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WITH UNIFORM VELOCITY GRADIENT

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by

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CHEMICAL REACTION IN A TURBULENT FLOW FIELD
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ABSTRACT

A simplified statistical theory for turbulent shear flows previously initiated by the present author is developed until it is completely self-containing in the flow region where the turbulence Reynolds number is sufficiently large. The theory, therefore, applies in the region outside of the laminar sublayers. The homologous flow and concentration fields are first analyzed for the chemically frozen case. From the analyses, the relationships between the mean velocity and concentration gradients, and the Reynolds stress, turbulence energy, turbulent transport of chemical species, and the mean square fluctuation of the species concentration are established. The comparison of the present results with the available experimental data is made, which shows a satisfactory agreement. The non-equilibrium chemical reaction was found to create an inhomogeneity in the concentration field which, among other things, causes the mean square fluctuation to vary nonuniformly with respect to the Damkohler number and the flow region.

I. INTRODUCTION

The conventional phenomenological approaches to the chemically reacting turbulent shear flow problems have been developed to near their maximum capacities. These approaches have been very useful in analyzing the nonequilibrium chemical reactions in various turbulent flow fields, in the absence of a better tractable method. A few of the examples are those given in Refs. 1-4.

It is known, however, that the correct description of a nonequilibrium chemical reaction in a turbulent flow field is outside the inherent limitations of the conventional phenomenological theories. Basically, it is because the rate of the chemical reaction in a turbulent flow field is coupled with the turbulent fluctuations so that the average chemical reaction rate at a given point is not equal to the rate based on the averaged properties, such as the averaged chemical species concentrations.

A simplified statistical theory[5] was developed by the present author in which the most important statistical quantities, such as the turbulent transport tensor, turbulence energy, and the average chemical reaction rate, were described in a tractable manner. The comparison of this theory with the other available theories, as well as a brief state-of-the-art review of the conventional phenomenological and the classical statistical theories in relation to the present theory, was included in that work.

In addition to the development of the general theory, the turbulent Couette flow of a chemically inert, single component fluid was analyzed in Ref. 5. The theory as applied to a chemically inert single component flow was shown to be self-containing up to the dissipation function. The experimentally available dissipation function was employed in the solution of the

Couette flow. A simple expression for the dissipation function, however, was derived[5] subsequently which was based on the "universal equilibrium" concept for high turbulence Reynolds numbers. This expression was shown to approximate the experimentally available dissipation functions closely for the Couette, pipe, and boundary layer flows, and the free jets. It was suggested[5] that an incorporation of this dissipation concept into the general theory would make the theory completely self-containing for the chemically inert, single component flows.

In the present paper, the simplified statistical theory developed previously[5] will first be made completely self-containing for the turbulent flows of binary mixture of chemically reacting fluids when the molecular Prandtl and Schmidt numbers are of order one, and when there is no laminar sublayer. The dissipation function derived from the "universal equilibrium" concept is valid only in the region where the turbulence Reynolds number is sufficiently large. Therefore, the self-contained theory to be developed herein is not valid in the laminar sublayers. As it was the case with the previous work,[5] we shall consider that the flow is incompressible.

The theory will be employed to analyze the following set of problems with no laminar sublayers.

We shall first analyze the stationary homogenous turbulence field with a uniform velocity gradient (homologous flow field). We shall establish the relationships between the mean velocity gradient, the Reynolds stress, and the turbulence energy. These results will be compared with the available experimental data obtained behind a nonuniform grid,[6] and at the center of a Couette flow[7,8,9] where the mean velocity gradient is uniform.

We shall then superimpose a uniform mean concentration gradient of a chemical species onto the homologous turbulence field in the absence of

chemical reactions. In analogy to the homologous flow field, we shall call the homogeneous concentration field with a uniform mean-concentration gradient a homologous concentration field. The relationships between the mean velocity and concentration gradients, mass transport, and the mean square fluctuation of the chemical species will be established. A qualitative comparison between the present results and the existing analyses[10,11] based on the classical statistical turbulence theories will be made. It is pointed out here at the outset that no quantitative comparison with the existing theoretical studies is possible because the momentum and mass (or heat) transports, as well as the other statistical properties, for the high Reynolds number, self-sustaining, stationary homologous field have never been actually computed in the previous studies.

The above described turbulent flow fields with the uniform velocity and concentration (or temperature) gradients are completely homogeneous so that there is neither a characteristic length nor a reference property. A chemical reaction, on the other hand, is a function of the real time and the absolute values of the properties. One must, therefore, define a reference length and the reference properties of the flow field before a chemical reaction can be studied.

We shall, therefore, consider a hypothetical mixing layer of finite width formed by two parallel, isotropic streams with the two different mean values of the velocity and the chemical species concentration. The flow and the species concentration in the mixing layer are made homologous in the absence of chemical reaction by assuming that the two bounding planes of the layer are completely pervious to momentum and mass. Then, the inhomogenieties of the concentration field caused by a nonequilibrium chemical reaction will be analyzed.

II. FORMULATION

This section will begin with a brief discussion of the general theory derived previously[5]. This theory will then be developed until it is completely self-containing for the turbulent flow fields with no mean pressure gradients and laminar sublayers, but containing a chemically reactive species.

General Theory

A physical turbulence field consists of all size eddies which are superimposed on each other and intermingled together. The fluctuating property of a fluid element observed at a point is a manifestation of the combined influence of the various size eddies of which the fluid element is a part. Another description of the turbulence field is that it is comprised of an infinite number (continuous spectrum) of degrees of freedom. The influences of these degrees of freedom on a fluid element at a point are determined through the spectrum analyses of the pertinent correlation functions derived from the Navier-Stokes equation.

The existing spectrum analyses of such as the two-point velocity correlation function[12,13] have established the following information of the dynamics of turbulence when the turbulence Reynolds number is sufficiently large. The turbulence energy and the other observable statistical properties caused by the turbulent fluctuation, such as the Reynolds stress, are predominantly due to the low-wave-number degrees of freedom. The high wave-number degrees of freedom are in equilibrium. The low wave number region of the spectrum is statistically separated from the high wave number region across an inertial subrange. The characteristic times of the low wave numbers

(low-wave-number degrees of freedom) are much longer than and separated from those of the high wave numbers.

In the simplified theory,[5] the turbulence field described above, which consists of a spectrum of degrees of freedom, is replaced by a nonequilibrium degree of freedom representing the low-wave number region of the spectrum and the high-wave number degrees of freedom which are always in equilibrium. The turbulence energy and the other important statistical properties caused by the fluctuation are considered to be due to the nonequilibrium degree of freedom only. The high-wave-number equilibrium degrees of freedom are considered to cause the evolution of the nonequilibrium degree of freedom toward an equilibrium state through their interactions with the nonequilibrium degree of freedom, and to dissipate the turbulence energy contained in this degree of freedom.

The model of the turbulence field described in the preceding paragraph satisfies precisely the generalized definition of the Brownian motion (see for instance Prigogine[14]). The Brownian motion describes[14] the evolution of a few degrees of freedom of a large system towards statistical equilibrium in situations in which all other degrees of freedom remain in equilibrium.

Before we begin the formulation of the present problem, according to the turbulence model after the Brownian motion, we shall briefly discuss the basic governing equation of the Brownian motion itself.

A rigorous statistical treatment of the Brownian motion, in particular the evolution of one degree of freedom representing an anharmonic oscillator in solid, was carried out by Prigogine[14]. A comparison of the results of the rigorous treatments and the classical Langevin's stochastic equations[14,15] is given therein. The two are shown to be identical in certain "weakly coupled"

cases, whereas they are found to be only approximately the same in other cases. The Langevin's stochastic equation is much simpler and is more amenable to solution than the equations resulting from the rigorous treatment. Furthermore, the Langevin's equation for the various simple cases has been solved[15] and the results have been shown to be essentially correct in comparison with the available experimental data.

Now returning to the present problem, the starting point of the present turbulence theory described earlier is the Langevin's stochastic equation,[15]

$$\frac{du_i}{dt} = -\beta(\vec{x})U_i + A_i(t) + K_i(\vec{U}) \tag{1}$$

where the index i denotes the first order Cartesian tensor. In view of the present turbulence model and the general discussion of the Brownian motion given before, the meanings of the various terms of Eq. (1) are as follows. The term on the left denotes the time rate of evolution (decay) of the instantaneous absolute velocity u_i at a point $\vec{x}$, where u_i is due to the nonequilibrium degree of freedom representing the low wave numbers of the turbulence spectrum. The first term on the fight represents the decay of u_i due to the long preferred memories of the low-wave-number nonequilibrium degree of freedom, whereas the second term denotes that due to the fast random fluctuations of the equilibrium degrees of freedom. In Eq. (1), U_i is $u_i - <u_i>$ where the symbol < > denotes the appropriate ensemble average. The function $\beta(\vec{x})$ is the characteristic decay rate of the nonequilibrium degree of freedom which will be defined more precisely later. Finally, K_i is the body force applied to the fluid element due to the molecular viscosity of the fluid, which will lead to the eventual dissipation of the turbulence energy.

Equation (1) thus describes the evolution of the nonequilibrium degree of freedom. Since it has been assumed in the present turbulence model that all the observable statistical properties are entirely due to this nonequilibrium degree of freedom, the mean velocity and the turbulence energy, for instance, are simply given by $<u_i>$ and $<U_k U_k>$ respectively.

