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QUARTERLY PROGRESS REPORT CONVOLUTIONAL CODING TECHNIQUES FOR DATA PROTECTION

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Submitted to:

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Research Period Reported: May 16, 1969 to August 15, 1969

1. Research on Inverse Systems

Previous work performed under this grant has demonstrated the fundamental connection between properties of convolutional codes and inverses of linear sequential circuits. A by-product of this research was the development of general inverses for linear sequential circuits (LSC's) and continuous dynamical systems (CDS's). This work is reported in the following journal article, preprints of which have been furnished to NASA but reprints of which are not yet available:

M. K. Sain and J. L. Massey, "Invertibility of Linear Time-Invariant Dynamical Systems," IEEE Trans. on Auto. Control, AC-14, April 1969, pp. 141-149.

This paper shows how to construct an inverse, where one exists, for an m-input LSC (or CDS) such that each output of the inverse when cascaded with the original system is one of the inputs to the original system delayed by L time units (integrated L times).

During the period reported here, the investigators have shown that the techniques in the above paper can be extended to apply to the case when each of the inputs is recovered with individually the least delay (the least integration) rather than with uniform delay (integration). This work will be published as:

M. K. Sain and J. L. Massey, "A Modified Inverse for Linear Dynamical Systems," to be presented at IEEE 8th Adaptive Processes Symposium, Penn. State Univ., Nov. 17-19, 1969.

Preprints of this article will be furnished to NASA in the near future.

2. Simulation of the Jelinek Sequential Decoding Algorithm

Our last Quarterly Progress Report reported some preliminary results

based on the computer program being developed for the UNIVAC 1107 computer to simulate the new sequential decoding algorithm proposed by F. Jelinek in the following report:

F. Jelinek, "A Stack Algorithm for Faster Sequential Decoding of Transmitted Information," Tech. Rpt., IBM T. J. Watson Research Center, Yorktown Heights, N. Y., 1969.

Our preliminary work had led us to pessimistic conclusions regarding the decoding speed for this algorithm compared to the Fano algorithm when both are simulated on a general purpose digital computer such as the UNIVAC 1107, However, further programming simplifications together with a valuable suggestion which the investigators received from F. Jelinek have caused us to change this conclusion considerably. A new version of our Jelinek simulation program has just been completed which we consider to be about the best possible that can be done on the UNIVAC 1107 computer. Comparison to the results of our Fano algorithm simulation program which we believe to represent comparable programming sophistication indicates that the Jelinek algorithm can be implemented on a general purpose computer with a considerable gain in decoding speed when the code rate is approximately R ______ In general, whenever the number of Fano computations exceeds about 2 or 3 times the number of decoding decisions on information bits, our results indicate that the Jelinek algorithm will decode the frame faster with greater savings as the computation is further increased. Production runs with the new algorithm are still in progress. As soon as these are completed within a few weeks, a detailed report will be made on this simulation. However, we would already encourage the idea that it may be very economical in computer time to convert from Fano sequential decoding to Jelinek sequential decoding.

3. Comparison of Various R = 1/2 Convolutional Codes for Sequential Decoding During the period reported here, Mr. D. J. Costello who has been a research assistant under this grant completed the requirements for his Ph. D. degree in electrical engineering. His thesis will be published as the following technical report under the grant which is now in preparation:

D. J. Costello, Jr., "Construction of Convolutional Codes for Sequential Decoding," U. of Notre Dame, Dept. of Elec. Engr. Tech. Rpt. EE-692, August 1969.

This report contains numerous new techniques for constructing good convolutional codes as well as several new upper and lower bounds on various distance measures defined for convolutional codes. The report will also include the detailed simulation results for 13 different R = 1/2codes of memory 35 (i.e. a constraint length of 1 + 35 = 36 branches or $2 \times 36 = 72$ bits.) The remainder of this report contains an extract from that section of Costello's report dealing with this simulation. This data is reported here since it may be of use to various personnel within NASA independently of the rest of the Costello report.

