated after one lag.

This concludes the presentation of the basic modulation systems. In the next chapter we present a comparison of their performance, and consider certain system aspects for 1 and N-bit modulators.

#### IV. SYSTEM COMPARISONS

In this chapter we compare the various delta modulation systems of Chapter 3, and show the tradeoffs between them. We consider the systems in the context of the television processing chain shown in Fig. 1.1. We assume an analog picture source as an input to an analog video sensor, e.g. a vidicon. The video output is a composite video signal, i.e., the picture video plus the synchronization signals. We will consider video bandwidths of 1 MHz to 20 MHz. The video is inputted to the digital modulator, the output of which is a binary data stream. This stream is suitably encoded for RF transmission, sent through the transmission channel, and decoded at the receiver. The decoded bit stream is inputted to the digital demodulator, the analog output of which is the composite video signal; this is displayed on an appropriate video display device.

In our comparison we will consider the five following systems:

- . 1 Bit Delta Modulation
- . 1 Bit Direct Feedback Modulation
- . N Bit Delta Modulation
- . N Bit Direct Feedback Modulation
- . Predictive Quantization

In the system comparisons we will consider:

- . Dynamic Range
- . Signal-to-Noise Ratio
- . Synchronization
- . Companding
- . Types of Quantization
- . Sampling Rate
- . Complexity
- . Multiplexing
- . Cost

We begin by summarizing the operation of the five systems. This is shown in tabular form in Table 4-1. Taking all factors into consideration, the recommended system is a one-bit direct feedback modulator. This is followed by the three-bit direct feedback and the three-bit delta modulator. The one-bit delta comes in next-to-last, and the N-bit predictive quantizer is last. The predictive quantizer is so ranked because the increase in its performance over the three-bit delta modulator is probably not equal to the increase in its complexity. The relative rankings of the one-bit delta and the three-bit direct feedback system could be interchanged if the user felt that the possible factor-of-two increase in bit rate was warranted by the simplicity of the system.

In the remainder of this chapter we will show the factors which led to Table 4-1.

#### 4.1 System Summaries

##### 4.1.1 Modulator Summary

The block diagrams of the five systems have been shown in detail in Chapter 3. Figure 4.1 summarizes the system modulators. It is seen that the N-bit and 1-bit modulators of a given type (i.e., delta and direct-feedback modulators) have the same block diagrams. In implementation they differ only by the forward loop quantizer and feedback loop converter required. For 1 bit systems, the quantizer can be a simple limiter; for N-bit systems, the quantizer must be a more complicated N-bit amplitude quantizer and analog-to-digital converter. The feedback loop for the N-bit systems requires a digital-to-analog converter, so that the feedback signal can be subtracted from the input signal. In the 1 bit systems this converter can be omitted.

A comparison of Figures 4.1(a) and (c) shows the close relation between a delta-modulator and a predictive quantizer. If the circuitry in the dashed box of 4.1(c) is considered as

Table 4-1

SYSTEM COMPARISON

	1 Bit Delta	1-Bit Direct Feedback	N-Bit Delta	3-Bit Direct Feedback	N-Bit Predictive Quantization
Signal-to-Noise Ratio (dB) 1*					
No companding	3 + 30 log <sub>10</sub> R	10 + 30 log <sub>10</sub> R	5.8 + 6N 2*	12 + 30 log <sub>10</sub> R	5.8 + 6N 3*
With companding	3 + 30 log <sub>10</sub> R	15 + 30 log <sub>10</sub> R		17 + 30 log <sub>10</sub> R	
Complexity	Least Complex		Intermediate Complexity		Most Complex
Bit Rate for "Good" 4.5 MHz TV Picture					
No companding	56 x 10 <sup>6</sup> bps	36 x 10 <sup>6</sup> bps	36 x 10 <sup>6</sup> bps	32 x 10 <sup>6</sup> bps	36 x 10 <sup>6</sup> bps 4*
With companding	unknown 5*	25 x 10 <sup>6</sup>		21 x 10 <sup>6</sup>	
Sync Requirements	bit synch required only if companding is used		bit and word synch required for all systems		
Recommended Companding Techniques	Digital operating off the transmitted bit stream				
Recommended Multiplexing Techniques	Multiplexing to be performed on outputs of modulators				
Estimated Maximum Bit Rate	Investigation of techniques for multiplexing of color signals				
System Ranking:					
1 is longest,					
8 is lowest					
No companding	7	2	5	4	8
With companding	6	1		3	

Note 1: Signal-to-Noise ratio is RMS video-to-granular noise power ratio

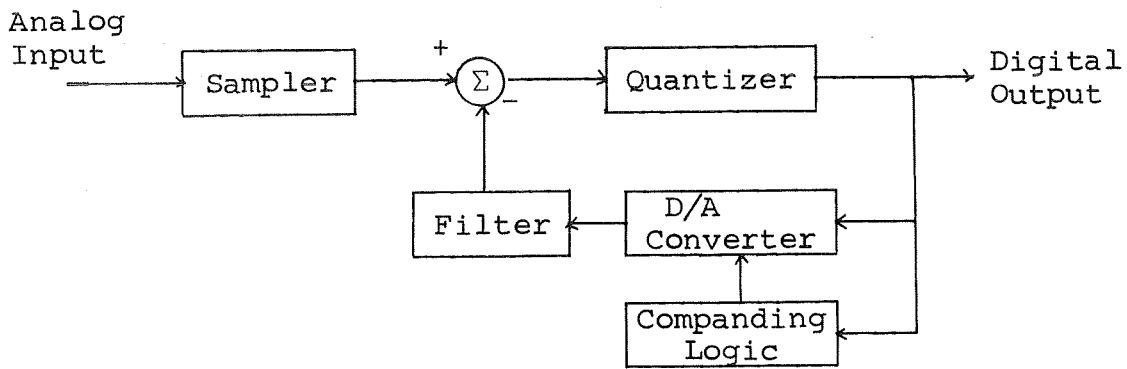
Note 2: N-Bit Delta assumed to be same as N bit Predictive quantization with previous element feedback

Note 3: Predictive Quantization may have ≈ 5 dB higher SNR if previous line and previous frame feedback is used

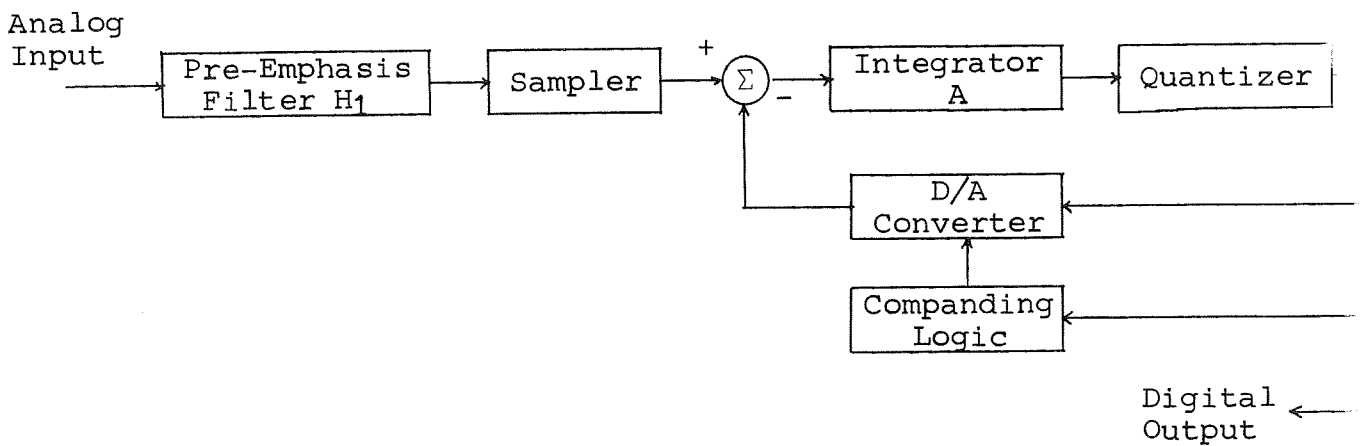
Note 4: If previous line and frame prediction is used, may be 24 x 10<sup>6</sup> bps.

Note 5: The only results available are those of Abate[] who obtained very little SNR improvement but did obtain an improved dynamic range. It is possible that different companding logic may yield an improvement in SNR.

