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Temperature Variation of the Resistivity of
Metallic Strain Gauge Materials

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Abstract

Some transition metal alloys used in strain gauges, have resistivities which are large and vary much more slowly with temperature than usual metals. In an attempt to understand the lack of temperature dependence of these high resistivity alloys the theory of the deformation potential interaction between an electron and a longitudinal phonon was extended into the high collision rate regime. It is demonstrated that if the deformation potential decreases with increasing collision rate then the resistivity should become independent of temperature. It was found however, that the deformation potential was even more independent of electron collision rate than previously reported. As a consequence this effect cannot account for the observed lack of temperature dependence of the alloys of interest.

I. Introduction

In a narrow concentration range $\text{Cu}_x\text{-Ni}_{1-x}$ ($x \cong .60$) alloys have a high resistance with a very small temperature variation. There are other transition metal alloys which behave in a similar fashion. These properties are just those required for good strain gauge materials. The alloys involved are concentrated and the constituents have quite different atomic potentials. Hence the "impurity" scattering is so strong, that it must be included in the band structure i.e. as part of the unperturbed Hamiltonian. This is a difficult problem, but the methods to attack it are beginning to be developed. (1), (2), (3), (4) In addition to this major problem, the combination of band structure and "impurity" scattering, it is also necessary to understand why the usual electron-phonon contribution to the resistivity is absent. The electron-phonon contribution normally leads to room temperature resistivities in the range of $1 \rightarrow 10 \mu\Omega\text{-cm.}$, and the observed resistivity of $\text{Cu}_{0.6}\text{-Ni}_{0.4}$ is $\sim 50 \mu\Omega\text{-cm.}$ But even so, the observed temperature variation of the Cu-Ni alloy (5) ($\sim 10^{-2} \mu\Omega\text{-cm./}^\circ\text{K}$) is much smaller than one would guess from the usual way of combining resistivities, i.e. adding the impurity resistivity ρ_I to the phonon resistivity ρ_P , $\rho = \rho_I + \rho_P$. The purpose of the work being reported here was to investigate a mechanism which would quench the electron-phonon interactions (the deformation potential (6), (7)) if scattering rates get sufficiently high. This would have the effect of reducing the temperature variation of the phonon part of the resistivity to bring our expectations into agreement with experiment. The effect we were seeking does occur, however due to a peculiarity in the make up of the deformation potential it does not occur for the range of parameters which are encountered. The virtue of the present calculations is that it extends the range of validity of the deformation potential (and

the Born-Oppenheimer approximation) far beyond the realm for which it was formerly thought to be valid.

II. The Deformation Potential

The deformation potential is the change of the energy of an electron in the presence of a phonon relative to its energy in the perfect undisturbed crystal. In a metal the major contribution to the deformation potential arises because the electrons do not follow the motion of the ions perfectly and as a consequence a longitudinal phonon produces an electrical potential, (in the Coulomb gauge a transverse phonon produces a vector potential). A single electron moves in this electrical potential caused by all the other electrons and the distorted ion lattice and the energy of interaction constitutes an extra term in the one electron Hamiltonian. We shall derive the deformation potential by generalizing arguments which have been developed for the theory of acoustic attenuation^{(7), (8), (9)}. The acoustic attenuation theory is modified here in two ways; first the effect of a phonon on band structure is formally introduced into the theory, and second the solution method previously used, i.e. the Champers trajectory method,⁽¹⁰⁾ is replaced by one in which the physical approximations are more evident. The theory is semi-classical and in outline proceeds as follows. A Boltzmann equation for the temporal and spacial dependence of the electron distribution function with a collision time approximation is introduced. A phonon acts as an external driving force in this equation. The Boltzmann equation is solved self consistently, with Maxwell's equations to yield a deformation potential. The resulting deformation potential is an explicit function of the collision time which was introduced into the Boltzmann equation. The deformation potential can then be used along with time dependent perturbation theory to calculate the collision time. This leads to an implicit expression for the collision time which can then be solved

self consistently. It turns out that as collision rates become faster, the deformation potential tends to decrease, causing the collision rates to saturate. Thus the temperature variation of the resistivity would tend to be quenched. Unfortunately, the deformation potential does not begin to decrease as a function of collision rate until the product of the electron mean free path Λ times the phonon wave vector q becomes of order of the ratio of velocity of sound v_s to the Fermi velocity v_F , $q\Lambda \sim v_s/v_F \sim 10^{-3}$ rather than the previously reported condition of $q\Lambda \sim 1$.⁽¹¹⁾ For the phonons which do most of the electron scattering $q\Lambda$ does get to be ~ 1 in transition metal alloys but never becomes $\sim 10^{-3}$. Hence, while this temperature quenching occurs in principle, it cannot be the cause of the observed lack of temperature dependence of some transition metal alloys.

III. Calculation of the Deformation Potential

Only the longitudinal phonon-electron interaction will be discussed. Most of the electron scattering is due to longitudinal phonons. We shall see that the interaction between the electrons and the longitudinal phonons does not change when $q\Lambda \sim 1$. The transverse phonon-electron interaction may change when $q\Lambda \sim 1$. However this is relatively uninteresting since it cannot account for the observed lack of temperature variation of the resistivity in transition metal alloys.

