MASS LOSS BY HOT STARS

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ABSTRACT

A mechanism is proposed to explain the mass-loss observed for luminous, hot stars. We show that the ultraviolet resonance lines of ions such as SiIV, CIV, NV, and SVI can give strongly negative effective gravities in the outer parts of the reversing layers of hot stars. We argue that a static reversing-layer is then no longer possible and that a continuous outflow of mass occurs. To calculate mass-loss rates resulting from this mechanism, we formulate the problem of steady-state, moving reversing-layers. From numerical solutions of the equations of this problem, the mass-loss rate is found to be $\lesssim L/c^2$; the evolution of the star is therefore not disturbed significantly. The domain in the ($\log T_{eff}$, $\log g$)-plane where the greatest mass-loss rates are expected is also approximately determined and found to be roughly consistent with observation. Finally, the problem of the supersonic flow to large distances from the star is briefly discussed.

I. INTRODUCTION

Recently Morton (1967a,b) has obtained spectra in the far ultraviolet for three early-type supergiants showing that the resonance lines of such ions as CIV, NV, and SiIV have P Cygni profiles (i.e., an undisplaced emission line with a blue-shifted absorption component). Carruthers (1968) has since reported similar observations for several stars including the O5f star ζ Puppis. Interpreted clasically, the P Cygni line profiles imply that these stars are losing mass and that the terminal velocity of the flow is $\sim$ 2500 km/sec. Morton's discovery therefore confirms other evidence, which he has reviewed (Morton 1967a), that luminous, hot stars are losing mass.

To explain this mass loss, one might first consider Parker's (1958) solar wind mechanism; that is, to suppose the outflow results from a hot stellar corona that cannot be contained by the star's gravitational field. This would require, however, that the enthalpy ($=5kT/m_H$ for ionized hydrogen) of the gas in the accelerating flow be sufficient to provide the kinetic energy per gm. of the gas moving with the terminal velocity. From this condition and the observed terminal velocity, it follows that the temperature in the accelerating flow must exceed 10^{7}°K, and we note that the ions CIV, NV, and SiIV would then be destroyed by collisional ionization. The considerable widths of the absorption components of the P Cygni line profiles prove, however, that these ions do indeed exist in the accelerating flow. Accordingly, we conclude that mass loss by hot stars is not a consequence of a hot corona.

In this paper, therefore, we develop the proposal (Lucy and Solomon 1967; "Paper I") that the mass loss results from the pressure exerted on the gas by absorption of radiation in resonance lines, especially those of abundant elements with wavelengths in the region where a hot star's continuum flux is greatest. We shall show that near a star's surface the force per gm. due to radiation pressure can exceed the star's surface gravity, thus making a static atmosphere unlikely. We are then led to formulate the problem of steady-state moving reversing-layers, the solution of which allows mass-loss rates to be predicted.

The suggestion that radiation pressure can result in mass loss was discussed over forty years ago by Johnson (1925) and Milne (1926, 1927). More closely related to the present investigation, however, is Pikelner's (1947) suggestion that atoms such as C, N, O, and S may be ejected from hot stars by the action of selective radiation pressure. The main difference between our work and these early investigations is our emphasis on the hydrodynamics of the problem as distinct from the expulsion of individual atoms.

II. THE MECHANISM

In stellar atmosphere calculations for hot stars the support provided by radiation pressure is commonly taken into account by writing the equation of hydrostatic equilibrium as

$$\frac{dP}{dx} = -g_{eff}\, \rho \equiv -(g-g_R)\, \rho \quad , \tag{1}$$

where P is the gas pressure, ρ is the density, $g = G\mathfrak{M}/R^2$ is the star's surface gravity, and g_{eff} is the effective surface gravity. If πF_ν is the radiative flux at frequency ν and β_ν is the total extinction coefficient per gm. (absorption plus scattering), then g_R, the amount by which the gradient of radiation pressure reduces g_{eff} below g, is given by

$$g_R = \frac{\pi}{c} \int_0^\infty \beta_\nu F_\nu \, d\nu \quad . \tag{2}$$

In order to drive material from the star by having $g_{eff} < 0$ near the surface, equation (2) suggests that frequencies where β_ν is large (e.g., within lines) should be considered. Normally, of course, F_ν is small when β_ν is large; in particular, $F_\nu \propto \beta_\nu^{-1}$ at large optical depths, so that the contribution of a given frequency interval to g_R does not then depend on β_ν. This behaviour no longer holds in the reversing layer, however. We shall show that absorption in resonance lines can indeed give $g_{eff} < 0$ close to the star's surface.

a) An Upper Limit to the Contribution of One Line

We may get an upper limit, $(g_R)^o_i$, to the contribution of one line to g_R by supposing that the continuum flux F_{ν_o} at the line frequency ν_o is not reduced by line absorption. This gives

$$(g_R)^o_i = \frac{\pi F_{\nu_o}}{c} \cdot \frac{n_i}{\rho} \int_0^\infty s_\nu \, d\nu \tag{3}$$

where n_i is the number density of absorbing ions and s_ν is the line absorption coefficient per ion. If n_a is the number density of the atom in question, including all stages of ionization, and n_H is the number density of hydrogen atoms, then equation (3) may be rewritten as

$$(g_R)^o_i = \frac{\pi F_{\nu_o}}{c} \cdot \frac{n_a}{n_H} \cdot \frac{n_i}{n_a} \cdot \frac{X}{m_H} \cdot \frac{\pi e^2}{m_e c} \cdot f \quad (4)$$

where X is the abundance of hydrogen by weight and f is the oscillator strength of the transition.

To have any possibility that $g_R > g$ as a result of line absorption, we must obviously have $\sum_i (g_R)^o_i >> g$. Let us therefore consider, as an example to be used throughout this section, $(g_R)^o_i$ for the stronger component of the CIV doublet at 1548 Å in an atmosphere with effective temperature T_{eff} = 25,120°K ($\log T_{eff} = 4.4$). At this effective temperature the line is in the wavelength region where the star's continuum flux is greatest; therefore, to a good approximation, we may set $F_{\nu_o} = B_{\nu_o}(T_{eff})$, where B_ν is the Planck function. If we also take $n_C/n_H = 3.3 \times 10^{-4}$ (Allen 1963), f = 0.2 (Varsavsky 1961), and X = 1, then

$$\log (g_R)^o_{1548} = 5.47 + \log \frac{n_i}{n_C} \quad (5)$$

Comparing this with the typical value log g = 3 for an early-type supergiant, we see that the upper limit for this one line exceeds g by a factor of 300 when all carbon is CIV ($n_i = n_C$).

b) Correction for Line Formation

The actual contribution of a line to g_R may be written as

$$(g_R)_i = \Psi \cdot (g_R)^o_i \quad , \qquad (6)$$

where Ψ is a correction factor allowing for the formation of the line. If $r_\nu = F_\nu / F_{\nu_o}$ is the residual intensity at frequency ν within the line, then Ψ is obviously given by

$$\Psi = \frac{\int_o^\infty r_\nu s_\nu \, d\nu}{\int_o^\infty s_\nu \, d\nu} \qquad (7)$$

