UNIVERSITY OF CONNECTICUT

SCHOOL OF ENGINEERING

STORRS, CONNECTICUT

# DYNAMIC ANALYSIS OF LUNAR GRAVITY SIMULATOR

C. H. Knapp

Department of Electrical Engineering

January 1970



This work has been sponsored by the
National Aeronautics and Space Administration
Research Grant NGL 07-002-002

# Dynamic Analysis of Lunar Gravity Simulator

by

C. H. Knapp

University of Connecticut
Storrs, Connecticut 06268

The control of carriage position and cable tension for the lunar gravity simulator involves time varying parameters, distributed parameters, and non-linear dynamics. Of particular interest is the feasibility of controlling cable tension and position in such a way that vertical and horizontal cable oscillations are kept satisfactorily small. Almost any development of a satisfactory control program will require modeling the cable in such a way that all or most of the states of the model are measurable.

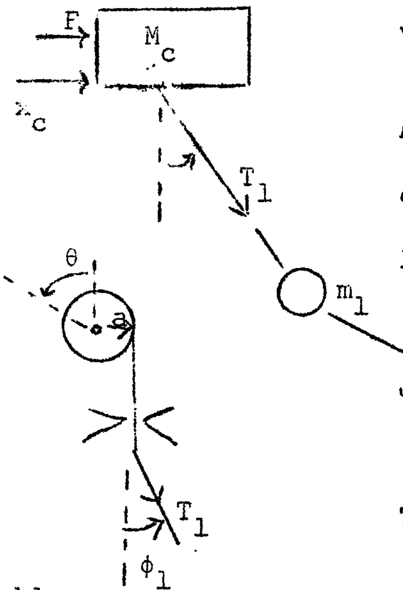
Some preliminary investigation of the modeling problem has been undertaken in which the cable is approximated by a number of point masses connected by springs. Each spring is constrained to move along the axis separating the point masses. This is similar to an approach used in the Kearfott report.* Rather than use a rectangular coordinate system, however, angle from vertical and distance between masses have been used as coordinates. The result is a non-linear system of equations, not requiring the approximations of the Kearfott report, and for which at least two states per mass, (angle and length change) can be measured.

It is intended that techniques will be developed for controlling the approximate model and comparing the performance with that achieved when the same controller is used with an essentially exact model.

* Lunar Landing Research Facility Study, "Kearfott Div. of General Precision Aerospace.

Equations for motion for the carriage, cable, and Module are developed below.

1.) Carriage



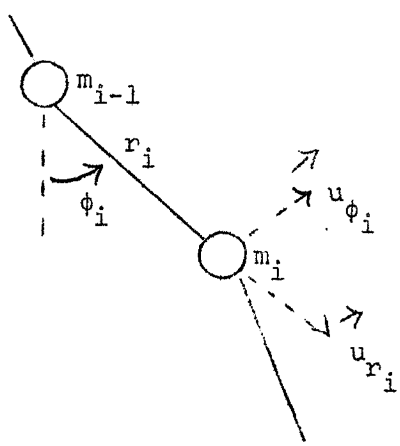

$$v_c = \dot{x}_c, \quad \dot{v}_c = \frac{F}{m_c} + \frac{T_i}{m_c} \sin \phi_1 \qquad (1)$$

Assumptions: 4. Mass of carriage >> mass of cable thus change of $m_c$ with change in cable length is negligible.

$$J \frac{d^2\theta}{dt^2} = T_1 \cos \phi_1 \, a + \text{Torque Developed} - D\frac{d\theta}{dt} \qquad (2)$$