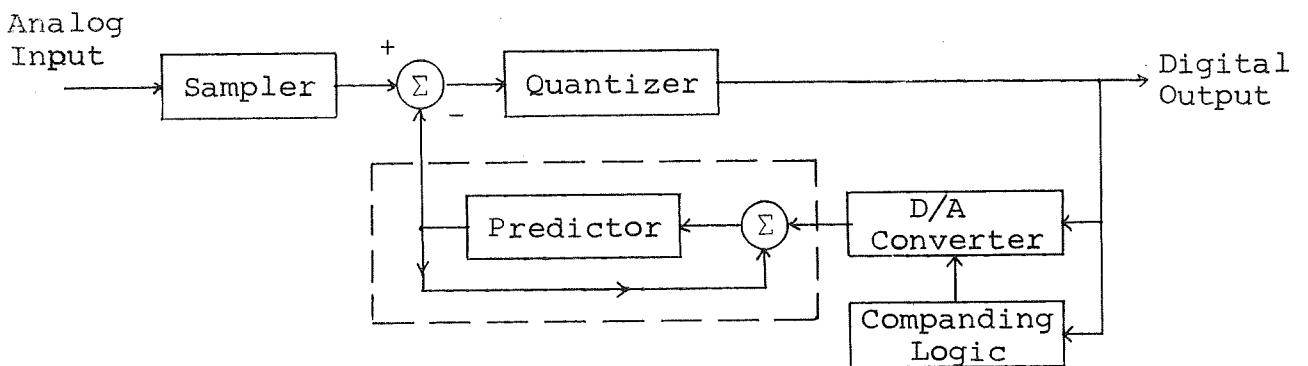




(a) 1-Bit and N-Bit Delta Modulation



(b) 1-Bit and N-Bit Direct Feedback Modulator



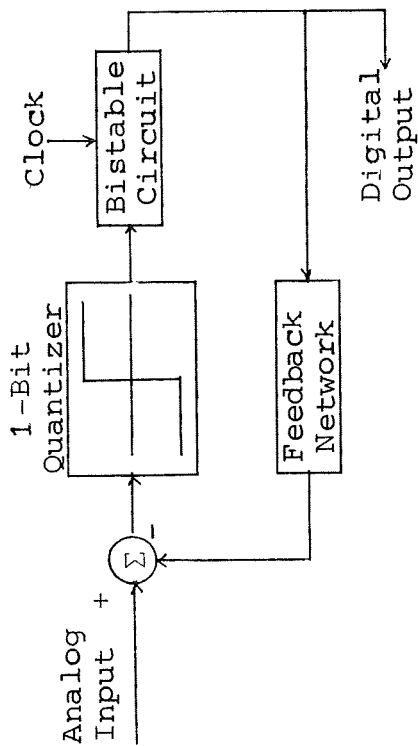
(c) Predictive Quantization Modulator

Fig. 4.1 Modulator Block Diagrams

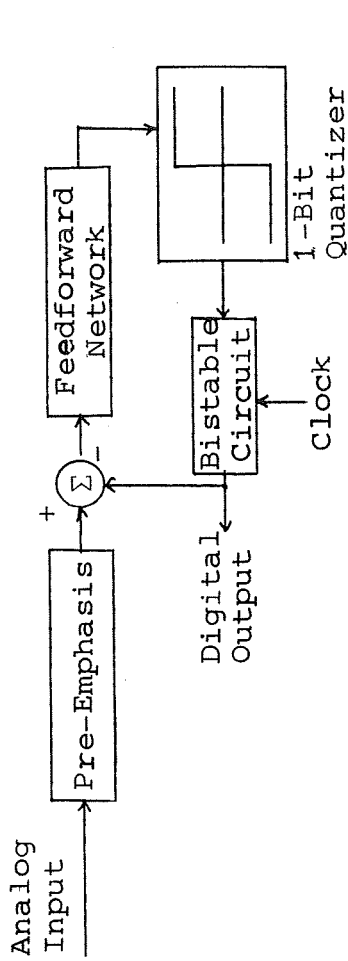
a filter network, the two systems are identical. The implementation of the delta modulator would use an analog filter while the implementation of a predictive quantizer could also use an analog predictive filter with its attendant analog delay lines, an implementation of a predictive quantizer might employ a digital filter with digital delays. Short delays, such as required for previous element feedback, could be implemented with digital delay lines; longer delays, such as required for previous line and previous frame feedback could be implemented with ultrasonic digital delay lines. Using current technology, fused quartz or glass delay lines having  $10^5$  bits/cu. in. packing densities can be built at 100 Mb/s bit rates [Sittig and Smits, 1969 Sittig, 1968]. If digital delays were used, the D/A converter could be placed at the output of the predictive network, instead of at the input. In this event, the dynamic range of the converter would have to be sufficient to handle the full range of the predictor output instead of simply the full range of the quantizer.

Companding is shown for all systems as a logic device which operates on the D/A converter and is controlled by the digital output. This configuration permits the signal to be recovered at the receiver without an auxiliary compander-control bit stream being sent. By operating on the D/A converter, the compander can change the feedback step size on a sample-by-sample basis. By being in parallel with the D/A converter, maximum speed can be achieved.

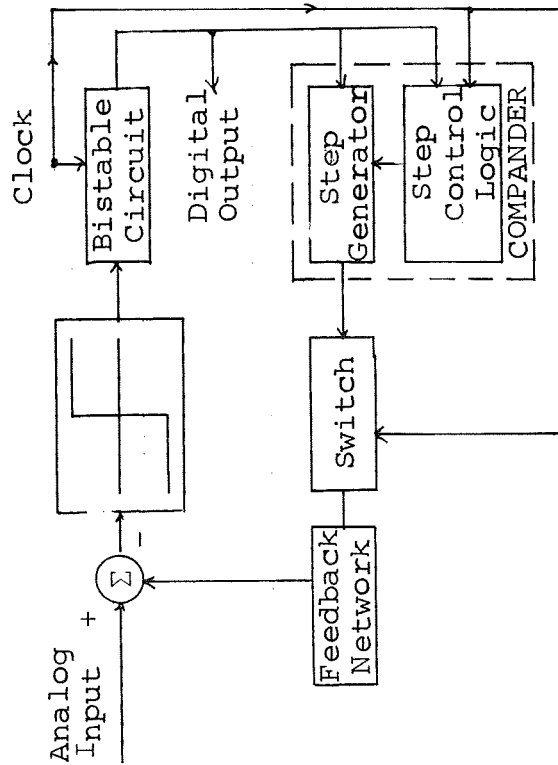
The block diagrams for 1-bit delta and direct feedback modulators are shown in Fig. 4.2. In this figure we again see the close similarity between the two systems as well as the simplifications introduced by utilizing a 1-bit code.



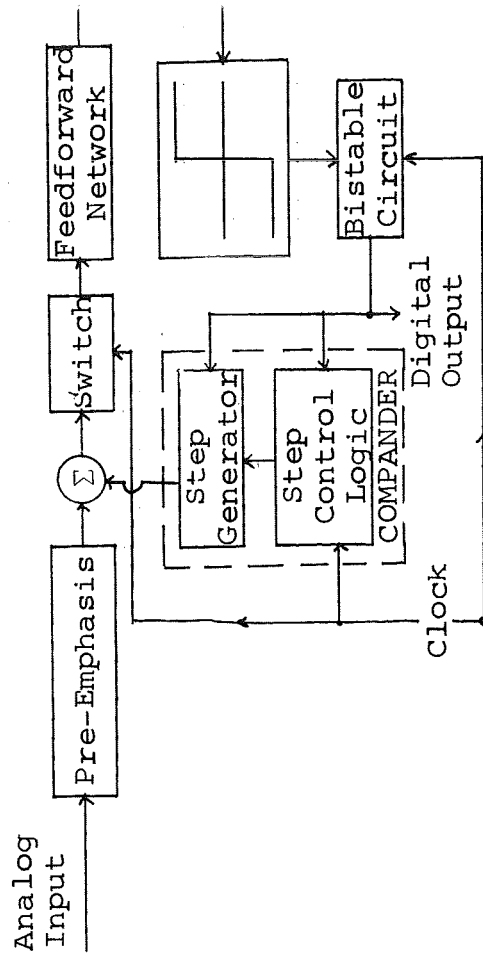
(a) Conventional Delta Modulation



(b) Conventional Direct Feedback Modulator



(c) Companded Delta Modulation



(d) Companded Direct Feedback Modulator

Fig. 4.2 One-Bit Modulators

The quantizer now reduces to a polarity sensor, e.g. a two-input voltage comparator, and the transmitted pulse train can be generated by using the comparator output to set a bistable circuit at time intervals determined by the clock rate. In the conventional delta modulator and direct feedback modulator, the pulse train is then fed back to the input differencing circuit—through the feedback network for the delta modulator, or directly for the direct feedback modulator. In the companded modulators the digital output is fed back to the compander logic, which generates a step amplitude and polarity based on the past digital outputs and the polarity of the current digital output. The logic for performing this is discussed in Section 4.3. It should be noted that time delays in performing the logical operations in the companding network may require the insertion of a switch before the feedback network of the delta modulator and the feed forward network of the direct feedback modulator. The purposes of these switches is to isolate the inputs of the networks during transient intervals, thus isolating them from incorrect feedback information. These switches can be implemented from diode gates and operational amplifiers.

In order for the companded systems to operate properly, the delay in feeding back a step must be less than one sampling interval. The maximum sampling rate will therefore be strongly influenced by the speed of the companding logic, and will be less than the maximum bit rate of a constant step, non-companded system.

#### 4.1.2 Demodulator Summary

The block diagram of the demodulator for all of the systems of Fig. 4.1 is shown in Fig. 4.3. The reconstruction filter will differ for each of the systems, but the remainder of the demodulator will be unchanged. In an N-bit system, both bit sync and word sync must be supplied to the D/A converter, while word sync must be supplied to the compander logic; word sync is required for the proper

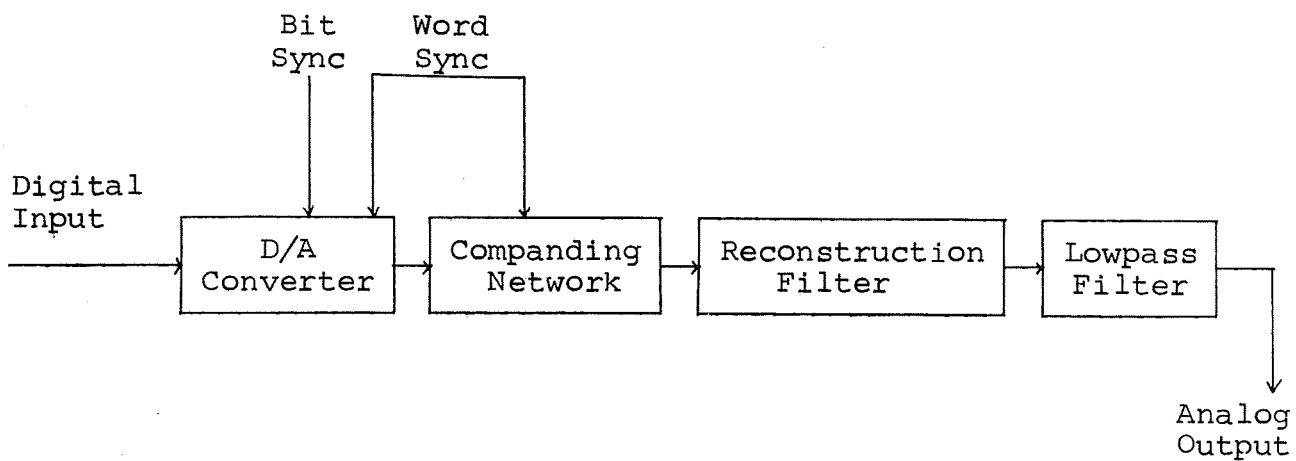


Fig. 4.3 Demodulator Block Diagram

framing of the digital signal and timing of the compander, while bit sync is required for synchronous operation of the D/A converter. In a 1 bit system, word and sync are the same, as all words are 1 bit in length. Synchronization is still required for operation of the compander logic. While the D/A converter is no longer required, a bit restorer might be included in order to have well defined digital pulses. Finally, if the system is a 1 bit system and companding is not used, no synchronization is required. As we are assuming the analog signal to be the composite video signal, line or frame synchronization is not required for any of the systems. The reconstruction filter for the delta demodulator is the same as the feedback filter of the delta modulator. The reconstruction filter for the direct feedback demodulator can have a transfer function which is the reciprocal of the pre-emphasis filter of the direct feedback modulator. The reconstruction filter for the predictive quantizer demodulator is the same as the network within the dashed box of Fig. 4.2(c).

