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Digital Spectral Analysis of Bistatic-Radar Echoes from Explorer XXXV

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RADIOSCIENCE LABORATORY
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ABSTRACT

Bistatic-radar echoes from Explorer XXXV have been received at Stanford using the Stanford Research Institute 150' dish antenna. Analog tape recordings are used to preserve narrow band receiver outputs for subsequent digital spectral analysis. With a proper choice of system parameters dynamic ranges in excess of 60 db are achieved. Use of the Cooley-Tukey algorithm in a real time computer program permits the computation of Fourier coefficients for a slightly greater than 1 KHz data bandwidth with 1 hz resolution. A subsequent operation combines sets of these coefficients to form reliable spectral estimates.

CONTENTS

ILLUSTRATIONS

1. Introduction

During the latter half of 1967 the lunar Explorer XXXV spacecraft and the Stanford Research Institute 150' dish antenna were used as the transmitting source and receiver, respectively, in a bistatic radar experiment to study the surface of the moon. This experiment was carried out as part of the Explorer XXXV flight program conducted by Goddard Space Flight Center. Officially, the experiment is known as the Stanford Telemetry Monitoring Experiment. Transmissions from the spacecraft, which is in a roughly equatorial lunar orbit, were received on the earth after reflection from the lunar surface. The reflected signals contain information regarding the physical shape and vertical electromagnetic structure of the lunar surface. Two other reports describe the design and operation of the receiving system (Tyler, 1968b) and the computer programs for predicting the strength and Doppler shift of the reflected signals (Tyler, 1968a). Complete descriptions of the bistatic radar concepts may be found in Fjeldbo (1964) and Tyler (1967). Preliminary results from the Explorer XXXV experiment appear in Tyler (1968c, d and 1970). This report describes the (digital) spectral analysis techniques used in the reduction of Explorer XXXV data for this experiment.

2. General Experimental Considerations

Explorer XXXV carries a 6 watt telemetry transmitter which operates in conjunction with an omnidirectional antenna system, and thus radiates equal power toward the earth and the moon. The downlink telemetry spectrum consists of a 136 MHz carrier phase modulated by either telemetry data from the on board experiments and engineering functions or ranging tones derived from an uplink signal transmitted from the ground. The index of modulation is chosen so that about 2 watts of the total radiated power remains in the carrier while the remaining 4 watts are symmetrically distributed among the modulation sidebands. With the exception of a telemetry synchronization tone at ±275 Hz from the carrier, the telemetry tones occupy the spectral region between 400 and 1000 Hz above and below the carrier signal. In the ranging mode the telemetry modulation is suppressed and the sidebands are displaced 900 kHz from the carrier signal.

The principal component of the echo, or carom, signal comes from a small region about the center of the first Fresnel zone on the mean lunar surface (Fjeldbo, 1964). Since this point continuously changes with the motion of the spacecraft about the moon, a Doppler effect is associated with the carom path, and may be calculated from the rate of change of the reflected path length to and from the mean surface.

If the moon were perfectly smooth, the echo signal would be an undistorted, albiet time delayed and Doppler shifted, replica of the transmissions that propagate directly to the earth. However, the roughness of the lunar surface modulates and spectrally broadens the echo. Since the telemetry sidebands are, *a priori*, randomly distributed over their range only the reflection of the cw telemetry carrier could be utilized. The extent of the echo broadening is correlated with the roughness of the surface and varies between about 0.1 Hz and about 250 Hz, while the differential Doppler shift between the direct and reflected signals varies between approximately ±1000 Hz. Both of these quantitites depend upon the spacecraft, moon, earth geometry and the velocity of the spacecraft.

The power in the carom signal varies with position in orbit and with the properties of the lunar surface. A maximum value for the total power in the echo was about 1/400th that of the direct. Signals several orders of magnitude weaker were detected in the course of the experiment.

The experimental problem then is to detect a weak, randomly fluctuating echo signal in the presence of the much stronger, directly propagating telemetry carrier and sidetones. Spectral analysis was chosen as the method. Observations were of necessity restricted to those times when the echo was separated from the directly propagating telemetry signals by Doppler effects. Digital techniques were required to achieve the necessary dynamic range (~60 db) stability.

