Second Quarterly Report

MHD BOUNDARY LAYERS WITH NON-EQUILIBRIUM IONIZATION AND FINITE RATES

(June 1, 1969 - September 1, 1969)

by

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I. INTRODUCTION

The present report is the second quarterly report under Contract NASw-1866 and covers the period June 1, 1969 to September 1, 1969. The problem being considered is the boundary layer that will develop over the segmented electrode wall in an MHD Channel when a non-equilibrium plasma is flowing.

As a result of work completed under NASw-1586 we have developed a theoretical analysis for a non-equilibrium boundary layer explicitely taking into account the electron temperature differential equation and the plasma sheath. This analysis was developed using a finite difference format so that finite electrodes along a wall could be treated in a non-similar manner.

The present program seeks to extend this earlier study to include finite ionization and recombination rates in order to make the analysis apply in a more realistic way to physically important situations.

I. INTRODUCTION

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II. REDUCTION OF GOVERNING EQUATIONS TO FINITE DIFFERENCE FORM

In our first Quarterly we reported on the proper development of the fundamental equations governing an MHD boundary layer with non-equilibrium and finite ionization rates. It was then shown how these equations could be transformed to new variables ξ and η to simplify the solution of the problem.

During the second quarter we have reduced these transformed equations to finite difference format. As before we write all four difference equations in the following compact notation.

$$A_n \, W_{n+1} + B_n \, W_n + C_n \, W_{n-1} = D_n$$

where

$$W_n = \begin{bmatrix} f'_{m+1,n} \\ g_{m+1,n} \\ \Theta_{m+1,n} \\ \alpha_{m+1,n} \end{bmatrix}$$

and

$$A_n = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}$$

$$B_n = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix}$$

$$C_n = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix}$$

$$D_n = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

The individual matrix elements are for the most part new and revised and are tabulated below:

<u>Momentum Equation</u>

$$A_{11} f'_{m+1,n+1} + B_{11} f'_{m+1,n} + B_{12} g_{m+1,n} + C_{11} f'_{m+1,n-1} = D_1$$

where

$$A_{11} = \frac{V \Delta \xi}{8 \xi f' \Delta \eta} - \frac{\ell \Delta \xi}{4 \xi f' (\Delta \eta)^2} - \frac{\ell' \Delta \xi}{8 \xi f' \Delta \eta}$$

$$B_{11} = 1 + \frac{\Delta \xi}{2 u_\infty} \frac{du_\infty}{d\xi} + \frac{\ell \Delta \xi}{2 \xi f' (\Delta \eta)^2}$$

$$B_{12} = - \frac{\Delta \xi}{2 u_\infty f'} \frac{du_\infty}{d\xi}$$

$$C_{11} = - \frac{V \Delta \xi}{8 \xi f' \Delta \eta} - \frac{\ell \Delta \xi}{4 \xi f' (\Delta \eta)^2} + \frac{\ell \Delta \xi}{8 \xi f' (\Delta \eta)}$$

$$D_1 = f'_{m,n} \left(1 - \frac{\Delta \xi}{2 u_\infty} \frac{du_\infty}{d\xi}\right) - \frac{V \Delta \xi}{4 \xi f'} f'_\eta + \frac{\Delta \xi}{2 u_\infty f'} \frac{du_\infty}{d\xi} g_{m,n}$$

$$+ \frac{\ell \Delta \xi}{4 \xi f'} f'_{\eta\eta} + \frac{\ell' (\Delta \xi)}{4 \xi f'} f'_\eta$$

Energy Equation

$$
\begin{aligned}
& A_{21}\, f'_{m+1,n+1} + A_{22}\, g_{m+1,n+1} + A_{23}\, \Theta_{m+1,n+1} + \\
& A_{24}\, \alpha_{m+1,n+1} + B_{21}\, f'_{m+1,n} + B_{22}\, g_{m+1,n} \\
& + B_{23}\, \Theta_{m+1,n} + B_{24}\, \alpha_{m+1,n} + C_{21}\, f'_{m+1,n-1} \\
& + C_{22}\, g_{m+1,n-1} + C_{23}\, \Theta_{m+1,n-1} + C_{24}\, \alpha_{m+1,n-1} \\
& = D_2
\end{aligned}
$$

$$A_{21} = - \frac{\Delta \xi}{2 \xi f'} \frac{u_\infty^2}{C_p T_\infty} \frac{\ell f'_\xi}{2 \Delta \eta}$$

$$A_{22} = L_1 \left[V - \frac{\ell'}{P_R} - \frac{2}{\Delta \eta} \left(\frac{\ell}{P_R} \right) \right]$$

$$A_{23} = - L_1 \left[\frac{2 \lambda T_{e\infty}}{\Delta \eta\, T_\infty} + \frac{\lambda' T_{e\infty}}{T_\infty} + \frac{5}{2} \frac{\sqrt{2\xi}\; k T_{e\infty} j y_\infty}{(\rho\mu)_r\, u_\infty C_p\, T_{e\infty} e} \right]$$

$$A_{24} = 0$$

$$B_{21} = L_2\, g_{m,n} \left[\frac{2\xi}{C_p\, T_\infty} \left(C_p \frac{dT_\infty}{d\xi} + u_\infty \frac{du_\infty}{d\xi} - \frac{j y_\infty B z}{(\rho\mu)_r \rho_\infty\, u_\infty} \right) \right]$$

$$B_{22} = 1 + 4 \left(\frac{\ell}{P_R} \right) \frac{L_1}{\Delta \eta} + \frac{2\xi}{C_p T_\infty} L_2 \left[\left(C_p \frac{dT_\infty}{d\xi} + u_\infty \frac{du_\infty}{d\xi} - \frac{j y_\infty B z}{(\rho\mu)_r \rho_\infty u_\infty} \right) f'_{m,n} \right.$$

$$- \frac{1}{(\rho\mu)_r \rho_\infty u_\infty^2} \frac{(\sigma\ Ex_\infty^2 + \frac{j_{y\infty}^2}{\sigma}(1+\beta e^2))}{2}$$

$$+ \frac{I \rho_\infty^2 (\tilde{\alpha}\ rec)_\infty}{(\rho\mu)_r\ U_\infty^2 M_c^3} \frac{\alpha_{m,n}}{g_{m,n}^3\ \Theta_{m,n}^{9/2}} (\alpha_{m,n}^2 - \frac{\alpha^3_{eq.\ m,n}}{2P - \alpha_{eq.\ m,n}}) \Bigg]$$

$$B_{23} = \frac{4}{\Delta\eta} L_1 \lambda \frac{T_{e\infty}}{T_\infty} + \frac{2\ \xi}{C_p\ T_\infty} L_2 \Bigg[\frac{2 I \rho_\infty^2}{(\rho\mu)_r\ U_\infty^2} \frac{(\tilde{\alpha}\ rec)_\infty}{m_c^3} \cdot \frac{\alpha_{m,n}}{g_{m,n}^2\ \Theta_{m,n}^{11/2}}$$

$$(\alpha^2_{eq.\ m,n} \left\{ \frac{2\ (P - \alpha_{eq.m,n})}{(2P - \alpha_{eq.m,n})} (\frac{3}{2} - \frac{I}{k\ T_{e\ \infty}\ \Theta_{m,n}}) - \frac{9}{4} \right\} + \frac{9}{4} \alpha^2_{m,n}) \Bigg]$$

$$B_{24} = \frac{2\ \xi}{C_p\ T_\infty} L_2 \frac{I \rho_\infty^2}{(\rho\mu)_r} \frac{(\alpha\ rec.)_\infty}{u_\infty^2\ m_c^3} \left(\frac{\alpha^2_{eq.\ m,n} - 3\alpha^2_{m,n}}{g_{m,n}^2\ \Theta_{m,n}^{9/2}} \right)$$

$$C_{22} = - L_1 \left[V - \frac{\ell'}{P_R} + \frac{2}{\Delta\eta} \frac{\ell}{P_R} \right]$$

$$C_{23} = L_1 \left[\lambda' \frac{T_{e\infty}}{T_\infty} + \frac{5}{2} \frac{\sqrt{2\ \xi}\ k T_{e\infty} j_{y\infty}}{(\rho\mu)_r\ u_\infty C_p\ T_\infty\ e} - \frac{2\lambda}{(\Delta\eta)} \frac{T_{e\infty}}{T_\infty} \right]$$

$$C_{24} = 0$$

$$D_2 = g_{m,n} + L_2 \Bigg[\left\{ \left(\frac{\lambda'}{P_R} \right) - V \right\} g_\eta + \frac{\lambda}{P_R} g_{\eta\eta} + \frac{\lambda T_{e\infty}}{T_\infty} \theta_{\eta\eta}$$

$$+ \left(\frac{\lambda' T_{e\infty}}{T_\infty} + \frac{5}{2} \frac{\sqrt{2\xi}}{(\rho\mu)_r} \frac{k T_{e\infty} \, j \, y_\infty}{u_\infty C_p T_\infty e} \right) \theta_\eta$$

