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NATIONAL GEODETIC SATELLITE PROGRAM
(NGSP)
STATION SOLUTIONS

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ABSTRACT

Presented in this report are the results and analysis derived from a short-arc, orbital solution of the twelve stations, Phase I, PAGEOS network and the results of a geometric reduction combining the BC-4 and Baker-Nunn networks in a simultaneous adjustment.

Comparisons of the derived station solutions from the short-arc reductions are made with some of the published NWL doppler derived station coordinates and with some of the Ohio State University solutions. It is believed that a station accuracy of $\pm$ 15 m. has been achieved. Only preliminary comparisons could be made with the geometric solution since the problem of equation instability prevented an adequate solution.

Tables showing the Root Mean Square (RMS) of the orbital solution indicates that there is no statistical contradiction with the results obtained from the orthogonal polynomial fitting process, which was done at an earlier date for the initial data screening prior to the geodetic station solutions. Other tables are presented which give the final position and velocity vectors for all orbits used in the 12 station, simultaneous, BC-4 orbital solution. A representative sample of the orbital correlation matrix is also presented in the report.

The simultaneous, BC-4 orbital solution involved a total of 426 parameters using approximately 24,000 observations. Since there were an insufficient number of common observations between the BC-4 and Baker-Nunn networks the two networks were tied together through local surveys by introducing four distance constraints. The results of the adjustment produced condition numbers from the normal equation matrix which were very large. The condition numbers indicate the degree of ill-conditioning inherent in the coefficient matrix of the normal equations. The lack of good results from this adjustment is attributed to weak geometry and excessively large weights used to hold some of the parameters.

The technique of correcting observations for parallactic refraction, phase angle, and satellite abberration are discussed. Chebyshev polynomials were utilized to produce a set of correction coefficients which could be used to interpolate for corrections at any time along the satellite trace.

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SECTION 1
INTRODUCTION

This report contains the results of two different geodetic solutions from optical observations of PAGEOS and from simultaneous observations collected by the Baker-Nunn network of the Smithsonian Astrophysical Observatory. The first solution involved the simultaneous short arc adjustment of the PAGEOS traces and the second adjustment was generated from the interstation directions of the BC-4 sites and interstation directions between the Baker-Nunn stations. The short arc adjustment produced excellent results but the surface adjustment failed to converge.

Detailed information concerning the BC-4, PAGEOS data used in these investigations is contained in a previously published report, "Orthogonal Polynomial Representation of BC-4 Traces", progress report Vol. 1, January 1969 [1]. Briefly, the results of that work revealed the following:

1) The majority of the PAGEOS traces were accurately represented by a 4th degree polynomial except for high declination passes above 60 degrees,

2) The shorter traces, for either BC-4-300 or BC-4-450 cameras, required only a second degree polynomial,

3) The mean standard deviation of all polynomial representations was 1.7 arc sec in right ascension and 1.6 arc sec in declination,

4) The average sigmas for the BC-4-450 camera was 1.5 sec in right ascension and 1.4 sec in declination,

5) The average sigmas for the BC-4-300 camera was 2.0 sec in right ascension and 2.0 sec in declination and

6) The sigmas for both cameras correspond to an RMS of $\pm$ 3 microns at the scale of the photographic plate.

SECTION 2
PAGEOS SHORT ARC SOLUTION

2.1 OBSERVATIONS

Observations used in this solution included all BC-4 camera data on the PAGEOS satellite available from the NASA Data Center. The observations were collected from the Phase I sites of the PAGEOS network, Figure 1, and were final reduced by the Coast and Geodetic Survey according to procedures given in Reference 2. The satellite directions are given for each image of the traces in terms of apparent right ascension and declination uncorrected for satellite parallax, phase angle, and aberration. The observational time is given in UT-1 system with corrections applied to refer the time to the adopted longitude of NAD relative to the Naval Observatory. The PAGEOS field work has progressed appreciably beyond the Phase I stage but the data from these other phases have not been deposited at the Data Center.

2.2 METHOD OF REDUCTION

The short arc solutions were obtained by using the NEO-EMBET (N-Epoch Orbital Error Model Best Estimate of Trajectory) approach which was developed by DBA Systems, Inc. [3]. Unlike those data reduction methods where the orbit model is Keplerian or where it is represented by polynomials, the NEO-EMBET technique is carried out in a rectangular, inertial coordinate system resulting in three second order differential equations. The orbital integrator is that developed by Hartwell [4]. Hartwell developed the recursive analytic continuation technique wherein each coefficient of the power series is formed in terms of its predecessor. The series solution to the system of differential equations truncates the gravitational potential at n=7, excluding non-zonal terms. This technique of handling the orbital solution precludes singularities due to small orbital eccentricities and instability due to very short orbits.

NEO-EMBET uses two categories of parameters; namely, the inner loop parameters and the outer loop parameters. In general, the outer loop

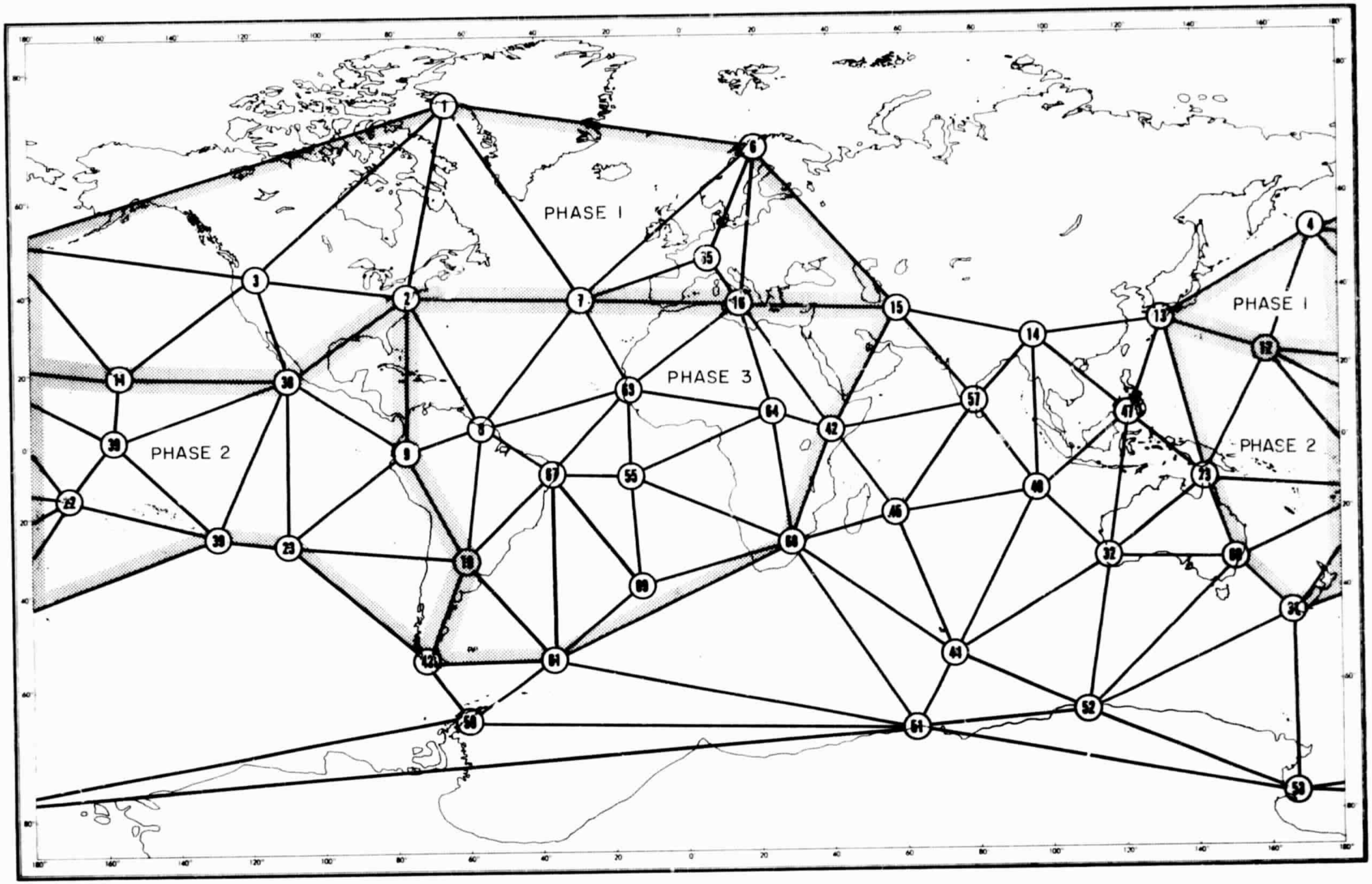

Figure 1

GEOMETRIC SATELLITE NETWORK (PAGEOS)

parameters are those which are common over all satellite passes, and the inner loop parameters are those which change from pass to pass. The coordinates of the observing sites are the most common set of outer loop parameters and the six orbital elements are typical inner loop parameters. A large scale, simultaneous adjustment of inner and outer loop parameters becomes practical by taking advantage of the highly patterned system of normal equations. The inner loop parameters in the normal equation system are 6 X 6 block diagonal matrices. The patterning makes it feasible to solve virtually an unlimited number of orbits simultaneously with the station coordinates.

2.3 RESULTS

2.3.1 Starting Coordinates

Since the BC-4 network was to be tied to the established Baker-Nunn net to produce a more extensive geodetic system, the local datum coordinates of the former were transformed to the SAO-C7 Geocentric System [5]. The quantities used to effect the datum shifts were the following:

$$\text{SAO-C7} = \text{NAD} + \quad [\,X = -26\text{m},\; Y = 155\text{m},\; Z = 185\text{m}\,] \tag{1}$$

$$\text{SAO-C7} = \text{ED} + \quad [\,X = -92\text{m},\; Y = -132\text{m},\; Z = -143\text{m}\,] \tag{2}$$

$$\text{SAO-C7} = \text{Old Haw.} + \quad [\,X = 59\text{m},\; Y = -263\text{m},\; Z = -203\text{m}\,] \tag{3}$$

The stations receiving datum shift (1) were Beltsville (6002), Moses Lake (6003), and Shemya (6004); shift (2) was applied to stations Catania, Sicily (6016), Tromso, Norway (6006), and Mashhad, Iran (6015); and shift (3) was applied to station Maiui, Hawaii (6011). Stations Thule, Greenland (6001), Gigedo Islands, Mexico (6038), Lajes AFB, Azores (6007), and Wake Island (6012) were either astronomic or map-scaled positions and received no shift to C7. The last station, Hohenpeissenberg, W. Germany, was defined on the Old Bavarian Geodetic Datum and it was shifted by X = 620m, Y = 4m, and Z = 418m, to place it on the SAO-C7 system. The local datum positions and the SAO-C7 starting positions for these stations are given in Tables 1 and 2. The C7 system was further enforced through the following SAO earth constants:

a_e = 6 378 142 m, ellipsoidal semi-major axis,

f^{-1} = 298.255, ellipsoidal flattening,

GM = 3.986009 X 10^{14} m^3 sec^{-2}, Constant of gravitation times the Earth's mass.

J_2 = 1082.639 X 10^{-6}

J_3 = -2.565 X 10^{-6}

J_4 = -1.608 X 10^{-6}

J_5 = -0.174 X 10^{-6}

J_6 = 0.542 X 10^{-6}

J_7 = -0.419 X 10^{-6}

Zonal coefficients of the Earth's gravitational field.

2.3.2 Initial Orbital Elements

The approximate position and velocity vectors for each orbit was obtained by selecting three simultaneous observations from two stations and geometrically intersecting these points to obtain the X, Y, Z space position of the satellite. One point was taken at the center of the satellite trace and the other two points were taken at the two ends of the shortest arc. The position of the mid-point was used as the position vector of the orbit and differences $\frac{\Delta X}{\Delta t}$, $\frac{\Delta Y}{\Delta t}$, $\frac{\Delta Z}{\Delta t}$, Δt = time increment, were used as average velocities. The station coordinates used for triangulating the orbit were the local datum positions, astros, or map-scaled locations as given in the NASA Station Directory and as shown in Table 1. The approximate orbital elements obtained in this manner were sufficiently well determined as not to require more than two or three iterations before converging to a final set.

2.3.3 Observation and Station Sigmas

The results of our orthogonal polynomial work [1] and results from other sources indicate that the RMS of the satellite traces should be 1.5 arc sec in the right ascension and declination for the BC-4 450mm and 2.0 arc

TABLE 1

LOCAL COORDINATES OF THE BC-4 PAGEOS SITES

(PHASE 1 STATIONS)

Sta. No.	Sta. Name	Latitude (N)	Longitude	h*(m)	Datum
6001	Thule, Greenland	76° 30' 00".000	291° 27' 30.000E	215/	Astro
6002	Beltsville, Maryland	39 01' 39".003	283 10 26.942E	44/45	NAD-27
6003	Moses Lake, Wash.	47 11 07".132	240 39 48.118E	369/358	NAD-27
6004	Shemya, Alaska	52 42 54".894	174 07 37".870E	35/-9	NAD-27
6006	Tromso, Norway	69 39 44".336	18 56 31.920E	106/	ED
6007	Azores	38 45 36".725	27 05 38.936W	52/-32	Local, Internat
6011	Hawaii; Maui	20 42 38".561	203 44 28.529E	3048/	Old Hawaii
6012	Wake Is.	19 17 23".227	166 36 39.780E	2/	Local Astro, Int.
6015	Mashhad, Iran	36 14 29".527	59 37 42.729E	989/953	ED 1950, Int.
6016	Catania, Italy	37 26 42".628	15 02 47.308E	8/46	ED, Int.
6038	Socorro, Mex.	18 43 44".93	110 57 20.72E	21/-13	Astro.
6065	Peisen, W. Germany	47 48 07".139	11 01 29.507E	943/	Old Bavarian

* h (m): elevations in meters above mean sea level and above the ellipsoid, respectively.

TABLE 2

STARTING COORDINATES

Station	Latitude (N)	Longitude	Altitude (m)
6001 Thule, Greenland	76° 30' 00.00"	291° 27' 30.00"	215
6002* Beltsville, Md.	39 01 39.33	283 10 27.36	10.4
6003* Moses Lake, Wash.	47 11 06.43	240 39 43.43	347
6004* Shemya, Alaska	52 42 50.02	174 07 29.80	78.4
6006* Tromso, Norway	69 39 44.77	18 56 23.14	97.5
6007 Azores	38 45 36.72	332 54 21.06	103.3
6011* Maui, Hawaii	20 42 26.70	203 44 37.66	3059
6012 Wake Island	19 17 23.23	166 36 39.78	23
6015* Mashhad, Iran	36 14 26.00	59 37 43.45	996
6016* Catania, Sicily	37 26 38.54	15 02 43.10	17
6038 Gigedo, Mex.	18 43 44.93	249 02 39.28	-8
6065* Hohenpeissenberg W. Germany	47 48 03.76	11 01 24.01	954

* On the C7 System

sec for the BC-4-300mm system. In view of this and due to the fact that only every other image of the satellite trace was being used in the short-arc adjustment, a mean standard deviation of one second of arc was adopted for the observations of both cameras.

The starting C7 coordinates of the BC-4 sites were constrained by modest amounts as compared to the actual estimates of station accuracy published by SAO. A priori sigmas of 300 meters in geodetic latitude and longitude and 100 meters in ellipsoidal height were applied to astronomic stations Thule, Azores, Wake Island and Gigedo; 80m in latitude, longitude and height were applied to stations Shemya, Tromso, Maui, Sicily, Mashhad, and Hohenpeissenberg; and 8 meters in the three position components was applied to Moses Lake. Station Beltsville (6002) served as the origin of the network and its coordinates were held fixed at the C7 values. The small sigmas of 8m for Moses Lake were chosen so that they would correspond to a scale of approximately one part in 400,000 between it and the Beltsville station. This scale is compatible with the scale of the orbit provided by GM = 398601 $\pm 1\ km^3\ sec^{-2}$.

The results of the adjustment proved that the above positional constraints were realistic. Only in two cases did the station corrections exceed one half of the value of the constraint. The exceptions were the astro station Gigedo which moved 399 meters northward and Mashhad which changed by 88m and 83m in geodetic latitude and longitude, respectively.

2.3.4 Preliminary Solutions

All PAGEOS orbits qualified through the orthogonal polynomial fitting process were used in a single adjustment of the 12 BC-4 stations. A total of 609 parameters were carried in this solution; 576 for the 96 orbits and 33 parameters for the eleven stations. The solution required approximately 3 hours of CDC-3800 computer time; including the on-line tabulations for two complete iterations. Approximately 250 observations per orbit were used for a total of 24,000 observations in the overall, preliminary adjustment.

The results of this adjustment revealed other problems with the data that were not uncovered by the polynomial fit. The fact that most stations in the network were being displaced by as much as one kilometer indicated that bad orbits were distributed through the net and the problem was not an isolated case. In order to systematically search out the bad data, it was decided to divide the network into three schemes and adjust each net individually. The first net included all NAD stations, Beltsville, Moses Lake, Thule, Hawaii, Wake Island and Shemya and the second network contained the ED sites, including station Azores (Figure 2). The solution of the North American Net resulted in removal of the following orbits:

Orbit (or event)	Stations co-observing
2421	6002, 6003
2677	6002, 6003
3561	6001, 6002
3935	6001, 6002
2666	6003, 6011
3574	6004, 6012
3781	6001, 6003
4292	6001, 6038
4398	6003, 6011, 6038

The criteria for rejecting these orbits was based on expected residuals consistent with present estimates of the station positions and camera performance. For example, the orbit residuals on the first iteration for station 6011, Hawaii on orbit 2666 above were consistently 100 arc sec or more while the residuals for other orbits for the same station averaged 1 or 2 seconds. Not all of the excluded orbits exhibited such high residuals; the average bad residual was more on the order of 10 seconds of arc.

A similar solution for the European Net and the use of the same rejection criteria resulted in the exclusion of the following orbits:

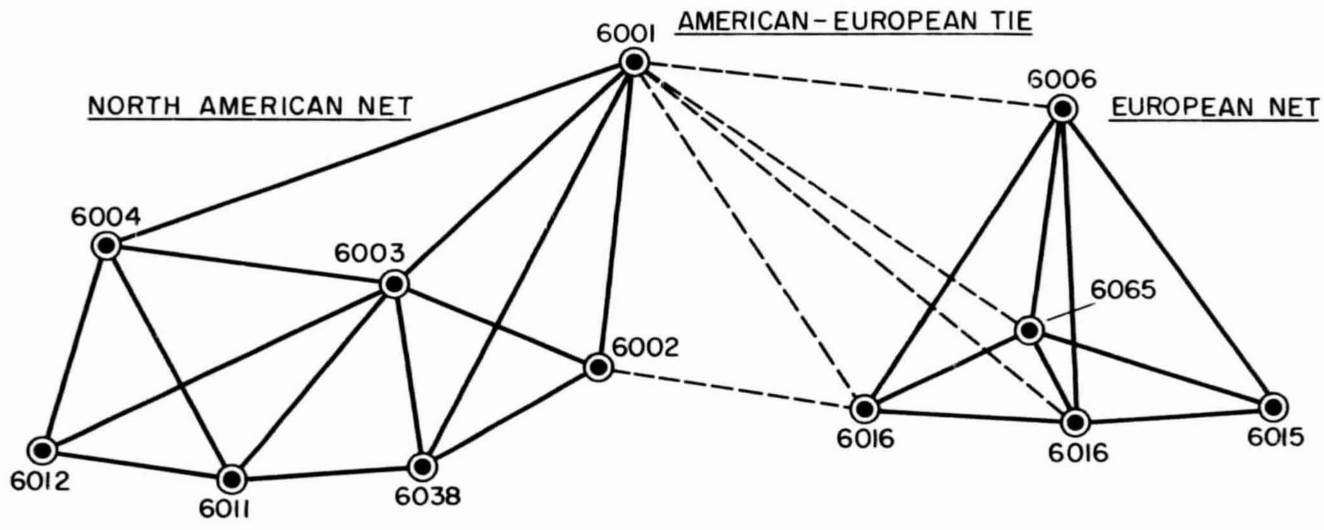

Figure 2

NETWORKS USED IN INDIVIDUAL ADJUSTMENTS

Orbit	Stations
2579	6007, 6016
3252	6006, 6015
3524	6016, 6065
3800	6006, 6007
3840	6006, 6065
3941	6006, 6065
3947	6006, 6065
2897	6006, 6015, 6016
4241	6007, 6016, 6065

After these two adjustments were completed, a North American-European tie was executed using stations Beltsville (6002) and Thule (6001) on the NA side with stations Tromso (6006), Azores (6007), Catania (6016) and Hohenpeissenberg (6065) on the European side (dotted lines Figure 2). Out of eleven orbtis available for this tie, three orbits had to be rejected:

Orbit	Stations
3250	6001, 6006
3446	6001, 6006
3949	6001, 6007

Orbit 3250 showed consistently high residuals of 14 seconds in right ascension for station Thule; orbit 3446 was off in both right ascension and declinations for both stations 6001 and 6006, and orbit 3949 showed right ascension discrepancies for both stations.

The elimination of the above orbits through these individual solutions left a total of 65 orbits for the final adjustment.

