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QUARTERLY PROGRESS REPORT
CONVOLUTIONAL CODING TECHNIQUES FOR DATA PROTECTION

REVISED

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1. Research on Inverse Systems

Previous work performed under this grant has demonstrated the fundamental connection between properties of convolutional codes and inverses of linear sequential circuits. A by-product of this research was the development of general inverses for linear sequential circuits (LSC's) and continuous dynamical systems (CDS's). This work is reported in the following journal article, preprints of which have been furnished to NASA but reprints of which are not yet available:

M. K. Sain and J. L. Massey, "Invertibility of Linear Time-Invariant Dynamical Systems," IEEE Trans. on Auto. Control, AC-14, April 1969, pp. 141-149.

This paper shows how to construct an inverse, where one exists, for an m-input LSC (or CDS) such that each output of the inverse when cascaded with the original system is one of the inputs to the original system delayed by L time units (integrated L times).

During the period reported here, the investigators have shown that the techniques in the above paper can be extended to apply to the case when each of the inputs is recovered with individually the least delay (the least integration) rather than with uniform delay (integration). This work will be published as:

M. K. Sain and J. L. Massey, "A Modified Inverse for Linear Dynamical Systems," to be presented at IEEE 8th Adaptive Processes Symposium, Penn. State Univ., Nov. 17-19, 1969.

Preprints of this article will be furnished to NASA in the near future.

2. Simulation of the Jelinek Sequential Decoding Algorithm

Our last Quarterly Progress Report reported some preliminary results based on the computer program being developed for the UNIVAC 1107 computer to simulate the new sequential decoding algorithm proposed by F. Jelinek in the following report:

F. Jelinek, "A Stack Algorithm for Faster Sequential Decoding of Transmitted Information," Tech. Rpt. , IBM T. J. Watson Research Center, Yorktown Heights, N. Y., 1969.

Our preliminary work had led us to pessimistic conclusions regarding the decoding speed for this algorithm compared to the Fano algorithm when both are simulated on a general purpose digital computer such as the UNIVAC 1107. However, further programming simplifications together with a valuable suggestion which the investigators received from F. Jelinek have caused us to change this conclusion considerably. A new version of our Jelinek simulation program has just been completed which we consider to be about the best possible that can be done on the UNIVAC 1107 computer. Comparison to the results of our Fano algorithm simulation program which we believe to represent comparable programming sophistication indicates that the Jelinek algorithm can be implemented on a general purpose computer with a considerable gain in decoding speed when the code rate is approximately R_{comp}. In general, whenever the number of Fano computations exceeds about 2 or 3 times the number of decoding decisions on information bits, our results indicate that the Jelinek algorithm will decode the frame faster with greater savings as the computation is further increased. Production runs with the new algorithm are still in progress. As soon as these are completed within a few weeks, a detailed report will be made on this simulation. However, we would already encourage the idea that it may be very economical in computer time to convert from Fano sequential decoding to Jelinek sequential decoding.

3. Comparison of Various R = 1/2 Convolutional Codes for Sequential Decoding

During the period reported here, Mr. D. J. Costello who has been a research assistant under this grant completed the requirements for his Ph. D. degree in electrical engineering. His thesis will be published as the following technical report under the grant which is now in preparation:

D. J. Costello, Jr., "Construction of Convolutional Codes for Sequential Decoding," U. of Notre Dame, Dept. of Elec. Engr. Tech. Rpt. EE-692, August 1969.

This report contains numerous new techniques for constructing good convolutional codes as well as several new upper and lower bounds on various distance measures defined for convolutional codes. The report will also include the detailed simulation results for 13 different R = 1/2 codes of memory 35 (i.e. a constraint length of 1 + 35 = 36 branches or 2 x 36 = 72 bits.) The remainder of this report contains an extract from that section of Costello's report dealing with this simulation. This data is reported here since it may be of use to various personnel within NASA independently of the rest of the Costello report.

