Reports of the Department of Geodetic Science

Report No. 125

THE NORTH AMERICAN DATUM IN VIEW OF GEOS I OBSERVATIONS

by
Ivan I. Mueller
James P. Reilly
Charles R. Schwarz

Prepared for

National Aeronautics and Space Administration
Washington, D. C.

Contract No. NGL 36-008-093
OSURF Project No. 2514

The Ohio State University
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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and it is under the technical direction of Jerome D. Rosenberg, Project Manager, Geodetic Satellites Program, NASA Headquarters, Washington, D.C. The contract is administered by the Office of University Affairs, NASA, Washington, D.C. 20546.

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1. INTRODUCTION

During the lifetime of the GEOS I satellite a large number of optical and electronic observations were made from stations located on the North American Datum (NAD). The NAD coordinates of most of these stations have been determined through precise geodetic ground ties to the first-order triangulation network. In this report an attempt is made to determine the coordinates of these stations from the available satellite observations and possible distortions present in the North American Datum. The coordinates were determined both from simultaneous observations (geometric mode) and from observations distributed along the orbital path (short-arc mode).

The observation stations involved are shown in Fig. 1. All coordinate computations (and results) are relative to the NAD coordinates (and variances) of Columbia, Missouri, the station nearest to the origin of the NAD: Meades Ranch, Kansas. On the other hand, the parameters, defining the distortions of the NAD with respect to the satellite determined system, refer to Meades Ranch.

2. OBSERVATIONS

The observations utilized in the calculations were taken by the United States Air Force using the PC-1000 cameras (15 stations), by NASA/Goddard Space Flight Center using mostly MOTS 40 cameras (15 stations), and by the U.S. Army Topographic Command (TOPOCOM)—sequential collation of range (SECOR) ranging system (4 stations). The optical observations were processed as described in [1, pp. 82-95, 129-137] and deposited in the National Space Science Data Center (NSSDC) by the agencies responsible for data reduction (Aeronautical Chart and Information Center and Goddard Space Flight Center).

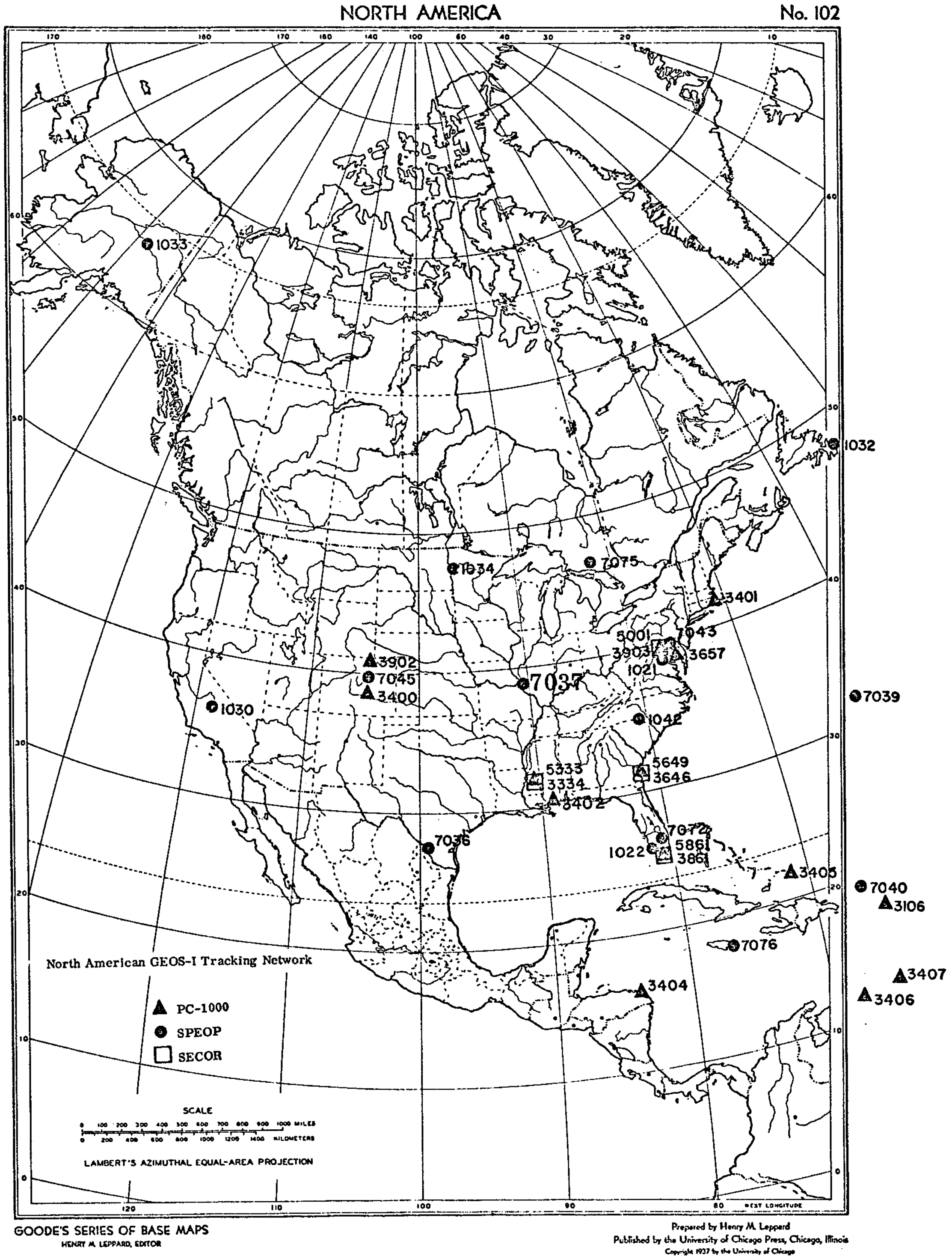
NORTH AMERICA
No. 102
North American GEOS-I Tracking Network
▲ PC-1000
● SPEOP
☐ SECOR
1033
1032
1034
7075
3401
5001
7043
3903
3657
1021
3902
7045
3400
7037
7039
1030
1042
5333
3334
5649
3646
3402
7072
5861
1022
3861
7036
3405
7040
3106
7076
3407
3404
3406
SCALE
LAMBERT'S AZIMUTHAL EQUAL-AREA PROJECTION
WEST LONGITUDE
GOODE'S SERIES OF BASE MAPS
HENRY M. LEPPARD, EDITOR
Prepared by Henry M. Leppard
Published by the University of Chicago Press, Chicago, Illinois
Copyright 1937 by the University of Chicago

Fig. 1

The SECOR data was directly obtained from U.S. Army TOPOCOM, where it was processed as given in [2, pp. 18-37] and [3].

2.1 Data Processing

The optical data (approximately 7000 observations) as received from the NSSDC required corrections as described below:

(1) The time given for each PC-1000 observation was in the system of the satellite clock; corrections to UTC were interpolated from graphs furnished by the Applied Physics Laboratory, Johns Hopkins University.

(2) The MOTS data was given in the UTC system, but the last digit (tenths of milliseconds) was rounded off. It was necessary to examine every MOTS data card and correct for this roundoff.

(3) The time correction from UTC to UT1 was performed by a computer program using the second difference Bessel formula to interpolate from tables provided by the U.S. Naval Observatory. Since the tables were for the differences UT2 - UTC and UT2 - UT1, the UT1 time was calculated by the formula:

$$\mathrm{UT1} = \mathrm{UTC} + (\mathrm{UT2} - \mathrm{UTC}) - (\mathrm{UT2} - \mathrm{UT1})$$

(4) Correction for parallactic refraction had to be applied to all PC-1000 and MOTS observations. It was computed using the formula described in [4, p. 93]. Since this formula requires the zenith distance and range to the satellite, it was necessary to write two computer programs: The first program separated all observations into simultaneous events, computed the Cartesian coordinates of the satellite as described in [5, pp. 17-18], and computed the range of the satellite from each observing station. The information on the data card and the range and the Local Apparent Sidereal Time (LAST) were written onto a magnetic tape. Another program was written that would read the tape, compute the parallactic refraction, and apply the corrections. A standard temperature of 10° C and a standard pressure of 760 mm Hg were used in the computation of the parallactic refraction.