3. Receiving System Characteristics

The details of the Stanford Explorer XXXV receiving system have been described in the references given in the introduction to this report. For purposes of data reduction the receiving system may be modeled as a narrow band tracking filter which maintain a fixed phase relationship with the telemetry carrier. The receiving system contains two identical coherent channels for each of orthogonal polarizations (right and left circular). The system output consists of in-phase and quadrature components of the filtered passbands, heterodyned to zero frequency. System design permits the filter passbands to be offset from the carrier frequency by fixed amounts. Filter passbands of 10, 50, 100 and 200 Hz were available. Frequency offsets could be set to a precision of 1 Hz.

This particular arrangement permitted certain economies in the data reduction which will become evident later. The data are preserved for subsequent reduction on frequency modulation tape recordings of the filter outputs with appropriate timing information. A schematic representation of the receiving system functions and recording process is given in Figure 1.

4. Formation of Spectral Estimates from Fourier Coefficients Use of the Cooley-Tukey Algorithm

A. Computational Efficiencies

The statistical relations between gaussian random processes, linear operations such as filtering, and Fourier coefficients are well understood (see Middleton, 1960) and it is assumed that the reader has a knowledge of the elementary concepts of such processes from statistical communications theory. It should be clear that the output spectrum of the Explorer XXXV receiving system consists of the telemetry signals in the selected passband, a random background component made up in part from the receiver noise and in part from the galactic background noise, and a weak narrowband component due to the reflection of the telemetry transmissions from the lunar surface.

We are concerned here with the digital formation of estimates (in the statistical sense) of this composite spectrum. It will be assumed that the data have been sampled at the Nyquist rate. In the past, the computationally most efficient method for obtaining these