To calculate Ψ , we suppose that the reversing layer is a region where only line absorption occurs and, as we are discussing resonance lines, we adopt coherent scattering as the mechanism of line formation. If we assume also that the scattering is isotropic and use Eddington's approximations, then the residual intensity throughout the reversing layer is (see, e.g., Ambartsumyan 1953, p.113)

$$r_\nu = \frac{1}{1 + \frac{3}{4}\tau_\nu} \quad , \tag{8}$$

where τ_ν is the optical thickness of the reversing layer at frequency ν within the line. (The residual intensity is independent of depth because for pure scattering problems flux is conserved at each frequency.) Substituting this expression for r_ν into equation (7) and considering only the Doppler core of the line profile, we obtain

$$\Psi = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dz}{\frac{3}{4}\tau_o + e^{z^2}} \tag{9}$$

where τ_o is the optical thickness of the reversing layer in the line center. This integral occurs in the theory of the curve of growth and may be expanded in an asymptotic series valid for $\ln\tau_o >> 1$ (Menzel 1936). Keeping only the leading term, we have

$$\Psi \sim \frac{8}{3\sqrt{\pi}} \cdot \frac{\sqrt{\ln(\frac{3}{4}\tau_o)}}{\tau_o} \quad , \tag{10}$$

which may be approximated by $3.5/\tau_o$. Combining this approximation with equations (5) and (6), we find that the CIV line alone gives $g_{eff} < 0$ for the model under consideration ($\log T_{eff} = 4.4$, $\log g = 3$) if

$$\frac{n_i}{n_c} > 10^{-3}\,\tau_o \quad . \tag{11}$$

Thus, provided $\tau_o < 10^3$, the effective gravity will be negative in those parts of the reversing layer where carbon is predominantly CIV. To see if this condition can be met in an atmosphere with these parameters, we must examine the ionization equilibrium in the reversing layer.

c) Ionization Equilibrium

The character of the ionization equilibrium is determined by the relative importance of photoionization and collisional ionization. For a given ionization potential, $\chi_r = h\nu_r$, the photoionization rate, Γ_R, is a function only of the radiation temperature, T_R; the collisional rate, Γ_C, on the other hand, is determined by the electron density, n_e, and the electron temperature, T_e. The ratio of the two rates is

$$\frac{\Gamma_R}{\Gamma_C} = \frac{\int_{\nu_r}^{\infty} \frac{B_\nu(T_R)}{h\nu} a_\nu \, d\nu}{n_e < \sigma_r v_e >} , \tag{12}$$

where σ_r is the cross-section for collisional ionization, v_e is the electron velocity, and a_ν is the photoionization cross-section. Böhm (1960, p. 101) has given approximations for these quantities from which we derive

$$\frac{\Gamma_R}{\Gamma_C} \simeq 6 \times 10^{10} \frac{T_e^{1/2} \chi_r^3}{n_e} , \tag{13}$$

where $T_R = T_e$, $h\nu_r / kT_R \gtrsim 1$, and the units of χ_r are electron volts.

Let us now apply this result to the ionization of CIII ions (χ_r = 47.9 eV) in the atmosphere with log T_{eff} = 4.4 and log g = 3. Taking T_e = 0.7. T_{eff} = 17,600° K and $n_e = 10^{14} cm^{-3}$, we find $\Gamma_R / \Gamma_C = 9 \times 10^3$, so that collisional ionization may be completely neglected. This conclusion applies for all the calculations reported in this paper.

The appropriate ionization equilibrium is therefore a balance between photoionization and radiative recombination in a manner familiar from studies of planetary nebulae, except that W, the geometrical dilution factor, is now not small. If we neglect photoionizations from excited states, the relative abundance of the r and r+1 stages of ionization is given by

$$\frac{n_{r+1}\, n_e}{n_r} = W.D. \frac{2\omega_{r+1}}{\omega_r} \cdot \frac{(2\pi m_e kT_R)^{3/2}}{h^3} \cdot \left(\frac{T_e}{T_R}\right)^{1/2} e^{-\frac{\chi_r}{kT_R}} , \tag{14}$$

where ω_r and ω_{r+1} are the statistical weights of the ground states, and D is the fraction of recombinations going to the ground state. If α_n is the recombination coefficient to state n and n^1 is the principal quantum number of the ground state, then

$$D = \frac{\alpha_{n^1}}{\sum_{n^1}^{\infty} \alpha_n} , \tag{15}$$

which we calculate assuming hydrogenic recombination rates (Bates and Dalgano 1962, Table I).

The justification for neglecting photoionizations from excited states and assuming hydrogenic recombination is the extreme sensitivity of n_{r+1}/ n_r to T_R - changing T_R by only 1000°K gives a factor ~ 4 in the ionization equilibrium. Therefore, since the brightness temperatures of hot stars in the extreme ultraviolet are not known accurately, our treatment of the ionization equilibrium is entirely adequate.

d) A Model for the Reversing Layer

Because the Doppler width, $\Delta\nu_D$, is large compared with the damping constant, the absorption coefficient at the line center is determined only by the Doppler effect. The optical thickness of the reversing layer at the line center is then given by

$$\tau_o = \frac{\sqrt{\pi}\, e^2}{m_e c} \cdot \frac{f}{\Delta\nu_D} \cdot N_{abs} \quad , \tag{16}$$

where N_{abs} is the column density of absorbing ions. For resonance lines, we may take the number density of absorbers n_i equal to n_{r+1}, the number density of atoms in the appropriate stage of ionization. This gives

$$N_{abs} = \int_0^\infty n_{r+1}\, dx \quad . \tag{17}$$

To evaluate this integral, we suppose that

$$n_e = (n_e)_p\, e^{-\frac{x}{H}} \text{ with } H = \frac{2kT_e}{m_H g} \tag{18}$$

where $(n_e)_p$ is the photospheric electron density and H is the scale-height of the reversing layer. This formula holds for a plane-parallel, isothermal, hydrogen atmosphere that is fully ionized and in which radiation pressure contributes negligibly to the support. If we further take T_e, T_R and W to be constant in the reversing layer, then equation (14) may be rewritten as

$$\frac{n_{r+1}\, n_e}{n_r} = \left(\frac{n_{r+1}\, n_e}{n_r}\right)_p \quad , \tag{19}$$

which shows that the degree of ionization increases with height in the reversing layer because of the decreasing electron density. Formulae (18) and (19) allow the integral (17) to be solved analytically; the result is

$$N_{abs} = \frac{n_a}{n_H}\, H \cdot \left(\frac{n_{r+1} n_e}{n_r}\right)_p \ln\left[\frac{n_r}{n_{r+1}} \cdot \frac{n_r}{n_r + n_{r-1}}\right]_p \quad , \tag{20}$$

assuming that $(n_{r+1}/n_r)_p \ll (n_r/n_{r-1})_p$. Notice that the factor $(n_{r+1}n_e/n_r)_p$ is independent of $(n_e)_p$, so that N_{abs} depends on $(n_e)_p$ only through the logarithmic term.