1. Brief Description of the Simulated Sequential Decoder.

In order to test the codes constructed in this chapter along with other known good codes, a sequential decoder was simulated on the Univac 1107 at the University Computer Center. Two simulations were made, one for a BSC and one for a Gaussian channel. Each program consists of four parts: a main program DECODE for reading in data and printing out results, a subprogram RANGEN for generating random noise, a subprogram TABSET for converting the random noise into tabular form suitable for the sequential decoder, and a subprogram SECO for the sequential decoding algorithm. Special thanks are due to Dr. K. Vairavan, who programmed both the RANGEN and TABSET subprograms, to Mr. John Geist and Mr. James Wruck, for their numerous contributions to the efficiency of the programs, and to Mr. J. Chang and Mr. John Brennan for the preparation of the Gaussian program.

Each subprogram was written in assembly language to make the program as fast as possible, while the main program was written in FORTRAN to facilitate the input-output. Input information needed for the operation of the BSC program is as follows (for a complete discussion of sequential decoding parameters, see Gallager [25]):

(1) channel error probability p;

(2) the memory m of the code;

(3) the generator of the code being tested;

(4) the threshold increment H of the sequential decoding search; H = CONMET x Δ where Δ is the true threshold increment of the Fano algorithm;

(5) a constant CONMET used to spread the difference between the metric values; CONMET = 8 was used in all the simulations reported here.

(6) bins for the number of computations.

R_{comp} and the metric values are then computed from p and CONMET. The threshold increment H used in the production runs was determined experimentally. The value of Δ which optimizes the bound on computation is known to be 2 [3]. Since CONMET was chosen as 8, the "optimum" H is 16. However, through testing a single code for different values of H, it was determined that choosing H to be 32 was a better choice from both a computational and probability of error standpoint. These results are shown in table 6.12.

Each production run consisted of 1000 frames of 256 branches(blocks of information digits) each for a particular code and a particular channel error probability p. A frame was cut off and considered to be "erased" if it reached 50,000 computations. If a frame was decoded perfectly, it took (256 + m) computations since 256 information blocks generate (256 + m) transmitted blocks and the algorithm would count one computation for each correctly decoded block. Hence the computational bins are just numbers inclusive between (256 + m) and 50,000 which record how many frames reached or exceeded that number of computations for decoding. Usually 13 computational bins were chosen for each production run.

In the Gaussian program the signal-to-noise ratio $\frac{E_b}{N_o}$ must be read in instead of p, where E_b is the energy per information digit and N_o is the noise power spectral density. Then the procedure outlined in Jacobs [33] is followed to compute the metric values needed by the sequential decoder.

Output information available from the BSC program includes the following:

(1) the actual branch metric values and R_{comp};

(2) for each decoded frame:

  (a) the number of computations;

  (b) the number of decoding errors;

  (c) the last branch decoded if the frame is erased;

  (d) the received sequence;

  (e) the decoded sequence;

(3) for the entire 1000 decoded frames:

  (a) the number of erased frames;

  (b) the number of incorrectly decoded frames;

  (c) the number of correctly decoded frames;

  (d) the distribution of computation into bins.