$$T_1 = \frac{r_1 - r_{10}}{c_1} \qquad \text{(see next section)} \qquad (3)$$

2.) Cable

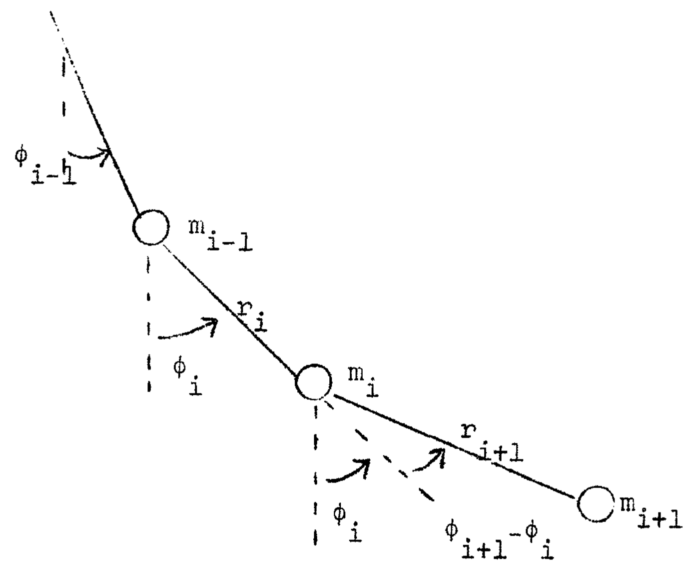


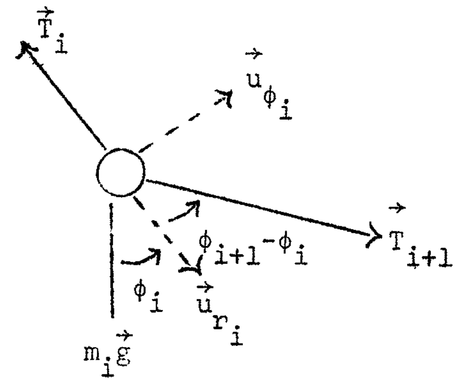

Assumptions: 1. Cable flexible - requires no bending moment. Cable always under tension.

2. Approximated by N segments (last being whiffle tree) of length $r_i$, connecting masses $m_i$.

3. Each segment rigid. Relative motion of $m_i$ with respect to $m_{i-1}$ is limited to two components: $\Delta r_i$ in direction $r_i$, $r_i\dot{\phi}_i$ in direction normal to $r_i$.

Latter assumption as well as other considerations of instrumentation suggest that velocity of $m_i$ be expressed in co-ordinates. $\vec{u}r_i$, $\vec{u}\phi_i$




$$\text{at } m_i: \frac{d}{dt}(m_i\vec{v}_i) = \frac{d}{dt}[m_i(v_{r_i}\vec{u}_{r_i} + v_{\phi_i}\vec{u}_{\phi_i})] = \vec{T}_i + \vec{T}_{i+1}+ m_i\vec{g} \tag{4}$$

$i = 1,2,\ldots n-1$

$$\text{Let } T_i = \frac{1}{c_i}[r_i - r_{i_o}],\ T_{i+1} = \frac{1}{c_{i+1}}[r_{i+1} - r_{(i+1)_0}] \tag{5}$$

$r_{i_0}$ = nominal "unstretched length" of cable.

Expansion of (4) using (5) gives

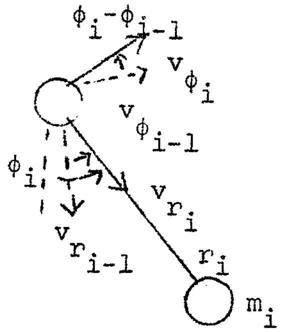
$$\dot{m}_i v_{r_i} + m_i(\dot{v}_{r_i} - \dot{\phi}_i v_{\phi_i}) = m_i g\cos\phi_i + \frac{1}{c_{i+1}}[r_{i+1} - r_{(i+1)_0}] \tag{6}$$

$$\cos(\phi_{i+1} - \phi_i) - \frac{1}{c_i}[r_i - r_{i_0}] \qquad i = 1,2,\ldots n-1$$

$$\dot{m}_i v_{\phi_i} + m_i(\dot{v}_{\phi_i} + \dot{\phi}_i v_{r_i}) = -g\sin\phi_i + \frac{1}{c_{i+1}}[r_{i+1} - r_{(i+1)_0}]\ \sin(\phi_{i+1}-\phi_i) \tag{7}$$

$i = 1,2,\ldots n-1$

(6), (7) give two differential equations at $m_i$, involving 4 unknowns $v_{r_i}$, $v_{\phi_i}$, $\phi_i$, $r_i$. Remaining equations, (2), are obtained by expressing $v_{r_i}$, $v_{\phi_i}$ in terms of corresponding components of $\vec{v}_{i-1}$ plus relative velocity terms as discussed in assumption 3. Thus

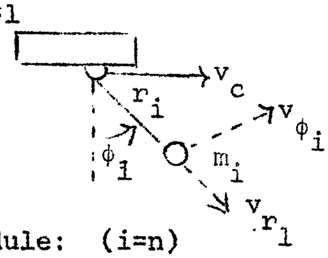



$$v_{\phi_i} = r_i\dot{\phi}_i + v_{\phi_{i-1}}\cos(\phi_i - \phi_{i-1}) - v_{r_{i-1}}\sin(\phi_i - \phi_{i-1}) \quad (8)$$

$$v_{r_i} = \dot{r}_i + v_{r_{i-1}}\cos(\phi_i - \phi_{i-1}) + v_{\phi_{i-1}}\sin(\phi_i - \phi_{i-1}) \quad (9)$$

$i = 2, \ldots n$

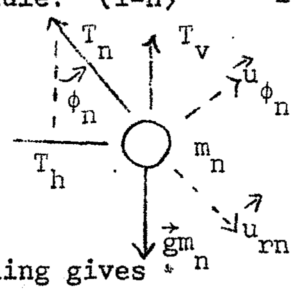
For i=1




$$v_{\phi_1} = r_1\dot{\phi}_1 + v_c \cos\phi_1 \quad (10)$$

$$v_{r_1} = \dot{r}_1 + v_c \sin\phi_1 \quad (11)$$

At module: (i=n)




