MASSACHUSETTS INSTITUTE OF TECHNOLOGY

MEASUREMENT SYSTEMS LABORATORY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE 39, MASSACHUSETTS

NASA CR 102005

RN-57

A Force-Measurement in Orbit to Check Scalar Gravitational Theory

by

John Hovorka

August, 1969

NAS9-8328

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Measurement Systems Laboratory

ABSTRACT

A study which included a Master's thesis* in Physics at M.I.T., completed in June, 1969, concludes that the Brans-Dicke theory of gravitation cannot readily be checked by means of a gravitational clock in eccentric orbit, even around Jupiter. Pertinent summarizing parts of the thesis are given. Consideration is then given to the use of a Pendulous Integrating Gyro Accelerometer to measure the force on an orbiting mass directly, to check the same theory. This analysis is also taken from the thesis. In the analytical comment following, the fundamental difficulties of the clock experiment are compared with those of the accelerometer experiment. A valid Earth-orbital experiment with the latter appears at the present time to be a good possibility, and a procedure for following this up is recommended, including suggestions for required accelerometer development.

---

* Blitch, M.G., "The Feasibility of a Gravitational Clock to Test the General Theory of Relativity", M.S. Thesis TE-32, M.I.T., June, 1969.

TABLE OF CONTENTS

Introduction to the Thesis Exerpts

In the pages immediately following are three exerpts from the cited thesis. The first, entitled Introduction, discusses the theoretical background of an experiment to measure possible variations in the "universal gravitational constant" G, as expected in the Brans-Dicke scalar gravitational theory. The experiment would consist in measuring the period-variations of an eccentrically-orbiting gravitational clock. These variations in G would also be detectable in principle in a direct force measurement made in orbit, and this is described in the second exerpt. While this exerpt is entitled "A Gyroscopic Gravitational Clock," the title is misleading; the device is not a clock, but an accelerometer, and will be so designated in the conclusions to this report.

In the last exerpt, the conclusion to the thesis is given; it is, on balance, unfavorable to the use of the gravitational clock for this purpose. However, the acceleromater measurement remains a good possibility.

Following these exerpts, the author of the present report comments on the thesis results and recommends a course of action.

I. INTRODUCTION

1.1 The Strong Principle of Equivalence and Its Possible Violation

This thesis is the result of an investigation of a possible test of the General Theory of Relativity. More particularly, the thesis examines a possible test of the Strong Principle of Equivalence of that theory. This principle deals with the equality of inertial frames for examining the numerical content of physical laws, and the extension of the concept of inertial frames to freely falling frames in gravitational fields. Specifically, the Strong Principle of Equivalence states "that at every space-time point in an arbitrary gravitational field it is possible to choose a locally inertial coordinate system such that, within a sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation. The Principle of Equivalence says that at any point in space-time we may erect a locally inertial coordinate system in which matter satisfies the laws of special relativity."[1] Consequently, the law of gravitational attraction between two massive bodies would hold true as they fell towards a third massive body if they were observed by a "co-moving" observer located in their immediate vicinity. This thesis deals with the possibility that this principle may not be true, and that a violation of the principle might be detected by such an observer. The law which would be altered as far as this thesis is concerned

1. Definition by Professor S. Weinberg in unpublished lecture notes on General Relativity and Cosmology, fall 1968 at MIT.

would be the gravitiational law mentioned, and the experiment and the observation would take place in a reference system freely falling in a gravitational field.

The experiment treated in the following text involves a harmonic oscillator whose elastic restoring force is gravitational, making its frequency a function of the Universal Gravitational Constant. According to the equivalence principal this restoring force, and, therefore, the oscillator frequency, would not change as far as a local and co-moving observer is concerned if the oscillator were placed in a reference frame freely falling toward some massive body. Then if the oscillator were synchronized with another time-keeping system, such as an atomic clock, and the second time-keeping system were made a companion to the oscillator, the synchronism would be maintained as the two fell from deep space toward some large planet. If the synchronism were not maintained, then the result could be ascribed to a violation of the Strong Principle of Equivalence. That is to say, the result would indicate such a violation if the oscillator did not change frequency due to some other cause. The specific problem of this thesis investigation is to determine if, assuming such a loss of synchronism occurs, this loss could be justifiably attributed to a violation of the Strong Principle of Equivalence rather than extraneous factors.

To this end reference is made to the current theory of C. Brans and R. Dicke[2] which predicts that a change in gravitational coupling would develop in the case cited above.