$$+ \frac{2\xi}{C_p T_\infty} \left(\frac{\sigma E x_\infty^2 + \frac{j y_\infty^2}{\sigma} (1 + \beta e)}{(\rho\mu)_r \; \rho_\infty \; u_\infty^2} \right) g_{m,n}$$

$$- \frac{2\xi}{C_p T_\infty} \frac{2 I \rho_\infty^2}{(\rho\mu)_r} \frac{(\alpha \text{ rec.})_\infty}{u_\infty^2 \, m_c^3} \frac{\alpha_{m,n}}{g_{m,n}^2 \; \theta_{m,n}^{9/2}} \left\{ \left[\frac{15}{4} - \frac{2(P - \alpha_{eq.m,n})}{(2P - \alpha_{eq.m,n})} \right. \right.$$

$$\left. \left. \left(\frac{5}{2} + \frac{I}{k T_{e\infty} \theta_{m,n}} \right) \right] \alpha^2_{eq.,m,n} - \frac{11}{4} \alpha^2_{m,n} \right\} \Bigg]$$

where

$$L_1 = \frac{\Delta \xi}{8 \xi f' \Delta \eta}$$

$$L_2 = \frac{\Delta \xi}{4 \xi f'}$$

Electron Energy Equation

$$A_{31}\, f'_{m+1,n+1} + A_{32}\, g_{m+1,n+1} + A_{33}\, \Theta_{m+1,n+1} + A_{34}\, \alpha_{m+1,n+1}$$

$$+ \quad B_{31}\, f'_{m+1,n} + B_{32}\, g_{m+1,n} + B_{33}\, \Theta_{m+1,n} + B_{34}\, \alpha_{m+1,n}$$

$$+ \quad C_{31}\, f'_{m+1,n-1} + C_{32}\, g_{m+1,n-1} + C_{33}\, \Theta_{m+1,n-1} + C_{34}\, \alpha_{m+1,n-1}$$

$$= \quad D_3$$

$$A_{31} = 0$$

$$A_{32} = L_1 \frac{\theta_{m,n}}{g_{m,n}} \left\{ \frac{k\, T_{e\infty} (\rho\mu)_r\, \rho_\infty u_\infty^2}{2\xi\, m_c} \alpha_{m,n}\, V - \frac{k\, j\, y_\infty \rho_\infty u_\infty T_{e\infty}}{e\sqrt{2\xi}} \right\}$$

$$A_{33} = L_1 \left\{ \frac{\frac{3}{2} k\, T_{e\infty} \rho_\infty u_\infty^2 (\rho\mu)_r}{2\xi\, m_c} \alpha_{m,n}\, V - \frac{\rho_\infty u_\infty^2 C_p (\rho\mu)_r T_{e\infty}}{2\xi} \right.$$

$$\left. \left(\frac{2\lambda}{\Delta\eta} + \lambda' \right) - \frac{\rho_\infty u_\infty}{\sqrt{2\xi}} \frac{3}{2} \frac{k\, T_{e\infty}}{e}\, j\, y_\infty \right\}$$

$$A_{34} = L_1 \frac{k\, j\, y_\infty \rho_\infty u_\infty T_{e\infty}}{\sqrt{2\xi}} \frac{\theta_{m,n}}{\alpha_{m,n}}$$

$$B_{31} = L_2 \frac{k\, T_{e\infty} \rho_\infty u_\infty^2 (\rho\mu)_r}{2\xi\, m_c} \alpha_{m,n}\, \theta_{m,n} \left\{ \frac{6\xi}{T_{e\infty}} \frac{d T_{e\infty}}{d\xi} - \frac{4\xi}{\rho_\infty} \frac{d\rho_\infty}{d\xi} \right\}$$

$$B_{32} = \frac{k\, T_{e\infty} (\rho\mu)_r \rho_\infty u_\infty^2}{2 \xi\, m_c} \frac{\alpha_{m,n}\, \theta_{m,n}}{g_{m,n}} + L_2 \Bigg[\left(\frac{3}{2} k\, T_{e\infty} \theta_{m,n} + I \right) \times$$

$$\frac{\rho_\infty (\tilde{\alpha}\ rec.)_\infty}{m_c} \times \frac{2\, \alpha_{m,n}}{g^3_{m,n}\, \theta^{9/2}_{m,n}} \left(\alpha^2_{m,n} - \left\{ \frac{\alpha_{eq.m,n}}{2P - \alpha_{eq.m,n}} \right\} \alpha^2_{eq.m,n} \right)$$

$$- \left\{ \sigma E^2_{x\infty} + \frac{j^2_{y\infty}}{\sigma} (1 + \beta_e^2) \right\} - \frac{3\, me}{m_c} k \rho_\infty T_\infty \alpha_{m,n} \sum_s \frac{\nu_{es}}{m_s} \Bigg]$$

$$B_{33} = \frac{2 k\, T_{e\infty} \rho_\infty u_\infty^2 (\rho\mu)_r}{2 \xi\, m_c} \alpha_{m,n} + L_2 \frac{k\, T_{e\infty} \rho_\infty u_\infty^2 (\rho\mu)_r}{2 \xi\, m_c} \alpha_{m,n}\, f'_{m,n} \times$$

$$\left\{ \frac{6 \xi}{T_{e\infty}} \frac{d T_{e\infty}}{d \xi} - \frac{4 \xi}{\rho_\infty} \frac{d \rho_\infty}{d \xi} \right\} + L_2 \left(\frac{3}{2} k\, T_{e\infty} \theta_{m,n} + I \right) \times$$

$$\frac{2 \rho_\infty (\tilde{\alpha}\ rec)_\infty}{m_c} \times \frac{\alpha_{m,n}}{g^2_{m,n}\, \theta^{11/2}_{m,n}} \Bigg[\left\{ \frac{2(P - \alpha_{eq.m,n})}{(2P - \alpha_{eq.m,n})} \left(\frac{3}{2} - \frac{I}{k\, T_{e\infty} \theta_{m,n}} \right) - \frac{9}{4} \right\} \times$$

$$\alpha^2_{eq_{m,n}} + \frac{9}{4} \alpha^2_{m,n} \Bigg] + \frac{L_1}{4 \Delta \eta} \frac{\rho_\infty u_\infty^2 C_p (\rho\mu)_r T_{e\infty} \lambda}{2 \xi}$$

$$+ L_2 \frac{3\, me}{m_c} k \rho_\infty T_{e\infty} \alpha_{m,n} \sum_s \frac{\nu_{es}}{m_s}$$

$$B_{34} = L_2 \Bigg[\frac{k\, T_{e\infty} \rho_\infty u_\infty^2 (\rho\mu)_r}{2 \xi\, m_c} f'_{m,n}\, \theta_{m,n} \left\{ \frac{6 \xi}{T_{e\infty}} \frac{d T_{e\infty}}{d \xi} - \frac{4 \xi}{\rho_\infty} \frac{d \rho_\infty}{d \xi} \right\}$$

$$+ \left(\frac{3}{2} k\, T_{e\infty} \theta_{m,n} + I \right) \frac{\rho_\infty (\tilde{\alpha}\ rec)_\infty}{m_c} \frac{1}{g_{m,n}\, \theta_{m,n}} \left(\alpha^2_{eq_{m,n}} - 3 \alpha^2_{m,n} \right)$$

$$+ 3 \frac{me}{m_c} k \rho_\infty T_{e\infty} \sum_s \frac{\nu_{es}}{m_s} \left(\theta_{m,n} - g_{m,n} \frac{T_\infty}{T_{e\infty}} \right) \Bigg]$$

$$C_{31} = 0$$

$$C_{32} = -A_{32}$$

$$C_{33} = -L_1 \left[\frac{\frac{3}{2} k\, T_{e\infty} \rho_\infty u_\infty^2 (\rho\mu)_r}{2\xi\, m_c} \alpha_{m,n} V + \frac{\rho_\infty u_\infty^2 C_p (\rho\mu)_r\, T_{e\infty}}{2\xi} \left(\frac{2\lambda}{\Delta\eta} - \lambda' \right) - \frac{\rho_\infty u_\infty}{\sqrt{2\xi}} \frac{3}{2} \frac{k\, T_{e\infty}}{e}\, j\, y_\infty \right]$$

$$C_{34} = -A_{34}$$

$$D_3 = \frac{\frac{3}{2} k\, T_{e\infty} \rho_\infty u_\infty^2 (\rho\mu)_r}{2\xi\, m_c} \alpha_{m,n} \left(\theta_{m,n} + L_2 V \theta_\eta \right)$$

$$+ L_2 \alpha_{m,n} f'_{m,n} \theta_{m,n} \frac{k\, T_{e\infty} \rho_\infty u_\infty^2 (\rho\mu)_r}{2\xi\, m_c} \left\{ \frac{6\xi}{T_{e\infty}} \frac{dT_{e\infty}}{d\xi} - \frac{4\xi}{\rho_\infty} \frac{d\rho_\infty}{d\xi} \right\}$$