Let us now apply these results to the atmosphere with log T_{eff} = 4.4 and log g = 3, assuming it to be pure hydrogen and fully ionized. If electron scattering is the dominant source of continuous absorption and we take $\tau = 2/3$ as the base of the reversing layer, and if we again neglect support by radiation pressure, then

$$(n_e)_p = \frac{2}{3} (H\sigma_e)^{-1} , \tag{21}$$

where σ_e is the scattering coefficient per electron. If we set $T_e = 0.7\ T_{eff}$, the scale height $H = 2.9 \times 10^9$ cm and $(n_e)_p = 3.5 \times 10^{14}$ cm $^{-3}$. Then, with $T_R = T_e$ in equation (14), the ionization equilibrium for CII-III-IV is $(n_{r+1}/n_r)_p = 2.0 \times 10^{-7}$ and $(n_r/n_{r-1})_p = 0.22$. Substitution of these values into equation (20) with $n_C/n_H = 3.3 \times 10^{-4}$ gives $N_{abs} = 9.4 \times 10^{14}$ cm $^{-2}$. We then find from equation (16) that $\tau_o = 88.3$, assuming that Doppler broadening arises only from thermal motions. The corresponding correction factor for line formation from equation (9) is $\Psi = 3.4 \times 10^{-2}$, so that from equations (5) and (6) we have

$$\log\ (g_R)_{1548} = 4.0 + \log\ \frac{n_{r+1}}{n_C} \tag{22}$$

This one line therefore gives $g_{eff} < 0$ for those layers where more than 10% of the carbon atoms are CIV ions. From equation (19), we find that this applies to those layers above the point where $\log n_e = 8.85$.

Thus, for this particular model, the layers above the point where $\log n_e = 8.85$ have $g_{eff} < 0$ because of absorption in this one line; consequently, these layers are gaining outward momentum from the radiation field at a greater rate than they are gaining inward momentum from the star's gravitational attraction. In this circumstance, these layers will be expelled from the star and we may reasonably suppose that a continuous outflow of material is then set up. This outflow we identify with the mass loss inferred from observation.

Above we have considered the CIV 1548 Å line in an atmosphere with $\log T_{eff} = 4.4$ and $\log g = 3$. For atmospheres with different parameters, there are other ions with resonance lines in the ultraviolet capable of giving $g_{eff} < 0$ in the outer parts of the reversing layer.

III. MOVING REVERSING-LAYERS

Having argued in §II that the hot, luminous stars observed to be losing mass do so because their reversing layers cannot be in hydrostatic equilibrium, we now study the outflow from their atmospheres. In particular, we wish to calculate mass-loss rates for these stars.

a) Hydrodynamics

We shall suppose that the flow is spherically symmetric and steady. Then, if v is the velocity at radius r, the mass-loss rate is

$$-\frac{d\mathfrak{M}}{dt} = 4\,\pi r^2\,\rho v \qquad (23)$$

and is a constant of the flow. Now, from solar-wind theory (Parker 1958; 1960a,b), we know that the only solution of the hydrodynamical equations satisfying the boundary condition of zero gas pressure at infinity is subsonic near the star and supersonic far from the star. For such a solution, any constant of the flow must be determined by the subsonic branch of the solution because in the supersonic flow information cannot be propagated upstream. Therefore, to calculate the mass-loss rate, we need only consider the flow up to the sonic point.

Assuming the sonic point to be close to the star's surface, we adopt plane-parallel geometry for the subsonic flow. The equation of motion for steady flow is then

$$v\frac{dv}{dx} = -\frac{1}{\rho}\frac{dP}{dx} - g_{eff} \quad , \tag{24}$$

and the continuity equation may be integrated to give

$$\rho v = J \quad , \tag{25}$$

where J is the constant mass flux through the atmosphere. To complete the system, we need an equation giving the rate of change of the entropy of the gas due to the emission and absorption of radiation. Here we avoid this complication by simply assuming the flow to be isothermal. Equations (24) and (25) may then be combined into the single equation

$$(v^2-a^2)\ \frac{1}{v}\ \frac{dv}{dx} = -g_{eff} \quad , \tag{26}$$

where $a = \sqrt{(P/\rho)}$ is the Newtonian speed of sound.

From equation (26), we see that a necessary condition for a finite velocity gradient at the sonic point ($v = a$) is

$$g_{eff} = 0 \text{ when } v = a \quad , \tag{27}$$

which is the analogue for this problem of the condition at the critical point in solutions for the solar wind (Parker 1960a,b). As we shall see, this condition plays a crucial role in determining the mass flux J.

b) Radiative Transfer

In the present problem, the radiation field acts as a source of momentum for the gas flow, the mechanism transferring momentum being absorption by ions with ultraviolet resonance lines. Such an ion sees a radiation field comprising an approximately isotropic component made up of photons scattered in the resonance line and a strongly anisotropic component made up of continuum photons. This suggests that the momentum transfer may be approximated by assuming that the gas gains momentum only by absorption of continuum (unscattered) photons. The transfer problem then reduces merely to the calculation of the extinction of the continuum radiation. Thus, if $I^0_\nu(\mu)$ is the specific intensity of the continuum radiation at angle $\cos^{-1}\mu$ to the local normal, the transfer equation to be solved in the moving reversing-layer is

$$\mu \frac{dI^o_\nu}{dx} = -l_\nu (\nu^1 - \nu_i) \rho I^o_\nu , \qquad (28)$$

where l_ν is the line absorption coefficient per gm., ν_i is the line frequency, and ν is the frequency of the radiation seen by the moving medium. If ν is the frequency in the rest frame of the star, then

$$\nu^1 = \nu \left(1 - \frac{\mu v}{c} \right) \qquad (29)$$

In terms of the solution of equation (28), the contribution of lines to g_R is given by

$$g_R = \frac{2\pi}{c} \int_0^\infty \int_0^1 l_\nu \, I_\nu^o \, \mu \, d\mu d\nu \qquad (30)$$

This double integral has to be calculated at every step in the solution of equation (26) since the right hand side of that equation is $-g_{eff} = g_R - g$.

Although the contributions of lines to g_R is the dominant effect, the contribution of electron scattering cannot be neglected for models with low surface gravities. We therefore write $g_{eff} = g_* - g_R$ with g_R given by equation (30) and g_* given by

$$g_* = g - \frac{\pi F}{c} \sigma \quad , \qquad (31)$$

where πF is the integrated flux and σ is the electron-scattering coefficient per gm.