---

2. C.Brans and R. Dicke, "Mach's Principle and a Relativisitc Theory of Gravitation", Phys. Rev.124 (3), Nov. 1961.

Calculations are made on the basis of this theory to determine how great a loss of synchronism could be expected. Then the possible uncertainties in the measurement are examined to see how they compare in magnitude with the effect to be measured. The criterion for experimental success used is that the effect to be measured should exceed the aggregate of the measurement uncertainties.

1.2 Calculation of the Expected Change in Gravitational Coupling

According to the Brans-Dicke Theory, the gravitational coupling between local masses is influenced by the rest of the mass in the universe, and the effect of the other masses in the universe in the immediate vicinity of a point mass can be expressed in the form:

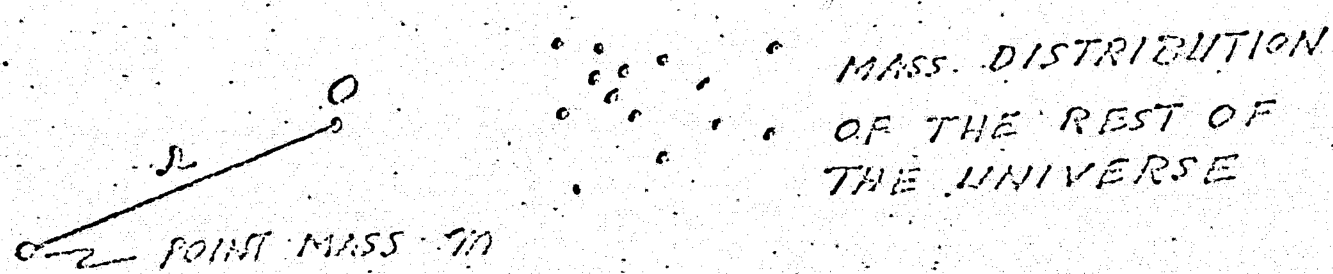
$$\Phi(r) = \Phi_0 + \frac{B'}{r} \qquad (1.2\text{-}1)$$

where $\Phi(r)$ = the scalar gravitational field at the observation point "O".

r = distance from the point mass to the observation point.

B' = a constant depending on the point mass

Φ_0 = the scalar gravitational field due to the mass distribution of the rest of the universe



These quantities are related, it is postulated, to the usual gravitational constant, G, (Newton's Universal Gravitational Constant) in the following way:

$$(1.2\text{-}2) \qquad \Phi_0 = \frac{B''}{G_0}$$

where B'' = a dimensionless constant

G_0 = the value of G that is due to the rest of the mass in the universe (besides the point mass, m)

$$(1.2\text{-}3) \qquad \Phi(r) = \frac{B''}{G(r)}$$

where $G(r)$ = the value of G that would be observed in the neighborhood of the point mass m.

Then from (1.2-1) the relation specifically involving G is:

$$(1.2\text{-}4) \qquad \frac{B''}{G(r)} = \frac{B''}{G_0} + \frac{B'}{r}$$

Now as an observation procedure, the change in G between two separate locations is what would be measured. Consider, then, the change, ΔG, in G as the radial distance is decreased from r_1 to r_2 in the sketch of (1.2-1). The gravitational coupling at the two different locations is:

$$(1.2\text{-}5) \qquad \frac{B''}{G(r_1)} = \frac{B''}{G_0} + \frac{B'}{r_1}$$

$$(1.2\text{-}6) \qquad \frac{B''}{G(r_2)} = \frac{B''}{G_0} + \frac{B'}{r_2}$$

Combining (1.2-5) and 1.2-6) gives the change in G:

$$(1.2\text{-}7) \qquad \frac{\Delta G}{G(r_1)} = - G(r_2)\,\frac{B'}{B''}\,\frac{\Delta r}{r_1\, r_2}$$

where $\Delta G = G(r_2) - G(r_1)$

$\Delta r = r_1 - r_2$

The form of (1.2-7) shows that the change, ΔG, would be a decrease in G as the observer moved from a larger to a smaller distance from the point mass m . This decrease in gravitational coupling would be manifested by a decrease in gravitational force between any two massive bodies.