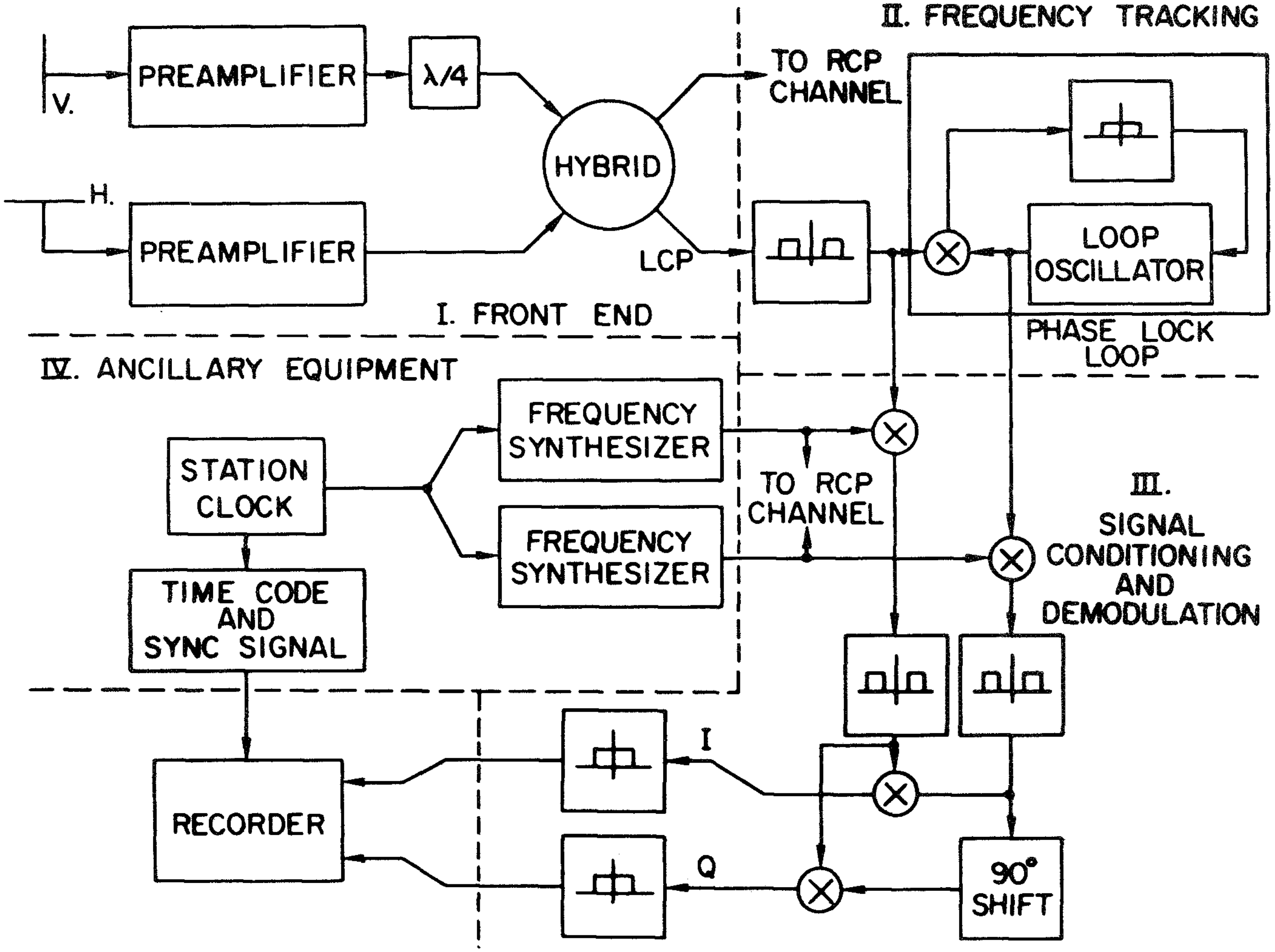

Figure 1. Functional block diagram of Stanford Explorer XXXV receiving system. The inphase and quadrature components of the filtered signal preserves the sense of rotation of phasors heterodyned to near d.c.

estimates was to compute the autocorrelation function of the data, and then use the fact that autocorrelation functions and power spectra are a Fourier transform pair to obtain the spectral estimates using only a single Fourier transform calculation (Blackman and Tukey, 1958). If one considers a collection of N data samples, then approximately $N \cdot \tau$ multiplications and additions are required to compute the autocorrelation function for τ lags. Using brute force techniques an additional τ^2 multiplication and additions must be performed to calculate corresponding Fourier series, assuming that the maximum number of independent frequency bins obtainable from this data is desired. If the number of degrees of freedom of the estimates is K, and the number of independent frequencies desired is M, then for this case $K \sim N/M$ and the number of calculations for a given K is proportional to KM^2, where it has been assumed that $K \gg 1$ is desired (see Blackman and Tukey). Precise arguments leading to the number of calculations are quite complex, and depend upon the details of the estimates formed. However, the results given here are good approximations for estimating the calculations required.

The Cooley-Tukey (1965) algorithm provides a method for computing complex Fourier series from a set of M complex data points with only $M \cdot \log_2 M$ complex multiplications and additions. That is, one computes

$$a_k = \sum_{l=0}^{M-1} x_l \, e^{-j2\pi\frac{lk}{M}} \quad k = 0, 1, \ldots, M-1$$

with considerably less than the M^2 operations required by brute force techniques. Since the Fourier coefficients a_k are independent random variables, spectral estimates with K degrees of freedom may be formed by summing successive $(a_k^m)^2$ with respect to m, where each (a_k^m) is calculated from a successive set of M data samples, x_j^m. The number of multiplications and additions to obtain estimates with K degrees of freedom is about $K \cdot M \cdot \log_2 M$, since the successive additions to form the sums of $(a_k^m)^2$ are negligible with respect to calculating the a_k^m themselves. Thus, for a given K, the ratio of computations required to form spectral estimates via the autocorrelation function and direct Fourier transform using the Cooley-Tukey algorithm approach is $M/\log_2 M$. The direct Fourier transform was chosen for this work because for large M it is the computationally much more efficient method.

B. Data Weighting Functions and Spectral Estimates

Fourier series, i.e., the set of Fourier coefficients, calculated directly from a sequence of unmodified data samples may be interpreted as the output of a set of filters with a frequency response such as

$$H(f) \sim \frac{\sin \pi f \cdot \Delta t}{\sin \pi f \cdot T} = \frac{\sin \pi f \cdot T/N}{\sin \pi f \cdot T}$$

here N = total number of samples in the data set

T = time in which the N samples were taken

f = frequency with inverse units of T

Δt = interval between samples

and it is assumed that the data samples are taken at a uniform rate. A continuous representation of frequency is used for convenience.