The Fokka-Planck equation appropriate for the present turbulent flow problems was derived in the previous paper[5] which is written below for the cases wherein there are no mean pressure gradients and laminar sublayers.

$$\frac{\partial F^{(1)}}{\partial t} + u_j \frac{\partial F^{(1)}}{\partial x_j} + \frac{\partial F^{(1)}}{\partial u_j} + \frac{\partial}{\partial u_j} (F^{(1)} K_j)$$

$$= \beta \left[\frac{\partial}{\partial U_j} (F^{(1)} U_j) - \frac{<U_k U_k>}{3} \frac{\partial^2 F^{(1)}}{\partial U_j \partial U_j} \right] + \gamma F^{(\alpha)} + Mf \qquad (2)$$

In the above equation, t is time, and $F^{(P)}$, where $P \geq 0$, is the P-th order distribution function of the chemical species defined by,

$$F^{(P)} d\vec{u} = n^P(\vec{x}, \vec{u}, t) \, f(\vec{x}, \vec{u}, t) du \qquad (3)$$

where $d\vec{u} = du \, dv \, dw$ with u, v, and w without indices denoting the x, y, and z Cartesian components of the velocity respectively. The symbol n represents the mass fraction of the chemical species. The function f is the distribution function of the fluid elements which expresses the occupation probability of a fluid element in the phase cell $d\vec{x} \, d\vec{u}$. Equation (3) gives, for instance,

$$\int F^{(P)} d\vec{u} = <n^P> \qquad (4)$$

where the symbol < > is used henceforth to denote the moment,

$$\langle Q \rangle = \int Q f \, d\vec{u} \tag{5}$$

The symbol γ denotes the chemical reaction rate constant, and the instantaneous chemical reaction rate in Eq. (2) is considered to be given by γn^α. The term, M, represents the molecular diffusion rate of the chemical species between the fluid element and the surroundings. The expression, $D\dfrac{\partial^2 n}{\partial x_k \partial x_k} f$, where D is the molecular diffusivity, was employed for M in the previous derivation[5]. This term as well as the molecular dissipation term k_j will be re-formulated in the following.

Dissipative Terms

Equation (2) as was formulated in Ref. 5 contains two unknown characteristic times. One is the characteristic time of the nonequilibrium degree of freedom defining β, and the other is the characteristic time of the equilibrium degrees of freedom which determines k_j and M.

The characteristic time of the nonequilibrium degree of freedom, $1/\beta$, which represents the low wave numbers or the larger eddies, is given as (see Ref. 5).

$$1/\beta = 2\Lambda/\langle U_k U_k \rangle^{\frac{1}{2}} \tag{6}$$

where Λ is the characteristic length of the larger eddies. Following the suggestion given at the end of Ref. 5, we let,

$$\Lambda = \frac{\lambda}{1 + \dfrac{L}{u_{0\infty}} \dfrac{\partial u_0}{\partial y}} \tag{7}$$

for the two dimensional flows wherein the only important velocity gradient is $\dfrac{\partial u_0}{\partial y}$. In Eq. (7), the subscript o denotes the averaged value, thus

$u_0 = <u>$, and L is the characteristic length of the flow field such as the half-width of the Couette flow and the boundary layer thickness. The subscript ∞ denotes the reference value, and λ is the characteristic length of the larger eddies in the limit of small $\partial u_0 / \partial y$.

The characteristic time of the equilibrium degrees of freedom, unlike that of the nonequilibrium degree of freedom, is not closely related to the characteristics of the flow field which generate the larger eddies. Hence, it cannot be defined in a simple manner as Eq. (7). The entire Brownian turbulence model, however, which has been formulated herein is based on the known dynamics[12,13] of the turbulence fields described in the second paragraph of the preceding subsection. The dynamics of turbulence, when pushed to its limit, leads to the "universal equilibrium" theory which states that the rate of dissipative processes which are due to the high wave numbers (equilibrium degrees of freedom) is in fact controlled by the characteristic time of the low wave numbers (nonequilibrium degree of freedom). The equilibrium degrees of freedom, in the present model, if we accept the "universal equilibrium" theory, dissipate the turbulence energy and the chemical species fluctuation only as fast as they receive them from the nonequilibrium degree of freedom. The nonequilibrium degree of freedom, therefore, is the rate-controlling degree of freedom in the dissipative processes.

We shall consider that this universal-equilibrium concept of dissipation holds for the homologous flow field to be studied herein.

We now express the dissipative terms K_j and M of Eq. (2) in terms of the characteristic time, $1/\beta$, when the molecular Prandtl and Schmidt numbers are of order one, as

$$K_j = -\beta(u_j - u_{0j}) = -\beta U_j$$

$$M = -\beta(n - n_0) = -\beta N$$

(8)

Governing Fokka-Planck Equations

With the use of Equations (3), (6), (7), and (8), Eq. (2) gives the following two self-contained equations governing the functions f and n respectively. The equation for f is derived from Eq. (2) by setting n = 1 and by discarding the chemical reaction term.

$$\frac{\partial f}{\partial t} + u_j \frac{\partial f}{\partial x_j} + \frac{\partial f}{\partial u_j}$$

$$= \frac{<U_k U_k>^{1/2}}{2\lambda} (1 + \frac{L}{u_{0\infty}} \frac{\partial u_0}{\partial y}) \left[2 \frac{\partial}{\partial U_j} (fU_j) - \frac{<U_k U_k>}{3} \frac{\partial^2 f}{\partial U_j \partial U_j} \right] \tag{9}$$

$$\frac{\partial nf}{\partial t} + u_j \frac{\partial nf}{\partial x_j} + \frac{\partial nf}{\partial u_j}$$

$$= \frac{<U_k U_k>^{1/2}}{2\lambda} (1 + \frac{L}{u_{0\infty}} \frac{\partial u_0}{\partial y}) [2 \frac{\partial}{\partial U_j} (nfU_j) - \frac{<U_k U_k>}{3} \frac{\partial^2 nf}{\partial U_j \partial U_j}]$$

$$+ \gamma n^\alpha f - \frac{<U_k U_k>^{1/2}}{2\lambda} (1 + \frac{L}{u_{0\infty}} \frac{\partial u_0}{\partial y}) (n - n_0) f \tag{10}$$

Equations (9) and (10) are integro-differential equations since $<U_k U_k>$, u_0, and n_0 represent the integration with respect to u, v, and w. These equations will be solved by an approximate moment method for the flow problem of the present interest subsequently. As a preparation of the solution, we derive the generalized moment equations in the following.

Generalized Moment Equations

We let Q be a general function of $\vec{U}$, $\vec{X}$, and t. Then as we multiply Eqs. (9) and (10) by Q and as we integrate them with respect to $d\vec{U} = dUdVdW$, there result after some manipulation,

$$\frac{\partial}{\partial t} \int Qf \, d\vec{U} - \int \frac{\partial Q}{\partial t} f d\vec{U} + \frac{\partial}{\partial x_k} \int (u_{ok} + U_k) \, Qf \, d\vec{U}$$

$$- \int (u_{ok} + U_k) \frac{\partial Q}{\partial x_k} f d\vec{U} + \int \frac{\partial Q}{\partial U_m} \left[(u_{ok} + U_k) \frac{\partial u_{om}}{\partial x_k} \right] f d\vec{U}$$

$$= - \frac{<U_k U_k>^{1/2}}{2\lambda} \left(1 + \frac{L}{u_{o\infty}} \frac{\partial u_o}{\partial y} \right) \left[2 \int U_k \frac{\partial Q}{\partial U_k} f d\vec{U} + \frac{<U_k U_k>}{3} \int \frac{\partial Q}{\partial U_m} \frac{\partial f}{\partial U_m} d\vec{U} \right] \qquad (11)$$

$$\frac{\partial}{\partial t} \int Qnf \, d\vec{U} - \int n \frac{\partial Q}{\partial t} f d\vec{U} + \frac{\partial}{\partial x_k} \int (u_{ok} + U_k) \, Qnf \, d\vec{U}$$

$$- \int (u_{ok} + U_k) \frac{\partial Q}{\partial x_k} \, nf \, d\vec{U} + \int \frac{\partial Q}{\partial U_m} \left[(u_{ok} + U_k) \frac{\partial u_{om}}{\partial x_k} \right] nf \, d\vec{U}$$

$$= - \frac{<U_k U_k>^{1/2}}{2\lambda} \left(1 + \frac{L}{u_{o\infty}} \frac{\partial u_o}{\partial y} \right) \left[2 \int U_k \frac{\partial Q}{\partial U_k} nf d\vec{U} + \frac{<U_k U_k>}{3} \int \frac{\partial Q}{\partial U_m} \frac{\partial nf}{\partial U_m} d\vec{U} \right]$$

$$+ \gamma \int Qn^\alpha f d\vec{U} - \frac{<U_k U_k>^{1/2}}{2\lambda} \left(1 + \frac{L}{u_{o\infty}} \frac{\partial u_o}{\partial y} \right) \int (n - n_o) \, Qf d\vec{U} \qquad (12)$$

As it was done in reference 5, one can readily show that the various particular moment equations derived from Eqs. (11) and (12) are essentially the same as the corresponding moment equations derived from the Navier-Stokes and the species conservation equations in the classical statistical turbulence studies[13].