(5) In early 1969 the U.S. Naval Observatory delivered revised values of A.1 - UT1 for the period January 1, 1956 to January 22, 1969. These values were given for each day, and they were punched on data cards. Also on these cards were the coordinates of the instantaneous pole with respect to the Conventional International Origin (CIO), and the value A.1 - UTC. In order to be able to use the revised values of UT1, it was necessary to use the time correction program mentioned in (3) above in the reverse to arrive at UTC, from which the revised UT1 was calculated by the formula:

$$\mathrm{UT1} = \mathrm{UTC} + (\mathrm{A.1} - \mathrm{UTC}) - (\mathrm{A.1} - \mathrm{UT1})$$

As before, a second difference Bessel formula was used to interpolate from the tables.

The SECOR data was used as received since it was previously corrected and screened in [3].

2.2 Data Screening and Rejection

The principle tool used for screening the optical data was the individual event adjustment. The observations are grouped by event (i.e., individual flash) and an adjustment is performed for the three components of the satellite position. The a posteriori variance of the observation of unit weight is computed and compared to a test value. If the computed variance is greater than the test value, the entire event (flash) is deleted from the data sample. A detailed description of the equations used in this process is given in [6, p. 57]. The purpose of the individual event adjustment is to detect blunders in the observational data, since these will generally cause large residuals and consequently a large a posteriori unit variance. On the other hand, the residuals and a posteriori unit variance also include some contribution from the errors in the station positions, which are held fixed during the event adjustment. Thus the test is efficacious only if the station positions are fairly well known. In the case of the MOTS and PC-1000 observations, the station positions were considered to be quite well known a priori. We expected the approximate station coordinates to be sufficiently accurate that their errors would seldom contribute more than a second of arc to any of the residuals in the individual event adjustments.

Altogether about 5000 MOTS and 2000 PC-1000 observations were investigated. We expected to find that the data had been thoroughly screened before it was deposited in the Data Center, so that it would not be necessary for us to delete any observations at all. However, the individual event adjustments showed that the unit variance was unacceptably large in a sizable number of cases. The values of the a posteriori unit variance fell over a wide range; only in some cases was this value large enough to indicate an obvious blunder, while in other cases this value indicated that the data probably contained a blunder of small magnitude. We were able to identify the actual observation containing the blunder in a few cases by examining the residuals of the individual event adjustment and in other cases by examining the residuals of the orbital mode adjustment, but in most cases the offending observation could not be identified and it was necessary to delete the whole event. We were not able to detect any correlation between bad observations or events and the tracking stations, so that we could not ascribe the existence of bad events to large errors in the coordinates of any station. On the other hand, we often found that all, or at least several, of the flashes of a sequence yielded poor event adjustments, which indicated the existence of an error in the plate reduction for at least one of the plates involved. Since we did not have access to the raw data and the plate reductions, we were not able to follow the possibility further.

The data suspected of containing blunders amounted to about 10% of the MOTS data and about 30% of the PC-1000 data. The PC-1000 data was the most troublesome, not only because of the large amount of suspected data but also because the value of the unit variance often fell into the "doubtful" range; the values of the a posteriori standard deviation of unit weight for the individual event adjustments were fairly continuously spread from 0" to 30". We were willing to accept that an individual event could yield a standard deviation of as much as 6" or even 10" due to the normal sample fluctuation of accidental errors. However, it seemed that a value greater than this amount indicated the probable presence of a small blunder that would be identifiable with sufficient investigative effort. In the case of the MOTS data, the a posteriori standard deviations were usually either less than 6" or greater than 20", thus allowing a fairly clear separation of good and erroneous data.

Since we were not in a position to search for the cause of the apparent blunders in the data, we were not able to determine which events actually contained small blunders and which only appeared bad due to normal sample fluctuation. If we were to consider the small blunders to be part of the population of accidental errors, the population standard deviation of the data would be so large as to render it practically useless for geodetic purposes. This meant that it was necessary for us to accept, a priori, some value for the standard deviation of the data and to rely on statistical methods to detect and delete suspect data.

Based on previous experience, we decided to ignore the standard deviations of the observations given on the data cards, since these were completely unrealistic, and to accept a value of 2".0 as the standard deviation of all optical observations, both MOTS and PC-1000. This value was used as the standard deviation for the declination and for the right ascension times the cosine of the declination. If this value is the true standard deviation of the accidental errors in the data, then the expected value of the a posteriori unit variance of an individual event adjustment is one. This statistic is distributed as chi-square so that we were able to construct a confidence interval in which it was expected to fall. Our final decision was to use a rejection criterion of 10, rejecting all events for which the unit variance was greater than this value. Since most of the event adjustments involved only two plates, and thus four observations and one degree of freedom, this rejection criterion corresponded in most cases to a probability level of .99843.

I.e., if the hypothesis that the true standard deviation is 2″.0 is correct, then only 0.157% of the events are deleted by this test when they are actually good events. This is a small price to pay for the rejection of most of the small blunders.

When combined with the a priori standard deviation of 2″.0, this rejection criterion corresponds to an observational standard deviation of 6″.3 for the sample of observations contained in the event. Thus the screening criterion used for optical data may be phrased as the rejection of all events for which the observational standard deviation, estimated from the individual event adjustment with the starting coordinates held fixed, is greater than 6″.3. As expected, this rejection criterion resulted in an overall unit variance of close to one, as seen in the results of the simultaneous adjustments of all nondeleted optical data (Section 3.1).

The SECOR range data had already been extensively screened during the processing described in [3]. Therefore it was not necessary to delete any range data. An a priori standard deviation of 1.7 m, obtained from [3], was used in forming normal equations from the range data.

2.3 Distribution of Observations

The number of simultaneous optical observations between the various stations and used in the adjustment is shown in Fig. 2. The station numbers along the borders of the matrix are the GOCC numbers described in [7]. The corresponding station names are given in Table 4. A number inside the matrix indicates the number of simultaneous events observed between the two stations appearing at the ends of the respective column and row intersecting at the number in question.

The time periods of optical passes used in the short-arc adjustment are listed in Table 1, and a matrix indicating the distribution of observations between the various stations and passes is shown in Fig. 3.

SECOR tracking of GEOS I was initiated on December 1, 1965, and terminated May 1, 1966. During this period the satellite was tracked more

Station	1032	3334	3902	1033	3400	3903	7039	3405	3407	3648	3404	3657	3406	7076	1021	3402	3401	3106	3861	7040	7043	7045	1042	7072	7036	1034	1030	7037	1022	Station
7075							14								20							11	48		24	25	31	30	16	7075
1032															1						6		3	1						1032
3334			3		3											3						3			6	3		2		3334
3902					5											3						5			8	4		9		3902
1033																						9				13	9			1033
3400																						11	8		11	4		3	5	3400
3903																					6	6	2			6	6	6	2	3903
7039								9		6		3	16	10	6	16	7	11	7			5	3	5	6	14		21	46	7039
3405									13		4	7	7		17	6		7	5	19	10			6	4			5	27	3405
3407												1	14		1		7	15	13	21	5									3407
3648											10	10			5	3	9	17	24	5			5	13	7	5		18	18	3648
3404												7		4		16	10	6	26	20	7		17	3		4	4	9	46	3404
3657															22	23	26	5	16	1	25		9	15	6	10	6	6	37	3657
3406														21	11	6		36	22	18	1		7	25	7			5	45	3406
7076																8	21	4	14	10	6	4	4	26	14	6		9	42	7076
1021																13	16	6	8	12	32	4	28	12	21	8	9	26	35	1021
3402																	11	10	36	5	20		47	11	20	20	22	22	61	3402
3401																		10	24	11	26		17	30	11	28	5	5	61	3401
3106																			10	58	8		9	19					23	3106
3861																				1	2		14	36	5	23	9	22	87	3861
7040																					13		17	15		5			79	7040
7043																						24	45	20	6	17	20	20	70	7043
7045																							22	9	37	48	97	29	31	7045
1042																								32	18	89	27	70	117	1042
7072																									16	9	5	16	193	7072
7036																										25	86	52	81	7036
1034																											101	93	74	1034
1030																												63	44	1030
7037																													63	7037
1022																														1022

Note: Station names may be found from Table 4.