$$+ L_2 \left(\frac{3}{2} k\, T_{e\infty} \theta_{m,n} + I \right) \frac{2\rho_\infty (\tilde{\alpha}\, rec)_\infty}{m_c} \frac{\alpha_{m,n}}{g^2_{m,n}\, \theta^{9/2}_{m,n}} \left\{ \frac{11}{4} \alpha^2_{m,n} - \left[\frac{15}{4} - \frac{2(P - \alpha_{eq.m,n})}{(2P - \alpha_{eq.m,n})} \times \left(\frac{5}{2} + \frac{1}{k\, T_{e\infty} \theta_{m,n}} \right) \right] \alpha^2_{eq_{m,n}} \right\}$$

$$+ \frac{k\, T_{e\infty} (\rho\mu)_r \rho_\infty u_\infty^2}{2\xi\, m_c} \alpha_{m,n} \theta_{m,n} \left[1 - \frac{L_2 V g_\eta}{g_{m,n}} \right]$$

$$+ \frac{\rho_\infty u_\infty^2 C_p (\rho\mu)_r\, T_{e\infty}}{2\xi} L_2 \left[\lambda\, \theta_{\eta\eta} + \lambda' \theta_\eta \right]$$

$$+ \frac{\rho_\infty u_\infty}{\sqrt{2\xi}} \frac{3}{2} \frac{k\, T_{e\infty}}{e}\, j\, y_\infty L_2 \theta_\eta + \left[\sigma E^2_{x_\infty} + \frac{j\, y_\infty^2}{\sigma} (1 - \beta_e^2) \right] L_2\, g_{m,n}$$

$$+ \frac{k\, j\, y_\infty}{e} \frac{\rho_\infty u_\infty T_{e\infty}}{\sqrt{2\xi}} \theta_{m,n} L_2 \left[\frac{g_\eta}{g_{m,n}} - \frac{\alpha_\eta}{\alpha_{m,n}} \right]$$

Electron Density Equation

$$A_{41}\, f'_{m+1,n+1} + A_{42}\, g_{m+1,n+1} + A_{43}\, \Theta_{m+1,n+1} + A_{44}\, \alpha_{m+1,n+1}$$

$$+ \; B_{41}\, f'_{m+1,n} + B_{42}\, g_{m+1,n} + B_{43}\, \Theta_{m+1,n} + B_{44}\, \alpha_{m+1,n}$$

$$+ \; C_{41}\, f'_{m+1,n-1} + C_{42}\, g_{m+1,n-1} + C_{43}\, \Theta_{m+1,n-1} + C_{44}\, \alpha_{m+1,n-1}$$

$$= \; D_4$$

$$A_{41} = A_{42} = A_{43} = 0$$

$$B_{41} = 0$$

$$C_{41} = C_{42} = C_{43} = 0$$

$$A_{44} = L_1 \, V$$

$$C_{44} = - A_{44}$$

$$B_{42} = L_2 \, \frac{4 \xi \rho_\infty^2 \, (\tilde{\alpha}\,\mathrm{rec})_\infty}{(\rho\mu)_r \, u_\infty^2 \, m_c^2} \left\{ \frac{\alpha_{m,n}}{g_{m,n}^3 \, \theta_{m,n}^{9/2}} \left(\frac{\alpha^3_{eq.m,n}}{2P - \alpha_{eq.m,n}} - \alpha^2_{m,n} \right) \right\}$$

$$B_{43} = L_2 \, \frac{4 \xi \rho_\infty^2 \, (\tilde{\alpha}\,\mathrm{rec})_\infty}{(\rho\mu)_r \, u_\infty^2 \, m_c^2} \left\{ \frac{\alpha_{m,n}}{g_{m,n}^2 \, \theta_{m,n}^{11/2}} \left[\left\{ \frac{9}{4} - \frac{2\,(P - \alpha_{eq.m,n})}{(2P - \alpha_{eq.m,n})} \left(\frac{3}{2} + \frac{1}{k\, T_{e\infty}\, \theta_{m,n}} \right) \right\} \alpha^2_{eq.m,n} - \frac{9}{4} \alpha^2_{m,n} \right] \right\}$$

$$B_{44} = 1 - L_2 \frac{2 \frac{e}{3} \rho_\infty^2 (\tilde{\alpha}\text{ rec.})_\infty}{(\rho\mu)_r \, u_\infty^2 \, m_c^2} \left\{ \frac{1}{g^2_{m,n} \, \theta^{9/2}_{m,n}} \left(\alpha^2_{eq\,m,n} - 3\alpha^2_{m,n} \right) \right\}$$

$$D_4 = L_2 \frac{4 \frac{e}{3} \rho_\infty^2 (\tilde{\alpha}\text{ rec})_\infty}{(\rho\mu)_r \, u_\infty^2 \, m_c^2} \left\{ \frac{\alpha^2_{eq.m,n} \, \alpha_{m,n}}{g^2_{m,n} \, \theta^{9/2}_{m,n}} \left\{ \frac{15}{4} - \frac{2(P - \alpha_{eq.m,n})}{(2P - \alpha_{eq.m,n})} \times \right. \right.$$

$$\left. \left. \times \left(\frac{5}{2} + \frac{I}{k \, T_{e\infty} \, \theta_{m,n}} \right) \right\} - \frac{11}{4} \frac{\alpha^3_{m,n}}{g^2_{m,n} \, \theta^{9/2}_{m,n}} \right\}$$

$$+ \alpha_{m,n} - L_2 \, V(\alpha_\eta)$$

III. PROGRAMMING

Reprogramming of our earlier computer program for the IBM 360/75 has been completed. Some additional checks of accuracy are being made as the 360/75 used a 32 bit word whereas the GE 635 used a 36 bit word. This results in a loss of one digit of accuracy, and in some cases may be significant.

The new computer program for the finite rate problem has been written and compiled, and is being checked out. A listing of this program is enclosed with this quarterly report.

IV. REVIEW OF SHEATH ANALYSIS

The boundary conditions that we have used so far are based on the matching of electron particle flux and electron energy flux at the interface between the continuum region and the free-fall region. This model is valid as long as the Debye length is much smaller than the smallest of the mean free paths, i.e. the electron-neutral mean free path. However, even in this case there still exists a transition region between the continuum and the free-fall regions. It is expected that, in this transition region, both the potential and the electron number density undergo a re-distribution; these quantities naturally match there values in the continuum and the free-fall regions in addition to the matching of particle and energy fluxes. Unfortunately, in order to investigate the transition region we have to use a kinetic theory approach and also take into account any change separation. The formulation is therefore extremely complicated. Bienkowski (Physics of Fluids, February, 1967) has concluded, on the basis of the asymptotic behavior of the equations, the existence of two transitional layers; but he gave no calculated values which can be only obtained by the numerical integration of a set of non-linear differential equations. We are exploring the possibility of solving several typical situations using the actual parameters of our problem in order to evaluate the accuracy of our assumed boundary conditions.

V. FUTURE EFFORTS

During the second quarter our principal effort was to formulate the new finite difference equations for the finite rate problem and to reprogram these new equations. This has now been completed.

A second effort was to review the sheath model that we have been using to establish our boundary condition on T_e .