III. FLOW FIELD

The turbulent flow field with uniform mean velocity gradient (homologous flow field) will be analyzed in this section(see Fig. 1).

Solution

Equation (9) will be solved by the moment method, after Mott-Smith[16], and Liu and Lees[17], as was done in the previous turbulence anlaysis[5]. For the details of this moment method, the readers are referred to the references 16 and 17.

We first approximate the distribution function f by the two half-Maxwellians as,

$$f = f_1 + f_2 \tag{13}$$

where,

$$f_1 = \frac{1}{(\frac{2}{3} \pi E_1)^{3/2}} \exp - \frac{(u-u_{01})^2 + V^2+W^2}{2E_1/3} \qquad \text{for } V > 0,$$

$$\tag{14}$$

$$f_2 = \frac{1}{(\frac{2}{3} \pi E_2)^{3/2}} \exp - \frac{(u-u_{02})^2 + V^2+W^2}{2E_2/3} \qquad \text{for } V < 0,$$

and f_1 and f_2 are considered to be zero for $V < 0$ and $V > 0$ respectively. The present moment method[5] of solution entails the determination of the four unknown functions $E_1(y)$, $E_2(y)$, $u_{01}(y)$, and $u_{02}(y)$ by substituting the assumed form of the distributions function, Eqs. (13), into a set of four particular moment equations to be deduced from Eq. (11) which is the generalized moment equation of Eq. (9). By approximating the distribution function by Eq. (13), we are conceptually[18] implying that the flow field is a shear layer of infinite extent formed between two isotropic turbulent parallel streams with the turbulence energies of $E_1(-\infty)$ and $E_2(\infty)$ respectively, and with the mean velocities of $u_{01}(-\infty)$ and $u_{02}(\infty)$ respectively.

We notice the fact that $v_0 = w_0 = \frac{\partial}{\partial t} = \frac{\partial}{\partial x} = \frac{\partial}{\partial z} = 0$ in a steady state (stationary) homologous flow field. Then, as we successively let $Q = 1$, $Q = U$, $Q = UV$, and $Q = U_k U_k = U^2 + V^2 + W^2$ in Eq. (11), there result the following equations.

$$\frac{d}{dy} \int f V \, d\vec{U} = 0 \tag{15}$$

$$\frac{d}{dy} \int f UV \, d\vec{U} = 0 \tag{16}$$

$$\frac{d}{dy} \int fUV^2 \ d\vec{U} + \left(\frac{du_0}{dy}\right) \int fV^2 \ d\vec{U}$$

$$= -\frac{2}{\lambda} (1 + \frac{L}{u_{0\infty}} \frac{du_0}{dy}) \left[\int f(U^2 + V^2 + W^2)d\vec{U}\right]^{1/2} \left(\int fUV \ d\vec{U}\right) \tag{17}$$

$$\frac{d}{dy} \int fV(U^2 + V^2 + W^2)d\vec{U} + 2 \frac{du_0}{dy} \int fUV \ d\vec{U}$$

$$= -\frac{1}{\lambda} (1 + \frac{L}{u_{0\infty}} \frac{du_0}{dy}) \left[\int f(U^2 + V^2 + W^2)d\vec{U}\right]^{3/2} \tag{18}$$

When Eqs. (13) and (14) are substituted in Eq. (15) for f, there results for the homologous flow, where $v_0 = 0$,

$$E_1 = E_2 = E \tag{19}$$

Equations (16), (17) and (18) become with the use of Eqs. (13), (14), and (19) as, after some manipulation and nondimensionalization,

$$\psi\theta = C_1 \tag{20}$$

$$\frac{d\phi}{dY} = -2\sqrt{\frac{6}{\pi}} \frac{(\psi\theta)\left[\frac{\psi^2}{4} + \theta^2\right]^{1/2}}{(\frac{\lambda}{L})\theta^2 + \left(\frac{6}{\pi}\right)^{\frac{1}{2}}(\psi\theta)\left[\frac{\psi^2}{4} + \theta^2\right]^{1/2}} \tag{21}$$

$$\frac{d\phi}{dy} = -\frac{\left[\frac{\psi^2}{4} + \theta^2\right]^{3/2}}{\frac{\psi\theta}{(6\pi)^{1/2}} (\frac{\lambda}{L}) + \frac{1}{2} (\frac{\psi^2}{4} + \theta^2)^{3/2}} \tag{22}$$

where $\quad \psi = \frac{u_{01} - u_{02}}{u_{0\infty}}$, $\qquad\qquad \theta = \frac{E^{1/2}}{u_{0\infty}}$,

$$\tag{23}$$

$$\phi = \frac{u_{01} + u_{02}}{u_{0\infty}}, \qquad\qquad Y = y/L,$$

and C_1 is a constant of integration of Eq. (16).

Equations (20), (21), and (22) comprise the governing equations for the three functions of ψ, θ, and ϕ, which together describe the flow field.

The function ϕ is related to the mean velocity u_0 through Eqs. (13) and (23) as,

$$\frac{u_0}{u_{0\infty}} = \frac{1}{u_{0\infty}} \int fu \, d\vec{U} = \frac{\phi}{2} \tag{24}$$

In a homologous flow field, therefore, $\frac{1}{2} \frac{d\phi}{dY} = \frac{du_0/u_{0\infty}}{dY}$ is constant. Hence, for a given mean velocity gradient, $\frac{1}{2} \frac{d\phi}{dY}$, Eqs. (21) and (22) constitute two coupled algebraic equations which would determine the unknowns ψ and θ which should be also constants. These values of ψ and θ will then give the constant C_1, which determines the Reynolds stress from Eq. (20), thus completely satisfying the three equations, Eqs. (20), (21), and (22).

For a given constant value of $d\phi/dY$, Eqs. (21) and (22) can be readily solved to give:

$$\psi = -\left[\frac{\pi}{6} \left(1 - \frac{\pi}{8} \right) \right]^{1/2} (\frac{\lambda}{L}) \frac{d\phi/dY}{2 + d\phi/dY} \tag{25}$$

and

$$\theta = \left(\frac{1 - \frac{\pi}{8}}{3^{1/2}} \right) (\frac{\lambda}{L}) \frac{d\phi/dY}{2 + d\phi/dY} \tag{26}$$

Equation (20) then gives, with the use of Eqs. (13), (25), and (26),

$$\frac{<UV>}{u_{0\infty}^2} = \frac{1}{u_{0\infty}^2} \int f \, UV \, d\vec{U} = \frac{C_1}{(6\pi)^{1/2}}$$

$$= - \frac{1}{6(3)^{1/2}} \left(1 - \frac{\pi}{8} \right)^{3/2} (\frac{\lambda}{L})^2 \left| \frac{d\phi/dY}{2 + d\phi/dY} \right|^2 \tag{27}$$

Solution of Eqs. (20), (21), and (22) is now complete. The turbulence energy is readily obtained from the solution as,

$$\frac{\langle U_k U_k \rangle}{u_{0\infty}^2} = \frac{1}{u_{0\infty}^2} \int f(U^2 + V^2 + W^2)\, d\vec{U} = \frac{\psi^2}{4} + \theta^2$$

$$= \frac{1}{3}\left(1 - \frac{\pi}{8}\right)\left(\frac{\lambda}{L}\right)^2\left(\frac{d\phi/dY}{2 + d\phi/dY}\right)^2 \qquad (28)$$

Discussion of the results and the comparison with the available experimental data will follow.

Discussion of the Flow Field

Equations (25) through (28) show that the relationship between the mean velocity gradient and $\langle UV \rangle/u_{0\infty}^2$, and $\langle U_k U_k \rangle/u_{0\infty}^2$ depend on the characteristic length ratio λ/L. As it was defined earlier, λ is the characteristic length of the nonequilibrium degree of freedom (large eddies) in the limit of the small mean velocity gradient, whereas L is the characteristic length of the flow field.

In the homogeneous field generated by two parallel plates, that is at the core region of a Couette flow, λ and L are fairly well defined, and we let $\lambda/L = 1$ where L is the half-width of the Couette flow field. The reference velocity, $u_{0\infty}$, is the velocity at the center of the Couette flow. The solid and the broken lines in Fig. 2 show the present solution, Eqs. (25) through (28), for $\lambda/L = 1$.

There are two published experimental works[7,8] on Couette flow which give the relationship between the mean velocity gradient at the center of the flow field and the surface shear which ideally should be equal to $\langle UV \rangle$ at the center. These experimental results are shown in Fig. 2.

Reichardt[8] showed only two experimental points, whereas Robertson[7] gave a rather large number of data.

Reichardt[8] employed two parallel belts running in the opposite directions.

In the Robertson work[7], the flow field was generated by using a belt

parallel to a fixed wall. Robertson noted that the velocity at the center, $u_{0\infty}$, was less than one half of the belt velocity $(u_0)_{belt}$, due to the side-wall effects (three dimensional effects). By employing a blower at the inlet to the flow region, $u_{0\infty}$ was brought up to half of the belt velocity. Figure 2, therefore, shows two sets of Robertson's data for the two different values of $u_{0\infty}/(u_0)_{belt}$. There is substantial scatter in the data.