Brief Description of the Simulated Sequential Decoder. 1. In order to test the codes constructed in this chapter along with other known good codes, a sequential decoder was simulated on the Univac 1107 at the University Computer Center. Two simulations were made, one for a BSC and one for a Gaussian channel. Each program consists of four parts: a main program DECODE for reading in data and printing out results, a subprogram RANGEN for generating random noise, a subprogram TABSET for converting the random noise into tabular form suitable for the sequential decoder, and a subprogram SECO for the sequential decoding algorithm. Special thanks are due to Dr. K. Vairavan, who programmed both the RANGEN and TABSET subprograms, to Mr. John Geist and Mr. James Wruck for their numerous contributions to the efficiency of the programs, and to Mr. J. Chang and Mr. John Brennan for the preparation of the Gaussian program.

Each subprogram was written in assembly language to make the program as fast as possible, while the main program was written in FORTRAN to facilitate the input-output. Input information needed for the operation of the BSC program is as follows (for a complete discussion of sequential decoding parameters, see Gallager [25]):

(1) channel error probability p;

(2) the memory m of the code;

(3) the generator of the code being tested;

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(5)

(4) the threshold increment H of the sequential

decoding search; H = CONMET x & where A is the true threshold increment of the Fano algorithm; a constant CONMET used to spread the difference

between the metric values; CONMET = 8 was used in all the simulations reported here.(6) bins for the number of computations.

 R_{comp} and the metric values are then computed from p and CONMET. The threshold increment Hused in the production runs was determined experimentally. The value of Δ which optimizes the bound on computation is known to be 2 [3]. Since CONMET was chosen as 8, the "optimum" H is 16. However, through testing a single code for different values of H, it was determined that choosing H to be 32 was a better choice from both a computational and probability of error standpoint. These results are shown in table 6.12.

Each production run consisted of 1000 frames of 256 branches(blocks of information digits) each for a particular code and a particular channel error probability p. A frame was cut off and considered to be "erased" if it reached 50,000 computations. If a frame was decoded perfectly, it took (256 \div m) computations since 256 information blocks generate (256 \div m) transmitted blocks and the algorithm would count one computation for each correctly decoded block. Hence the computational bins are just numbers inclusive between (256 \div m) and 50,000 which record how many frames reached or exceeded that number of computations for decoding. Usually 13 computational bins were chosen for each production run.

 $\frac{E_b}{N_o}$ must be read in instead of p, where E_b is the energy per information digit and N_o is the noise power spectral density. Then the procedure outlined in Jacobs [33] is followed to compute the metric values needed by the sequential decoder.

In the Gaussian program the signal-to-noise ratio

Output information available from the BSC program includes the following:

- (1) the actual branch metric values and R_{comp} ;
- (2) for each decoded frame:
 - (a) the number of computations;
 - (b) the number of decoding errors;
 - (c) the last branch decoded if the frame is erased;
 - (d) the received sequence;
 - (e) the decoded sequence;

(3) for the entire 1000 decoded frames:

- (a) the number of erased frames;
- (b) the number of incorrectly decoded frames;
- (c) the number of correctly decoded frames;

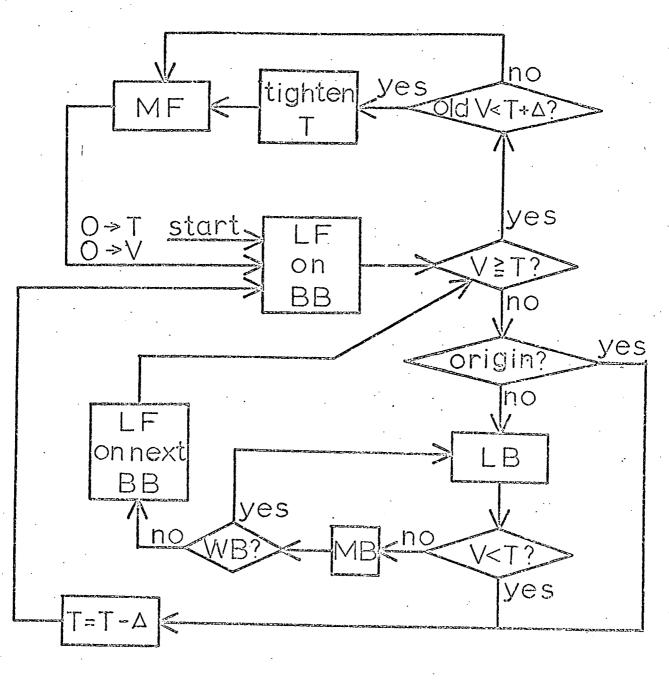
(d) the distribution of computation into bins.