Assume horizontal ($T_h$ and vertical ($T_v$) components of thrust. For thrusting vehicle:

$$m_n \frac{d\vec{v}_n}{dt} = m_n\vec{g} + \vec{\text{Thrust}} + \vec{T}_n$$

Expanding gives

$$m_n[\dot{v}_{r_n} - \dot{\phi}_n v_{\phi_n}] = m_n g\cos\phi_n - \frac{1}{c_n}[r_n - r_{n_0}] + Th\sin\phi_n - Tv\cos\phi_n \quad (12)$$

$$m_n[\dot{v}_{\phi_n} + \dot{\phi}_n v_{r_n}] = -gm_n\sin\phi_n + Th\cos\phi_n + Tv\sin\phi_n \quad (13)$$

Summarizing (1) - (13)

$$\dot{v}_{r_i} - \phi_i v_{\phi_i} = g\cos\phi_i + \frac{\cos(\phi_{i+1} - \phi_i)}{m_i c_{i+1}}[r_{i+1} - r_{(i+1)_0}] - \frac{1}{m_i c_i}[r_i - r_{i_0}] - \frac{\dot{m}_i v_{r_i}}{m_i} \quad (A)$$

$$\dot{v}_{\phi_i} + \dot{\phi}_i v_{r_i} = -g\sin\phi_i + \frac{1}{c_{i+1}}[r_{i+1} - r_{(i+1)_0}]\sin(\phi_{i+1} - \phi_i) - \frac{\dot{m}_i}{m_i}v_{\phi_i} \quad i = 1, 2, \ldots n-1 \quad (B)$$

$$\dot{v}_{r_n} - \dot{\phi}_n v_{\phi_n} = g\cos\phi_n - \frac{1}{m_n c_n}[r_n - r_{n_0}] + \frac{Th}{m_n}\sin\phi_n - \frac{Tv}{m_n}\cos\phi_n \quad (AN)$$

$$\dot{v}_{\phi_n} + \dot{\phi}_n v_{r_n} = -g\sin\phi_n + \frac{Th}{m_n}\cos\phi_n + \frac{Tv}{m_n}\sin\phi_n \quad (BN)$$

$$v_{\phi_i} = r_i\dot{\phi}_i + v_{\phi_{i-1}}\cos(\phi_i - \phi_{i-1}) - v_{r_{i-1}}\sin(\phi_i - \phi_{i-1}) \quad (C)$$

$$v_{\phi_i} = \dot{r}_i + v_{r_{i-1}}\cos(\phi_i-\phi_{i-1})+v_{\phi_{i-1}}\sin(\phi_i-\phi_{i-1}) \qquad (D)$$

$$v_{\phi_1} = r_1\dot{\phi}_1+v_c\cos\phi_1 \qquad (C1)$$

$$v_{r_1} = \dot{r}_1 + v_c\sin\phi_1 \qquad (D1)$$

$$\dot{v}_c=F/m_c+\frac{T1}{m_c}\sin\phi_1 = \frac{F}{m_c} + \frac{r_1-r_{10}}{c_1 m_c} \quad \sin\phi_1 \qquad (E)$$

$$J\frac{d^2\theta}{dt^2} = - \frac{[r_1-r_{10}]}{c_1} \cdot \text{ a } \cos\phi_1+ \text{Torque developed-Viscous dumping} \qquad (F)$$

Note that $m_n$ changes with time in a known way as fuel is burned. IF as cable plays out $r_{i_0}$, $m_{i_0}$, $c_i$ all change proportional to increase in nominal length $[-a\theta]$, then $r_{i_0} = r_i^0[1-a\theta/\ell_0]$, $m_i=m_i^0\,[1-a\theta/\ell_0]$, $c_i= c_i^0[1-a\theta/\ell_0]$

$$\ell =\sum_i r_{i_0} = \sum_i r_i\,[1-a\theta/\ell_0] = \ell_0-a\theta, \ \ell_0=\text{length with } \theta=0.$$

The equilibrium state found for one set of conditions satisfying $\dot{v}_{r_c} = \dot{v}_{\phi_i} = \dot{r}_i= \dot{\phi}_i=\dot{v}_c=\dot{\theta}=\dot{m}_i=0$ is by examination of (A)-(F)

$$\phi_i=0, \ i=1,\ldots n \qquad (14)$$

$$\frac{r_i-r_{i_0}}{c_i} = m_i g + \frac{r_{i+1}-r_{i_0}}{c_{i+1}}, \quad \frac{r_n-r_{n_0}}{c_n}= m_n g-Tv(>0) \qquad (15)$$

$$Th=0 \qquad (16)$$

$$v_{\phi_i} = v_{\phi_{i=1}}, \ v_{\phi_1} = v_c= \text{constant} = v_0; \text{ or } v_{\phi_i} = v_0 \qquad (17)$$

$$v_{r_i} = v_{r_{i-1}}, \ v_{r_1}=0, \text{ or } v_{r_i}=0 \qquad (18)$$

$$F=0, \ \frac{r_1-r_{1_0}}{c_1} \text{ a } = \text{Torque development} \qquad (19)$$