The readers are referred to the original papers[7,8] for the detailed description of the experiments as well as the analyses of the data.

As to the comparison of the theoretical results with the experimental data, Fig. 2 shows that the present theory predicts the correct order and variation, with respect to du_0/dy, of $<UV>$. Beyond that, we may say that the present theory seems to agree with the Reichardt's results much more closely than the Robertson's data.

The experimental results of a homologous flow field generated by a non-uniformly spaced grid located in a channel are given by Rose[6]. Here, the analysis of the flow field is much more complicated than that of the core region of the Couette flow. In the Couette flow, λ/L is clearly one since there is only one characteristic length of the flow configuration. In the problem of Rose[6], the characteristic length and velocity governing the overall flow, L and $u_{0\infty}$, are, as were the case with the Couette flow, the half-width of the channel and the center-line velocity, respectively. However, the characteristic length, λ, is governed by both the grid spacings and the channel width. In a flow behind a uniform grid without the confining walls, λ is equal to the grid spacing. In the Rose's work, the grid spacing varies across the channel, and also the effect of the wall is increasingly felt by the eddies as the flow developes through the channel.

Even if we consider that the effect of the walls on the eddies is negligible in the flow region where the measurements[6] were taken, the value of

λ at a given point is affected by all different values of the nonuniform spacings of the grid. The fact that the measured values of $\langle U_k U_k \rangle$ and $\langle UV \rangle$ were relatively uniform across the channel at the regions sufficiently downstream of the grid indicates that all size eddies produced by the various grid spacings were sufficiently mixed in these regions, producing a uniform value of λ. Therefore, for these regions, we take λ equal to the maximum value of the grid spacing outside the wall boundary layer, where the mean velocity gradient is uniform. This value according to the discription[6] of the grid is about 0.17L.

By using $\lambda/L = 0.17$, the present theory, Eqs. (25) through (28), gives $\langle UV \rangle / u_{0\infty}^2 = -4.6 \times 10^{-5}$ and $\langle U_k U_k \rangle / u_{0\infty}^2 = 2.5 \times 10^{-4}$ for the only experimentally employed value[6] of $(L/u_{0\infty})(du_0/dy) = 0.135$. For the same value of the mean velocity gradient, Rose measured $\langle UV \rangle / u_{0\infty}^2$ of between about -3.5×10^{-5} and -4.2×10^{-5}, and $\langle U_k U_k \rangle / u_{0\infty}^2$ of about 3×10^{-4}. Therefore, the agreement between the present theory and Rose's experimental data is satisfactory.

Analysis of the flow field is now completed. We shall now study the transport of chemical species with homologous flow field.

IV. HOMOGENEOUS SCALAR FIELD

We shall study the transport and fluctuation of a chemical species in the absence of chemical reaction (chemically frozen case). This case is equivalent to the heat transfer problem if we consider the species mass fraction n to represent the temperature instead. The chemically reacting case will be analyzed in the subsequent section.

Analysis of Concentration Field

The mixing layer for n is homogeneous only if dn_0/dY is uniform and the chemical reaction is frozen.

We first seek the solution of Eq. (10) for n subject to the constant dn_0/dy and frozen chemistry. Consistent with the moment method we employed for the flow field, we shall obtain this solution by solving an appropriate number of particular moment equations to be derived from Eq. (12) which is the generalized moment equation of Eq. (10).

In order to be consistent with Eq. (13), we first express n by,

$$n(y,\vec{U}) = n_1 + n_2 \tag{29}$$

where,

$$n_1 = n_1(y,\vec{U}) \qquad\qquad \text{for } V \geq 0$$
$$\tag{30}$$
$$n_2 = n_2(y,\vec{U}) \qquad\qquad \text{for } V \leq 0$$

and n_1 and n_2 are zero for $V < 0$ and $V > 0$ respectively. With the expression of Eq. (29), the transport and the mean square fluctuation of n are given by,

$$<NV> = <nV> = \int_{-\infty}^{\infty} dU \int_{0}^{\infty} dV \int_{-\infty}^{\infty} dW\, n_1\, Vf_1 + \int_{-\infty}^{\infty} dU \int_{-\infty}^{0} dV \int_{-\infty}^{\infty} dW\, n_2\, Vf_2 \tag{31}$$

$$<N^2> = <n^2> - n_0^2$$

$$= \int_{-\infty}^{\infty} dU \int_{0}^{\infty} dV \int_{-\infty}^{\infty} dW\, n_1^2\, f_1 + \int_{-\infty}^{\infty} dU \int_{-\infty}^{0} dV \int_{-\infty}^{\infty} dW\, n_2^2\, f_2$$

$$- \left[\int_{-\infty}^{\infty} dU \int_{0}^{\infty} dV \int_{-\infty}^{\infty} dW\, n_1\, f + \int_{-\infty}^{\infty} dU \int_{-\infty}^{0} dV \int_{-\infty}^{\infty} dW\, n_2\, f \right]^2 \tag{32}$$

As it is seen from the solution of the flow field, Eqs. (25) through (28), the various statistical quantities caused by the fluctuations, such as $<UV>$ and $<U_k U_k>$, depend only on the mean velocity gradient and not on the mean velocity itself in a homologous flow field. Otherwise, since the mean velocity varies linearly with y, the statistical properties such as $<UV>$ will vary

with y and the flow field will no longer be homologous.

Similarly, in a homogeneous concentration field of infinite extent with a constant dn_0/dy, the statistical quantities caused by the fluctuations, such as $<NV>$ and $<N^2>$, should be uniform. A study of Eq. (12) in the light of Eqs. (29) and (30) shows that n_1 and n_2 must be independent of $\vec{U}$, other than they depend on the direction of V as defined by Eq. (30), in order that the concentration field may be homogeneous. This point will be made more clear later.

We shall now derive two particular moment equations from Eq. (12) in order to determine the functions $n_1(y)$ and $n_2(y)$. Although the present section is concerned only with the chemically frozen case, $\gamma = 0$, we shall first derive the equations for the general γ for the later use. As we successively put $Q = 1$ and $Q = V$ into Eq. (12), with $v_0 = w_0 = (\partial/\partial t) = (\partial/\partial x) = (\partial/\partial z) = 0$, there result the equations,

$$\frac{d}{dy} \int V \, n \, f \, d\vec{U} = \gamma \int n^\alpha \, f \, d\vec{U} \tag{33}$$

$$\frac{d}{dy} \int V^2 \, n \, f \, d\vec{U} = -3 \frac{<U_k U_k>}{2\lambda} \left(1 + \frac{L}{u_{0\infty}} \frac{du_0}{dy}\right) \int V \, n \, f \, d\vec{U}$$

$$+ \gamma \int v \, n^\alpha \, f \, d\vec{U} \tag{34}$$

Above two equations become, with the aid of Eqs. (13) and (29), and after a nondimensionalization,

$$\frac{d}{dY} (\alpha_1 - \alpha_2) = \Gamma \frac{(6\pi)^{1/2}}{2\theta} (\alpha_1^2 + \alpha_2^2) \tag{35}$$

$$\frac{d}{dY} (\alpha_1 + \alpha_2) = -\frac{3}{4} \left(\frac{6}{\pi}\right)^{1/2} \left(\frac{L}{\lambda}\right) \left(\frac{\psi^2}{4} + \theta^2\right)^{1/2} \frac{(2 + d\phi/dY)}{\theta} (\alpha_1 - \alpha_2)$$

$$+ \Gamma \left(\frac{6}{\pi}\right)^{1/2} \frac{1}{\theta} (\alpha_1^2 - \alpha_2^2) \tag{36}$$

where

$$\alpha_1 = n_1/n_{0\infty}, \quad \alpha_2 = n_2/n_{0\infty}, \quad \Gamma = \gamma L n_{0\infty}/u_{0\infty}. \tag{37}$$

Now we consider the frozen chemistry of $\Gamma = 0$.

For the present case wherein n_1 and n_2 are functions of y only, we have,

$$\frac{d(\frac{n_0}{n_{0\infty}})}{dY} = \frac{1}{n_{0\infty}} \frac{d}{dY} \int n\, f\, d\vec{U} = \frac{1}{2} \frac{d}{dY} (\alpha_1 + \alpha_2) \tag{38}$$

Therefore, for a given uniform mean concentration gradient, Eqs. (35) and (36) can be readily solved, and there results,

$$\alpha_1 - \alpha_2 = -\frac{8}{3} \left(\frac{\pi}{6}\right)^{1/2} \left(\frac{\lambda}{L}\right) \frac{\theta}{\left(\frac{\psi^2}{4} + \theta^2\right)^{1/2}} \left[\frac{dn_0/n_{0\infty}}{\frac{dY}{2+d\phi/dY}}\right] \tag{39}$$

The turbulent transport and the mean square fluctuation are obtained from Eqs. (25), (26), (31), (32), and (39), as,

$$\frac{\langle NV \rangle}{n_{0\infty} u_{0\infty}} = \frac{1}{(6\pi)^{1/2}} \theta\, (\alpha_1 - \alpha_2) \left(\frac{\lambda}{L}\right)$$

$$= -\frac{4}{9(3)^{1/2}} \left(1 - \frac{\pi}{8}\right)^{3/2} \left(\frac{\lambda}{L}\right)^2 \frac{(d\phi/dY)}{(2 + d\phi/dY)^2} \frac{d(n_0/n_{0\infty})}{dY} \tag{40}$$

$$\frac{\langle N^2 \rangle}{n_{0\infty}^2} = \frac{1}{4} (\alpha_1 - \alpha_2)^2$$

$$= \frac{8\pi}{27} \left(1 - \frac{\pi}{8}\right) \left(\frac{\lambda}{L}\right)^2 \left[\frac{1}{2 + d\phi/dY}\right]^2 \left[\frac{d(n_0/n_{0\infty})}{dY}\right]^2 \tag{41}$$

Discussion of Homogeneous Concentration Field

Above solutions show that the turbulent transport, in a homologous flow and concentration field, is porportional to the mean concentration gradient, whereas the mean square fluctuation varies with the square of the gradient. Also, the increasing mean velocity gradient, for $d\phi/dY \lesssim 2$, increases the

transport whereas it decreases the mean square fluctuation.