Clearly the total number of error digits can be easily calculated from (2b). When the number of computations reached 50,000, decoding was terminated and the frame declared "erased". The output then recorded how far the search had progressed in the code tree when decoding was terminated. The printout of the received sequence and the decoded sequence for each frame is optional in the program. For each computational bin, the number of frames which reached or exceeded that amount of computation is recorded. For example, the bin labeled 50,000 always contains the number of "erased" frames, and the bin labeled (256 + m) always contains the total number of frames.

In the Gaussian program, additional outputiinformation about the channel is available.

In the RANGEN subprogram, a library subroutine is used to generate a noise sequence distributed according to the channel error probability p for the BSC program. In the Gaussian program, the noise sequence is distributed according to the quantized channel model given by Jacobs [33]. TABSET merely converts the noise sequence into tabular data for use by SECO.

SECO is the actual sequential decoding algorithm. The version used is thoroughly discussed by Gallager [25]. A flow chart for SECO is shown in figure 6.7. It is always assumed that the all-zero sequence has been transmitted. Since this was known to the programmer, SECO was always biased to look out on a 1 branch before looking out on a 0 branch in case the metric values on the two branches were tied. (Here the discussion pertains only to $R = \frac{1}{N}$ codes, in which there are only two branches emanating from each node.) This undoubtedly resulted in slightly more computation than would be required normally, but of course this deficiency was common to all runs and would be expected to have no effect on the comparison between different codes.



LF = look forward BB = best branch V = node value T = threshold Δ = threshold increment MF = move forward LB = look back MB = move back WB = worst branch

Fig. 6.7. SECO flow chart.

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A computation was counted as a "forward look", i.e., every time the decoder looked forward on a branch, and at no other time, a single computation was counted. Each computation, including the calculation of the parity digits, took about 100 µ sec of computer time.

The SECO algorithm is capable of handling both systematic and non-systematic codes with $m \leq 72$. Programs actually available are for $R = \frac{1}{2}$, $R = \frac{1}{3}$, and $R = \frac{1}{4}$ only. However, only results on $R = \frac{1}{2}$ codes will be reported here, since they are sufficiently representative of all rates. Also, data was taken for only three values of p and one value of $\frac{E_b}{N_0}$. These values are very typical, though, of a practical randomly distributed space channel. For p = .033, i.e., $R = \frac{1}{2} = (0.9) R_{comp}$, each production run of 1000 frames took about 2 minutes of computer time. For p = .045, i.e., $R = \frac{1}{2} = R_{comp}$, each run took about 4 minutes. For p = .057, i.e., $R = \frac{1}{2} = (1.1) R_{comp}$, each run took about 20 minutes. And for $\frac{E_b}{N_0} = 2$ or 3 db, each run took about 5 minutes.

2. Comparative Analysis of Codes.

In appendix A charts are given which have complete information on 13 different codes. A name and number is assigned to each code for identification purposes, and the means of construction for each code is briefly explained. Simulation results are given for the four channels described above. Not all the codes were tested with p = .057, since the computation time was so long.

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An interesting comparison can be drawn between code 1 (from algorithm Al) and code 3 (from algorithm A6). Note that there are fewer error frames for code 3. This appears to be due to the fact that d_{FREE} is larger for code 3, since d_{FD} is the same for both codes, and substantiates the previous statement that d_{FREE} is a more important parameter than d_{FD} for sequential decoding.

Also compare code 11 (the non-systematic code from algorithm A9) with code 12 (from Forney [28]). The non-systematic code is clearly superior in number of error frames, although it has more erased frames. For the noisiest BSC, p = .057, code ll makes no decoding errors while code 12 incorrectly decodes about 10% of the frames. However code 11 erases about 15% more frames than does code 12. But of these frames it appears that about half of them were incorrectly decoded by code 12. [Massey [32] has termed this a "fools rush in where angels fear to tread" phenomenon. The slight computational advantage of code 12 over code 11 is clearly due to this phenomenon. Since code 11 is more easily implemented than code 12 and it has the "quick look" property, we can conclude that it is far superior to code 12 in system performance as well as system complexity. In fact, code 11 did not make a single decoding error in all four simulations. To the author's knowledge, code 12 is generally considered the best m = 35 systematic code available for sequential decoding The performance of code 11 verifies the earlier statement that better results can be obtained for non-systematic codes than

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for systematic codes when used with sequential decoding (since more free distance is available for non-systematic codes).