Fig. 2 Distribution of simultaneous MOTS and PC-1000 observations.

Table 1

Time Periods of GEOS I Passes Used in the Short-Arc Adjustment

Pass No.	Date	From	To
1	24 Nov 65	$5^h 12^m$	$5^h 18^m$
2	24 Nov 65	7 13	7 19
3	25 Nov 65	5 17	5 23
4	26 Nov 65	5 18	5 24
6	27 Nov 65	5 17	5 23
7	14 Dec 65	2 22	2 29
8	15 Dec 65	0 24	0 31
9	18 Dec 65	0 34	0 40
10	18 Dec 65	2 42	2 48
12	21 Dec 65	9 28	9 34
13	3 Jan 66	6 15	6 20
14	3 Jan 66	8 17	8 22
15	4 Jan 66	6 20	6 25
151	12 Jan 66	4 49	4 54
152	13 Jan 66	4 52	4 58
16	14 Jan 66	4 52	5 00
16-A	14 Jan 66	5 00	5 04
17	14 Jan 66	6 48	6 56
18	15 Jan 66	4 58	5 03
181	16 Jan 66	3 01	3 06
19	17 Jan 66	3 05	3 12
20	17 Jan 66	5 07	5 13
201	18 Jan 66	3 10	3 17
202	19 Jan 66	3 14	3 23
22	19 Jan 66	5 12	5 17
23	22 Jan 66	3 19	3 24
24	24 Jan 66	3 29	3 36
26	28 Jan 66	1 43	1 49
26-A	28 Jan 66	1 49	1 55
261	29 Jan 66	1 55	2 01
262	3 Feb 66	0 14	0 21
263	5 Feb 66	0 16	0 22
264	6 Feb 66	0 25	0 29
265	26 Feb 66	8 52	8 58
266	27 Feb 66	9 00	9 07

Note: All times given are UT.

Table 1 (continued)

Pass No.	Date	From	To
28	9 Mar 66	$7^h 37^m$	$7^h 44^m$
29	9 Mar 66	9 41	9 46
30	12 Mar 66	7 44	7 49
33	14 Mar 66	7 56	8 06
35	16 Mar 66	5 57	6 02
35-A	16 Mar 66	6 02	6 09
36	16 Mar 66	8 03	8 07
36-A	16 Mar 66	8 07	8 14
39	18 Mar 66	6 08	6 17
40	19 Mar 66	6 13	6 19
40-A	19 Mar 66	6 19	6 25
42	21 Mar 66	4 12	4 21
43	21 Mar 66	6 21	6 27
44	22 Mar 66	4 18	4 25
45	22 Mar 66	6 29	6 35
46	24 Mar 66	6 39	6 47
49	27 Mar 66	4 48	4 53
50	27 Mar 66	6 53	6 59
51	28 Mar 66	4 51	4 57
52	28 Mar 66	6 57	7 03
53	29 Mar 66	4 50	4 56
54	29 Mar 66	7 00	7 06
55	30 Mar 66	2 53	2 59
56	31 Mar 66	2 56	3 03
57	17 Apr 66	8 38	8 44
58	18 Apr 66	8 37	8 43
581	22 Apr 66	6 56	7 02
59	24 Apr 66	6 58	7 06
60	26 Apr 66	7 05	7 12
61	27 Apr 66	7 15	7 24
70	14 Jan 66	0 39	0 49
71	6 Apr 66	11 47	11 55
72	8 Apr 66	9 53	10 01
73	9 Apr 66	9 58	10 05
74	10 Apr 66	10 02	10 09
75	13 Apr 66	10 15	10 20
76	15 Apr 66	10 18	10 24

Table 1 (continued)

Pass No.	Date	From	To
77	10 May 66	4 01	4 07
78	11 May 66	2 01	2 07
79	11 May 66	4 00	4 06
80	13 May 66	4 11	4 18
81	15 May 66	2 11	2 20
82	15 May 66	2 24	2 31
83	16 May 66	2 22	2 28
83-A	16 May 66	2 32	2 38
84	11 Jul 66	5 06	5 11
85	11 Jul 66	5 15	5 20
86	11 Jul 66	7 16	7 21
88	20 Jul 66	3 42	3 48
89	22 Jul 66	3 54	4 00
90	25 Jul 66	4 08	4 14

Note: Station names may be found from Table 4; pass descriptions from Table 1

Station / Pass	7075	1032	1033	3903	7074	7039	3405	3407	7071	3648	3404	3649	3657	3406	7076	1021	3402	3401	3106	3861	7040	7043	7045	1042	7072	7036	1034	1030	7037	1022
1																2						7		6						9
2																						7				9				10
3																4					11									7
4																									6	7				13
6																									5	7				12
7																									8					8
8						10																								10
9					1																13	4								11
10																6									5			11		
12					5		4		12	5	4		6							5	6				6				6	11
13						12				6				6															7	12
14											5																4		10	6
15						6								6		7			6										4	
151													7				7	7	5											12
152												13						6		7					3		3			2
16							3					3								3								6	11	4
16-A							3					3		6					6	3	7								3	3
17			7																				7					7		
18					6				6			1	1			3									6	11		7	11	6
181							7					4							7	4										
19						5		5				4					4	7		4	7						5			11
20					6				6											6			7	5	6			7		6
201								7					6				7	6		14										
202								7	4	5		7						6		12							5		5	
22																							6				13	7	7	
23													6										7				11	13		

Fig. 3 Number of observations used in the short-arc adjustment by pass and station.

Station Pass	7075	1032	1033	3903	7074	7039	3405	3407	7071	3648	3404	3649	3657	3406	7076	1021	3402	3401	3106	3861	7040	7043	7045	1042	7072	7036	1034	1030	7037	1022
24										6						3				6							7	4	13	7
26						1						2	3			6	5							3		4	6		10	4
26-A						3				6	6	9	3				6		3	6	7					3			2	10
261							13	7					7	7												1				
262							13	5				7									13									
263										6		13		6						7	6				6					6
264								7	2	6		7	7						7						5					5
265												10	4												4					1
266												9	6								3	12			6					12
28							7						5			6	6				6	7								13
29																												13		13
30															7									1	7					14
33						5																	13			7		7	12	
35																	5					2			10					10
35-A															6							3			9					9
36																							11		4			11		4
36-A																							9		5			3		9
39																						7		7		7				14
40														5	7							4	4						4	11
40-A		6																				10	3						10	10
42				7										6	5			5						13						18
43																		6				6		12			6			12
44						7								4	14			7		7					7	6				11
45																		5							6	4		4		11
46																							7				7		7	7

Fig. 3 (continued)