#### 4.2 Signal-to-Noise Ratio and Sampling Rate

The maximum RMS video signal-to-noise ratios for the modulation systems are given below. The signal-to-noise ratios are based on the transmitted signal being a 4.5 MHz bandwidth composite video signal for monochrome television. The N-bit delta modulator performance is taken to be the same as that of a predictive quantizer when the input is a Markoff process with 0.97 correlation function. The performance of the predictive quantizer may be improved by about 2 to 5 dB if previous line and previous frame prediction is used. The SNR for other signals can be derived by the formulas of the previous chapter.

1 bit Delta Modulation; (with or without companding) [Abate, 1967]

$$S/N = 3 + 30 \log_{10} R \text{ dB}$$

1 bit Direct Feedback Modulation [Brainard and Candy, 1969],  
[Bosworth and Candy, 1969]

$$\begin{aligned} S/N &= 10 + 30 \log_{10} R \text{ dB (without companding)} \\ &= 15 + 30 \log_{10} R \text{ dB (with companding)} \end{aligned}$$

3 bit Direct Feedback Modulation [Brainard and Candy, 1969]

$$\begin{aligned} S/N &= 12 + 30 \log_{10} R \text{ dB (without companding)} \\ &= 17 + 30 \log_{10} R \text{ dB (with companding)} \end{aligned}$$

N bit Delta Modulation and Predictive Quantization [O'Neal, 1966]

$$S/N = -6.5 + 6N + 10 \log_{10} 1/(1 - \rho^2)$$

$$\begin{aligned} \cong 5.8 + 6N \text{ dB; sampling at Nyquist} \\ \text{assuming } \rho = 0.97, \\ N \geq 3, \text{ Markoff Process} \end{aligned}$$

N bit PCM [O'Neal, 1966] (Note: for PCM, S/N does not depend on spectrum)

$$S/N \cong -6.5 + 6N$$

These equations are plotted as a function of rate in Fig. 4.4. The data of Fig. 4.4 for a 4.5 MHz bandwidth television signal, are summarized in Table 4-2, which shows the required number of samples per second as well as the sampling rate, R, required for a 28 dB signal-to-noise ratio.

The data of Table 4-2 indicate that the direct feedback systems are significantly superior to all of the other systems except the N-bit delta. The data also show that the companding of the direct feedback systems provides a significant performance improvement. However, the improvement of a 3-bit system over a

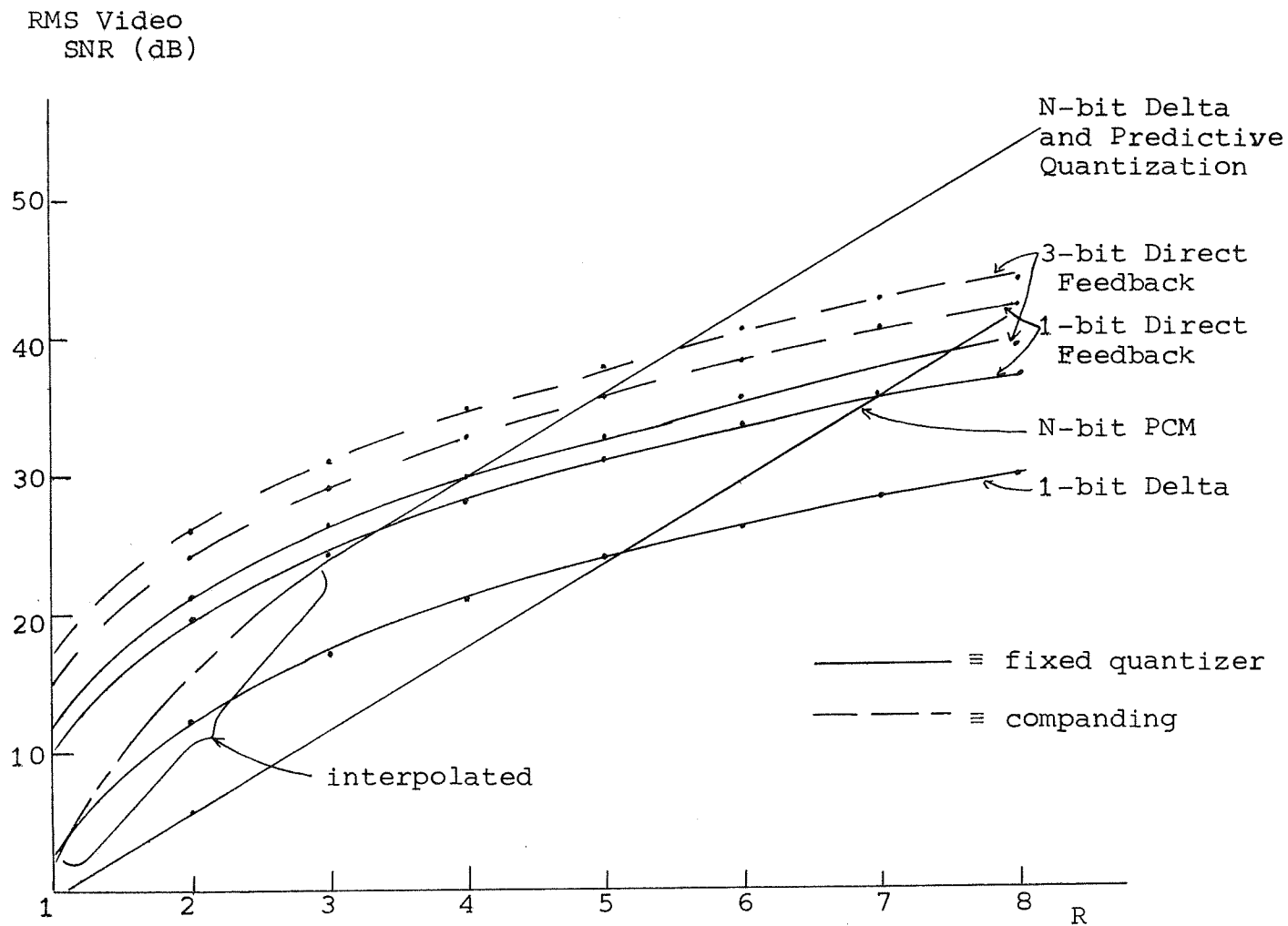


Fig. 4.4 Comparison of Systems:  
RMS Monochrome Video SNR vs Sampling Rate as  
Multiples of Nyquist Rate



Table 4-2

BIT RATES REQUIRED FOR 28 dB RMS VIDEO SNR.  
 COMMERCIAL MONOCHROME TV SIGNAL, 4.5 MHz VIDEO BANDWIDTH

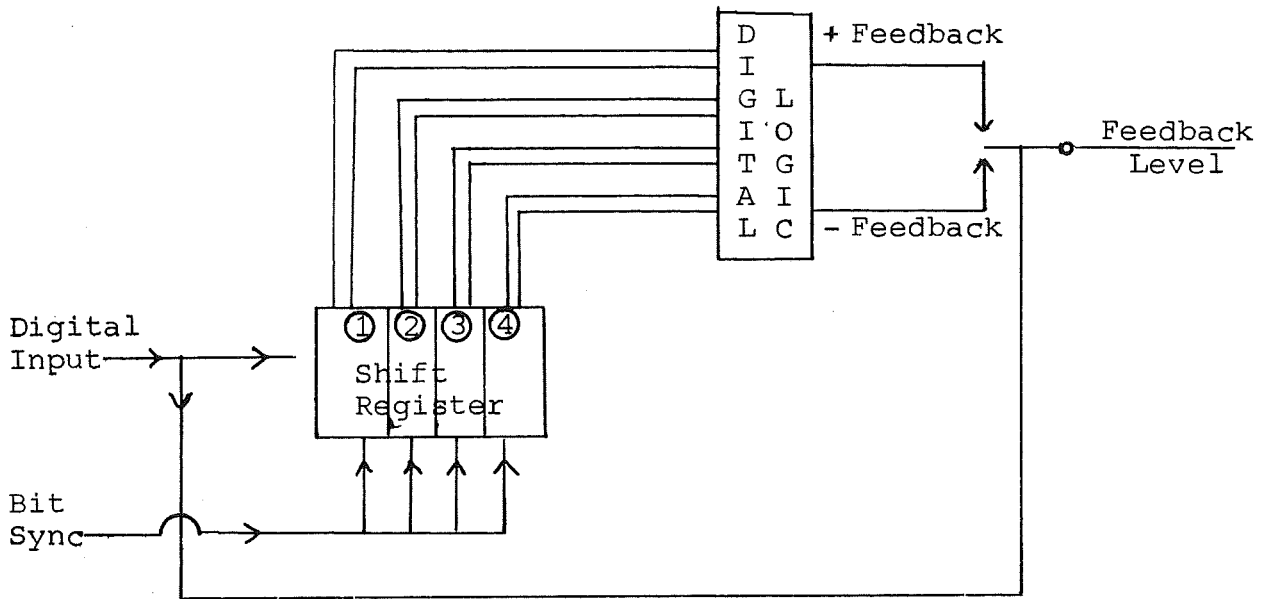
<u>System</u>	<u>Bit Rate</u>	
	R (multiples of Nyquist rate)	Bits/sec (4.5 MHz TV)
PCM	6	$54 \times 10^6$
1-bit Delta	6.3	$56 \times 10^6$
N-bit Delta and Predictive Quantizer	4	$36 \times 10^6$
1 bit Direct feedback without companding	4	$36 \times 10^6$
with companding	2.75	$25 \times 10^6$
3 bit Direct feedback without companding	3.5	$32 \times 10^6$
with companding	2.35	$21 \times 10^6$

1 bit system is probably not great enough to justify the use of a 3-bit system. This is especially so when one considers the complexity involved in building a 3 bit quantizer, as well as the added requirement for word synchronization. Similarly, the 3 bit delta may be sufficiently complex vis-a-vis the 1 bit direct feedback system as to make its consideration unwarranted.

#### 4.3 Companding

In this section we outline the structure of a compander which can be used with the various feedback quantization systems. While the discussion will be oriented towards companding of one-bit systems, the principles can readily be applied to N-bit systems. Various companding techniques have been introduced in Chapter 3. Of these, we consider the class of companders which change the feedback step size in accordance with the polarity of the received bit stream as the most promising. One of the advantages of this technique is that an auxiliary bit stream is not required to carry the companding data. A second advantage is that high-speed digital techniques can be used to form the feedback levels. Our discussion will therefore be of this class.