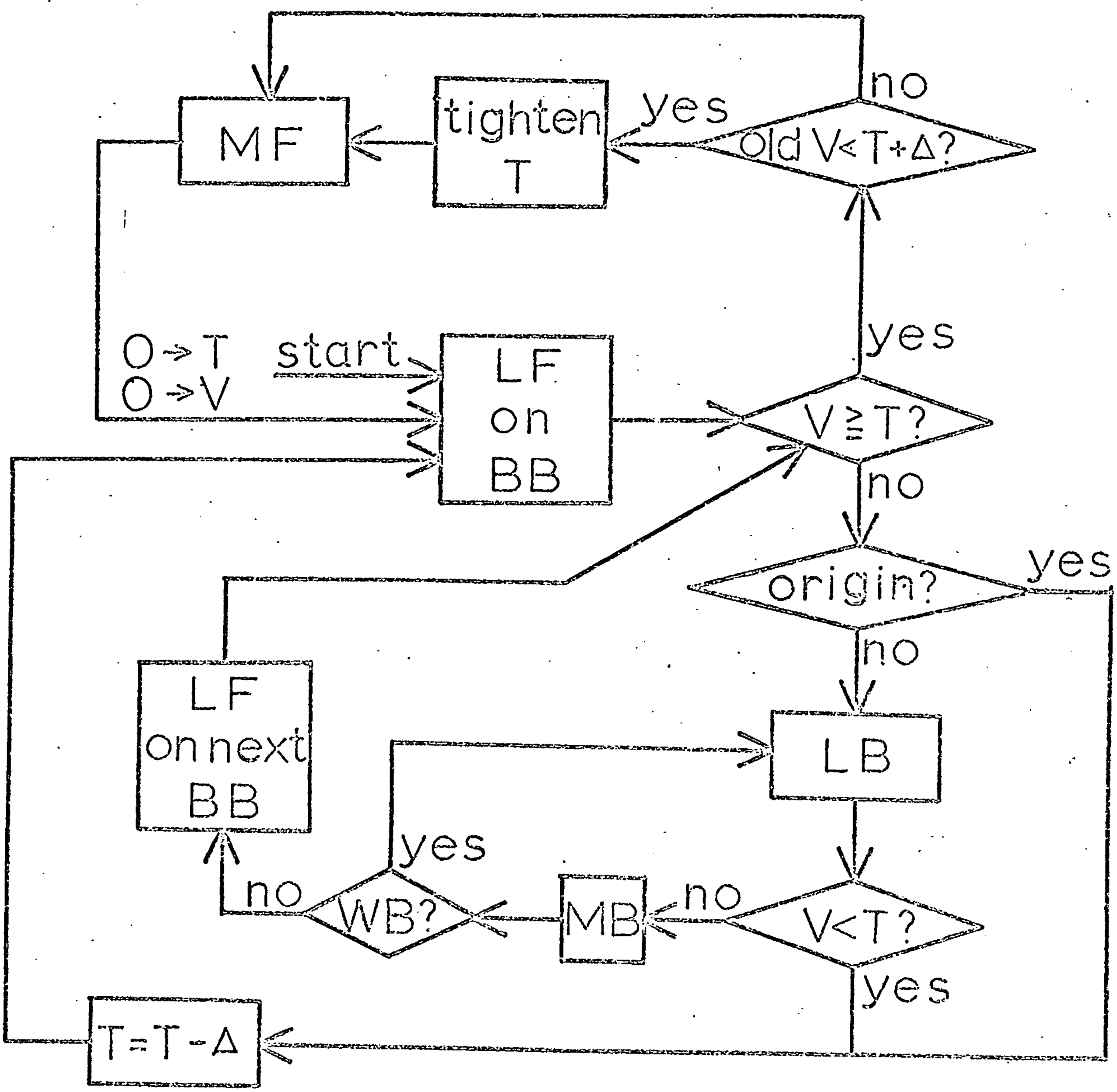
Clearly the total number of error digits can be easily calculated from (2b). When the number of computations reached 50,000, decoding was terminated and the frame declared "erased". The output then recorded how far the search had progressed in the code tree when decoding was terminated. The printout of the received sequence and the decoded sequence for each frame is optional in the program.

For each computational bin, the number of frames which reached or exceeded that amount of computation is recorded. For example, the bin labeled 50,000 always contains the number of "erased" frames, and the bin labeled (256 + m) always contains the total number of frames.

In the Gaussian program, additional output information about the channel is available.

In the RANGEN subprogram, a library subroutine is used to generate a noise sequence distributed according to the channel error probability p for the BSC program. In the Gaussian program, the noise sequence is distributed according to the quantized channel model given by Jacobs [33]. TABSET merely converts the noise sequence into tabular data for use by SECO.

SECO is the actual sequential decoding algorithm. The version used is thoroughly discussed by Gallager [25]. A flow chart for SECO is shown in figure 6.7. It is always assumed that the all-zero sequence has been transmitted. Since this was known to the programmer, SECO was always biased to look out on a 1 branch before looking out on a 0 branch in case the metric values on the two branches were tied. (Here the discussion pertains only to $R = \frac{1}{N}$ codes, in which there are only two branches emanating from each node.) This undoubtedly resulted in slightly more computation than would be required normally, but of course this deficiency was common to all runs and would be expected to have no effect on the comparison between different codes.