Solution of (15) gives

$$\frac{r_i - r_{i_0}}{c_i} = g \sum_{j=i}^{n} m_i - T_v \qquad (20)$$

Now assign $r_{i_e}$ to solution of above. Note $\frac{r_{i_e} - r_{i_0}}{c_i}$ = [Effective Weight] x g Below

Nominally we set $T_v = m_n g/6$. Thus

$$T_{i_e} = \frac{r_{i_e} - r_{i_0}}{c_i} = g \sum_{j=i}^{n-1} m_j + 5 m_n g/6$$

Note if $m_n/6 >> \sum_{i=1}^{n-1} m_i$ [i.e. cable whiffle tree weight much less that 5/6 module weight]

then to a good approximation

$$T_{i_e} = \frac{r_{i_e} - r_{i_0}}{c_i} = 5 m_n g/6 \quad \text{(This is only the equilibrium value)}$$

Linearizing (A)-(F) about the equilibrium point, i.e. about

$$\dot{v}_{r_i} = \dot{v}_{\phi_i} = \dot{\phi}_i = \phi_i = \dot{r}_i = \dot{m}_i = T_h = F = v_{r_i} = 0,\ v_{\phi_i} = v_0 = v_c,\ T_v = m_n g/6$$

$$r_i = r_{i_e} - r_{i_0} + c_i \left[g \sum_{j=i}^{n} m_j - m_n g/6\right] \qquad i \neq 1$$

gives

$$\dot{v}_{r_i} - v_0 \dot{\phi}_i = \frac{1}{m_i c_{i+1}} [r_{i+1} - r_{i+1_e}] - \frac{1}{m_i c_i} [r_i - r_{i_e}] \qquad i=1,\ldots n-1 \qquad (A)$$

$$\dot{v}_{\phi_i} = -g\phi_i + \frac{(r_{(i+1)_e} - r_{(i+1)_0})}{m_i c_{i+1}} [\phi_{i+1} - \phi_i] - \frac{\dot{m}_i}{m_i} v_0 \qquad i=1,\ldots n-1 \qquad (B)$$

$$v_{\phi_i} - v_0 = v_{\phi_{i-1}} - v_0 + r_{i_e} \dot{\phi}_i \qquad i=2,n \qquad (C)$$

$$v_{r_i} = v_{r_{i-1}} + v_0[\phi_i - \phi_{i-1}] + \dot{r}_i \qquad i=2,n \qquad (D)$$

$$(v_{\phi_1} - v_0) = r_{1_e} \dot{\phi}_1 + (v_c - v_0), \ \text{(D1)}. \quad v_{r_1} = v_0 \phi_1 + \dot{r}_1 \qquad (C1)$$

$$\dot{v}_{r_n} - \dot{\phi}_n v_0 = -\frac{1}{m_n c_n}(r_n - r_{n_e}) - \frac{1}{m_n} (T_v - m_n g/6) \qquad (An)$$

$$\dot{v}_{\phi_n} = -g\phi_n + \frac{T_h}{m_n} + \frac{g}{6}\ \phi_n = -\frac{5}{6}g\phi_n + \frac{T_h}{m_n} \qquad \text{(Bn)}$$

$$\dot{v} = \frac{F}{m_c} + \frac{\phi_1}{m_c c_1}\ (r_{ie} - r_{i_0}) \qquad \text{(E)}$$

$$J\frac{d^2\theta}{dt^2} = -\ \frac{a}{c_1}\ [r_1 - r_{1e}] + \Delta \text{ Torque developed- Viscous damping} \qquad \text{(F)}$$

Note: It may be more meaningful to consider equilibrium with $\dot{\phi} = \dot{m}_i =$ constant.

For convenience let

$$\Delta r_i = r_i - r_i\ ,\ \Delta r_{i_0} = r_{ie} - r_{i_0},\ \Delta T_v = T_v - m_n g/6$$

$$\Delta v_{\phi_i} = v_{\phi_i} - v_0,\ \Delta v = v_c - v_0$$

And re-examine (A)-(F)

$$\dot{v}_{r_i} = v_0\dot{\phi}_i + \frac{\Delta r_{i+1}}{m_i c_{i+1}} - \frac{\Delta r_i}{m_i c_i} \qquad i=1,\ldots.n-1 \qquad \text{(A)}$$

$$\Delta\dot{v}_{\phi_i} = -g\phi_i + \frac{\Delta r_{(i+1)_0}}{m_i c_{i+1}}\ [\phi_{i+1} - \phi_i] - \frac{\dot{m}_i}{m_i}\ v_0 \qquad i=1,\ldots n-1 \qquad \text{(B)}$$