Various weighting sequences have been used in the past. Winkler's HIDM [Winkler, 1963] used an exponential weighting of 1,1,2,4,8,16. Bosworth and Candy [Bosworth and Candy, 1969], direct feedback coder used 1,1,2,3,5...5. Abate [Abate, 1967] used both linear and exponential series for weights. These, and others, can be implemented by the technique shown in Fig. 4.5. This figure shows a mechanization of the compander for the 1,2,2,3,5...5 weights. The extension of this to other weights is straightforward. A four stage shift register is used to store the four most recent digital transmissions. Based on their polarities, two possible feedback levels are set up. These two levels are chosen from the set  $\{+5, +3, +2, +1\}$  according to the truth table shown in Fig. 4.5.



Stage				+ Feedback	- Feedback
1	2	3	4		
1	1	1	1	+5	-1
1	1	1	-1	+3	-1
1	1	-1	1	+2	-1
1	1	-1	-1	+2	-1
1	-1	1	1	+1	-1
1	-1	1	-1	+1	-1
1	-1	-1	1	+1	-1
1	-1	-1	-1	+1	-1
-1	1	1	1	+1	-1
-1	1	1	-1	+1	-1
-1	1	-1	1	+1	-1
-1	1	-1	-1	+1	-1
-1	-1	1	1	+1	-2
-1	-1	1	-1	+1	-2
-1	-1	-1	1	+1	-3
-1	-1	-1	-1	+1	-5

Fig. 4.5 Compander Technique and Truth Table

The choice between the positive and negative feedback signal is based on the polarity of the next digital transmission. It is noteworthy that the entire companding process is a choice between preset voltage levels. Thus it can be implemented by high-speed digital switching logic. Furthermore, it can be implemented in two stages. The first stage, the setting up of the two feedback levels, can be started as soon as the new bit transmission occurs. The second stage, the selection between the two feedback levels, is started as soon as the polarity of the succeeding transmitted signal is known.

The amount of companding is given by the ratio of the largest to the smallest step size. In this case the ratio is 5, corresponding to 14 dB of companding. Depending on the variation expected among the classes of transmitted pictures, this may or may not be an adequate range. If a wider range is required, it can be achieved by simply multiplying the weights by the desired scaling factor.

It should be noted that those level generating systems which have the structure given in Fig. 4.5 are special cases of the structure considered by Fine [Fine, 1968].

#### 4.4 Multiplexing

The purpose of multiplexing is to combine several television channels or several sequences of digital signaling elements in such a way that:

- (1) the set of sequences is suitable for subsequent transformation into RF by carrier modulation, and
- (2) the set of sequences may be separated (demultiplexed) and the receiver without undue crosstalk.

Multiplexing can therefore be applied to either several video signals going into the modulator, or to the digital outputs of several different modulators. We will consider both of these techniques.

##### 4.4.1 Video Multiplexing

The two types of multiplexing which are normally performed on baseband data are time division multiplexing (TDM) and frequency division multiplexing (FDM). In TDM PCM if M signals,

each of bandwidth  $W$  Hz, are to be transmitted, they are sampled sequentially in time at a rate of  $2W$  samples/sec. If each of the samples is encoded into a  $N$  bit digital word, the digital data rate is  $2NMW$  bits/second. The encoding is conventionally performed with a zero-memory PCM encoder. This would correspond to the block diagram of Fig. 4.2(a) with the feedback loop opened. In order to use TDM on the input to any of the feedback modulators which we are considering, it would be necessary to also multiplex the parts of the modulator which have memory. This would be:

- (a) the filter and companding logic of the delta modulator
- (b) the integrator and companding logic of the direct feedback modulator
- (c) the delay portions of the predictor loop and the companding logic for the predictive quantization modulator.

TDM on the input is thus a practical technique at low data rates, and becomes increasingly more difficult as the data rate increases.

In FDM the baseband signals are shifted in frequency relative to each other, so that the  $M$  signals occupy a baseband of  $MW$  Hz. If this composite signal is to be sent by  $N$ -bit PCM, it is sampled at a  $2 MW$  rate, again resulting in  $2NMW$  bits/second. With a zero-memory encoder this results in a signal which can be recovered at the receiver. However the filter characteristics of the feedback modulators which we are considering would affect each of the shifted baseband signals in a different manner if FDM were used to multiplex the input to a feedback modulator. Due to the non-linear nature of the modulations, it would be necessary to experimentally determine the quality of the resulting video signals. It might be expected, however, that the quality of the pictures would be degraded.

#### 4.4.2 Digital Multiplexing

From the discussion of Section 4.4.1 one is led to the conclusion that probably the most practical means of multiplexing several feedback modulators is to operate on their output bit streams. This can be done in several ways. One method is to time division multiplex their respective bit streams. The resulting bit rate is  $M$  times the bit rate for each modulator. The multiplexed digital data stream can then be used to modulate an RF carrier by any of the conventional means, e.g., frequency or phase modulation. A second possibility is to frequency division multiplex the video signals, and use the composite data stream to modulate a carrier. If frequency modulation is used, the result is the familiar FDM/FM. A third form of multiplexing which can be employed is subcarrier modulation, which is analogous to FDM. In this technique, each of the digital modulator outputs is used to frequency or phase modulate a subcarrier oscillator. The separate oscillator outputs are then combined and used to modulate an RF carrier, which is the final transmitted signal. Frequency modulation is commonly used on the final signal as well, resulting in an FM/FM system.

The digital multiplexing techniques should be capable of operating at higher rates than the video multiplexing techniques, as there is no requirement of switching energy storing networks. The TDM technique is simply the problem of generating a short sampling pulse, while the FDM/FM and FM-FM modulation techniques pose no difficult hardware problems assuming that modulators with the required bandwidth are required.

#### 4.4.3 Demultiplexing Synchronization

The demultiplexing of the digital data poses no synchronization problems if FDM or subcarrier multiplexing techniques are used. This is so because the signals are combined in the frequency domain, where signal separation can be performed by frequency translation filtering, and frequency selective demodulators. Once the signals are demodulated, however, the

appropriate bit and word synchronizations must be established if they are required for proper video demodulator operation. As we have shown previously, a binary demodulator without companding requires no synchronization, a binary demodulator with companding requires bit synchronization, while a M'ary demodulator with or without companding requires both bit and word synchronization.

If TDM is utilized, means must be provided for separating the digital data streams and identifying their respective analog sources. Assuming that bit synchronization exists, this can be performed by interleaving a low data rate framing sequence along with the desired television data. Recognition of the framing sequence will allow the channels to be separated.

No separate signals are required to provide line and field synchronization of the television signal, as the input to the modulator is the composite video signal, which is made up of the picture video plus the synchronization waveforms.

#### 4.4.4 Color Television Multiplexing

The multiplexing of color pictures presents a separate, and interesting problem. The picture in color television is conventionally defined in terms of three primary signals,  $E_R$ ,  $E_G$ ,  $E_B$ , the red, green and blue primaries. The luminance signal,  $E_Y$ , is derived from these by the transformation:

$$E_Y = 0.30 E_R + 0.59 E_G + 0.11 E_B$$

The chrominance signals,  $E_I$  and  $E_Q$  are defined as:

$$E_I = 0.74 (E_R - E_Y) - 0.27 (E_B - E_Y)$$

$$E_Q = 0.48 (E_R - E_Y) + 0.41 (E_B - E_Y) .$$

Thus, given luminance and chrominance, the three primaries can be uniquely recovered by linear transformations. In United States color television, the luminance has a full 4.5 MHz bandwidth, while  $E_I$  is bandlimited to 1.5 MHz and  $E_Q$  to 0.5 MHz.

In forming the final transmitted signal,  $E_Y$ ,  $E_I$  and  $E_R$  are gamma corrected, the resulting chrominance signals are used as the in-phase and quadrature modulations for a quadrature RM signal, and the resulting signal is interleaved with the luminance signal to form a low-visibility pattern.

The operations described above are another form of multiplexing. In transmitting a color television signal via feedback quantization, one could consider at least three techniques. These are:

- (a) Transmit the red, blue and green signals, each over separate channels, and each at full bandwidth. This would require three times the bit rate of a single monochrome channel.
- (b) Form the band-limited, gamma-corrected in-phase and quadrature chrominance signals of bandwidths 0.5 and 1.5 MHz respectively. Transmit the luminance and the two chrominance signals as multiplexed video signals. This would require 1.45 times the bandwidth of a simple monochrome channel.
- (c) Form the standardized composite color picture signal and transmit it with no modifications.

Method (c) should require the lowest bandwidth if no significant distortions of the color information are introduced by the feedback quantization process. If distortions are introduced, a higher sampling rate than is required for monochrome transmission will be required. In this case, method (b) might prove to be a competitive technique. Method (a) is in all likelihood non-competitive, although it potentially provides the highest color fidelity. To our knowledge, studies of feedback quantizer modulation of color television has not been reported in the literature. The field is worthy of study. Obviously, many other techniques of color coding and color multiplexing can be devised. In particular, recent work by Gronemann [Gronemann, 1969], which



indicates that the color content of a picture may be less than 1 bit per element, indicates that digital color television should be achieved with essentially the same bit rate as monochrome television.

#### 4.5 Transmission Rates

In this section we consider the timing necessary for operation of one-bit delta modulators and direct feedback modulators, and show the factors which determine the maximum bit rate. For this discussion we refer to Figs. 4.2 and 4.5, and their related discussions. The timing cycle of a direct feedback modulator can be broken down into two distinct time-sequential phases. These are:

##### Phase 1

- . Feed forward switch closed
- . Integration performed in feed forward network
- . Current transmitted signal stored at step-control logic input

##### Phase 2

- . Feed forward network switch opened
- . Companding registers shifted
- . New threshold decision made
- . Step generator logic sets next feedback step.

Under ideal circumstances, Phase 1 should use the entire interpulse period, with Phase 2 taking zero time. However, the shifting in the storage registers, setting of the thresholds and the threshold decision each take a non-zero amount of time, as do the opening and closing of the feed forward switch, the switching between the positive and negative step, and the logical decisions necessary for the setting up of the two possible step amplitudes.

Using currently available components, it is estimated that a switch can be built which will change state in about 2 ns. Operational amplifiers such as would be used in the input circuit, the

quantizer, and the feed-forward integrator are now available with similar delays. Even utilizing high speed logic circuits, the companding register shifting and associated logical operations would probably take the order of 10 ns. Thus there are basic loop propagation times in the order of 15 ns, which indicate a maximum bit rate of the order of 67 Mbps for a companded system. It is likely that other considerations would lower the maximum rate, so a more conservative limit would be 50 Mbps. This is a sufficiently high rate for a 4.5 MHz signal to be transmitted by delta modulation, or a 9 MHz signal to be transmitted by a direct feedback modulator. In principle, still higher bit rates are possible if parallel companding logic is provided, so that Phase 2 can be performed in parallel with Phase 2. The limiting case would achieve the same bit rate as an uncomanded system, which is estimated to be the order of 100 Mbps.