For the continuum flux at the line frequencies, we take $F_\nu = B_\nu(T_{eff})$. Thus, if we ignore limb - darkening, $I_\nu^o(\mu) = B_\nu(T_{eff})$ is the obvious boundary condition for the transfer equation at x = 0, the base of the reversing layer. However, the contribution of each line to g_R at x = 0 is then $(g_R)_i^o$ (see § IIa), so that an abundant ion at x = 0 gives $g_R \gg g$. We avoid this by taking

$$I_\nu^o(\mu) = \frac{\sigma}{\sigma+1_\nu} B_\nu(T_{eff}) \quad \text{at } x = 0 \ , \tag{32}$$

which allows approximately for a reduced flux in the line already at $x = 0$ and always gives $g_R < g$ at $x = 0$. The important ions in a given solution only become abundant high in the reversing layer so that for them equation (32) reduces to $I_\nu^o = B_\nu(T_{eff})$.

c) An Upper Limit for the Mass Flux

In Paper I, we briefly reported calculations of mass-loss rates made under the assumption that the contribution of each line to g_R was $(g_R)_i^0$ (see § IIa). Since $(g_R)_i^0$ is a function only of n_e when T_e, T_R and W are fixed, the condition $g_{eff} = 0$ at the sonic point then reduces to an algebraic equation for n_e at $v = a$, which in turn determines the mass flux $J = m_H n_e a$. Because attenuation of the continuum radiation is ignored in such calculations, there is no limit to the momentum that may be transferred to the gas. This can result in such a high mass flux that the momentum flux at the sonic point (=Ja) is greater than that available by the absorption of continuum radiation in the lines. When this occurs, the calculation is not consistent.

This consistency check for the calculations of Paper I may be used to derive an approximate upper limit for the mass flux. Let us consider an ion with a resonance line at $\nu = \nu_i$. As the gas accelerates from small velocities up to the sonic velocity, the rest frequency of the continuum radiation absorbed in the line center shifts from ν_i to $\nu_i+\mu\Delta\nu$, where $\Delta\nu = \frac{a}{c}\nu_i$. Thus, if the line width is << $\Delta\nu$, the frequency interval from which this line transfers momentum to the gas in the subsonic flow is $(\nu_i, \nu_i+\Delta\nu)$. The total momentum available in this interval is $\pi F_\nu \Delta\nu/c$, which must therefore exceed the momentum flux Ja of the gas flow at the sonic point. After substitution for $\Delta\nu$, this condition gives

$$J < \frac{\pi F_{\nu_i} \cdot \nu_i}{c^2} , \qquad (33)$$

which may also be derived from the basic equations of the problem.

For black-body radiation, the frequency of maximum emissions ν_m is such that $B_{\nu_m} \cdot \nu_m = 0.58 \times (\sigma T^4/\pi)$. Therefore, since the lines we are discussing are in the frequency range where the star's flux is greatest, we expect that $F_{\nu_i} \cdot \nu_i \simeq F$, the integrated flux. We then have

$$J \lesssim \frac{\pi F}{c^2} \quad . \qquad (34)$$

We now note that the star's luminosity is $4\pi R^2 \times \pi F$ and that the mass-loss rate is $4\pi R^2 J$. An approximate upper limit to the mass-loss rate when only one line is considered is therefore given by

$$- \frac{d\mathfrak{m}}{dt} \lesssim \frac{L}{c^2} \quad . \qquad (35)$$

Thus we have the interesting result that the mass-loss rates expected from this mechanism are comparable with the rate at which a star loses mass because of the mass defect of the nuclear reactions in the stellar interior.

IV. RESULTS

In this section we give numerical solutions of the equations of the simplified theory of moving reversing-layers formulated in §III. Additional calculations of the effect of resonance line absorption on static reversing-layers, using the simple model described in §IId, are also reported.

a) Important Lines

The resonance lines included in our calculations are listed in Table 1. The ionization potentials of these ions are appropriate for the atmospheres of hot stars and the ions are from abundant elements. The oscillator strengths for these lines were taken from Varsavsky (1961) and the abundances of the elements from Allen's (1963) compilation. Lines from excited states were entirely neglected because the reduced population of the excited state relative to the ground state and the smaller f-values usually combine to make them unimportant. For NIII and SIV, however, there are low-lying excited states (excitation potentials 0.02 and 0.12 eV) that are not unimportant, but which were not included.

Note that the wavelength separations of the components of the doublets in Table 1 are sufficiently large that they act independently in the subsonic flow. Beyond the sonic point, however, the velocity may well become large enough that the long wavelength components are deprived of continuum radiation as a result of absorption in the short wavelength components at smaller velocities.

b) Approximations

The following approximations were made in the computations:

(i) The line absorption coefficient, $l_\nu(\nu-\nu_i)$, was approximated by a pure Doppler profile when $|\nu-\nu_i| \leq 2\Delta\nu_D$ and by the sum of the Doppler profile and the pure damping profile when $|\nu-\nu_i| > 2\Delta\nu_D$. This approximation for the Voigt profile is excellent if the damping constant, γ, is such that $\gamma/4\pi << \Delta\nu_D$ (see, e.g., Anbartsumyan 1953, p. 138). For most of the calculations the damping constant was taken to be γ_0, the classical value, which should be a good approximation for resonance line absorption at low densities.

(ii) The electron temperature and the radiation temperature throughout the moving reversing-layer were set by the formulae: $T_e = T_R$ and $T_e = 0.7\ T_{eff}$. These formulae are intended to allow roughly for the result from model atmosphere calculations (see, e.g., Mihalas 1965, Fig. 2) that the surface temperatures of hot stars are somewhat lower than the grey atmosphere result: $T_0 = 0.81\ T_{eff}$.

(iii) The degree of ionization in the flow was calculated assuming ionization equilibrium (eq. [14]) with the dilution factor W = 1/2.

(iv) The double integral for g_R (eq. [30]) was evaluated with a 25-point trapezoidal qaudrature formula spanning the frequency of each line and a one-point formula for the integration over μ (with the point at μ=2/3).

(v) The electron pressure, P_e, at x = 0, the base of the moving reversing-layer, was calculated from the formula: log P_e = log g - 0.5, which corresponds roughly to τ = 0.25. (The results are quite insensitive to this choice.)

c) Details of the Calculations

A solution of the equations for a moving reversing-layer is obtained with the following steps:

(i) The parameters of the problem, log T_{eff} and log g, are specified and a guess made for the mass flux J.

(ii) The velocity at x = 0 is calculated from equation (25) using the guessed value for J and the density calculated from the specified values of

T_e and P_e (assuming pure hydrogen).

(iii) With the velocity at x = 0 now known and the specific intensity at x = 0 given by equation (32), the differential equations (26) and (28) may be integrated as an initial value problem. (The order of the system depends on how many lines are included; if five lines are important in a given model, the order is 5 x 25 + 1 = 126.)

(iv) In general the value of J will be wrong; the integration will therefore not fulfill the condition: $g_{eff} = 0$ at v = a (eq. [27]). The mass flux J is then decreased if $g_{eff} > 0$ as v → a and increased if $g_{eff} \rightarrow 0$ when v < a. (Notice that decreasing J increases the degree of ionization in the flow because of the smaller electron densities.)

(v) The steps (ii) - (iv) are then repeated until the mass flux J is determined with sufficient accuracy.

d) Solutions

From a large number of models, calculated in this way, we have constructed Figure 1, which shows lines of constant log J (the units of J being gm · $cm^{-2} \cdot sec^{-1}$) in the (log T_{eff}, log g)-plane. We see that the mass flux is high for the parameters (log T_{eff} = 4.4 - 4.5, log g = 3.0 - 3.5) corresponding to the early-type supergiants and also for the parameters (log T_{eff} ∿ 4.6, log g ∿ 4) corresponding to the Of stars. The mass flux is also high, however, for early-type main-sequence stars (log T_{eff} = 4.35 - 4.5, log g = 4.2) for which there are no observations indicating mass loss.