Finally, compare the performance of code 2 with code 1. This indicates the advantage of using longer codes. However, encoder complexity increases with code length, which is an important consideration in many applications.

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EFFECT OF VARYING THE T	THRESHOLD INCREMENT H
	Iame Minimum Weight Code
	Type <u>Systematic</u>
Generators: Normal Form (Octal)	Read-in Form (Octal)
400000000000000000000000000000000000000	
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Nature of Construction: Algorithm	A1
Simulation Results: (1) Channel <u>BSC: p=.033</u> H <u>4</u> Computation:	Total Error Ditos ou
N $292 310 350 400 475 50$ #Frameswith #C \geq N1000 1000 1000 1000 1000 99	50 700 1250 2500 5000 10K 20K 50K
(2) Channel <u>BSC: p=033</u> H <u>8</u> $\frac{1}{1000}$ <u>N 292 310 350 400 475 55</u> #Frames with #C \ge N 1000 1000 1000 991 95	0 700 1250 2500 5000 10K 20K 50K
(3) Channel <u>BSC: p.033 H <u>IG</u> F Computation:</u>	otal Error Erased rames 1000 Frames 11 Frames 0
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Standard and the standard and sta	otal 1000 Error 16 Erased O ram s1000 Frames 16 Frames O Total Error Bits: 45
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	Type Systematic
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	Total Error Bits: 91 20 850 11000 1200 1500 4000 10K 25K
with $\#C \ge N$ 1000 969 906 822 681 55	52 426 352 275 203 62 16 7
(2) Channel <u>BSC</u> <u>p=.033</u> H <u>32</u> <u>Computation:</u> <u>N</u> <u> 292 310 350 400 475 55</u> #Frames	otal 1000 Error 16 Erased 0 Frames 1000 Frames 6 Frames 0 Total Error Bits: 45
with $\#C \ge N$ 000 000 992 880 585 39	7 206 53 16 4 1 0 0
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with $\#C \ge N$ 1000 1000 999 991 900 76	7 551 261 110 40 17 6 1
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(5) Channel H T Computation:	rames Frames Frames Frames Frames Frames Frames
#Frames with $\#C \ge N$	
(6) Channel H To Computation:	otal Error Erased Tames Frames Frames Total Error Bits:
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with $\#C \ge N$	
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#Frames with $\#C \ge N$												
(6) Channel Computation:			H	Tot Fra	al iges		Eri Fr. Tot	ror al	Erro	F1	ased ames ts:	
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		J		•	-	•	• • -			•••	•	•
			· : ·		· · ·		·, · . ·		•		•	
Simulation Res	ults: E	the or	घ	ጥለተ የ		. : 	mm~~			F	·	
(1) Channel G Computation:							0 t o l	- h'm2	ົດກໍ	Eras Fram Bits	ed es	7
N #Frames	292 400	1450 500	1600	700 8	50 1100	0 1200	1500	14000	110k	25k		1 -
with $\#G \ge N$	1060 971					1	•			ļ Ņ		
(2) Channel <u>B</u> Computation:	SC: p=.	. 033	H 32	Tota	1 160	VV E	rror		, I	Eras	ed	,
Computation:	1000 11100			- Fran	les los	$\frac{1}{1}$	rame otal	SErr	or	Frame Bits	25 0	0
#Frames	1292 1900	12201100	1820	1000115	00/200	2 2500	15000	10K	120 h	150K		
with $\#C \ge N$	1000 880	<u> </u>	1		4 21	17	8	2	0	0		
(3) Channel <u>B</u> S Somputation:	<u>sc: p=</u>	. 045	H <u>32</u>	Tota Fram	1 es <u>JO(</u>	O F	ror	S /		rase	2.5	4
N	1292 1.00	5501700	1850	11000115	00/2000	12500	5000	Err	or 1 20k	Bits:	72	1
$\frac{1}{10} \frac{1}{10} \frac$	1000 491	757 534	427	354 21	0 151	1	53	28	12	4		<u> </u>
(4) Channel			I	Tota Fram		Eri	ror	-		rase	e di	<u> </u>
Computation: N	1 1			1		To	cal :	Erro	r Bi	rame		
$\frac{\text{\#Frames}}{\text{with } \#C \ge N}$		· · · · · · · · · · · · · · · · · · ·										
۹۳۹-۰۰۰ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰۰۹ - ۲۰ ۱	1			met	<u></u>	· ·		·		l		
(5) Channel Computation:	1997 - 1977 - 1986 - 1977 - 1977 - 1976 - 1976 - 1976 - 1976 - 1976 - 1976 - 1976 - 1976 - 1976 - 1976 - 1976	I I	ί	Tota Franc	ès	FI	ror	s Err	Ę	rase rame its	ds	
N #Frames								<u>лла</u> Т	<u> </u>	LUS		
with #C 2 N		·					.	·				
(6) Channel	-	F		Tota] Frame	s	Er	ror	~~~.L	ـــــــ <u>ب</u>	rase] d	
Computation: N	I. I.		r			To	anes [a]	Erro	r Bi	rame. ts:	s	
Frames												
with #C≧N		1 1	1	1	1 1		1		1	1	. 1	1