From (1.2-7) the maximum change to be observed when approaching a point mass from an initially remote locale is:

$$(1.2\text{-}8) \qquad \left.\frac{\Delta G}{G(r_1)}\right|_{MAX} = \lim_{r_1 \to \infty} \left[-\frac{B'}{B''}\, G(r_2)\, \frac{\Delta r}{r_1 r_2} \right] = -\frac{B'}{B''}\,\frac{G(r_2)}{r_2}$$

For an observer approaching Earth, for example, the expected shift would be calculated using the value of $G(r_2)$ observed on Earth, which is 6.7×10^{-8} dyne cm^2/gm^2 at a distance from the center of mass of $r_2 = 6.4 \times 10^8$ cm (i.e., on the surface of the Earth). The constants B' and B'' in (1.2-.) and (1.2-2) are, according to the theory being considered:

$$(1.2\text{-}9) \qquad B' = \frac{m}{c^2} \, \frac{2}{3+2\omega'}$$

$$(1.2\text{-}10) \qquad B'' = \frac{4+2\omega'}{3+2\omega'}$$

so that in (1.2-8) their ratio is:

$$(1.2\text{-}11) \qquad \frac{B'}{B''} = \frac{m}{c^2} \, \frac{1}{2+\omega'}$$

The constant, ω', that has been introduced is a number whose value is not definitely established at present. A lower limit of "6" is set in the Brans-Dicke Theory by the observed precession of the perihelion of Mercury. The upper bound is not so definitely limited. A value of approximately "9" is favored by the authors of the theory. In this thesis a value for ω' of "8" will be used. Then, with the help of (1.2-11) and this value of ω', (1.2-8) becomes:

$$(1.2\text{-}12) \qquad \left.\frac{\Delta G}{G(R_1)}\right|_{MAX} = -\frac{1}{10} \, \frac{m}{c^2} \, \frac{G(R_2)}{R_2}$$

for an initially remote observer approaching Earth, the maximum shift in G is:

$$(1.2\text{-}13) \qquad \left.\frac{\Delta G}{G(R_1)}\right|_{MAX,\ EARTH} = -7 \times 10^{-11}$$

The tiny magnitude of the shift is not likely to be discerned in the ordinary sort of laboratory. However, the size of the shift depends upon the magnitude of the mass, m, and going to the vicinity of a more massive planet can produce a greater effect. For instance, Jupiter has over 300 times the mass of the earth. It is not as dense as the earth (about 1 gm/cm^3 as compared to 6 gm/cm^3) and the increase in mass is partly offset by an increase in minimum distance, r_2. However, the gravitational shift would still be appreciably larger:

$$\left.\frac{\Delta G}{G(r_1)}\right|_{MAX,\ JUPITER} = -2 \times 10^{-9} \qquad (1.2\text{-}14)$$

The sun gives a still greater effect. For an observer moving from deep space to 10 Solar radii the shift in G would be:

$$\left.\frac{\Delta G}{G(r_1)}\right|_{MAX,\ SUN} = -2 \times 10^{-8} \qquad (1.2\text{-}15)$$

The numerical values given above would be different, of course, for different values of ω', such as might be found in other gravitational theories where a scalar field varies as $\frac{m}{r c^2}$ (as in the Brans-Dicke Theory). The numerical value of the change in coupling may therefore be larger or smaller than the results obtained above. In order to have a numerical criterion for judging the experiment, considering this possibility (and in anticipation that the coupling shift will be small if it exists at all), it is assumed in this thesis that the coupling shift observed will be of the order of parts in 10^9. It is shown in the next chapter that the clock frequency would change by the same order of magnitude as the gravitational coupling change. That is to say, the size of the effect to be measured is of the order of parts in 10^9. The numerical criterion to judge the experiment is, then, that the uncertainties in clock frequency due to all causes should not exceed parts in 10^{10}.

It is to be noted that accepting uncertainties of that order rules out Earth as a suitable point mass for the experiment, according to the Brans-Dicke Theory. To give the numerical values discussed some perspective, it may be observed that there are about 3×10^7 seconds in a year, so that an ordinary clock keeping time to 3 seconds over a year would be good to about a part in 10^7. Such a clock would have to keep time to within 3 seconds over 100 years for an accuracy comparable to the gravitational clock contemplated. Thus it is apparent that a high degree of frequency stability would be required of the gravitational clock.