Appendix A  
NEW TECHNOLOGY

After a diligent review of the work performed under this contract, no new innovation, discovery, improvement or invention was made.

## Appendix B

### EQUIVALENCE OF DIRECT FEEDBACK AND CONVENTION DELTA-MODULATOR STRUCTURES

The purpose of this Appendix is to show that the Direct Feedback modulator, which is illustrated in Fig. A1, can be made equivalent to a conventional delta-modulator (Fig. A2) by appropriate selection of the filters  $H_1$  and  $H_2$ .

In reference to Fig. A1, let  $x$  be the modulator input,  $y$  be the modulator output and  $z$  the input to the filter  $H_2$ . We can write

$$z = H_1 x - y$$

where  $H_1 x$  denotes the output of filter  $H_1$  with  $x$  as input.

Referring now to Fig. A3, the input to filter  $H_2$  is given by

$$z = H_1 x - H_1 (H_1^{-1} y)$$

where  $H_1^{-1}$  is the inverse filter of  $H_1$ . This can be written as

$$z = H_1 x - y$$

which is the same as that obtained from the direct feedback model. If we now set

$$H_1^{-1} = H_2 = H_3$$

the structure given in Fig. A3 collapses to that shown in Fig. A2.

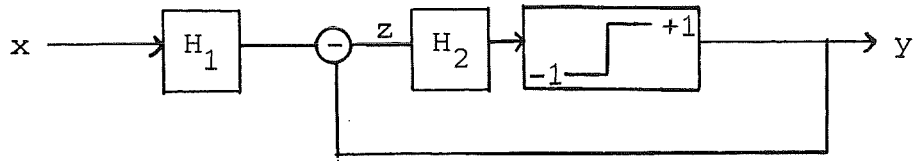


Fig. A1. Direct Feedback

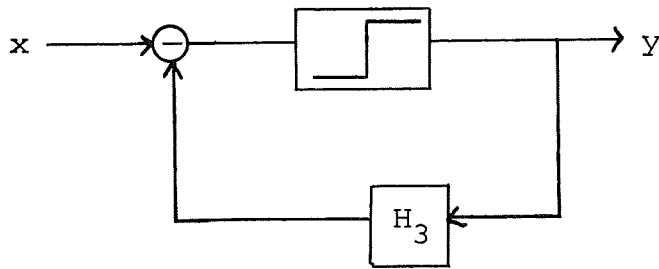


Fig. A2. Conventional

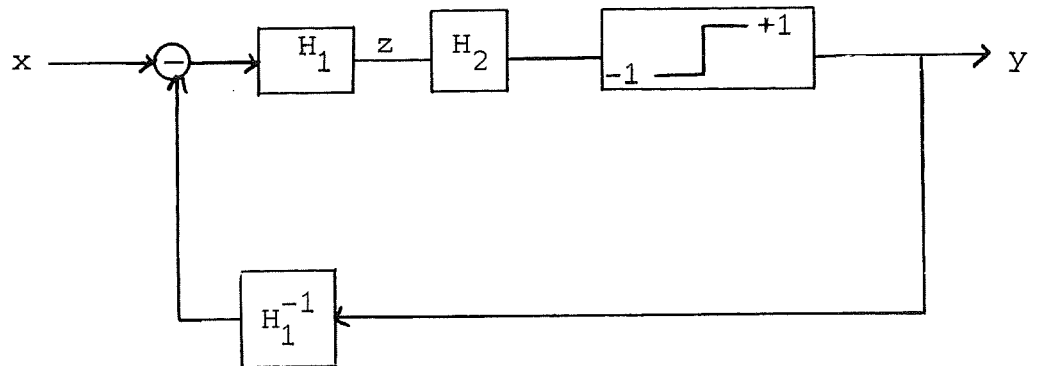


Fig. A3.

## BIBLIOGRAPHY

- Aaron, M.R., Fleischman, J.S., McDonald, R.A., and Protonotarios, E.N., "Response of Delta Modulation to Gaussian Signals", BSTJ, Vol. 48, No. 5, May-June 1969, pp. 1167-1196.
- Abate, J.E., "Linear and Adaptive Delta Modulation", Proc. IEEE, Vol. 55, No. 3, March 1967, pp. 298-307.
- Ando, H., see Fukushima, K., Dec. 1963.
- Ando, H., and Yamaguchi, K., "TV Bandwidth Compression System by Contour Interpolation", J. Inst. Elect. Commun. Engrs., Japan, Vol. 49, No. 6, June 1966, pp. 1107-1113 (in Japanese).
- Aoki, T., see Inose, H., June 1963.
- Aoki, T., see Inose, H., March 1966.
- Arter, F.W., "Simple Delta Modulation System", Proc. IRE (Australia), Vol. 23, No. 9, Sept. 1962, pp. 517-523.
- Aughenbaugh, G.W., Irwin, J.D., and O'Neal, J.B., "Delayed Differential Pulse Code Modulation", Proc. of the Second Annual Princetone Conference on Information Sciences & Systems, March 1968.
- Baikovskii, V.M., "On the Noise Immunity of the Discrete Incremental Method of Information Transmission", Avtomat. i Telemeth (USSR), Vol. 21, No. 3, 1960, pp. 344-358 (in Russian).
- Baikovskii, V.M., "Transmission Capacity of a Channel with Discrete Differential Modulation", Radiotekhnika (USSR), Vol. 16, No. 7, July 1961, pp. 11-19 (in Russian).
- Balder, J.C., and Kramer, C., "Video Transmission by Delta Modulation Using Tunnell Diodes", Proc. IRE, Vol. 50, No. 4, April 1962, pp. 428-431.
- Balder, J.C., see Kramer, C., Aug. 1963.
- Balder, J.C., and Kramer, C., "Analog-to-Digital Conversion by Means of Delta Modulation", IEEE Trans. Space Elect. and Telemetry, Vol. SET-10, No. 3, Sept. 1964, pp. 87-90.
- Barton, M.P., see Cherry, C., 1963.
- Beddoes, M.P., "Experiments with a Slope-Feedback Code for Television Compression", IRE Trans. Broadcasting, Vol. BC-7, No. 2, March 1961, pp. 12-28.
- Beddoes, M.P., "A Two-Channel Method for Compressing the Bandwidth of Television Signals", Proc. IEE (Gt. Grtn), Vol. 110, No. 2, Feb. 1963, pp. 369-374.

- Bellman, A., and Vagliani, F., "Optimal Quantization in Pulse Code Modulation Systems", Alta Frequenza (Italy), Vol. 36, No. 10, Oct. 1967, pp. 942-951.
- Bello, P.A., Lincoln, R.N., and Gish, H., "Statistical Delta Modulation", Proc. IEEE, Vol. 55, No. 3, March 1967, pp. 308-319.
- Benz, F., "Pulse Code Modulation and Possibilities of Its Further Development", Elektrotech, u. Maschinenhan (EuM), Vol. 73, No. 4, Feb. 1956, pp. 73-78 (in German).
- Bisignani, W.T., Richards, G.P., and Whelan, J.W., "The Improved Gray Scale and the Coarse-Fine PCM Systems, Two New Digital TV Bandwidth Reduction Techniques", Proc. IEEE, Vol. 54, No. 3, 1966, pp. 376-390.
- Bosworth, R.H., and Candy, J.C., "A Companded One-Bit Coder for Television Transmission", BSTJ, Vol. 48, No. 5, May-June 1969, pp. 1459-1480.
- Bowers, F.K., "What Use is Delta Modulation to the Engineers", Trans. AIEE I, Vol. 76, 1957, pp. 142-147, also Comm. and Elec. No. 30, May 1957.
- Bowers, F.K., "Delta Modulation for Cheap and Simple Telemetering", IRE Wescon Conv. Rec., Vol. 3, pt. 5, 1959, pp. 63-67.
- Brainard, R.C., Mounts, F.W., and Prasada, B., "Low-Resolution TV: Subjective Effects of Frame Repetition and Picture Replenishment", BSTJ, Vol. 46, No. 1, Jan. 1967, pp. 261-271.
- Brainard, R.C., "Low Resolution TV: Subjective Effects on Noise Added to a Signal", BSTJ, Vol. 46, No. 1, Jan. 1967, pp. 233-260.
- Brainard, R.C., "Subjective Evaluation of PCM Noise-Feedback Coder for Television", Proc. IEEE, Vol. 55, No. 3, March 1967, pp. 346-352.
- Brainard, R.C., and Candy, J.C., "Direct-Feedback Coders: Design and Performance with Television Signals", Proc. IEEE, Vol. 57, No. 5, May 1969, pp. 776-786.
- Brockelsby, C.F., and Palfreeman, J.S., "Ultrasonic Delay Line and Their Applications to Television", Philips Tech. Review, Vol. 25, No. 9, 1963-1964, pp. 234-252.
- Brolin, S.J., see Brown, J.M., 1967.
- Brolin, S.J., and Brown, J.M., "Companded Delta Modulation for Telephony", IEEE Trans. Commun. Tech., Vol. COM-16, No. 1, Feb. 1968, pp. 157-162.