2.3.5 Final Adjustment by Short Arcs

The final short arc adjustment was generated with essentially no constraint on the orbital elements. The standard deviations of the position and velocity vector of the orbits were set at 10^8 meters in all six components so that the orbit would adjust freely. As was done in the preliminary adjustment, only every other data point was used in the solution and each observation was assigned a one arc second standard deviation. The solution involved 423 unknowns; 390 orbital elements and 33 station parameters, using approximately 16,250 observations. Table 3 shows the orbits (events) and other information pertinent to the data used.

The running time on the CDC-3800 for 3 full iterations amounted to 153 minutes. Iteration 3 produced a solution vector identical to the second iteration and iteration two changed the parameters by only 15% of iteration one.

The results of the station correction relative to the starting C7 coordinates are shown in Table 4 in terms of geodetic latitude, longitude, and height. Aside from the initial astro stations, most of the station movements look fairly good in view of the amount of data available. The standard deviations are a bit smaller than expected but they certainly should not be larger than twice their listed values. One of the more surprising aspects of the results was the uniformity and the relative low sigma values in station height. It had been expected, based on previous geometric solutions and various simulation studies of geometric networks, that these sigmas would be 1.5 to 2 times higher than the sigmas in the latitude and longitude components. As it turned out, the magnitude of σ_h was the same as σ_φ and σ_λ - a fact probably attributable to the uniform scale provided by GM over the whole network.

From a broad inspection of Table 4 we can make the following general remarks regarding the adjustment: 1) the rather large movement of station Thule was to be expected in view of its initial map-scaled position, 2) Moses Lake, assigned a σ_φ, σ_λ, σ_h of 8 meters, has changed consistent with the NAD positional accuracy relative to Beltsville, 3) Shemya's position on NAD has never been considered more accurate than 50 meters in its horizontal position, consequently a shift $\Delta\varphi$ = -57m, $\Delta\lambda$ = -68m, and Δh = -38 should be expected, 4) the shift to Tromso are essentially within the estimated accuracy of the SAO-C7 system, 5) Azores is an astro and its geodetic shift is difficult to estimate but the values listed are acceptable, 6) Hawaii is definitely within the C7 uncertainties, 7) Wake is an astro and its corrections look valid, 8) Mashhad's corrections appear large based on our present knowledge of the

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC of ARC)		POLY DEG.		INITIAL DEC. (DEG)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
* 6001	3185	B688	248	1.0	1.5	4	3	45	12	228	450
* 6001	3538	B748	299	1.0	1.4	2	3	18	-15	284	450
6001	2965	B650	220	1.8	2.3	3	3	62	-16	203	450
6001	2855	B635	272	1.4	1.7	4	4	43	13	254	450
6001	3781	B790	175	0.9	1.1	4	4	48	-12	154	450
* 6001	3787	B794	200	1.6	2.2	3	3	51	-15	178	450
* 6001	3535	B746	297	1.2	1.5	4	3	38	-15	273	450
* 6001	3795	B799	173	1.1	1.4	4	3	44	-11	154	450
* 6001	3436	B713	225	0.9	1.4	4	3	45	-13	203	450
* 6001	3448	B719	223	1.2	1.3	4	4	56	-15	203	450
6001	3027	B654	298	1.2	1.6	4	4	38	15	273	450
* 6001	3481	B737	225	1.4	1.2	8	6	78	-8	203	450
6001	3458	B724	274	1.4	1.8	4	3	31	-13	261	450
6001	3520	B739	125	0.7	1.6	5	3	46	-15	113	450
6001	3933	B834	198	1.1	1.1	4	4	53	-15	182	450
* 6001	4061	B870	200	1.2	1.5	4	4	46	-13	185	450
6001	3949	B841	200	1.1	1.4	4	4	54	-16	185	450
* 6001	3772	B784	222	1.2	1.7	4	3	43	-14	205	450
* 6001	2893	B639	317	1.0	1.2	4	4	31	16	302	450
* 6001	3837	B825	199	1.3	1.4	4	4	53	-15	178	450
6001	3935	B835	197	1.3	1.8	5	4	66	-18	186	450
* 6001	2894	B640	375	1.1	1.5	4	3	20	14	351	450
* 6001	3775	B787	246	1.1	1.3	4	4	39	-13	226	450
6001	4292	B893	350	1.2	2.0	4	3	22	-16	328	450
6001	3941	B837	196	1.0	1.2	5	4	60	-17	185	450
6001	3687	B768	167	1.6	1.4	5	6	78	-7	154	450
* 6001	4251	B890	298	1.2	1.5	4	3	20	-13	278	450
6001	3782	B791	245	1.0	1.2	4	4	45	-13	227	450
* 6001	3173	B686	298	1.5	1.9	4	4	33	13	278	450
6001	3524	B741	274	1.1	1.6	4	3	30	-13	261	450
6001	3446	B718	224	1.3	1.5	5	5	58	14	226	450
* 6001	3560	B753	272	1.0	1.5	3	3	25	-16	261	450
6001	3250	B696	266	1.6	2.1	4	4	37	15	251	450
* 6001	3483	B738	273	1.1	1.8	5	4	49	15	252	450
6001	3561	B754	246	0.8	1.0	4	4	51	-14	234	450

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC) R.A. cos δ	SIGMA (SEC. of ARC) DEC.	POLY DEG. R.A.	POLY DEG. DEC.	INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
6002	2452	8319	291	•	1•1		3	20	10	277	450
6002	2481	8323	223	0•7	0•6	4	3	39	14	223	450
6002	2420	8308	423	21•9	2•4	10	10	73	8	397	300
* 6002	3539	E393	200	0•9	0•9	4	3	58	-14	180	450
* 6002	3538	E392	199	1•1	1•2	4	3	52	-14	192	450
6002	2965	D388	291	•	•						450
6002	2677	B365	272	•9	1•2	3	3	2	13	255	450
* 6002	3787	E406	275	1•9	1•2	6	6	72	-17	250	450
6002	2574	B346	271	2•0	2•2	4	3	48	15	252	450
6002	2421	S309	472	2•6	2•3	10	9	86	-21	471	300
6002	2492	B326	225	0•8	0•9	5	4	54	12	528	450
* 6002	2531	B334	298	1•7	1•4	3	3	35	12	281	450
6002	2614	B354	250	0•8	1•2	[illegible]	3	15	14	228	450
6002	2419	S307	350	•	94•	4	2	50	21	230	300
* 6002	2497	B328	249	1•0	0•9	4	3	35	14	233	450
* 6002	2694	B639	347	1•5	1•6	3	3	-8	16	326	450
* 6002	2893	D375	325	1•3	1•0	5	5	63	9	304	450
6002	2892	D374	275	1•3	1•1	4	4	55	12	254	450
6002	3935	E416	275	1•1	1•6	6	4	66	14	256	450
* 6002	4267	E450	174	1•8	1•7	3	3	36	-11	154	450
* 6002	4244	E444	272	1•1	1•4	4	4	30	-19	251	450
* 6002	2909	D379	297	1•6	1•6	3	3	1	15	277	450
* 6002	4251	E446	200	1•1	1•1	4	4	60	-14	178	450
* 6002	2523	B333	297	1•0	1•3	4	3	23	13	281	450
6002	2517	B331	248	•	1•0	1	3	38	12	238	450
6002	2491	B352	294	•	3•1	0	3	51	11	277	450
* 6002	2542	B340	249	0•8	0•9	4	3	29	15	228	450
* 6002	3560	E394	225	1•1	1•2	4	4	66	-14	203	450
* 6002	4259	E448	250	1•2	1•4	4	2	16	-14	228	450
* 6002	4236	E422	224	1•2	1•5	3	3	9	-13	213	450
* 6002	4182	E436	249	1•5	1•3	4	4	41	-13	228	450
6002	3561	E395	223	1•2	1•1	4	3	61	-9	202	450

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
6003	2452	4385	250	1.0	1.1	4	4	17	10	227	450
6003	2481	4397	273	.	1.1		3	21	10	253	450
6003	2420	4369	324	1.2	1.1	4	4	42	11	302	450
* 6003	3185	A479	273	3.5	1.1	9	10	83	-4	255	450
* 6003	3539	A491	199	1.0	1.2	4	3	50	-13	177	450
6003	2666	A443	274	.9	1.3	3	3	6	14	251	450
6003	2855	A460	257	.	1.2	8	9	79	3	276	450
6003	2677	A446	344	1.2	1.2	3	3	-6	12	327	450
6003	3781	A510	199	2.0	1.5	5	5	74	-4	177	450
* 6003	3795	A513	200	2.1	1.2	6	7	79	-4	181	450
6003	2587	A431	295	1.2	1.5	4	3	20	12	1107	450
6003	2421	A370	306	1.1	1.2	5	4	58	3	302	450
6003	2492	A402	247	0.9	0.9	4	4	39	13	568	450
* 6003	4061	A544	223	1.5	1.5	4	3	53	-9	208	450
* 6003	2672	A445	298	0.9	1.2	3	3	-5	14	275	450
6003	2614	A436	344	1.0	1.2	4	4	4	16	326	450
6003	2419	A368	317	1.4	1.3	4	4	24	13	302	450
* 6003	2497	A404	275	1.0	1.0	4	4	24	14	250	450
* 6003	3837	A521	223	1.8	1.7	4	4	62	-8	203	450
* 6003	4267	A574	200	1.1	1.2	3	3	26	-13	182	450
* 6003	3978	A531	248	1.7	1.9	7	5	66	6	231	450
* 6003	2661	A441	304	1.1	1.4	4	3	26	14	301	450
* 6003	2678	A447	295	1.0	1.6	3	3	0	15	275	450
6003	4398	A583	244	0.9	1.1	4	4	20	-13	238	450
* 6003	4276	A756	195	1.1	1.2	4	4	43	-15	184	450
6003	3782	A511	250	1.5	1.5	5	4	64	-1	226	450
* 6003	2523	A413	225	1.0	1.5	3	3	24	13	203	450
6003	2517	A411	225	1.0	1.1	4	3	39	14	203	450
* 6003	3173	A478	273	1.2	1.1	5	3	59	8	255	450
6003	2609	A435	271	1.0	1.5	3	3	7	15	1020	450
* 6003	2542	A417	299	1.3	1.4	4	3	14	14	277	450
* 6003	4182	A557	148	1.3	1.6	3	3	47	-13	138	450
* 6003	4212	A567	200	1.0	1.5	3	3	22	-14	184	450
* 6003	4196	A562	198	1.2	1.9	3	3	33	-16	184	450
6003	3817	B812	275	1.2	1.4	4	4	42	-14	250	450
* 6003	4236	A571	318	1.8	2.0	3	3	3	-14	302	450

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
6004	4178	1070	284	•	2•4	0	4	40	-22	275	300
* 6004	2866	1009	431	1•9	2•1	4	4	5	19	419	450
* 6004	3483	1036	371	1•8	2•4	6	5	76	-17	352	450
6004	3817	1055	148	2•0	1•6	4	4	73	-3	134	450
* 6004	3409	1033	320	1•9	2•5	4	4	25	-23	302	300
6004	3574	1040	267	1•5	1•8	4	4	24	-19	251	450
6004	3752	1052	243	2•9	2•9	5	4	63	-11	275	450
* 6004	3775	1051	246	1•8	1•6	4	4	57	-12	227	450
* 6004	3978	1062	272	1•5	1•7	5	4	58	-9	254	450
* 6004	3488	1038	246	1•6	2•1	3	3	12	-17	255	450

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC) R.A. cos δ	SIGMA (SEC. of ARC) DEC.	POLY DEG. R.A.	POLY DEG. DEC.	INITIAL DEC. (DEG)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
* 6006	2883	C240	319	1.5	1.8	4	3	24	13	302	450
6006	3252	C262	256	2.4	2.9	3	2	28	-13	258	450
* 6006	2958	C248	240	1.6	2.2	2	2	28	13	234	450
* 6006	3436	C298	224	2.8	3.1	4	4	49	-13	205	450
* 6006	3448	C303	196	1.7	2.0	4	4	55	-11	180	450
* 6006	2891	C243	324	1.5	2.4	4	3	20	16	302	450
* 6006	3481	C313	248	1.9	1.9	5	5	63	12	228	450
6006	2897	C245	325	1.7	1.9	3	3	13	14	303	450
* 6006	2646	C227	249	1.2	1.8	4	3	40	14	227	450
* 6006	2818	C235	273	1.0	1.6	4	2	28	14	254	450
6006	3520	C331	124	1.2	1.7	6	4	44	15	107	450
6006	3933	C386	223	1.4	1.5	4	4	53	-13	203	450
6006	3800	C376	196	1.6	1.7	4	4	52	-13	177	450
6006	3034	C251	277	.	1.8	0	3	23	15	273	450
6006	3840	C377	149	1.6	2.2	4	4	61	-12	134	450
6006	3145	C256	224	1.3	1.8	4	4	59	-14	204	450
6006	2890	C242	260	1.7	2.3	4	3	35	14	254	450
6006	2892	C244	337	1.9	2.7	4	3	19	14	353	450
6006	4021	C399	387	2.4	2.6	4	4	52	-15	186	450
* 6006	4020	C398	172	1.5	2.2	4	3	60	-16	158	450
6006	3941	C389	197	1.4	2.2	4	4	54	-11	181	450
6006	3687	C353	227	2.8	4.2	4	3	58	16	228	450
6006	3446	C302	261	2.2	3.1	4	3	42	13	254	450
* 6006	3429	C294	199	1.6	2.1	4	3	43	12	180	450
6006	3250	C261	267	1.6	1.9	4	4	35	13	255	450
6006	3939	C387	195	1.6	1.7	4	4	57	-15	181	450
6006	3947	C390	175	1.4	1.6	4	4	57	-14	160	450
* 6006	4083	C401	149	1.0	1.8	4	3	50	-13	138	450
6006	3483	C314	271	2.1	2.4	4	4	44	10	254	450

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
* 6007	2505	X013	359	1.5	1.5	4	4	10	22	397	300
6007	2622	X040	353	1.4	1.4	4	4	26	21	370	300
6007	2579	X034	357	1.9	2.3	4	3	22	24	370	300
* 6007	2883	X076	421	2.7	2.0	5	5	50	19	422	300
6007	3027	X082	243	2.3	2.2	4	3	59	6	238	300
6007	2466	X004	342	1.9	1.8	4	4	33	17	401	300
* 6007	2675	X051	475	1.8	2.0	4	4	2	21	440	300
6007	2574	X033	403	2.2	1.8	4	4	34	20	433	300
6007	3933	X094	330	3.2	2.9	9	8	68	18	316	300
* 6007	2531	X023	321	1.6	1.9	6	3	52	22	341	300
6007	3800	X090	275	2.1	1.7	9	8	74	9	250	300
6007	3949	X095	135	2.1	2.6	4	4	71	7	144	300
* 6007	2694	X054	423	1.6	1.8	3	4	-9	19	394	300
6007	4021	X097	188	3.0	2.5	5	5	74	4	181	300
6007	4201	X102	294	.	1.7	1	10	81	-7	274	300
6007	4241	X104	323	1.8	1.8	5	4	63	-13	296	300
* 6007	2909	X078	467	2.4	2.3	4	4	-2	20	440	300
* 6007	2520	X018	360	2.1	2.3	6	3	53	23	370	300
6007	2491	X010	261	1.6	1.4	5	4	58	14	307	300
* 6007	4083	X098	273	2.9	2.8	4	4	60	-8	254	300
* 6007	4210	X103	245	2.0	2.2	5	4	71	-20	231	300
* 6007	2472	X006	366	1.7	1.5	4	4	35	18	399	300
6007	2515	X015	248	.	1.3	0	4	13	21	307	300

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
6011	2666	V157	473	1.9	2.1	4	3	22	20	444	300
6011	2587	V142	320	1.5	1.9	5	2	56	19	1492	300
6011	2640	V153	272	1.5	1.6	4	4	31	19	1252	300
* 6011	2672	V158	372	1.5	1.7	4	3	16	16	349	300
* 6011	2894	V187	384	3.8	2.4	8	7	76	5	374	300
* 6011	3488	V218	321	1.7	1.7	4	4	52	-22	298	300
* 6011	2661	V156	490	2.6	2.4	7	5	61	18	466	300
6011	4618	U499	215	1.5	1.8	3	3	-1	20	202	300
6011	4618	V278	262	1.8	1.8	3	4	-5	19	242	300
6011	4398	V244	214	2.2	2.7	4	3	57	-17	258	300
* 6011	4406	V246	313	1.3	1.4	4	3	51	-21	298	300
* 6011	2703	V164	424	1.9	2.2	4	3	29	17	397	300
* 6011	2678	V160	400	1.8	1.8	4	3	21	20	374	300
* 6011	4212	V236	372	1.2	1.3	4	4	47	-19	350	300
* 6011	2736	V166	422	2.0	2.4	5	3	47	20	397	300
* 6011	2679	V161	347	1.8	1.7	3	3	16	18	324	300
* 6011	4245	V239	373	2.8	2.8	4	3	51	-23	348	300
6011	4605	V276	210	1.0	1.3	3	4	-2	19	195	300
6011	2609	V147	336	1.7	1.8	4	3	27	20	1596	300

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
6012	4178	Z072	275	1.3	1.1	6	5	78	-13	254	300
6012	4605	Z118	245	1.9	2.4	3	4	-1	19	250	300
* 6012	2866	Z026	515	1.8	1.8	4	3	32	22	510	300
* 6012	2679	Z005	371	1.6	1.7	4	3	18	21	350	300
* 6012	3409	Z053	261	1.5	1.7	5	4	74	-24	324	300
6012	3574	Z066	317	1.8	1.9	6	5	74	-22	322	300
* 6012	2703	Z007	368	2.9	3.4	3	3	44	20	351	300
* 6012	2736	Z012	494	1.9	1.9	4	4	36	20	462	300
6065	3818	D021	250	1.0	1.1	3	4	24	-16	226	450
* 6012	2661	Z002	348	2.0	1.4	3	4	53	13	374	300

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
6015	3252	Y082	316	2.3	2.1	6	4	74	-17	298	300
6015	3535	Y108	354	.	3.2	1	5	67	5	350	300
6015	2897	Y061	435	1.8	1.6	4	4	36	20	420	300
* 6015	2646	Y034	370	2.3	2.0	5	4	53	12	404	300
6015	3145	Y080	434	.	40.	1	10	69	19	490	300
6015	2890	Y060	317	3.3	2.9	5	4	62	11	305	300
* 6015	2611	Y027	208	1.4	1.5	3	3	12	13	211	300
* 6015	3545	Y110	316	1.5	1.5	4	4	47	-22	298	300
* 6015	2626	Y030	175	1.4	2.0	3	3	10	10	175	300
6015	2494	Y004	403	1.9	1.8	3	3	15	17	2000	300
6015	2499	Y005	431	2.3	2.4	4	3	29	19	2600	300
* 6015	4233	F144	223	1.3	1.6	6	5	77	-10	203	450
6015	3939	Y127	279	4.2	4.6	9	6	68	18	323	300
6015	3947	Y128	328	2.9	3.3	9	7	68	16	319	300

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
* 6016	2505	S178	422	1.1	1.3	4	4	9	22	400	300
6016	2622	S213	374	1.5	1.7	4	3	29	20	350	300
* 6016	3352	S306	474	1.5	1.6	4	4	24	20	439	300
6016	3279	D002	223	1.2	1.6	4	3	56	-18	201	450
6016	2579	S198	442	2.2	2.4	4	4	16	20	420	300
6016	2883	S259	221	.	3.0	1	3	56	11	232	300
* 6016	2958	S271	325	2.1	2.4	7	5	63	18	326	300
* 6016	3535	S327	316	3.1	2.6	5	6	69	-3	350	300
* 6016	3447	S315	268	1.2	1.9	4	4	64	-24	252	300
* 6016	2891	S261	423	1.9	1.9	4	4	46	18	397	300
6016	2897	S262	418	1.4	1.4	5	4	39	22	397	300
6016	2466	S163	337	1.4	1.9	5	4	55	21	326	300
6016	3458	S316	325	.	52.	10	10	83	-7	298	300
6016	2646	S222	438	3.7	1.8	10	10	72	10	418	300
* 6016	2675	S229	374	1.3	1.8	4	4	5	21	350	300
6016	3034	S275	347	.	2.0	0	5	56	19	338	300
* 6016	2818	S250	315	3.4	2.8	5	5	62	16	304	300
* 6016	2611	S208	171	1.5	1.8	3	2	9	8	167	300
* 6016	3545	S329	296	1.8	1.6	4	3	49	-22	274	300
* 6016	2626	S214	175	1.5	1.6	3	3	10	9	167	300
6016	2494	S173	342	.	1.6	0	4	11	18	2600	
6016	2499	S175	347	1.4	1.4	4	4	28	19	2600	300
6016	4241	S399	274	1.9	1.3	5	6	74	-12	249	300
* 6016	4233	S398	250	3.6	3.0	4	3	58	-11	227	300
6016	3818	S350	325	1.7	1.7	5	5	76	-20	299	300
6016	3946	S368	515	1.4	1.5	4	4	23	18	490	300
* 6016	2520	S184	456	1.7	1.7	4	4	37	18	444	300
6016	3524	S326	321	3.2	2.2	6	5	71	-12	298	300
* 6016	2472	S166	335	1.6	2.1	6	3	54	21	326	300
6016	2515	S182	447	.	2.2	1	4	9	21	419	300

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
* 6038	4267	U423	322	1.2	1.2	5	4	67	-18	298	300
* 6038	4244	U417	321	2.1	2.0	4	3	52	-18	298	300
6038	4292	U427	396	9.1*	2.4	10	10	84	-12	374	300
6038	4398	V447	394	2.6	2.5	4	4	47	-18	374	300
* 6038	4406	U449	445	1.4	1.4	4	4	40	-21	423	300
* 6038	4251	U419	375	1.8	1.3	5	5	73	-12	350	300
* 6038	4276	U425	375	2.7	2.2	7	5	79	-17	349	300
* 6038	4259	U421	298	2.0	1.8	4	3	45	-19	275	300
* 6038	4182	U406	197	1.9	1.8	5	5	82	-11	181	300
* 6038	4245	U418	386	2.0	2.0	4	4	50	-23	374	300
* 6038	4212	U411	350	3.0	2.9	4	4	62	-22	324	300
* 6038	4196	U408	300	2.6	2.0	5	4	72	-19	287	300
6038	4236	U416	223	1.9	1.6	3	3	31	-15	227	300

TABLE 3

SUMMARY OF BC-4 PAGEOS DATA AND POLYNOMIAL FIT (Continued)

STATION	EVENT	PLATE	TOTAL IMAGES	SIGMA (SEC. of ARC)		POLY DEG.		INITIAL DEC. (DEG.)	DELTA DEC.	TIME SPAN SECS.	CAMERA BC-4
				R.A. cos δ	DEC.	R.A.	DEC.				
* 6065	3352	D005	271	1.7	2.0	3	3	21	13	255	450
6065	3279	S298	275	1.8	2.4	5	3	73	-20	252	300
* 6065	3447	D008	170	1.5	1.6	3	3	42	-15	154	450
6065	3933	D034	275	1.3	1.2	5	4	57	-5	249	450
6065	3840	D029	198	1.5	1.4	4	4	61	-6	185	450
* 6065	3772	D020	222	1.3	0.9	5	6	72	-3	200	450
6065	4021	D056	199	1.3	1.1	5	4	65	-5	183	450
* 6065	4020	D005	198	1.2	0.9	6	5	73	-6	183	450
6065	4201	D088	225	1.4	1.3	4	3	50	-9	203	450
6065	4241	D095	219	1.1	0.9	5	5	70	-16	203	450
* 6065	4233	D093	224	1.0	0.9	4	4	49	-12	203	450
6065	3941	D038	273	1.6	1.4	5	4	56	-3	250	450
6065	3946	D040	350	1.6	1.6	4	3	5	9	335	450
6065	3524	D010	223	1.1	1.0	5	4	69	-14	200	450
* 6065	3429	D007	223	1.3	1.3	6	4	68	9	205	450
* 6065	3939	D037	200	1.6	1.6	4	3	56	-7	181	450
6065	3947	D041	200	1.2	1.3	4	3	57	-7	179	450
* 6065	4210	D092	263	1.0	1.3	4	4	37	-14	251	450

* Orbits (or events) used in the short arc orbital adjustment

Note: A number larger than 5 or a number omitted from columns 5 and 6 indicate that the polynomial representation of the trace was inadequate. Generally this problem will also be reflected in columns 7 and 8 where the degree of the polynomial is either 0 or 1.