$$\Delta v_{\phi_i} = \Delta v\phi_{i-1} + r_{ie}\ \dot{\phi}_i,\ i=2,\ldots n;\ \Delta v_{\phi_1} = r_{1e}\dot{\phi}_1 + \Delta v \qquad \text{(C)}$$

$$v_{r_i} = v_{r_{i-1}} + v_0\ [\phi_i - \phi_{i-1}] + \Delta\dot{r}_i,\ i=2,\ldots n;\ v_{r_1} = v_0\phi_1 + \Delta\dot{r}_1 \qquad \text{(D)}$$

$$\dot{v}_{r_n} = \dot{\phi}_n v_0 - \frac{\Delta r_n}{m_n c_n} - \frac{1}{m_n}\ \Delta T_v \qquad \text{(An)}$$

$$\Delta\dot{v}_{\phi_n} = -\ \frac{5}{6}g\phi_n + T_h/m_n \qquad \text{(Bn)}$$

$$\Delta\dot{v} = F/m_c + \frac{\phi_1}{m_c c_1}\ \Delta r_{1_0} \qquad \text{(E)}$$

$$J\frac{d^2\theta}{dt^2} = -\frac{a}{c_1}\ \Delta r_1 + \text{Torque} - D\frac{d\theta}{dt} \qquad \text{(F)}$$

Now $v_{r_i}$, $v_{\phi_i}$ are readily eliminated. One way to proceed is to use (C) to obtain

$$\Delta v_{\phi_i} = r_{ie}\dot{\phi}_i + r_{(i-1)e}\dot{\phi}_{i-1} + \ldots r_{1e}\dot{\phi}_1 + \Delta v = \sum_{k=1}^{i} r_{ke}\,\dot{\phi}_k + \Delta v \quad i=1,2,\ldots n \tag{C1}$$

Likewise from (D)

$$v_{r_i} = \Delta\dot{r}_i + v_0[\phi_i - \phi_{i-1}] + \Delta\dot{r}_{i-1} + v_0[\phi_{i-1} - \phi_{i-2}] + \ldots \Delta\dot{r}_2 + v_0[\phi_2 - \phi_1] + \Delta\dot{r}_1 + v_0\phi_1 \tag{D1}$$

$$= \sum_{k=1}^{i} \Delta\dot{r}_k + v_0\phi_i \qquad i=1,2,\ldots n$$

Now combining (B), (C1) gives

$$\sum_{k=1}^{i} r_{ke}\ddot{\phi}_k + \Delta\dot{v} = -g\phi_i + \frac{\Delta r_{(i+1)_0}}{m_i c_{i+1}}(\phi_{i+1} - \phi_i) - \frac{\dot{m}_i v_0}{m_i} \quad i=1,2,\ldots n-1 \tag{B1}$$

while (A), (D1) give

$$\sum_{k=1}^{i} \Delta\ddot{r}_k = \frac{\Delta r_{i+1}}{m_i c_{i+1}} - \frac{\Delta r_i}{m_i c_i} \qquad i=1,2,\ldots n-1 \tag{A1}$$

(c1), (Bn) yield

$$\sum_{k=1}^{n} r_{ke}\ddot{\phi}_k + \Delta\dot{v} = -\frac{5}{6}g\phi_n + T_h/m_n \tag{B1n}$$

(D1), (An) give

$$\sum_{k=1}^{n} \Delta\ddot{r}_k = \frac{\Delta r_n}{m_n c_n} - \Delta T_v/m_n \tag{A1n}$$

These four equations can be used to replace (A), (B), (An), (Bn) and illustrate that $\phi$, $r$ have become uncoupled. A more convenient form involving only a single second derivative for $\ddot{\phi}$, $\Delta\ddot{r}$ can be obtained by subtracting i-1 equation from i equation in each case. This gives

$$r_{ie}\ddot{\phi}_i = -g[\phi_i - \phi_{i-1}] + \frac{\Delta r_{(i+1)_0}}{m_i c_{i+1}}(\phi_{i+1} - \phi_i) - \frac{\Delta r_{i_0}}{m_{i-1} c_i}(\phi_i - \phi_{i-1}) + \left[\frac{\dot{m}_{1-1}}{m_{i-1}}\right] v_0 \tag{1-i}$$

$$\ddot{\phi} = -\left[g + \frac{\Delta r_{(i+1)_0}}{m_i c_{i+1}} + \frac{\Delta r_{i_0}}{m_{i-1} c_i}\right] \frac{\phi_i}{r_{ie}} + \left[\frac{\Delta r_{(i+1)_0}}{r_{ie} m_i c_{i+1}}\right] \phi_{i+1}$$

