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AN ANALYSIS OF A MANUFACTURING PROCESS USING THE GERT APPROACH

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AN ANALYSIS OF A MANUFACTURING PROCESS USING THE GERT APPROACH

Introduction

In this research, a manufacturing process is analyzed using the GERT (Graphical Evaluation and Review Technique) Approach. Although GERT normally is used for analysis in the time domain, it has been adapted for a production system where the parameter of interest is cost [9]. This research will demonstrate the use of GERT in solving production problems.

Several references are available describing the background and mechanics of GERT [5,6,7,8,11]. These topics are not presented in this paper. However, the steps of the GERT Approach will be used in this research, particularly as applied in the cost domain. These steps are [10]:

1. Convert a qualitative description of a system or problem to a model in stochastic network form.
2. Collect the necessary data to describe the branches of the network.
3. Determine the equivalent function or functions of the network.
4. Convert the equivalent function into performance measures associated with the network. Examples of performance measures are:
    a. the probability that a specified node is realized;
    b. the average cost to realize the specified node;
    c. an estimate of the standard deviation of the cost to realize the specified node;
    d. the minimum cost observed to realize the specified node;
    e. the maximum cost observed to realize the specified node; and,
    f. a histogram of the costs to realize the specified node.
5. Make inference concerning the system under study from the information obtained in Item 4 above.

Each of these steps will be applied in subsequent sections of this paper.

The purpose of this research is to demonstrate the worth of the GERT Approach in modeling production systems, and to emphasize the importance of the network formulation and data collection steps in the GERT Approach.

Much of the research associated with GERT has been theoretical. Many interesting problems have been considered but little has been done to bridge the gap between the theoretical and the practical. This research is an attempt to bridge that gap by demonstrating the usefulness of the GERT Approach as a management tool in industry.

Perhaps the most difficult problems the researcher faces in solving realistic problems with GERT (or with any technique) is determining the level of complexity required of a model. In GERT, this involves a determination of the complexity to be included in the network formulation and the data collection process. Although the network formulation and data collection steps of the GERT Approach are very important to the analysis, they have received little attention. Therefore, these steps were considered in detail in this research.

A GERT Approach to Manufacturing Problems

A series of operations required to produce a gear at AiResearch/ Phoenix was selected as the manufacturing process to be studied. The past history of this particular gear indicated a high incidence of rework, or scrap, or both. In fact this part was singled out as the one that has caused the most problems in recent months.

The Manufacuturing Process

The process begins with the release of a work order for a batch of gears and ends at final stores. The process itself can be separated into four basic groups of operations: (1) milling, (2) heat treatment, (3) grinding, and (4) inspection and final processing. The milling operations are interspersed with heat treatments and include such operations as turning and hobbing. After completing the milling operations, the parts are subjected to an extensive heat treatment cycle where operations correspond to stages in the cycle. It is here that the gears are hardened to the desired specifications. Finally, the gears are ground to obtain the proper finish and dimensions. Also there are some operations in this group which require additional machining and heat treatment. On completion of the grinding operations, a final inspection is performed. Acceptable gears are sent to finish stores while those rejected are either reworked or scrapped.

In most cases, non-conforming gears are tagged when discovered and continue to flow through the shop with those gears which have no visible defects. All gears are inspected at the final inspection station; however, those which are tagged receive special attention. In many cases these

discrepancies are so minor that a tagged gear can be accepted without further machining. Nevertheless, there are those parts which must be reworked at the appropriate station, or scrapped, or both. Of course there is a greater incidence of rework and scrapping among tagged gears than among untagged gears.

The final operation of the heat treatment cycle is to inspect each gear for "white spots" caused by an inproper heat treatment. If this problem occurs, the defective gears are immediately reprocessed through the heat treatment cycle. It is assumed that only one reprocessing will be required. This is the only case in which rework is accomplished prior to determining the disposition of the gear at the final inspection station.

Although this is a brief discussion of the process, it should be sufficient for the purposes of this report. Now, the problem can be defined.

Problem Definition

The past history of this particular gear indicates a definite need for an analysis of the production process. Recent work order releases have encountered problems in the heat treatment cycle and in grinding operations. In one instance some form of rework was required on all gears in the original release.

In this process rework and scrappage are costly items which significantly increase the unit cost of the finished product. Therefore, the production manager is interested in alternative methods to reduce the unit cost while maintaining a smooth product flow in the shop. Analyses of this type can be accomplished using the GERT Approach.

Development of the GERT Network

The first step in the GERT Approach is to convert a qualitative definition of the manufacturing process to a GERT network model. Using the four basic groups of operations in this process, a simplified initial network is shown in Figure 1. The network represents the possible paths a single part can follow in the course of the process. The network is drawn for a part as opposed to being a description of the manufacturing operations through which parts flow, i.e., a queueing model.

The qualitative network is not complete until the difference between tagged (non-conforming) and untagged gears is resolved. Since tagged gears have a higher incidence of rework or scrap at the final inspection station, the network must reflect this difference to adequately model the process. This difference can be resolved in the network by defining an alternate set of operations which are identical to the operations being performed on the untagged parts. The final qualitative GERT network is shown in Figure 2 where the alternative set of operations are designated by the 200 series nodes. Once a gear is tagged, it proceeds along the alternate path; hence, the higher incidence of rework and scrap can be reflected on the output side of node 209 in Figure 2.