In the immediate future we plan to concentrate on numerical calculations using our two computer programs. We will also formulate the problem for the insulator wall as contrasted to the electrode wall which we are studying now.

```
      COMMON/COM1/BK,EM,EC,C1,PI,AKO,R,Y(1000),TE,DUM(10),TW,EKSIS,EKSIM
     1,DUM1(7),MEDGE,EKSI
      COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,
     2 NTIMES,NPRINT,NTAB,NVMN
C     RETURN HERE FOR START OF NEXT CASE
   10 MEDGE=1
      NRTST = 1
      NTIMES = 0
      NK = 1
      BK = 1.38E-23
      EM = 9.107E-31
      EC = 1.602E-19
      C1 = 2.42E21
      PI = 3.1416
      AKO = 8.854E-12
      Y(1) = 0.
      R = 8.317E3
      CALL READIN
   40 CALL EDGE1
      SMLHH(2) = TW/TEEKSI(MEDGE)
   50 N = 0
      CALL NXTLST
   60 N = 1
      CALL NXTLST
   70 N= N+1
      CALL ABCD
      CALL EKPK
      IF (NRTST-N) 80,80,90
   80 CALL TEST
      NRTST = N
      IF (CONST) 90,200,200
   90 IF(N-999) 70,110,110
C     CONVERGENCE NOT ATTAINED, PRINT AND GO TO NEXT CASE
  110 WRITE (6,903)
      NWRIT = NPL1
      CALL WOSUB
      GO TO 10
C     CONVERGENCE ATTAINED, GO TO NEXT PROFILE
  200 CALL WNSUB
  202 IF (KK-NK) 220,220,210
C     ITERATE UP TO (KK-1) TIMES
  210 NVMN = 2
      NK = NK+1
      CALL VMNSUB(&50)
C     INITIALIZE FOR NEXT PROFILE
  220 NK = 1
      NVMN = 1
      NWRIT = NPL1
      CALL VMNSUB(&50)
      IF (EKSIM-EKSI) 260,260,40
  260 WRITE (6,902)
      N=NWRIT-1
      PUNCH 900,((W(I,J),I=1,N),J=1,4)
      PUNCH 900,(V(I),I=1,N)
      WRITE(7,901) N
      GO TO 10
  900 FORMAT (4E18.8)
```

```
901 FORMAT (7H NWRIT=,I4)
902 FORMAT (16H THATS ALL FOLKS)
903 FORMAT (36H CONVERGENCE NOT ATTAINED, TRY AGAIN)
    END
```

```
CREADIN
      SUBROUTINE READIN
      DIMENSION ARAKSI(1  )
      COMMON/COM1/DUM1(1 ,18),TW,EKSIS,EKSIM,DKSI,DUM2(2),DETA,E2ESI,DELT
     1A,MEDGE,EKSI
      COMMON/COM2/ EDGE(1 ,12),TEMPRA(5 ),QEAT(5 ),QECST(5 )
      COMMON/COM3/ SMLHH(4),V(1   ),WLCT(1   ,4),W(1   ,4),TEEKSI(1 )
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,EPR,N,NPL1,NWRIT,KK,NK,NRTST,
     ? NTIMES,NPRINT,NTAB,NVMN
      COMMON/COM6/ ELLS,EL,ELNX,YAMLS,YAM,YAMNX,PRNLS,PRN,PRNNX,S,GNX,
     ? THENX,ENX,CONX,BETAX,QEI,QEA,QECS,C2,PER,CP,ROMR,CM,TIW,TIWC,PIZ
     ?,SM,BZ,ALFANX
      DIMENSION FF(12)
      NAMELIST/NAME1/DELTA,EKSIS,EKSIM,DKSI,DETA,CM,SM,TIWC,CP,PER,P
     1IZ,C2,BZ,EKR,EP1,EP2,EP3,EP4
      NAMELIST/NAME2/TEMPRA,QEAT,QECST,EDGE,SMLHH,TEEKSI,NWRIT
      NAMELIST/NAME3/KK,NTAB
      READ (5,NAME1)
      READ (5,NAME2)
    7 READ (5,NAME3)
      WRITE (6,666)
  666 FORMAT (' ***** THE FOLLOWING IS A LIST OF THE INPUT *****')
      WRITE (6,NAME1)
      WRITE (6,NAME2)
      WRITE (6,NAME3)
      IF (NWRIT  .EQ.  0)   GO TO 8
      READ(5,899) ((W(I,J),I=1,NWRIT),J=1,4)
      READ(5,899) (V(I),I=1,NWRIT)
  899 FORMAT (4E18.8)
      GO TO 9
    8 NWRIT =N
    9 CONTINUE
      WRITE (6,667)
  667 FORMAT (' ***** END OF INPUT LISTING *****')
      ROMR=3.1E-7*EDGE(1,7)*EDGE(1,2)**.75
      WRITE (6,900)EKSIS,EKSIM,DKSI,DETA
  900 FORMAT (7H0EKSIS=E16.8,3X,6HEKSIM=E16.8,3X,5HDKSI=E16.8,3X,5HDETA=
     1E16.8,3X//)
      WRITE (6,901)DELTA,ROMR,CM,SM,CP
  901 FORMAT (7H0DELTA=E16.8,3X,6HROMR =E16.8,3X,5HCM = E16.8,3X,
     15HSM = ,E16.8,3X,4HCP =,E16.8//)
      WRITE (6,902)TIW,PER,PIZ,C2,BZ
  902 FORMAT(7H0TIWC =E16.8,3X,6HPER = E16.8,3X,5HPIZ = E16.8,3X,
     15HC2 = ,E16.8,3X,4HBZ =,E16.8//)
      CALL INITIL
      CALL WCSUB
      WRITE (6,905)
  905 FORMAT (1H1/14X6HTEMPRA20X4HQEAT21X5HQECST/)
      WRITE (6,906)(TEMPRA(I),QEAT(I),QECST(I),I = 1,NTAB)
  906 FORMAT (1H 3E25.8)
      NN = (EKSIM-EKSIS)/DKSI+1.5
      WRITE (6,907)
  907 FORMAT (1H1/1X4HEKSI15X2HUE16X2HTE16X3HETE15X3HDUE15X3HDTE,
     ?13X5HALFAE)
      ARAKSI(1) = EKSIS
      DO 10  I = 2,NN
   10 ARAKSI(I) = ARAKSI(I-1)+DKSI
      WRITE(6,908)(ARAKSI(I),(EDGE(I,J),J = 1,5),I = 1,NN)
```

```
  908 FORMAT (1H 7E16.8)
      WRITE (6,909)
  909 FORMAT (1H1/7X4HAKSI13X4HDETE13X4HRHOE13X5HDRHOE12X4HAJYE14X3HEXE1
     13X4HPRES/)
      WRITE (6,910)(APAKSI(I),(EDGE(I,J),J = 6,12),I = 1,NN)
  910 FORMAT (1H 7E17.8)
  911 FORMAT (12A6)
 9999 RETURN
      END
```

```
      SUBROUTINE INITIL
      COMMON/COM1/DUM1(1007),TE,DUM2(10),TW,DUM3(3),TDA,TDDA,DETA2,DETA,
     1DUM4(4)
      COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,
     2 NTIMES,NPRINT,NTAB,NVMN
      DO 10 I = 1,NWRIT
      DO 11 J = 1,4
   11 WLST(I,J) = W(I,J)
   10 CONTINUE
      IF (NWRIT-1000) 9,14,14
C     EXTEND PROFILES
    9 NPL1 = NWRIT+1
      DO 12 I = NPL1,1000
      V(I) = V(I-1)-DETA
      DO 13 J = 1,4
      WLST(I,J) = 1.
   13 W(I,J) = 1.
   12 CONTINUE