TABLE 4

CORRECTIONS TO PROVISIONAL COORDINATES

Station No.	Name	Correction (m)	Sigmas (m)
6001	Thule, Greenland	$\Delta\phi$ (m) = 141.8 $\Delta\lambda$ (m) = 176.1 Δh (m) = -26.4	± 8 ± 2 ± 4
6003	Moses Lake, Wash.	$\Delta\phi$ (m) = 5.1 $\Delta\lambda$ (m) = -14.8 Δh (m) = -12.4	± 1 ± 7 ± 3
6004	Shemya, Alaska	$\Delta\phi$ (m) = -57.4 $\Delta\lambda$ (m) = -68.4 Δh (m) = -38.4	± 8 ± 10 ± 9
6006	Tromso, Norway	$\Delta\phi$ (m) = 16.2 $\Delta\lambda$ (m) = 26.7 Δh (m) = 19.5	± 5 ± 10 ± 7
6007	Azores	$\Delta\phi$ (m) = -37.8 $\Delta\lambda$ (m) = 58.8 Δh (m) = 16.3	± 4 ± 9 ± 5
6011	Hawaii	$\Delta\phi$ (m) = 1.1 $\Delta\lambda$ (m) = 0.5 Δh (m) = 24.0	± 7 ± 10 ± 9
6012	Wake Island	$\Delta\phi$ (m) = 152.6 $\Delta\lambda$ (m) = 1.2 Δh (m) = -4.6	± 10 ± 9 ± 15
6015	Mashhad	$\Delta\phi$ (m) = -83.3 $\Delta\lambda$ (m) = 88.1 Δh (m) = 5.0	± 12 ± 9 ± 15
6016	Catania, Sicily	$\Delta\phi$ (m) = -19.5 $\Delta\lambda$ (m) = 0.2 Δh (m) = 24.9	± 8 ± 10 ± 10
6038	Gigedo	$\Delta\phi$ (m) = 399.2 $\Delta\lambda$ (m) = 49.6 Δh (m) = 12.3	± 5 ± 5 ± 4
6065	Peisen, Germany	$\Delta\phi$ (m) = -26.0 $\Delta\lambda$ (m) = -8.1 Δh (m) = -4.3	± 7 ± 10 ± 8

ED extension to that area, 9) station movements for Catania and Hohenpeissenberg are of the order expected, and 10) Gigedo is an astro and could well receive a shift of $\Delta\varphi$ = 399m, $\Delta\lambda$ = 50m and Δh = 12m.

The final positions of the solution are listed in Table 5.

2.3.6 Orbit Residuals

Table 6 shows the root mean square (RMS), about the mean of the residuals for each orbit in the adjustment. The average RMS from all entries in this Table is 1.7 arc sec in right ascension and 1.8 arc sec in declinations. These values are almost identical to the average RMS of the orthogonal polynomial fit; 1.7" and 1.6" respectively. The slight RMS difference in declination, (1.8" - 1.6" = 0.2"), between the orbit residuals and the polynomial fit is probably due to the larger number of orbits used in obtaining the mean from the polynomial results. In any case, one should expect these values to agree with each other unless the adjustment had distorted the spatial network due to additional bad data left in the solution. Apparently, this was not the case since the RMS for most orbits matched the polynomial RMS very closely.

Table 7 shows the residuals of Table 6 grouped according to observing stations and camera systems; the 300mm FL and 450mm FL camera. Notable in this table are the slightly larger mean RMS for the BC-4-300 system. The average RMS for each camera compute to,

BC-4-450: R.A. cos δ = 1.6", Dec = 1.6"
BC-4-300: R.A. cos δ = 2.0", Dec = 2.1"

These RMS' are within 0.1 arc seconds of the corresponding mean from the Orthogonal Polynomial fit.

2.3.7 Correlation

The final orbital elements from the adjustment are given in Table 8. The position and velocity vectors are in earth fixed coordinates (referred to the Greenwich meridian) defined on the C7 system.

TABLE 5

FINAL COORDINATES OF SHORT ARC ORBITAL ADJUSTMENT

(a_e = 6 378 142, f^{-1} = 298.255)

Station	φ (N) / X (m)	λ (E) / Y (m)	h (m) / Z (m)
6001	76° 30' 04.73" 546 554m	291° 27' 54.43" -1 389 990m	188.6 6 180 202m
6002*	39 01 39.33 1 130 773	283 10 27.36 -4 830 833	10.4 3 994 706
6003	47 11 06.60 -2 127 831	240 39 42.70 -3 785 842	334.6 4 656 029
6004	52 42 48.11 -3 851 788	174 07 26.04 396 420	40.0 5 051 319
6006	69 39 45.31 2 102 913	18 56 25.69 721 648	78.0 5 958 139
6007	38 45 35.46 4 433 660	332 54 23.57 -2 268 179	119.6 3 971 641
6011	20 42 26.71 -5 465 988	203 44 37.69 -2 404 386	3035.0 2 242 199
6012	19 17 28 32 -5 858 557	166 36 39.79 1 394 511	18.4 2 093 808
6015	36 14 23.22 2 604 337	59 37 47.09 4 444 269	1001.0 3 750 279
6016	37 26 37.89 4 896 430	15 02 43.10 1 316 145	41.9 3 856 647
6038	18 43 58.24 -2 160 983	249 02 41.02 -5 642 717	4.3 2 035 369
6065	47 48 02.89 4 213 588	11 01 24.01 820 820	949.7 4 702 735

* 6002 Beltsville, was held fixed on the SAO, C7 System; the shifts applied for North American Datum to C7 system were:

X (C7) = X (NAD) - 26m

Y (C7) = Y (NAD) + 155m

Z (C7) = Z (NAD) + 185m

TABLE 6

ORBIT RESIDUAL

Orbit	Station	Residuals (RMS) Sec of Arc		Orbit	Station	Residuals (RMS) Sec of Arc	
		RA cos δ	Dec			RA cos δ	Dec
2472	6007	1.6	1.8	2891	6006	1.6	2.4
	6016	1.6	2.2		6016	1.9	2.2
2497	6002	0.9	0.8	2893	6001	1.1	1.8
	6003	1.1	1.2		6002	1.2	1.4
2505	6007	1.8	1.4	2894	6001	1.3	1.8
	6016	1.2	1.4		6011	3.0	2.4
2520	6007	2.0	2.4	2909	6002	1.6	1.6
	6016	1.4	1.8		6007	2.2	2.6
2523	6002	1.1	1.0	2958	6006	1.7	2.4
	6003	1.0	1.4		6016	2.2	2.2
2531	6002	1.6	1.2	3173	6001	1.7	1.8
	6007	1.8	1.8		6003	1.0	1.0
2542	6002	0.8	0.8	3185	6001	1.0	1.6
	6003	1.2	1.2		6003	2.0	1.2
2611	6015	1.6	1.4	3352	6016	1.5	1.6
	6016	1.6	1.8		6065	1.7	1.8
2626	6006	1.6	2.2	3409	6004	2.0	2.6
	6016	1.4	1.8		6012	1.4	1.8
2646	6006	1.3	1.8	3429	6006	1.6	2.4
	6015	2.7	1.8		6065	1.3	1.2
2661	6003	1.2	1.2	3436	6001	1.6	1.4
	6011	2.1	2.2		6006	2.9	2.8
	6012	1.4	1.6	3447	6016	1.4	2.2
2672	6003	1.0	1.2		6065	1.7	1.8
	6011	1.5	1.6	3448	6001	2.2	1.4
2675	6007	1.8	2.2		6006	1.6	2.0
	6016	1.4	1.8	3481	6001	4.2	1.4
2678	6003	1.0	1.8		6006	2.1	2.0
	6011	1.6	2.0	3483	6001	3.5	1.6
2679	6011	1.7	1.8		6004	1.9	2.2
	6012	1.6	1.8		6006	2.2	2.8
2694	6002	1.2	1.2	3488	6004	1.7	1.8
	6007	1.4	1.2		6011	1.9	1.8
2703	6011	2.1	2.2	3535	6001	1.2	1.4
	6012	2.8	3.2		6016	3.1	3.2
2736	6011	2.1	2.4	3538	6001	1.2	1.4
	6012	1.7	2.0		6002	1.2	1.4
2818	6006	1.2	1.4	3539	6002	1.0	0.8
	6016	3.0	2.8		6003	1.0	1.0
2866	6004	1.5	2.0	3545	6015	1.5	1.6
	6012	1.9	1.8		6016	1.8	1.6
2883	6006	1.4	1.8	3560	6001	1.2	1.6
	6007	2.5	2.2		6002	1.2	1.4

TABLE 6

ORBIT RESIDUAL (Continued)

Orbit	Station	Residuals (RMS) Sec of Arc		Orbit	Station	Residuals (RMS) Sec of Arc	
		RA cos δ	Dec			RA cos δ	Dec
3772	6001	1.5	1.6	4259	6002	1.4	1.4
	6065	1.2	1.0		6038	2.4	2.2
3775	6001	1.2	1.4	4267	6002	2.0	1.8
	6004	1.8	1.8		6003	1.0	1.6
3787	6001	1.8	1.6		6038	1.6	1.4
	6002	1.8	1.6	4276	6003	1.5	1.2
3795	6001	1.5	1.6		6038	3.2	2.8
	6003	2.1	1.2	4406	6011	1.4	1.8
3837	6001	1.3	1.6		6038	1.8	1.4
	6003	1.9	1.6				
3939	6006	1.5	1.8				
	6065	1.8	1.4		Average	= 1.7	= 1.8
3978	6003	1.8	1.6				
	6004	2.1	1.4				
4020	6006	1.7	2.0				
	6065	1.2	0.8				
4061	6001	1.7	2.0				
	6003	1.6	1.8				
4083	6006	1.4	1.8				
	6007	4.2	2.8				
4182	6002	1.9	1.2				
	6003	2.5	1.4				
	6038	1.5	2.6				
4196	6003	1.3	1.8				
	6038	2.1	2.2				
4210	6007	2.3	2.2				
	6065	1.3	1.2				
4212	6003	1.0	1.8				
	6011	1.3	2.0				
	6038	2.9	3.2				
4233	6015	1.7	2.0				
	6016	3.0	2.6				
	6065	1.1	0.8				
4236	6002	1.1	1.6				
	6003	1.2	2.6				
4244	6002	1.2	1.4				
	6038	2.4	2.0				
4245	6011	2.8	3.2				
	6038	2.4	2.2				
4251	6001	1.2	1.4				
	6002	1.0	1.2				
	6038	1.9	1.6				

TABLE 7

ORBIT RESIDUALS GROUPED ACCORDING TO STATIONS

Total Orbits Observed	Station	(Sec.) RA x cos δ	(Sec.) Dec	Camera F.L.
18	6001	1.7	1.6	450
17	6002	1.3	1.3	450
19	6003	1.4	1.5	450
6	6004	1.8	2.0	450
14	6006	1.7	2.1	450
10	6007	2.2	2.1	300
11	6011	2.0	2.1	300
6	6012	1.8	2.0	300
4	6015	1.9	1.7	300
14	6016	1.9	2.1	300
10	6038	2.2	2.2	300
7	6065	1.4	1.3	450

TABLE 8

FINAL ORBITAL ELEMENTS

(Earth Fixed Coordinates)

ORBIT	X (m)	Y (m)	Z (m)	$\dot{X}$ (m/sec)	$\dot{Y}$ (m/sec)	$\dot{Z}$ (m/sec)
2472	7 049 830	1 385 083	8 392 169	-4385.734	- 901.5624	3783.485
2497	- 755 255	-8 141 022	7 487 072	196.4129	3954.058	4303.619
2505	9 229 907	-1 883 133	5 861 268	-3055.597	300.0903	4983.593
2520	6 484 564	-2 871 367	8 555 988	-4142.655	1806.400	3674.581
2523	-3 547 537	-7 773 697	7 123 650	1353.257	3483.310	4473.550
2531	6 128 857	-3 766 086	8 490 429	-3831.831	2307.642	3718.460
2542	873 300	-8 741 952	6 871 239	- 598.2044	3532.913	4579.614
2611	6 452 184	7 490 912	5 329 312	-1475.861	-2280.298	5104.675
2626	7 817 650	6 148 849	5 257 734	-1844.473	-1930.438	5118.146
2646	5 255 651	2 180 922	9 665 249	-4727.477	-1753.039	2781.916
2661	-7 484 341	-1 763 222	8 251 398	4113.155	1083.556	3849.456
2672	-6 182 757	-8 117 276	4 844 949	1131.770	2155.302	5208.548
2675	9 948 376	912 211	5 289 274	-2593.627	- 622.5420	5085.857
2678	-6 538 610	-7 476 576	5 403 410	1493.815	2280.654	5051.920
2679	-10 264 350	- 662 075	4 678 597	2277.630	562.3783	5247.430
2694	6 589 994	-8 054 496	4 430 754	-1705.834	1409.471	5299.772
2703	-9 036 251	1 855 531	6 581 496	3320.789	- 376.7105	4642.528
2736	-8 542 980	-2 384 375	7 083 636	3390.755	1212.529	4426.660
2818	6 187 434	2 586 192	9 116 192	-4350.763	-1783.559	3236.094
2866	-6 842 283	6 163 251	6 709 750	2713.012	-2054.484	4554.675
2883	6 879 000	- 744 584	8 961 563	-4603.137	504.0284	3335.032
2891	7 113 161	-1 225 638	8 736 361	-4448.991	729.4020	3492.216
2893	-1 015 521	-5 526 119	9 758 717	1135.115	4960.135	2639.105
2894	-5 274 844	-3 546 45U	9 336 894	4073.267	2626.408	3033.457
2909	4 986 533	-9 026 973	4 871 374	-1523.646	1913.945	5137.307
2958	6 637 326	1 222 363	9 056 250	-4642.259	- 817.727	3239.968
3173	-4 071 854	-1 898 596	10 059 818	5156.306	1862.533	2066.957
3185	-1 586 864	-3 800 649	10 183 643	2682.619	4883.831	1864.558
3352	7 461 357	-3 443 719	7 442 686	-3787.398	1590.149	4154.319
3409	-7 781 089	2 146 982	6 002 112	-3520.974	1097.969	-5279.234
3429	5 061 389	- 353 846	9 525 244	-5386.904	752.1126	2505.002
3436	4 247 966	-2 014 653	9 078 624	5127.514	-1921.596	-3155.473
3447	7 017 427	2 219 516	6 893 543	4050.815	1257.478	-4818.565

TABLE 8

FINAL ORBITAL ELEMENT

(Earth Fixed Coordinates)

(continued)

ORBIT	X (m)	Y (m)	Z (m)	$\dot{X}$ (m/sec)	$\dot{Y}$ (m/sec)	$\dot{Z}$ (m/sec)
3448	3 433 622	-1 978 835	9 457 727	5230.515	-2245.887	-2695.989
3481	883 979	- 391 424	10 345 079	5955.562	1577.998	- 783.4561
3483	-1 690 907	240 406	10 378 282	5383.986	-2883.809	590.7615
3488	-8 484 710	- 707 472	5 304 171	-3330.299	- 101.8149	-5534.237
3535	3 588 105	2 658 181	9 136 180	4169.467	3744.639	-2990.060
3538	4 236 484	-5 626 514	7 197 039	2722.465	-3571.051	-4596.158
3539	- 990 723	-6 625 576	7 532 727	- 614.3132	-4648.631	-4380.264
3545	6 322 855	4 308 018	6 516 245	3417.22[illegible]	2255.800	-4973.765
3560	3 197 304	-5 600 240	7 746 942	2501.763	-4137.629	-4217.440
3772	4 294 466	- 567 857	8 977 760	5832.058	- 183.0737	-2802.277
3775	-3 575 432	-2 624 202	8 924 286	-4331.573	-3867.564	-2864.259
3787	2 211 007	-3 815 266	8 923 297	3395.708	-4736.393	-2838.349
3795	-1 426 061	-4 179 007	8 911 963	-1343.036	-5677.537	-2835.772
3837	405 574	-4 194 380	8 964 286	1188.904	-5815.650	-2674.099
3939	2 172 326	3 303 354	9 003 259	2730.730	5439.166	-2431.210
3978	-2 589 121	-2 269 356	9 166 654	-4058.326	-4741.338	-2076.694
4020	3 820 240	2 106 351	8 793 029	4969.014	3403.358	-2614.039
4061	1 302 228	-4 438 268	8 664 972	2161.626	-5557.036	-2739.272
4083	4 809 500	718 927	8 549 445	5749.393	1314.098	-2859.874
4182	-2 992 993	-6 530 041	7 095 201	-2037.551	-4485.616	-4091.542
4196	-4 007 298	-6 951 666	6 339 527	-2298.294	-3782.886	-4527.085
4210	5 544 373	-3 811 118	7 479 603	4319.847	-2866.104	-3792.853
4212	-4 925 318	-6 959 778	5 816 765	-2495.544	-3236.593	-4765.530
4233	3 918 405	4 378 505	8 057 990	3548.304	4311.168	-3273.989
4236	305 782	-9 707 090	4 194 090	- 212.6856	-3065.899	-5309.870
4244	- 55 381	-8 843 892	5 491 985	- 230.7657	-3867.024	-4876.037
4245	-6 340 861	-6 190 922	5 471 468	-2904.004	-2543.728	-4883.200
4251	- 321 435	-6 878 189	7 405 257	- 177.9642	-5121.758	-3821.560
4259	- 936 262	-9 355 612	4 762 125	- 610.0718	-3360.279	-5122.160
4267	-1 120 224	-8 105 591	6 290 443	- 719.3053	-4316.748	-4497.403
4276	-1 303 297	-7 140 611	7 145 710	- 880.1460	-4859.965	-3990.790
4406	-7 808 720	-5 126 250	5 308 306	-3176.297	-1779.737	-4814.278

Statistics on each orbit resulting from the short arc solution are too voluminous to include in the report; however, the overall results can be adequately illustrated by two orbits (Tables 9, 10 and 11).

Orbit 4236 in these tables shows appreciably larger sigmas and a higher degree of correlation than orbit 2472. If we also look at Table 12 we note that orbit 2472 represents a fairly strong geometric situation. Both satellite traces are fairly long, both traces are about the same length, and the excursion in elevation angle is also good from both observing stations.