From the results given in Eqs. (27) and (40), one may readily derive the ratio of the turbulent mass (or heat) to momentum diffusivities, ϵ_D/ϵ_m, where these diffusivities are defined in the conventional manner by the equations,

$$<NV> = -\epsilon_D \frac{dn_0}{dy}$$

$$<UV> = -\epsilon_m \frac{du_0}{dy}$$ (42)

The ratio simply becomes

$$\epsilon_D/\epsilon_m = 4/3$$ (43)

The transient, decaying, low Reynolds number turbulence field with the uniform velocity and temperature gradients was studies by Deissler[10,11]. As it was mentioned in section 1, the problems considered by Deissler, as well as the similar problems considered by others, are basically different from the present one, and no direct comparison can be made. However, a qualitative comparison can be made of the ratio ϵ_D/ϵ_m. Deissler[11] showed that a large mean velocity gradient has the effect on the turbulence spectrum somewhat analogous to that of a large Reynolds number, and that the ratio ϵ_D/ϵ_m approaches one from above as the velocity gradient is continuously increased, when the molecular Prandtl number is order one. This is essentially consistent with present ratio given in Eq. (43).

We have now completed the analysis of the homogeneous concentration field which itself has no characteristic length. The ratio λ/L in Eqs. (40) and (41) is arbitrary as it was in Eqs. (27) and (28). As we have seen in section III, these characteristic lengths become defined only as we consider a homologous flow and concentration field generated by a particular physical configuration.

A concentration field with chemical reaction cannot be homogeneous because a chemical reaction depends on the real time. Therefore, all the characteristic lengths must be defined a priori.

The inhomogeneous concentration field generated by a chemical reaction will be considered in the next section.

V. CHEMICAL REACTIONS

As it was mentioned at the end of the preceding section, a characteristic length must be defined before a chemical reaction can be analyzed. We shall in the following, first define a physical system such that the flow and scalar fields are both homologous in the absence of a chemical reaction. The solutions obtained in the preceding section, therefore, describe the flow and chemically frozen concentration fields of this system. The effect of chemical reaction on the scalar field will then by analyzed.

Chemically Frozen Field

In order to provide a characteristic length without creating laminar sublayers, we consider the hypothetical homologous flow field of width L created between two isotropic[19] streams with the mean velocities $u_{o1}(0)$ and $u_{o2}(L)$ respectively, and with the equal turbulence energy of E. The flow is sketched in Fig. 3. We are assuming that the planes at 1 and 2 (see Fig. 3) are perfectly pervious to momentum and mass (slippery).

The solution of the flow field proceeds exactly as it did for the infinite homologous field of section III, and we obtain Eqs. (27) and (28) with $d\phi/dY$ given in terms of the present boundary conditions, $u_{o1}(0)$ and $u_{o2}(L)$, as,

$$\frac{d\phi}{dY} = 2 \frac{L}{u_{o\infty}} \frac{du_o}{dy} = -\left\{1 + \left[\frac{\pi}{6}\left(1 - \frac{\pi}{8}\right)\right]^{1/2} - \Delta\right\} + \left[\left\{1 + \left[\frac{\pi}{6}\left(1 - \frac{\pi}{8}\right)\right]^{1/2} - \Delta\right\}^2 + 4\Delta\right]^{1/2} \quad (44)$$

$$\phi = \left(\frac{d\phi}{dY}\right) Y + 2 \frac{u_{o1}(0)}{u_{0\infty}} + \left[\frac{\pi}{6}\left(1 - \frac{\pi}{8}\right)\right]^{1/2} \tag{45}$$

where

$$\Delta = \left[u_{o2}(L) - u_{o1}(0)\right]/u_{0\infty} \tag{46}$$

In the above solution, we set $\lambda/L = 1$. The $u_{0\infty}$ is an arbitrary reference velocity, which we may set equal to $u_{o1}(0)$ for convenience. Since the planes are slippery, there are velocity slips at these planes, instead of laminar sublayers, as seen from Eq. (45).

Now, we must prescribe the necessary conditions on n for the two isotropic streams. Since the values of n at y = 0 and y = L may be still functions of $|\vec{U}|$ [20], it is not sufficient just to prescribe values of <n>. There are infinitely many function of $|\vec{U}|$ which would give the same <n>.

Physically, this means that the concentration field between the two planes is dependent upon the *fluctuations of n*, as well as upon the averaged values of n, in the two bounding streams.

Again proceeding exactly as we have done for the infinite homologous concentration field of section IV, we find that the boundary conditions on n at y = 0 and y = L must be specified as constants in order to obtain a homogeneous field with a uniform dn_0/dy between the two planes. Therefore, in the present problem, the constant values of $\alpha_1(0)$ and $\alpha_2(1)$ are specified. The solution of the scalar field is then, again, those given by Eqs. (40) and (41) with $d(n_0/n_{0\infty})/dY$ given in terms of the present boundary conditions as,

$$\frac{d(n_0/n_{0\infty})}{dY} = \frac{9}{8}\left\{\left[\frac{\pi}{24\left(1 - \frac{\pi}{8}\right)} + \frac{1}{3}\right]\frac{2}{\pi}\right\}^{1/2} (2 + \phi')[\alpha_2(1) - \alpha_1(0)] \tag{47}$$

$$n_0/n_{0\infty} = \frac{dn_0/n_{0\infty}}{dY}\left[Y + \frac{8}{3}\left(\frac{\pi}{6}\right)^{1/2}\left(1 - \frac{\pi}{8}\right)^{1/2}\frac{1}{2 + d\phi/dY}\right] + 2\alpha_1(0) \tag{48}$$

The homologous flow field and the chemically frozen, homologous concentration field have now been described.

The condition that n must be constant in the two isotropic bounding streams implies the following. Each fluid element of each stream must contain exactly the same mass fraction of the chemical species, and therefore, there must be no fluctuation of the species[21]. In an usual isotropic stream with a scalar quantity, say temperature, such as that generated by a uniformly heated grid, there is a spectral distribution of the temperature field which results in a fluctuation of the temperature. This is because the transfer of the scalar quantity to the fluid elements at the grid is not uniform but is related to the generation of the turbulence itself. If, on the other hand, a chemical species (or heat) is uniformlay mixed into a fluid in, say, a settling chamber, and if the resulting uniform mixture is then allowed to become turbulent by passing through a grid, then the resulting isotropic stream will have a constant n.

The n-fluctuations given by Eqs. (40) and (41) are caused by the mixing of the two streams with constant but different n's in the mixing layer between the two planes.

When n in the two isotropic bounding streams are made functions of $|\vec{U}|$, the results show that the scalar field between the two planes is inhomogeneous. Quantities such as $<NV>$ and $<N^2>$ are found to be different from the present results, Eqs. (40) and (41), and they vary across the mixing layer. The case of the mixing layers formed by two streams with the initial n-fluctuations will be considered in a future paper.

Now, we shall see the effect of chemical reactions on the concentration field which is homologous (homogeneous with a uniform dn_0/dy) when the chemical reaction is frozen.

Chemically Reacting Field

The equations governing the concentration field between the two planes with chemical reaction have been already derived. These are Eqs. (35) and (36) with $\lambda/L = 1$ for the present system of Fig. 3. For convenience, we let the reference concentration $n_{0\infty}$ be equal to the average concentration of the lower bounding stream, $n_1(o)$.

Solutions of Eqs. (35) and (36) have been obtained in the present study by the use of a digital computer, for the second order chemical reaction, $\alpha = 2$.

The solutions are shown in Figs. 4 through 7 for the two different flow fields of $(L/u_{0\infty})du_0/dy$ of 1/4 and 1/2 respectively. Each figure shows the solutions for the two Damkohler numbers of $\Gamma = 0$ and $\Gamma = -0.0281$. The boundary conditions for all cases shown are:

$$\alpha_1(0) = 1$$
$$\alpha_2(1) = 5.81$$

Figures 4 thorugh 7 show first of all that the overall reaction in the mixing layer is substantially reduced as mean velocity gradient is increased from 1/4 to 1/2. As it can be inferred from the chemically frozen solution, Eqs. (40) and (41), an increase in the velocity gradient tends to increase the species transport, whereas it tends to decrease the mean square fluctuation of the species. The chemical reaction rate is proportional to the mean square fluctuation, as well as to the sqare of the mean, of the species concentration in the present second order reaction. Therefore, an increasing velocity gradient tends to reduce the chemical reaction rate and, at the same time, tends to reduce the residence time by increasing $|<NV>|$. These two tendencies together, therefore, tend to freeze the overall reaction in the flow field as

the velocity gradient is increased.