18

Code No. 7	Code	Name Balanced	Code
Memory = 35	Rate = $\frac{1}{2}$	Type Syste	
Generators: Normal Form (<u>()</u>		
Rollia, Pola (Uccal)	Read-in Form	(Octal)
400000000	· ·		
Known Distanc	e Properties:		
•	a _{min} = 13	16 4	^d free ≦ 20
Nature of Cons	struction: Algorithm	n Ag	
Simulation Reg	sults:	Total Los Error	o" Proced
Canad and a week and a second data and a	auss: Eb/No=2.0 H 32	HOTAL	krror Rite oo
N #Frames	292 400 400 500 600	700 850 1000 1200 15001	4000/10K 25h
with $\#C \ge N$	1000 971 903 814 665	537 410 339 263 193	68 18 10
(2) Channel B Computation:	SC: p=. 033 H 32	Total Frames 1000 Error Total	<u>O</u> Erased. O Error Hits. O
N #Frames	1292 1400 1550 700 850 11	00011500 2000 2500 50001	10K 20K 50K
with $\#C \ge N$	1000 880 378 193 126	85 33 21 15 10	300
(3) Channel <u>B</u> <u>Computation:</u> N	SC: p=.045 H 32	Total	Error Bitseinos
#Frames with #C \geq N	1000 991 743 516 417	000/1500/2000/2500/5000/	10k 20k 50k
(4) Channel	H	Total Error	Franci
Computation:		Francs Francs Total E	Frames rror Bits:
#Frames with $\#C \ge N$			
(5) Channel Computation:	H	Total Error Frames Frames Total	Erased Frames Error Bits:
#Frames with #C \geq N			
(6) Channel Computation:	1	Total Error Frames Total E	Erased Frames rror Bits:
$\frac{1}{\text{#Frames}}$ with #C \geq N			

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Code No. 8				Code	e Na	me _	Bal	and	ced	A	die	sin t		· · · · ·
Memory = 35	· · · ·	Ra	ite =		•	-	•		· _	ma	-			• •
Generators: Normal. Form (••••••••••••••••••••••••••••••••••••••	· · · · · · · · · · · · · · · · · · ·	2	· · ·		-	· ·	••••				 	
			· · ·	• •		. <u>R</u>	ead-	<u>in 1</u>	orm	(0c	tal.)		· .
400000000 732453703							-						2	
Known Distance Nature of Cons	d	in =1	3	int	0	f a	zode	2 n	18 <u>≤</u> o. 7	đ fr	e.e [≦]	22		
				• •••••		· · · · ·	· · ·	· . :					•••	
Simulation Res (1) Channel <u>G</u> Computation:	sults:	Eb/No =	<u>2.0</u> F	I <u>3</u> 2	C To	tal	s <u>10</u> 0	70 <u>E</u>	rroi ranc	s	<u>></u>	Eras Fram	ed es _	3
N	1292 140													
#Frames with $\#C \ge N$	1000 9-	73 900	800	649	536	412	324	26.6	211	59	21	8	1	
Current and a surger of the su	<u>SC:p</u>								0501	1	O for	Eras Franc Bits	ed es	0
#Frames	1292140	01550	700 1	8.50	1000	1500	2000	2500	5000	10K.	201	504		1
with $\#C \ge N$	1000 87	3 376	191	114	87.	37	20	15	7	2	0	0		
(3) Channel BS Computation:					-		100	.11	rror Came	S		Erase Frame Bits	28	6
N #Frames	1292 140	0 550	17001	850	1000	11500	2000	2500	5000	1 i0h	20k	50K		<u>†</u>
with $\#C \ge N$	1000 99	2 746	523	410	334	203	144	116	54	30	15	6		
(4) Channel Computation:			H		701 Fr;	tal Bunds	***	Eri Fra Tol	ror unes	Erro		irase Tame	ů s	l
N #Frames]									
with $\#C \ge N$														
(5) Channel Computation: N			H	63+*****	Tot Fra	al mes		- Fr To	ror	⁵ Err	F or I	rase rame lits:	d s	
#Frames				···							·		Ī	
with $\#C \ge N$												· [•	
(6) Channel Computation: N			H	••••••	Tot Fra	al mes		Er Fr Tot	ror anes Ial	Erro	E F r Bi	rased rame	1 5	
Frames with $\#C \ge N$			-											
•••				20		1					ــلــــــــــــــــــــــــــــــــــ	• • • • • • • • • • • • • • • • • • • •	<u>-</u> -	l