$$+\left[g + \frac{\Delta r_{i_0}}{m_{i-1} c_i}\right] \frac{\phi_{i-1}}{r_{ie}} + \frac{v_0}{r_{ie}} \left[\frac{\dot{m}_{i-1}}{m_{i-1}} - \frac{\dot{m}_i}{m_i}\right] \qquad i=2,\ldots n-1$$

$$\ddot{\phi}_1 = -\frac{\Delta\dot{v}}{r_{1e}} - \left[g + \frac{\Delta r_{2_0}}{m_1 c_2}\right] \frac{\phi_1}{r_{1e}} + \frac{\Delta r_{2_0}}{m_1 c_2 r_{1e}} \phi_2 - \frac{\dot{m}_1}{m_1} v_0 \qquad (1\text{-}1)$$

$$\ddot{\phi}_n = \left[-\frac{5}{6} g\phi_n + T_h/m_n + g\phi_{n-1} - \frac{\Delta r_{n_0}}{m_{n-1} c_n} (\phi_n - \phi_{n-1}) + \frac{\dot{m}_{n-1}}{m_{n-1}} v_0\right] \frac{1}{r_{ne}}$$

$$= \left(g + \frac{\Delta r_{n_0}}{m_{n-1} c_n}\right) \frac{1}{r_{ne}} \phi_{n-1} - \left[\frac{5}{6} g + \frac{\Delta r_{n_0}}{m_{n-1} c_n}\right] \frac{\phi_n}{r_{ne}} + \frac{T_h m_n}{r_{ne}} + \frac{\dot{m}_{n-1} v_0}{m_{n-1} r_{ne}} \qquad (1\text{-}n)$$

Likewise

$$\Delta\ddot{r}_i = \frac{\Delta r_{i+1}}{m_i c_{i+1}} - \frac{\Delta r_i}{m_i c_i} - \frac{\Delta r_i}{m_{i-1} c_i} + \frac{\Delta r_{i-1}}{m_{i-1} c_{i-1}} \qquad (2\text{-}i)$$

$$= \frac{\Delta r_{i-1}}{m_{i-1} c_{i-1}} - \left[\frac{1}{m_i} + \frac{1}{m_{i-1}}\right] \frac{\Delta r_i}{c_i} + \frac{\Delta r_{i+1}}{m_i c_{i+1}} \qquad i=2,\ldots n-1$$

$$\Delta\ddot{r}_1 = \frac{\Delta r_2}{m_1 c_2} - \frac{\Delta r_1}{m_1 c_1} \qquad (2\text{-}1)$$

$$\Delta\ddot{r}_n = -\frac{\Delta r_n}{m_n c_n} - \frac{\Delta T_v}{m_n} - \frac{\Delta r_n}{m_{n-1} c_n} + \frac{\Delta r_{n-1}}{m_{n-1} c_{n-1}} \qquad (2\text{-}N)$$

$$= \frac{\Delta r_{n-1}}{m_{n-1} c_{n-1}} - \left[\frac{1}{m_n} + \frac{1}{m_{n-1}}\right] \frac{\Delta r_n}{c_n} - T_v/m_n$$

These equations may also be obtained as follows:

From (D)

$$\Delta\ddot{r}_i = \dot{v}_{r_i} - \dot{v}_{r_{i-1}} - v_0 \left[\dot{\phi}_i - \dot{\phi}_{i-1}\right]$$

Whereas (A) gives

$$\dot{v}_{r_i} - \dot{v}_{r_{i-1}} = v_0\,[\dot{\phi}_i - \dot{\phi}_{i-1}] + \frac{\Delta r_{i+1}}{m_i c_{i+1}} - \frac{\Delta r_i}{m_i c_i} - \frac{\Delta r_i}{m_{i-1} c_i} + \frac{\Delta r_{i-1}}{m_{i-1} c_{i-1}}$$

Combining these gives (2-i)

From (C)

$$r_{ie}\,\ddot{\phi}_i = \Delta\dot{v}_{\phi_i} - \Delta\dot{v}_{\phi_{i-1}}$$

whereas (B) gives

$$\Delta\dot{v}_{\phi_i} - \Delta\dot{v}_{\phi_{i-1}} = -g[\phi_i - \phi_{i-1}] + \left(\frac{\dot{m}_{i-1}}{m_{i-1}} - \frac{\dot{m}_i}{m_i}\right) v_0 + \frac{\Delta r_{(i+1)_0}}{m_i c_{i+1}} (\phi_{i+1} - \phi_i) - \frac{\Delta r_{i_0}}{m_{i-1} c_i} (\phi_i - \phi_{i-1})$$

The latter two combine to yield (1-i).