- Brown, E.F., "Low-Resolution TV: Subjective Comparison of Interlaced and Noninterlaced Pictures", BSTJ, Vol. 46, No. 1, Jan. 1967, pp. 199-232.
- Brown, E.F., "A Sliding-Scale Direct-Feedback PCM Coder for Television", BSTJ, Vol. 48, No. 5, May-June 1969, pp. 1537-1553.
- Brown, J.M. and Brolin, S.J., "Companded Delta Modulation for Telephoning", Proc. IEEE Region III Convention, Jackson, 1967, pp. 161-164.
- Brown, J.M., see Brolin, S.J., 1968.
- Buck, A.L., "Reduction of Bandwidth in Television Transmission Systems", IRE Trans. Broadcasting, Vol. BC-8, No. 1, April 1962, pp. 29-33.
- Budrikis, Z.L., "Upper Bound to the Information Rate of the Television Source", National Radio and Electronics Engineering Convention, Sydney, Australia, 1967, pp. 26-27.
- Bylanski, P., "Second-Order Redundancy of Coarsely Quantized Video Signals", Proc. IEE (Gr. Brtn), Vol. 114, No. 12, Dec. 1967, pp. 1831-3.
- Capon, J., "A Probabilistic Model for Run-Length Coding of Pictures", IRE Trans. Inf. Th., Vol. IT-5, No. 4, Dec. 1959, pp. 157-163.
- Carbrey, R.L., "Video Transmission Over Telephone Cable Pairs by Pulse Code Modulation", Proc. IRE, Vol. 48, Sept. 1961, pp. 1546-1561.
- Chatterjee, P.K., see Das, J., 1967.
- Chatterjee, P.K., see Das., J., 1968.
- Cherry, E.C., "The Bandwidth Compression of Television and Facsimile", J. Televis. Soc. (Gt. Brtn), Vol. 10, No. 2, April-June 1962, pp. 40-49.
- Cherry, E.C., "An Experimental Compression System for Facsimile and Television", IEE Conf. Rep. Ser. (Gt. Brtn), No. 5, 1963, pp. 16-18, Television Engineering Conference, London, 1962.
- Cherry, E.C., Kubba, M.H., Pearson, D.E., and Barton, M.P., "An Experimental Study of the Possible Bandwidth Compression of Visual Image Signals", Proc. IEE (Gt. Brtn), Vol. 51, No. 11, Nov. 1963, pp. 1507-1517.
- Ciscato, D., and DiCamillo, P., "Delta Modulating System Analysis", Automaz. e Strum (Italy), Vol. 14, No. 11, Nov. 1966, pp. 518-525 (in Italian).



Cotton, R.V., et al, "Demonstration of the Feasibility of Using Delta Modulation for Pictorial Transmission", Final Report, May 1963, Contract AF33(657)-8478, ASD-TDR-63-528; also Phase 1 Report, ASD-TDR-62-1037, Nov. 1962.

Cotton, R.V., see Remm, R.L., 1966.

Cotton, R.V., Remm, R.L., and Strohmeyer, G.R., "Analysis of Delta Modulation Encoding Using Computer Simulation", IEEE Trans. Aero-Space Elect. Sys., Vol. AES-2, No. 4, (supp), July 1966, pp. 380-387 (1966 Aerospace Systems Conference, Washington, 1966).

Daae, N., "Delta Modulation", Elektrotek Tidsskrift, (Norway), Vol. 74, No. 4, Feb. 5, 1961, pp. 51-55 (in Norwegian).

Das, J., and Chatterjee, P.K., "Optimized  $\Delta$ - $\Delta$  Modulation System", Electronics Letters (Grt. Brtn), Vol. 3, No. 6, June 1967, pp. 286-287.

Das, J., Farugui, N.M., Sharma, P.D., and Chatterjee, P.K., "A Comparison of Synchronous and Asynchronous Pulse Modulation Systems", J. Inst. Engrs., (India), Vol. 48, No. 9, May 1968, pp. 666-688.

Davies, R. E., see Wharton, W., June 1966.

Debart, H., "A Study of the "Delta" Modulation System from the Point of View of Information Theory", Cables et Transm. (France), Vol. 15, No. 3, July 1961, pp. 198-204 (in French).

DeJager, F., see Schouten, J.F., June 1952.

DeJager, F., "Delta Modulation, A Method of PCM Transmission Using the 1-Unit Code", Philips Res. Reports, Vol. 7, Dec. 1952, pp. 442-468.

DeJager, F., and Greefkes, J.A., "Continuous Delta Modulation", IEEE International Symposium on Global Commun., Vol. 6, June 1964, pp. 38-39.

DeJager, F., see Greefkes, J.A., 1968.

Deloraine, E.M., VanMerlo, S., and Derjavitch, B., "Method and System of Impulse Transmission", French Patent 932-140, Aug. 10, 1946.

Deregnaucourt, J., "Systems Engineering Aspects of High Capacity PCM Systems", IEEE Internat. Convention Record, Vol. 14, Pt. 1, 1966, pp. 175-179.

Deriugin, N.G., "The Power Spectrum and Correlation Function of the Television Signal", Telecommunications, 7, 1957, pp. 1-12.

- DiCamillo, P., see Ciscato, D., 1966.
- Dite, W., see Panter, P.F., Jan. 1951.
- Drapkin, P.E., "Video Data Compression", IEE Trans., Aerospace Electronics Syst., Vol. AES-2, No. 4, July 1966, pp. 392-400, (1966 Aerospace Systems Conf., Washington, 1966).
- Edson, J.O., and Henning, H.H., "Broadband Coders for an Experimental 224Mb/s PCM Terminal", BSTJ, Vol. XLIV, No. 9, Nov. 1965, pp. 1887-1940.
- Ehrman, L., "Analysis of Some Redundancy Removal Bandwidth Compression Techniques", Proc. IEE, Vol. 55, No. 3, March 1967, pp. 278-286.
- Fadely, J. K., see Knight, J.M., Dec. 1964.
- Farugui, M.N., "A Slope-Quantized Binary Pulse Code Modulation", Electronic Engng (Grt. Brtn), Vol. 38, Jan. 1966, pp. 31-35.
- Farugui, N.M., see Das, J., 1968.
- Fine, T., "Properties of an Optimum Digital System and Applications", IEEE Trans. on Inf. Th., Vol. IT-10, Oct. 1964, pp. 287-296.
- Fine, T., U.S. Patent No. 3,393,364, Statistical Delta Modulation System, July 1968.
- Franks, L.E., "A Model for the Random Video Process", BSTJ, Vol. 45, No. 4, April 1966, pp. 609-630.
- Fujita, H., see Inose, H., Jan. 25, 1963.
- Fukushima, R., "Prediction Theorem in Mutli-Dimensional Space", J. Inst. Elect. Commun. Engrs. Japan, Vol. 46, No. 2, Feb. 1963, pp. 169-178 (in Japanese).
- Fukushima, K. and Ando, H., "TV Band Compression by Multi-Mode Interpolation - Experiment by Computer Simulation", Tech. J. Japan Broadcasting Corp., Vol. 15, No. 12, Dec. 1963, pp. 1-17.
- Gabor, D., and Hill, P.C.J., "Television Band Compression by Contour Interpolation", Proc. IEE (Grt. Brtn), paper 3507E, Vol. 108B, May 1961, pp. 308-315.
- Geddes, W.K.E., see Newell, G.F., 1961.
- Gerber, S.E., "The Intelligibility of Delta Modulated Speech", IEEE Trans. Audio Electroacoustics, Vol. AU-14, No. 2, June 1966, pp. 93-96.
- Gish, H., see Bello, P.A., 1967.
- Gish, H., "Optimum Quantization of Random Sequences", Tech. Rep. 529, Harvard University Division of Engineering and Applied Physics, May 1967.

- Golding, L.S., and Schultheiss, A.M., "Study of an Adaptive Quantizer", Proc. IEEE, Vol. 55, No. 3, March 1967, pp. 293-297.
- Goodman, D.J., "The Application of Delta Modulation to Analog-to-PCM Encoding", BSTJ, Vol. 48, No. 2, Feb. 1969, pp. 321-344.
- Goodman, D.J., "Delta Modulation Granular Quantizing Noise", BSTJ, Vol. 48, No. 5, May-June 1969, pp. 1197-1218.
- Gouriet, G.G., "Bandwidth Compression of a Television Signal", Proc. IEEE, Paper 2357R, Vol. 104B, May 1957, pp. 265-272.
- Graham, R.E., "Predictive Quantization of Television Signals", IRE Wescon Conv. Rec., Vol. 2, pt. 4, 1958, pp. 147-157.
- Graham, R.E., U.S. Patent No. 2,905.756, Method and Apparatus for Reducing Television Bandwidth, Sept. 2, 1959.
- Gray, J. R., and Kitsopoulos, S.C., "A Precision Sample-and-Hold Circuit with Subnanosecond Switching", IEEE Trans. Circ. Th., CT 11, Sept. 1964, pp. 389-396.
- Greefkes, J.A., see Schouten, J.F., June 1952.
- Greefkes, J.A., and DeJager, F., "Continuous Delta Modulation", Philips Res. Rep. (Netherlands), Vol. 23, No. 2, April 1968, pp. 233-246.
- Gronneman, U.F., "Coding Color Pictures", Symposium on Bandwidth Compression, MIT, Cambridge, Mass., April 2-4, 1969.
- Habara, K., " $\Delta$ -M Demodulator Using Esaki Diodes", J. Inst. Elect. Commun. Engrs., Japan, Vol. 46, No. 6, June 1963, pp. 785-794 (in Japanese).
- Halijak, C.A., and Tripp, J.S., "A Deterministic Study of Delta Modulation", Fourth Joint Automatic Control Conference, (New York, A.I. Ch. E, 1963), pp. 376-385.
- Halijak, C.A., and Tripp, J.S., "A Deterministic Study of Delta Modulation", IEEE Int. Conv. Rec., Vol. 11, Pt. 8, 1963, pp. 247-259.
- Heiter, G.C., and Young, E.H., "A 100 MHz, 1024 Bit Recirculating Ultrasonic Delay Line Store", Digest Solid-State Circuits Conf., Paper FAM 10-3, Feb. 16, 1968.
- Hill, P.C.J., see Gabor, D., 1961.
- Hosokawa, S., see Tanaka, Y., 1963.
- Hosokawa, S., see Tanaka, Y., Dec. 1963.