LF = look forward BB = best branch V = node value

T = threshold Δ = threshold increment

MF = move forward LB = look back MB = move back

WB = worst branch

Fig. 6.7. SECO flow chart.

A computation was counted as a "forward look", i.e., every time the decoder looked forward on a branch, and at no other time, a single computation was counted. Each computation, including the calculation of the parity digits, took about 100 μsec of computer time.

The SECO algorithm is capable of handling both systematic and non-systematic codes with $m \leqq 72$. Programs actually available are for $R = \frac{1}{2}$, $R = \frac{1}{3}$, and $R = \frac{1}{4}$ only. However, only results on $R = \frac{1}{2}$ codes will be reported here, since they are sufficiently representative of all rates. Also, data was taken for only three values of p and one value of $\frac{E_b}{N_o}$. These values are very typical, though, of a practical randomly distributed space channel. For $p = .033$, i.e., $R = \frac{1}{2} = (0.9)\ R_{comp}$, each production run of 1000 frames took about 2 minutes of computer time. For $p = .045$, i.e., $R = \frac{1}{2} = R_{comp}$, each run took about 4 minutes. For $p = .057$, i.e., $R = \frac{1}{2} \approx (1.1)\ R_{comp}$, each run took about 20 minutes. And for $\frac{E_b}{N_o} = 2$ or 3db, each run took about 5 minutes.

2. Comparative Analysis of Codes.

In appendix A charts are given which have complete information on 13 different codes. A name and number is assigned to each code for identification purposes, and the means of construction for each code is briefly explained. Simulation results are given for the four channels described above. Not all the codes were tested with $p = .057$, since the computation time was so long.

An interesting comparison can be drawn between code 1 (from algorithm A1) and code 3 (from algorithm A6). Note that there are fewer error frames for code 3. This appears to be due to the fact that d_{FREE} is larger for code 3, since d_{FD} is the same for both codes, and substantiates the previous statement that d_{FREE} is a more important parameter than d_{FD} for sequential decoding.

Also compare code 11 (the non-systematic code from algorithm A9) with code 12 (from Forney [28]). The non-systematic code is clearly superior in number of error frames, although it has more erased frames. For the noisiest BSC, p = .057, code 11 makes no decoding errors while code 12 incorrectly decodes about 10% of the frames. However code 11 erases about 15% more frames than does code 12. But of these frames it appears that about half of them were incorrectly decoded by code 12. Massey [32] has termed this a "fools rush in where angels fear to tread" phenomenon. The slight computational advantage of code 12 over code 11 is clearly due to this phenomenon. Since code 11 is more easily implemented than code 12 and it has the "quick look" property, we can conclude that it is far superior to code 12 in system performance as well as system complexity. In fact, code 11 did not make a single decoding error in all four simulations. To the author's knowledge, code 12 is generally considered the best m = 35 systematic code available for sequential decoding. The performance of code 11 verifies the earlier statement that better results can be obtained for non-systematic codes than for systematic codes when used with sequential decoding (since more free distance is available for non-systematic codes).

Finally, compare the performance of code 2 with code 1. This indicates the advantage of using longer codes. However, encoder complexity increases with code length, which is an important consideration in many applications.

EFFECT OF VARYING THE THRESHOLD INCREMENT H

Code No. 1 **Code Name** Minimum Weight Code

Memory = 35 **Rate** = $\frac{1}{2}$ **Type** Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
40000 0 000 000	
65 11.02 104421	

Known Distance Properties: $d_{min} = 13$ $d_{free} = 13$

Nature of Construction: Algorithm A1

Simulation Results:

(1) Channel BSC: p = .033 H 4 Total Frames 1000 Error Frames 10 Erased Frames 0
Computation: Total Error Bits: 34

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	1000	1000	1000	999	986	607	154	42	15	4	0		