At this point the branches of the network are specified only in qualitative terms. In this analysis the branch parameters of interest are probability and cost; hence data must be obtained to determine the frequency of occurrence for each branch and the cost incurred if this branch of the network is realized. This initiates the data collection phase of the GERT Approach.

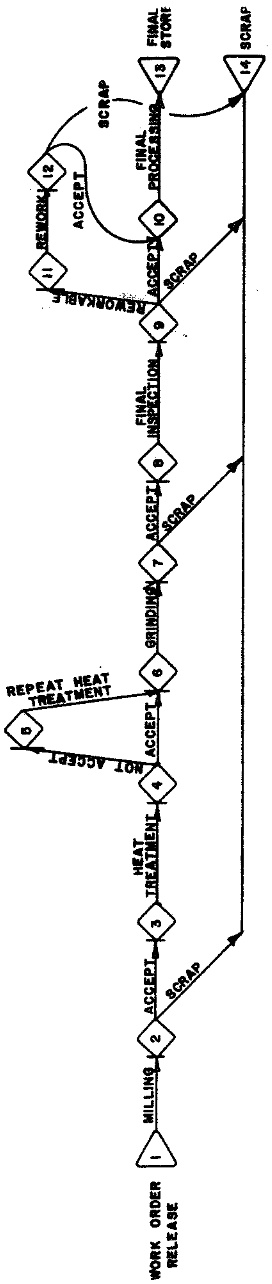


Figure 1. Initial Qualitative Network of the Manufacturing Process

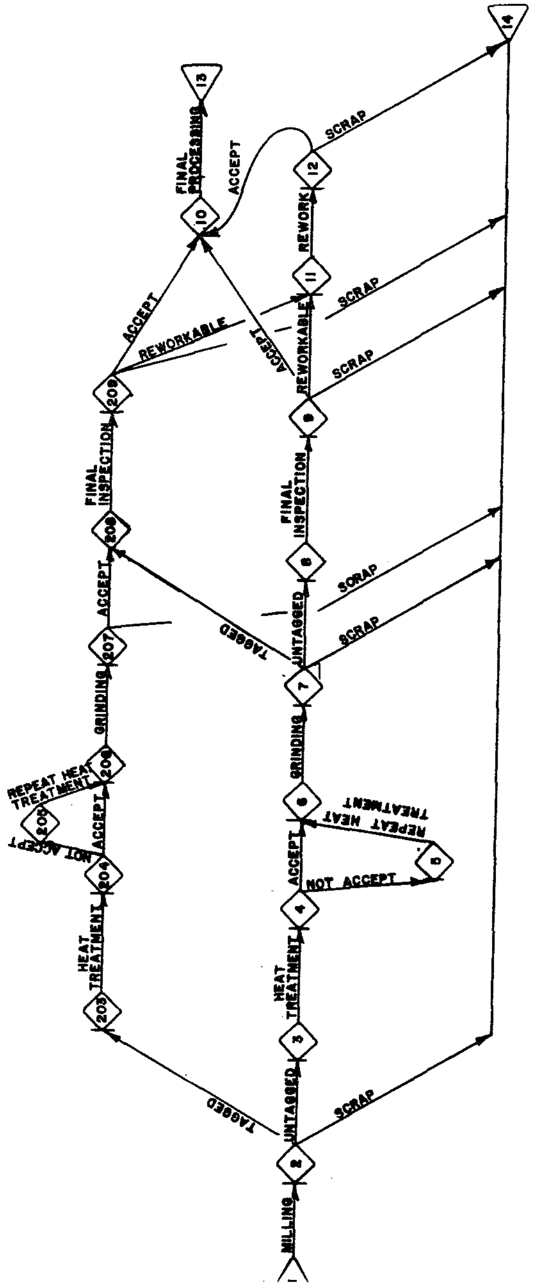


Figure 2. Final Qualitative GERT Network of the Manufacturing Process

Data Collection

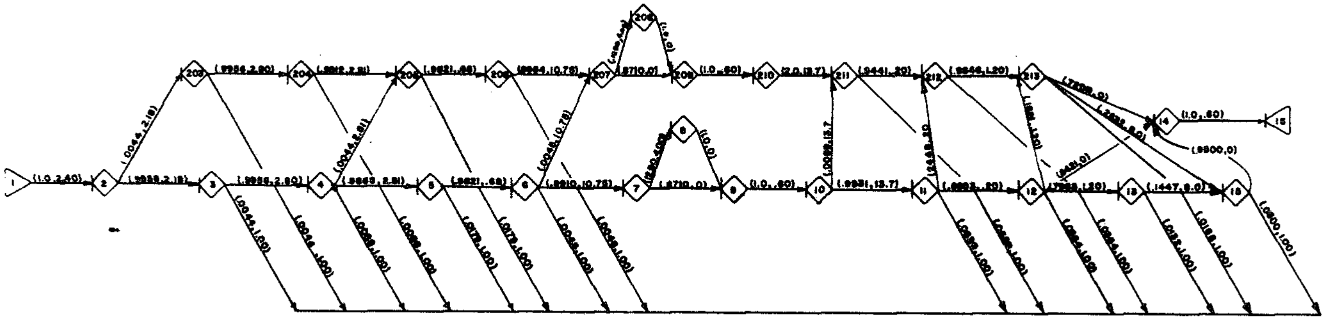
Obtaining good estimates of the branch parameters is possibly the most difficult phase of the GERT Approach. The analysis is only as good as the input data; hence, the importance of accurate data should be emphasized.