- Hosokawa, S., Yamashita, K., and Tanaka, Y., "Delta Modulation Coder Having Instantaneous Companded Characteristics", Mem. Fac. Engng. Osaka City Univ. (Japan), Vol. 7, Dec. 1965, pp. 55-60.
- Hosokawa, S. and Tazaki, S., "Delta Modulation With Instantaneous Companding of Signal Slope", Mem. Fac. Engng. Osaka City Univ. (Japan), Vol. 8, Dec. 1966, pp. 123-131.
- Huang, T., "PCM Picture Transmission", IEEE Spectrum, Vol. 2, No. 12, Dec. 1965, pp. 57-63.
- Huang, T., "Digital Picture Coding", Proc. of the Nat. Elect. Conf., Chicago, Vol. 22, 1966, pp. 783-787.
- Huang, T.S., Tretiak, O.J., Prasada, B., and Yamaguchi, Y., "Design Considerations in PCM Transmission of Low Resolution Monochrome Still Pictures", Proc. IRE, Vol. 55, No. 3, March 1967, pp. 331-335.
- Inose, H., Yasuda, Y., and Murakami, J., "A Communication System by Code Modulation ( $\Delta$ - $\Sigma$  Modulation)", J. Inst. Elect. Commun. Engrs., Japan, Vol. 44, No. 11, Nov. 1961, pp. 1775-1780 (in Japanese).
- Inose, H., Yasuda, Y., and Murakami, J., "A Telemetry System by Code Modulation -  $\Delta$ - $\Sigma$  Modulation", IRE Trans. Space Elect. and Telemetry, Vol. SET-8, No. 3, Sept. 1962, pp. 204-209.
- Inose, H., Yasuda, Y., Kawai, Y., and Takazi, M., "The Subscriber-Line Circuit and the Signaling and Tone System for an Experimental Time-Division Exchange Featuring Delta-Modulation Techniques", IRE Trans. Comm. Sys., Vol. CS-10, No. 4, Dec. 1962, pp. 397-407.
- Inose, H., Yasuda, J. Murakami, and Fujita, H., "New Modulation Technique Simplifies Circuits", Electronics, Vol. 36, No. 4, Jan. 25, 1963, pp. 52-55.
- Inose, H., Tagaki, M., and Aoki, T., "A Method of Synchronization in Time-Division Multiplexed Delta Modulation System", J. Inst. Elect. Commun. Engrs., Japan, Vol. 46, No. 6, June 1963, pp. 785-794 (in Japanese).
- Inose, H., and Yasuda, Y., "A Unity Bit Coding Method by Negative Feedback", Proc. IEE (Gr. Brtn), Vol. 51, No. 11, Nov. 1963, pp. 1524-1535.
- Inose, H., Aoki, T., and Watanabe, K., "Asynchronous Delta Modulation System", J. Inst. Elect. Commun. Engrs., Japan, Vol. 49, No. 3, March 1966, pp. 9-11 (in Japanese).

- Itosokawa, S., "Ternary Delta Modulation", J. Inst. Elect. Commun. Engrs., Japan, Vol. 49, No. 11, Nov. 1966, pp. 2103-2109.
- Johnson, F.B., "Calculating Delta Modulator Performance", IEEE Trans. Audio and Electroacoustics, Vol. MU-16, No. 1, March 1968, pp. 121-129.
- Kaneko, H., see Tomozawa, A., 1966.
- Kaneko, H., see Tomozawa, A., Feb. 1968.
- Kawai, Y., see Inose, H., Dec. 1962.
- Kelley, R.A., "An Experimental High-Speed Digital Transmission System", Bell Lab. Record, Vol. 45, No. 2, Feb. 1967, pp. 45-48.
- Kellogg, W.C., "Information Rates in Sampling and Quantization", IEEE Trans. Inf. Th., Vol. IT-13, No. 3, July 1967, pp. 506-511.
- King, B.C., see Knight, J.M., Dec. 1964.
- Kitsopoulos, S.C., and Kretzmer, E.R., "Computer Simulation of a Television Coding Scheme", Proc. IRE, Vol. 49, No. 6, June 1961, pp. 1076-1077.
- Knapp, C.F., see Schreiber, W.F., 1958.
- Knight, J.M., Fadely, J.K., Raga, G.L., and King, B.C., "Digital Television: Shrinking Bulky Bandwidth", Electronics, Vol. 37, No. 31, 14 Dec. 1964, pp. 77-84.
- Koehler, D., "A 110 Megabit Gray Code to Binary Code Serial Translator", 1965 Int. Solid-State Circuits Conf. Digest Techn. Papers.
- Korn, G.A., "Analog/Hybrid Storage and Pulse Modulation", IEE Trans. Electronic Comp., Vol. EC-12, No. 4, Aug. 1963, pp. 411-412.
- Korobkov, L.A., "A Method of Transmitting Electrical Signals", Author's Certificate No. 106036 (USSR), Byulleten Izobretenii, No. 4, 1957.
- Kovanic, E.F., "A High Accuracy Nine-Bit Digital-to-Analog Converter Operating at 12 MC.", IEEE Trans. Commun. Electron., March 1964, pp. 185-191.
- Kozuch, M., see Lender, A., 1961.
- Kramer, C., see Balder, J.C., 1962.
- Kramer, C., and Balder, J.C., "Television Waveguide Link", Electronics, Vol. 36, No. 31, Aug. 2, 1963, pp. 51-52.
- Kramer, C., see Balder, J.C., 1964.

Kretzmer, E.R., see Kitsopoulos, S.C., June 1961.

Kretzmer, E.R., "Statistics of Television Signals", BSTJ., Vol. 31, July 1952, pp. 751-763.

Kubba, M.H., see Cherry, C., 1963.

Kubba, M.H., "Automatic Picture Detail Detection in the Presence of Random Noise", Proc. IEE (Grt. Brtn), Vol. 51, No. 11, Nov. 1963, pp. 1518-1523.

Kubba, M.H., see Cherry, C., 1963.

Lender, A., and Kozuch, M., "Single-Bit Delta Modulating Systems", Electronics, Vol. 34, No. 46, Nov. 17, 1961, pp. 125-129.

Libois, L.J., "New Method of Code Modulation- $\Delta$ -Modulation", Onde Elect., Vol. 32, Jan. 1952, pp. 26-31 (in French).

Limb, J.O., and Sutherland, I.G., "Run-Length Coding of Television Signals", Proc. IEEE, Vol. 53, No. 2, Feb. 1965, pp. 169-170.

Limb, J.O., "Differential Quantization of Television Signals", National Radio and Electronics Engineering Convention, Sydney, 1967, pp. 98-9.

Limb, J.O., "Source-Receiver Encoding of Television Signals", Proc. IEEE, Vol. 55, No. 3, March 1967, pp. 364-379.

Limb., J.O., "Entropy of Quantized Television Signals", Proc. IEE (Grt. Brtn), Vol. 115, No. 1, Jan. 1968, pp. 16-20.

Linocln, R.N., see Bello, P.A., 1967.

MacDonald, R.A., "Signal-to-Noise and Idle Channel Performance of Differential Pulse Code Modulation Systems — Particular Applications to Voice Signals", BSTJ, Vol. 45, No. 7, Sept. 1966, pp. 1123-1151.

Magnuski, H., "Wide-Band Channel for Emergency Communication", IRE Int. Conv. Rec., Vol. 9, pt. 8, 1961, pp. 80-84, see also IRE Trans. Veh. Comm., Vol. VC-10, No. 2, Aug. 1961, pp. 40-45.

Malling, L.R., "Video Data Compression", Instrum. Control Syst., Vol. 40, No. 4, April 1967, pp. 94-5.

Manasse, F.K., "Directional Correlation — A Technique to Reduce Bandwidth in PCM Television Transmissions", IEEE Trans. Commun. Tech., Vol. COM-15, No. 2, April 1967, pp. 204-208.

Martin, A.V.J., "Two-Dimensional Predictive Redundancy in a Television Display", IRE Trans. Comm. Sys., CS-7, No. 1, May 1959, pp. 57-61.

- Maschoff, R.H., "Delta Modulation", Electro-Technology, Vol. 73, No. 1, Jan. 1964, pp. 91-98.
- Matsuoka, T., see Miyakaua, H., Nov. 1966.
- Mazo, J.E., "Quantizing Noise and Data Transmission", BSTJ, Vol. 47, No. 8, Oct. 1968, pp. 1737-1754.
- Manshikov, G.G., "Investigation of Error in Double Delta Modulation", Cybernetics, Vol. 2, No. 2, March-April 1966, pp. 14-20.
- Mertens, H., "The General Principles of Frequency Compression", Rev. HF (Belgium), Vol. 7, No. 6, 1968, pp. 161-170 (in French).
- Miyakaua, H., and Matsuoka, T., "Predictive Transformation of Binary Sequences Obtained by Delta Modulation", J. Inst. Elect. Commun. Engrs., Japan, Vol. 49, No. 11, Nov. 1966, pp. 2210-2216.
- Mounts, F.W., "Low-Resolution TV: An Experimental Digital System for Evaluating Bandwidth-Reduction Techniques", BSTJ, Vol. 46, No. 1, Jan. 1967, pp. 167-198.
- Mounts, F.W., see Brainard, R.C., 1967.
- Mounts, F.W., and Pearson, D.E., "Apparent Increase in Noise Level When Television Pictures are Frame-Repeated", BSTJ, Vol. 48, No. 3, March 1969, pp. 527-540.
- Mounts, F.W., "Frame-to-Frame Digital Processing of TV Pictures to Remove Redundancy", Paper 6.2 of Symposium on Picture Bandwidth Compression, April 2-4, 1969, MIT, Cambridge, Mass.
- Murakami, J., see Inose, H., Nov. 1961.
- Murakami, J., see Inose, H., Sept. 1962.
- Murakami, J., see Inose, H., Jan. 25, 1963.
- Newell, G.F., and Geddes, W.K.E., "Tests of Three Systems of Bandwidth Compression of Television Signals", Proc. IEE (Gr. Brtn), paper 3613, July 1961, 13 pp., see also Vol. 109B, 1962.
- Niatdori, K., "Statistical Analysis of DPCM", J. Inst. Elec. Commun. Engrs., Japan, Vol. 48, No. 2, Feb. 1965, pp. 4-6.
- Norvell, B., See Varsos, S.G., 1964.

- Oliver, B.M., "Efficient Coding Monograph", No. 2009, 1952.
- O'Neal, J.B., Jr., "Delta Modulation Quantizing Noise Analytical and Computer Simulation Results for Gaussian and Television Input Signals", BSTJ, Vol. 45, No. 1, Jan. 1966, pp. 117-142.
- O'Neal, J.B., Jr., "Predictive Quantizing Systems (Differential Pulse Code Modulation) for the Transmission of Television Signals", BSTJ, Vol. 45, No. 5, May-June 1966, pp. 689-721.
- O'Neal, J.B., "A Bound on Signal to Quantizing Noise Ratios for Digital Encoding Schemes", Proc. IEEE, Vol. 55, No. 3, March 1967, pp. 287-291.
- O'Neal, J.B., "Noise in Delta Modulation Systems Caused by Errors in the Transmission Medium", Proc. 1968 IEE National Telemetry Conf., Houston, 1968, pp. 125-130.
- Palfreeman, J.S., see Brockelsby, C.F., 1963-64.
- Pallari, R., "Perturbations and Signal Regeneration", Alta Frequenza, Vol. 23, Feb. 1954, pp. 16-36 (in Italian).
- Panter, P.F., and Dite, W., "Quantization Distortion in Pulse-Count Modulation with Non-Uniform Spacing of Level", Proc. IRE, Vol. 39, Jan. 1951, pp. 44-48.
- Pearson, D.E., see Cherry, C., 1963.
- Picardi, G., "Delta Modulation of Speech", Note Recens. Not. (Italy), Vol. 14, No. 5-6, Sept.-Dec. 1965, pp. 317-332 (in Italian).
- Prasada, B., "Some Possibilities of Picture Signal Bandwidth Compression", IEEE Trans. Comm. Sys., Vol. CS-11, No. 3, Sept. 1963, pp. 315-328.
- Prasada, B., see Brainard, R.C., 1967.
- Pratt, W.K., "Coding Compression of a Television Bandwidth Compression Reduction System", Proc. IEEE, Vol. 54, No. 6, June 1966, pp. 914-916.
- Pratt, W.K., "A Bibliography on Television Bandwidth Reduction Studies", IEEE Trans. Inf. Th., Vol. IT-13, No. 1, Jan. 1967, pp. 114-115.
- Protonotarios, E.N., "Slope Overload Noise in Differential Pulse Code Modulation Systems", BSTJ, Vol. 46, No. 9, Nov. 1967, pp. 2119-2161.
- Protonotarios, E.N., see Aaron, 1969.
- Raga, G.L., see Knight, J.M., 1964.
- Raga, G.L., "A Unified Approach to Digital Television Compression", 1965 National Telemetry Conference, pp. 29-34.