IV. A GYROSCOPIC GRAVITATIONAL CLOCK

4.1 Description of the Clock

A scheme for building a gravitational clock which employs a single-degree-of-freedom pendulous integrating gyroscope affixed to a massive sphere is shown in Figure 19. The gravitational attraction of the sphere on the pendulous mass of the gyro produces a torque on the gyro of the magnitude:

$$(4.1\text{-}1) \qquad M_{GRAV} \cong \frac{G m m'}{(r_b + y)^2} l$$

This torque is balanced by a gyroscopic torque developed by precession of the gyro. The gyroscopic torque has the magnitude:

$$(4.1\text{-}2) \qquad M_{GYRO} = \omega_p H$$

where: ω_p = precession speed

H = gyro spin angular momentum

The relation between the precession speed and the gravitational constant when the torques are balanced is:

$$(4.1\text{-}3) \qquad \omega_p = G \frac{m'}{(r_b + y)^2} \frac{m l}{H}$$

where $\left(\frac{m l}{H}\right) = S$ = sensitivity of the gyro in rad/sec per cm/sec^2

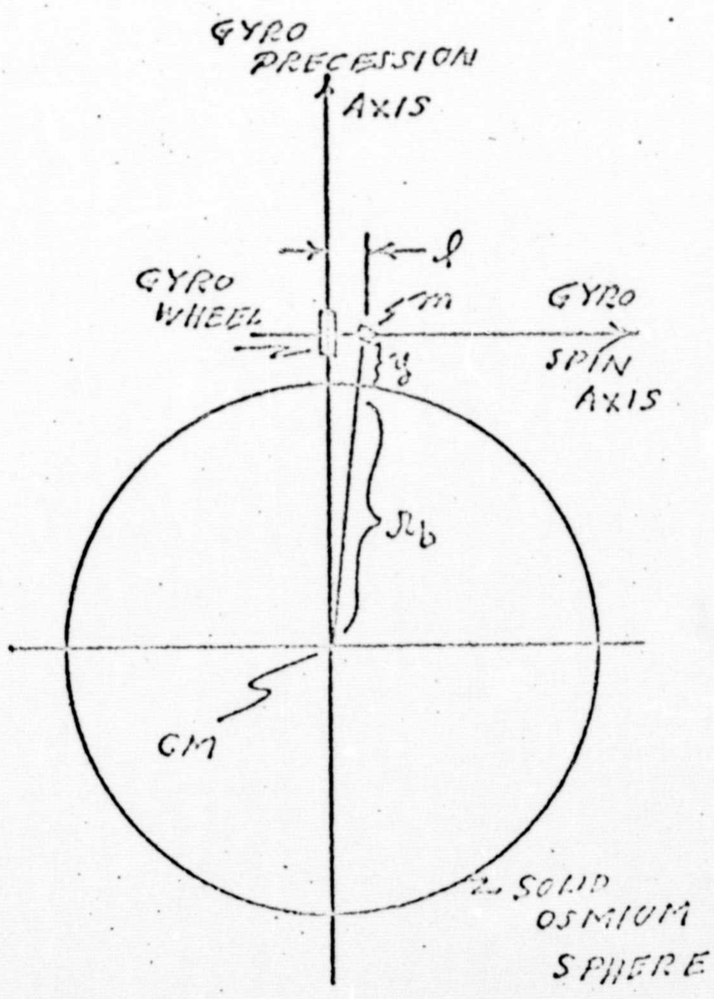


FIGURE 19.

SCHEME FOR GYROSCOPIC MEASUREMENT

OF GRAVITATIONAL COUPLING

As a numerical example, consider a 20 cm radius for an Osmium sphere (mass = 750 kg) and a gyro mounting height of 1 cm. Then (4.1-3) becomes:

(4.1-4) $$\omega_p = (1.1 \times 10^{-4} \text{ cm/sec}^2)\left(\frac{ml}{H}\right)$$

The sensitivity of pendulous gyros used for inertial guidance is usually about 1 rad/sec for a "1 gravity" acceleration or about 1 rad/sec for 10^3 cm/sec^2. For this sensitivity, the precession rate in (4.1-4) becomes:

(4.1-5) $$\omega_p = 1.2 \times 10^{-7} \text{ rad/sec}$$

This is an impractically slow rate to measure small (parts in 10^9) changes in G as it would take 2 years for the gyro to make one revolution. But the fact that the gyro is usually designed for the sensitivity above does not mean that other sensitivities are not possible. ... If

(4.1-9) $$H = 10^{-4} \text{ g cm}^2\text{/sec}$$

This angular momentum together with a pendulosity of 10 gm-cm for the gyro produces a sensitivity:

$$S = \left(\frac{ml}{H}\right) = 10^5 \frac{rad/sec}{cm^2/sec} \qquad (4.1\text{-}10)$$

For this sensitivity the precession rate in (4.1-4) becomes 10 rad/sec. The period for one complete revolution is thus 0.6 sec, or about 10^4 less than for one complete cycle of the gravitational oscillator.