In this case data was obtained from the most recent gear production history. The data base was limited to the three most recent work order releases which represented a total of 229 gears. As production systems are subject to periodic change, older data may not truly reflect the current process.

The frequency of occurrence of each branch was relatively easy to obtain. Work orders at AiResearch/Phoenix indicate at what point in the process a gear was scrapped or designated as non-conforming (tagged). Therefore, with the aid of a production control specialist, an operation by operation history of discrepancies was developed. The manufacturing process was further simplified by grouping series of operations which had no discrepancies. The GERT network representation of this simplified version of the process is shown in Figure 3. A description of the operations and the frequency of discrepancies can be found in Table I.

The network presented in Figure 3 is essentially the same as the network given in Figure 2. However, there are some differences which need clarification.

The groupings of operations used in the network of Figure 3 do not correspond to the four main groupings of operations discussed previously. For example, the branch joining nodes 1 and 2 of the network in Figure 2



Branch Parameters (p,c): p = probability that branch is realized

c = cost incurred if branch is realized

Figure 3. Quantitative GERT Network of the Manufacturing Process

TABLE I.

BRANCH PARAMETERS FOR A GERT NETWORK OF THE MANUFACTURING PROCESS

Source Node	Sink Node	Operation Number	General Description of Operations	Probability Fraction	Probability Decimal	Cost ($)
1	2	10-30	Initial processing and milling	229/229	1.0000	2.40
2	3	40	Milling	228/229	.9956	2.18
2	203	40*[1]	Milling	1/229	.0044	2.18
3	4	50-70	Heat treatment and milling	227/228	.9956	2.80
3	99	SCRAP	Scrap after Operation 40	1/228	.0044	1.00
203	204	50-70*	Heat treatment and milling	227/228	.9956	2.80
203	99	SCRAP	Scrap after Operation 40*	1/228	.0044	1.00
4	5	80	Milling	224/227	.9868	2.81
4	205	80*	Milling	1/227	.0044	2.81
4	99	SCRAP	Scrap after Operation 70	2/227	.0088	1.00
204	205	80*	Milling	225/227	.9912	2.81
204	99	SCRAP	Scrap after Operation 70*	2/227	.0088	1.00
5	6	90	Milling	219/223	.9821	0.68
5	99	SCRAP	Scrap after Operation 80	9/223	.0179	1.00
205	206	90*	Milling	219/223	.9821	0.68
205	99	SCRAP	Scrap after Operation 80*	4/223	.0179	1.00
6	7	100-365	Milling and heat treatment	217/219	.9910	10.75
6	207	100-365*	Milling and heat treatment	1/219	.0045	10.75
6	99	SCRAP	Scrap after Operation 90	1/219	.0045	1.00
206	207	100-365*	Milling and heat treatment	218/219	.9954	10.75
206	99	SCRAP	Scrap after Operation 90*	1/219	.0046	1.00
7	9	DUMMY	Heat treatment O.K.	189/217	.8710	0.00
7	8	REWORK	Heat treatment repeat	28/217	.1290	4.00
8	9	DUMMY	Precedence relationship	28/28	1.0000	0.00
9	10	370	Milling	217/217	1.0000	0.60
207	209	DUMMY	Heat treatment O.K.	189/217	.8710	0.00
207	208	REWORK	Heat treatment repeat	28/217	.1290	4.00
208	209	DUMMY	Precedence relationship	28/28	1.0000	0.00
209	210	370*	Milling	217/217	1.0000	0.60
10	11	380-500	Milling and grinding	143/144	.9931	13.70
10	211	380-500*	Milling and grinding	1/144	.0069	13.70
210	211	380-500*	Milling and grinding	144/144	1.0000	13.70
11	12	510	100% nital etch	100/143	.6993	0.20
11	212	510*	100% nital etch	35/143	.2448	0.20
11	99	SCRAP	Scrap after Operation 500	8/143	.0559	1.00
211	212	510*	100% nital etch	135/143	.9441	0.20
211	99	SCRAP	Scrap after Operation 500*	8/143	.0559	1.00
12	13	520-550	Heat treatment and inspection	90/113	.7965	1.20
12	213	520-550*	Heat treatment and inspection	19/113	.1681	1.20
12	99	SCRAP	Scrap after Operation 510	4/113	.0354	1.00
212	213	520-550*	Heat treatment and inspection	109/113	.9646	1.20
212	99	SCRAP	Scrap after Operation 510*	4/113	.0354	1.00
13	14	DUMMY	Accepted at final inspection	64/76	.8421	0.00
13	15	REWORK	Miscellaneous rework	11/76	.1447	8.00
13	99	SCRAP	Scrap after final inspection	1/76	.0132	1.00
213	13	DUMMY	Accepted at final inspection	18/19	.7200[2]	0.00
213	15	REWORK	Miscellaneous rework	5/19	.2632[2]	8.00
213	99	SCRAP	Scrap after final inspection	-	.0168	1.00
15	14	DUMMY	Final processing	-	.9500**	0.00
15	99	SCRAP	Scrap after rework	-	.0500**	1.00
14	16	560-590	Final processing	-	1.0000	0.60