The ratio of sine functions plays the same role in analysis of sample data as the sinc function in the analysis of continuous functions (Bracewell, 1965). H(f) is simply the envelope of the transform of a sampled pulse of length T. From another point of view, the result of calculating the Fourier series of a finite set of (band-limited) data samples is the discrete convolution of the infinite Fourier series with the function H(f). Thus, in the case of a pure sinusoidal variation, the resulting spectrum is simply H(f).

For many applications, H(f) decays so slowly that the presence of a strong signal may cause large (compared with the inherent computation noise) effects some distance away. For the case of the Explorer XXXV observations the carrier and telemetry sidetones frequently have 3 to 4 orders of magnitude more power than the echo signals we wish to detect. The minimum value of the normalized power transfer function $H(f)^2/H(0)^2$ is $1/N^2$, while for small values of f this ratio goes as $\frac{T^2}{f^2}$, or 6 db for every octave of $\frac{1}{T}$.

These unwanted responses at some distance from the carrier may be reduced by a multiplicative weighting of the data samples prior to taking the Fourier transform. If we let $d(n\Delta t)$ represent the sampled data, and $w(n\Delta t)$ be the weighting function, then the resulting Fourier series is the convolution of the transforms of d and w. That is

$$S(f) = D * W$$

where D and W are the Fourier series of d and w, respectively and the symbol $*$ denotes convolution. The function W replaces the function H