(2) Channel BSC: p = .033 H 8 Total Frames 1000 Error Frames 10 Erased Frames 0
Computation: Total Error Bits: 34

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	1000	1000	991	957	756	226	55	16	5	0	0		

(3) Channel BSC: p . 033 H 16 Total Frames 1000 Error Frames 11 Erased Frames 0
Computation: Total Error Bits: 37

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	1000	989	869	665	369	85	23	10	0	0	0		

(4) Channel BSC: p = .033 H 32 Total Frames 1000 Error Frames 16 Erased Frames 0
Computation: Total Error Bits: 45

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	992	880	585	397	206	53	16	4	1	0	0		

(5) Channel BSC: p = .033 H 64 Total Frames 1000 Error Frames 26 Erased Frames 0
Computation: Total Error Bits: 62

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	993	886	642	502	315	85	29	7	3	1	0		

(6) Channel BSC: p = .033 H 128 Total Frames 1000 Error Frames 177 Erased Frames 0
Computation: Total Error Bits: 488

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	1000	997	982	954	891	565	227	86	22	9	1		

Code No. 1 **Code Name** Minimum Weight Code

Memory = 35 **Rate** = $\frac{1}{2}$ **Type** Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400 000 000 000 651 102 104 421	

Known Distance Properties:

$d_{min} = 13$ $d_{free} = 13$

Nature of Construction: Algorithm A1

Simulation Results:

(1) Channel Gauss: $E_b/N_o = 2.0$ H 32 Total Frames 1000 Error Frames 16 Erased Frames 3
Computation: Total Error Bits: 91

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	969	906	822	681	552	426	352	275	203	62	16	7		

(2) Channel BSC: p = .033 H 32 Total Frames 1000 Error Frames 16 Erased Frames 0
Computation: Total Error Bits: 45

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	1000	992	880	585	397	206	53	16	4	1	0	0		

(3) Channel BSC: p = .045 H 32 Total Frames 1000 Error Frames 73 Erased Frames 1
Computation: Total Error Bits: 355

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	1000	999	991	900	769	551	261	110	40	17	6	1		

(4) Channel BSC: p = .057 H 32 Total Frames 1000 Error Frames 428 Erased Frames 9
Computation: Total Error Bits: 6406

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	1000	938	846	775	719	587	508	450	287	156	56	9		

(5) Channel ______ H ___ Total Frames ___ Error Frames ___ Erased Frames ___
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel ______ H ___ Total Frames ___ Error Frames ___ Erased Frames ___
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 2 Code Name Minimum Weight Code

Memory = 71 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000000000000000 651102104421022041101101	

Known Distance Properties:

$d_{min} = 21$ $d_{free} = 21$

Nature of Construction: Algorithm A1

Simulation Results:

(1) Channel Computation: Gauss: $E_b/N_o = 2.0$ H 32 Total Frames 1000 Error Frames 9 Erased Frames 4 Total Error Bits: 26

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	995	970	904	727	607	455	372	282	212	57	13	5		

(2) Channel Computation: BSC: p = .033 H 32 Total Frames 1000 Error Frames 11 Erased Frames 0 Total Error Bits: 16

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	978	507	249	144	81	37	23	16	5	1	0	0		

(3) Channel Computation: BSC: p = .045 H 32 Total Frames 1000 Error Frames 62 Erased Frames 1 Total Error Bits: 320

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	1000	832	589	458	363	217	146	103	29	11	5	1		

(4) Channel Computation: H Total Frames Error Frames Erased Frames Total Error Bits:

N															
#Frames with #C ≧ N															

(5) Channel Computation: H Total Frames Error Frames Erased Frames Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel Computation: H Total Frames Error Frames Erased Frames Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 3 Code Name Minimum Free Weight Code

Memory = 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000 732460703401	

Known Distance Properties:

$d_{min} = 13$ $d_{free} = 17$

Nature of Construction: Algorithm A6

Simulation Results:

(1) Channel Gauss: $E_b/N_0 = 2.0$ H 32 Total Frames 1000 Error Frames 5 Erased Frames 1
Computation: Total Error Bits: 27