[1] Non-conforming gears are tagged and allowed to travel with acceptable gears
[2] Adjusted to allow for scrap
* Operations performed on tagged gears
** Estimated

represents the basic group of milling operations. However, for the network in Figure 3, this same group of milling operations correspond to the series of branches between nodes 1 and 6. Hence, discrepancies occurred at several of the milling operations as shown in Table I.

Another difference between the two networks concerns the representation preliminary in-process inspections. Since these inspections are performed directly after many operations, the inspection cost is included as part of the processing cost. At each point in the process where this occurs, the GERT network would involve a processing branch and inspection branch. Since these operations are in series, a branch representing the inspection cost is not required. Combining operations at this level reduces the size of the network.

For illustration purposes, a detailed description of the process represented by the branch between nodes 3 and 4 of the quantitative network is given in Figure 4. By combining the costs associated with processing and inspection, node 3' can be eliminated. The branches emanating from node 3" represent the decision alternatives following inspection and no costs are associated with any of these branches. Hence, the costs of the subsequent processing operation can be combined with the probabilities associated with the inspection decision. In other words, node 4 could replace node 3", thus, again reducing the number of branches required. Therefore, the detailed partial network in Figure 4 has been reduced to the partial network shown in Figure 5. This partial network is equivalent to the corresponding section of the quantitative network in Figure 3.

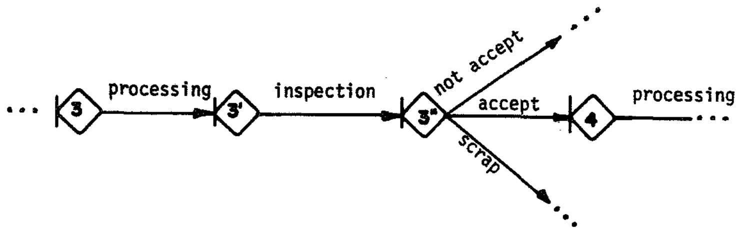


Figure 4. Qualitative Description of a Partial Network

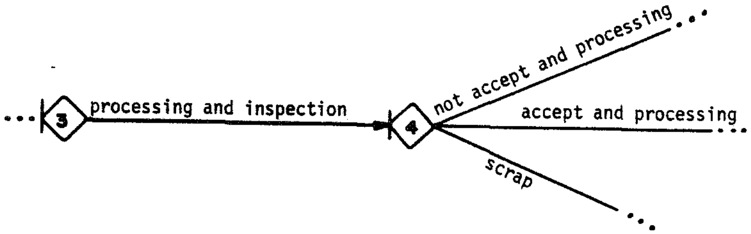


Figure 5. Refinement of the Partial Network

The cost parameter associated with each branch of the network is often a nebulous quantity. Before proceeding further the researcher must decide the level of complexity required for the analysis of the network.

This is equivalent to the level of abstraction decision in other model building activities. If only a low-level network is required, the cost parameter can be related to the set-up and processing times ignoring overhead costs. An average cost/hour can then be assigned to each operation based on actual performance or standard data. However, several operations such as heat treatment involve mainly equipment and equipment depreciation costs. For these operations it was necessary to estimate a cost per part based on management policies. For the stated objectives of this report, costs are assumed to be deterministic. Further refinements to the network will be discussed in a subsequent section.

For this network standard data was used to obtain the set-up and processing times of most operations. An average cost/hour of $4.00 was used to relate cost to time. A thorough analysis of overhead costs is beyond the scope of this paper; therefore, estimates were made for those operations which are considered as overhead items. The GERT network in Figure 3 contains the costs associated with each branch in the network.

Equivalent Network, Performance Measures and Inferences

The final three steps in the GERT Approach are: (1) to determine the equivalent network and total cost distribution; (2) to convert the total cost distribution into performance measures associated with the network; and (3) to make inferences concerning the production system.

The GERT EXCLUSIVE-OR computer program [3,4] provides a means for obtaining the information upon which inferences can be made. The input data required by the program can be obtained directly from the GERT network given in Figure 3. Output from the program includes the following information:

1. The probability and cost associated with each path in the network.
2. The probability that a part is accepted or scrapped.
3. The mean and variance of the cost incurred for accepted or acrapped parts.