The advantage of the shorter period is that more measurements (i.e., more complete cycles) of the gravitational constant are made in a given amount of time so that a more effective averaging of the results can be obtained. The ability to average over many measurements means that random errors may be "averaged out" or, at least, made less significant. The precession rate arrived at in (4.1-10) is not necessarily a maximum value. It is a value which may be attainable with instrumentation currently available or being constructed, such as the NASA SFIR instrument. The stability of the sensitivity in (4.1-3) of that instrument is above a part in 10^7. The significance of this stability for the measurement of changes in the gravitational constant is as follows.

4.2 Consideration of Uncertainties and Comparative Advantages of the Gyroscopic Gravitational Clock

If the gravitational constant changes by a small amount, ΔG, then the precession rate in (4.1-3) changes by a small amount, $\Delta\omega$, according to:

$$(4.2\text{-}1) \qquad \frac{(\Delta\omega)_{GRAV}}{\omega} = \frac{\Delta G}{G}$$

But if the instrument sensitivity S in (4.1-3) changes by a small amount, ΔS, the precession rate changes according to:

$$(4.2\text{-}2) \qquad \frac{(\Delta\omega)_{\Delta S}}{\omega} = \frac{\Delta S}{S}$$

Now the change in gravitational constant to be expected is of the order of parts in 10^9 as given in Chapter I. Therefore, the resultant change in gyro precession rate given in (4.2-1) would be of that order. But the change in the gyro precession rate because of a change in sensitivity would be of the order of a part in 10^7 for instruments like the one mentioned above. The uncertain changes in the gyro rate would thus mask the rate change to be measured by two orders of magnitude. However, the incertainty in 10^6 measurements would be only 10^{-3} of the uncertainty in one measurement (assuming random errors). Therefore, in 10^6 cycles of the instrument the overall measurement uncertainty would be a part in 10^{10}. The measurement time required for 10^6 cycles would be 174 hours or about a week. Therefore, the pendulous gyro having a sensitivity stability of a part in 10^7 designed for the sensitivity in (4.1-10) of 10^5 rad/sec per cm/sec^2

would allow measurements of changes in gravitational constant to parts in 10^{10} without an inordinate amount of time-on-station for the space probe. That is to say, the space probe transporting the system in Figure 19 would be stationed in orbit about Jupiter (or the sun) for about a week so as to obtain a good average of the gyro readings.

It is possible that the stability of the gyro would be better than a part in 10^7 for this application. This application is almost a free-fall situation whereas the gyro stability is usually tested under "1 gravity" conditions. But it is known that some of the uncertainties in gyros diminish with decreasing acceleration. This has been shown for similar instrumentation in the recent Apollo missions. Therefore, in an environment of very low acceleration such as aboard a space probe the gyro stability could improve. The inertial guidance applications for which most gyros are used require stable gyro performance at high (over 1 gravity) accelerations, with not much design effort specifically directed at measurements of very low (10^{-7}) accelerations. But experience indicates that the stability could be improved if desired and quite possibly made adequate for this application (The gyro stability may already be adequate, but it has not been tested for this particular application).

In addition to the advantage of quicker and more numerous measurements, certain other benefits would accrue from the employment of such a gyro. One benefit would be the alleviation of the attitude stability requirement necessary for the oscillator. The necessity for attitude stabilization for the sphere-gyro system shown in Figure 19 arises from the centrifugal force on the gyro pendulous mass if the sphere rotates about its CM. The centrifugal force from the rotation would be opposite in direction to the gravitational force and would produce a torque on the gyro counter to the torque in (4.1-1). The ratio of the centrifugal force to the gravitational force is:

$$(4.2\text{-}3) \qquad \frac{F_c}{F_g} = \frac{(r_b + y)^3}{GM'} \omega_c^2$$

where

F_c = centrifugal force on gyro

ω_c = rotation speed of sphere

F_g = gravitational force on gyro pendulous mass

As a numerical example, consider a rotation rate of 10^{-2} deg/hr, and the parameters used for (4.1-4). The ratio in (4.2-3) becomes:

$$(4.2\text{-}4) \qquad \frac{F_c}{F_g} = 4 \times 10^{-10}$$

Now a rotation rate of 10^{-2} deg/hr is about 10 times the drift rate of a good inertial gyro. Thus the rotation rate of the sphere in Figure 19 could be controlled well enough by a stabilized platform to keep uncertainties due to centrifugal