   14 TW = SMLHH(2)*TEEKSI(1)
      TDA = 2.*DETA
      TDDA = 2./DETA
      DETA2 = DETA**2
 9999 RETURN
      END
```

```
CEDGE1
      SUBROUTINE EDGE1
      COMMON/COM1/DUM1(10,7),TE,ETE,PRES,AJYE,UE,DUE,DTE,DETE,RHOE,DRHOE
     1,EXE,TW,EKSIS,EKSIM,DKSI,DUM2(4),TSQSI,DELTA,MEDGE,EKSI
      COMMON/COM2/ EDGE(10,12),TEMPRA(50),QEAT(50),QECST(50)
      COMMON/COM6/ ELLS,EL,ELNX,YAMLS,YAM,YAMNX,PRNLS,PRN,PRNNX,S,GNX,
     2 THENX,ENX,CONX,BETAX,QEI,QEA,QECS,C2,PER,CP,ROMR,CM,TIW,TIWC,PIZ
     2,SM,BZ,ALFANX
      UE = EDGE(MEDGE,1)
      TE = EDGE(MEDGE,2)
      ETE = EDGE(MEDGE,3)
      DUE = EDGE(MEDGE,4)
      DTE = EDGE(MEDGE,5)
      DETE = EDGE(MEDGE,6)
      RHOE = EDGE(MEDGE,7)
      DRHOE = EDGE(MEDGE,8)
      AJYE = EDGE(MEDGE,9)
      EXE = EDGE(MEDGE,10)
      PRES = EDGE(MEDGE,11)
      ALFAE = EDGE(MEDGE,12)
      ROMR=RHOE*3.1E-7*TE**.75
      IF (AJYE) 1,2,2
    1 TIW=TIWC
      GO TO 3
    2 TIW=0.
    3 AA=MEDGE-1
      EKSI=EKSIS+AA*DKSI
      MEDGE=MEDGE+1
      TSQSI=SQRT(2.*EKSI)
 9999 RETURN
      END
```

```
CNXTLST
      SUBROUTINE NXTLST
      COMMON/COM1/BK,EM,EC,C1,PI,AKO,R,Y(1000),TE,ETE,PRES,AJYE,UE,DUM(5
     1),EXE,DUM1(7),DETA,TSQSI,DUM2(3)
      COMMON/COM2/ EDGE(10,12),TEMPRA(50),QEAT(50),QECST(50)
      COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
      COMMON/COM4/ E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),
     ?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,
     ? NTIMES,NPRINT,NTAB,NVMN
      COMMON/COM6/ ELLS,EL,ELNX,YAMLS,YAM,YAMNX,PRNLS,PRN,PRNNX,S,GNX,
     ? THENX,ENX,CONX,BETAX,QEI,QEA,QECS,C2,PER,CP,ROMR,CM,TIW,TIWC,PIZ
     ?,SM,BZ,ALFANX
      QEAQAA = .1
      NPL1 = N+1
      ELLS = EL
      EL = ELNX
      YAMLS = YAM
      YAM=YAMNX
      PRNLS = PRN
      PRN = PRNNX
      GNX = .5*(W(NPL1,2)+WLST(NPL1,2))
      THENX = .5*(W(NPL1,3)+WLST(NPL1,3))
      ALFANX =.5*(W(NPL1,4)+WLST(NPL1,4))
      ET = ETE*THENX
      PRNNX = 2./3.
      ELNX = GNX**(-.25)
    3 ENX = (ALFANX*PRES)/(BK*GNX*TE)
      CALL TABLE(TEMPRA,QEAT,QECST,ET,NTAB,N,QEA,QECS)
      QEI = PI*((EC/AKO)*(EC/(16.*BK*ETE*THENX)))**2
      QEI = QEI*ALOG(32.*(BK*ETE)**1.5/EC/EC*AKO**1.5/(EC*ENX**.5)
     1*(THENX**1.5))
      ENUE = PRES*QEA/(BK*GNX*TE)+(PER*PRES/BK/TE/GNX-ENX)*QECS+ENX*QEI
      SQT = SQRT(8.*BK/EM*ETE*THENX/PI)
      ENUE = ENUE*SQT
      CONX = EC/EM*EC*ENX/ENUE
      BETAX = BZ*EC/ENUE/EM
      A1 = 1.24E7*ET**1.5/ENX**.5
      A2 = 1.8E5*ET/ENX**(1./3.)
      GAM = ENX*(BK*TE/PRES)*GNX
      BKSKA = 3.E-6*ET**2.5/(TE*GNX)**.75/ALOG(55.+A1**4+A2**4)
      BKEKA = BKSKA/(1.+BETAX**2)/(1.+1.414*(1.-GAM)/GAM*BKSKA*
     1(QEAQAA)*SQRT(EM/CM*GNX*TE/ET))
      YAMNX = ELNX/PRNNX*BKEKA
      IF (NPL1-1) 5,5,9999
    5 GRP = YAMNX*CP*ROMR*UE*(EC/BK)/TSQSI/DETA
      AJEYW = ALF1NX*AJYE+ALF2NX*EXE
      SCRB = (AJYE+TIW)/EC/ENX*SQRT(2.*PI*EM/(BK*ETE*THENX))+SQRT((2.*PI
     1*EM)/SM)
      B2 = 2.*GRP/((2.-ALOG(SCRB))*(AJYE+EC*ENX*SQRT(BK*ETE*THENX/SM
     1))+1.5*GRP-2.5*AJEYW+2.*TIW)
      B3 = -.25*B2
      DO 10 J = 1,4
      DO 11 I = 1,4
      SMLHH(I) = 0.
      H(I,J) = 0.
   11 EF(I,J) = 0.
   10 CONTINUE
```

```
      B(3,3) = B2
      BF(3,3) = B3
      SMLHH(3)=B2*WLST(2,3)+B3*WLST(3,3)-WLST(1,3)+4.*TE*GNX*TIW/ETE
     1/(2.*GRP/F2)
      S = C1*(ET)**1.5 *EXP(-C2/ET)
      GRP = 4.*PER/(BK*S)*PRES/TE/GNX
      IF (.01 - GRP) 2,1,1
    1 ENEQX = GRP/4.*(1.-GRP/4.)*S
      GO TO 119
    2 ENEQX = S/2.*(SQRT(1.+GRP)-1.)
  119 SMLHH(4) = ENEQX
 9999 RETURN
      END
```

```
.CTABLE
      SUBROUTINE TABLE(TEMP,QEA,QECS,ARG5,KTAB,N,XQEA,XQECS)
      DIMENSION TEMP(50),QEA(50),QECS(50)
      XTEMP = ARG5
    9 IF (N) 10,101,10
   10 IF (XTEMP-XTEMPL) 101,11,11
  101 J = 1
      NTAB = KTAB
   11 NTAB1 = NTAB+1
      K = J-1
      CALL TLU1(XTEMP,NTAB,TEMP(J),J,IERR)
      IF (IERR) 13,14,13
   13 WRITE (6,901)XTEMP
      GO TO 9999
   14 NTAB = NTAB1-J
      J = J+K
      IF (NTAB) 9999,9999,15
15    XQEA=QEA(J)+(XTEMP-TEMP(J)/(TEMP(J+1)-TEMP(J)))*(QEA(J+1)-QEA(J))
      XQECS=QECS(J)+(XTEMP-TEMP(J)/(TEMP(J+1)-TEMP(J)))*(QECS(J+1)-
     : QECS(J))
   16 XTEMPL = XTEMP
  901 FORMAT (34H THIS TEMPERATURE IS NOT IN TABLE-,E16.8)
 9999 RETURN
      END
```

```
      SUBROUTINE TLU1(X,N,TABLE,J,IERR)
      REAL TABLE(N)
      IF(TABLE(1)-X) 10,20,30
   20 J=1
      IERR=0
      RETURN
   30 J=1
      IERR=-1
      RETURN
   10 NN=N-1
      DO 40  J=2,NN
      IF(TABLE(J)-X) 40,60,70
   60 IERR=0
      RETURN
   70 J=J-1
      IERR=0
      RETURN
   40 CONTINUE
      IF(TABLE(N)-X) 80,90,100
   80 J=N
      IERR=1
      RETURN
   90 J=N
      IERR=0
      RETURN
  100 J=N-1
      IERR=0
      RETURN
      END
```