Figures 4 through 7 show that both mean square fluctuations, $<N^2>/n_{0\infty}$ and $<N^2>/n_0$, are drastically changed when there is a nonequilibrium chemical reaction. Inferring from the frozen solution, Eqs. (40) and (41), we see that $<N^2>$ tends to vary with the square of the mean concentration gradient. The present results show that the variations with respect to Y of the mean square fluctuations, $<N^2>/n_{0\infty}$ and $<N^2>/n_0$, qualitatively reflect that of the mean concentration gradient in the chemically nonequilibrium case also. However, the magnitudes of changes in these fluctuations caused by the chemical reaction are rather surprising.

Let us consider the recombination process given by the present chemical kinetics γn^2 where γ is a negative constant. The equilibrium state as $\Gamma \to -\infty$ is $n = 0$, as can be seen from Eqs. (35) and (36). Therefore, in that limit, $<N^2>$, as well as n_0, vanishes. Therefore, in the upper region of the present flow field, say $Y > .75$, $<N^2>$ increases as $|\Gamma|$ is first increased. The continuous increase in $|\Gamma|$, however, will eventually cause $<N^2>$ to pass through a maximum and will cause it to decrease toward zero. For the other values of Y, on the other hand, the chemical reaction causes a monotonic decrease of $<N^2>$.

No general conclusion, of course, can be drawn from the present limited study. We may say, however, that the chemical reaction will affect $<N^2>$ nonuniformly with respect to the Damkohler number and the flow field.

VI. CONCLUDING REMARKS

A simplified statistical theory was initiated in the previous paper[5] which can be employed to analyze the chemically reacting, turbulent shear flows. This theory, which was self-containing only up to the dissipation

function, is developed in the present work until it is completely self-containing for the molecular Prandtl and Schmidt numbers of order one.

This self-contained theory is first employed to analyze the homologous flow and chemical-species concentration fields in the absence of chemical reaction. The effect of chemical reaction on the concentration field is then enalyzed.

The relationship between the Reynolds stress and the mean velocity gradient obtained in the present analysis is compared to those experimentally measured at the center of the Couette flows, and behind a nonuniform grid. The present theoretical results agree closely with Reichardt's data[7] of Couette flow. With an appropriate extimate of the characteristic length, the present results also agree satisfactorily with the nonuniform grid data of Rose.[6]

From the analysis of the chemically frozen concentration field, the ratio of the mass to momentum diffusivities has been determined to be 4/3. The diffusivities are defined in accordance with the conventional phenomenological theory. This ratio is only for the homologous flow and concentration fields. The turbulent transport, as well as the Reynolds stress, will vary from the homologous case when the fields are inhomogeneous. A second order nonequilibrium chemical recombination is considered in the present study. The reaction creates an inhomogeniety in the concentration field such that the mean square fluctuation of concentration varies nonuniformly with respect to the Damkohler number and flow field.

In certain region of the flow field, the fluctuation initially increases with the Damkohler number. With the continuous increase in the Damkohler number, the fluctuation will pass through a maximum and then will decrease toward the chemically equilibrium state which, for the present particular chemical

kinetics, is zero. In the other flow regions, on the other hand, the fluctuation decreases monotonically as the Damkohler number is increased.

ACKNOWLEDGEMENT

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18.   Only conceptually because, in reality, ther is no shear layer of infinite extent.

19.   The distribution functions for the two streams are Maxwellian with

respect to the mean velocities u_{o1} and u_{o2}, respectively.

20. The values of n for the two streams are functions of the absolute values of $\vec{U}$ because both the flow and the concentration fields are isotropic.

21. It can be readily shown that $\langle N^2 \rangle = \int F^{(2)} \, d\vec{U} - \left[\int F^{(1)} \, d\vec{U} \right]^2 = 0$ if n is constant.

FIGURE CAPTIONS

Fig. 1.   Homologous Flow and Concentration Fields of Infinite Extent

Fig. 2.   Homologous Flow at the Center of Couette Flow
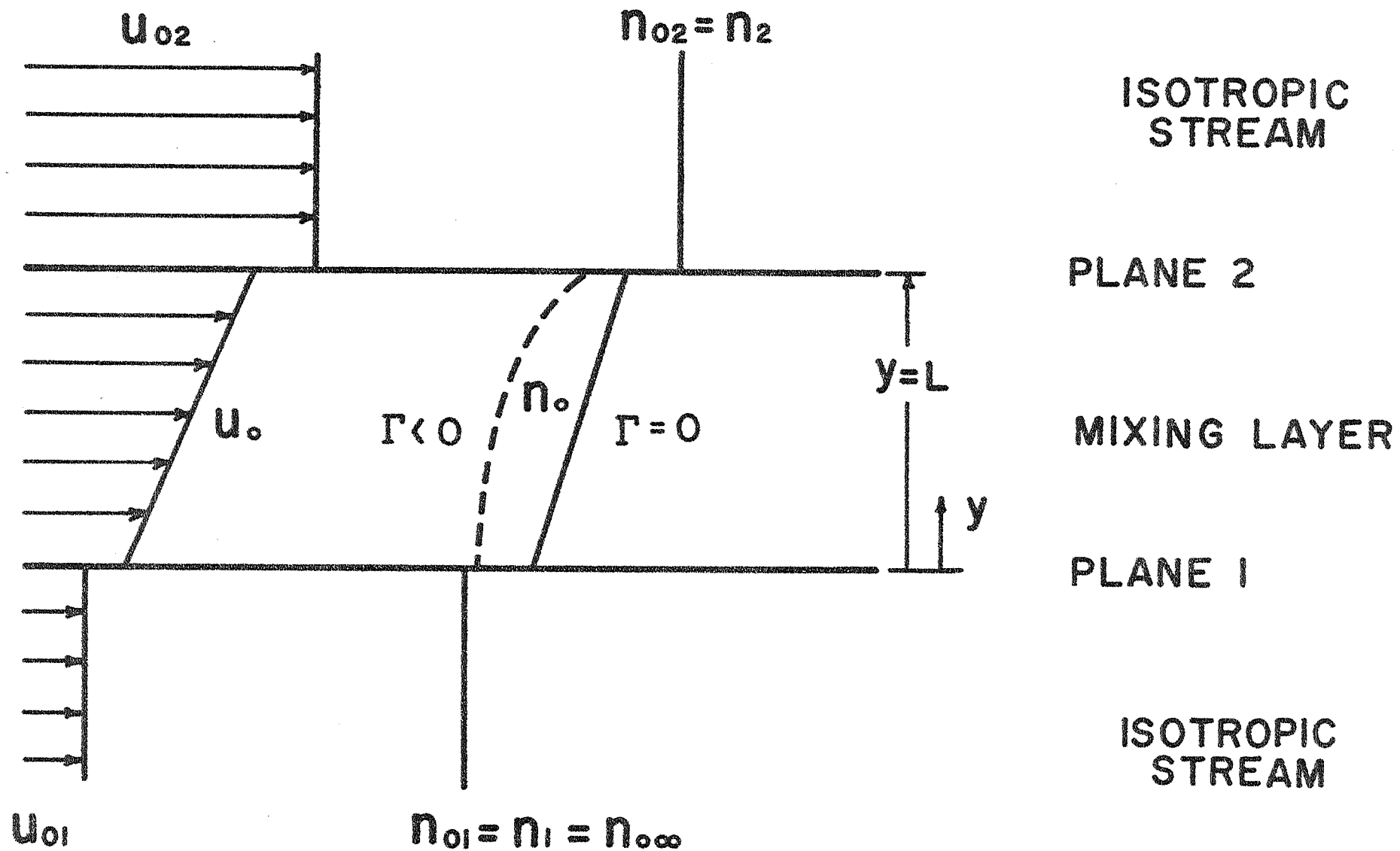
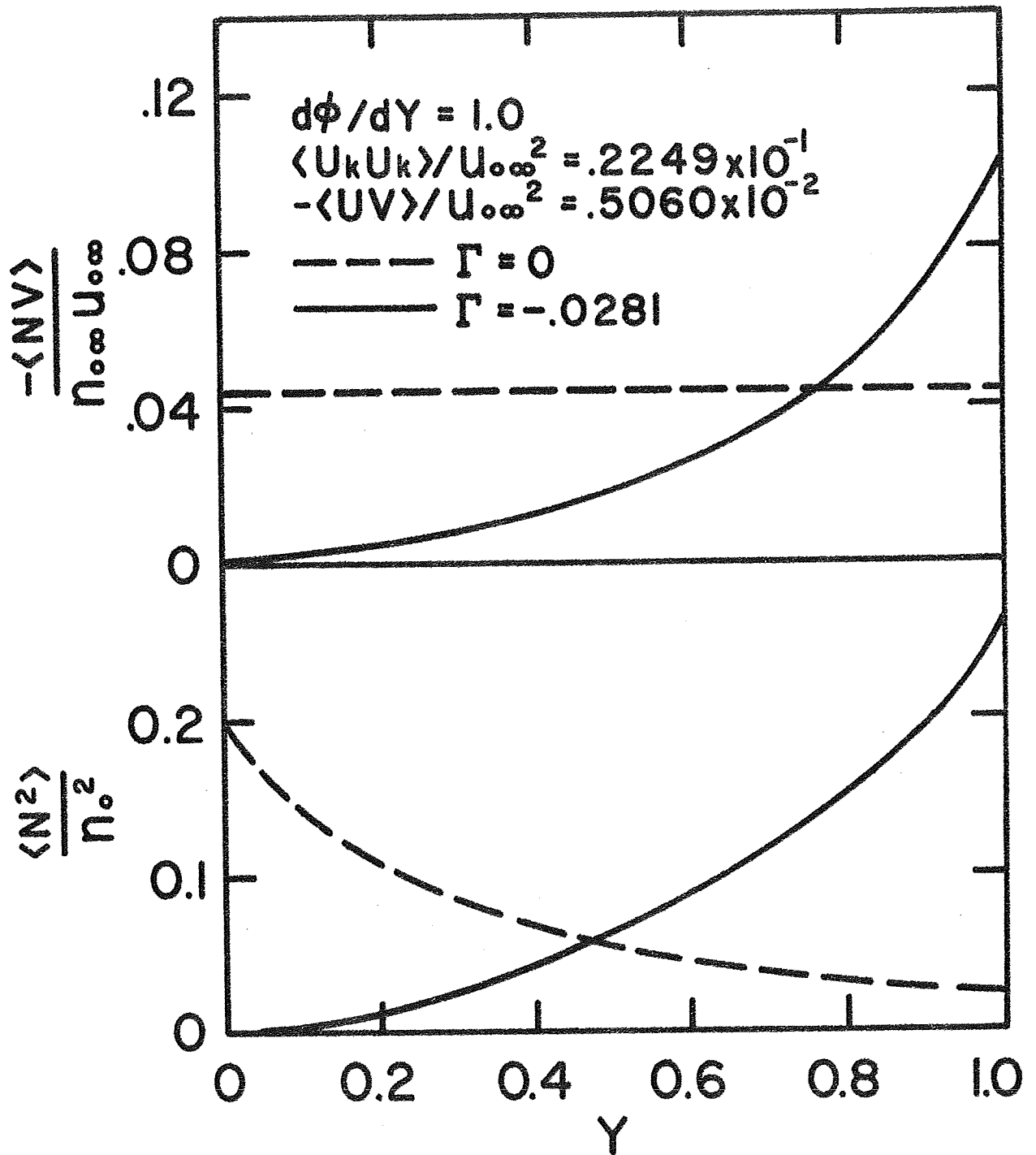Fig. 3.   Shear and Mixing Layers Formed Between Two Isotropic Streams