The Boltzmann equation governs the temporal and spatial variation of the electron concentration distribution $f(\vec{r}, \vec{k}; t)$. The function $f(\vec{r}, \vec{k}; t)$ is the number of electrons per unit volume per unit (wave number)³ at point \vec{r} , with wave vector \vec{k} (crystal momentum $\vec{p} = \hbar\vec{k}$), at time t . The Boltzmann equation in the collision time approximation is:

$$1) \quad \frac{df(\vec{r}, \vec{k}; t)}{dt} = - \frac{f(\vec{r}, \vec{k}; t) - f_{l.e.}(\vec{r}, \vec{k}; t)}{\tau}$$

$$= \frac{\partial f}{\partial t} + \frac{\vec{F}}{\hbar} \cdot \frac{\partial f}{\partial \vec{k}} + \frac{\hbar \vec{k}}{m^*} \cdot \frac{\partial f}{\partial \vec{F}}$$

where $f_{l.e.}(\vec{r}, \vec{k}; t)$ is the local equilibrium distribution to which electron collisions tend to drive the system, and \vec{F} is the external force on the electrons caused by the phonon. The Boltzmann equation is only expected to be valid in the limit of long wave length phonons. For phonons whose wave lengths are comparable to interatomic spacings, this picture cannot yield accurate answers, however it should indicate trends. At any point in space the electron concentration $n(\vec{r}, t)$ is related to $f(\vec{r}, \vec{k}, t)$ through the expression

$$2) \quad n(\vec{r}, t) = \int d^3k f(\vec{r}, \vec{k}; t)$$

In thermodynamic equilibrium and in the absence of a phonon the distribution function $f_e(\vec{k})$ is a Fermi distribution:

$$3) \quad f_e(\vec{k}) = \frac{1}{4\pi^3} \frac{1}{e^{\beta(\epsilon_{\vec{k}}^0 - \mu)} + 1}$$

and the equilibrium electron concentration n_0 is:

$$4) \quad n_0 = \int d^3k f_e(\vec{k})$$

Eq. (4) determines μ^0 , the Fermi energy. $\epsilon_{\vec{k}}^0$ is the unperturbed energy as a function of \vec{k} , i.e., the band structure.

Suppose a longitudinal phonon with which the electron interacts has wave number q and frequency ω . Then the displacement (complex) $\vec{\xi}(\vec{r}, t)$ of an ion

relative to its equilibrium position, and velocity $\vec{u}(\vec{r}, t)$ of this ion located at \vec{r} and at time t can be expressed as

$$\vec{\xi}(\vec{r}, t) = \delta\vec{\xi} e^{i(\vec{q}\cdot\vec{r} - \omega t)}$$

5)

$$\vec{u}(\vec{r}, t) = \delta\vec{u} e^{i(\vec{q}\cdot\vec{r} - \omega t)}$$

where $\delta\vec{u} = -i\omega\delta\vec{\xi}$,

and the unit vectors $\delta\hat{\xi}$, $\delta\hat{u}$ and \hat{q} are equal, $\delta\hat{\xi} = \delta\hat{u} = \hat{q}$. The local equilibrium distribution $f_{l.e.}$ into which collisions (either with other phonons or with impurities) tends to drive the electrons is a drifted Fermi distribution:

$$6) \quad f_{l.e.}(\vec{r}, \vec{k}; t) = \frac{1}{4\pi^3} \frac{1}{e^{\beta[\epsilon_{\vec{k}}(\vec{r}, t) - \mu(\vec{r}, t) - m\vec{v}_{\vec{k}} \cdot \vec{u}(\vec{r}, t)]} + 1}$$

with

$$7) \quad n(\vec{r}, t) = \int d^3k f_{l.e.}(\vec{r}, \vec{k}; t)$$

$\epsilon_{\vec{k}}(\vec{r}, t)$ is the band structure at \vec{r}, t as changed by the presence of the phonon. $\epsilon_{\vec{k}}(\vec{r}, t)$ can be calculated from band structure theory⁽⁶⁾ and we shall accept it as an input to this calculation. The term $-m\vec{v}_{\vec{k}} \cdot \vec{u}(\vec{r}, t)$, with $\vec{v}_{\vec{k}} = \frac{1}{\hbar} \frac{\partial \epsilon_{\vec{k}}}{\partial \vec{k}}$, is introduced to account for the fact that the electrons move relative to a fixed reference frame (\vec{k} is in the fixed reference frame) and in the presence of a phonon the lattice is in motion. The energy in the Fermi function, $f_{l.e.}$, has been linearized with respect to small quantities, so terms proportional to u^2 have been dropped. Thus collisions will try to

drive the electron distribution into one which moves with the lattice. Finally the distribution is called "local equilibrium" because the Fermi energy level $\mu(\vec{r}, t)$ to which collisions drive the system is determined from the restriction imposed by Eq. 7. So the Fermi level is the appropriate one for the local band structure and the local electron concentration. The external force on an electron \vec{F} , in Eq. (1), is given by a Hamiltonian equation,

$$8) \quad \vec{F} = - \frac{\partial \epsilon_{\vec{k}}(\vec{r}, t)}{\partial \vec{r}} + e\vec{E}$$

The first term in Eq. (8) - $\frac{\partial \epsilon_{\vec{k}}(\vec{r}, t)}{\partial \vec{r}}$ arises because the band structure energy is position dependent. The second term $e\vec{E}$ is the electric force. The field \vec{E} is calculated self-consistently from Poisson's equation

$$9) \quad \frac{\partial}{\partial \vec{r}} \cdot \vec{E} = 4\pi e [n(\vec{r}, t) - n_I(\vec{r}, t)]$$

All the equations needed to solve the problem have now been introduced.