To get the actual mass-loss rate from J, we must also specify the star's mass, radius, or luminosity. In terms of the star's mass, the mass-loss rate is

$$-\frac{d\mathfrak{M}}{dt} = \frac{4\pi GJ}{g} . \tag{36}$$

Applying this formula to the model with log T_{eff} = 4.4 and log g = 3.0 for which log J = -7.69, we find the mass-loss rate is 1.1 x $10^{-8}\mathfrak{M}_{\odot}$/yr if the star's mass is 20 $\mathfrak{M}_{\odot}$.

Solutions are not given in the region of the (log T_{eff}; log g)-plane where g_* (see eq. [31]) is negative, for then not only is a static atmosphere impossible but also a static envelope. This follows because both F and g are $\propto r^{-2}$ deep into the star; consequently, if $g_* < 0$ in the atmosphere, it is also negative throughout the stellar envelope. In this circumstance, we would expect the entire envelope to be expelled on a short time-scale. The region where $g_* < 0$ is therefore obviously not relevant for stars that are steadily losing mass with imperceptible motions in their photospheres.

Some understanding of the behaviour of the solutions may be obtained from Figure 2, which shows various properties of the solutions as a function of log T_{eff} when log g = 3.5. The fractional contribution to g_R of the lines of the various ions at a point in the flow just below the sonic point is plotted in the upper diagram; the logarithm of the fractional abundance of each ion at the same point is plotted in the middle diagram; and log J is plotted in the lower diagram.

Figure 2 shows that the ions primarily responsible for the outflow are: SiIV for log T_{eff} =4.23-4.36, CIV for log T_{eff} = 4.38-4.51, and SVI for log T_{eff} =4.52-4.64. Taking CIV as an example, we see that its lines first become effective when $n(CIV)/n_C \approx 0.01$ at the sonic point and cease to be effective as soon as $n(CIV)/n_C \approx 1$. When the lines first become effective, the column density of CIV ions is so small that the contribution of each component of the doublet is $\approx (g_R)_i^0$ (see §IIa); consequently, only a small abundance fo CIV ions at the sonic point gives $g_R = g_*$. With increasing T_{eff}, the column density, and therefore also the optical thickness, increase so that a greater abundance of ions at v = a is needed to give $g_{eff} = 0$. Finally, the column density reaches a point where the continuum radiation with rest frequency $\nu_i(1+\frac{a}{c})$, which is absorbed in the line centre at v = a, has already been absorbed in the damping wings at smaller velocities; the condition $g_R = g_*$ can then no longer be fulfilled by the CIV ions. When this happens, the mass flux adjusts until the line(s) of some other ion gives $g_{eff} = 0$ at v = a. Thus, when CIV drops out at log T_{eff} = 4.51, the mass flux drops, thereby increasing the degree of ionization, until the abundance of the SVI ion is sufficient for its resonance doublet to become an important contributor to g_R. This drop is J when CIV ceases to be effective shifts to lower effective temperatures if the damping constant is increased (see lower diagram in Fig. 2) as we would expect from the above discussion.

In §IIIc, we derived the approximate upper limit L/c^2 for the mass-loss rate when only one line is considered. The corresponding upper limit for the mass flux is

$$J_*^* = \frac{\sigma}{c^2} T_{eff}^4 , \tag{37}$$

where σ is now the Stefan-Boltzmann constant. In the lower diagram of Figure 2, J_* is compared with the calculated values of J. We see that J_* is a rather good approximation to the mass flux near the maxima of J.

e) Mass-Loss Domain

As we have seen, moving reversing-layers with large mass fluxes are found for stars showing no evidence of mass loss, which suggests that moving solutions might exist when static solutions are also possible. To investigate this point, we make further calculations of $(g_R)_i$ (see equation [6]) for the model reversing-layer described in §IId. All the lines listed in Table 1 are now included and g is replaced by g_* in the expression (eq. [18]) for the scale height in the reversing layer.

We regard the reversing layer as non-stationary if the condition $\sum_i (g_R)_i > g_*$ holds above some point in the layer. If this condition holds only when these lines with $\tau_o > 1$ (see eq. [16]) are included, we expect most of the momentum available to the lines to be transferred to the gas, so that the mass flux will be close to the upper limit given in the inequality (32). If, on the other hand, the condition holds only when those lines with

$\tau_o < 1$ are included, the mass flux is likely to be substantially below this upper limit.

Figure 3 contains the results of such calculations. The cross-hatched areas define the domain of non-stationary reversing-layers where large mass fluxes are expected ($\tau_o < 1$). The hatched areas show how the mass-loss domain is extended if lines with $1 > \tau_o > 0.01$ are included. Outside these areas, we expect the mass flux to be negligible or the reversing layer to be static.

It must be emphasized that the calculations used to construct this diagram are not rigorous; in particular, the effect of lines on g_{eff} is not allowed for in the stratification of the reversing layer. Thus our identification of the mass-loss domain in the (log T_{eff}, log g)-plane is rather tentative.

Figure 3 also contains an evolutionary track for a star of $30\mathfrak{M}_o$ (Stothers 1966) with composition X = 0.70, Z = 0.03 and tracks for stars of $9\mathfrak{M}_o$ and $5\mathfrak{M}_o$ (Hofmeister 1967) with composition X = 0.739, Z = 0.021. (To simplify the diagram, rapid phases of evolution are omitted for the $30\mathfrak{M}_o$ stars.) The diagram is then seen to be roughly in agreement with the observational result that mass loss occurs for early-type supergiants and not for main-sequence stars, except for those of high mass. These simple calculations do, however, indicate the possibility of significant mass loss for main-sequence stars with log T_{eff} = 4.38 - 4.44.

These calculations have shown that we do indeed find moving solutions when static solutions are also possible. The reason for this is readily understood. In the example of §IId, we found that the CIV line at 1548Å ceases to give $g_{eff} < 0$ when $\tau_o \gtrsim 10^3$. When this happens, however, the optical thickness of the reversing layer at frequency $\nu_i + \Delta\nu$, where $\Delta\nu = \frac{\ddot{a}}{c}\nu_i$, is still small because $\Delta\nu \simeq 3.5\Delta\nu_D$ for the CIV ion. Thus ions moving out with the sonic speed still see sufficient continuum radiation for the condition $g_R = g_*$ to be satisfied.

f) Consistency Checks

Having obtained models of moving reversing-layers, we now check our assumptions for self-consistency. We do this for the model with log $T_{eff} = 4.4$ and log g = 3, the details of which are given in Table 2. This table gives the velocity, electron density, electron pressure, and effective gravity as functions of height above the starting point. In addition, the logarithmic abundance of CIV ions, which are almost wholly responsible for the outflow, is given. The last line of the table gives the boundary condition: $g_{eff} = 0$ at v = a.

The major assumptions of the theory are: (i) plane-parallel geometry; (ii) ionization equilibrium; (iii) isothermal flow; (iv) negligible drift velocities; (v) momentum transfer by continuum photons only.