Using the output from the program, the probability of an acceptable part was found to be 0.857 with a mean cost of $39.94 and a variance of 11.66. In other words if a lot of 1000 gears were released, it is expected that 857 gears would be accepted at an average cost of $39.95. Similarly, the probability of a scrapped part is 0.143 with an expected cost of $31.37 and a variance of 158.33. Since the cost of acceptable gears does not consider scrappage, it does not reflect the true cost of producing an acceptable gear. Adjusting this cost to allow for scrappage yields a total expected cost of $44.25.

At this stage in the GERT analysis, it is possible to construct an equivalent network. Thus the GERT network in Figure 3 can be reduced to the network in Figure 6 (where the expected values are only displayed.) Now, it is desired to determine the probability mass function associated with each of the branches in Figure 6. This procedure is referred to as the <u>inversion process</u>.

The probabilities and costs associated with each path can be used to obtain the total cost distribution for both acceptable and scrapped parts. All paths with a probability of less than .001 where ignored. The cost distribution for acceptable parts is shown in Figure 7. The cost distribution for scrapped parts is shown in Figure 8.

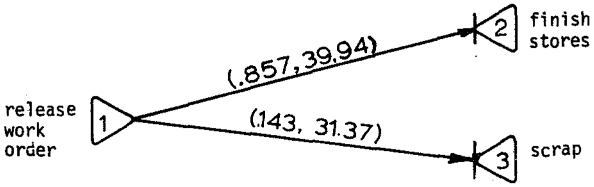


Figure 6. Equivalent Network of the Manufacturing Process.

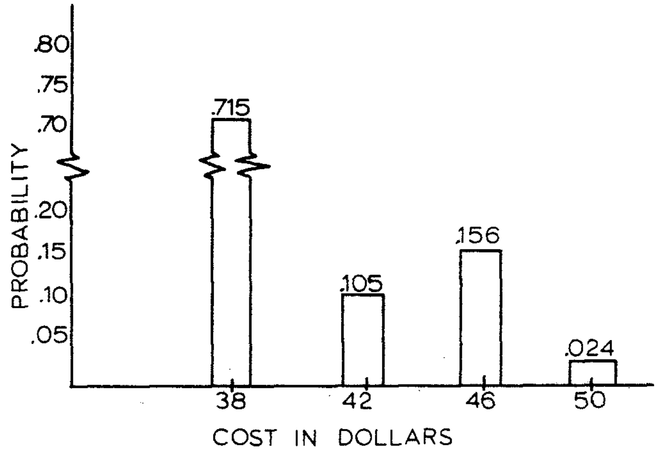


Figure 7. Probability Mass Function of Cost for Acceptable Gears

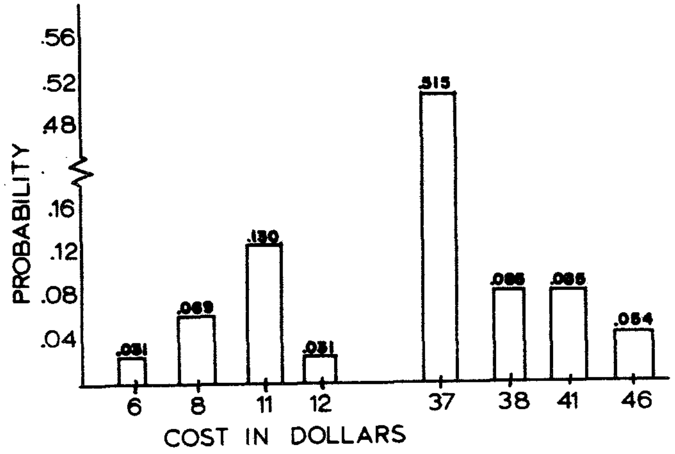


Figure 8. Probability Mass Function of Cost for Scrapped Gears

Probability statements can now be made about the production cost of both acceptable and scrapped gears. For example, the probability that an acceptable gear will cost more than $46 is 0.024.

An improvement analysis is considered as part of the review process. However, the discussion is included in the next section because of the importance of this technique as a management tool in solving production problems.

Improvement Analysis

An improvement analysis allows the manager to evaluate the effect of various alternatives on the total production system. Initially, each alternative must be defined and analyzed to determine the branches of the network affected. The network is then modified to reflect these changes in either network structure or network parameters. The GERT computer program can then be used to obtain the effect of the change. This process is repeated for all alternatives and a comparison can then be made to assess the worth of the proposed alternatives.

Three alternatives are considered in this research. Two of these are actual alternatives proposed by AiResearch/Phoenix. The last is a hypothetical alternative for illustrating a feature of the model.

For an initial cost of $5,000, an additional quench die can be purchased for use in the heat treatment cycle. It is estimated that this capital expenditure would increase the cost per gear by $.50. As a result, it is anticipated that the probability of a successful heat treatment cycle would be increased from 0.871 to 0.950.

A second alternative for improving the effectiveness of the heat treatment cycle is to purchase a carburizing fixture for an initial cost of $2000. In this case it is estimated that the increased cost would be $.25 per gear. For this alternative the probability of a successful heat treatment cycle would be increased to 0.920.