Station / Pass	7075	1032	1033	3903	7074	7039	3405	3407	7071	3648	3404	3649	3657	3406	7076	1021	3402	3401	3106	3861	7040	7043	7045	1042	7072	7036	1034	1030	7037	1022
49																								14			14			14
50																							3	5			12		6	
51																								7			14		14	
52																							4	6				6	12	
53																						7	14			7	14	14	7	
54																							6				13	7		
55														1		1		4	4		1	5		5						
56															6						4			4		2				4
57											7						7					14		14						
58																	9							13			13			6
581														6			13			14									7	7
59											4													14			14			7
60																		6							7	7	12	5		5
61											5				5						6		6					3		12
70		7																									6		6	
71			14																									6		
72			7																				7							
73			7																				11						7	
74			7																				7				12	7	7	
75			5																									7		
76			10																									6		
77	5																									14			7	14
78	14															5								7						
79	14															5								13		7		14		

Fig. 3 (continued)

Station / Pass	7075	1032	1033	3903	7074	7039	3405	3407	7071	3648	3404	3649	3657	3406	7076	1021	3402	3401	3106	3861	7040	7043	7045	1042	7072	7036	1034	1030	7037	1022
80	14														7								13					14		6
81	20																													7
82															6						14									7
83	14															7					7									13
83-A															14						14									7
84	1					6									14						7					14			7	
85	14					14																					7		7	
86	7															11								12			7	7	7	
88	7															14										14	7		4	7
89	14																							14		7			14	
90	13																						5	7			13	7	14	

Fig. 3 (continued)

than 230 passes. Data from 47 of these were selected at random at the AMS as part of a 1966 study. The original 20 events per second had been edited and reduced in number to one event (four ranges) per forty seconds. Table 2 shows the distribution of the 480 events along the various orbits. Each event was observed from the same four tracking sites at Homestead AFB, Florida; Herndon, Virginia; Stoneville, Mississippi; and Hunter AFB, Georgia.

Table 2

Distribution of SECOR Events

Orbit	Date	No. of Events	Orbit	Date	No. of Events
1657	3-24-66	10	1945	4-18-66	13
1682	3-27-66	3	1946	4-18-66	3
1693	3-28-66	3	1958	4-19-66	8
1704	3-29-66	8	1982	4-21-66	6
1728	3-31-66	11	1993	4-22-66	15
1740	4-01-66	10	1994	4-22-66	5
1752	4-02-66	4	2004	4-23-66	7
1753	4-02-66	8	2005	4-23-66	8
1755	4-02-66	10	2006	4-23-66	14
1763	4-03-66	8	2017	4-24-66	10
1871	4-12-66	4	2018	4-24-66	4
1874	4-12-66	4	2028	4-25-66	15
1875	4-12-66	7	2029	4-25-66	13
1883	4-13-66	11	2040	4-26-66	20
1886	4-13-66	12	2041	4-26-66	14
1887	4-13-66	8	2052	4-27-66	9
1898	4-14-66	18	2053	4-27-66	9
1899	4-14-66	9	2064	4-28-66	14
1907	4-15-66	16	2065	4-28-66	14
1910	4-15-66	10	2076	4-29-66	16
1911	4-15-66	12	2077	4-29-66	10
1919	4-16-66	14	2088	4-30-66	17
1922	4-16-66	8	2100	5-01-66	15
1934	4-17-66	13			

3. NETWORK ADJUSTMENT

3.1 Geometric Adjustment

The network adjustment in the geometric (simultaneous) mode was carried out as described in [17], or in more detail in [5], [6], and [8], with minor modifications. Four adjustments (NA-1, NA-2, NA-3, and NA-4) were performed with characteristics as follows:

NA-1	Only MOTS and PC-1000 data was used. The coordinates of Columbia, Missouri (7037), were given a weight of 10 (which kept the station coordinates effectively fixed). The scale was determined by imposing a chord constraint between Homestead, Florida (3861) and Greenbelt, Maryland (7043). These two stations were on the precise traverse of the U.S. Coast and and Geodetic Survey (USCGS). This chord distance is 1,531,562.9 m and was constrained to ±2 m (1: 750,000), as estimated by the USCGS.
NA-2	Same as NA-1 but the coordinates of Columbia, Missouri, were given a weight of 0.11 (which corresponds to the standard deviation computed by Simmons' formula [18]).
NA-3	The SECOR data was included along with the MOTS and PC-1000 data. The coordinates of Columbia, Missouri, were given a weight of 0.11. There was no chord constraint; the scale was determined from the SECOR ranges.
NA-4	Same as NA-3 but the chord distance, (3861) - (7043), was constrained in the same way as in the NA-1.

Table 3

General Information on the Geometric Adjustments

	NA-1	NA-2	NA-3	NA-4
No. of PC-1000 stations	15	15	15	15
No. of MOTS stations	15	15	15	15
No. of SECOR stations	—	—	4	4
σ (a priori)	2"	2"	optical:2".0 range:1.7 m	optical:2".0 range:1.7 m
Rejection criteria	6".3	6".3	optical:6".3 range: none	optical:6".3 range: none
Weight assigned to coordinates of Columbia, Mo.	10	0.11	0.11	0.11
Chord constraint between Homestead and Greenbelt (1,531,562.9 m)	1:750,000	1:750,000	—	1:750,000
Relative positions of the 4 SECOR stations and the 4 collocated PC-1000 stations constrained by adding 999 to the diagonal elements of the corresponding 3×3 matrices	—	—	X	X
No. of known and unknown ground stations	30	30	34	34
No. of unknown ground stations	29	29	33	33
No. of obs. minus 3 times No. of events	4306	4306	5281	5281
No. of spatial chord equations	1	1	0	1
No. of unknowns in normal equations	87	87	99	99
No. of degrees of freedom	4220	4220	5182	5183
Quadratic sum of all the residuals (VPV)	3951.4	3951.4	7818.3	7825.8
Variance of unit weight	0.9	0.9	1.5	1.5
Standard deviation of unit weight	1.0	1.0	1.2	1.2

Table 4

Coordinates of the North American GEOS I Tracking Stations from the Geometric Adjustment

GOCC #	Name		NAD	σ	NA-1	σ	NA-2	σ	NA-3	σ	NA-4	σ
7075	Sudbury, Ontario	X	692646.1	6.0	0.7	4.8	0.7	5.6	- 12.9	8.1	- 1.1	7.0
	MOTS 40	Y	-4347225.8	5.0	9.6	5.8	9.6	6.4	- 5.4	8.3	2.1	7.9
		Z	4600298.3	4.9	8.6	5.0	8.6	5.8	0.7	7.7	10.7	6.8
1032	St. Johns, Newfoundland	X	2602802.5		-81.8	68.8	-81.8	68.8	-130.4	88.5	-89.2	87.4
	MOTS 40	Y	-3419301.2		-10.7	97.3	-10.7	97.4	- 50.4	124.1	-20.6	123.7
		Z	4697477.3		-43.8	37.8	-43.8	37.9	- 52.2	48.2	-41.3	48.0
3334	Greenville, Miss.	X	- 84957.5	4.1	10.3	15.8	10.3	16.1	- 17.2	8.9	- 1.8	7.1
	PC-1000	Y	-5328100.8	3.1	-13.0	13.0	-13.0	13.3	0.0	10.3	- 1.5	10.3
		Z	3493285.2	3.7	-38.9	10.4	-38.9	10.8	8.9	9.0	- 2.9	8.0
3902	Cheyenne, Wyo.	X	-1234668.8	2.9	39.2	31.7	39.2	31.9	56.8	40.6	44.7	40.4
	PC-1000	Y	-4651355.3	2.4	15.3	34.6	15.3	34.7	6.7	44.0	12.0	44.0
		Z	4174612.3	2.5	-40.2	11.5	-40.2	11.8	- 40.3	14.9	-36.6	14.8
1033	College, Alaska	X	-2299238.0	10.7	6.3	15.0	6.3	15.3	40.0	21.5	12.3	19.2
	MOTS 40	Y	-1445840.4	11.0	19.3	73.3	19.3	73.3	- 42.5	94.3	3.6	93.0
		Z	5751629.0	7.8	-20.1	40.0	-20.1	40.2	- 45.6	51.6	-20.5	50.9
3400	Colorado Springs, Colo.	X	-1275173.6	2.7	-21.5	15.5	-21.5	15.8	- 2.5	20.6	-16.8	20.0
	PC-1000	Y	-4798165.6	2.3	-21.1	11.6	-21.1	12.0	- 26.9	15.1	-25.3	15.1
		Z	3994037.6	2.4	7.0	8.0	7.0	8.5	10.3	10.5	12.5	10.4
3903	Herndon, Va.	X	1089023.7	6.4	- 7.3	11.6	- 7.3	11.9	- 21.0	11.0	- 4.7	9.4
	PC-1000	Y	-4843194.9	5.1	113.7	13.9	113.7	14.2	10.4	5.7	4.4	5.3
		Z	3991564.7	5.6	- 3.1	9.9	- 3.1	10.3	- 1.6	7.5	-13.7	6.2
7039	Bermuda Island	X	2308230.2	8.8	18.9	8.5	18.9	9.0	- 21.8	15.8	11.3	11.0
	MOTS 40	Y	-4873765.5	7.2	31.9	5.4	31.9	6.1	24.9	7.3	24.9	7.3
		Z	3394389.0	8.3	- 8.5	5.4	- 8.5	6.2	3.4	7.4	- 2.0	7.1