We see immediately that assumption (i) is justified. The height of the sonic point above the starting point is 0.57 R_o, which is small compared to the radii ($\sim$20 R_o) of stars for which this solution is appropriate.

Assumption (ii) is justified if the time-scale for the removal of deviations from ionization equilibrium, t_I, is less than the time-scale of the flow, t_F. For the latter we take $t_F = (\frac{dv}{dx})^{-1}$, which decreases with height and has the value $t_F = 2 \times 10^3$ sec at the sonic point. For CIV ions we may show that $t_I^{-1} = \alpha n_e$, where α is the recombination coefficient for the CIV ion, provided that n(CIV) << n(CIII) and n(CIII) ≈ n_C. This gives $t_I = 50 \times (10^{10}/n_e)$ sec, so that $t_I \ll t_F$ throughout the subsonic flow and assumption (ii) is justified. We may note, however, that departures from ionization equilibrium will become important in the supersonic flow.

Assumption (iii) is justified if the cooling of the gas by adiabatic expansion, which occurs on the time-scale t_F, is counteracted by heating due to photoelectrons. If photoionization of hydrogen atoms is the dominant process, then, because hydrogen is highly ionized, the important time-scale is t_R, the recombination time for protons. Taking $t_R^{-1} = \alpha n_e$, we find that $t_R = 300 \times (10^{10}/n_e)$ sec, so that $t_R \ll t_F$ and assumption (iii) is justified. Again we note that this assumption will break down in the supersonic flow.

Assumption (iv), in this case, asserts that the CIV ions have a negligible drift velocity with respect to the rest of the gas. A necessary condition for this to be true is that $t_D \ll t_F$, where t_D is the relaxation time for a CIV ion with thermal speed (≈ 5 km/sec). Applying the formula given by Spitzer (1964, p. 132), we find $t_D = 10^{-4} \times (10^{10}/n_e)$ sec, so that $t_D \ll t_F$. It is also necessary that in the time t_D the velocity change, Δv, of a CIV ion, due to the scattering of 1548Å continuum photons, should be small compared to the thermal speed. Neglecting attenuation of the continuum radiation, we find that in 10^{-4} sec a CIV ion scatters ∿2000 photons giving $\Delta v \sim 0.5$ km/sec << thermal speed. Thus, in the subsonic flow, we may indeed assume that all components of the gas move together. Beyond the sonic point, however, the velocity increment in time t_D may become large enough that drift velocities are no longer negligible.

Assumption (v) fails when the attenuation of the continuum radiation becomes very large, or, equivalently, when $(g_R)_i \ll (g_R)_i^o$. At the sonic point in this model, we find that $\log (g_R)_{1548} = 2.56$ and $\log (g_R)^o_{1548} = 3.46$, so that the validity of neglecting the flux in the scattered photons is becoming doubtful. However, were we to include the scattered photons by adding a source function to equation (28), the difficulty of getting solutions would greatly increase. Moreover, the argument of §IIIc suggests that the mass flux would not change significantly.

V. SUMMARY AND DISCUSSION

The main purpose of this paper has been to explain the mass-loss observed for luminous, hot stars. We have shown that absorption in the ultraviolet resonance lines of SiIV, CIV, NV and SVI can give strongly negative effective gravities in the outer parts of a simple model for a static reversing-layer. Interpreting this as a failure of hydrostatic equilibrium, we have suggested that the star would then set up a continuous mass flow and that this outflow is the observed phenomenon. Support for this suggestion comes from the rough agreement between the properties of stars known to be losing mass and the domain in the ($\log T_{eff}$, $\log g$)-plane where static reversing-layers have this problem (Fig.3).

In addition to identifying the mechanism responsible for the outflow, we have calculated mass-loss rates for these stars. With a simple theory of moving reversing-layers, we have found that the mass-loss rate is $\lesssim L/c^2$, a result that may also be obtained with a simple physical argument (§IIIc). On the basis of this result, we therefore assert that the mass loss observed for OB supergiants and Of stars is of no consequence for their evolution.

Because of our interest in mass-loss rates, we have restricted this paper to the subsonic flow. It may be of interest, however, to report without details some limited results for the supersonic flow. For a star with $\mathfrak{M} = 20\, \mathfrak{M}_o$, $R = 23\, R_o$, $L = 1.9 \times 10^5\, L_o$, the velocity reaches 1000 km/sec at $r/R = 1.3$, 2000 km/sec at $r/R = 1.9$, and the terminal velocity is 3,300 km/sec. In this calculation, we included only the CIV resonance doublet and computed the abundance of the ion from the appropriate rate equations assuming an isothermal, steady flow. A further result of interest is that the extinction of the continuum radiation gives $r_\nu \approx 0.3$ for the residual intensity in the absorption component of the P Cygni profile for the CIV doublet.

Although these limited results are encouraging, a major difficulty remains: If the outflow is steady, there seems to be no way of getting an abundance of NV ions sufficient to produce the strong line observed at 1240 Å. [Note that the low electron densities in the supersonic flow do not increase the degree of ionization significantly because the time-scale of the flow ($\sim 10^4$ sec) is much shorter than the time required to establish ionization equilibrium (cf. §IV f).] A possible answer to this problem is that the flows are unlikely to be steady. If we increase the velocity of a fluid element with respect to the surrounding gas, the ions in this element then see a greater flux of continuum radiation. The force per gm on this element is therefore increased and this in turn increases the perturbation in velocity. We suggest that the growth of this instability results, through dissipation at shock fronts, in a small fraction of the kinetic energy available in the mean flow being used to heat the gas. If temperatures $\sim 2.10^5$ °K arise in this way, NV ions will be created by collisional ionization on a time-scale comparable to the time-scale of the flow. We might add that temperatures at which NV and OVI would be destroyed by collisional ionization are unlikely to be reached, for then no ions capable of absorbing continuum radiation with $\lambda > 911$ Å would remain, so that the source of the instability and heating would be removed. Clearly further investigations of the supersonic flows are required before we can claim to understand the spectra of these stars.