CABCD

```
      SUBROUTINE ABCD
      COMMON/COM1/BK,EM,EC,C1,PI,AKO,R,Y(1000),TE,ETE,PRES,AJYE,UE,DUE,D
     1TE,DETE,RHOE,DRHOE,EXE,TW,EKSIS,EKSIM,DKSI,TDA,TDDA,DETA2,DETA,TSQ
     1SI,DELTA,MEDGE,EKSI
      COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
      COMMON/COM4/ E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),
     ?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,
     ? NTIMES,NPRINT,NTAB,NVMN
      COMMON/COM6/ ELLS,EL,ELNX,YAMLS,YAM,YAMNX,PRNLS,PRN,PRNNX,S,GNX,
     ? THENX,ENX,CONX,BETAX,QEI,QEA,QECS,C2,PER,CP,ROMR,CM,TIW,TIWC,PIZ
     ?,SM,BZ,ALFANX
      FP = .5*(W(N,1)+WLST(N,1))
      G = GNX
      THETA = THENX
      BETAE = BETAX
      COND = CONX
      ENE = ENX
      ALFA = ALFANX
      DO 10 I=1,4
   10 D(I) = 0.
      DO 11 I=1,4
      DO 11 J=1,4
      AA(I,J)=0.
      B(I,J)=0.
   11 C(I,J)=0.
      SNUS = (PRES/BK*QEA)/TE/G/CM+(QECS/SM)*(PER*PRES/TE/BK/G-ENE)+(EN
     1E*QEI)/CM
      SNUS = SNUS*DELTA*SQRT(8.*BK*ETE*THETA/PI/EM)
      FPA = (WLST(N+1,1)-WLST(N-1,1))/TDA
      FPAA = (WLST(N+1,1)-2.*WLST(N,1)+WLST(N-1,1))/DETA2
      GA = (WLST(N+1,2)-WLST(N-1,2))/TDA
      GAA = (WLST(N+1,2)-2.*WLST(N,2)+WLST(N-1,2))/DETA2
      THA = (WLST(N+1,3)-WLST(N-1,3))/TDA
      THAA = (WLST(N+1,3)-2.*WLST(N,3)+WLST(N-1,3))/DETA2
      ALFAA = (WLST(N+1,4) - WLST(N-1,4))/TDA
      RHOS = R/PRES/40.*TE*(WLST(N,2)+WLST(N-1,2))
      Y(N) = Y(N-1)+TSQSI*RHOS*.5*DETA/UE
      CALL NXTLST
      ELA = (ELNX-ELLS)/TDA
      YAMA = (YAMNX-YAMLS)/TDA
      PRA = (PRNNX-PRNLS)/TDA
      V1 = V(N)
      Q1 = DKSI/(4.*EKSI*FP*TDA)
      Q2 = 2.5*BK*ETE/EC*TSQSI/(ROMR*UE*CP*TE)
      Q3 = DKSI*DETE
      Q4 = PIZ/(RHOE*CP*TE)
      Q5 = 1.5*BK*ETE*THETA+PIZ
      Q6 = 2.5*BK*ETE*THETA+PIZ
      Q7 = 5.*BK/EC*RHOE*ETE*UE/TSQSI/G
      Q9 = ROMR*UE**2
      Q3 = RHOE*CP*ETE*Q9
      Q8 = RHOE*CP*TE*Q9
      Q10 = 1.5*BK*ENE
      F1 = FP
      Q = G
      FP = WLST(N,1)
```

```
      G = WLST(N,2)
      THETA = WLST(N,3)
      ALFA = WLST(N,4)
      ET = ETE*THETA
      S=C1*(ET)**1.5*EXP(-C2/ET)
      GRP = 4.*PER/(BK*S)*PRES/TE/GNX
      IF (.01-GRP) 2,1,1
    1 ENEQX = GRP/4.*(1.-GRP/4.)*S
      GO TO 3
    2 ENEQX = S/2.*(SQRT(1.+GRP)-1.)
    3 ALFAEQ = ENEQX*BK*TE*G/PRES
      ALFAF = (5.6E-27)*(BK*ETE/EC)**(-4.5)
      L2 = DKSI/(4*EKSI*FP)
      L1 = L2/(2*DETA)
      AA(1,1) = Q1*(V1-2.*EL/DETA-ELA)
      AA(1,2) = 0.
      AA(1,3) = 0.
      AA(1,4) = 0.
      B(1,1) = 1.+DKSI*DUE/2./UE+4.*Q1*EL/DETA
      B(1,2) = -DKSI*DUE/(2.*UE*F1)
      B(1,3) = 0.
      B(1,4) = 0.
      C(1,1) = Q1*(-V1-2.*EL/DETA+ELA)
      C(1,2) = 0.
      C(1,3) = 0.
      C(1,4) = 0.
      D(1) = FP*(1.-DKSI*DUE/2./UE)+(DKSI/4./EKSI/F1)*(-FPA*V1+2.*EKSI*G
     1*DUE/UE+ELA*FPA+EL*FPAA)
      AA(2,1) = -2.*Q1*UE**2/CP/TE*EL*FPA
      XTEMP = (2.5*SQRT(2*EKSI)*BK*ETE*AJYE)/(ROMR*UE*CP*ETE*EC)
      AA(2,2) = L1*(V1 - ELA/PRN - TDDA*EL/PRN)
      AA(2,3) = -L1*((TDDA*YAM*ETE/ET) + YAMA*ETE/ET + XTEMP)
      AA(2,4) = 0.
      B(2,1) = L2*G*(2*EKSI/(CP*ET)*(CP*DTE+UE*DUE - AJYE*BZ/
     : (ROMR*RHOE*UE)))
      XTEMP = (CP*DTE + UE*DUE - AJYE*BZ/(ROMR*RHOE*UE))*FP
      YTEMP = -1.0/(ROMR*RHOE*UE**2)*.5*(COND*EXE**2 +
     % AJYE**2/COND*(1.0 + BETAE**2))
      ZTEMP = (PIZ*RHOE**2*ALFAR)/(ROMR*UE**2 *CM**3)*(ALFA/(G**3
     % *THETA**4.5))*(ALFA**2 - ALFAEQ**3/(2.*PER - ALFAEQ))
      B(2,2) = 1. + 4.*EL/PRN*L1/DETA + 2.*EKSI/(CP*TE)*L2*XTEMP+YTEMP+
     % ZTEMP
      UTEMP = 4.*L1*YAM*ETE/(DETA*TE)
      VTEMP = 2.*PIZ*RHOE**2 *ALFAR/(ROMR*UE**2 *CM**3)*ALFA/(G**2 *
     : THETA**5.5)
      WTEMP = ALFAEQ**2*(2.*(PER - ALFAEQ)/(2.*PER - ALFAEQ)*(1.5 - EC*
     : PIZ/(BK*ETE*THETA)) - 2.25) + 2.25*ALFA**2
      B(2,3) = UTEMP + 2.*EKSI/(CP*TE)*L2*VTEMP*WTEMP
      B(2,4) = 2.*EKSI/(CP*TE)*L2*PIZ*RHOE**2 *ALFAR/(ROMR*UE**2 *CM**3)
     : *(ALFAEQ**2 - 3.*ALFA**2)/(G**2 *THETA**4.5)
      C(2,1) = -AA(2,1)
      C(2,2) = -L1*(V1 - ELA/PRN + TDDA*EL/PRN)
      UTEMP = 2.5*SQRT(2.*EKSI)*BK*ETE*AJYE/(ROMR*UE*CP*TE*EC)
      C(2,3) = L1*(YAMA*ETE/TE + UTEMP - 2.*YAM*ETE/(DETA*TE))
      C(2,4) = 0.
      AA(3,1) = 0.
      VTEMP = (ELA/PRN - V1)*GA + EL/PRN*GAA + YAM*ETE/TE*THAA
      UTEMP = (YAMA*ETE/TE + UTEMP)*THA
```

```
      WTEMP = 2.*EKSI/(CP*TE)*((COND*EXE**2 + AJYE**2/COND*
     : (1. + BETAE**2))/(ROMR*RHOE*UE**2))*G
      XTEMP = -2.*EKSI/(CP*TE)*((3.75-2.*(PER - ALFAEQ)/(2.*PER -
     :  ALFAEQ)*(2.5 + EC*PIZ/(BK*ETE*THETA)))*ALFAEQ**2 - 2.75*ALFA**2)
      YTEMP = 2.*PIZ*RHOE**2 *ALFAR*ALFA/(ROMR*UE**2 * CM**3 * G**2
     :  *THETA**4.5)
      D(2) = G + L2*(VTEMP + UTEMP +WTEMP + XTEMP*YTEMP)
      UTEMP = (BK*ETE*ROMR*RHOE*UE**2)/(2.*EKSI*CM)*ALFA*V1
      VTEMP = (BK*AJYE*RHOE*UE*ETE)/(EC*SQRT(2.*EKSI))
      AA(3,2) = L1*THETA/G*(UTEMP - VTEMP)
      UTEMP = (1.5*BK*ETE*RHOE*UE**2 *ROMR*ALFA*V1)/(2.*EKSI*CM)
      VTEMP = ((RHOE*UE**2 *CP*ROMR*ETE)/(2.*EKSI))*(YAM*TDDA + YAMA)
      WTEMP = (RHOE*UE)/SQRT(2.*EKSI)*1.5*(BK*ETE)/EC*AJYE
      AA(3,3) = L1*(UTEMP - VTEMP - WTEMP)
      AA(3,4) = L1*(BK*AJYE*RHOE*UE*ETE)/SQRT(2.*EKSI)*THETA/ALFA
      UTEMP = (BK*ETE*RHOE*UE**2 *ROMR)/(2*EKSI*CM)
      VTEMP = (6.*EKSI)/ETE*DETE - 4.*EKSI*DRHOE/RHOE
      B(3,1) = L2*UTEMP*ALFA*THETA*VTEMP
      UTEMP = (BK*ETE*ROMR*RHOE*UE**2)/(2.*EKSI*CM)*ALFA*THETA/G
      VTEMP = (1.5*BK*ETE*THETA + PIZ)*RHOE*ALFAR/CM*(2.*ALFA/(G**3 *
     % THETA**4.5))
      WTEMP = ALFA**2 - ALFAEQ/(2*PER - ALFAEQ)*ALFAEQ**2
      XTEMP = COND*EXE**2 + AJYE**2 /COND*(1 + BETAE**2)
      YTEMP = 3*EM/CM*BK*RHOE*TE*ALFA*SNUS
      B(3,2) = UTEMP + L2*(VTEMP*WTEMP - XTEMP - YTEMP)