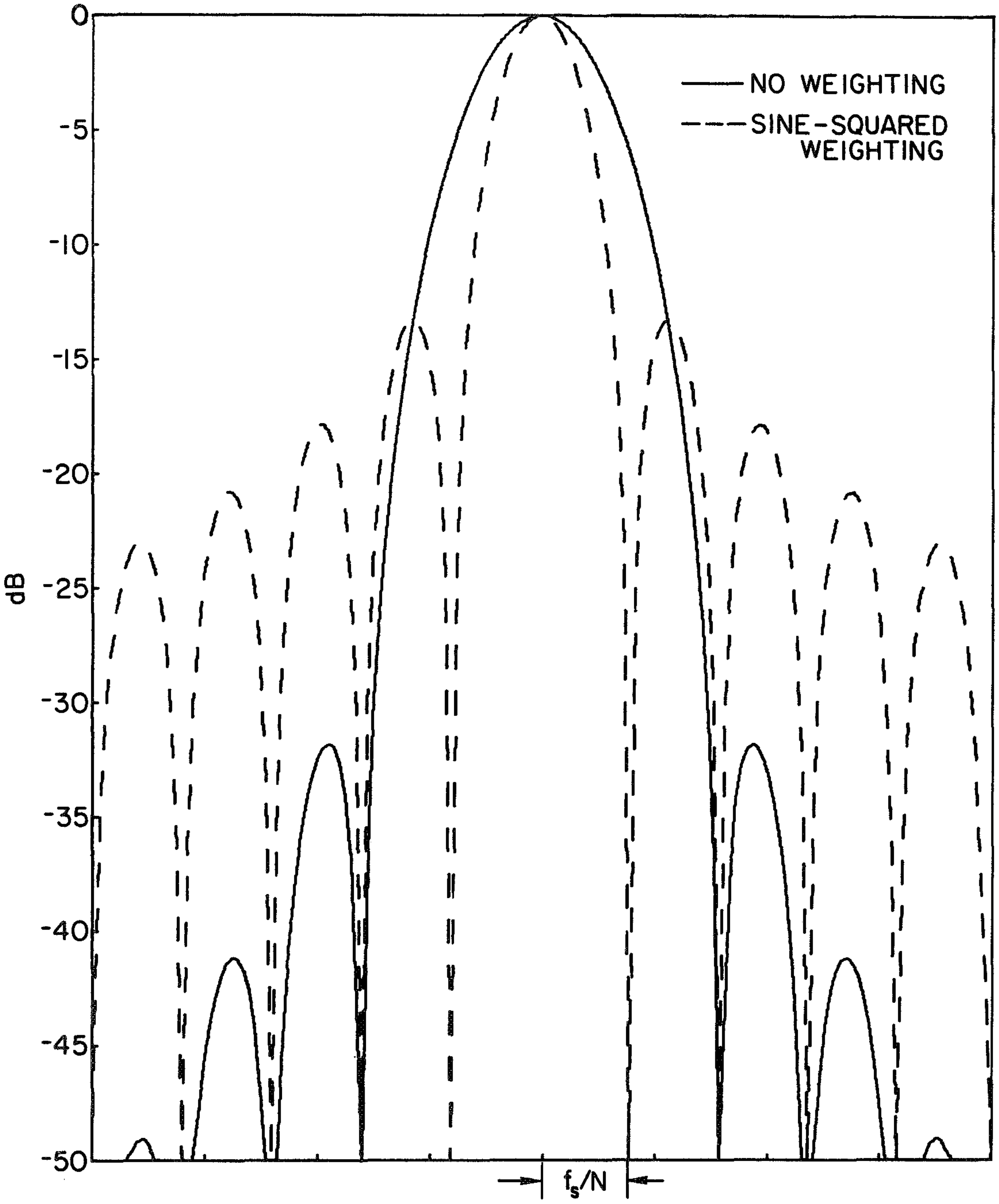

Figure 2. Comparison of equivalent filter functions for an unweighted Fourier transform and sine-squared weighting.

from the unweighted case as the filter response.

The choice of W function heavily depends upon the experimental situation. A number of "standard" choices exist and have been discussed elsewhere. For the Explorer XXXV data we chose

$$w = \sin^2 (t)$$

both because of its simplicity and because it represents a good compromise between convolutional broadening of the analysis bandwidth and sidelobe response. This is equivalent to a "hanning" window. The power transfer function of the equivalent filter is $|W|^2$. The two filter functions corresponding to the sine-squared weighting and the unweighted transform are compared in Figure 2. It is clear that an improvement of approximately 20 db in side lobe response is achieved through use of the sine-squared weighting.

5. The Spectral Analysis Computer Programs

A. Quantization, Data Weighting, and Calculations of Fourier Coefficients

Analog-to-digital conversion of the Explorer XXXV data tapes and computation of weighted Fourier coefficients was accomplished with a single real time computer program run on an SDS 930 computer system at the Stanford Research Institute (SRI), Menlo Park California*. A block diagram of the system is given in Figure 3. The Fourier coefficients

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*This program was written under subcontract from Stanford to SRI by A. Larsen, using programming techniques developed in his own work at SRI.