It is coincidence rather than a requirement of the model that both of these alternatives affect the same branches of the network in Figure 3. An important feature of the improvement analysis is that any or all of the branches of the network can be affected by any alternative. This feature is illustrated by the third alternative.

Suppose that a heat treatment which improves the forging's machinability can be applied prior to the first machining operation.

Estimates are that this improved machinability would increase the acceptance at final inspection 0.842 to 0.942 for untagged gears and from 0.720 to 0.820 for tagged gears. It is estimated that a cost of $.20 per gear would be incurred for the process.

The description of the alternatives can now be converted into network terminology. The modifications to the network required for each alternative are shown in Table II. Hence, three new networks are created for evaluation purposes.

TABLE II
NETWORK CHANGES FOR EACH ALTERNATIVE IN THE SENSITIVITY ANALYSIS

Alternative	Source Node	Sink Node	Probability		Cost	
			From	To	From	To
Quench Die	6	7			10.75	11.25
	6	207			10.75	11.25
	206	207			10.75	11.25
	7	9	.871	.950		
	7	8	.129	.050		
	207	209	.871	.950		
	207	208	.129	.050		
Carburizing	6	7			10.75	10.95
	6	207			10.75	10.95
	206	207			10.75	10.95
	7	9	.871	.920		
	7	8	.129	.080		
	207	209	.871	.950		
	207	208	.129	.050		
Improved Machinability	1	2			2.40	2.60
	13	14	.842	.942		
	13	15	.144	.044		
	213	14	.720	.820		
	213	15	.263	.163		

It is not necessary to use the GERT computer program for evaluating he first two alternatives. A closer look at each of these alternatives eveals that only the input and output sides of nodes 7 and 207 of the etwork in Figure 3 would be affected. These nodes are associated with ntagged and tagged gears respectively. Since the tagged and untagged ections of the network are mutually exclusive (only one will be realized n any realization of the network), an analysis of the proposed alternatives eed only consider the effect in either section.

The section of the network affected by the proposed alternatives s shown in Figure 9.

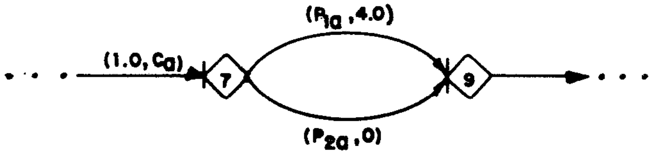


Figure 9. Partial Network for an Improvement Analysis

In this partial network, C_a represents the increased cost of the eat treatment cycle for alternative a, p_{1a} is the probability that the ear must repeat the heat treatment cycle at a cost of $4, and p_{2a} is he probability that the gear can proceed directly to the subsequent peration. The equivalent w-function for this partial network is

$$W_{Ea} = P_{2a}e^{C_aS} + p_{1a}e^{(4+C_a)S}$$

where

$$p_{2a} = 1 - p_{1a}$$

By differentiating w_{Ea} with respect to s and setting s = 0, the expected increase in total cost (TC_a) is

$$E[TC_a] = C_a + 4P_{1a}$$

For the original network of Figure 3, $C_o = 0$ and $p_{10} = 0.1290$ which gives a standard comparison of

$$E[TC_o] = [.1290][4.0] = .516.$$

Therefore, if $E[TC_a] < .516$, the alternative is preferred to the original network. Now, the two alternatives can be evaluated with the results shown in Table III. Based on these results, both alternatives should be rejected. The expectation decision criterion is used in making these decisions.

TABLE III
RESULTS OF IMPROVEMENT ANALYSIS FOR ALTERNATIVES 1 AND 2

a	C_a	p_{1a}	$4p_{1a}$	$E[TC_a]$	Decision
1	$.50	.05	.20	$.70	Reject
2	.20	.08	.32	.52	Reject

A similar analysis could be performed for the third alternative; however, the calculations would be laborious. This is because two sections of the original network are affected by the alternative. Therefore, the simplest method for evaluating this alternative is to use the GERT computer program.

Using the computer output for alternative 3, the probability of an acceptable gear was found to be 0.862 with a mean cost of $39.33. The probability of a scrapped part was 0.138 with an expected cost of $31.07. Adjusting the average cost of acceptable gears to include scrappage, yields $43.64 as the expected cost of an acceptable gear. This is a reduction in cost of $.51 per gear for this alternative, over the original process given in Figure 3. Therefore, alternative 3 is preferred to the original network.

This concludes the improvement analysis. The worth of GERT as a management tool in production control has been demonstrated. Perhaps the greatest benefit of the technique is that it forces the analyst to think of each alternative in terms of a network. Therefore, it is possible to gain insights into the problem perhaps unattainable by conventional means. This was particularly true of the improvement analysis for alternatives 1 and 2.

Refinements of the Model

Thus far the analysis of the manufacturing process has been based on deterministic (constant) cost parameters assigned to branches of the network. In practice, these costs are not deterministic but are random variables.