The solution to the set of equations, Eq. (1), (2), (6), (7), (8), (9), in the linear approximation is characterized by all variables oscillating about their equilibrium with changes in values proportional to the same spatial and temporal factor $e^{i(\vec{q} \cdot \vec{r} - \omega t)}$. Thus the various quantities can be written as:

$$10) \quad f(\vec{r}, \vec{k}; t) = f_e(\vec{k}) + \delta f(\vec{k}) e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$n(\vec{r}, t) = n_0 + \delta n e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$n_I(\vec{r}, t) = n_0 + \delta n_I e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$f_{l.e.}(\vec{r}, \vec{k}; t) = f_e(\vec{k}) + \delta f_{l.e.}(\vec{k}) e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$\vec{E}(\vec{r}, t) = \delta \vec{E} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$\epsilon_{\vec{k}}(\vec{r}, t) = \epsilon_{\vec{k}}^{\circ} + \delta \epsilon_{\vec{k}} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

Eqs. (3), (6), (7), (10) combine to yield:

$$11) \quad \delta f_{l.e.} = \frac{\partial f_e}{\partial \epsilon_{\vec{k}}} (\delta \epsilon_{\vec{k}} - \delta \mu - m v_{\vec{k}} \cdot \delta \vec{u})$$

where because the Fermi distribution cuts off sharply

$$12) \quad \frac{\partial f_e}{\partial \epsilon_{\vec{k}}} \cong -\frac{1}{4\pi^3} \delta(\epsilon_{\vec{k}}^{\circ} - \mu^{\circ})$$

and $\delta \mu$ is fixed by

$$13) \quad \delta n = \int \delta f_{l.e.} d^3 k = -\mathcal{N}(\epsilon_F) [\overline{\delta \epsilon_F} - \delta \mu]$$

The quantity

$$14) \quad \mathcal{N}(\epsilon_F) \equiv \frac{1}{4\pi^3} \int \frac{dS_F}{h v_{\vec{k}_F}}$$

is the density of states per unit energy range at the Fermi surface, dS_F is an element of area in \vec{k} space on the Fermi surface and $v_{\vec{k}_F}$ is the speed of the electrons with wave number \vec{k}_F , and the energy

$$15) \quad \overline{\delta \epsilon_F} \equiv \frac{1}{S_F} \int \delta \epsilon_{\vec{k}_F} dS_F$$

is the electron energy change averaged over the Fermi surface. In the special case of a parabolic band, i.e. $\epsilon_{\vec{k}}^0 = \frac{\hbar^2 k^2}{2m^*}$, then the Fermi surface is spherical and

$$17) \quad \mathcal{N}(\epsilon_F) = \frac{m^* k_F}{\pi^2} = \frac{3n_0}{2\epsilon_F}$$

$$18) \quad \overline{\delta\epsilon_F} = \delta\epsilon_{\vec{k}_F}$$

Eq. 13 relates the shift in the Fermi energy $\delta\mu$ to the locally shifted electron density δn and an average band structure change $\overline{\delta\epsilon_F}$. If Eqs. (10), (13) are inserted into Eqs. (1), (9), and only linear terms retained i.e. $\frac{\vec{F}}{\hbar} \cdot \frac{\partial f}{\partial \vec{k}} \cong \frac{\vec{F}}{\hbar} \cdot \frac{\partial f_e}{\partial \vec{k}}$

$$19) \quad \delta f(\vec{k}) = \frac{\partial f_e}{\partial \epsilon_{\vec{k}}^0} \frac{[\delta\epsilon_{\vec{k}} + \frac{q^2}{2} (\frac{\delta n}{\mathcal{N}} - \frac{\delta n_I}{\mathcal{N}})] i\vec{q} \cdot \vec{\Lambda} + [\delta\epsilon_{\vec{k}} - \overline{\delta\epsilon_F} - \frac{\delta n}{\mathcal{N}} - m\vec{v}_{\vec{k}} \cdot \delta\vec{u}]}{1 - i\omega\tau + i\vec{q} \cdot \vec{\Lambda}}$$

where

$$20) \quad \vec{\Lambda} \equiv \vec{v}_{\vec{k}} \tau = \text{the mean free path of an electron with velocity } \vec{v}_{\vec{k}}$$

and

$$21) \quad \alpha \equiv (4\pi e^2 \mathcal{N})^{1/2} \equiv (\text{Fermi-Thomas shielding length})^{-1}$$

The term $\frac{\partial f_e}{\partial \epsilon_{\vec{k}}^0}$ guarantees that only electrons near the Fermi surface contribute to δf . For the remainder of the derivation all quantities will be evaluated at the Fermi surface. We shall drop the subscripts "F" everywhere.