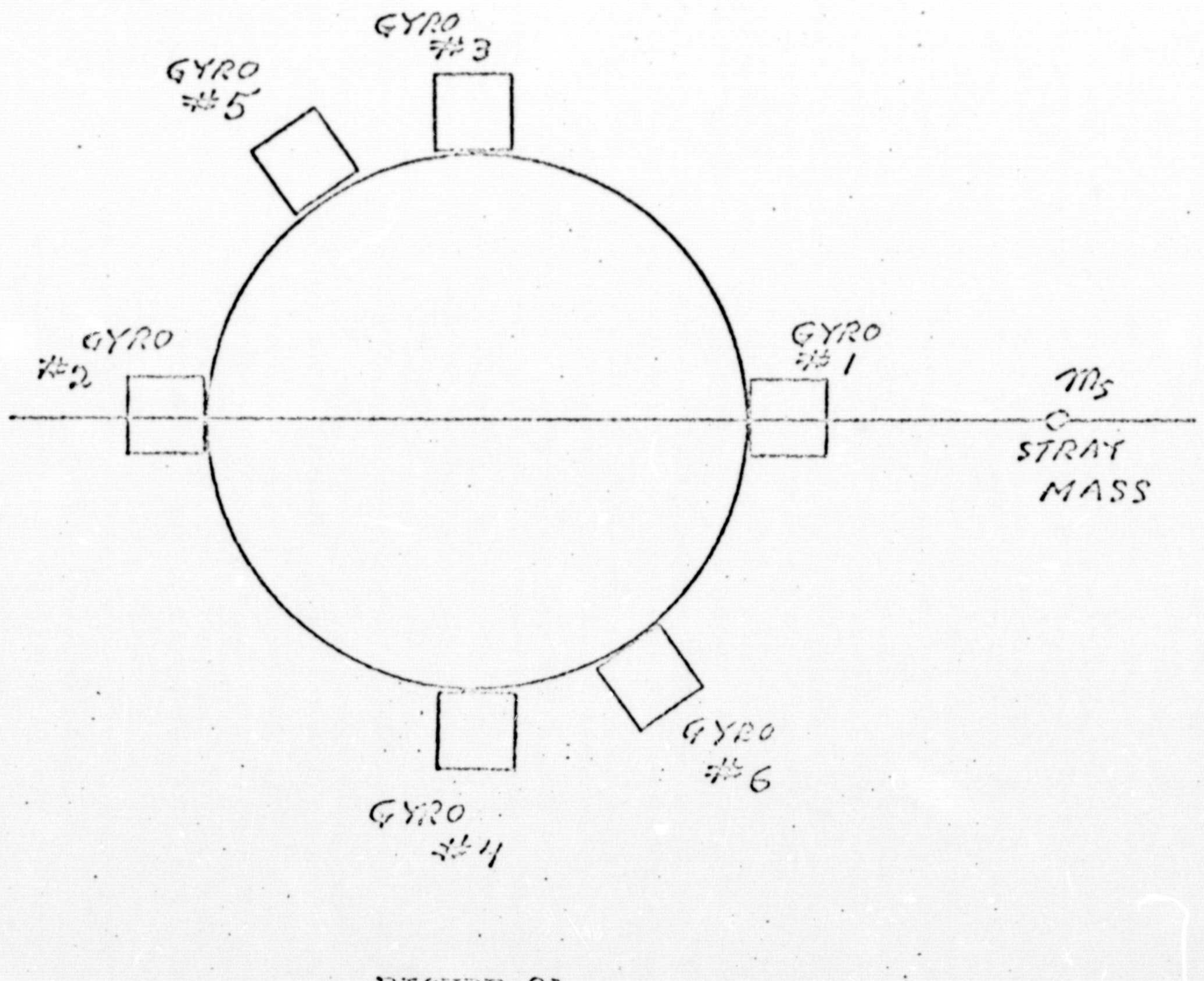


FIGURE 21.

PLAN FOR NULLIFYING EFFECT OF STRAY MASSES force at or below the level of (4.2-4). The level given in (4.2-4) is pessimistic in any event because the rotation rate of a 750 kg ball would not be that uncertain over the period of one measurement (46 seconds). No other attitude stabilization would be required for this system. It could be allowed to tumble slowly in space if that were a convient procedure.

Another benefit from the use of the gyro is that the problem of stray masses from dirt in the interplanetary region might be alleviated if not eliminated. Figure 21 illustrates how this might be done. The gravitational force on the pendulous mass of gyro #1 due to the stray mass shown is opposite to the attraction of the sphere. But the force on the mass of gyro #2 is in the same direction as the attraction of the sphere. Therefore if the two precession rates of these gyros are added together and averaged, the combined effect of the stray mass will be less than on either gyro separately.