It can also be seen from Table 12 that orbit 4236 has a less amount of observational overlap and shorter range in elevation angles. These conditions lead to higher correlation among certain orbital parameters than we had for orbit 2472. Orbit 4236 represents the more extreme case of correlation rather than a representative case. The correlation matrices for most of the orbits are very similar to the results of orbit 2472, Table 10.

There is no evidence in the results which might suggest a problem of ill-conditioning. The degree of correlation would have undoubtedly been higher if the data spans had been restricted to only the overlap portion of the traces. In such instances, 30% of the orbit co-observed by the two camera systems would have been lost due to the smaller field of view of the BC-4-450 camera.

2.3.8 Comparison of Results

Since the final positions of the short-arc solution should represent geocentric coordinates, it is desirable to check its values with another set also derived by the dynamic method. The two stations to be compared below are two nearby stations of the TRANET and PAGEOS net; the TRANET station coordinates having been solved for by SAO and NWL in two independent solutions. The local survey information tying the stations is available from the NASA Station Directory so that the position of the PAGEOS site can be reconstructed from the TRANET station.

The comparison for stations Hawaii (6011) and Shemya (6004), with respect to the NWL [6] and SAO results are as follows:

6011, Hawaii, (NWL & BC-4 Comparison)

	Latitude (N)	Longitude (E)	Height (m)
NWL Position (7100), C7	21° 31' 15.49"	202° 00' 09.04	405
Local Survey	- 48 48.30	1 44 27.92	--
Position of 6011	20 42 27.19	203 44 36.96	--
Short Arc Solution	26.71	37.69	3035
Difference	0.48"	-0.73"	--
Difference (m)	12m	-20m	--

6004, Shemya, (NWL & BC-4 Comparison)

	Latitude (N)	Longitude (E)	Height (m)
NWL Position (7739), C7	52 42 55.37	174 06 38.46	46
Local Survey	-6.63	46.44	-4
Position of 6004	52 42 48.74	174 07 24.90	42
Short Arc Solution	48.11	26.04	40
Difference	0.63	-1.14"	2m
Difference (m)	18m	-17m	2m

6011, Hawaii, (SAO & BC-4 Comparison)

	Latitude (N)	Longitude (E)	Height (m)
SAO Position (Sta. 7100), C7	21 31' 14.37"	202 00 08.19"	428m
Survey	- 48' 48.30"	1 44 27.92	--
Position of 6011	20 42 26.07	203 44 36.11	
Short Arc Solution	20 42 26.71	203 44 27.69	
Difference	-0.64"	-1.58"	
Difference (m)	-19m	44m	

6004, Shemya, (SAO & BC-4 Comparison)

	Latitude (N)	Longitude (E)	Height (m)
SAO Position (Sta. 7739), C7	52 42 55.88"	174 06 38.04"	114
Survey	-6.63	46.44	-9
Position of 6004	52 42 49.25	174 07 24.48	105
Short Arc Solution	52 42 48.11	174 07 26.04	40
Difference	1.14"	-1.56	60m
Difference (m)	34m	29m	60m

TABLE 9
ORBITAL SIGMAS
(Meters and Meters/Sec.)

Orbit Elements / Orbit No.	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	σ_x	σ_y	σ_z	$\sigma_{\dot{x}}$	$\sigma_{\dot{y}}$	$\sigma_{\dot{z}}$
2472	7049830	4385083	8392169	-4386	- 902	3783	2	1	4	.02	.01	.04
4236	305782	-9707089	4194090	- 213	-3065	-5310	20	113	8	.17	.94	.08

TABLE 10
CORRELATION MATRIX: ORBIT 2472
Stations 6007, 6016

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
X	1	.4	.6	-.3	-.2	-.1
Y		1	.3	-.2	-.4	-.1
Z			1	-.2	-.1	-.2
$\dot{X}$				1	.4	.5
$\dot{Y}$					1	.3
$\dot{Z}$						1

TABLE 11

CORRELATION MATRIX: ORBIT 4236

Stations 6002, 6003

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
X	1	-.96	-.72	.97	.94	-.41
Y		1	-.75	.90	.99	-.44
Z			1	-.68	-.74	-.80
$\dot{X}$				1	.87	-.38
$\dot{Y}$					1	-.46
$\dot{Z}$						1

TABLE 12

ORBITAL SPAN

Orbit	Station	Time (Sec.)			Elevation Angles (Deg.)		
		Start	End	Span	Start	End	Span
2472	6007	290	685	394	34	53	19
	6016	332	652	320	53	74	21
4236	6002	12875	13004	129	9	1	8
	6003	12842	12891	49	2	0	2

The agreement with the NWL solution is quite good inasmuch as the NWL estimated accuracy for Hawaii and Shemya are 15m and 25m, respectively. The larger differences between SAO and this solution is probably due to the fact that the SAO coordinates for 7100 and 7739 are only a provisional set [7, p. 42].

A more direct and perhaps a more valid comparison can be made with respect to an OSU solution,[8], which also employed the short-arc method in the adjustment and also used BC-4 PAGEOS data. The OSU solution held the Beltsville station as the origin of its triangulation and solved for the coordinates of three other stations (6003, 6001, 6038) on the C5 system.

After converting the OSU C5 coordinates to the C7 a_e = 6378 142 and f (inverse) = 298.255, the agreement for station Gigedo (6038) is as follows:

OSU	18°	43'	58.43"	249°	02'	41.38"	19m
Short-Arc	18	43	58.24	249	02	41.02	4m
Difference			0.19			0.36	15m
Difference (m)			6m			10m	15m

In view of the fact that the scale of the OSU solution was provided by the chord distance between 6002 and 6003 as derived from their NAD coordinates, the agreement is as good as can be expected.

As a final test, the twelve BC-4 station coordinates were also used in a least squares solution to compute the ellipsoidal semi-major axis, a_e, and the semi-minor axis, b_e. This was accomplished by computing the total geocentric radius for each station, subtracting the mean sea-level height from it, and fitting the resulting X, Y, Z coordinates at mean sea-level to the standard ellipsoidal expression. As expected, the results for both a_e and b_e were not very good, the flattening computing to 1/297.60 with a correlation between a_e and b_e of 0.7. However, when the flattening was inforced to 1/298.255, the resulting semi-major axis was a_e = 6378 141 meters.

A value of a_e = 6378 141.5m was achieved when the Baker-Nunn stations on page 87 of Reference 7 were added to the solution with the BC-4 positions.

SECTION 3
GEOMETRICAL SOLUTION

3.1 GENERAL

BC-4 Camera data for the geometrical solution were derived from the Orthogonal Polynomial constants. An efficient method for correcting PAGEOS data for parallactic refraction, phase of the satellite, and planetary aberration is to utilize Chebyshev polynomials. A series of computer programs were written for the purpose of preparing BC-4 data for input into a geometrical triangulation adjustment with the Baker-Nunn Network. Figure 3 illustrates the computational process for the adjustment. The treatment of BN data has been previously described [9].

3.2 GENERATING SIMULTANEOUS OBSERVATIONS

The first step is the fitting of orthogonal polynomials in right ascension and declination. This procedure was described in [1]. After these results were screened and some events eliminated, corrections for phase angle, parallactic refraction, and planetary aberration were generated in the following manner. Six synthetic simultaneous observations (uncorrected) were generated for each simultaneously observed arc at certain key values of time from each station. From the six simultaneous pairs of observations, the range from each station to the six synthetic points and the sum of the parallactic refraction correction and the phase angle corrections were computed. Fifth degree Chebyshev polynomials with time as the independent variable was then fitted to the six ranges and combined corrections. The six polynomials were used at a later stage to correct the synthetic simultaneous observations. The ranges and sum of corrections were fitted well by the polynomials; in fact, they were fitted to a higher accuracy than required by the circumstances of the problem.

The correction polynomials generated can be utilized to apply corrections to the original data for any data included within the time of overlap of the satellite traces. The use of interpolation polynomials to effect these corrections is efficient, requiring a minimum of computer time.

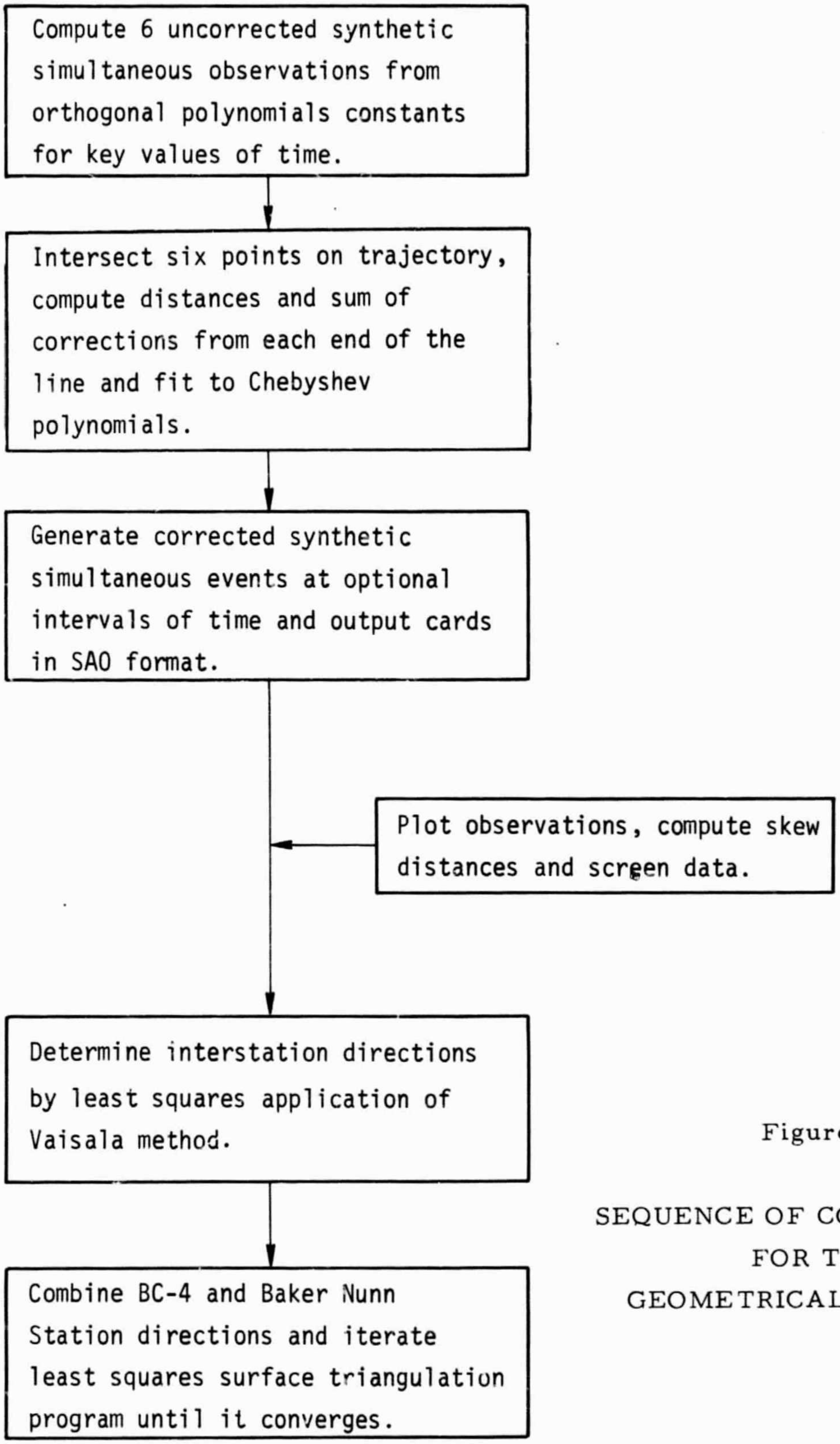

Figure 3

SEQUENCE OF COMPUTATIONS FOR THE GEOMETRICAL SOLUTION

The punched cards containing the Chebyshev polynomial coefficients were merged with the punched cards containing the Orthogonal least squares polynomials in right ascension and declination for input to a small program which computes corrected synthetic simultaneous observations for arbitrary times. As output the program produces cards in SAO format for use in an interstation direction adjustment program and a plotting program. For the geometrical adjustments, synthetic simultaneous observations were produced every 30 seconds. Plots for 24 of the 30 BC-4 lines are shown in the Appendix.

Besides giving a graphical picture of the observational geometry, the plotting program is also useful as a screening device to check events which are patently in error. The principal means of judging poor events was the "skew" distance discussed in [9]. All events whose skew distances exceeded reasonable values were eliminated. Comparison of these events and those eliminated from the short arc solution showed that in every case those eliminated by the excess skew distance criteria were also eliminated from the short arc solution. However, 9 orbits eliminated from the short arc solution passed the skew distance test. These were border-line cases which were eliminated from the short arc solution to avoid costly repetitious computer runs.

3.3 INTERSTATION DIRECTIONS

After the data was qualified, the interstation direction solution, [10], was made from corrected simultaneous observations. This program produces cards punched in the format required for input to an already existing surface triangulation program. The interstation directions so produced appeared to be in excellent agreement with those obtained from the short arc solution with the exception of two lines whose geometrical circumstances were poor. These two lines had a spread in reference angle of less than 5°, [11]. (See an error analysis of interstation direction solutions by Lambeck [12]). The result was that the computed values of the two lines were in error by many times their standard deviations. In all other cases, the solutions and variances appear to be reasonable. Table 13 shows a summary of the direction cosines obtained for the 30 lines in the solution.

TABLE 13

BC-4 INTER-STATION DIRECTION SOLUTION RESULTS

Station No.		Direction Cosines			Number of Events	Number of Points
6001	6002	.14187538	-.83558373	-.53072706	6	36
6001	6003	-.68561413	-.61420744	-.39074606	5	31
6001	6004	-.90136438	.36608980	-.23134502	2	16
6001	6006	.59118707	.80211212	-.08434448	6	47
6001	6007	.85321222	-.19276914	-.48463282	1	5
6001	6011	-.82835639	-.13976770	-.54248565	1	11
6001	6016	.77327358	.48108098	-.41304848	2	18
6001	6038	-.41484947	-.65159520	-.63507764	2	14
6001	6065	.80957320	.48807676	-.32614769	3	21
6002	6003	-.93493604	.29982055	.18974255	9	63
6002	6007	.79005866	.61300633	-.00552785	3	29
6002	6038	-.84062580	-.20733223	-.50036148	5	33
6003	6004	-.37965516	.92102763	.08700616	1	7
6003	6011	-.76829682	.31794381	-.55554633	4	36
6003	6012	-.54234983	.75307795	-.37246512	1	11
6003	6038	-.01031802	-.57809871	-.81590160	5	33
6004	6006	.98717427	.05392274	.15026407	1	9
6004	6011	-.37689758	-.65398844	-.65593242	1	9
6004	6012	-.54078299	.26896497	-.79700163	2	22
6006	6007	.54463998	-.69857080	-.46407558	2	16
6006	6015	.11504939	.85436106	-.50678479	2	18
6006	6016	.78781355	.16765820	-.59265549	4	38
6006	6065	.85875781	.04034678	-.51079073	6	32
6007	6016	.12797870	.99126692	-.03180167	8	92
6007	6065	-.06918905	.97079994	.22969622	2	14
6011	6012	-.10270412	.99395368	-.03883228	5	55
6011	6038	.71356594	-.69916366	-.04465235	3	31
6015	6016	.59083551	-.80632596	.02742000	5	39
6015	6065	.39466066	-.88864262	.23357496	1	7
6016	6065	-.57150665	-.41459393	.70816102	6	42

TABLE 14

RESIDUALS FROM INTER-STATION ADJUSTMENT LINE 6007-6016

EVENT (ARC) NO.	STATION 6007		STATION 6016	
	V1 (sec)	V2 (sec)	V3 (sec)	V4 (sec)
2466	.2721	.1773	-.3842	-.0914
	1.1596	.8220	-1.0172	-.1806
	1.0902	.4740	-1.1715	-.3982
	.5515	.2622	-.5717	-.0525
	-.2207	-.1151	.2198	.0219
	-.9956	-.5718	.9487	.1027
	-1.5941	-1.0129	1.4456	.1712
	-1.8583	-1.3136	1.5937	.2079
	-1.6829	-1.3321	1.3550	.7844
	-1.0074	-.9002	1.3417	.4893
	-.7261	-.7399	1.3909	.8279
2472	.1786	.0548	-.8993	-.0724
	.4610	.1536	-1.5721	-.0839
	.6057	.2197	-1.2859	-.1109
	.4676	.1850	-.9630	-.0856
	.3173	.1373	-.6318	-.0144
	.2236	.1061	-.4288	-.0100
	.2416	.1262	-.4444	-.0105
	.3791	.2190	-.6654	-.0630
	.5722	.3673	-.9530	-.0889
	.5236	.3758	-1.2839	-.0735
	.3940	.3184	-1.3001	-.1062
2505	-.5256	-.0358	.0640	-.0036
	-.0810	-.0061	.0891	-.0065
	-.1454	-.0119	.1604	-.0145
	-.3158	-.0283	.3494	-.0380
	-.4853	-.0475	.5390	-.0691
	-.5823	-.0620	.6492	-.0965
	-.5833	-.0674	.6530	-.1114
	-.4929	-.0619	.5541	-.1074
	-.3583	-.0488	.4047	-.0885
	-.2428	-.0359	.2755	-.0676
	-.2312	-.0371	.2637	-.0723
	-.4084	-.0712	.4683	-.1428
	-1.7703	-.3358	.2269	-.0767
2520	3.2632	-.0951	-.1869	.0511
	.4451	-.0086	-.0645	.0193
	-.4497	.0180	.1313	-.0428
	-.8986	.0382	.0676	-.0965
	-1.0381	.0467	.0807	-.1263
	-1.0691	.0126	.0861	-.1481

TABLE 14 (continued)

EVENT (ARC) NO.	STATION 6007		STATION 6016	
	VI (sec)	V2 (sec)	V3 (sec)	V4 (sec)
2520	-.8568	.0106	.2872	-.1361
	-.6845	.0088	.2398	-.1257
	-.3550	.0186	.0327	-.0760
	.4143	-.0217	-.0403	.1045
	1.2163	-.0621	-.5031	.3664
	1.9493	-.0467	-.4413	.3633
	-.0216	.0007	.0021	-.0020
2579	-1.3783	-.0049	.4509	-.1080
	.3451	.0004	-.4550	.1224
	.4552	-.0006	-.6049	.1816
	.1325	-.0005	-.1776	.0148
	-.2863	.0017	.3871	-.0357
	-.5276	.0011	.7201	-.2922
	-.5400	.0013	.7445	-.3318
	-.3121	.0009	.4350	-.2124
	.1332	-.0017	-.0470	.0251
	.4526	-.0062	-.6461	.3769
	.6490	-.0091	-.9389	.5975
	.4361	-.0060	-.6400	.4440
	-.6020	.0075	.5046	-.3815
2622	.3932	.0296	-.3698	.0580
	-.5392	-.0439	.5070	-.0225
	-.2754	-.0970	1.0356	-.0517
	-.3437	-.1309	.3230	-.0725
	.1313	.0541	-.1233	.0310
	.5764	.2570	-.5412	.1521
	.8721	.4212	-.8183	.2569
	.9782	.5123	-.9173	.3214
	.4142	.2355	-1.5524	.1517
	.4151	.2566	-1.5541	.1694
	1.3053	.2197	-1.2204	.5937
2675	.9296	.0920	-.7491	-.1214
	.9467	.1113	-.7613	-.0320
	.8109	.1109	-.6507	-.0282
	.5637	.0882	-.4512	-.0200
	.2633	.0465	-.2102	-.0095
	-.0284	-.0056	.0226	.0010
	-.2662	-.0585	.2113	.0097
	-.4193	-.1017	.3317	.0151
	-.4808	-.1280	.3790	.0170
	-.4630	-.1347	.3636	.0635
	-.3643	-.4619	.2848	.0479

TABLE 14 (continued)

EVENT (ARC) NO.	STATION 6007		STATION 6016	
	V1 (sec)	V2 (sec)	V3 (sec)	V4 (sec)
4241	.5093	1.4257	-2.1982	1.7712
	.0883	.1676	-.1535	.0887
	.1967	.1233	-.4494	.0627
	.1642	.0806	-.3806	.0393
	-.2093	-.0832	.3438	-.0389
	-.1718	-.0566	.4164	-.0254
	-.0615	-.0171	.1062	-.0074
	-.2703	-.2564	.9383	-.1060
	.1611	.0742	-.9481	.0662

An examination of the residuals resulting from the interstation direction solutions to the corrected synthetic simultaneous observation for the BC-4's reveal a definite tendency to serial correlation. An inspection of the residuals for the more heavily observed lines show long strings of residuals as being either all positive or all negative. In addition to serial correlation from point to point, the pattern of the residuals strongly suggests the presence of small systematic errors in the data as analyzed. Table 14 is a printout of the residuals in right ascension and declination for line 6007-6016.

3.4 TEST TRIANGLE

As a test prior to the combined solution, the triangle Beltsville, Moses Lake, Gigedo was adjusted. This triangle was also adjusted with NEO-EMBET and by Ohio State University which used a lesser number of observations [8]. The synthetic simultaneous observations for this triangle comprise approximately 25% of the total for the whole BC-4 network. The coordinates of Beltsville were effectively fixed by preassigning small variances to them. The chord distance from Beltsville to Moses Lake which was obtained from geodimeter traverses was given a weight of 1/1,000,000 of the distance. The weight on the distance is pessimistic with respect to the internal accuracy of the geodimeter traverse but may be optimistic considering possible systematic errors.