Eq. (19) relates two unknowns $\delta f(\vec{k})$ and δn . Note that δn does not depend on \vec{k} . Thus if $\delta f(\vec{k})$ from Eq. (19) along with the expressions in

Eq. (10) were substituted into Eq. (2), then the resulting equation can be solved for δn in terms of known quantities. This procedure leads to the answer:

$$22) \quad \delta n = \frac{\delta n_I - \frac{\delta \epsilon}{q} \frac{q^2}{2} - i \frac{m v_k^+ \delta u}{q \Lambda} \frac{q^2}{2}}{1 + (1 + I(\tau)) \frac{q^2}{2}}$$

where

$$I(\tau) = \frac{i\omega\tau}{1 - i\omega\tau - \frac{i2q\Lambda}{\ln\left(\frac{1-i\omega\tau + iq\Lambda}{1-i\omega\tau - iq\Lambda}\right)}}$$

Note that $q\Lambda = \left(\frac{\omega}{v_s}\right)(v_F\tau) = \frac{v_F}{v_s} \omega\tau \gg \omega\tau$. Thus in a metal, where the Fermi velocity $v_F \approx 10^8$ cm/sec is always large compared to the velocity of sound $v_s \approx 10^5$ cm/sec., the terms $(\omega\tau)$ which arise from the phonon oscillations ($\propto \frac{\partial f}{\partial t}$) are always small compared to those $(\omega q\Lambda)$, caused by electron motion ($\propto \frac{\partial f}{\partial \epsilon}$). Thus to lowest order in $(\omega\tau/q\Lambda) = (v_s/v_F)$, $I(\tau)$ is:

$$24) \quad I(\tau) \approx \frac{i\omega\tau}{1 - \frac{2q\Lambda}{\tan^{-1}\left(\frac{2q\Lambda}{1-q^2\Lambda^2}\right)}}$$

In the two interesting limits $I(\tau)$ becomes

$$25) \quad I(\tau) \approx \begin{cases} \frac{-i\omega\tau}{2} \ll -i & ; \quad q\Lambda \ll 1 \\ \frac{-3i\omega\tau}{q^2\Lambda^2} = \frac{-3i}{q\Lambda} \frac{v_s}{v_F} & ; \quad q\Lambda \ll 1 \end{cases}$$

Thus $|I(\tau)|$ is small compared to unity until $q\Lambda$ becomes small enough so that $q\Lambda \ll \frac{v_s}{v_F}$. As a consequence shielding does not begin to be affected by collisions until $q\Lambda$ becomes of order 10^{-3} . The numerator of Eq. (22) contains three terms which arise respectively from coulomb coupling between the electrons and ions, the shift in the Fermi level in the local equilibrium distribution, and the drift term in the local equilibrium. Note that the first two terms in the numerator are independent of the collision rate but the drift term does depend on $q\Lambda$.

The electric field $\vec{E}(\vec{r}, t)$ and the electrical potential $\phi(\vec{r}, t)$ are related by the usual expression

$$26) \quad \vec{E}(\vec{r}, t) = - \frac{\partial \phi(\vec{r}, t)}{\partial \vec{r}}$$

or using $\phi(\vec{r}, t) = \delta\phi e^{i(\vec{q} \cdot \vec{r} - \omega t)}$ then

$$27) \quad \phi(\vec{r}, t) = \frac{iE(\vec{r}, t)}{q}$$

Eq. (27), (9), (10) combine to yield:

$$28) \quad \phi(\vec{r}, t) = \frac{4\pi e}{q} [n(\vec{r}, t) - n_I(\vec{r}, t)]$$

Finally the deformation energy caused by the longitudinal phonon with wave vector \vec{q} can be written as:

$$29) \quad D_{\vec{q}}(\vec{r}, t) = e\phi(\vec{r}, t) + (\epsilon_{\vec{k}}(\vec{r}, t) - \epsilon_{\vec{k}}^0) \\ = \delta D_{\vec{q}} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

This can be expressed in terms of known quantities by substituting from Eqs. (28), (10), (22), (21).

$$30) \quad \delta D_{\vec{q}} = \frac{\left(\frac{-\delta n_I}{\mathcal{N}} + \frac{\delta \epsilon_{\vec{k}}}{k} \frac{q^2}{2} \right) (1 + I(\tau)) + (\delta \epsilon_{\vec{k}} - \overline{\delta \epsilon}) - i \frac{m v_{\vec{k}} \delta u}{q \Lambda}}{1 + (1 + I(\tau)) \frac{q^2}{q^2}}$$

The total deformation energy H_{e-p} i.e. the electron-phonon interaction, from all longitudinal phonons is:

$$31) \quad H_{e-p} = \sum_{\vec{q}} [\delta D_{\vec{q}} e^{i(\vec{q} \cdot \vec{r} - \omega t)} + D_{\vec{q}}^* e^{-i(\vec{q} \cdot \vec{r} - \omega t)}]$$