From equilibrium conditions (see (15) page 5)

$$\frac{\Delta r_{i_0}}{m_i c_i} = g + \frac{\Delta r_{(i+1)_0}}{m_i c_{i+1}}, \quad \frac{5}{6} g = \frac{\Delta r_{n_0}}{c_n m_n}$$

Thus (1-1), (1-i), (i-n) can be simplified to give

$$\ddot{\phi} = -\frac{\Delta r_{i_0}}{c_i}\left[\frac{1}{m_i} + \frac{1}{m_{i-1}}\right]\frac{\phi_i}{r_{ie}} + \frac{\Delta r_{(i+1)_0}}{r_{ie} m_i}\,\frac{\phi_{i+1}}{c_{i+1}} + \frac{\Delta r_{(i-1)_0}}{m_{i-1} c_{i-1}}\,\frac{\phi_{i-1}}{r_{ie}} \qquad (1\text{-}i)$$

$$+ \frac{v_0}{r_{ie}}\left[\frac{\dot{m}_{i-1}}{m_{i-1}} - \frac{\dot{m}_i}{m_i}\right] \quad i=2,\ldots n-1$$

$$\ddot{\phi}_n = \frac{\Delta r_{(n-1)_0}}{m_{n-1} c_{n-1}}\,\frac{\phi_{n-1}}{r_{ne}} - \frac{\Delta r_{n_0}}{c_n r_{ne}}\left[\frac{1}{m_n} + \frac{1}{m_{n-1}}\right]\phi_n + \frac{T_h m_n}{r_{ne}} + \frac{m_{n-1} v_0}{m_{n-1} r_{ne}} \qquad (1\text{-}n)$$

$$\ddot{\phi}_1 = -\frac{\Delta\dot{v}}{r_{ie}} - \frac{\Delta r_{1_0}}{m_1 c_1 r_{1e}}\,\phi_1 + \frac{\Delta r_{2_0}}{m_1 c_2 r_{1e}}\,\phi_2 - \frac{\dot{m}_1}{m_1} v_0 \qquad (1\text{-}1)$$

Thus in general

$$\Delta\ddot{r}_i = c_{ii-1}\Delta r_{i-1} + c_{ii}\Delta r_i + c_{ii+1}\Delta r_{i+1} + d_i \Delta T_v \qquad (2\text{-}i)$$

where $c_{i,i-1} = \frac{1}{m_{i-1}c_{i-1}}$ , $c_{ii} = -\frac{1}{c_i}\left[\frac{1}{m_i} + \frac{1}{m_{i-1}}\right]$, $c_{ii+1} = \frac{1}{m_i c_{i+1}}$ (4)

$$c_{10} = 0,\ c_{n,n+1} = 0,\ d_i = 0,\ i \neq n,\ d_n = -\frac{1}{m_n}$$

also

$$\ddot{\phi}_{ii} = a_{ii-1}\phi_{i-1} + a_{ii}\phi_i + a_{ii+1}\phi i+1 + \frac{v_0}{r_{ie}}\left[\frac{\dot{m}_{i-1}}{m_{i-1}} - \frac{\dot{m}_i}{m_i}\right] \qquad i=2,\ldots n \qquad (1\text{-}i)$$

$$a_{i,i-1} = \frac{\Delta r_{(i-1)_0}}{m_{i-1}c_{i-1}r_{ie}} = \frac{\Delta r_{(i-1)_0}}{r_{ie}}\ c_{ii-1}$$

$$a_{ii} = -\frac{\Delta r_{i_0}}{c_i r_{ie}}\left[\frac{1}{m_i} + \frac{1}{m_{i-1}}\right] = \frac{\Delta r_{i_0}}{r_{ie}}\ c_{ii}$$

$$a_{i,i+1} = \frac{\Delta r_{(i+1)_0}}{r_{ie}m_i c_{i+1}} = \frac{\Delta r_{(i+1)_0}}{r_{ie}}\ c_{ii+1} \qquad i=2,\ldots n-1$$

Simplifications:

If 5/6 weight of module, $m_n 5/6g$ greatly exceeds cable weight (as it does in lunar simulator), equilibrium tension throughout cable (except possibly for whiffle tree) is same.

That is

$$\frac{\Delta r_{i_0}}{c_i} = T_{ie} = \underbrace{\frac{5}{6}m_n g}_{\text{module}} + \underbrace{m_{n-1}g}_{\text{whiffle tree}} = T,\ i=1,\ldots n-1$$

$$r_{ie} = r_e,\ i=1,2,\ldots n-1$$

If cable lengths are equal, except for $r_n$ regardless of total length $\ell$,

$$c_i = c,\ i=1,\ldots n-1$$

$$m_i = m,\ i=1,\ n-2$$

If $m_{n-1}$ = whiffle tree mass, $\dot{m}_{n-1}=0$

In this case :

$$c_{ii} = -\frac{2}{mc}, \; c_{ii-1} = c_{ii+1} = \frac{1}{mc} \qquad i=2,n-2$$

$$c_{n-1,n-1} = -\frac{1}{c}\left[\frac{1}{m} + \frac{1}{m_{n-1}}\right], \; c_{nn} = -\frac{1}{c_n}\left[\frac{1}{m_{n-1}} + \frac{1}{m_n}\right]$$