      UTEMP = (1.5*BK*ETE*RHOE*UE**2 *ROMR)/(2.*EKSI*CM)*ALFA
      VTEMP = L2*(BK*ETE*RHOE*UE**2 *ROMR)/(2.*EKSI*CM)*ALFA*FP*
     % (6.*EKSI/ETE*DETE - 4.*EKSI/RHOE*DRHOE)
      WTEMP = L2*(1.5*BK*ETE*THETA + PIZ)*(2.*RHOE*ALFAR)/CM*ALFA/(G**2
     % *THETA**5.5)
      XTEMP = (2.*(PER - ALFAEQ)/(2.*PER - ALFAEQ)*(1.5 - EC*PIZ/
     % (BK*ETE*THETA)) - 2.25)*ALFAEQ**2 + 2.25*ALFA**2
      YTEMP = L1/(4.*DETA)*RHOE*UE**2 *(CP*ROMR*ETE*YAM)/(2.*EKSI)
      ZTEMP = L2*3.*EM/CM*BK*RHOE*ETE*ALFA*SNUS
      B(3,3) = UTEMP + VTEMP + WTEMP*XTEMP + YTEMP + ZTEMP
      UTEMP = (BK*ETE*RHOE*UE**2 *ROMR)/(2.*EKSI*CM)*FP*THETA
      VTEMP = 6.*EKSI/ETE*DETE - 4.*EKSI/RHOE*DRHOE
      WTEMP =(1.5*BK*ETE*THETA + PIZ)*RHOE*ALFAR/CM
      XTEMP = 1./(G**2 *THETA**4.5)*(ALFAEQ**2 - 3.*ALFA**2)
      YTEMP = 3.*EM/CM*RHOE*ETE*SNUS*(THETA - G*TE/ETE)*BK
      B(3,4) = L2*(UTEMP*VTEMP + WTEMP*XTEMP + YTEMP)
      C(3,1) = 0.
      C(3,2) = -A(3,2)
      UTEMP = 1.5*BK*ETE*RHOE*UE**2 *ROMR/(2.*EKSI*CM)*ALFA*V1
      VTEMP = RHOE*UE**2 *CP*ROMR*ETE/(2.*EKSI)*(2.*YAM/DETA - YAMA)
      WTEMP = RHOE*UE/SQRT(2.*EKSI)*1.5*BK*ETE/EC*AJYE
      C(3,3) = -L1*(UTEMP + VTEMP - WTEMP)
      C(3,4) = -AA(3,4)
      UTEMP = 1.5*BK*ETE*RHOE*UE**2 *ROMR/(2.*EKSI/CM)*ALFA*(THETA +
     : L2*V1*THA)
      VTEMP = L2*ALFA*FP*THETA*((BK*ETE*UE**2 *RHOE*ROMR)/(2.*EKSI*CM))*
     : (6.*EKSI/ETE*DETE - 4.*EKSI/RHOE*DRHOE)
      WTEMP = L2*(1.5*BK*ETE*THETA + PIZ)*2.*RHOE*ALFAR/CM*ALFA/
     : (G**2 *THETA**4.5)*(2.75*ALFA**2 - (3.75 - 2.*(PER - ALFAEQ)/
     : (2.*PER - ALFAEQ)*(2.5 + EC*PIZ/(BK*ETE*THETA)))*ALFAEQ**2)
      XTEMP = (BK*ETE*ROMR*RHOE*UE**2)/(2.*EKSI*CM)*ALFA*THETA*
     : (1. - (L2*V1*GA)/G)
      YTEMP = RHOE*UE**2 *CP*ROMR*ETE/(2.*EKSI)*L2*(YAM*THAA + YAMA*THA)
```

```
     : ZTEMP = RHOE*UE/SQRT(2.*EKSI)*1.5*BK*ETE/EC*AJYE*L2*THA +
     : (COND*EXE**2 + AJYE**2/COND*(1. + BETAE**2))*L2*G
      ATEMP = PK*AJYE/EC*RHOE*UE*ETE/SQRT(2.*EKSI)*THETA*L2*
     : (GA/G - ALFAA/ALFA)
      D(3) = UTEMP + VTEMP + WTEMP + XTEMP + YTEMP + ZTEMP + ATEMP
      B(4,1) = 0.
      AA(4,1) = 0.
      AA(4,2) = 0.
      AA(4,3) = 0.
      C(4,1) = 0.
      C(4,2) = 0.
      C(4,3) = 0.
      AA(4,4) = L1*V1
      C(4,4) = -AA(4,4)
      UTEMP = L2*(4.*EKSI*RHOE**2 *ALFAR)/(ROMR*UE**2 *CM**2)
      VTEMP = ALFA/(G**3 *THETA**4.5)
      WTEMP = ALFAEQ**3/(2.*PER - ALFAEQ) - ALFA**2
      B(4,2) = UTEMP*VTEMP*WTEMP
      VTEMP = ALFA/(G**2 *THETA**5.5)
      WTEMP = (2.25 - 2.*(PER - ALFAEQ)/(2.*PER - ALFAEQ)*(1.5 + EC*PIZ/
     : (BK*ETE*THETA)))*ALFAEQ**2 - 2.25*ALFA**2
      B(4,3) = UTEMP*VTEMP*WTEMP
      UTEMP = L2*(2.*EKSI*RHOE**2 *ALFAR)/(ROMR*UE**2 *CM**2)
      VTEMP = 1./(G**2*THETA**4.5)*(ALFAEQ**2 - 3.*ALFA**2)
      B(4,4) = 1. - UTEMP*VTEMP
      UTEMP = L2*(4.*EKSI*RHOE**2 *ALFAR)/(ROMR*UE**2 *CM**2)
      VTEMP = ALFAEQ**2 *ALFA/(G**2 *THETA**4.5)
      WTEMP = 3.75 - 2.*(PER - ALFAEQ)/(2.*PER - ALFAEQ)*
     :(2.5 + EC*PIZ/(BK*ETE*THETA))
      XTEMP = 2.75*ALFA**3/(G**2 *THETA**4.5)
      D(4) = UTEMP*(VTEMP*WTEMP - XTEMP) + ALFA - L2*V1*ALFAA
 9999 RETURN
      END
```

```
CEKPK
      SUBROUTINE EKPK
      COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(25)
      COMMON/COM4/ E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),
     ?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NPTST,
     ? NTIMES,NPRINT,NTAB,NVMN
      DIMENSION SCRACH(3,3)
      IF (N-2) 10,10,140
   10 DO 40 I = 1,4
      DO 30 J = 1,4
      A(I,J) = B(I,J)
      TEMP(I,J) = AA(I,J)
      DO 20 K = 1,4
      A(I,J) = C(I,K)*H(K,J)+A(I,J)
   20 TEMP(I,J) = C(I,K)*EF(K,J)+TEMP(I,J)
   30 CKMAT(I,J) = A(I,J)
   40 CONTINUE
   50 CALL MXINV
      DO 80 I = 1,4
      DO 70 J = 1,4
      AA(I,J) = 0.
      DO 60 K = 1,4
   60 AA(I,J) = A(I,K)*TEMP(K,J)+AA(I,J)
   70 CONTINUE
   80 CONTINUE
      DO 110 I = 1,4
      E(5,I) = - AA(1,I)
      E(6,I) = - AA(2,I)
      E(7,I) = - AA(3,I)
      E(8,I) = -AA(4,I)
      TEMPP(I) = 0.
      DO 100 J = 1,4
  100 TEMPP(I) =C(I,J)*SMLHH(J)+TEMPP(I)
  110 D(I) = D(I)-TEMPP(I)
      DO 130 I = 1,4
      PHI(2,I) = 0.
      DO 120 J = 1,4
  120 PHI(2,I) = A(I,J)*D(J)+PHI(2,I)
  130 CONTINUE
      GO TO 9999
  140 MN = (N-1)*4-3
      DO 160 J = 1,4
      TEMP(4,J) = E(MN + 3,J)
      A(4,J) = B(4,J)
      TEMP(3,J) = E(MN+2,J)
      A(3,J) = B(3,J)
      TEMP(2,J) = E(MN+1,J)
      A(2,J) = B(2,J)
      TEMP(1,J) = E(MN,J)
  160 A(1,J) = B(1,J)
      DO 190 I = 1,4
      DO 180 J =1,4
      DO 170 K = 1,4
  170 A(I,J) = C(I,K)*TEMP(K,J)+A(I,J)
  180 CONTINUE
  190 CONTINUE
  200 CALL MXINV
```

```
      DO 230 I = 1,4
      DO 220 J = 1,4
      TEMP(I,J) = 0.
      DO 210 K = 1,4
  210 TEMP(I,J) = A(I,K)*AA(K,J)+TEMP(I,J)
  220 CONTINUE
  230 CONTINUE
      M = 4*N-3
      DO 250 I = 1,4
      E(M + 3,I) = -TEMP(4,I)
      E(M+2,I) = -TEMP(3,I)
      E(M+1,I) = -TEMP(2,I)
  250 E(M,I) = -TEMP(1,I)
      DO 270 I = 1,4
      TEMPP(I) = 0.
      PHI(N,I) = 0.
      DO 260 J = 1,4
  260 TEMPP(I) = C(I,J)*PHI(N-1,J)+TEMPP(I)
  270 D(I) = D(I)-TEMPP(I)
      DO 290 I = 1,4
      DO 280 J = 1,4
  280 PHI(N,I) = A(I,J)*D(J)+PHI(N,I)
  290 CONTINUE
      P1=PHI(N,1)
      P2=PHI(N,2)
      P3=PHI(N,3)
      P4 = PHI(N,4)
 9999 RETURN
      END
```

```
CTEST
      SUBROUTINE TEST
      COMMON/COM3/ EMLHH(4),V(1000),WEST(1000,4),W(1000,4),TEEKSI(100)
      COMMON/COM4/ E(5000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),
     ?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWPIT,KK,NK,NRTST,
     ? NTIMES,NPRINT,NTAB,NVMN
      M = 4*N - 3
      TERM1 = ABS(1.-E(M,1)-E(M,2)-E(M,3)-E(M,4)-PHI(N,1))
      TERM2 = ABS(1.-E(M+1,1)-E(M+1,2)-E(M+1,3)-E(M+1,4)-PHI(N,2))