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Code No. 9		• • •		•	Cod	e Na	me	Balc	ince	d	Bus	sa	ana	2	Col
Memory = 35	•	-	Ra	ate =			7	'ype	Sy	ste	ma	+;;	<u></u>		
Generators:			· · ·	•			· ·		· · ·		*		2		
Normal Form (·				F	Read-	in	Form	(Oc	tal)		
4000000000 732443151							· · · ·				· · · ·	مربع قرار ما			
Known Distance Nature of Cons	•	dmi	n =)3	3	cod	6.0	bta		Г. h	18≦	đ fr	ee ≦	20	• 11	
to extend or	ie ç	of K	3055	gar	1 g' s	op	tIm	al	. 0 . 00	y O. les.) ~	Igor	י Th	.m 41
imulation Res (1) Channel <u>G</u> computation:	avs	, E								01.31	H 777	-10	Eras Fram Bits		3
$\frac{N}{\text{Frames}}$	1000	973	899	790	1648	3 52	1 395	5 326	262	199	14000 57	19	125K		
2) Channel <u>B</u> computation:	<u>SC:</u>	p=.	033		H <u>3</u>		ame:	<u>s 100</u>		rror rame otal	s <u>C</u> Err) or	Eras Fram Bits	ed es 6	0
Frames th #C ≧ N	1212	1400	371	1700	1850	11000	30	12000	14	5006	1015	20k	50h		
3) Channel <u>B</u> computation:	30:	-P==	049	5 1	H <u>32</u>	To Fr	tal ames	<u> </u>		rror rame	s L	1	Eraso France Bits	5 C .	3
N Franes ith #C ≥ N		992	1550 739	1700	1	1	1	2000	2500	5000 43	10k 25	20k 13	150K		
4) Channer		· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	I	1.	To	tal ames	L	Er	l <u> </u>			3 Grase trame	a	
N . Frames											Erro	r B			
ith $\#C \ge N$ 5) Channel		!		H		To	tal ames		Ęŗ	ror		Ę	rase	đ	
omputation: N Frames									<u> </u>		Err	ōr î		s	
ith #C ≥ N									l	· ·					
5) Channel				H		Toi Fra	al imes			ror anes Lal 1	Erro	– E F r B	rase rame its:	d s	
N N Tames	i i	· · · · ·	1		1	J		:	- T						