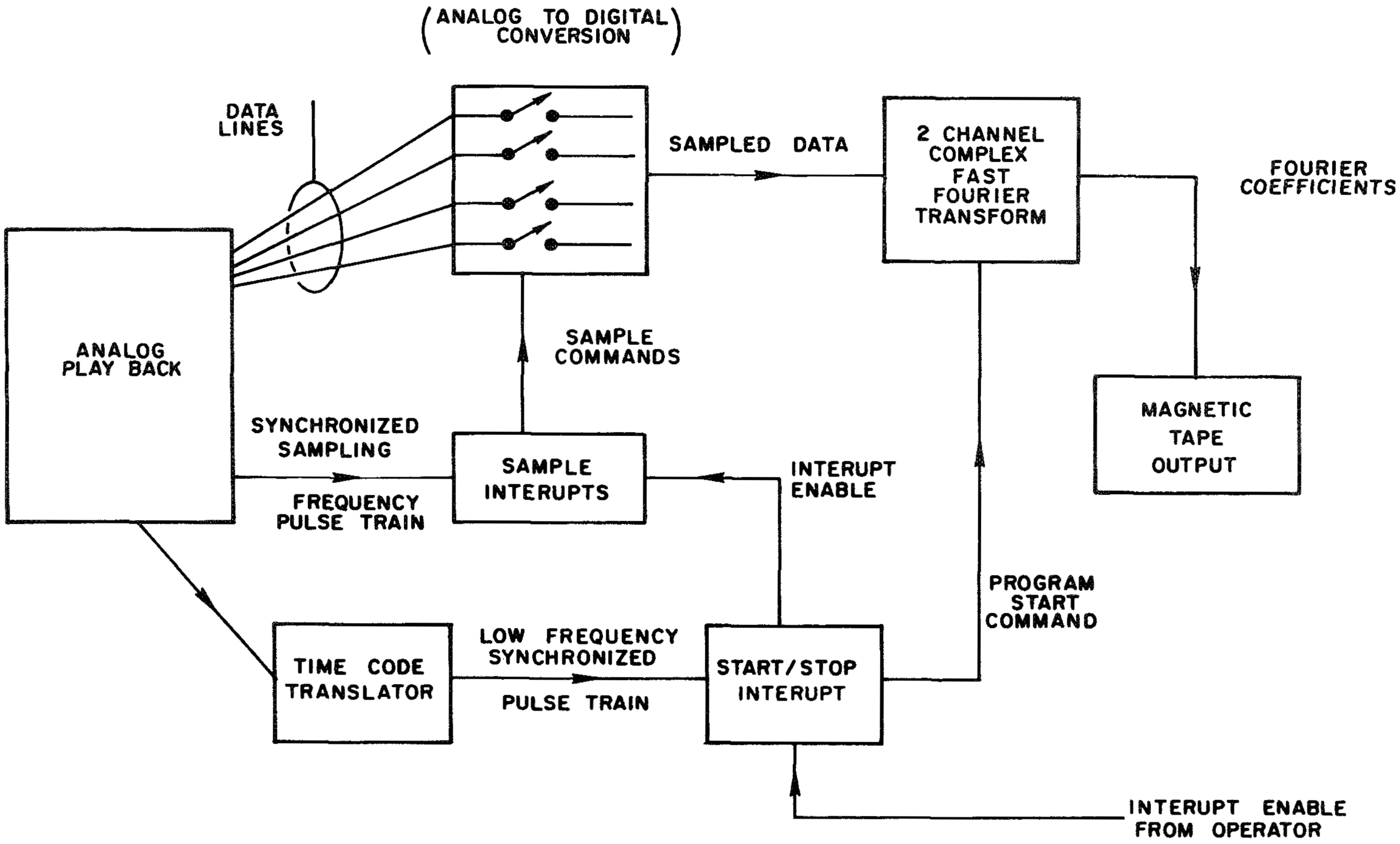

Figure 3. Block diagram of data reduction system for analog-to-digital conversion, data weighting and calculation of Fourier coefficients.

are combined into spectral estimates in a separate step which is described below in the section 5B of this report.

There are four separate inputs to the system:

1) an arming signal supplied by the operator;
2) a start pulse derived from a data time code channel;
3) a timing pulse train; and
4) a set of four analog data channels consisting of in-phase and quadrature signal components for each of two polarizations.

The program is run by an operator who sets up the analog equipment, loads the computer programs and then monitors the equipment and the program during its operation. The time code translator provides a visual decimal indication of the time at which the data were recorded and pulse trains which are synchronized with decimal changes in the clock reading (e.g., secs, ten seconds, minutes...) One of these pulse trains (usually one pulse per ten seconds) was used to initiate the sampling and computation sequence. When first loaded, the program remains in a wait state until armed and a start pulse has been received, after which it immediately begins sampling in synchronism with the timing pulse train. All four data channels are sampled simultaneously in order to preserve phase and the phase relationships between the pairs of quadrature components and the two data channels. The complex Fourier coefficients corresponding to both input polarizations are formated and written on a single magnetic tape. Data weighting, computation of the Fourier coefficients and the output operations are time-shared with the sampling and with each other. Fourier series of 512 complex samples were computed for each data channel. The maximum rate at which the program could complete the necessary computations and output operations before it was time to begin the analysis of the next data block was 560 Hz. Thus in real time an 1120 Hz data bandwidth was analyzed with approximately 1 Hz resolution.

Figure 4 is a block diagram which shows the sequencing and buffering procedures used to manipulate the data and calculate the Fourier coefficients. The procedure is quite straightforward and consists of setting up buffer areas in core for data input and output while computations are carried out elsewhere. At the completion of the computations the tape writing sequence is initiated and the program waits for the active buffer area to be filled before continuing. Output onto magnetic tape procedes in parallel with computation. "Old" and "new" buffers are continuously interchanged. The size of core available is the limiting factor in the number of Fourier coefficients which can be efficiently computed at one time.