all units are in meters

Table 4 (continued)

GOCC #	Name		NAD	σ	NA-1	σ	NA-2	σ	NA-3	σ	NA-4	σ
3405	Grand Turk	X	1919530.5	9.2	- 6.2	7.6	- 6.2	8.1	-40.0	13.8	-12.3	9.9
	PC-1000	Y	-5621245.2	6.6	27.8	5.8	27.8	6.5	32.9	8.4	23.9	7.8
		Z	2315617.1	9.0	-28.0	7.4	-28.0	8.0	2.0	11.5	-17.6	9.4
3407	Trinidad	X	2979925.0	11.4	- 5.3	11.4	- 5.3	11.7	-57.1	20.4	-15.3	14.4
	PC-1000	Y	-5513746.9	9.2	94.7	7.1	94.7	7.6	98.3	9.6	90.6	9.2
		Z	1180994.8	12.1	-25.1	12.0	-25.1	12.3	24.3	19.1	-10.1	14.9
3648	Hunter AFB, Ga	X	832594.6	6.3	2.8	5.1	2.8	5.9	-14.9	8.4	0.1	6.6
	PC-1000	Y	-5349690.7	4.6	17.5	4.9	17.5	5.7	12.7	4.7	5.5	4.0
		Z	3360414.7	5.7	- 1.2	5.3	- 1.2	6.0	15.9	6.1	3.8	4.4
3404	Swan Island	X	642541.2	8.5	-20.0	5.1	-20.0	5.9	-32.4	8.3	-21.4	7.4
	PC-1000	Y	-6054109.5	5.4	48.8	5.9	48.8	6.6	61.1	9.3	46.1	7.7
		Z	1895518.2	8.2	- 5.1	7.6	- 5.1	8.2	31.5	12.7	6.7	9.4
3657	Aberdeen, Md.	X	1186826.8	6.6	- 7.3	5.2	- 7.3	5.9	-28.7	9.6	-10.4	7.2
	PC-1000	Y	-4785340.8	5.3	20.8	5.2	20.8	6.0	11.8	7.1	14.0	7.0
		Z	4032705.0	5.7	4.3	4.0	4.3	4.9	6.1	5.6	8.9	5.5
3406	Curacao	X	2251837.6	10.7	- 1.0	8.2	- 1.0	8.7	-41.2	15.2	- 9.3	10.5
	PC-1000	Y	-5817069.3	7.8	24.5	5.8	24.5	6.5	32.5	8.7	20.5	7.6
		Z	1327016.0	11.0	11.8	10.1	11.8	10.5	58.6	16.7	26.4	12.4
7076	Jamaica, B.W.I.	X	1384188.1	9.1	4.0	6.5	4.0	7.1	-20.8	11.3	- 0.1	8.7
	MOTS 40	Y	-5905826.8	6.1	15.4	6.9	15.4	7.5	25.9	10.1	12.8	9.0
		Z	1966367.6	8.9	9.1	8.0	9.1	8.5	44.3	12.9	20.6	9.9
1021	Blossom Point, Md	X	1118061.3	6.5	- 7.3	5.0	- 7.3	5.8	-27.8	9.3	-10.3	7.1
	MOTS 40	Y	-4876472.4	5.1	20.9	4.4	20.9	5.2	13.8	6.1	14.6	6.1
		Z	3942793.7	5.6	0.0	3.9	0.0	4.8	3.1	5.4	4.9	5.4
3402	Semmes, Ala.	X	167291.0	5.2	- 1.3	4.0	- 1.3	4.9	- 5.9	6.4	- 1.1	6.1
	PC-1000	Y	-5482121.9	3.8	20.2	4.1	20.2	5.0	23.5	6.3	15.9	5.7
		Z	3244863.3	4.7	- 1.5	4.5	- 1.5	5.4	13.7	6.5	6.3	6.0

Table 4 (continued)

GOCC #	Name		NAD	σ	NA-1	σ	NA-2	σ	NA-3	σ	NA-4	σ
3401	L. G. Hanscom Field, Mass.	X	1513184.2	7.4	-14.5	5.9	-14.5	6.6	-41.9	11.2	-19.4	8.0
	PC-1000	Y	-4463730.2	6.2	33.1	5.9	33.1	6.6	18.1	8.1	24.1	7.8
		Z	4282975.7	6.3	3.4	4.1	3.4	5.0	0.9	6.0	7.1	5.6
3106	Antigua Island	X	2881872.3	10.6	0.2	10.1	0.2	10.5	-50.0	18.9	- 9.5	12.7
	PC-1000	Y	-5372329.3	8.5	29.4	5.1	29.4	5.8	29.8	7.1	23.8	6.8
		Z	1868346.8	11.1	19.7	7.9	19.7	8.5	57.5	13.1	32.2	9.8
3861	Homestead AFB, Fla.	X	961792.9	7.4	8.6	4.6	8.6	5.5	- 6.9	8.2	7.4	6.5
	PC-1000	Y	-5679312.7	5.1	22.2	3.9	22.2	4.9	30.4	6.4	19.2	5.1
		Z	2729707.6	7.0	3.2	4.1	3.2	5.0	23.9	5.9	12.1	4.3
7040	San Juan, P. R.	X	2465090.5	10.1	- 2.6	8.7	- 2.6	9.2	-45.5	16.4	-10.6	11.1
	MOTS 40	Y	-5535082.5	7.7	15.5	5.1	15.5	5.9	19.0	7.4	11.0	6.8
		Z	1985346.2	10.3	- 4.1	7.1	- 4.1	7.7	31.4	12.1	7.7	8.8
7043	GSFC, Greenbelt, Md.	X	1130742.6	6.5	- 2.4	4.9	- 2.4	5.7	-21.8	9.4	- 3.6	7.0
	PTH-100	Y	-4831487.8	5.2	36.1	4.6	36.1	5.4	26.3	6.5	29.7	6.4
		Z	3993952.9	5.6	2.2	4.2	2.2	5.1	4.9	5.8	11.0	5.4
7045	Denver, Colo.	X	-1240449.5	2.6	12.1	5.2	12.1	6.0	28.9	8.8	15.2	7.5
	MOTS 40	Y	-4760379.7	2.2	1.4	4.2	1.4	5.1	- 7.3	6.2	- 4.9	6.2
		Z	4048804.6	2.4	7.1	4.2	7.1	5.1	7.8	6.0	10.6	5.9
1042	Rosman, N. C.	X	647539.6	5.5	- 9.8	3.8	- 9.8	4.8	-22.7	7.1	-11.6	5.9
	MOTS 40	Y	-5178083.5	4.2	14.7	3.3	14.7	4.4	12.6	5.1	9.3	5.0
		Z	3656534.4	4.9	- 1.6	3.7	- 1.6	4.7	6.1	5.2	4.1	5.2
7072	Jupiter, Fla.	X	976297.2	7.2	- 0.2	4.6	- 0.2	5.4	-18.0	8.5	- 2.7	6.6
	MOTS 40	Y	-5601549.2	5.0	13.1	4.1	13.1	5.1	18.4	6.5	9.0	5.7
		Z	2880071.8	6.7	- 2.1	4.5	- 2.1	5.4	18.2	7.1	6.2	5.8
7036	Edinburg, Tex.	X	- 828463.9	5.3	2.7	3.9	2.7	4.9	12.6	6.6	4.8	6.0
	MOTS 40	Y	-5657604.0	3.8	- 3.3	4.1	- 3.3	5.1	4.2	6.8	- 5.7	5.9
		Z	2816639.7	5.0	4.6	5.0	4.6	5.7	25.6	8.1	12.1	6.6