Let's investigate $D_{\vec{q}}$ to see where it changes character as τ decreases. To do this, express $\delta \epsilon_{\vec{k}}$, $\overline{\delta \epsilon}$ and δu in terms of δn_I . First find relations between each of these quantities and the lattice displacement $\delta \xi$. Eq. (5) already contains the relation between δu and $\delta \xi$. The other quantities δn_I , $\delta \epsilon_{\vec{k}}$, $\overline{\delta \epsilon}$ are each proportional to the lattice dilation $\frac{\partial}{\partial \vec{r}} \cdot \vec{\xi} = i \vec{q} \cdot \vec{\xi} = i q \delta \xi e^{i(\vec{q} \cdot \vec{r} - \omega t)}$. Thus we find

$$32) \quad \delta n_I = -i n_0 q \delta \xi$$

$$33) \quad \delta \epsilon_{\vec{k}} = i U_{\vec{k}} q \delta \xi = -U_{\vec{k}} \frac{\delta n_I}{n_0}$$

$$34) \quad \overline{\delta \epsilon} = i \overline{U} q \delta \xi = -\overline{U} \frac{\delta n_I}{n_0}$$

where $U_{\vec{k}}$ the energy shift of the \vec{k}^{th} electron state per unit dilation and its average over the Fermi surface \overline{U} are inputs to this theory⁽⁶⁾. Substitute

Eqs. (5), (32), (33), (34) into Eq. (30).

$$35) \quad \delta D_{\vec{q}} = \frac{-\delta n_I}{n_o} \left[\frac{\left(\frac{n_o}{\delta} + U_{\vec{k}} \frac{q^2}{q^2} \right) (1 + I(\tau)) + (U_{\vec{k}} - \bar{U}) + \frac{ihkv_s}{q\Lambda}}{1 + (1 + I(\tau)) \frac{q^2}{q^2}} \right]$$

If this equation is now specialized to the case of a parabolic band then using Eqs. (17), (18) it becomes

$$36) \quad \delta D_{\vec{q}} = \frac{-\delta n_I}{n_o} \left[\frac{\left(\frac{2\epsilon_F}{3} + U_{\vec{k}} \frac{q^2}{q^2} \right) (1 + I(\tau)) + \frac{i2\epsilon_F}{q} \frac{v_s}{v_F}}{1 + (1 + I(\tau)) \frac{q^2}{q^2}} \right]$$

In the usual limit $q\Lambda \gg 1$ which applies to most metals it reduces to the customary answer⁽⁶⁾

$$37) \quad \delta D_{\vec{q}} = \frac{-\delta n_I}{n_o} \left[\frac{\frac{2\epsilon_F}{3} + U_{\vec{k}} \frac{q^2}{q^2}}{1 + \frac{q^2}{q^2}} \right]$$

where Eq. (25) has been used. In the other limit where the collisions become fast $q\Lambda \lesssim 1$ which can be reached in transition metal alloys, then Eq. (36) along with Eq. (25) gives:

$$38) \quad \delta D_{\vec{q}} = \frac{-\delta n_I}{n_o} \left[\frac{\left(\frac{2\epsilon_F}{3} + U_{\vec{k}} \frac{q^2}{q^2} \right) - i \frac{3v_s}{v_F} \frac{1}{q\Lambda} U_{\vec{k}} \frac{q^2}{q^2}}{1 + \left(1 - i \frac{3v_s}{v_F} \frac{1}{q\Lambda} \right) \frac{q^2}{q^2}} \right]$$

The rigid ion model predicts $U_k^+ = \frac{2}{5} \epsilon_F$ and somewhat better theory due to Bardeen has $U_k^+ \ll \epsilon_F$. (11)(12) In either case δD_q^+ remains essentially unchanged until $q\Lambda$ becomes of order $\frac{v_s}{v_F} \cong 10^{-3}$.

If we simplify still further to the Bardeen case where $U_k^+ \ll \epsilon_F$ (nearly free electron case) and remember (Eq. 32) that δn_I is a negative pure imaginary number then the real and imaginary parts of δD_q^+ become:

$$39) \quad \text{Re}[\delta D_q^+] = \frac{\frac{2}{3}\epsilon_F |\delta n_I|}{1 + \frac{q^2}{q^2}} \left[\frac{\frac{3v_s}{v_F} \frac{1}{q\Lambda} \frac{q^2}{q^2}}{1 + \left(\frac{3v_s}{v_F} \frac{1}{q\Lambda}\right)^2 \frac{(q^4/q^4)}{\left(1 + \frac{q^2}{q^2}\right)^2}} \right]$$

$$40) \quad \text{Im}[\delta D_q^+] = \frac{\frac{2}{3}\epsilon_F |\delta n_I|}{1 + \frac{q^2}{q^2}} \left[\frac{1}{1 + \left(\frac{3v_s}{v_F} \frac{1}{q\Lambda}\right)^2 \frac{(q^4/q^4)}{\left(1 + \frac{q^2}{q^2}\right)^2}} \right]$$

A sketch of these functions from Eqs. (37), (39), (40) is in Figure 1. The $\text{Re}[\delta D_q^+]$ reaches its peak at the same place that the $\text{Im}[\delta D_q^+]$ is half its initial value. This occurs for $\frac{1}{q\Lambda} = \frac{v_F}{3v_s} \left(\frac{1 + q^2/q^2}{q^2/q^2} \right)$. Hence for $q/q \ll 1$ ($q \sim 10^{-7}$ cm for most metals) the $\text{Re}[\delta D_q^+]$ can be ignored and the $\text{Im}[\delta D_q^+]$ practically doesn't change. For $q/q = 1$, then $\text{Im}[\delta D_q^+]$ reaches half of its initial value at $\frac{1}{q\Lambda} = \frac{2v_F}{3v_s} \sim 10^3$. This means $\frac{1}{\tau} = \frac{2}{3} \left(\frac{v_F}{v_s} \right)^2 \omega \sim 10^{+6} \times 10^{13} [\text{sec}^{-1}]$ or $\tau \sim 10^{-19}$ [sec]. This τ is far faster than any found even in the highest resistivity alloys e.g. for $\sigma \sim 100\mu\Omega\text{-cm}$. then $\tau \sim 10^{-15}$ sec.