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	973	900	800	651	521	407	336	264	210	60	20	8		

(2) Channel BSC: p=.033 H 32 Total Frames 1000 Error Frames 1 Erased Frames 0
Computation: Total Error Bits: 5

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	873	379	183	121	84	33	21	14	8	2	0	0		

(3) Channel BSC: p=.045 H 32 Total Frames 1000 Error Frames 17 Erased Frames 3
Computation: Total Error Bits: 174

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	992	746	522	400	327	196	147	107	45	23	13	3		

(4) Channel BSC: p=.057 H 32 Total Frames 1000 Error Frames 234 Erased Frames 33
Computation: Total Error Bits: 5226

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	1000	927	823	746	683	547	481	437	309	202	107	33		

(5) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 4 Code Name Minimum Free Weight Adjoint

Memory = 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000 653110162117	

Known Distance Properties:

$d_{min} = 13$ $d_{free} = 18$

Nature of Construction: Adjoint of code no. 3

Simulation Results:

(1) Channel Gauss : $E_b/N_o = 2.0$ H 32 Total Frames 1000 Error Frames 3 Erased Frames 4
Computation: Total Error Bits: 25

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	972	904	815	668	537	407	328	263	194	66	22	10		

(2) Channel BSC : p = .033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	880	378	190	116	83	36	18	13	7	2	0	0		

(3) Channel BSC : p = .045 H 32 Total Frames 1000 Error Frames 6 Erased Frames 4
Computation: Total Error Bits: 65

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	991	744	515	410	337	207	141	116	58	23	14	4		

(4) Channel ______ H ___ Total Frames ___ Error Frames ___ Erased Frames ___
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(5) Channel ______ H ___ Total Frames ___ Error Frames ___ Erased Frames ___
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel ______ H ___ Total Frames ___ Error Frames ___ Erased Frames ___
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 5 Code Name Maximum Weight Code

Memory = 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400 000 000 000 736 677 773 575	

Known Distance Properties:

$d_{min} = 13$ $d_{free} = 16$

Nature of Construction: Algorithm A7

Simulation Results:

(1) Channel Gauss: $E_b/N_o = 2.0$ H 32 Total Frames 1000 Error Frames 3 Erased Frames 2
Computation: Total Error Bits: 10

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	974	902	809	652	539	414	344	278	202	65	26	8		

(2) Channel BSC: p=.033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	878	372	195	121	89	36	19	17	9	3	0	0		

(3) Channel BSC: p=.045 H 32 Total Frames 1000 Error Frames 8 Erased Frames 5
Computation: Total Error Bits: 40

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	992	763	534	415	327	205	140	110	56	25	15	5		

(4) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(5) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 6 Code Name Maximum Weight Adjoint

Memory = 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000 656374423651	

Known Distance Properties:

$d_{min} = 13$ $18 \leq d_{free} \leq 22$

Nature of Construction: Adjoint of code no. 5

Simulation Results:

(1) Channel Gauss: $E_b/N_o = 2.0$ H 32 Total Frames 1000 Error Frames 0 Erased Frames 7
Computation: Total Error Bits: 0

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≥ N	1000	971	902	818	662	541	421	335	267	207	65	17	11		

(2) Channel BSC: p = .033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	880	385	193	122	86	34	21	17	8	2	0	0		

(3) Channel BSC: p = .045 H 32 Total Frames 1000 Error Frames 7 Erased Frames 4
Computation: Total Error Bits: 72

N	292	[illegible]	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	491	757	534	427	354	210	151	114	53	28	12	4		

(4) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≥ N															

(5) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≥ N															

(6) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≥ N															

Code No. 7 Code Name Balanced Code

Memory = 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000 653125446515	

Known Distance Properties:

$d_{min} = 13$ $16 \leq d_{free} \leq 20$

Nature of Construction: Algorithm A8

Simulation Results:

(1) Channel Gauss: $E_b/N_0 = 2.0$ H 32 Total Frames 1000 Error Frames 3 Erased Frames 5
Computation: Total Error Bits: 33