Table 4 (continued)

GOCC #	Name		NAD	σ	NA-1	σ	NA-2	σ	NA-3	σ	NA-4	σ
1034	E. Grand Fork, Minn.	X	- 521678.9	4.3	2.3	3.0	2.3	4.2	8.4	5.5	4.0	5.3
	MOTS 40	Y	-4242198.1	3.7	1.1	4.2	1.1	5.1	-15.0	6.9	- 5.8	6.2
		Z	4718543.6	3.5	1.3	4.0	1.3	4.9	- 9.0	6.9	2.7	5.6
1030	Mojave, Calif.	X	-2357214.3	5.7	- 0.9	8.2	- 0.9	8.7	34.0	14.5	5.5	10.6
	MOTS 40	Y	-4646475.7	4.8	7.1	4.0	7.2	4.9	- 3.2	6.0	0.6	5.9
		Z	3668134.6	5.2	- 4.5	4.2	- 4.5	5.1	2.5	5.8	0.3	5.8
7037	Columbia, Mo.	X	- 191260.6	3.0	0.0	0.3	0.0	2.9	0.6	3.7	0.7	3.7
	MOTS 40	Y	-4967428.4	2.5	0.0	0.3	0.0	2.9	- 3.3	3.5	- 3.6	3.5
		Z	3983084.5	2.7	0.0	0.3	0.0	2.9	0.8	3.3	1.9	3.3
1022	Ft. Myers, Fla.	X	807883.1	7.3	2.0	3.9	2.0	4.8	-13.3	7.5	0.0	5.9
	MOTS 40	Y	-5652136.5	5.1	10.7	3.6	10.7	4.7	16.5	6.2	6.6	5.1
		Z	2833327.2	6.8	1.1	4.2	1.1	5.1	22.2	7.0	9.6	5.4
5861	Homestead, Fla.	X	963493.6	7.4					- 6.9	8.2	7.4	6.5
	SECOR	Y	-5679877.4	5.1					30.4	6.4	19.2	5.1
		Z	2727946.6	7.0					23.9	5.9	12.1	4.3
5001	Herndon, Va.	X	1088883.5	6.4					-21.0	11.0	- 4.7	9.4
	SECOR	Y	-4843079.7	5.1					10.4	5.7	4.4	5.3
		Z	3991674.3	5.6					- 1.6	7.5	-13.7	6.2
5333	Stoneville, Miss.	X	- 84975.8	4.1					-17.2	8.9	- 1.8	7.1
	SECOR	Y	-5328098.4	3.1					0.0	10.3	- 1.5	10.3
		Z	3493294.2	3.7					8.9	9.0	- 2.9	8.0
5649	Hunter AFB, Ga.	X	832512.0	6.3					-14.9	8.4	0.1	6.6
	SECOR	Y	-5349730.3	4.6					12.7	4.7	5.5	4.
		Z	3360369.8	5.7					15.9	6.1	3.8	4.4

3.2 Short-Arc Orbital Mode Adjustment

In addition to the geometric solutions described in the previous section, one adjustment was performed in the short-arc mode using the program described in [9]. Only the optical tracking stations were involved in this adjustment since no timing information was available for the SECOR observations. The results of this adjustment are given in Table 5.

The orbital arcs used in the short-arc adjustment were limited to about 10^{m}. These arcs are too short to afford a strong determination of the scale of the network through the adopted value of the GM. Therefore, scale was furnished by constraining the spatial chord distance between Homestead, Florida, and Greenbelt, Maryland, as had been done in the geometric adjustments. This distance and its a priori uncertainty were computed again from the geodetic coordinates of these two stations on the Cape Canaveral datum (i.e., the USCGS high-precision traverse).

The geocentric coordinates of Columbia, Missouri were constrained in order to define the origin of the coordinate system. These geocentric coordinates together with their associated covariance matrix were taken from [10]:

$X = -191\,290\,\mathrm{m} \pm 3.8$

$Y = -4967\,274\,\mathrm{m} \pm 4.1$

$Z = 3983\,255\,\mathrm{m} \pm 4.1$

The differences between the NAD coordinates and the short-arc solution coordinates of Columbia were (NAD - short arc):

$dx = 32.1\ \mathrm{m}$

$dy = -158.8\ \mathrm{m}$

$dz = -171.3\ \mathrm{m}$

In order to be able to compare the short-arc solution coordinates to the NAD coordinates, these shifts were added to the short-arc solution. The resulting coordinate differences appear under the heading "Orbital" in Table 5. The uncertainty of these coordinates was obtained by quadratically removing the uncertainties of the geocentric coordinates (3.8 m, 4.1 m, 4.1 m) of Columbia from the standard deviations of the short-arc solution, and quadratically adding the uncertainties of the NAD coordinates of Columbia (3.0 m, 2.5 m, 2.7 m).

The geometric mode adjustment that most nearly resembles the orbital mode adjustment in terms of data used and constraints applied is the one designated NA-2. The short-arc solution and the standard deviation between the two was computed by removing the variance of the coordinates of Columbia from the variances of the two solutions and adding the resulting variances. These also appear in Table 5. From the table it is evident that the agreement between the geometric adjustment and the short-arc adjustment is satisfactory at all stations, except at San Juan, Puerto Rico; St. Johns, Newfoundland; and College, Alaska. The blame should probably be placed on the insufficient amount of data available and/or on the poor geometry.