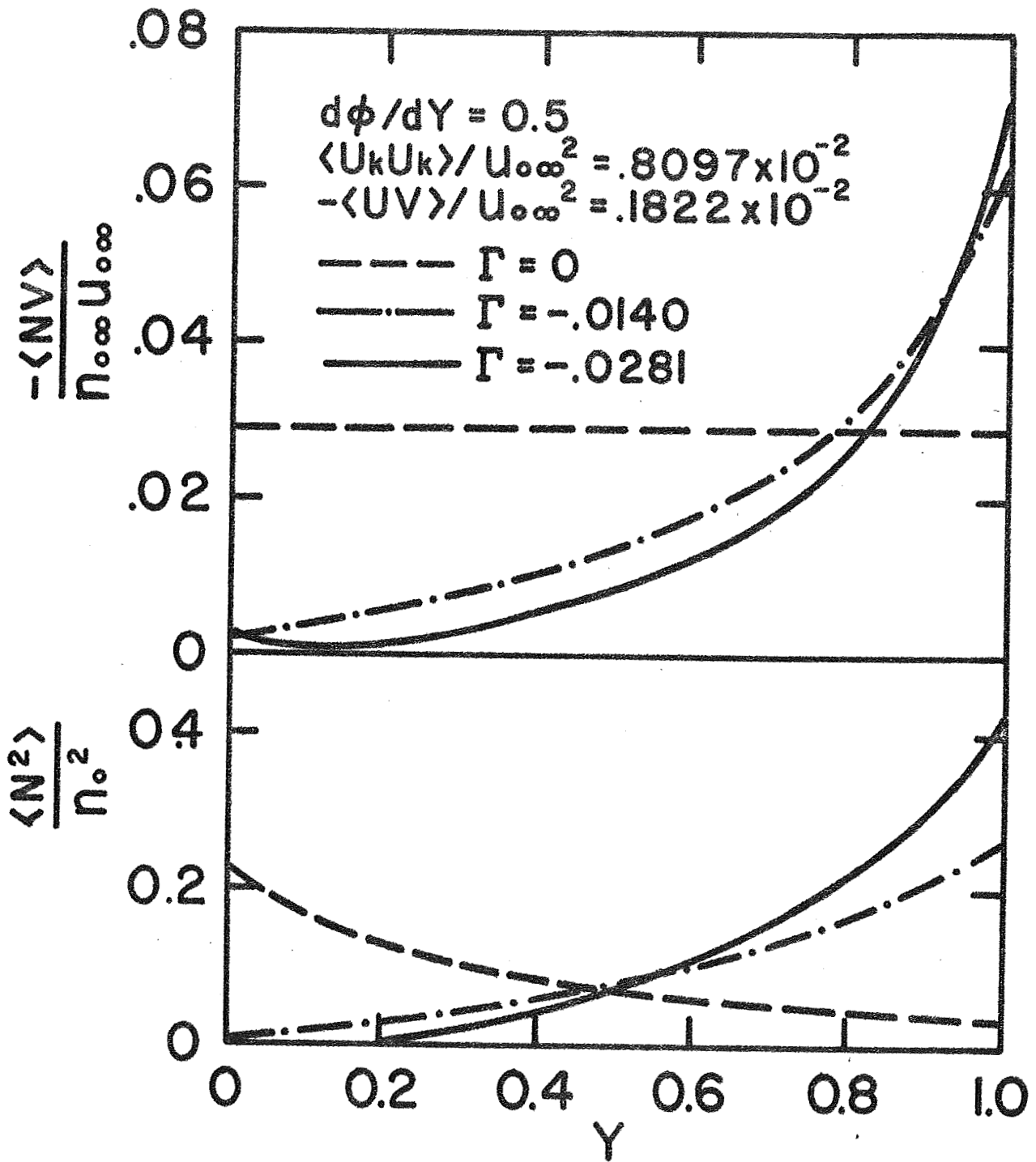Fig. 4.   Effect of Chemical Reaction on Concentration Field

Fig. 5.   Effect of Chemical Reaction on Concentration Field

Fig. 6.   Effect of Chemical Reaction on Concentration Field

Fig. 7.   Effect of Chemical Reaction on Concentration Field
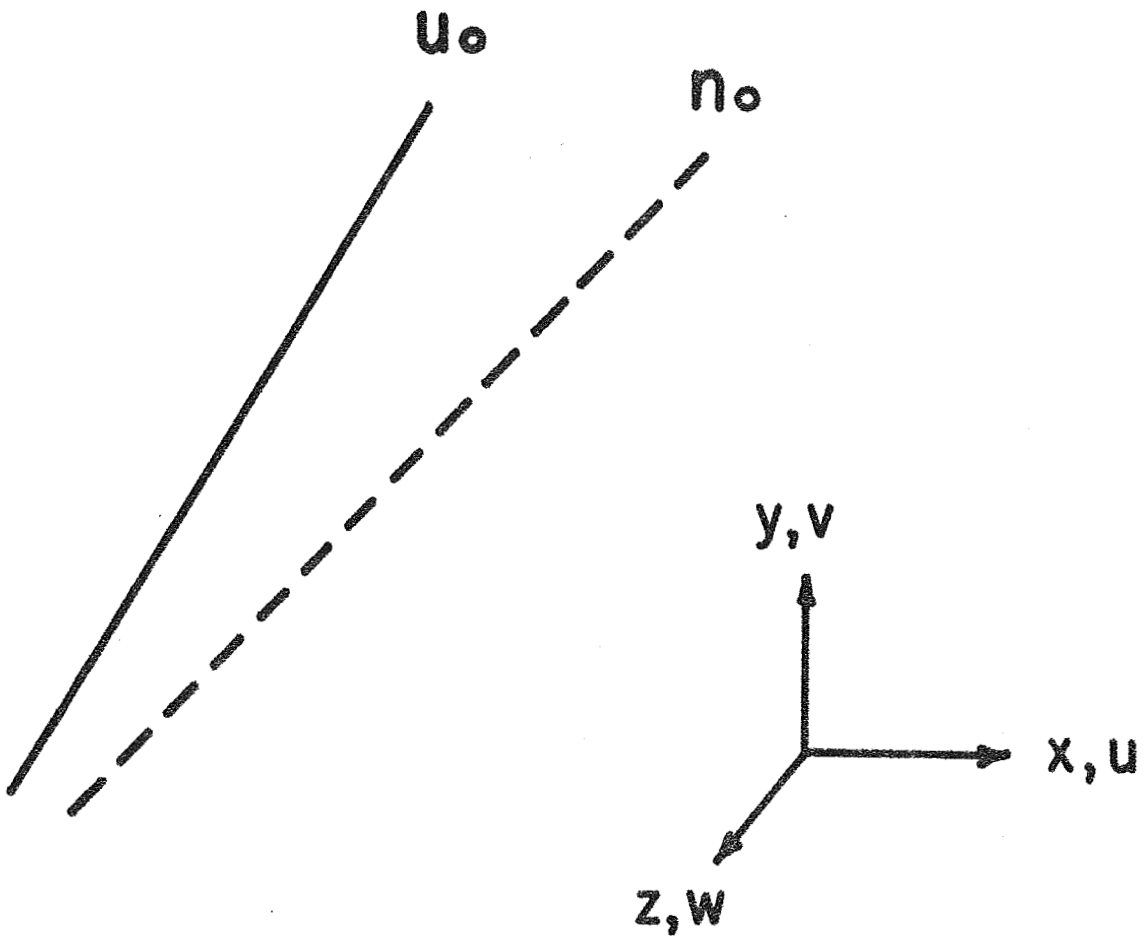
FIG. 1   Homologous Flow and Concentration Fields of Infinite Extent