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TABLE 1

RESONANCE LINES

Ion	λ(Å)	Ion	λ(Å)
CIII	977.0	SiIV	1393.7
CIV	1548.2	"	1402.7
"	1550.7	SIII	1190.2
NIII	989.8	SIV	1062.7
NV	1238.8	SVI	933.4
"	1242.8	"	944.5

TABLE 2

A MOVING REVERSING-LAYER

($\log T_{eff} = 4.4$, $\log g = 3.0$; $\log J = -7.69$)

v (km/sec)	$\log n_e$	$\log P_e$	$\log \frac{n(CIV)}{n_C}$	x (10^{10} cm)	g_{eff} (cm sec^{-1})
0.001	14.115	2.500	-6.64	0.000	700.6
0.04	12.483	0.868	-4.65	1.556	700.6
0.61	11.300	-0.315	-3.45	2.684	700.0
1.54	10.895	-0.720	-3.05	3.070	698.7
2.72	10.650	-0.965	-2.80	3.301	695.7
3.89	10.494	-1.121	-2.65	3.445	690.0
5.06	10.379	-1.235	-2.53	3.549	679.8
6.23	10.289	-1.326	-2.44	3.629	663.6
7.40	10.214	-1.401	-2.37	3.693	638.9
8.57	10.150	-1.464	-2.30	3.747	604.2
9.74	10.095	-1.520	-2.25	3.792	557.7
10.91	10.045	-1.569	-2.20	3.831	499.1
12.08	10.001	-1.614	-2.15	3.866	427.7
13.25	9.961	-1.654	-2.11	3.897	344.1
14.43	9.924	-1.691	-2.08	3.925	249.2
15.36	9.897	-1.718	-2.05	3.946	165.4
16.30	9.871	-1.744	-2.02	3.965	75.3
16.97	9.854	-1.761	-2.01	3.978	6.9
17.03					0.0

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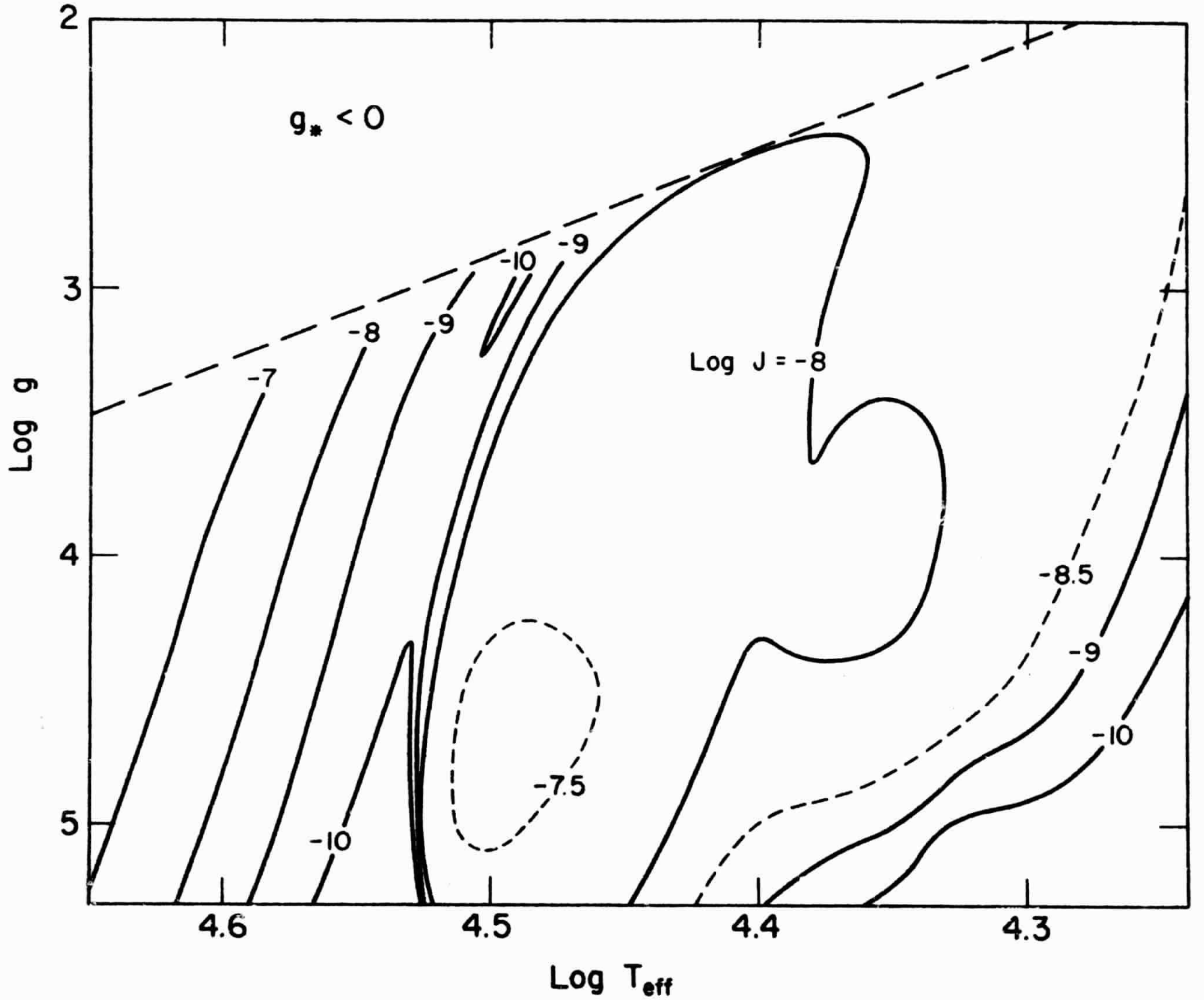
FIGURE CAPTIONS

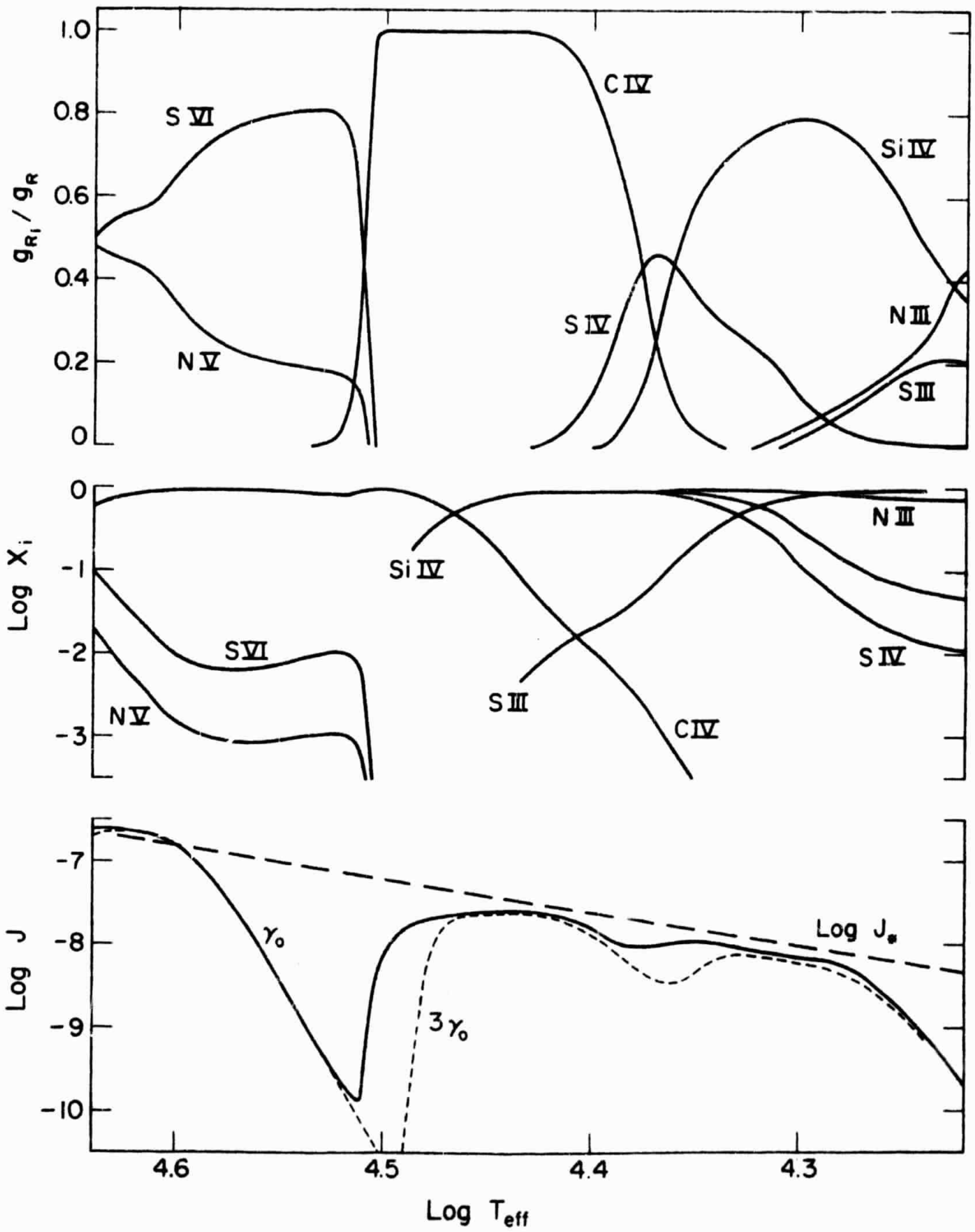
FIG. 1--Lines of constant mass flux, J, in the (log T_{eff}, log g)-plane.