What happened? Why is it the less restrictive condition $q\Lambda \sim \frac{v_s}{v_F}$ rather than the previously reported condition $q\Lambda \sim 1$, which must be

satisfied before the deformation potential begins to change its character. The reason is that the magnitude of the deformation potential is dominated by the electrical forces which cause shielding. These forces are quite large compared to the effective forces on the electron distribution due to collisions, even for relatively small $q\Lambda \sim 1$. The physical situation which must be reached before the collisions begin to compete with the electric forces is that collisions must become sufficiently fast so that in a phonon period ($\frac{1}{\omega}$) an electron cannot diffuse a distance equal to a phonon wave length ($1/q$). In this case collisions force the electrons to remain firmly attached to their moving ions. The root mean square distance $d(t)$ that an electron diffuses in a time t is $d(t) = \Lambda \sqrt{(t/\tau)}$. We require $d(\frac{1}{\omega}) \approx \frac{1}{q}$, or $\Lambda \sqrt{1/\omega\tau} \approx (\frac{1}{q})$, or finally

$$41) \quad q\Lambda \approx \frac{v_s}{v_F}$$

Thus collisions cannot influence the electron distribution, and so the deformation potential, until $q\Lambda \approx \frac{v_s}{v_F}$. However, acoustic energy loss begins to vary with τ when $q\Lambda \sim 1$. This occurs because in the expression for acoustic energy loss the large, τ independent term $\sim \delta n$ which arises from the electrical forces causing shielding is multiplied by a small term which varies with τ . Hence the processes which begin to vary with τ when $q\Lambda \sim 1$ are not competing with large constant process in this case. Similarly, one also expects the transverse phonon-electron interaction which does not compete with the electric forces causing shielding to change character when $q\Lambda \sim 1$.

For completeness, H_{e-p} can be expressed in terms of longitudinal phonon creation and annihilation operators $a_{\vec{q}}^+$ and $a_{\vec{q}}$ by remembering⁽¹³⁾ that

$$42) \quad \vec{\xi} = \hat{q}(2\sigma_M\omega)^{\frac{1}{2}} a_{\vec{q}} e^{i(\vec{q}\cdot\vec{r} - \omega t)}$$

where σ_M is the mass density of the solid. Then from Eqs. (31), (32), (35) the electron phonon interaction is:

$$43) \quad H_{e-p} = i \sum_{\vec{q}} \frac{\vec{q}}{q} (2\delta\omega)^{\frac{1}{2}} \left\{ \left[\frac{\left(\frac{n_o}{N(\epsilon_F)} + U_{k_F} \frac{q^2}{q^2} \right) (1+I(\tau)) + (U_{k_F} - \bar{U}_F) + \frac{imv_{k_F} v_s}{q\Lambda_F}}{1+(1+I(\tau)) \frac{q^2}{q^2}} \right] a_{\vec{q}} e^{i(\vec{q}\cdot\vec{r}-\omega t)} \right. \\ \left. - \left[\frac{\left(\frac{n_o}{N(\epsilon_F)} + U_{k_F} \frac{q^2}{q^2} \right) (1+I^*(\tau)) + (U_{k_F} - \bar{U}_F) - \frac{imv_{k_F} v_s}{q\Lambda_F}}{1+(1+I^*(\tau)) \frac{q^2}{q^2}} \right] a_{\vec{q}}^+ e^{-i(\vec{q}\cdot\vec{r}-\omega t)} \right\}$$

where for emphasis the subscript "F" has been replaced on the appropriate symbols.

Conclusions

The deformation potential is remarkably independent of collisions. The main reason is that the deformation potential is controlled by the same electrical forces which cause shielding. Collisions do not begin to affect the electron distributions established by these electrical forces until the distance an electron diffuses in a phonon period becomes shorter than a phonon wave length. Under these circumstances the electrons cannot move far enough from "their ions" in a phonon period to set up the shielded distribution. When the lattice disturbance is a longitudinal phonon this means the electrons are "rigidly" attached to the ions by collisions so there is no electrical potential established. In fact, for longitudinal phonons the ions are super-shielded since for these very high collision rates the electrons follow the

ions almost perfectly. If this phenomenon occurred in a physical range of collision rates it would cause the collision rate to saturate and become temperature independent.

Since this effect does not account for the observed phenomena, what else could cause it? The experimental correlation between high resistivities and low temperature coefficients is still suggestive. The calculation done here is a semi-classical and single scattering theory, which may be the source of the difficulty. The scattering rates typically encountered in these alloys are so high that mean free paths are approaching interatomic spacings, so that electron momentum is not a very good quantum number (or classical concept). A better theory will need to start from electron states which are quite different from Bloch states. The most promising new technique^{(1), (2)} to attack this problem starts from multiple scattering theory and then uses the "coherent potential approximation" (C.P.A.) to solve the problem. So far, the effect of phonons on the electron transport properties have not been incorporated into the C.P.A. theory. Next we plan to investigate the effect of multiple electron scattering from concentrated "impurities" on the electron-phonon interaction and so on the temperature variation of the resistivity.