Now if another pair of gyros, #3 and #4, is added as shown, the gravitational force from the stray mass will not produce a torque on those gyros. Consequently, if the precession rates of gyros #1 through #4 are added and their average taken, the effect of the stray mass will be less than with an average based on just gyros #1 and #2. In general, then, any pair of gyros added to the system and included in the above averaging will reduce the effect of a stray mass (for example, gyros #5 and #6 in Figure 21).

Placing a large number of gyros symmetrically on the surface of the sphere and averaging over their summed precession rates will thus greatly reduce the effect of any stray mass. For a sphere of 20 cm in radius (5×10^3 cm^2 in surface area) and a mounting surface area of 50 cm^2 for each gyro, 10^2 gyros could be mounted for the averaging process.

It is to be noted that such a symmetrical arrangement of gyros over the spherical surface would not reduce the gravitational force on the pendulous mass of any gyro, but instead, would actually increase it. Therefore, the precession rate would not be decreased by making the system less sensitive to gravitational gradients from stray masses. This option is not available to the gravitational oscillator since adding additional masses to it so as to reduce gradient sensitivity reduces the oscillator amplitude (see Chapter II). Furthermore, if this arrangement was employed, the averaging of measurements over 100 instruments would be of material benefit for the case of any uncertainty such as has been mentioned. The measurement time given in connection with (4.2-2) would be less than 2 hrs if 100 instruments were used instead of just one.

A third benefit stemming from the use of the gyro is that extremely low temperature operation (e.g., superconducting temperatures) is not required. In this regard, the change in precession rate per degree centigrade is 6 parts in 10^6 for the 20 cm Osmium sphere postulated for (4.1-4). Therefore, temperature control to 10^{-4} °C would be required to hold the precession rate constant to parts in 10^{10}. However, the heat capacity of the sphere would be 10^5 joule/$^{\circ}C$ so that about 10^{1} joules would be needed to change the temperature of the sphere by 10^{-4} $^{\circ}C$. Considering the 0.6 second rotation period for the gyros, the sphere would have to absorb or lose about 20 watts in order to change the precession rate by parts in 10^{10} on one rotation period. Such a power flow would almost certainly be prevented, so that temperature excursions would be appreciably less than 10^{-4} $^{\circ}C$ for one measurement period. The temperature sensitivity given above was calculated considering a steel casing for the gyro (The casing was considered to be screwed into the Osmium sphere and mechanically pinned there). It therefore appears that ordinary temperatures used, as are commonly specified for such gyros (i.e., in the 20° to 40° C range), would be permissible for the gravitational constant measurements and that the need for liquid Helium and associated apparatus is avoided. Also avoided is the uncertainty reported in Chapter III concerning magnetization of superconductors. Superconductors would be unnecessary for either suspension or magnetic shielding for the gyros.

The other problems discussed for the gravitational oscillator such as anharmonicity and the necessity for corrective measurements, and impulses for energy replenishment of the oscillator, are not present in the case of the gyro-sphere arrangement. The problem of electrical contamination could be solved for the gyro in a way similar to that used for the oscillator. Thermal agitation should not be a limiting factor for the gyroscopic measurements because of the stiffness of the restraint on the motion of the pendulous mass.

V. CONCLUSIONS

In Chapter I the expected shift in the gravitational oscillator frequency was calculated to be of the order of parts in 10^9. The criterion established to judge the accuracy of the oscillator was that the aggregate of all uncertainties in the oscillator frequency should not exceed the order of parts in 10^{10}. The research for this thesis indicates that those uncertainties in the measurement of the gravitational oscillator frequency which arise from dimentional changes due to temperature variations, cosmic rays incident on the oscillator rotor, measurement of the angular position of the rotor at its zero crossing, magnetic and electrical contamination of the rotor, and oscillator anharmonic effects should be of the order of parts in 10^{10} or less. The investigations also indicate that the uncertainties due to thermal agitation of the rotor, null uncertainty due to the practical limitations in current attitude control systems for space craft, and stray masses (dirt) near the massive bodies of the solar system could be expected to exceed the order of parts in 10^{10}.

It is possible, of course, that convincing evidence may be found that thermal agitation does not limit the zero-crossing measurements of high Q (10^6 or higher). It is also possible that the capability for attitude control to 10^{-10} radians may be demonstrated. It is further possible that space exploration in the vicinity of the most massive bodies of the solar system (Jupiter and the sun) will show that the oscillator disturbances from dirt particles accumulated there would be sufficiently rare events that they would not constitute an uncertainty in the oscillator frequency measurement. However, on the basis on information available at present it is the conclusion of the research that the results from frequency measurements by the gravitational oscillator could not be justifiably used to prove or disprove that the Strong Principle of Equivalence had been violated.