Table 5

Coordinates of the North American GEOS I Optical Tracking Stations from the Short-Arc Orbital Mode Adjustment

GOCC#	Name		NAD	Orbital	σ	Orbital - NA-2	σ
7075	Sudbury, Ontario	X	692 646.1	5.5	6.1	4.8	7.1
	MOTS 40	Y	-4347 225.8	13.7	6.1	4.1	8.2
		Z	4600 298.3	14.0	5.4	5.4	6.9
1032	St. Johns, Newfoundland	X	2602 802.5	170.4	87.4	252.2	111.1
	MOTS 40	Y	-3419 301.2	621.9	275.1	632.6	291.8
		Z	4697 477.3	-475.6	231.2	431.8	234.2
1033	College, Alaska	X	-2299 238.0	94.1	24.5	87.8	28.6
	MOTS 40	Y	-1445 840.4	820.0	31.5	800.7	79.7
		Z	5751 629.0	-546.3	51.0	-526.2	64.7
3903	Herndon, Va.	X	1689 023.7	- 13.8	13.8	- 6.5	17.7
	PC-1000	Y	-4843 194.9	94.5	16.1	- 19.2	21.2
		Z	3991 564.7	13.8	11.6	16.9	15.0
7039	Bermuda	X	2308 230.2	16.9	11.7	- 2.0	14.1
	MOTS 40	Y	-4873 765.5	31.7	6.1	- 0.2	7.9
		Z	3394 389.0	- 3.4	6.3	5.1	8.0
3405	Grand Turk	X	1919 530.5	- 30.8	10.9	- 24.6	12.9
	PC-1000	Y	-5621 245.2	9.5	8.0	- 18.3	9.7
		Z	2315 617.1	7.4	11.1	- 20.6	13.1
3407	Trinidad	X	2979 925.0	- 50.2	17.1	- 46.9	20.4
	PC-1000	Y	-5513 746.9	100.0	10.5	5.3	12.5
		Z	1180 994.8	3.9	18.4	29.0	21.8

all units are in meters

Table 5 (continued)

GOCC#	Name		NAD	Orbital	σ	Orbital - NA-2	σ
3648	Hunter AFB, Ga.	X	832 594.6	4.9	6.9	2.1	8.0
	PC-1000	Y	-5349 690.7	7.7	6.2	- 9.8	7.6
		Z	3360 414.7	3.2	4.9	7.1	8.5
3404	Swan Island	X	642 541.2	-17.9	8.2	2.1	9.2
	PC-1000	Y	-6054 109.5	45.3	8.8	- 3.5	10.4
		Z	1895 518.2	26.9	11.8	32.0	13.4
3657	Aberdeen, Md.	X	1186 826.8	- 7.4	7.6	- 0.1	8.6
	PC-1000	Y	-4785 340.8	19.1	8.5	1.7	9.8
		Z	4032 705.0	5.8	6.1	1.5	6.8
3406	Curacao	X	2251 837.6	-22.7	11.8	-21.7	14.1
	PC-1000	Y	-5817 069.3	37.0	9.0	12.5	10.6
		Z	1327 016.0	28.1	15.6	16.3	18.4
7076	Jamaica, B.W.I.	X	1384 188.1	-17.1	8.8	-21.1	10.4
	MOTS 40	Y	-5905 826.8	- 1.8	8.3	-17.2	10.6
		Z	1966 367.6	31.7	10.3	22.6	12.8
1021	Blosson Point, Md.	X	1118 061.3	-17.3	7.5	-10.0	8.5
	MOTS 40	Y	-4876 472.4	9.4	5.5	-11.5	6.7
		Z	3942 793.7	6.6	5.0	6.6	5.8
3402	Semmes, Ala.	X	167 291.0	- 8.0	6.5	- 6.7	7.0
	PC-1000	Y	-5482 121.0	20.2	6.2	0.0	7.1
		Z	3244 863.3	-10.0	7.6	8.5	8.5

Table 5 (continued)

GOCC #	Name		NAD	Orbital	σ	Orbital - NA-2	σ
3401	L. G. Hanscom Field, Mass.	X	1513 184.2	-28.7	8.9	-14.3	10.2
	PC-1000	Y	-4463 730.2	5.8	9.5	-27.3	11.0
		Z	4282 875.7	21.3	6.2	17.9	7.0
3016	Antigua Island	X	2881 872.3	-27.1	15.9	-27.3	18.6
	PC-1000	Y	-5372 329.3	26.5	11.4	- 2.9	12.3
		Z	1868 346.8	56.2	14.9	36.5	16.7
3861	Homestead AFB, Fla.	X	961 792.9	12.2	6.9	3.6	7.7
	PC-1000	Y	-5679 312.7	11.9	5.6	-10.3	6.6
		Z	2729 707.6	9.4	6.1	6.2	7.0
7040	San Juan, P.R.	X	2465 090.5	-43.8	12.4	-41.2	14.8
	MOTS 40	Y	-5535 082.5	- 9.7	6.5	-25.2	8.1
		Z	1985 346.2	36.2	10.2	40.3	12.2
7043	GSFC, Greenbelt, Md.	X	1130 742.6	7.8	7.1	10.2	8.0
	PTH-100	Y	-4831 487.8	46.4	6.3	10.3	7.6
		Z	3993 952.9	- 5.9	6.1	8.1	7.0
7045	Denver, Colo.	X	-1240 449.5	- 2.8	6.4	-14.9	7.7
	MOTS 40	Y	-4760 379.7	10.2	5.0	8.8	6.2
		Z	4048 804.6	7.6	5.6	0.5	6.5
1042	Rosman, N. C.	X	647 539.6	- 2.1	6.2	7.7	6.6
	MOTS 40	Y	-5178 083.5	21.6	4.6	6.9	5.3
		Z	3656 534.4	-12.4	5.5	-10.8	6.1

Table 5 (continued)

GOCC #	Name		NAD	Orbital	σ	Orbital - NA-2	σ
7072	Jupiter, Fla.	X	976 297.2	-16.4	6.5	-16.2	7.3
	MOTS 40	Y	-5601 549.2	-11.7	5.0	-24.8	6.2
		Z	2880 071.8	-11.7	6.4	- 9.6	7.5
7036	Edinburg, Tex.	X	- 828 463.9	- 6.4	5.1	- 9.1	5.7
	MOTS 40	Y	-5657 604.0	-11.3	5.7	- 8.0	6.8
		Z	2816 639.7	5.6	7.2	1.0	8.4
1034	E. Grand Forks, Minn.	X	- 521 678.9	- 2.9	4.2	- 5.2	4.2
	MOTS 40	Y	-4242 198.1	11.6	5.4	10.5	6.5
		Z	4718 543.6	7.4	5.1	6.1	6.0
1030	Mohave, Calif.	X	-2357 214.3	-35.6	10.2	-34.7	12.7
	MOTS 40	Y	-4646 475.7	15.1	5.0	8.0	6.1
		Z	3668 134.6	-15.7	5.8	-11.3	6.7
7037	Columbia, Mo.	X	- 191 260.6	0.0	3.0	0.0	0.0
	MOTS 40	Y	-4967 428.4	0.0	2.5	0.0	0.0
		Z	3983 084.5	0.0	2.7	0.0	0.0
1022	Ft. Myers, Fla.	X	807 883.1	4.9	5.8	2.9	6.2
	MOTS 40	Y	-5652 136.5	2.6	4.9	- 8.1	5.8
		Z	2833 327.2	7.4	6.4	6.3	7.3

General Information:	No. of unknown stations	30
	No. of observations	6247
	No. of orbital arcs	86
	Degrees of freedom	5641
	Standard deviation of the observation of unit weight	1.0

The chord distance between Homestead, Fla., and Greenbelt, Md., as obtained from their Cape Canaveral Datum coordinates (USCGS high precision traverse) was constrained to one part in 750,000.