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Appendix A - Temperature Variation of the Collision Time

Since the effect to be discussed here does not occur in a physical range of parameters only an outline of a theory will be developed. The detailed theory could easily be constructed following this outline. We have shown, Eqs. (27), (32), that the electron-phonon interaction H_{e-p} can be symbolically written as

$$A-1) \quad H_{e-p} = H_{ep}^{\circ} g(\tau)$$

where the function $g(\tau)$ contains the effect of collisions on the deformation potential and is a function whose magnitude starts at unity and decreases as τ decreases. Then the "golden rule" of time dependent perturbation theory leads to the result

$$A-2) \quad \frac{1}{\tau} = K(T) |H_{e-p}^{\circ}|^2 \overline{|g(\tau)|^2}$$

where the appropriate average $\overline{|g|^2}$ is over all phonon wave numbers. The function is sketched in Figure II. $K(T)$ contains all the temperature dependence and we can define a τ_0 as:

$$A-3) \quad \frac{1}{\tau_0(T)} = K(T) |H_{ep}^{\circ}|^2$$

where $\tau_0(T)$ is the temperature variation of the collision time if the deformation potential were independent of τ . Combining Eqs. (A-2) and (A-3) yields

$$A-4) \quad \frac{\tau_0(T)}{\tau} = \overline{|g(\tau)|^2}$$

In Figure II, Eq. (A-4) is solved graphically. Clearly as long as $|\overline{g(\tau)}|^2$ remains unity $\tau = \tau_0(T)$ but when $\tau_0(T)$ begins to intercept the curve $|\overline{g(\tau)}|^2$ near $\frac{1}{q\Lambda} = \frac{v_F}{v_S}$ then τ becomes practically independent of $\tau_0(T)$. Thus under these circumstances τ saturates and becomes almost independent of temperature. Unfortunately this never occurs in a physical range of parameters and so cannot account for the lack of temperature dependence of some transition metal alloys e.g. copper constantan.