The following table (Table 15) is a comparison of the coordinates of Gigedo. As can be seen, the results agree quite well.

TABLE 15

	Geometric	Short Arc
Latitude:	18 - 43 - 58.84 N	18 - 43 - 58.24 N
Longitude:	249 - 02 - 41.61 E	249 - 02 - 41.02 E
Height:	-6.3m	4.3m

3.5 COMBINED BC-4, BN GEOMETRIC SOLUTION

Attempts to combine the interstation directions of the BC-4 net and the interstation directions of the Baker-Nunn cameras to obtain a geometrical worldwide solution failed because of ill-conditioning. The ill-conditioning is due in large part to the geometrical configuration of the triangulation network, Figure 4. However, part of the difficulty was due to the triangulation program which has no provision for fixing or constraining the coordinates of a station other than by assigning to them small variances. Modification of the program to either fix station coordinates (removing them from the solution) or to constrain them by the matrix bordering techniques of Stearn [13], was impossible in the time available.

The best index for judging the extent of the ill conditioning in a particular problem is the condition number of the matrix; that is, the ratio of the largest to smallest eigenvalue in the normal equations or inverse matrix. For large matrices, the eigenvalues are difficult to obtain; however, the application of the Gerschgorin disc theorem [14] enabled us to determine crude but effective estimates of the condition number of the matrices for various adjustments.

3.5.1 First Adjustment

In one solution where the system would not converge, the conditions of the adjustment were as follows:

1) The coordinates of Beltsville were fixed to 0.03m, in latitude and longitude and 0.001m in height by assigning variances of 1×10^{-6} $(\text{seconds})^2$ in latitude and longitude and 1×10^{-6} $(\text{meters})^2$ in height.

2) Chord distance between Jupiter-Beltsville, Beltsville-Moses Lake, Jupiter-Moses Lake, obtained from USC&GS geodimeter traverse were given variances corresponding to $((1/1,000,000) \times \text{chord distance})^2$.

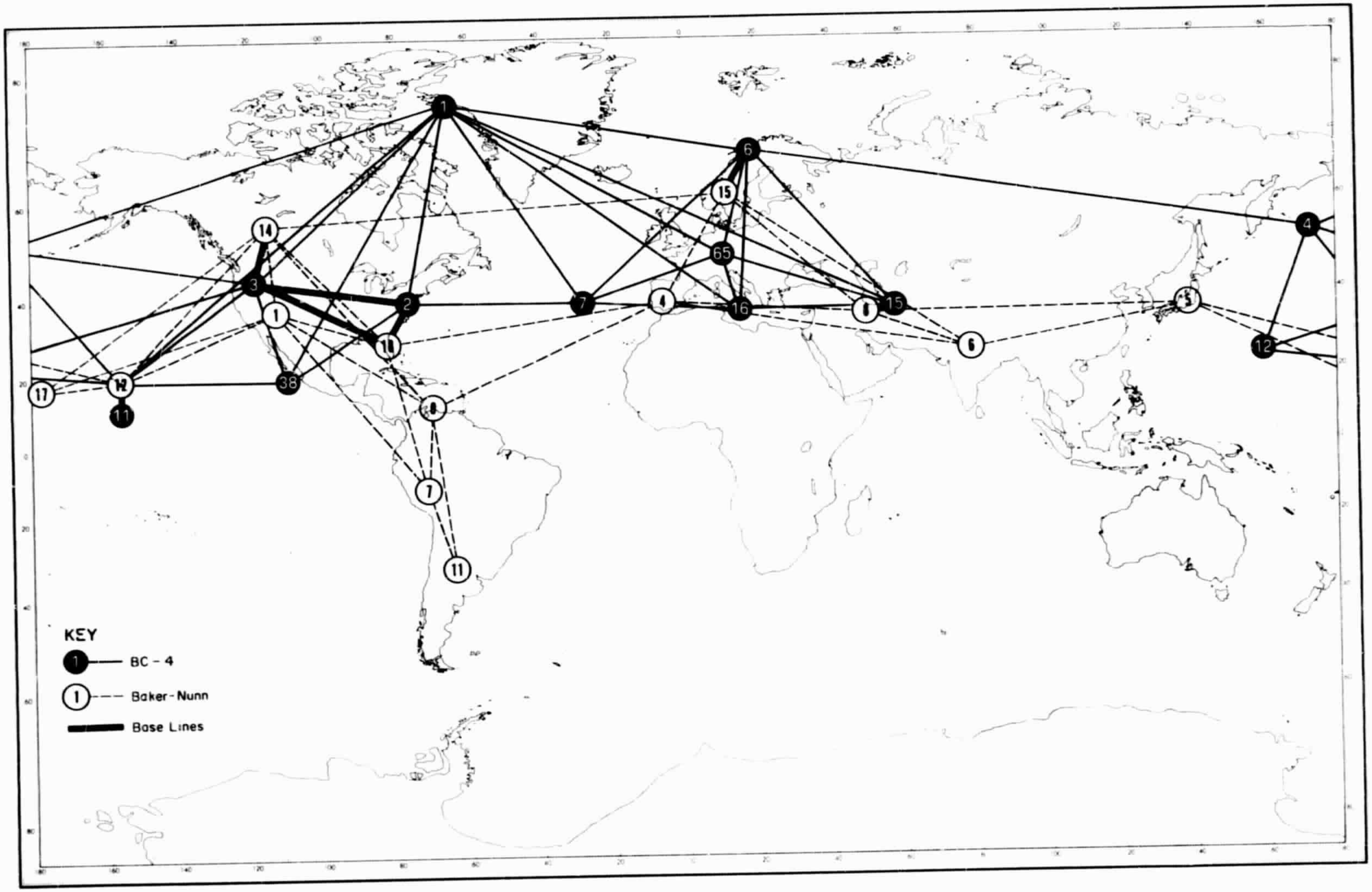

Figure 4

COMBINED SAO/BC-4 (Phase I) NETWORKS

3) Chord distances from conventional triangulation between Moses Lake-Cold Lake, and Tromso-Olso were given variances of $((1/100,000) \times \text{chord distance})^2$.

4) The chord distance from Mashhad to Shiraz was given a variance of $((1/10,000) \times \text{chord distance})^2$.

5) The survey distance of 130 meters between the Hawaii BN and BC-4 cameras was given a variance of $(1 \text{ meter})^2$.

6) Large variances were assigned to other station coordinates.

The preliminary coordinates used for the stations were the C7 coordinates for the BN cameras and the final coordinates of the BC-4 cameras from the short arc orbital solution. The system diverged. The condition number of the matrix was on the order of 2,000,000.

A study of the normal equations of the solution which diverged indicated the following circumstances.

1) The variances applied to the coordinates of Beltsville caused the condition number to increase two orders of magnitude over that which would have occurred without the coordinates of Beltsville in the solution.

2) The eigenvalues corresponding to the unknowns for the BN stations were typically an order of magnitude or more below those of the BC-4 network.

3.5.2 Other Adjustments

Accordingly, the input variances of the coordinates of Beltsville were given variances of $((.01) \text{ sec})^2$ in latitude and longitude and $(.1 \text{ meter})^2$ in height. The input variances of Baker-Nunn camera in the geometrically worst situation (e.g., Tokyo) was given variances of $(1 \text{ sec})^2$ in latitude and longitude and a variance of $(5 \text{ meters})^2$ in height and the height of Villa Dolores was given a variance of $(10 \text{ meters}).^2$ The solution converged but gave absurdly distorted results due to ill-conditioning.

A number of other similar adjustments gave results of the same type. The only solution that produced results reasonably in accord with common sense was one in which small variances were assigned to the input station positions (e. g., in effect the station positions were known in advance). More time is required to study the geometrical solution in order to produce more good results.

SECTION 4
SUMMARY AND CONCLUSIONS

The coordinates of the BC-4 (Phase I) sites from the short arc solution are determined to an average standard deviation of ± 8 meters in each positional component based on the assumption that each satellite direction was good to 1 sec of arc and by using every other data point of each trace. If a mean observational sigma of 1.7 sec in both right ascension and declination had been used (as suggested by the RMS of the station residuals) the resulting sigma in position would have been about ±12 meters. The 12m also appears to be a more realistic value from comparisons with Doppler at stations Hawaii and Shemya which show an agreement of 14 meters in each coordinate, and the comparison with OSU for Gigedo is also within the 12 meter value. Based on these comparisons and for reasons given below, it is felt than an accuracy of ±15 meters is a valid estimate for the final coordinates. Future large-scale determinations incorporating more PAGEOS stations and more data should improve this accuracy by a factor of two.

A careful review of all orbit residuals revealed that there is still a residual bias in the data, probably amounting to about one or two tenths of a second. Part of this bias is attributed to the fact that the orbit was made to absorb the satellite corrections but part of it is in the observations themselves, as revealed in the inter-station direction adjustment where these corrections were applied. A comprehensive residual analysis was not made at this time. It was felt that such an analysis would prove more productive if it were made on readjusted observational data of Phase I (to be provided by USC&GS) and on data from subsequent phases when all cameras had been converted to a common 450mm FL system.

The station corrections resulting from the short arcs (Table 4) are all realistic except for stations Shemya, 6004 and Mashhad, 6015. Since the comparison of Shemya with the Doppler solution is in good agreement, the magnitude of the corrections must be due to a weak geodetic connection of that area relative to NAD and hence to C7. The large corrections for Mashhad, however, cannot be attributed to a similar cause. The fact that this station is at the edge of the triangulation network tends to suggest this as a possible cause but the results for other outlying stations do not confirm it.

The final coordinates of the BC-4 stations (Table 5), including those for Mashhad, were used to compute an equatorial radius by removing the mean sea level heights from each geocentric radii and enforcing a meridional flattening of 1/298.255. The results of that computation produced an earth radius of 6378 141m. A similar solution using the C7 coordinates of the Baker-Nunn sites with the BC-4 stations produced a radius of 6378 142m. As expected, a computation of both axes, equatorial and polar, produced inferior results due to the small number and distribution of these stations.

The combined surface, three dimensional adjustment of BC-4 and Baker-Nunn stations did not produce good results. Several solutions were attempted, but in each instance it resulted in ill-conditioned normal equation matrices with excessively large ratios in eigenvalues. Further work is required to establish the exact causes producing instability. In particular, it is suggested that the two networks be first adjusted individually and if similar results are obtained with the PAGEOS network, this net should be split in three subnets and readjusted independently like the short arc solutions. Concurrently, a spatial triangulation solution should also be generated using the same data. The latter solution should be relatively inexpensive since several operating computer programs are available for use.

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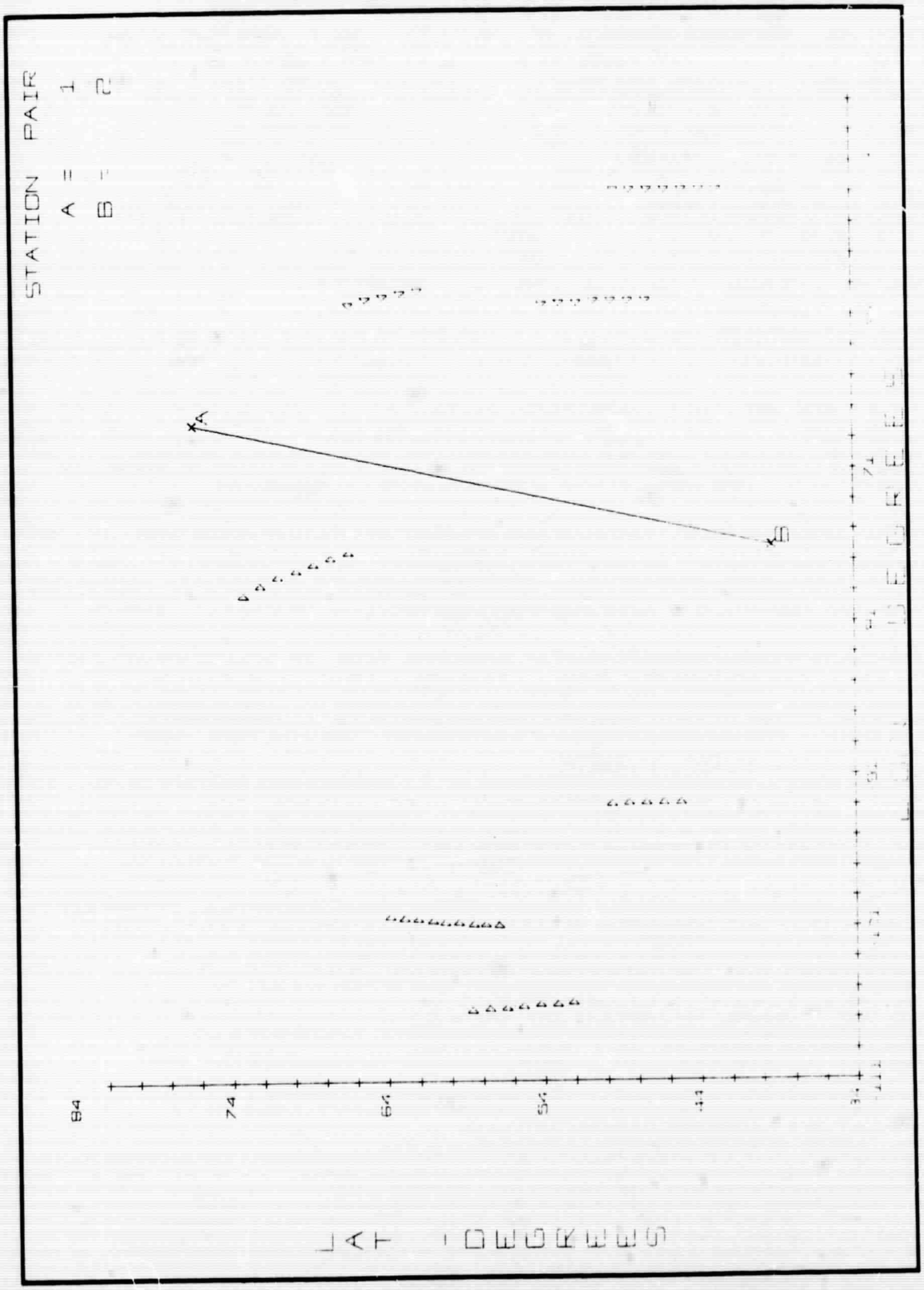
APPENDIX

The following graphs show the observational geometry of the simultaneous points generated for the geometrical adjustment. The time interval between each point is 30 seconds.

The leading figures of the station numbers have been omitted in the graphs. Station 1 is actually station 6001, etc.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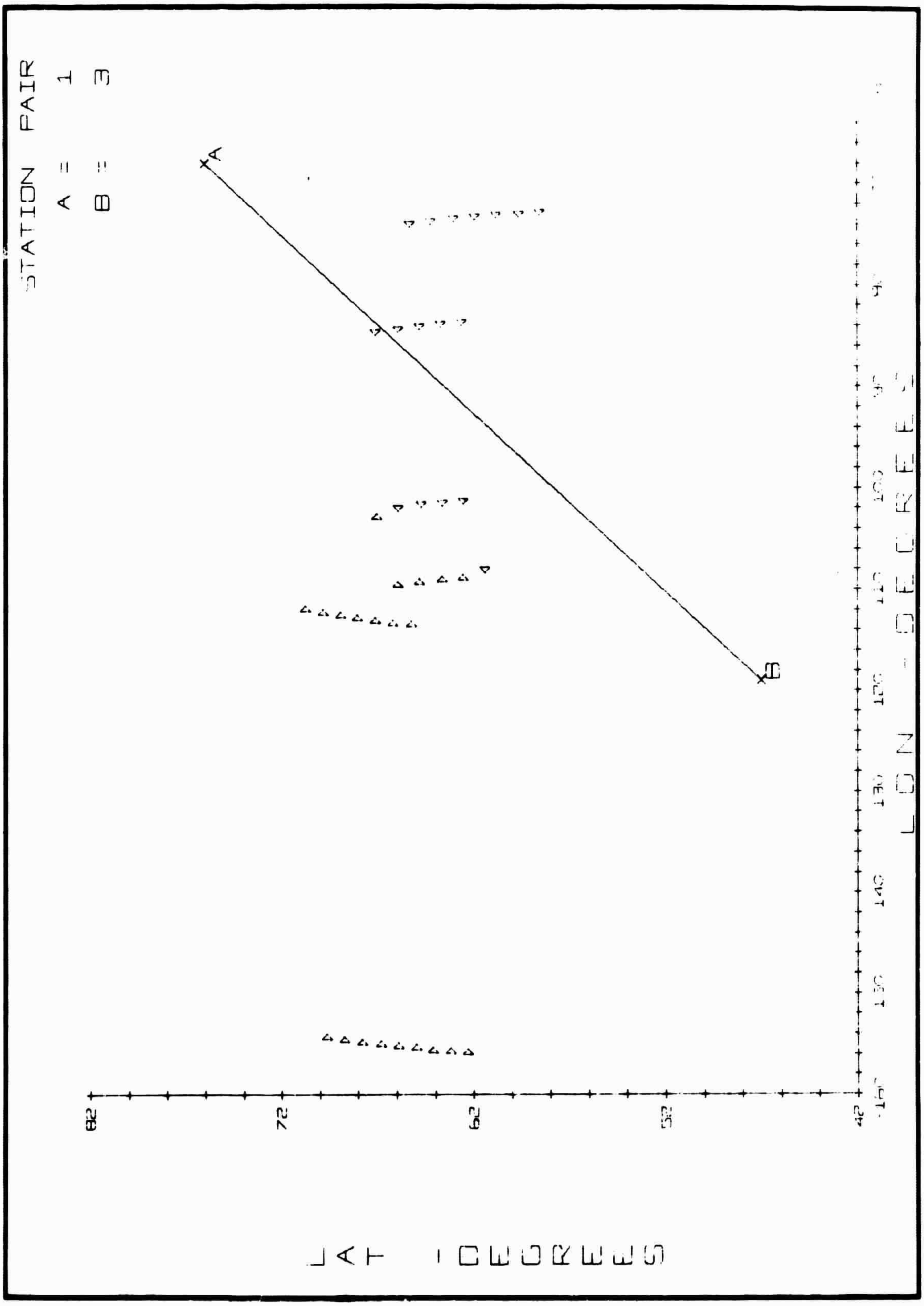
B = 2
A
B
84
74
64
54
44
34
LAT - DEGREES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B = 3
A
B
LON - DEGREES
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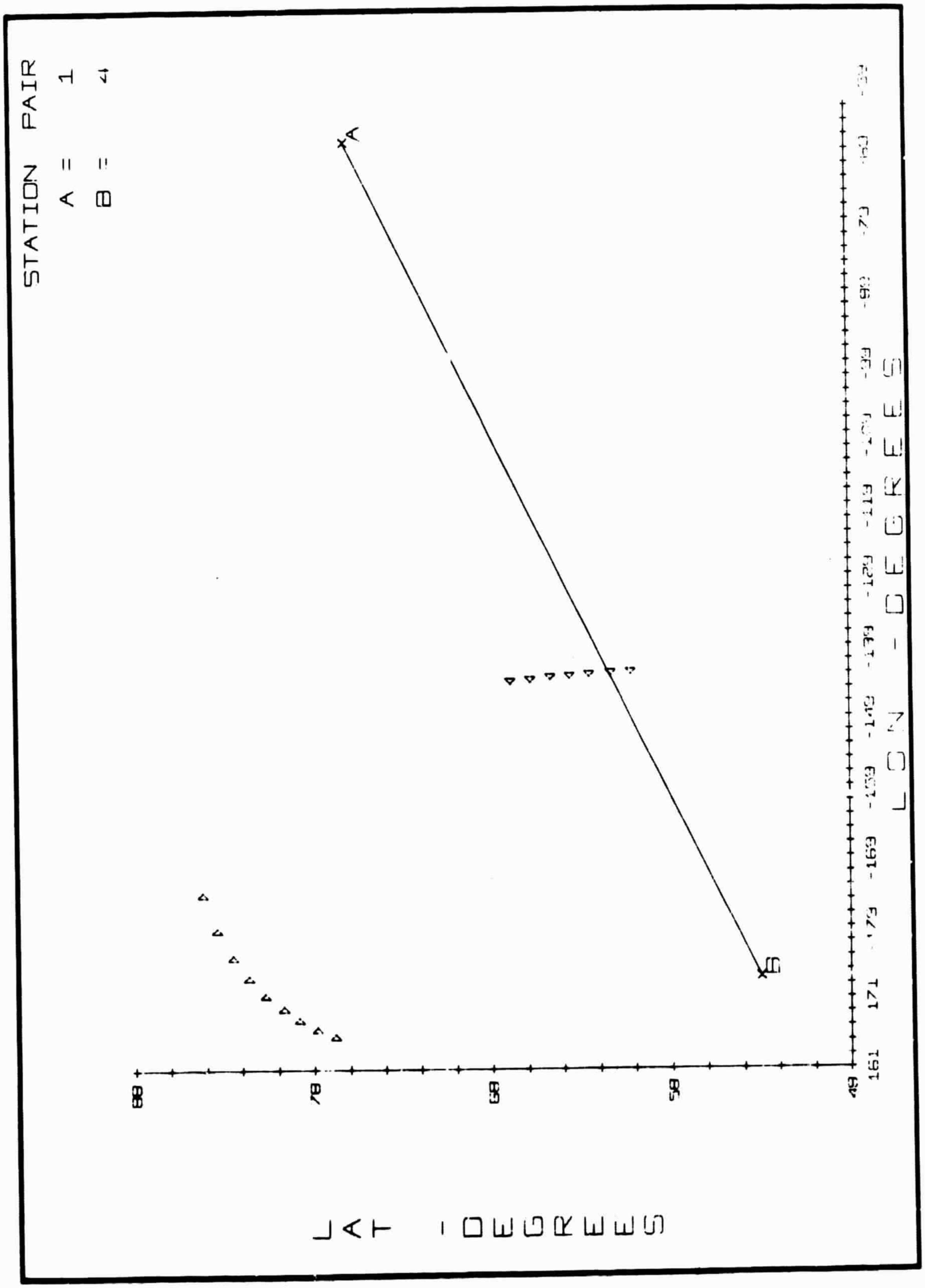
72
62
52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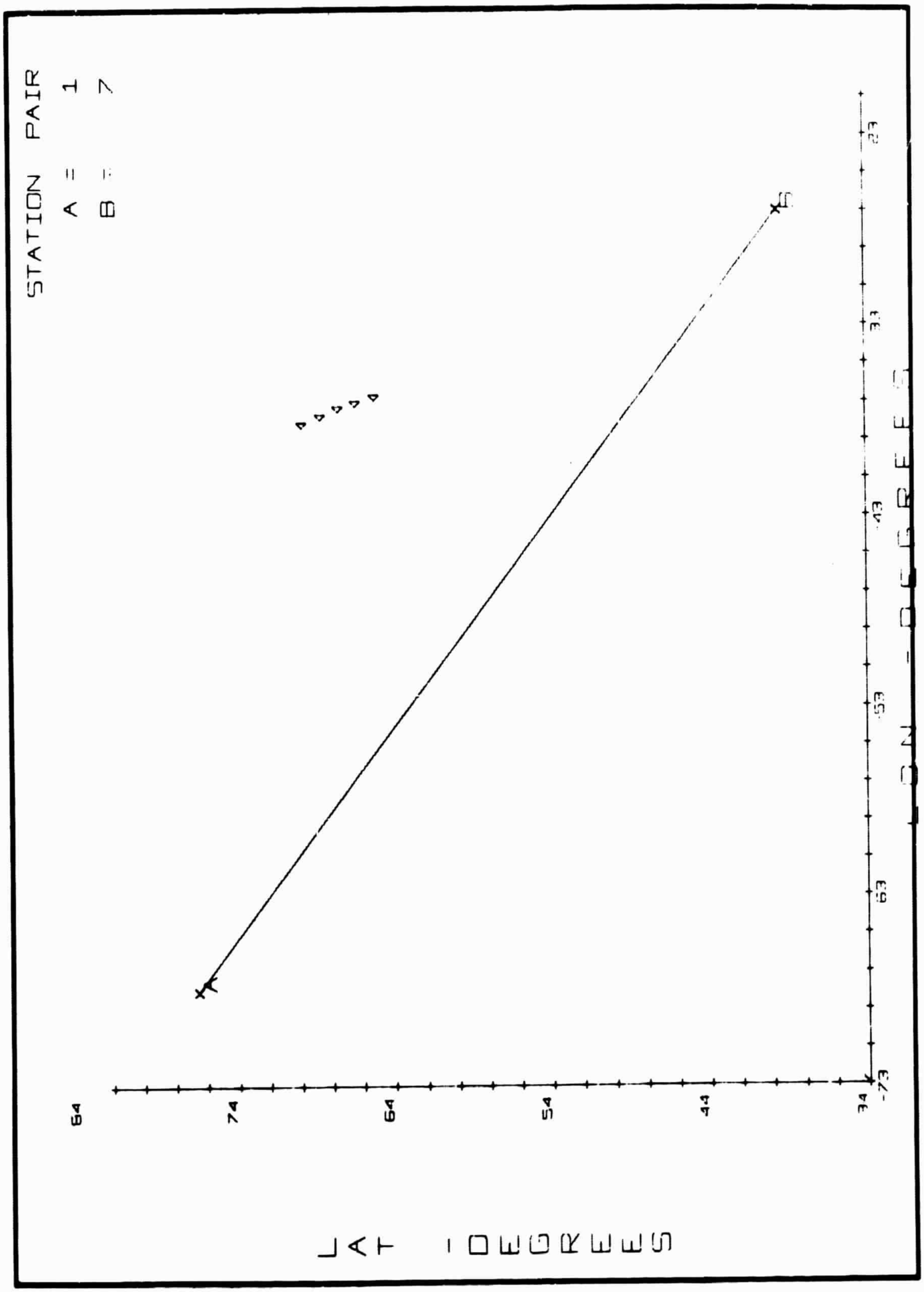
B = 4
A
B
LAT - DEGREES
LON - DEGREES