B. Combination of Fourier Coefficients in Spectral Estimates

Except for the small amount of smoothing introduced by the data weighting function w the Fourier coefficients are very poor estimates of the power spectrum. The variance of the magnitude squared of the Fourier coefficients $|a|^2$ is on the order of the mean of the same quantity. Therefore some additional smoothing is necessary if meaningful spectral estimates are to be obtained.

A second computer program is used to read the Fourier coefficients from the digital tape and perform the smoothing operation. The

sums

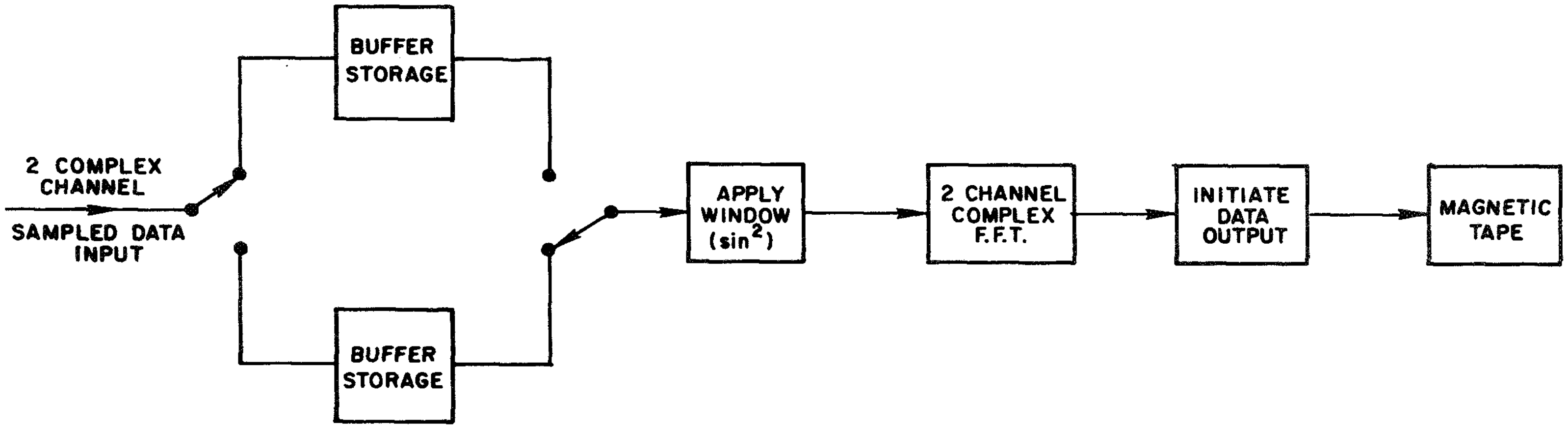

Figure 4. Block diagram of procedure for sequencing and buffering sampled data and Fourier coefficients.

$$A_j = \sum_{\ell = 1}^{L} \sum_{m = 1}^{M} {a_{j+\ell}^{m}}^{2} \qquad j = 0, L, 2L, \ldots$$

are formed, and the results are displayed on an on-line plotter and printed out. In general, the ratio of variance of the spectral estimates to the mean of A will decrease by $(LM)^{-\frac{1}{2}}$. The values of M and L are inserted by the operator and may be varied during the analysis of a single tape to accomodate changing experimental conditions.

6. Comments on Digital Spectral Analysis

The techniques and systems described in this report evolved from many attempts to find a workable method for processing a great deal of data under rather stringent conditions. Dynamic ranges of over 50 db were frequently required as were narrow equivalent filter bandwidths. For quantitative analyses it is necessary to have results in a convenient numerical form. These requirements could have been met using analog techniques only with great difficulty and at considerably more expense than encountered here, if at all.

In our experience there was no need to store the sampled data, so a considerable economy is achieved in only writing the Fourier coefficients on the digital tapes. No information is lost through this procedure since, if necessary, the sampled data can be recovered through an inverse transform.

The bulk of the computations occur in the initial calculation of the Fourier coefficients. Once these have been obtained it is a very simple matter to modify the characteristics of the spectral analysis by changing the summation parameters in the averaging program. Although we have not done so, fundamental alterations to the analysing filters may be accomplished by convolution of the Fourier coefficients with appropriate functions and by coherently combining successive sets of coefficients.

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