- Remm, R.L., Cotton, R.V., and Stronmeyer, G.R., "Analysis and Implementation of a Delta Modulation Pictorial Encoding System", Proceedings 1966 National Telemetry Conference, Boston, Mass., pp. 27-34.
- Remm, R.L., see Cotton, R.V., 1966.
- Richards, G.P., see Bisignani, W.T., 1966.
- Richards, G.P., and Bisignani, W.T., "Redundancy Reduction Applied to Coarse-Fine Encoded Video", Proc. IEEE, Vol. 55, No. 10, pp. 1707-1717.
- Roberts, L.G., "Picture Coding Using Pseudo-Random Noise", IRE Trans. Inf. Th., Vol. IT-8, No. 2, Feb. 1962, pp. 145-154.
- Rosenfeld, A., "Bandwidth Reduction Bibliography", IEEE Trans. Inf. Th., Vol. IT-14, No. 4, July 1968, pp. 601-602.
- Sassa, K., "Computer Simulation of the Variable Interval System for Bandwidth Compression", Tech. J. Japan Broadcasting Corp., Vol. 19, No. 2, 1967, pp. 84-92.
- Schouten, J.F., DeJager, F., and Greefkes, J.A., "Delta Modulation, A New Modulation System for Telecommunication", Philips Tech. Rev., Vol. 13, March 1952, pp. 237-248.
- Schouten, J.F., DeJager, F., and Greefkes, J.A., "Delta Modulation, A New Modulation System for Telecommunication", Communications News, Vol. 12, June 1952, pp. 115-124.
- Schreiber, W.F., "The Measurement of Third-Order Probability Distributions of Television Signals", IRE Trans. on Inf. Th., Vol. IT-2, Sept. 1956, pp. 94-105.
- Schreiber, W.F., and Knapp, C.F., "TV Bandwidth Reduction by Digital Coding", IRE Nat. Conv. Record., Vol. 6, pt. 4, 1958, pp. 88-99.
- Schreiber, W.F., "Picture Coding", Proc. IEEE, Vol. 55, No. 3, March 1967, pp. 320-331.
- Schroter, F., "Bandwidth Economy in Television", Tech. Mitt. PTT (Switzerland), Vol. 40, No. 10, 1962, pp. 354-363 (in French and German).
- Schroter, F., "Storage and Frequency Band Compression in TV", Bull. Assoc. Suisse Elect., (Switzerland), Vol. 51, No. 20, Oct. 8, 1960, pp. 999-1004 (in German).
- Schultheiss, A.M., see Golding, L.S., 1967.

- Seyler, A.J., "Bandwidth Reduction in Television Relaying", Proc. IRE Australia, Vol. 16, No. 7, pp. 218-225, No. 8, pp. 271-276, July-Aug. 1955.
- Seyler, A.J., "The Coding of Visual Signals to Reduce Channel Capacity Requirements", Proc. IEEE (Grt. Brtn), Vol. 109C, July 1962, pp. 676-684.
- Seyler, A.J., "An Experimental Frame Difference Signal Generator for the Analysis of Television Signals", Proc. IRE Australia, Vol. 24, No. 11, Nov. 1963, pp. 797-807.
- Seyler, A.J., "Probability Distributions of Television Frame Differences", Proc. IRE Australia, Vol. 26, No. 11, Nov. 1965, pp. 355-365.
- Sharma, P.D., "Characteristics of Asynchronous Delta Modulation and Binary Slope Quantized PCM Systems. Second in the Series of Articles on Feedback Modulation Systems", Electronic Engng. (Grt. Brtn), Vol. 40, Jan. 1968, pp. 32-37.
- Sharma, P.D., see Das, J., 1968.
- Sittig, E.K., "High Speed Ultrasonic Delay Line Design: A Restatement of Some Basic Considerations", Proc. IEEE, Vol. 56, No. 7, July 1968, pp. 1194-1202.
- Sittig, E.K., and Smits, F.M., "Recirculating Ultrasonic Stores: An Economical Approach to Sequential Storage with Bit Rates Beyond 100 MHz", BSTJ, Vol. 48, No. 3, March 1969, pp. 659-674.
- Stampfl, R., "Remarks on Pulse Delta Modulation", Oster Z. Telegr.-Teleph.-Funk-Fernsehtech., [OTF], Vol. 8, No. 5-6, May-June 1954, pp. 58-63 (in German).
- Stampfl, R., "Notes on Pulse Delta Modulation", Oster Z. Telegr.-Teleph.-Funk-Fernsehtech., [OTF], Vol. 8, No. 7-8, July-Aug. 1954 (in German).
- Steele, R., and Thomas, M.W.S., "Two Transistor Delta Modulator", Electronic Engineering (Grt. Brtn), Vol. 40, Sept. 1968, pp. 513-516.
- Strohmeier, G.R., see Cotton, R.V., 1966.
- Stronmeyer, G.R., see Remm, R.L., 1966.
- Sutherland, I.G., see Limb, J.O., Feb. 1965.
- Tagaki, M., see Inose, H., June 1963.
- Takazi, M., see Inose, H., Dec. 1962.

- Tanaka, Y., Yamashita, and Hosokawa, S., "A New Decoding Method of Delta Modulation Code, Taking the Redundancy of Codes into Consideration", Mem. Fac. Engng., Osaka City Univ. (Japan), Vol. 5, Dec. 1963, pp. 43-55.
- Tanaka, Y., Yamashita, K., and Hosokawa, S., "Instantaneous Companding for  $\Delta$ -PCM Communication", Mem. Manchester Lit. Phil. Soc., Vol. 4, 1963, pp. 71-78.
- Tanaka, Y., see Hosokawa, S., 1965.
- Tazaki, S., See Hosokawa, S., 1966.
- Thomas, M.W.S., see Steele, R., Sept. 1968.
- Tomozawa, A., and Kaneko, H., "Companded Delta Modulation for Telephone Transmission", Proc. Nat. Elect. Conf., Chicago, Vol. 22, 1966, pp. 463-468.
- Tomozawa, A., and Kaneko, H., "Modulation Characteristics of Companded Delta Modulation", J. Inst. Elect. Comm. Engrs., Japan, Vol. 49, No. 11, Nov. 1966, pp. 2186-2193.
- Tomozawa, A., and Kaneko, H., "Companded Delta Modulation for Telephone Transmission", IEEE Trans. Comm. Tech., Vol. COM-16, No. 1, Feb. 1968, pp. 149-157.
- Tripp, J.S., see Halijak, C.A., Ch. E, 1963.
- Tripp, J.S., see Halijak, C.A., 1963.
- Tsybakov, B.S., "Linear Coding of Images", Radio Engineering Electronic Phys., Vol. 7, No. 3, March 1962, pp. 351-360.
- Vagliani, F., See Bellman, A., 1967.
- VanDeWeg, N., "Quantizing Noise of a Single Integration Delta Modulation System with an N-Digit Code", Philips Research Rev., Vol. 8, Oct. 1953, pp. 367-385.
- Varsos, S.G., and Norvell, B., "Datec-Digital Adaptive Techniques for Efficient Communications", IEEE Internat. Conv. Rec., Vol. 12, Pt. 6, 1964, pp. 435-445.
- Vasseur, J.P., "Comparison of Code Modulation and Classical Modulation", Ann. Radioelect., Vol. 9, April 1954, pp. 137-149 (in French).
- Want, P.P., "Idle Channel Noise of Delta Modulation", IEEE Trans. on Comm. Tech., Vol. COM-16, No. 5, Oct. 1968, pp. 737-742.

Watanabe, K., see Inose, H., March 1966.

Wharton, W., and Davies, R.E., "Field Store Standards Conversion, Conversion Between Television Signals With Different Field Frequencies Using Ultrasonic Delay Lines", Proc. IEE (Grt. Brtn), Vol. 113, No. 6, June 1966, pp. 989-996.

Whelan, J.W., see Bisignani, W.T., 1966.

Williams, B.E., "A Delta Modulation System Using Junction Transistors", Electronic Engineering, Vol. 31, Nov. 1959, pp. 74-80.

Winkler, M.R., "High Information Delta Modulation", IEEE Int. Conv. Rec., Vol. 11, Pt. 8, 1963, pp. 260-265.

Winkler, M.R., "Pictorial Transmission with HIDM", IEEE Int. Conv. Rec., Vol. 13, Pt. 1, 1965, pp. 285-291.

Wolf, J.K., "Effects of Channel Errors on Delta Modulation", IEEE Trans. Comm. Tech., Vol. COM-14, No. 1, Feb. 1966, pp. 2-7.

Yamaguchi, K., see Ando, H., 1966.

Yamashita, K., see Tanaka, Y., 1963.

Yamashita, K., see Tanaka, Y., Dec. 1963.

Yamashita, K., see Hosokawa, S., 1965.

Yasuda, Y., see Inose, H., Nov. 1961.

Yasuda, Y., see Inose, H., Sept. 1962.

Yasuda, Y., see Inose, H., Dec. 1962.

Yasuda, Y., see Inose, H., Jan. 25, 1963.

Yasuda, Y., see Inose, H., Nov. 1963.

Zetterberg, L., "A Comparison Between Delta and a Pulse Code Modulation", Ericsson Technics, Vol. 11, No. 1, Jan. 1955, pp. 95-154.