STATION PAIR
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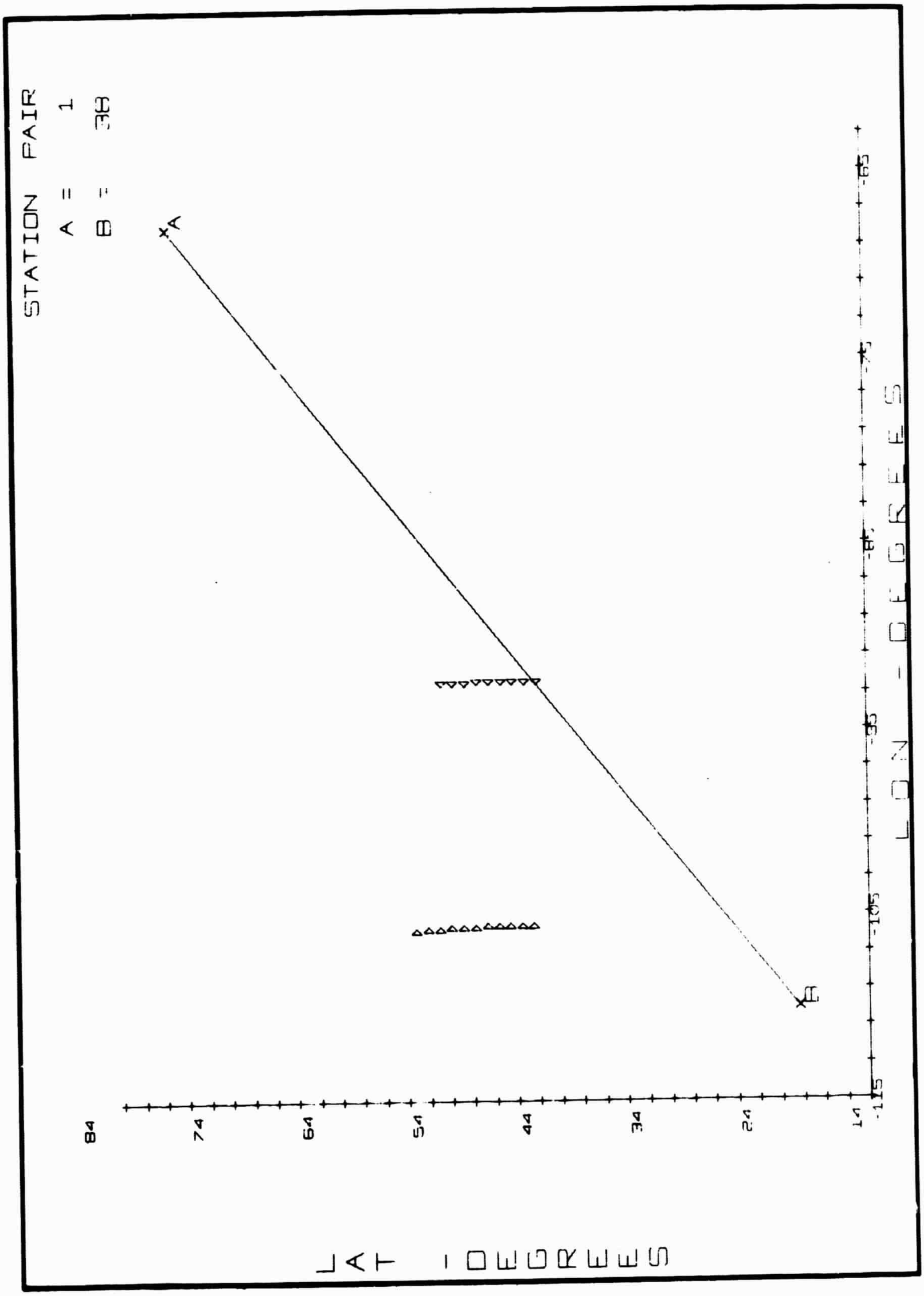
B = 7
LAT - DEGREES
LON - DEGREES
74
64
54
44
34
-73
-63
-53
-43


STATION PAIR
A = 1
B = 38
A
B
LAT - DEGREES
84
74
64
54
44
34
24
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LON - DEGREES
-115
-105
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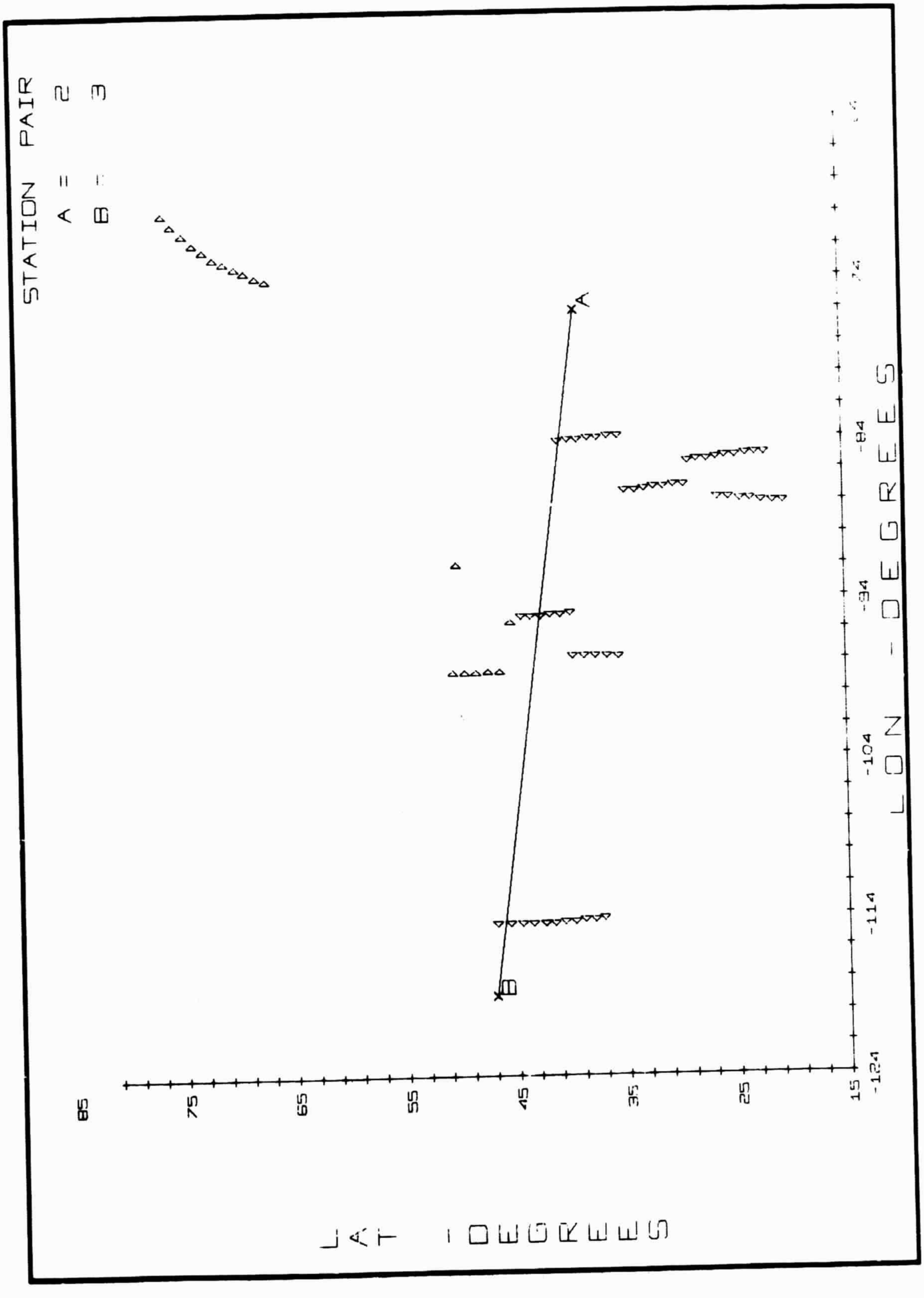
-65


STATION PAIR
A = 2
B = 3
A
B
LAT - DEGREES
LON - DEGREES
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75
65
55
45
35
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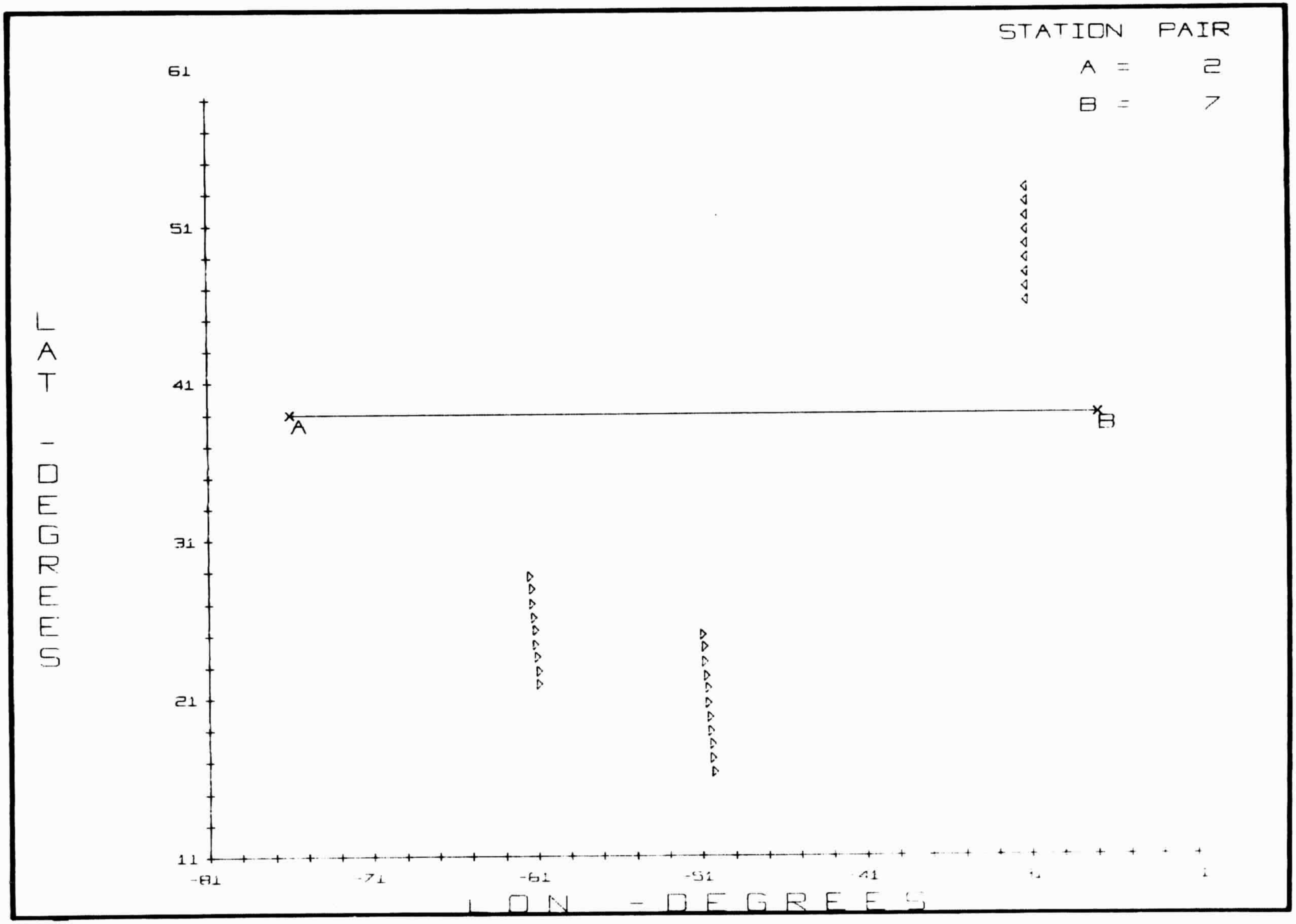
-114
-104
-94
-84


STATION PAIR
A = 2
B = 7
LAT - DEGREES
LON - DEGREES
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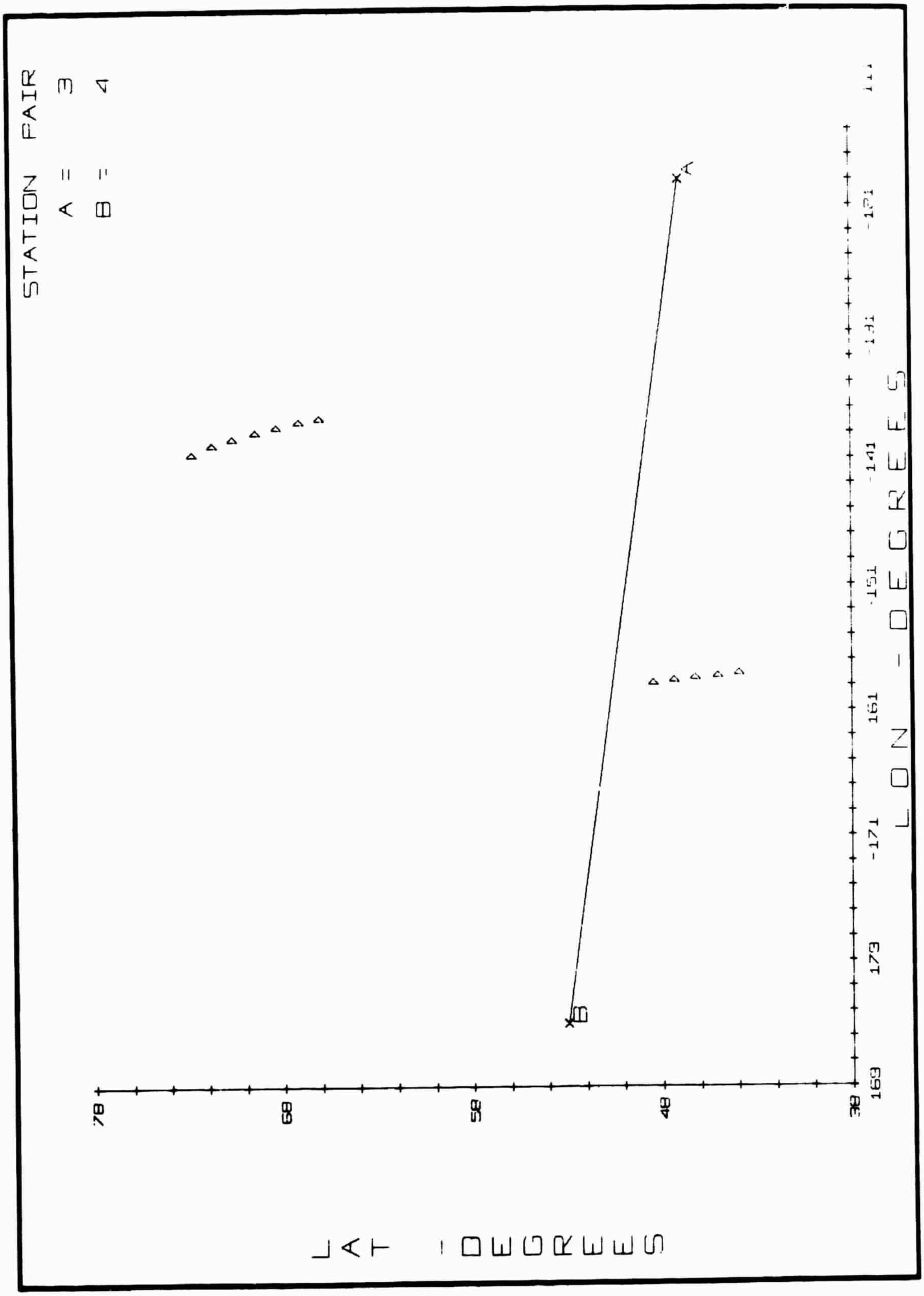
51
41
31
21
11
-81
-71
-61
-51
-41
A
B