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	971	908	814	665	537	410	339	263	193	68	18	10		

(2) Channel BSC: p=.033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	880	378	193	126	85	33	21	15	10	3	0	0		

(3) Channel BSC: p=.045 H 32 Total Frames 1000 Error Frames 12 Erased Frames 2
Computation: Total Error Bits: 103

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	991	743	516	417	344	206	151	116	58	25	14	2		

(4) Channel ____ H ____ Total Frames ____ Error Frames ____ Erased Frames ____
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(5) Channel ____ H ____ Total Frames ____ Error Frames ____ Erased Frames ____
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel ____ H ____ Total Frames ____ Error Frames ____ Erased Frames ____
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 8 Code Name Balanced Adjoint

Memory = 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000 732453703632	

Known Distance Properties:

$d_{min} = 13$ $18 \leq d_{free} \leq 22$

Nature of Construction: Adjoint of code no. 7

Simulation Results:

(1) Channel Gauss : $E_b/N_0 = 2.0$ H 32 Total Frames 1000 Error Frames 0 Erased Frames 3
Computation: Total Error Bits: 0

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	973	900	800	649	536	412	324	266	211	59	21	8		

(2) Channel BSC : p = .033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	873	376	191	114	87	37	20	15	7	2	0	0		

(3) Channel BSC : p = .045 H 32 Total Frames 1000 Error Frames 5 Erased Frames 6
Computation: Total Error Bits: 64

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	992	746	523	410	334	203	144	116	54	30	15	6		

(4) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(5) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 9 Code Name Balanced Bussgang 2 Code

Memory = 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000 732443151623	

Known Distance Properties:

$d_{min} = 13$ $18 \leq d_{free} \leq 20$

Nature of Construction: The code obtained by using Algorithm A8 to extend one of Bussgang's optimal codes.

Simulation Results:

(1) Channel Computation: Gauss: $E_b/N_0 = 2.0$ H 32 Total Frames 1000 Error Frames 1 Erased Frames 3 Total Error Bits: 7

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	973	899	799	648	521	395	326	262	199	57	19	10		

(2) Channel Computation: BSC: p = .033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0 Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	873	371	181	114	78	30	17	14	7	2	0	0		

(3) Channel Computation: BSC: p = .045 H 32 Total Frames 1000 Error Frames 4 Erased Frames 3 Total Error Bits: 32

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	992	739	512	391	307	192	133	105	43	25	13	3		

(4) Channel Computation: H Total Frames Error Frames Erased Frames Total Error Bits:

N															
#Frames with #C ≧ N															

(5) Channel Computation: H Total Frames Error Frames Erased Frames Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel Computation: H Total Frames Error Frames Erased Frames Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 10   Code Name Balanced Bussgang 2 Adjoint

Memory = 35   Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000 653137244673	

Known Distance Properties:

$d_{min} = 13$ $17 \leq d_{free} \leq 23$

Nature of Construction: Adjoint of code no. 9.

Simulation Results:

(1) Channel Gauss: $E_b/N_0 = 2.0$ H 32 Total Frames 1000 Error Frames 1 Erased Frames 4
Computation: Total Error Bits: 19

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	971	906	814	662	550	408	334	256	185	69	22	14		

(2) Channel BSC: p = .033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	880	370	189	118	78	32	21	14	9	2	0	0		

(3) Channel BSC: p = .045 H 32 Total Frames 1000 Error Frames 3 Erased Frames 7
Computation: Total Error Bits: 22

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	991	740	506	402	317	187	138	117	46	24	12	7		

(4) Channel ____ H ____ Total Frames ____ Error Frames ____ Erased Frames ____
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(5) Channel ____ H ____ Total Frames ____ Error Frames ____ Erased Frames ____
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel ____ H ____ Total Frames ____ Error Frames ____ Erased Frames ____
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 11 Code Name Non-systematic Code

Memory = 35 Rate = $\frac{1}{2}$ Type Non-systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
733 533 676737 533 533 676737	

Known Distance Properties:

$d_{min} = 11$ $17 \leq d_{free}$

Nature of Construction: Algorithm A9

Simulation Results:

(1) Channel Gauss: $E_b/N_o = 2.0$ H 32 Total Frames 1000 Error Frames 0 Erased Frames 5
Computation: Total Error Bits: 0

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	968	909	835	676	567	445	358	292	225	70	17	9		

(2) Channel BSC: p = .033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	883	405	223	135	92	47	26	18	5	2	0	0		

(3) Channel BSC: p = .045 H 32 Total Frames 1000 Error Frames 0 Erased Frames 8
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	991	785	581	477	382	240	167	134	63	36	23	8		

(4) Channel BSC: p = .057 H 32 Total Frames 1000 Error Frames 0 Erased Frames 249
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	1000	949	863	802	753	640	585	543	440	358	303	249		

(5) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 12 Code Name NASA Code

Memory ≈ 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400 000 000 000 715 473 701 317	

Known Distance Properties:

$d_{min} = 14$ $d_{free} = 18$

Nature of Construction: The adjoint of the code Forney obtained by using the Lin-Lyne algorithm to extend one of Bussgang's optimal codes.

Simulation Results:

(1) Channel Gauss: $E_b/N_0 = 2.0$ H 32 Total Frames 1000 Error Frames 0 Erased Frames 4
Computation: Total Error Bits: 0

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≧ N	1000	969	900	810	652	523	404	327	254	188	60	19	9		

(2) Channel BSC: p = .033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	884	337	189	111	79	26	18	16	8	2	2	0		

(3) Channel BSC: p = .045 H 32 Total Frames 1000 Error Frames 2 Erased Frames 4
Computation: Total Error Bits: 12

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	991	756	510	403	320	187	138	104	48	31	11	4		

(4) Channel BSC: p = .057 H 32 Total Frames 1000 Error Frames 87 Erased Frames 108
Computation: Total Error Bits: 2071

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≧ N	1000	1000	932	817	734	673	532	455	412	319	237	181	108		

(5) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

(6) Channel H Total Frames Error Frames Erased Frames
Computation: Total Error Bits:

N															
#Frames with #C ≧ N															

Code No. 13 Code Name Lin-Lyne Code

Memory = 35 Rate = $\frac{1}{2}$ Type Systematic

Generators:

Normal Form (Octal)	Read-in Form (Octal)
400000000000 653134307713	

Known Distance Properties:

$d_{min} = 14$ $19 \leq d_{free} \leq 22$

Nature of Construction: Forney's extension of the Lin-Lyne algorithm.

Simulation Results:

(1) Channel Gauss: $E_b/N_o = 2.0$ H 32 Total Frames 1000 Error Frames 0 Erased Frames 5
Computation: Total Error Bits: 0

N	292	400	450	500	600	700	850	1000	1200	1500	4000	10k	25k		
#Frames with #C ≥ N	1000	971	907	815	665	538	403	334	262	189	65	20	11		

(2) Channel BSC: p=.033 H 32 Total Frames 1000 Error Frames 0 Erased Frames 0
Computation: Total Error Bits: 0

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	992	880	565	370	185	44	14	7	2	0	0		

(3) Channel BSC: p=.045 H 32 Total Frames 1000 Error Frames 3 Erased Frames 3
Computation: Total Error Bits: 31

N	292	310	350	400	475	550	700	1250	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	999	991	895	739	506	232	111	50	22	9	3		

(4) Channel BSC: p=.057 H 32 Total Frames 1000 Error Frames 80 Erased Frames 118
Computation: Total Error Bits: 1824

N	292	400	550	700	850	1000	1500	2000	2500	5000	10k	20k	50k		
#Frames with #C ≥ N	1000	1000	927	809	738	666	542	477	432	330	272	206	118		

(5) Channel ____ H ____ Total Frames ____ Error Frames ____ Erased Frames ____
Computation: Total Error Bits:

N															
#Frames with #C ≥ N															

(6) Channel ____ H ____ Total Frames ____ Error Frames ____ Erased Frames ____
Computation: Total Error Bits:

N															
#Frames with #C ≥ N															