Code No. 10	a			Code	e Na	me _	Bal	anc	ed	Bus	squ	ang	2	Ad
Memory = 35	-	Ra	te =	$\frac{1}{2}$	-	T	ype	<u>Sy</u>	ste	ma	tis	ر	· · ·	
Generators:						· ·		:	`	- :	•.		•	· .
Normal Form (C	ctal)		·		<u> </u> -	R	ead-	in	Form	(00	tal)		
4000000000 6531372441			·		-				 *		-			
known Distance	Proper	ties	•	• • • • • •	· :: .•	. :		•					•	
	. d _{mi}	n = 13	3						17 ≦	d fr	ee	≦ 23		
Nature of Cons	tructio	n: Ą	o j t	oin t	0	f c	ode	nc	; c	 		0		
						:				·				
imulation Rest 1) Channel <u>G</u> Computation:	auss							T	otal	Er		Eras Frai Bits	nes	4
N Frames	292 400	1450	1500	1600	700	850	11001	1200	1500	1400	dior	(251	5 10	
ith $#C \ge N$	1000 971	906	814	662	2 550	408	334	256	185	69	22	14.		
2) Channel BC Computation:	<u>;c;p=</u>	.03	<u>} </u>	H <u>32</u>	2 To Fr	tal ames	, 100		rron	S	<u>0</u>	Eras		0
N Frames	1292 400	15 50	700	1850	1000	1500	2000	2500	5000	104	120k	1.2.01		.}
with $\#C \ge N$	1000 880										0	0		
3) Channel <u>BS</u> omputation:								-T	rror rame	Err	$\frac{2}{10r}$	Eras Fran Bits	les,	7_
N Frames	292 400	550	700	1850	11000	1501.	2000	2500	15000	llok	20k	150k		1
	1000 991	740	506	402	317	187	138	117	46	24	12	7		
4) Channel			H	[1: - 1:	tol mee	·····	Eri	ror	Frrc		Eras Fram its:	ed les _	
N Frames												1		1
ith $\#C \ge N$													1 :	
5) Channel			H		To Fra	tal ames			ror Tame	s _{Err}	· · or	Eras Fram Bits	ed es	
N Frames					ļ									1 1
th $\#C \ge N$										·				·
b) Channel			H		Tot Fra	al mes		Er - Fr To	ror ames	Erre	Ł	rase rame	2S	· · · ·
N	1	• • • • •			ŀ	Ī	T	T			<u></u>			·

•	Code No. 1) Code 1	Name Non-systematic Code
	Memory = 35 Rate = $\frac{1}{2}$	Type Non-systematic
	Generators:	
	Normal Form (Octal)	Read-in Form (Octal)
	7 3 3 5 3 3 6 7 6 7 3 7 5 3 3 5 3 3 6 7 6 7 3 7	
	Known Distance Properties:	
	$d_{\min} = 1$	$17 \leq d_{free}$
, , , , , , , , , , , , , , , , , , ,	Nature of Construction: Algorithm	n A9
•		
	Simulation Results: (1) Channel Gauss: EMNo: 2.0 H 32 Computation:	Total 1000 Error O Erased 5
:		Total Error Bits: 0 00 850 1000 1200 1500 4000 10k 25k 1
	with #C ≥ N 1000 968 909 835 676 5	67 445 358 292 225 70 17 9
	(2) Channel <u>BSC: p=.033</u> H <u>32</u> Computation:	Total Error Bits o
	N 29214001550 7001850110	001500 2000 2500 5000 10k 20k 50k
	with $\#C \ge N$ 1000 883 405 223 135 9	2 47 26 18 5 2 0 0
	GOMPTICAL LUIT.	Frames 1000 Frames O Frames 8 Total Error Bits: 0
	# Trales 1997 400155017001850110	00/1500/2000/2500/5000/10K 20K 50K
	With #C ≥ N 1000 991 785 581 477 38	32 240 167 134 63 36 23 8
		Total 1000 Error O Erased 249 Total Error Bits: 0
	- Eromon	2011500/2000/2500/5000/10K 20K SOK
	with $\#C \ge N$ 1000 1000 949 863 802 75 (5) Channel H T	
	Computation:	otal Error Erased rames Frames Frames Total Error Bits:
	N #Frames	
1	with $\#C \ge N$	
	(6) Channel H T Computation:	otal Error Erased rames Frames Frames Total Error Bits:
	$\frac{\pi}{r} rames$ with $\pi C \ge N$	
• •		
	23	