      TERM3 = ABS(1.-E(M+2,1)-E(M+2,2)-E(M+2,3)-E(M+2,4)-PHI(N,3))
      TERM4 = ABS(1.-E(M+3,1)-E(M+3,2)-E(M+3,3)-E(M+3,4)-PHI(N,4))
      IF (TERM1-EP1) 1,100,100
    1 IF (TERM2-EP2) 2,100,100
    2 IF (TERM3-EP3) 3,100,100
    3 IF (TERM4-EP4) 4,100,100
  100 CONST = -1.
      GO TO 9999
    4 CONST = 0.
 9999 RETURN
      END
```

```
CWNSUB
      SUBROUTINE WNSUB
      DIMENSION AAA(3),BBB(3)
      COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
      COMMON/COM4/ E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),
     ?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,
     ? NTIMES,NPRINT,NTAB,NVMN
      NPL1=N
      DO 10 I = 1,4
   10 W(N,I) = 1.
   35 N = N-1
      MN = 4*N-3
      DO 50 I = 1,4
      TEMP(4,I) = E(MN+3,I)
      TEMP(3,I) = E(MN+2,I)
      TEMP(2,I) = E(MN+1,I)
   50 TEMP(1,I) = E(MN,I)
      DO 70 I = 1,4
      W(N,I) = 0.
      DO 60 J = 1,4
   60 W(N,I) = TEMP(I,J)*W(N+1,J)+W(N,I)
   70 W(N,I) = W(N,I)+PHI(N,I)
      IF (N-2) 75,75,35
   75 DO 90 I = 1,4
      AAA(I) = 0.
      BBB(I) = 0.
      DO 80 J = 1,4
      AAA(I) = H(I,J)*W(2,J)+AAA(I)
   80 BBB(I) = EF(I,J)*W(3,J)+BBB(I)
   90 W(1,I) = BBB(I)+AAA(I)+SMLHH(I)
 9999 RETURN
      END
```

```
CVMNSUB
      SUBROUTINE VMNSUB(*)
      COMMON/COM1/DUM1(1019),EKSIS,EKSIM,DKSI,TDA,TDEA,DETA2,DETA,DUM2(3
     11),EKSI
      COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
      COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,
     2 NTIMES,NPRINT,NTAB,NVMN
      TERM1 = EKSI*DETA/DKSI
      TERM2 = DETA/4.
      DO 10 I = 2,1000
   10 V(I) = V(I-1)-(TERM1+TERM2)*(W(I,1)+W(I-1,1))+(TERM1-TERM2)*(WLST(
     1I,1)+WLST(I-1,1))
      GO TO (29,99),NVMN
   29 DO 11 I = 1,1000
      DO 12 J = 1,4
   12 WLST(I,J) = W(I,J)
   11 CONTINUE
      CALL WCSUB
      GO TO 9999
   99 RETURN 1
 9999 RETURN
      END
```

```
CWOSUB
      SUBROUTINE WCSUB
      COMMON/COM1/DUM(7),Y(1000),DUM1(12),EKSIS,FKSIM,DKSI,DUM2(6),MEDGE
     1,EKSI
      COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
           COMMON/COM5/ CONST,EP1,EP2,EP3,ERR,N,NPL1,NWRIT,KK,NK,NRTST,
     2 NTIMES,NPRINT,NTAB,NVMN
      IF (1-MEDGE) 1,2,1
    2 EKSI = EKSIS-.5*DKSI
      DO 10 I = 2,NWRIT
   10 Y(I) = Y(I-1)+1.
      GO TO 3
    1 EKSI = EKSI+.5*DKSI
    3 WRITE (6,902) EKSI
  902 FORMAT (1H1/5HEKSI=E16.8/)
      WRITE (6,903)
  903 FORMAT (1HC11X2HFP19X1HG16X5HTHETA18X1HV19X1HY/)
      WRITE (6,904)((W(I,J),J = 1,3),V(I),Y(I),I = 1,NWRIT)
  904 FORMAT (1H 5E20.8)
 9999 RETURN
      END
```

```
CMXINV
      SUBROUTINE MXINV
      COMMON/CLM4/ F(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),
     ?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)
      DIMENSION S(4,4)
      S(1,1) = (A(2,2)*A(3,3)*A(4,4) + A(2,3)*A(3,4)*A(4,2) +
     : A(2,4)*A(4,3)*A(3,2) - A(2,2)*A(4,3)*A(3,4) -
     : A(2,3)*A(3,2)*A(4,4) - A(2,4)*A(3,3)*A(4,2))
      S(2,1) =-(A(2,1)*A(3,3)*A(4,4) + A(2,3)*A(3,4)*A(4,1) +
     : A(2,4)*A(4,3)*A(3,1) - A(2,1)*A(4,3)*A(3,4) -
     : A(2,3)*A(3,1)*A(4,4) - A(2,4)*A(3,3)*A(4,1))
      S(3,1) = (A(2,1)*A(3,2)*A(4,4) + A(2,2)*A(3,4)*A(4,1) +
     : A(2,4)*A(4,2)*A(3,1) - A(2,1)*A(4,2)*A(3,4) -
     : A(2,2)*A(3,1)*A(4,4) - A(2,4)*A(3,2)*A(4,1))
      S(4,1) =-(A(2,1)*A(3,2)*A(4,3) + A(2,2)*A(3,3)*A(4,1) +
     : A(2,3)*A(4,2)*A(3,1) - A(2,1)*A(4,2)*A(3,3) -
     : A(2,2)*A(3,1)*A(4,3) - A(2,3)*A(3,2)*A(4,1))
      S(1,2) =-(A(1,2)*A(3,3)*A(4,4) + A(1,3)*A(3,4)*A(4,2) +
     : A(1,4)*A(4,3)*A(3,2) - A(1,2)*A(4,3)*A(3,4) -
     : A(1,3)*A(3,2)*A(4,4) - A(1,4)*A(3,3)*A(4,2))
      S(2,2) = (A(1,1)*A(3,3)*A(4,4) + A(1,3)*A(3,4)*A(4,1) +
     : A(1,4)*A(4,3)*A(3,1) - A(1,1)*A(4,3)*A(3,4) -
     : A(1,3)*A(3,1)*A(4,4) - A(1,4)*A(3,3)*A(4,1))
      S(3,2) =-(A(1,1)*A(3,2)*A(4,4) + A(1,2)*A(3,4)*A(4,1) +
     : A(1,4)*A(4,2)*A(3,1) - A(1,1)*A(4,2)*A(3,4) -
     : A(1,2)*A(3,1)*A(4,4) - A(1,4)*A(3,2)*A(4,1))
      S(4,2) = (A(1,1)*A(3,2)*A(4,3) + A(1,2)*A(3,3)*A(4,1) +
     : A(1,3)*A(4,2)*A(3,1) - A(1,1)*A(4,2)*A(3,3) -
     : A(1,2)*A(3,1)*A(4,3) - A(1,3)*A(3,2)*A(4,1))
      S(1,3) = (A(1,2)*A(2,3)*A(4,4) + A(1,3)*A(2,4)*A(4,2) +
     : A(1,4)*A(4,3)*A(2,2) - A(1,2)*A(4,3)*A(2,4) -
     : A(1,3)*A(2,2)*A(4,4) - A(1,4)*A(2,3)*A(4,2))
      S(2,3) =-(A(1,1)*A(2,3)*A(4,4) + A(1,3)*A(2,4)*A(4,1) +
     : A(1,4)*A(4,3)*A(2,1) - A(1,1)*A(4,3)*A(2,4) -
     : A(1,3)*A(2,1)*A(4,4) - A(1,4)*A(2,3)*A(4,1))
      S(3,3) = (A(1,1)*A(2,2)*A(4,4) + A(1,2)*A(2,4)*A(4,1) +
     : A(1,4)*A(4,2)*A(2,1) - A(1,1)*A(4,2)*A(2,4) -
     : A(1,2)*A(2,1)*A(4,4) - A(1,4)*A(2,2)*A(4,1))
      S(4,3) =-(A(1,1)*A(2,2)*A(4,3) + A(1,2)*A(2,3)*A(4,1) +
     : A(1,3)*A(4,2)*A(2,1) - A(1,1)*A(4,2)*A(2,3) -
     : A(1,2)*A(2,1)*A(4,3) - A(1,3)*A(2,2)*A(4,1))
      S(1,4) =-(A(1,2)*A(2,3)*A(3,4) + A(1,3)*A(2,4)*A(3,2) +
     : A(1,4)*A(3,3)*A(2,2) - A(1,2)*A(3,3)*A(2,4) -
     : A(1,3)*A(2,2)*A(3,4) - A(1,4)*A(2,3)*A(3,2))
      S(2,4) = (A(1,1)*A(2,3)*A(3,4) + A(1,3)*A(2,4)*A(3,1) +
     : A(1,4)*A(3,3)*A(2,1) - A(1,1)*A(3,3)*A(2,4) -
     : A(1,3)*A(2,1)*A(3,4) - A(1,4)*A(2,3)*A(3,1))
      S(3,4) =-(A(1,1)*A(2,2)*A(3,4) + A(1,2)*A(2,4)*A(3,1) +
     : A(1,4)*A(3,2)*A(2,1) - A(1,1)*A(3,2)*A(2,4) -
     : A(1,2)*A(2,1)*A(3,4) - A(1,4)*A(2,2)*A(3,1))
      S(4,4) = (A(1,1)*A(2,2)*A(3,3) + A(1,2)*A(2,3)*A(3,1) +
     : A(1,3)*A(3,2)*A(2,1) - A(1,1)*A(3,2)*A(2,3) -
     : A(1,2)*A(2,1)*A(3,3) - A(1,3)*A(2,2)*A(3,1))
      DETER = A(1,1)*S(1,1) + A(1,2)*S(2,1) + A(1,3)*S(3,1) +
     : A(1,4)*S(4,1)
      DO 11 I=1,4
      DO 10 J=1,4
   10 A(I,J)=S(I,J)/DETER
```

```
   11 CONTINUE
 9999 RETURN
      END
```