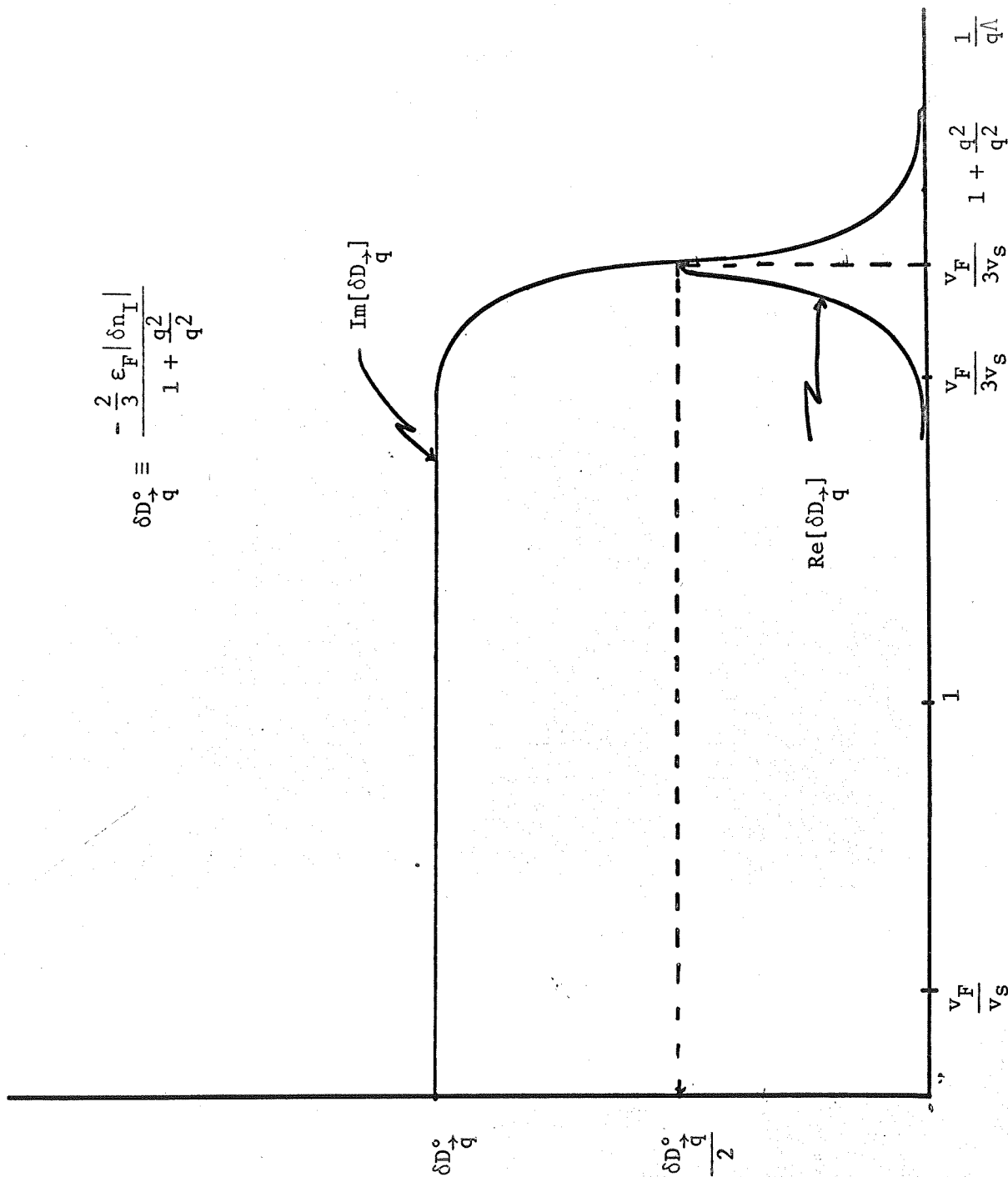


Figure 1

Real and imaginary parts of the deformation potential associated with a phonon of wave vector \vec{q} as a function of $\frac{1}{q\Lambda}$.

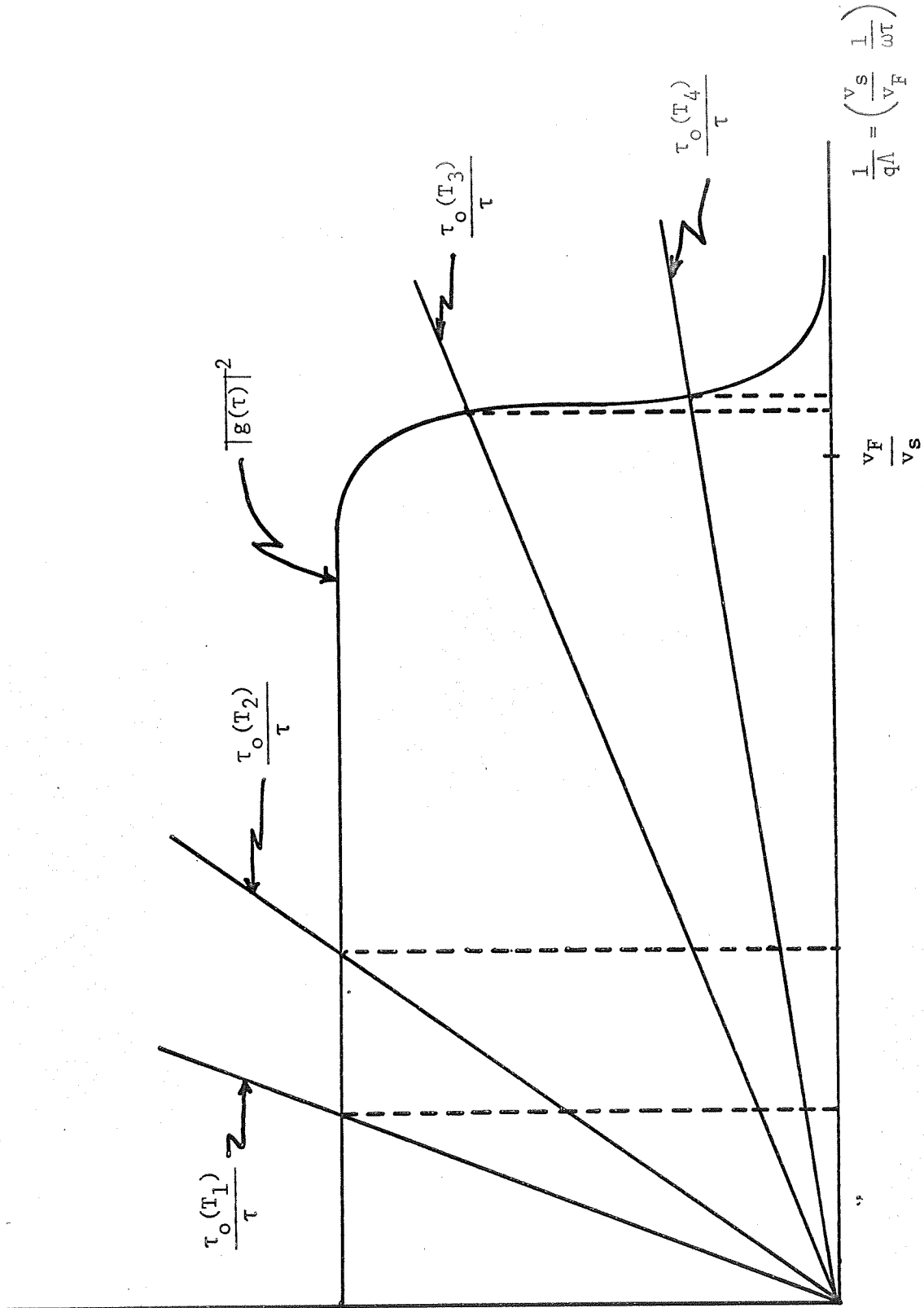


Figure II

The intercepts of $\frac{\tau_o(T)}{\tau}$ and $|g(\tau)|^2$ are shown for four temperatures chosen so $\tau_o(T_2) - \tau_o(T_1) = \tau_o(T_4) - \tau_o(T_3)$. Clearly, in the region where $|g(\tau)|^2$ changes sharply τ becomes nearly independent of $\tau_o(T)$.