$$c_{n,n+1} = 0, \; c_{n,n-1} = \frac{1}{m_{n-1}c_{n-1}}, \; c_{n-1,n} = \frac{1}{m_{n-1}c_n}$$

$$c_{11} = -\frac{1}{mc}, \; c_{10} = 0$$

$$a_{ii} = -\frac{2T}{mr_e}, \; a_{ii-1} = \frac{T}{mr_e} \qquad i=2,n-2$$

$$a_{11} = -\frac{T}{mr_e}, \; a_{10}=0$$

$$a_{n-1n-1} = -\frac{T}{r_e}\left[\frac{1}{m} + \frac{1}{m_{n-1}}\right], \; a_{nn} = -\frac{\Delta r_{n_0}}{c_n r_{ne}}\left[\frac{1}{m_n} + \frac{1}{m_{n-1}}\right]$$

$$a_{n-1,n} = \frac{\Delta r_n}{r_e m_{n-1} c_n}, \; a_{n,n+1} = 0$$

$$\ddot{\phi}_i = -\frac{2T}{mr_e}\phi_i + \frac{T}{mr_e}\phi_{i+1} + \frac{T}{mr_e}\phi_{i-1} \qquad i=2,\ldots n-2$$

$$\ddot{\phi}_1 = \frac{\Delta\dot{v}}{r_e} - \frac{T}{mr_e}\phi_1 + \frac{T}{mr_e}\phi_2 - \frac{\dot{m}_1}{m_1}v_0$$

$$\ddot{\phi}_{n-1} = \frac{T}{mr_e}\phi_{n-2} - \frac{T}{r_e}\left[\frac{1}{m} + \frac{1}{m_{n-1}}\right]\phi_{n-1} + \frac{\Delta r_n}{c_n r_e}\;\frac{\phi_n}{m_{n-1}}$$

$$\ddot{\phi}_n = \frac{T}{m_{n-1}r_{ne}}\phi_{n-1} - \frac{\Delta r_{n_0}}{c_n r_{ne}}\left[\frac{1}{m_n} + \frac{1}{m_{n-1}}\right]\phi_n + \frac{T_h m_n}{r_{ne}}$$

$$\Delta\ddot{r}_i = \frac{1}{mc}\Delta r_{i-1} - \frac{2}{mc}\Delta r_i + \frac{\Delta r_{i+1}}{mc} \qquad i=2,\ldots n-2$$

$$\Delta\ddot{r}_1 = -\frac{1}{mc}\Delta r_1 + \frac{\Delta r_2}{mc}$$

$$\Delta\ddot{r}_{n-1} = \frac{\Delta r_{n-2}}{mc} \left[ \frac{1}{m} + \frac{1}{m_{n-1}} \right] \frac{\Delta r_1}{c} + \frac{\Delta r_n}{m_{n-1} c_n}$$

$$\Delta\ddot{r}_n = \frac{\Delta r_{n-1}}{m_{n-1} c} - \left[ \frac{1}{m_n} + \frac{1}{m_{n-1}} \right] \frac{\Delta r_n}{c_n} - \frac{\Delta T_v}{m_n}$$

If cable lengths $r_{i_0}$ are unequal, but $m_n + m_{n-1} >>$ cable mass

$$\frac{\Delta r_{i_0}}{c_i} = T_{ie} = T \qquad i-1, \ldots n-1$$

Also

$\frac{\dot{m}_i}{m_i} = \frac{\dot{r}_{i_0}}{r_{i_0}}$ = constant if $\Delta r_{i_0}$ is proportional to $r_{i_0}$ as cable lengthens. That is, the proportion $\frac{r_{i_0}}{\ell}$ is maintained as $\ell$ changes. Now the last term of (1-i) drops out, $\frac{\Delta r_{i_0}}{c_i} = T$. $\dot{m}_{n-1} = 0$ if whiffle tree weight is constant. Thus many troublesome terms drop out even when cable lengths are unequal.

Constants

| | |
|---|---|
| $w' = .8$ lbs/ft. | linear cable weight |
| $m' = 2.485 \times 10^{-2}$ slugs/ft. | linear cable mass |
| $k = 80000$ lbs/in. | cable spring constant |
| $c = 1/k = 1.25 \times 10^{-3}$ in/lb. | cable spring compliance |
| $t_e = 5/6\ w_n + w_{n-1} = 10{,}340$ lbs. | cable tension at equilibrium |
| $w_{n-1} = 2000$ lbs. | whiffletree weight |
| $m_{n-1} = 62.0$ slugs | whiffletree mass |
| $T_{n-1} = 5/6\ w_n = 8{,}340$ lbs. | whiffletree tension |
| $w_n = 10{,}000$ lbs. | vehicle weight |
| $m_n = 310.0$ slugs | vehicle tension |
| $T_v = 1/6\ m_n = 1{,}660$ lbs. | vertical thrust on vehicle |