4. IMPLICATIONS REGARDING THE NORTH AMERICAN DATUM

4.1 Coordinate Transformations

The general relationship between a right-handed coordinate system defined by a certain geodetic datum (geodetic system, e.g., NAD) and one which is defined by the origin in the geocenter, the Z axis in the direction of the CIO, and the X axis in the plane of the Greenwich Mean Astronomic Meridian as defined by the Bureau International de l'Heure (average terrestrial system) is as follows [11]:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} dx_0 \\ dy_0 \\ dz_0 \end{vmatrix} + \begin{vmatrix} x_0 \\ y_0 \\ z_0 \end{vmatrix} + M \begin{vmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{vmatrix} + \epsilon \begin{vmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{vmatrix}$$

where

X, Y, Z	are the coordinates of a point in the average terrestrial system
x, y, z	are the coordinates of the same point in the geodetic system
M	is the rotation matrix of three rotations $(\theta_x, \theta_y, \theta_z)$ to rotate the geodetic system parallel to the average terrestrial system
x_0, y_0, z_0	are the geodetic coordinates of a point P which is kept fixed during the rotation
dx_0, dy_0, dz_0	are the coordinates of the origin of the geodetic system in the average terrestrial system, after the former has been made parallel with respect to the latter
ϵ	is the scale factor

In practice three main systems have been proposed:

(1) Bursa [12] and Wolf [13] select the point P at the origin of the geodetic coordinate system (i.e., $x_0 = y_0 = z_0 = 0$) and rotate about the axes x, y, z.

(2) Molodensky [14] selects the point P at the origin of the geodetic datum (e.g., at Meades Ranch on the NAD) and rotates about axes parallel to x, y, z.

(3) Veis [15] selects the point P at the origin of the geodetic datum and rotates about axes pointing to the geodetic zenith (z), to the south (x), and to the east (y) in the geodetic horizon.

In our case, since we are not concerned with translations, (1) and (2) are equivalent, and the transformation parameters are restricted to three rotation angles (either in the Bursa/Molodensky or Veis systems) and to the scale factors (all referred to the origin at Meades Ranch). These are determined from the satellite-determined coordinates (X, Y, Z) and the NAD coordinates (x, y, z) of the tracking stations.

The rotation angles are defined as customary: In the right-handed coordinate systems they are positive for counterclockwise rotation as viewed looking toward the origin from the positive end of the rotation axis. For example, in the Veis system, when θ_x is positive the rotation is from the east to the west in the prime vertical plane; when θ_y is positive the rotation is from the north to the south in the meridian plane; and when θ_z is positive the rotation is from the east to the west in the horizon plane (in azimuth).

4.2 Results

In order to detect systematic differences between the satellite-determined station coordinates from the NA-2 solution and the NAD coordinates, transformation parameters (rotations and the scale) were computed from a least squares adjustment utilizing a developed form of the transformation equation above [16]. The stations were considered independent from each other and only those were used for which it was possible to verify that the NAD coordinates were based on direct ground ties to the first-order triangulation net. Nineteen stations were used in three solutions:

(1) 14 stations in the eastern half of the U. S.

(2) 5 stations in the western half of the U. S.

(3) 19 stations in both the eastern and western halves of the U. S.

The first two solutions were made to detect possible differences between the two halves since they were adjusted separately in the original NAD adjustment. The results are summarized in Table 6. It seems evident that the western data is insufficient to allow meaningful quantitative conclusions. However, it seems evident that significant distortions are present. The eastern parameters, on the other hand, indicate a need for a rotation in the order of one second in azimuth (to the east) and one in the order of two seconds in the prime vertical plane (to the west), together with a possible reduction in

Table 6

Datum Transformation Parameters: NA-2 - NAD

		Eastern Half (14 stations)*	Western Half (5 stations)**	Combination (19 stations)
Veis	θ_z ('')	-1.2 ± 0.4	0.4 ± 0.9	-0.9 ± 0.3
	θ_y ('')	-0.2 ± 0.5	0.2 ± 1.2	-0.2 ± 0.4
	θ_x ('')	2.0 ± 0.6	-1.4 ± 1.2	1.2 ± 0.3
Molodensky	θ_z ('')	-2.4 ± 0.5	1.4 ± 1.1	-1.5 ± 0.3
	θ_y ('')	-0.3 ± 0.5	0.5 ± 1.1	0.0 ± 0.3
	θ_x ('')	-0.3 ± 0.5	0.3 ± 1.2	-0.2 ± 0.4
	ϵ ($\times 10^{-6}$)	-1.3 ± 2.1	0.0 ± 4.7	-1.8 ± 1.6

*Eastern stations: 1021, 1022, 1034, 1042, 3334, 3401, 3402, 3648, 3657, 3861, 3037, 7043, 7072, 7075

**Western stations: 1030, 3400, 3902, 7036, 7045 scale in the order of 1×10^{-6}. Applying these transformation parameters to the NAD would make it conform better to the satellite data. It is likely, however, that these small systematic distortions do not arise from actual systematic errors in the observations but are due rather to errors in the data reduction and adjustment methods utilized in the original NAD adjustment.

Table 7 shows the transformation parameters computed from the NA-3 solution. The resulting rotations are of the same order of magnitude as before, but the scale reduction is about a factor of ten larger. The latter obviously is the influence of the scale enforced through the SECOR ranges which were put in with a standard deviation of 1.7 m, corresponding on the average to approximately 1:1,800,000. (The average range was 3000 km.)

Table 7

Datum Transformation Parameters: NA-3 – NAD

		Eastern Half	Western Half	Combination
Veis	θ_z (")	- 1.1 ± 0.5	0.3 ± 1.1	- 0.8 ± 0.3
	θ_y (")	- 0.1 ± 0.6	0.0 ± 1.5	- 0.1 ± 0.5
	θ_x (")	1.8 ± 0.6	- 1.6 ± 1.5	1.0 ± 0.3
Molodensky	θ_z (")	- 2.1 ± 0.6	1.4 ± 1.3	- 1.3 ± 0.4
	θ_y (")	- 0.3 ± 0.5	0.8 ± 1.3	0.0 ± 0.3
	θ_x (")	- 0.2 ± 0.6	0.1 ± 1.4	- 0.1 ± 0.5
	ϵ ($\times 10^{-6}$)	-17.1 ± 2.5	-17.0 ± 5.9	-17.4 ± 2.0

Table 8 shows the transformation parameters from the NA-4 solution. The rotations are again of the same magnitude. The overriding effect of the chord constraint over SECOR is evident from the $\sim 1:4$ ratio of the ϵ's taken from the NA-3 and NA-4 solutions.

Table 8

Datum Transformation Parameters: NA-4 - NAD

		Eastern Half	Western Half	Combination
Veis	θ_z (")	-1.2 ± 0.5	0.3 ± 1.0	-0.9 ± 0.3
	θ_y (")	-0.2 ± 0.5	0.1 ± 1.4	-0.1 ± 0.5
	θ_x (")	1.5 ± 0.6	-1.4 ± 1.4	1.0 ± 0.3
Molodensky	θ_z (")	-1.9 ± 0.6	1.3 ± 1.3	-1.3 ± 0.3
	θ_y (")	0.0 ± 0.5	0.7 ± 1.3	0.1 ± 0.3
	θ_x (")	-0.2 ± 0.5	0.2 ± 1.4	-0.1 ± 0.5
	ϵ ($\times 10^{-6}$)	-4.1 ± 2.3	-2.9 ± 5.3	-4.4 ± 1.7

5. CONCLUSIONS

Adopting the NA-4 geometric solutions as a standard (since it is based on most of the available data), it seems evident that at least the eastern half of the North American triangulation system contains inherent systematic distortions expressed by the rotations at Meades Ranch

θ_x = 1.″5 ±0.5 (in the prime vertical plane to the west)

θ_z = 1.″2 ±0.6 (in azimuth to the east)

and by the scale factor

$\epsilon = 4.1 \times 10^{-6} \pm 2.3$ (reduction)

No quantitative conclusion should be drawn on the western half since the available data (the number of stations) seems insufficient.

The same parameters corresponding to the whole NAD are

θ_x = 1.″0 ± 0.3 (rotation in the prime vertical to the west)

θ_z = 0.″9 ± 0.3 (rotation in azimuth to the east)

$\epsilon = 4.4 \times 10^{-6} \pm 1.7$ (reduction)

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