FIG. 2   Homologous Flow at the Center of Couette Flows

FIG. 3 Shear and Mixing Layers Formed Between Two Isotropic Streams

FIG. 4   Effect of Chemical Reaction on Concentration Field

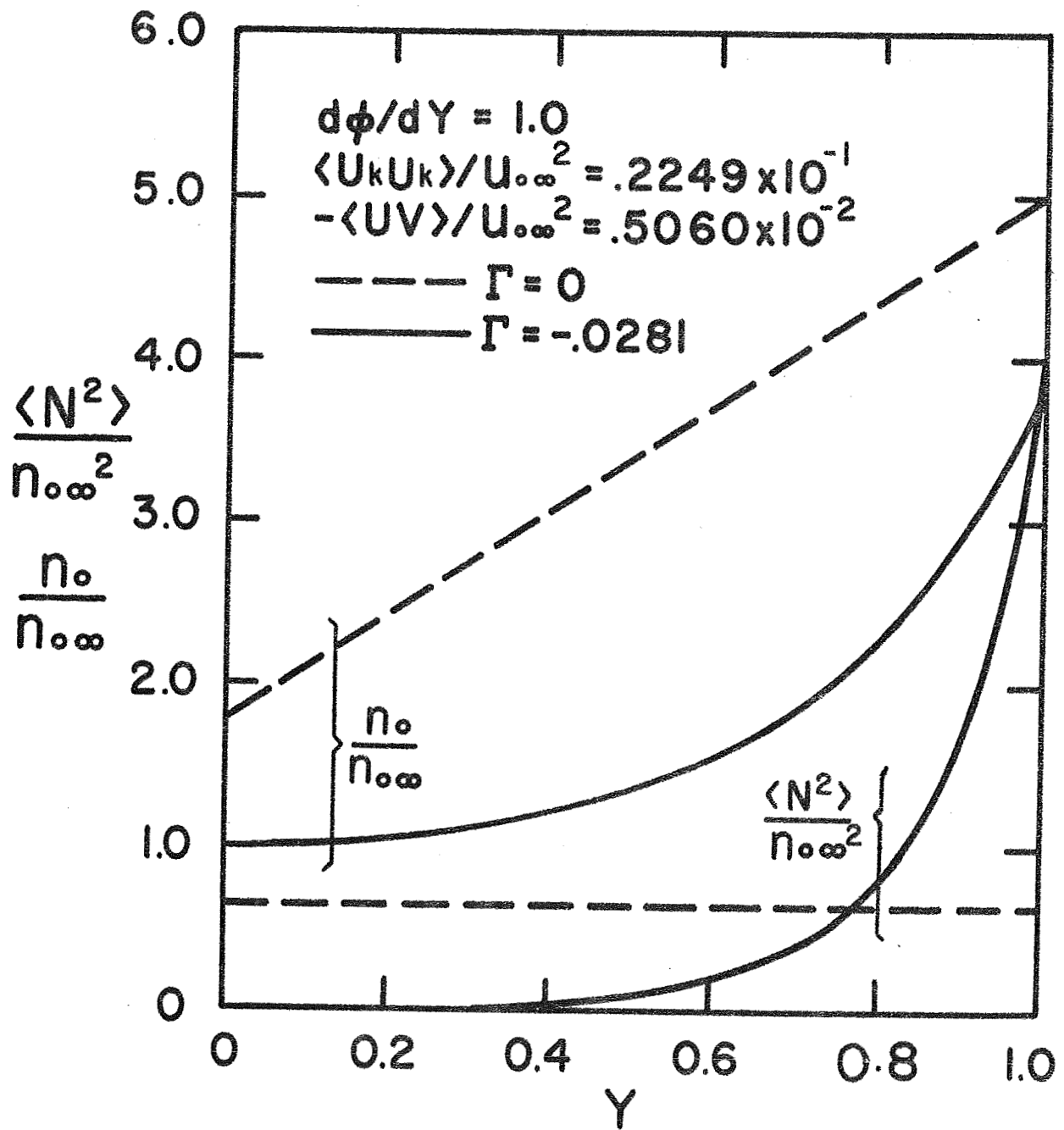FIG. 5   Effect of Chemical Reaction on Concentration Field
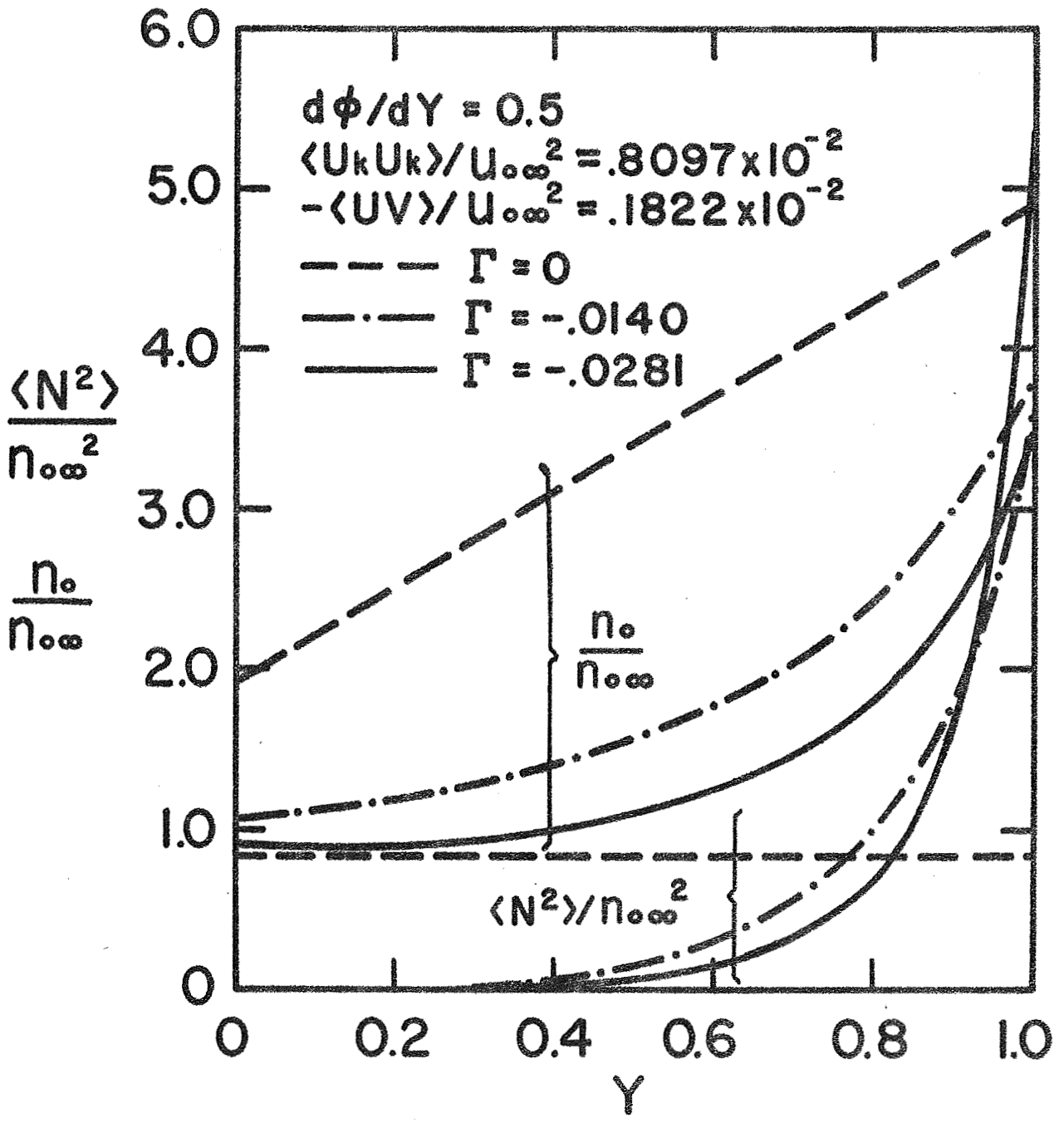
FIG. 6 Effect of Chemical Reaction on Concentration Field

FIG. 7   Effect of Chemical Reaction on Concentration Field