When randomness is introduced into the model, the GERT Approach becomes even more useful than before. Even for a simple problem it is a laborious task to obtain system performance measures by analytical methods. Using GERT, however, the analysis can proceed as before. Once the equivalent network is obtained, it is at least theoretically possible to obtain the cost distribution of the equivalent network. Unfortunately, for most networks, this is very difficult. The interested reader is referred to Hill [2] for an extensive treatment of this subject.

Treating the cost parameter as a random variable requires data to form probability distributions for each branch of the network. At AiResearch/Phoenix the necessary information was gleaned from daily tabulation runs of actual processing times charged against the operations. Costs are obtained by multiplying the processing times by an average cost/hour. In many cases, actual processing times were for a batch of gears. When this occurred an average processing time per gear was calculated. In these cases a deterministic cost parameter was assumed as equal to this average value. For those operations which were considered as overhead items, the parameter estimates were also assumed to be deterministic.

Next, the underlying probability distributions of cost for the remaining operations were determined. This is no simple task; particularly if curves must be fitted for many operations and more than one type of distribution is considered. A curve-fitting technique developed by Dineen [1] simplifies this problem. Using the available data as input to this computer program, attempts to fit as many as nine types of distributions to the data can be made. The program makes a goodness-of-fit test and the resulting statistics from this test and the parameter estimates are printed. In this study, three distributions were considered: (1) exponential, (2) normal, and (3) uniform. In some cases more than one distribution could be statistically fitted to the data. In these situations, the curve with the best fit was used.

Three types of modifications are made to the network of Figure 3. These are:

1. Incorporating the newly obtained average costs for existing branches of the network;
2. Incorporating probability distributions for existing branches as determined by the curve fitting program, and
3. Expanding existing branches which represent groups of operations, some of which have an associated probability distribution for cost.

The revised network is shown in Figure 10 where the branch parameters are represented by their respective w-functions. Using Table IV, this network can be related to the network presented in Figure 3.

Using the GERT computer program for the modified network, the probability of an acceptable gear was found to be 0.857 with a mean cost of $39.14 and a variance of 17.68. The probability of a scrapped part is 0.143 with a mean cost of $29.89 and a variance of 192.37. Adjusting the average cost of acceptable gears to include scrappage, yields $43.42 as the expected cost of an acceptable gear.

For GERT networks containing only EXCLUSIVE-OR nodes, the n^{th} moment of the equivalent network depends only on the first n moments of the branches of the network [5]. Thus the probability and expected costs for the deterministic and random variable cases should be the same. The expected cost differed slightly due to the revised method of computing the average costs associated with the branches. Since the branch variances are increased when the cost is considered as a random variable, the vari-