Furthermore, it is recommended that alternative designs which may sucessfully circumvent the limitations of the oscillator be considered. One such alternative, the pendulous gyroscopic clock mentioned in this thesis, merits further investigation.

Analytical Comment on the Gravitatonal Clock's Drawbacks

In the thesis conclusions immediately preceding, the gravitational clock is considered infeasible, on balance; in contrast, the Pendulous Integrating gyro accelerometer is considered potentially feasible and the basis for a possible alternative experiment. The measured variation in G was expected to be of the order of one part in 10^9, so that gravitational clock was considered effectively precise enough if it had errors within one part in 10^{10}. The figures apply to an eccentric orbit around Jupiter; around the Earth, a precision ten times better is required. After examining what appears to be an exhaustive list of pertinent error sources, the author of the thesis concludes that errors larger than one part in 10^{10} may be expected from the following sources:

1. Thermal agitation of the rotor. This point would require further investigation if the gravitational clock experiment were pursued.
2. Attitude control (to 10^{-10} radian).
3. Stray masses (dirt) in and around the gravitational oscillator.

Note that the thermal-agitation conclusion is based on the measurement of time intervals between zero-crossings of the pendulum. Amplitude information, except for the indicated zero amplitude at crossover is not taken from the pendulum during its swing. The point was to reduce the measurement to a time-measurement only, on the general principle that time-interval measurement is the most accurate known form. Here the principle was applied by measuring a force (and thus G) by measuring the period of a pendulum.

While it would seem that a great deal of information would be thrown away in not measuring the amplitude during the entire cycle, there would be great difficulty in doing this to one part in 10^{10}. However, the thesis implicitly answers this, as will be shown below.

<u>The Accelrometer Force-Measurement Viewed as a Zero-Frequency Pendulum</u>

The difficulty with the amplitude measurement can be reduced, probably to within acceptable limits, by reducing the period of te pendulum to zero. At once the time between zero-crossings is known (it is infinite) and the "amplitude" measurement--the force between two masses--measures (along with some of the disturbances mentioned in the thesis in connection with the gravitational clock) the variations in G. But this is just what the author of the thesis is talking about in suggesting that a Pendulous Integrating Gyro Accelerometer be mounted on a test mass and orbited. The experiment is then changed from a <u>dynamic</u> one, measuring a time interval between the zero-crossings of the pendulum, into a <u>static</u> one, measuring the slowly varying force between a seismic mass in an accelerometer and a large test mass. The remaining questions are, can an accelerometer be made precisely enough to do the job? And also, as a practical question, can the experiment be performed in Earth orbit?

<u>State of the Accelerometer Art, and Recommendations</u>

The most promising accelerometer for the present purpose appears to be the Pendulous Integrating Gyro Accelerometer. The chances of making it sufficiently precise are good, chiefly because it is the only kind of force-measurer which does not depend on the elastic property of a material, i.e., it does not render force in terms of distance. The principle of the accelerometer has been in the literature for many years*. It has been and continues to be an object of research at the M.I.T. Instrumentation Laboratory, most recently in connection with the NASA SFIR-research program. Briefly, the accuracy requirement is this. If such an accelerometer were Earth-orbited, we would need absolute accuracies of the order of one part in 10^{11}. With 10^6 measurements averaged, however, we would only need about one part in 10^8. This is the goal for the new generation of these devices being built for NASA at M.I.T. To get more accuracy from present instruments, (which are good to about one part in 10^7) we need to increase the pendulosity (and thus the sensitivity). We could keep the angular momentum at its present level, thus retaining present torque-uncertainty levels, which are satisfactory.

Given the gyro wheel angular momentum for minimum torque-uncertainty effect the major trade-off appears at present to be between sensitivity (and thus pendulosity), and the measurement of fractions of a revolution of the accelerometer float to high enough accuracy. Both of these aspects have already been studied at M.I.T. in other connections; their application to the present case is being made now.

We have discussed this with members of the M.I.T. Instrumentation Laboratory; and projected accelerometer development appears to be eminently feasible, and would take, it is estimated, three years.

---

* Draper, C.S., W. Wrigley and J. Hovorka, "Inertial Guidance" (Pergamon, N.Y., 1960) p. 94 ff.