STATION FAIR
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B = 4
LON - DEGREES
LAT - DEGREES
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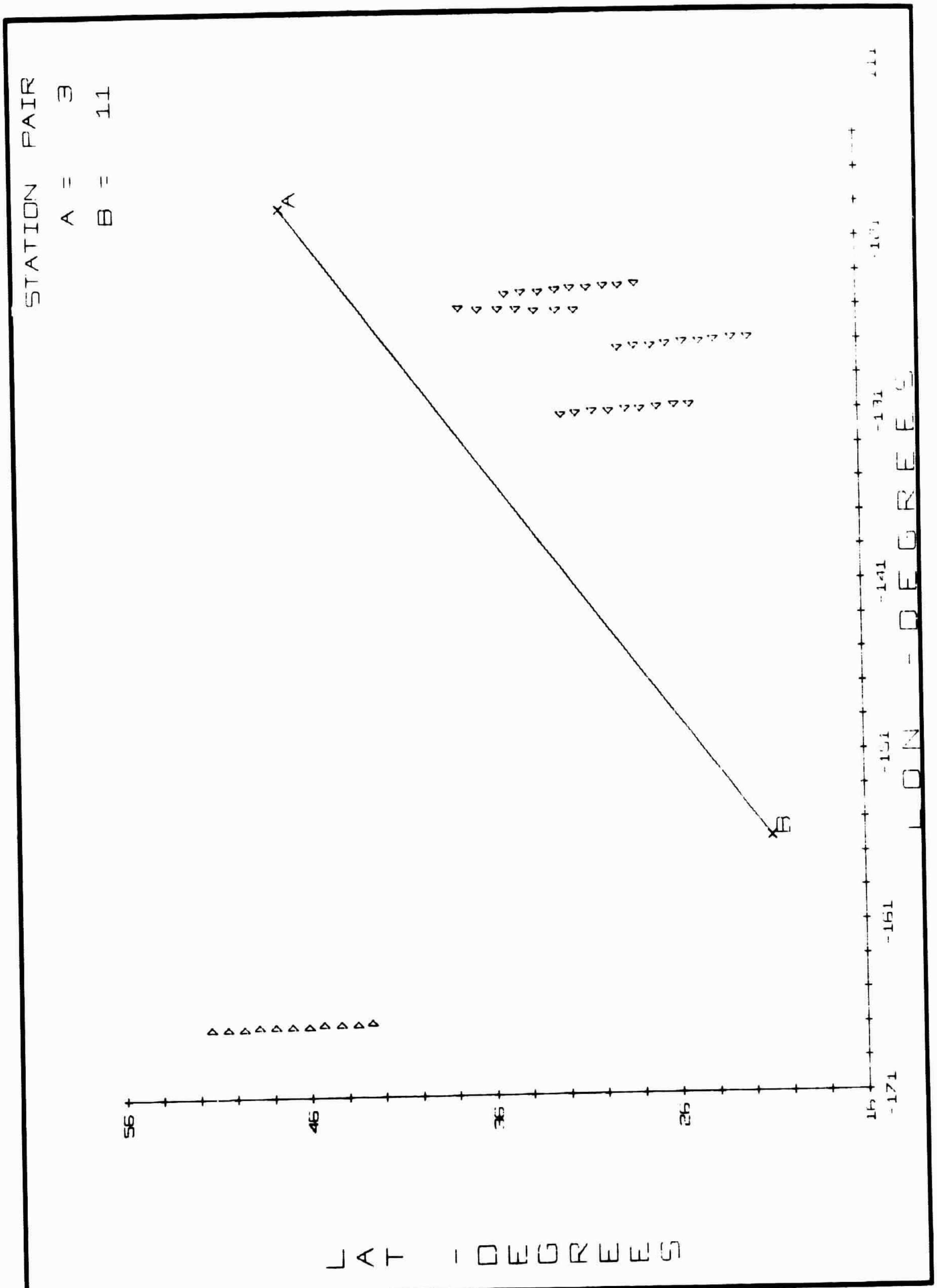
68
58
48
38


STATION PAIR
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A
B
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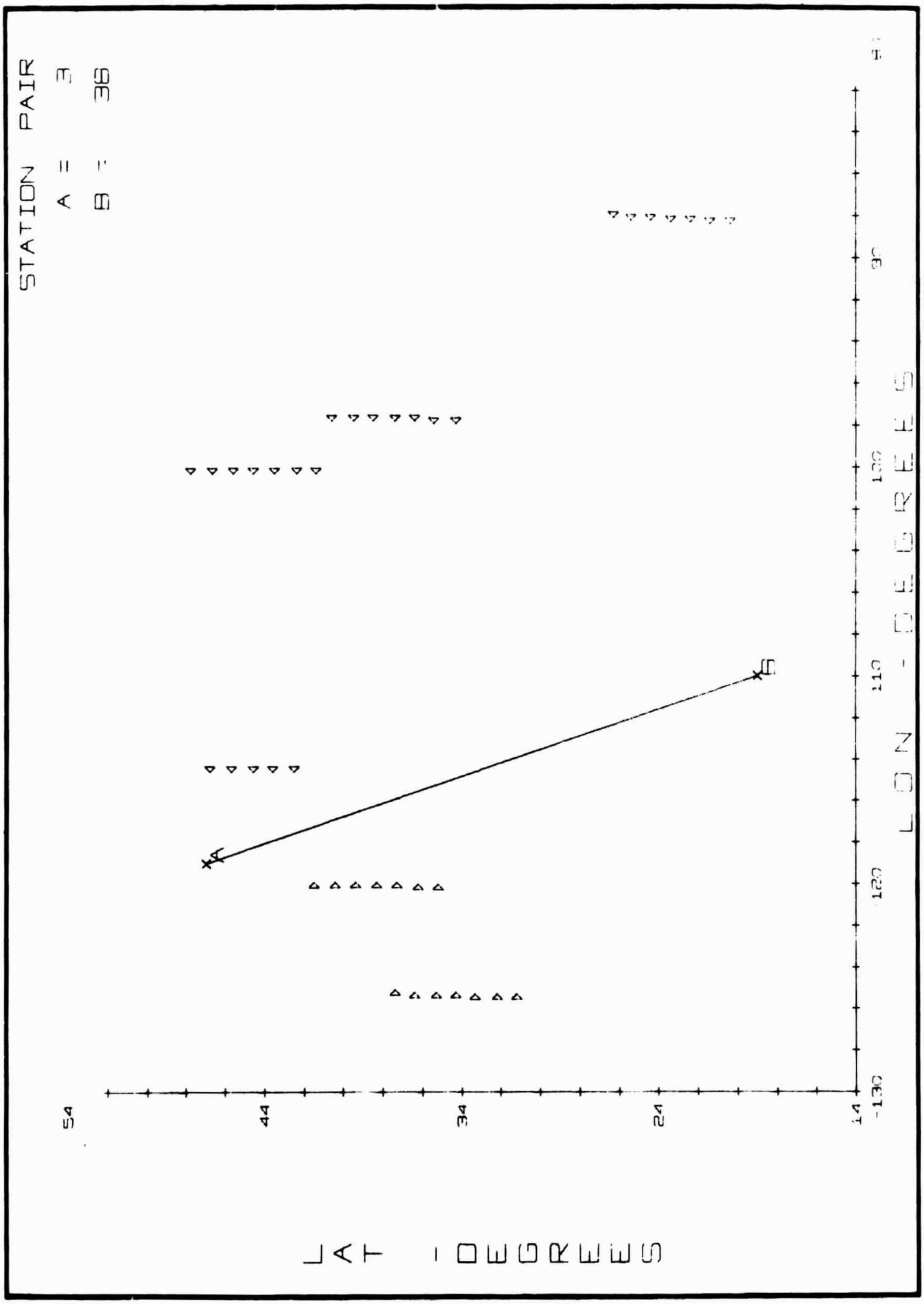
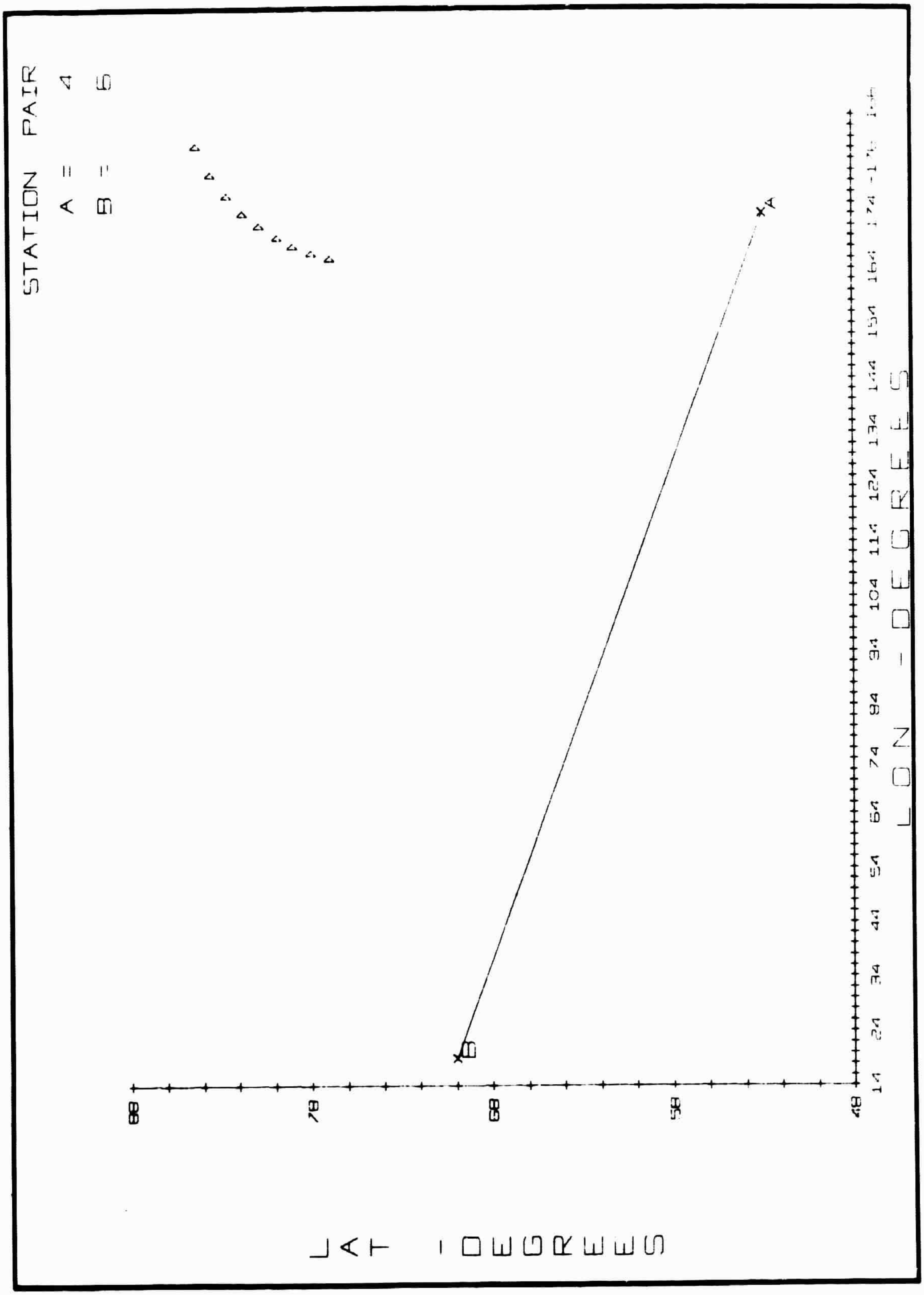
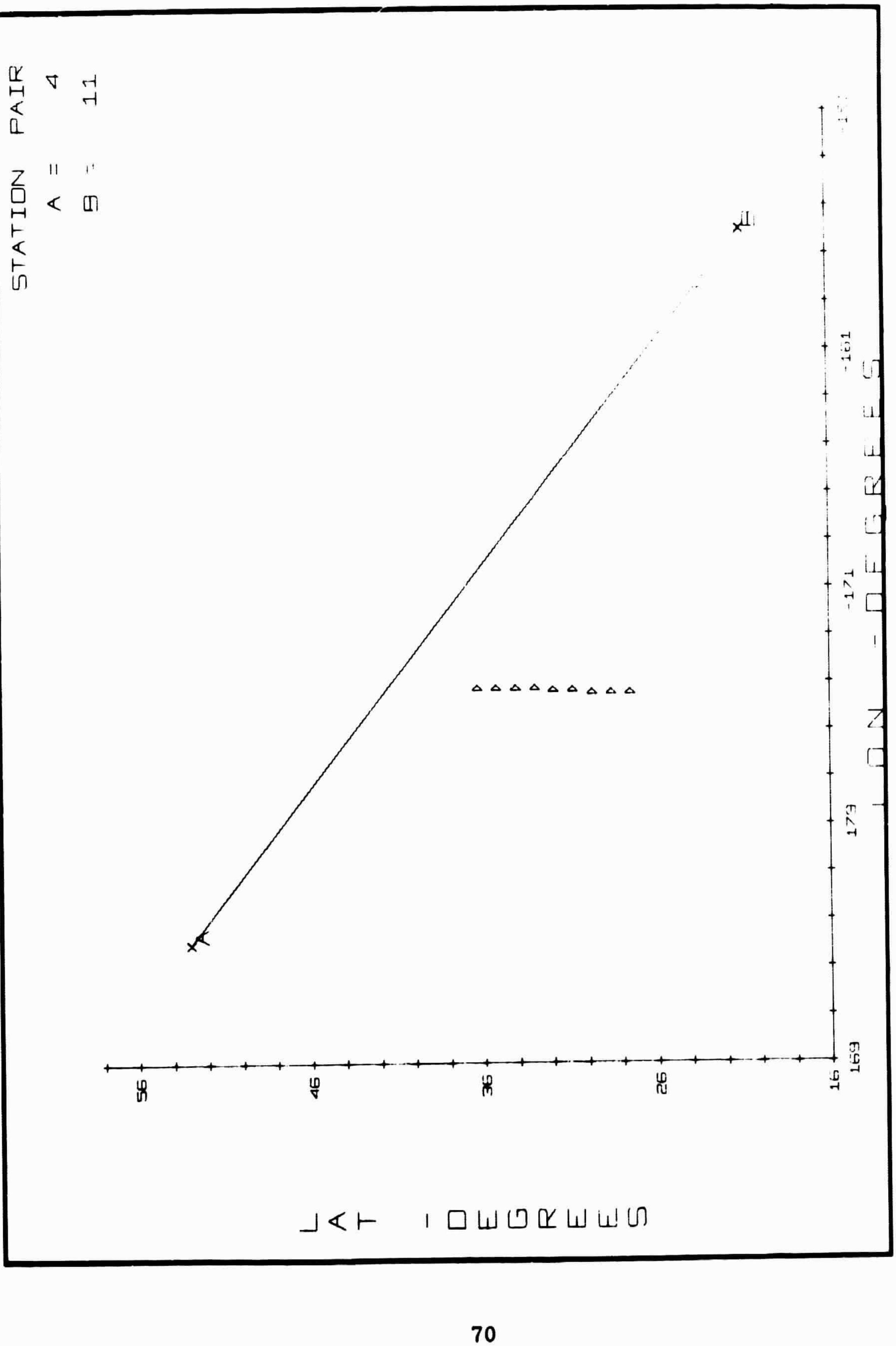
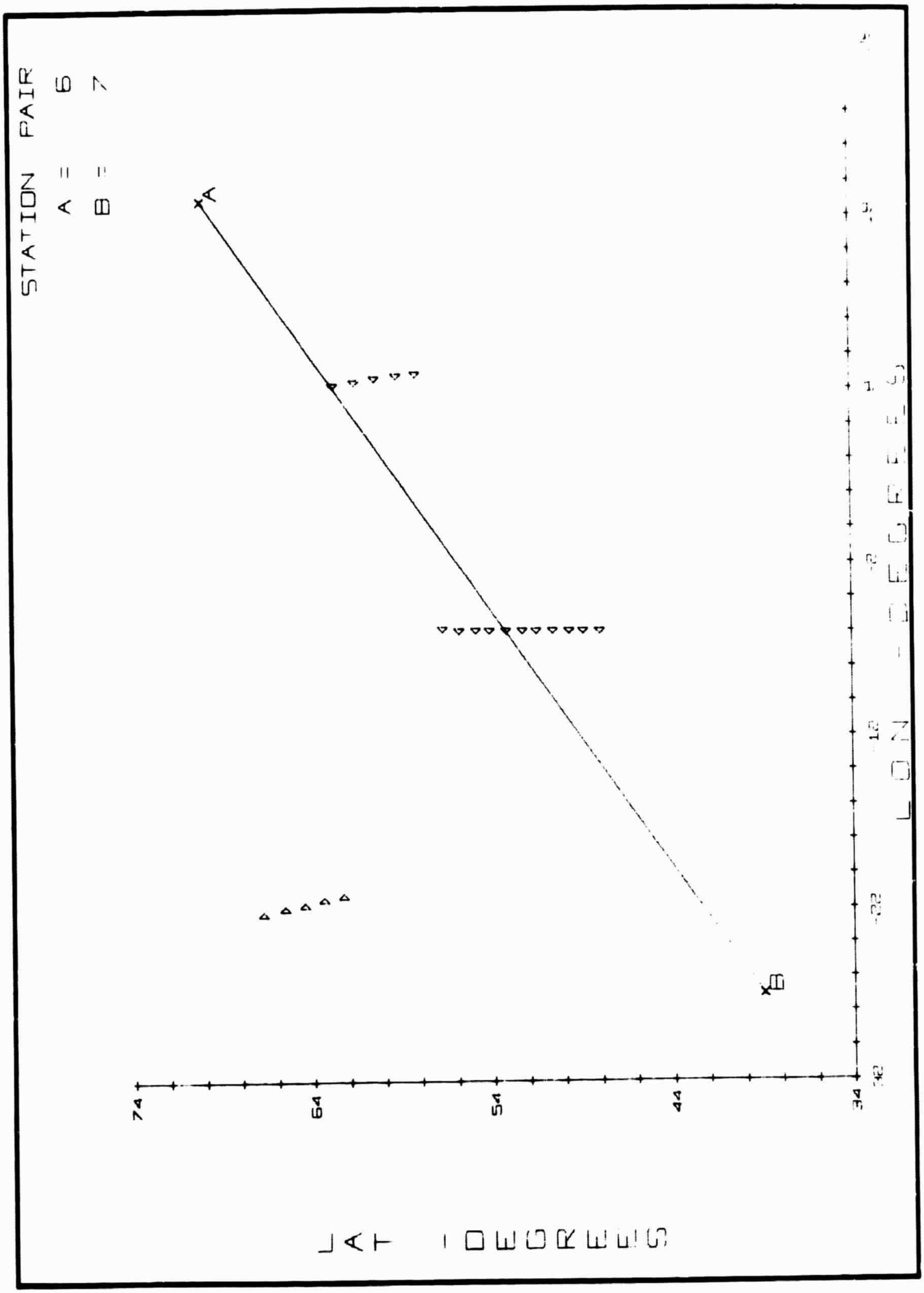
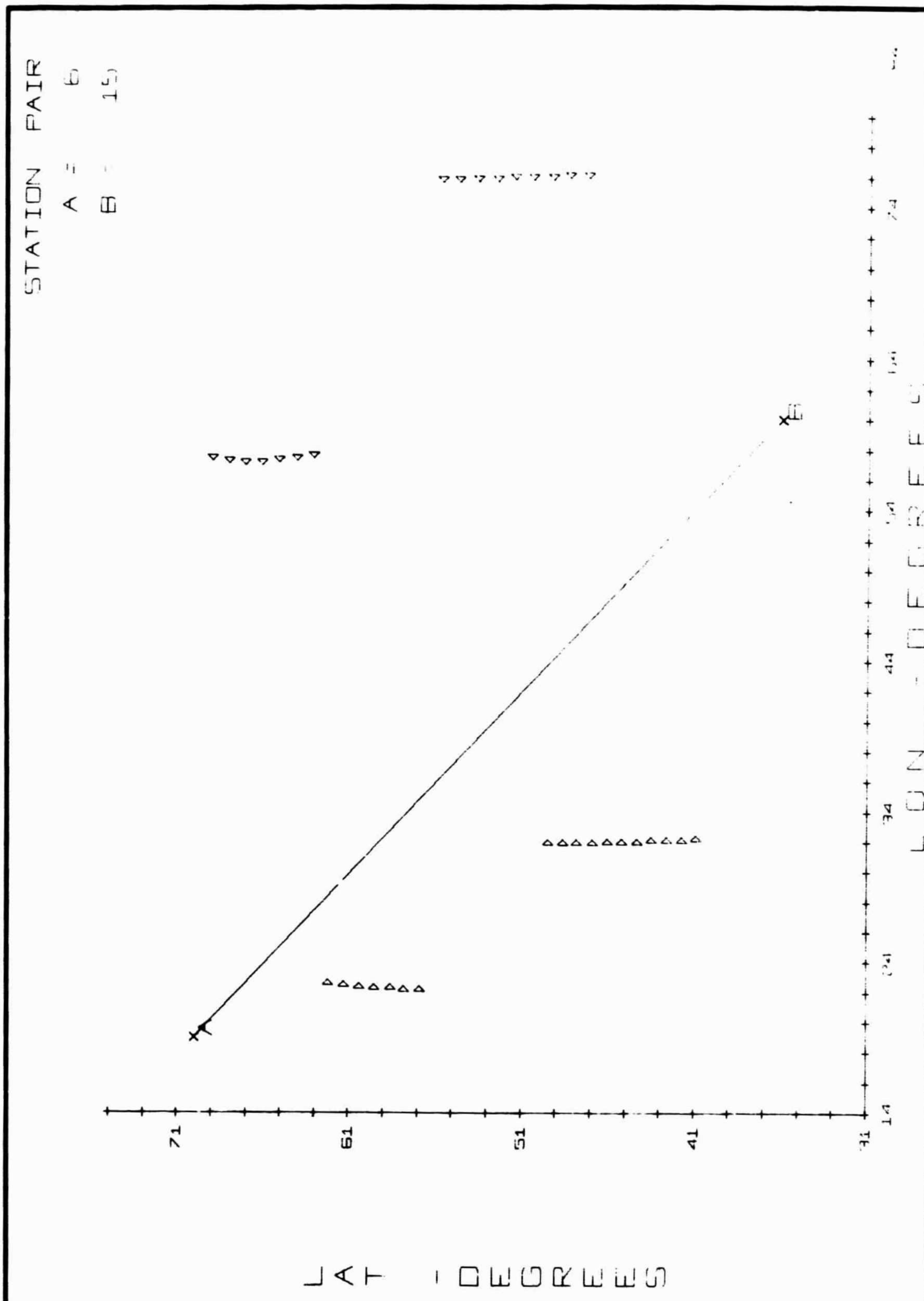
26
-171
-161
-141
-131
LON - DEGREES
LAT - DEGREES