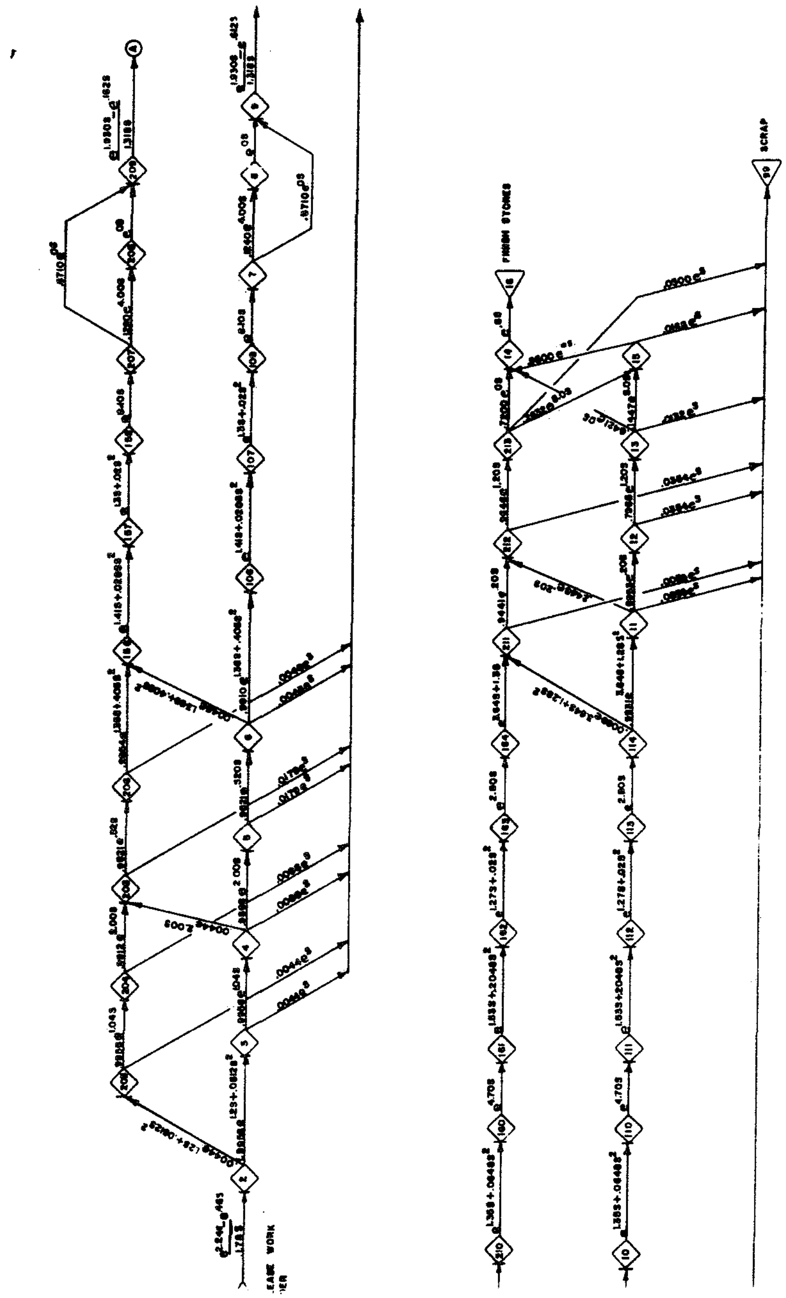


Figure 10. Extension and Refinement of the GERT Network for a Manufacturing Process

TABLE IV

COST PARAMETER ESTIMATES FOR THE REVISED GERT NETWORK

Mod. Type	Source Node	Sink Node	Operation Number	General Description of Operations	Dist.	Parameter Estimates ($)
2	1	2	10-30	Initial Processing & Milling	U	a = .46, b = 2.24
2	2	3	40	Milling	NΦ	μ = 1.20, σ = .32
2	2	203	40*[1]	Milling	NΦ	μ = 1.20, σ = .32
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
3	6	106	100-105	Milling	NΦ	μ = 1.36, σ = .90
3	6	156	100-105*	Milling	NΦ	μ = 1.36, σ = .90
-	6	99	SCRAP	Scrap after Operation 90	D	μ = 1.00
3	206	156	100-105*	Milling	NΦ	μ = 1.36, σ = .90
-	206	99	SCRAP	Scrap after Operation 90'	D	μ = 1.00
3	106	107	110	Milling	NΦ	μ = 1.41, σ = .24
3	156	157	110*	Milling	NΦ	μ = 1.41, σ = .24
3	075	108	120	Milling	NΦ	μ = 1.30, σ = .20
3	157	158	120*	Milling	NΦ	μ = 1.30, σ = .20
3	108	7	130-365	Milling & Heat Treatment	D	μ = 8.10
3	158	207	130-356*	Milling & Heat Treatment	D	μ = 8.10
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
2	9	10	370	Milling	U	a = .612, b = 1.93
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
2	209	210	370*	Milling	U	a = .612, b = 1.93
3	10	110	380	Grinding	NΦ	μ = 1.35, σ = .36
3	210	160	380*	Grinding	NΦ	μ = 1.35, σ = .36
3	110	111	390-430	Milling & Grinding	D	μ = 4.70
3	160	161	340-430*	Milling & Grinding	D	μ = 4.70
3	111	112	440	Grinding	NΦ	μ = 1.53, σ = .64
3	161	162	440	Grinding	NΦ	μ = 1.53, σ = .64
3	112	113	450	Grinding	NΦ	μ = 1.27, σ = .20
3	162	163	450*	Grinding	NΦ	μ = 1.27, σ = .20
3	113	114	460-480	Grinding	D	μ = 2.80
3	163	164	460-480*	Grinding	D	μ = 2.80
3	114	11	500	Grinding	NΦ	μ = 3.64, σ = 1.6
3	114	211	500*	Grinding	NΦ	μ = 3.64, σ = 1.6
3	164	211	500*	Grinding	NΦ	μ = 3.64, σ = 1.6
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.

Distribution Key

U - Uniform
NΦ - Normal
D - Discrete

[1] Non-conforming years are tagged and allowed to travel with acceptable gears.

*Operations performed on tagged gears.

NOTE: Only branches which were affected by modifications of types 2 and 3 are included in this table. Type 1 modifications are incorporated in the w-functions of the network presented in Figure 10.

ance of the equivalent network should also increase. From the results given above this increase was 6.02 for acceptable gears and 34.04 for scrapped gears. This gives management an indication of the variations they can expect in gear production cost. If the probability density function of the equivalent network cost was obtained, probability statements regarding the percentage of time the cost/gear exceeded specified dollar amounts could be made.

Summary and Conclusions

The GERT Approach was used to analyze a manufacturing process at AiResearch/Phoenix. The process selected for study was one which has caused many production problems in recent months. The analysis begins at the point where the problem is recognized and progresses to a solution by applying the steps of the GERT Approach.

In this report the network formulation and data collection steps of the GERT Approach were emphasized. A detailed description was given of how to convert a physical manufacturing process into a GERT network form. The form of the branch parameters were developed from written descriptions of the operations represented by each branch. The importance of the data collection phase of the analysis was stressed. In addition, the sources and procedures used in obtaining the necessary data were discussed.

A GERT analysis was performed for the manufacturing process. As part of the review process, Three alternatives were proposed and evaluated by means of an improvement analysis. In addition it was shown how to expand the basic model to include greater detail including the incorporation of costs as random variables.

Based on this research, it can be concluded that GERT can be used as a method for analyzing manufacturing processes. It provides a convenient mechanism for discussing, designing, and improving such processes.

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