Code No. 12		Ced	e Name N	ASA Cod	٤	
Memory $= 35$		Rate = $\frac{1}{2}$	·	e <u>System</u>		میں میں محمد میں محمد میں محمد م
Generators: Normal Form (Octal)		l Poo	d in Form ((No. 1. N	
400 000 00			Nea	d-in Form ((octal)	•••••••••••••••••••••••••••••••••••••••
71547370						
Nature of Con	d min struction:	= 14 The adje	oint of t	d _f he code F	orner offer	ned
by using the optimal code	es.	le d'aori	ithm to	extend on	e of Bussg	ang's
Simulation Res (1) Channel <u>G</u> <u>Computation:</u> N	auss: Eb/N	<u>= ?.0</u> H 3:	2 Total Frames L	000 Error Frames Total E	O Erased Frames - Fror Bits: 0	Ľ,
#Frames	1000 969 90	0 810 652	1523 4AU 2.	00 1205 1500 140 27 254 188 60		
(2) Channel <u>B</u> Computation:	<u>5C: p=.0</u>	<u>33 н 32</u>	2 Total Frames 1	000 Error	O Erased	0
#Frames			10001200120	00/2500 5000 10	K 20K 50K	
with $\#C \ge N$	1006 884; 30				20	
(3) Channel <u>B</u> <u>Computation</u> :	• •			Toto1 27	2 Erased Frames	4
$\frac{1}{\text{#Frames}}$ with #C \geq N	1000 991 75	6 510 403	320 187 13	8 104 48 3		
(4) Channel $\underline{B} \subseteq$ Computation:			Trance 100	6 Error	27 Erased 1	08
N #Frames	1292 1400 155	• • •	•	Total Err 0/2500/5000/101	or Bitos -	1
with $\#C \ge N$	1000 1000 93	2 817 734	673 532 45	5 412 319 237		
(5) Channel Computation:		ĨĬ	Total Frames	Error Frames Total Er	Emagod	
$\frac{N}{\#Frames}$ with $\#C \ge N$					rer Bits:	
(6) Channel Computation:		H	Total Frances	Error Frames Total Err	Erased Frames	
$\frac{1}{\text{#Frames}}$ with $\text{#C} \ge \mathbb{N}$						
		24				d

.

•	Code No. 13			Code	Name	Li	<u>n - 1</u>	Lyn	e (od	e			
	Memory = 35		Rate	$=\frac{1}{2}$	-	Type	<u></u>	<u>yste</u>	ma	tic		: •		
	Generators: Normal Form (Octal)		· · · · · · · · · · · · · · · · · · ·		Read-	in 1	Form	(Oct	al)				
*.	400000000	_										· · · ·		•
	Known Distance Natale of Cons	d mir	L = 14	· -· .	1.		R.C.	19 ≦						
	algoritism.		1 011	iey s	ex i	r 210	n	0 }	The	<i>۲</i> – ۱	n- 1	-yn	و	
	(1) Channel <u>G</u> (1) Channel <u>G</u> (1) Channel <u>C</u>	auss						of al	Erre	nr R	raso	ed es	5	
	$\frac{N}{\text{#Trames}}$ with #C $\geq N$	292 1400 1000 971	907 81	5 665	538 4	03 331	+ 262	189	4000 65	10 K 20	25K 11			
	(2) Channel <u>B</u> Computation:	<u>SC: p=.</u>	033	H <u>32</u>	Tota Fram	1 es 100		rror rame otal	s <u>0</u> Erro	r B	rase rame its:	ेते इ	0	
	$\frac{1}{\text{#Frames}}$ with #C \geq N	292 310 1000 1000	350 400	21475	550 70	0 1250	2500	12000	lok	20K	50 K			
	(3) Channel <u>B</u> Computation:					1 es <u>100</u>	o F	rror rames otal	3 3	Ē	rase	20	3	
	$\frac{N}{\# \text{Franes}}$ with #C $\geq N$	1000 1000	350 401	1 895	550 70	0 1250	2500	5000	10K 22	20K	50K			
	(4) Charnel B Computation:	المتحصير البور مرد ومغ	in the second	<u></u>				ror ames tal H			l	i S I	18	
· ·	#Frames	12921400	550 700	1850	1000/150	0/2000	2500	5000	10K12	20K 5	SOKI	1821	+ 	
•	with $\#C \ge N$	1000 1000	927 800					المستحمد الم	272 2	06	118			·· ·
	(5) Channel Computation:		· · · ·	H	Total Frame	s		cror cames otal	Erro	Fr Fr B	ase ame its:	s		- · ·
	#Frames with #C ≥ N			• -	-				· · · ·	·		 .		
	(6) Channel Computation:			H	Total Frame	s	Er Fr To	ror anes tal I	Error	Er Fr Bi	ased ame ts:	di s		•
	Frames with $\#C \ge N$										· 			
·		· · ·		2	5 1	•				2 12 - 14. 2 12 - 14.	••••••••••••••••••••••••••••••••••••••	·.		