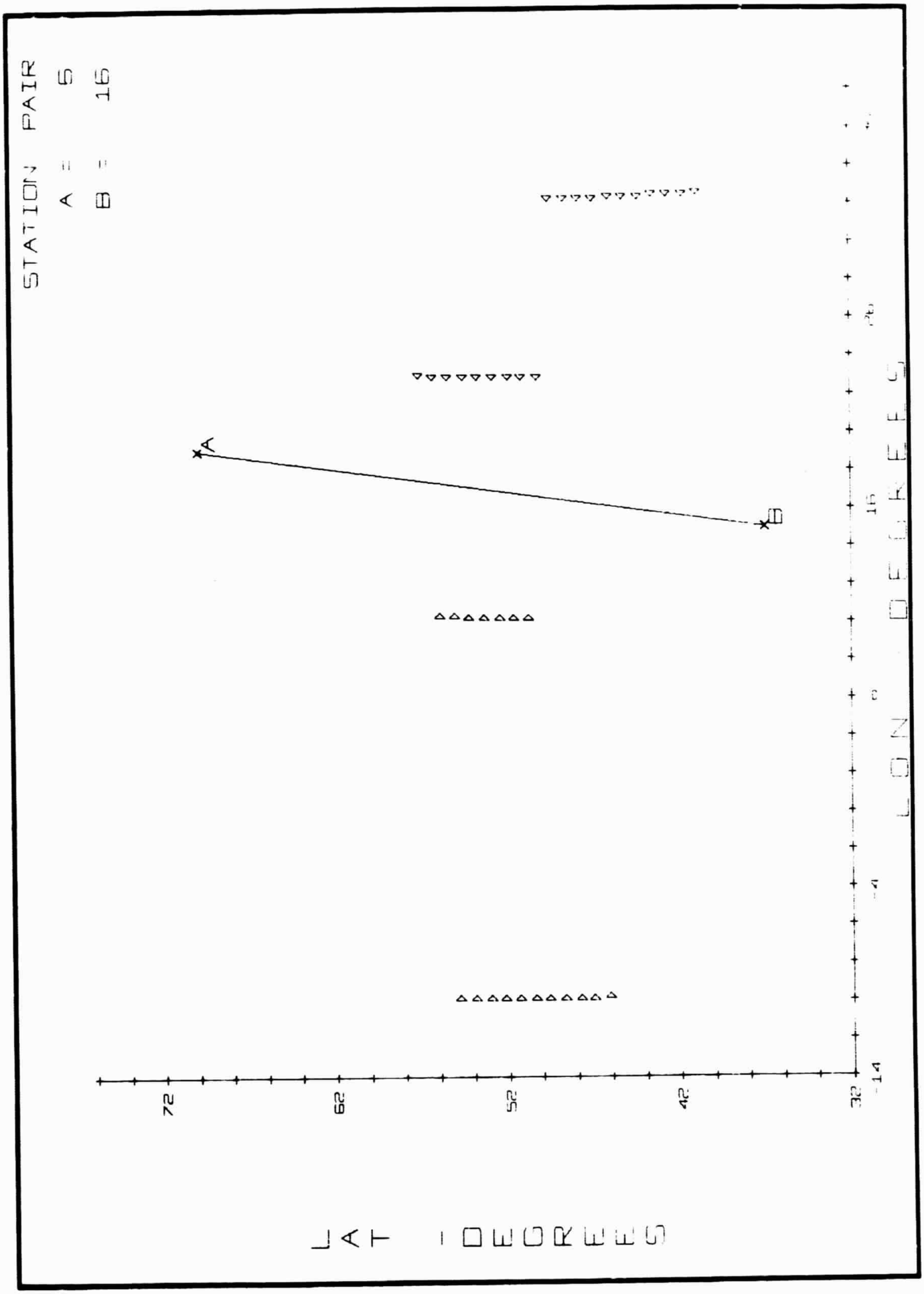

STATION PAIR
A = 3
B = 38
LAT - DEGREES
LON - DEGREES
54
44
34
24
14
-130
120
110
100
90


STATION PAIR
A = 4
B = 6
LON - DEGREES
LAT - DEGREES
14 24 34 44 54 64 74 84 94 104 114 124 134 144 154 164 174
48 58 68 78 88
A
B

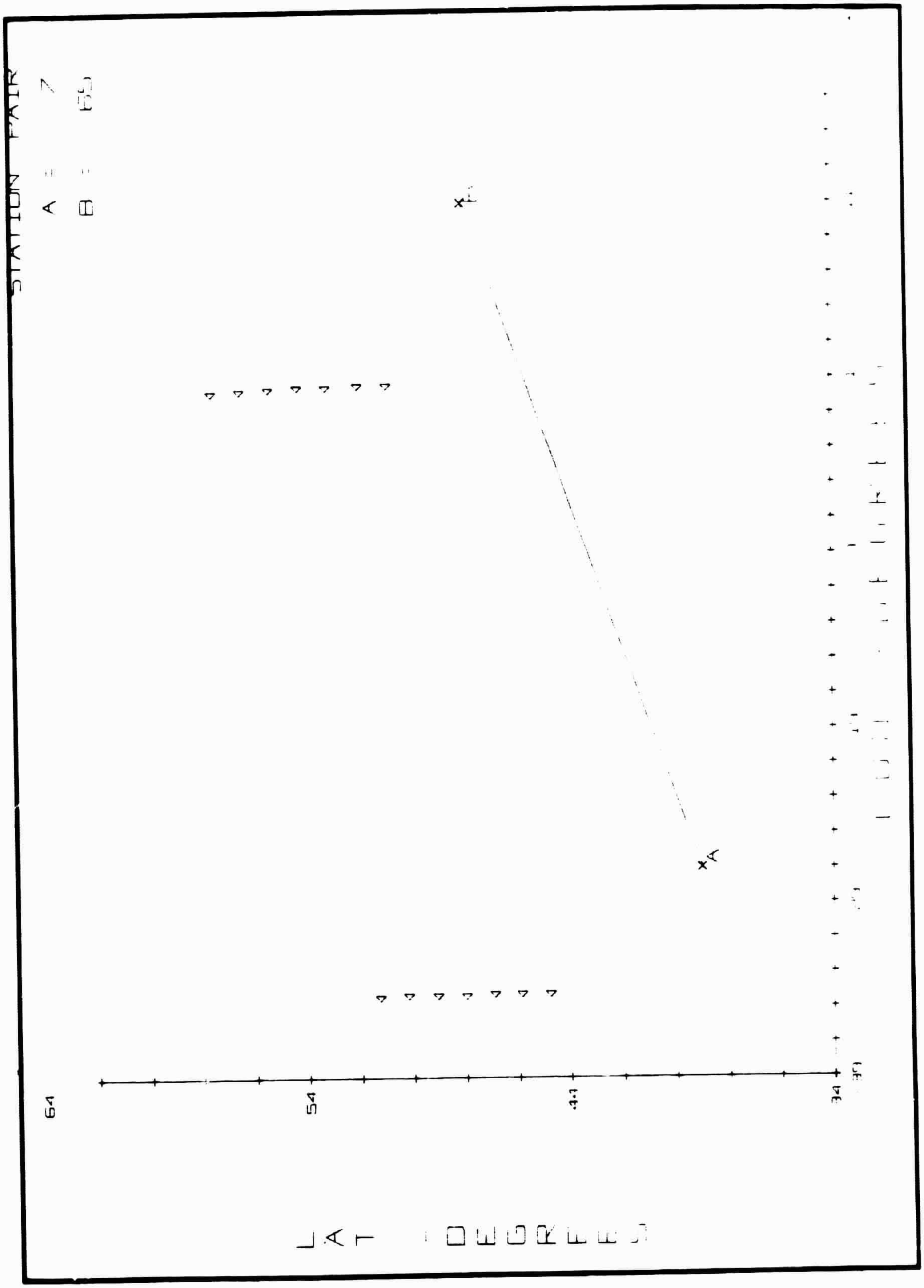

STATION PAIR
A = 4
B = 11
LAT - DEGREES
LON - DEGREES
56
46
36
26
16
169
179
-171
-161


STATION PAIR
A = 6
B = 7
A
B
74
64
54
44
34
LAT (DEGREES)
LON (DEGREES)

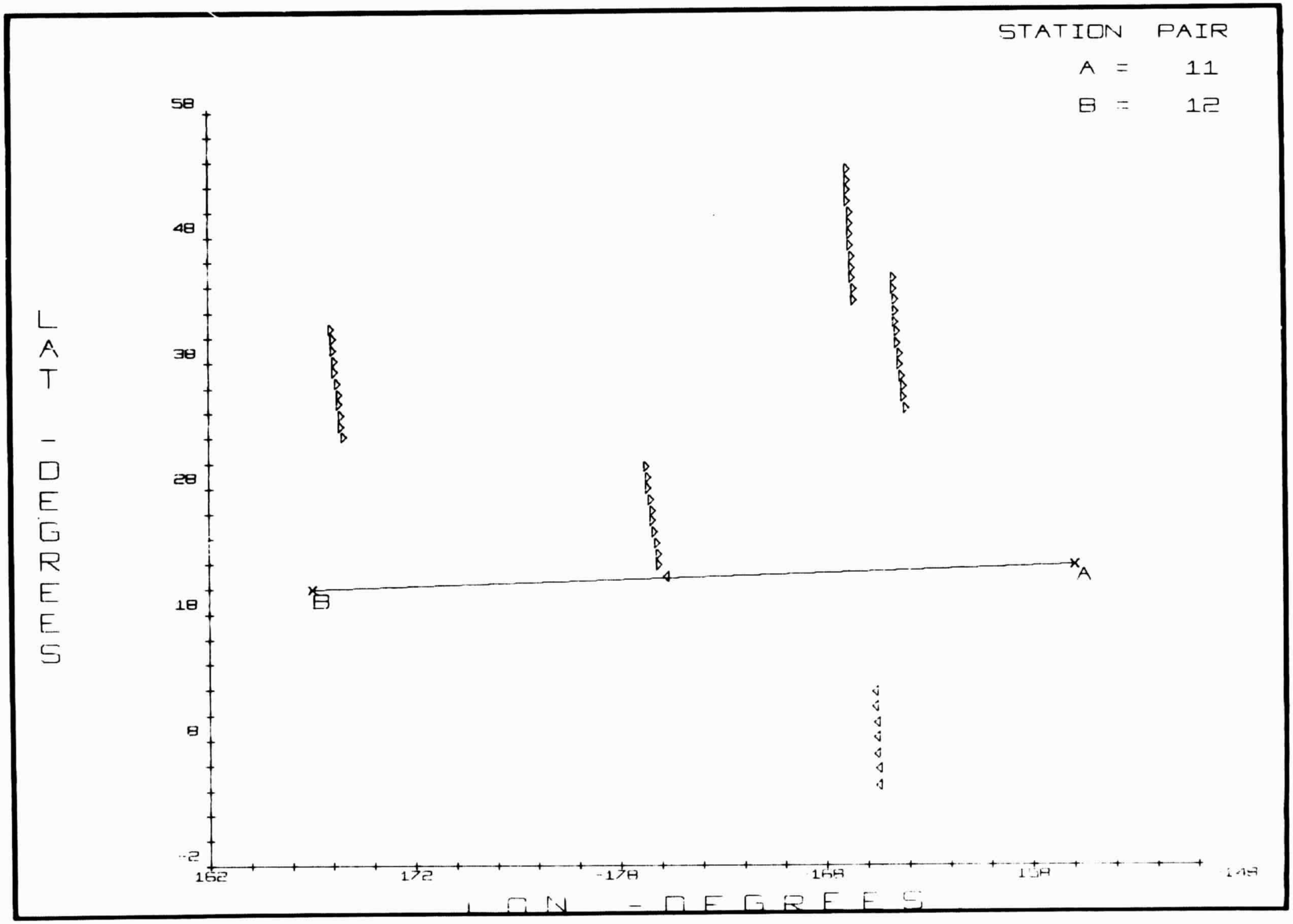

STATION PAIR
A = 6
B = 15
LAT - DEGREES
71
61
51
41
LON - DEGREES
14
34
44
74

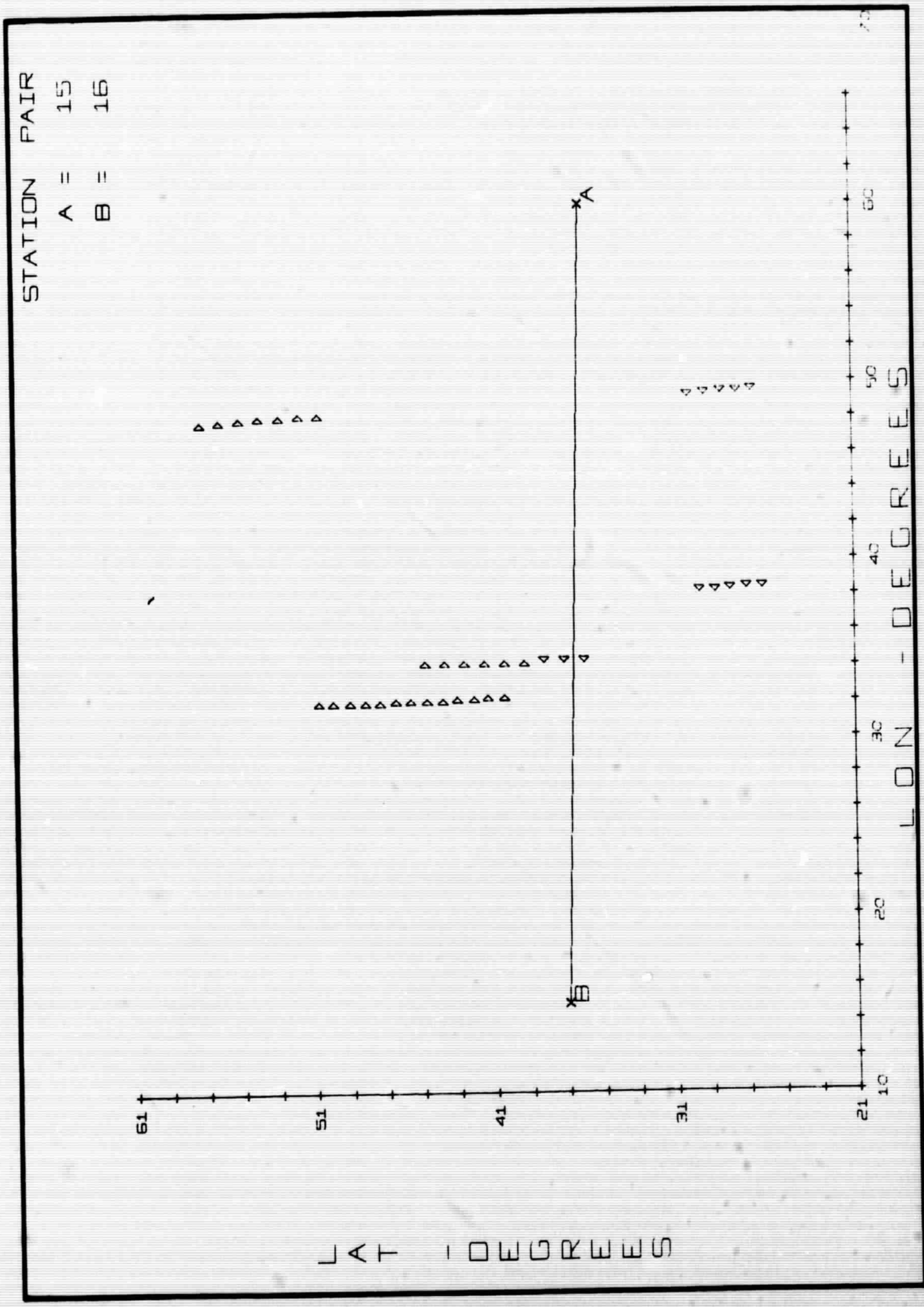


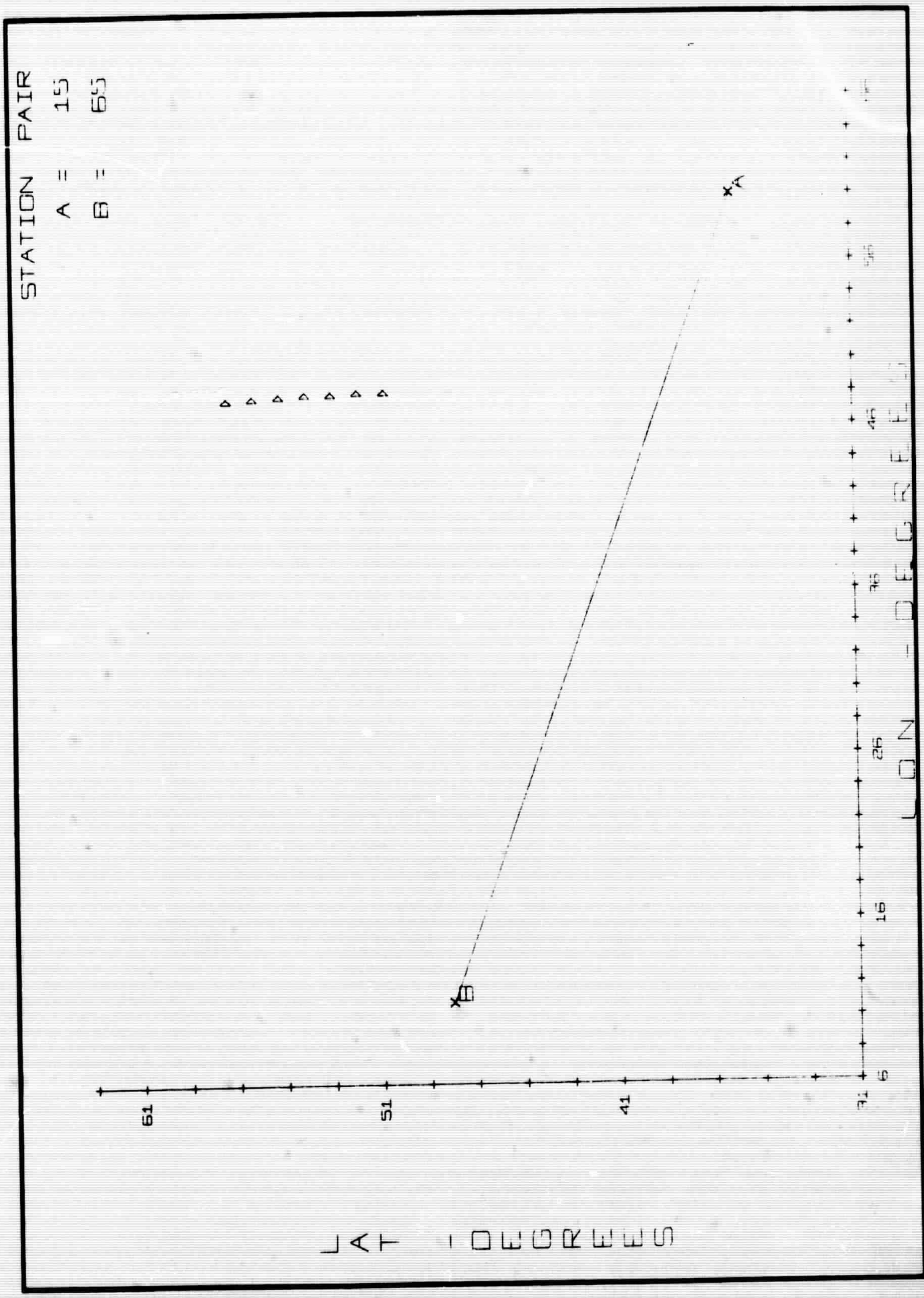
STATION PAIR
A = 5
B = 16
A
B
72
62
52
42
32
-14
16
LAT (DEGREES)
LON DEGREES



STATION PAIR
A = 7
64
54
44
34
LAT


STATION PAIR
A = 11
B = 12
LAT - DEGREES
LON - DEGREES
58
48
38
28
18
8
-2
162
172
-178
-168
-158
-148
A
B


STATION PAIR
A = 15
B = 16
LAT - DEGREES
LON - DEGREES
61
51
41
31
21
10
20
30
40
50
60
A
B


STATION PAIR
A = 15
B = 65
LAT - DEGREES
LON - DEGREES
61
51
41
31
6
16
26
36
46
56