FIG. 2--Properties of models with log g = 3.5 as functions of log T_{eff}. Upper diagram shows fractional contribution to g_R of various ions near the sonic point. Middle diagram shows the fractional abundance, x_i, of the ions at the same point in the flow. Lower diagram shows mass flux, J, and also limiting mass flux, J_* (see eq. [36]).

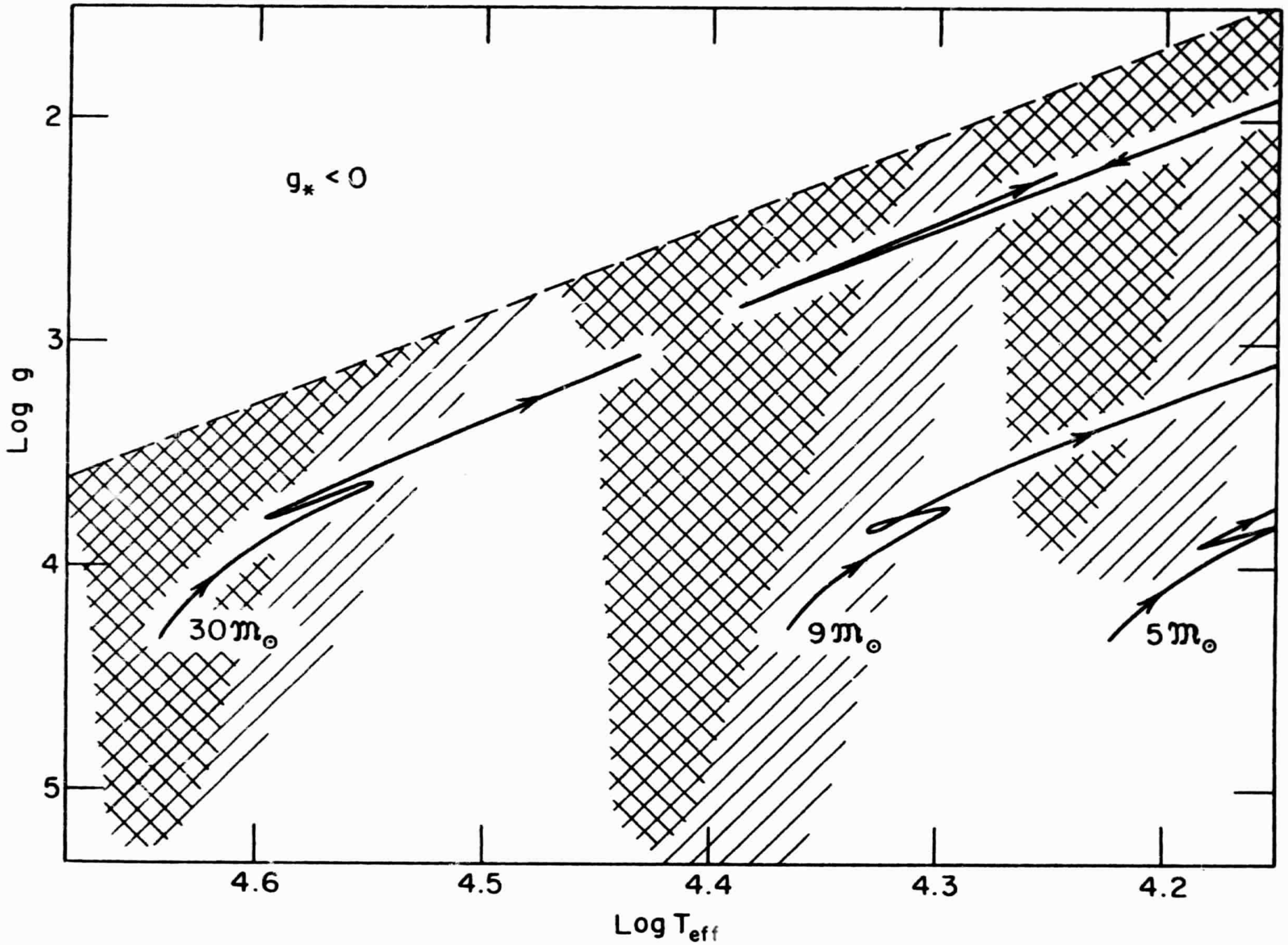
FIG. 3--Mass-loss domain in the (log T_{eff}, log g)-plane. Cross-hatched areas are regions where the greatest mass fluxes are expected. The evolutionary track for 30 $\mathfrak{M}_o$ is from Stothers (1966) and the tracks for 9 $\mathfrak{M}_o$ and 5 $\mathfrak{M}_o$ are from Hofmeister (1967).



Log g
2
3
4
5
$g_* < 0$
-7
-8
-9
-10
-9
-10
Log J = -8
-7.5
-8.5
-9
-10
4.6
4.5
4.4
4.3
Log T_{eff}



1.0
0.8
0.6
0.4
0.2
0
g_{R_i} / g_R
S VI
N V
C IV
Si IV
S IV
N III
S III
0
-1
-2
-3
Log X_i
Si IV
S VI
N V
S III
C IV
N III
S IV
-7
-8
-9
-10
Log J
γ_0
$3\gamma_0$
Log J_*
4.6
4.5
4.4
4.3
Log T_{eff}



$g_* < 0$
$30\mathfrak{M}_\odot$
$9\mathfrak{M}_\odot$
$5\mathfrak{M}_\odot$
Log g
2
3
4
5
4.6
4.5
4.4
4.3
4.2
Log T_{eff}

END

DATE

FILMED