NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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POINTING OF THE 16 FOOT ANTENNA

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Technical Memorandum No. NGL-006-69-3

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MILLIMETER WAVE SCIENCES

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Abstract

This paper developes a comprehensive pointing theory for the 16-foot antenna. The pointing of the antenna anywhere in the sky is related to the servo encoder readings through eight error parameters. It is shown that these eight parameters may be determined by an orderly sequence of simple experiments.

Table of Contents

I. Introduction

The position of an object in the sky is usually given in equitorial coordinates. The equitorial coordinate system is a spherical system in which the pole is parallel with the rotational axis of the earth. The equitorial coordinates are declination, δ, and hour angle, t. The declination of an object is the complement of the conventional polar angle. The hour angle of an object is the angle measured in the equitorial plane between the meridian and the projection of the ray to the object into the equitorial plane. This coordinate system is shown in Figure A1.

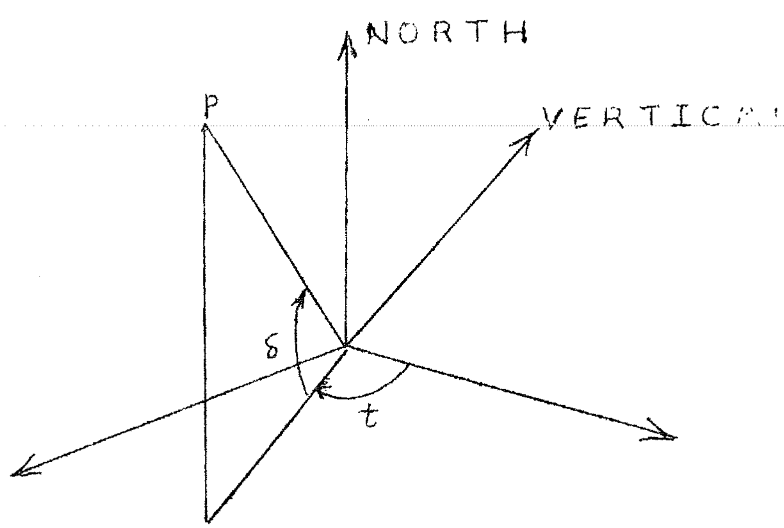


Figure A1

The sixteen foot antenna is supported by a mount which is intended to be an equitorial coordinate system. One axis, the polar axis, is suspended between two piers. This axis is intended to be parallel to the celestial pole. A second axis, the declination axis, is mounted on the polar axis and is perpendicular to it. The antenna is then mounted on the declination axis so that it points in a direction perpendicular to the declination axis. Thus, rotating the structure about the polar axis changes the hour angle of the antenna while rotating the antenna about the declination axis changes the declination of the antenna. The relative angles of these rotations are sensed by servo-encoders and displayed to one-one thousandth of a degree.

II. The Pointing Problem

In determining the antenna pointing for a given celestial object one usually starts with the geocentric coordinates. The geocentric coordinates are the coordinates at which the object would be seen if viewed from the center of the earth. The antenna is located at the surface of the earth, and coordinates referred to the sight of the antenna are called topocentric coordinates. Geocentric and topocentric coordinates differ because of refraction and parallax. However, the servo-encoders do not read topocentric coordinates because their readings are affected by several small but important imperfections in the antenna. These are:

1. Encoder Bias - The angles read by the servo-encoders are only relative angles.
2. Sag - Sag is caused by the elasticity of the spars that support the feed. Since the effect of sag is to move the feed in the vertical plane, it will be associated with refraction and parallax, which also occur in the vertical plane.
3. Orthogonality - This is the group of three errors which are best thought of as errors of orthogonality. These are as follows:

a. The polar axis may not be perpendicular to the equitorial plane.

b. The declination axis may not lie in a plane perpendicular to the polar axis.

c. The antenna beam may not lie in the plane perpendicular to the declination axis.

One can see that the servo-encoder readings differ from the geocentric coordinates of the object by several small but important effects. The purpose of this paper is to give a method with which the servo-encoder readings can be obtained from the geocentric coordinates. First, simple approximate equations are derived that correct the geocentric coordinates for the various errors discussed above. Second, methods are given to measure all of the error parameters. The results of some of these measurements are presented.

III. Correction Equations

Detailed consideration of all the error effects is rather tedious and is given in Appendix AA. The correction equations will be simply stated here with only heuristic justification. Only those terms that are of greatest significance are retained, so that the correction equations should be regarded as approximations. If first order accuracy is not enough, then the exact expressions of Appendix AA can be used. However, unless the errors are large the first order approximation is quite good in most of the sky.

One is interested in the pointing of both the radio frequency beam and the optical telescope mounted on the antenna backup structure. The pointing of these two axes are similar but they do exhibit important differences, thus, there is a pair of equations relevant to each. The radio frequency beam correction equations are

$$D = \delta + \epsilon \cos(t - \varphi) - \{S(E) + Z - R(E)\} \cos \mu + K_{Rf} \quad (1a)$$

$$P = t + [\epsilon \sin(t - \varphi) - \Gamma] \tan \delta - [\{S(E) + Z - R(E)\} \sin \mu + L_{Rf}] \sec \delta + K_p, \quad (1b)$$

while the appropriate equations for the optical telescope are

$$D = \delta + \epsilon \cos(t - \varphi) - \{Z - R_o(E)\} \cos \mu + K_o \quad (2a)$$

$$P = t + [\epsilon \sin(t - \varphi) - \Gamma] \tan \delta - (\{Z - R_o(E)\} \sin \mu + L_o) \sec \delta + K_p \quad (2b)$$

where:

D = Declination servo reading

P = Polar servo reading

δ = geocentric declination

t = geocentric hour angle

$R(E)$ = Refraction correction

$S(E)$ = sag correction. Sag is positive if the beam is rotated toward the vertical.

Z = parallax correction

ϵ = co-declination of polar axis

φ = hour angle of polar axis

Γ = angle declination axis makes with plane perpendicular to polar axis. Γ is positive if the western tip of the declination axis is north of this plane.

L_o = angle telescope axis makes with plane perpendicular to declination axis. L_o is positive if the telescope axis is west of this plane.

L_{Rf} = angle radio frequency beam makes with plane perpendicular to declination axis when the beam is exactly vertical.

K_o = Declination additive constant for optical telescope

K_{Rf} = Declination additive constant for R.f. axis

K_p = Polar encoder additive bias.

μ = angle at the object in the vertical - celestial pole - object spherical triangle. The cosine and sine of this angle are plotted in Figure A2 and A3 for convenient reference.

First let us consider the declination equation term by term. One can see that the declination servo reading must first be corrected for polar axis misalignment. Clearly the error in declination will be maximum when one looks along the hour angle of the polar axis, φ. Likewise it will be zero when one looks at an hour angle perpendicular to φ. The correct interpolation between these two extremes is the cosine law. The term $\{Z - R(E)\} \cos \mu$ is the standard correction for refraction and parallax, and the term L_o or L_{Rf} appears because the servo-encoders only sense relative angles.

The polar equation is only slightly more difficult. The first correction term may be thought of as a declination axis tilt term. From A4 one can see that a tilt in the declination axis will make an arc error on the unit sphere

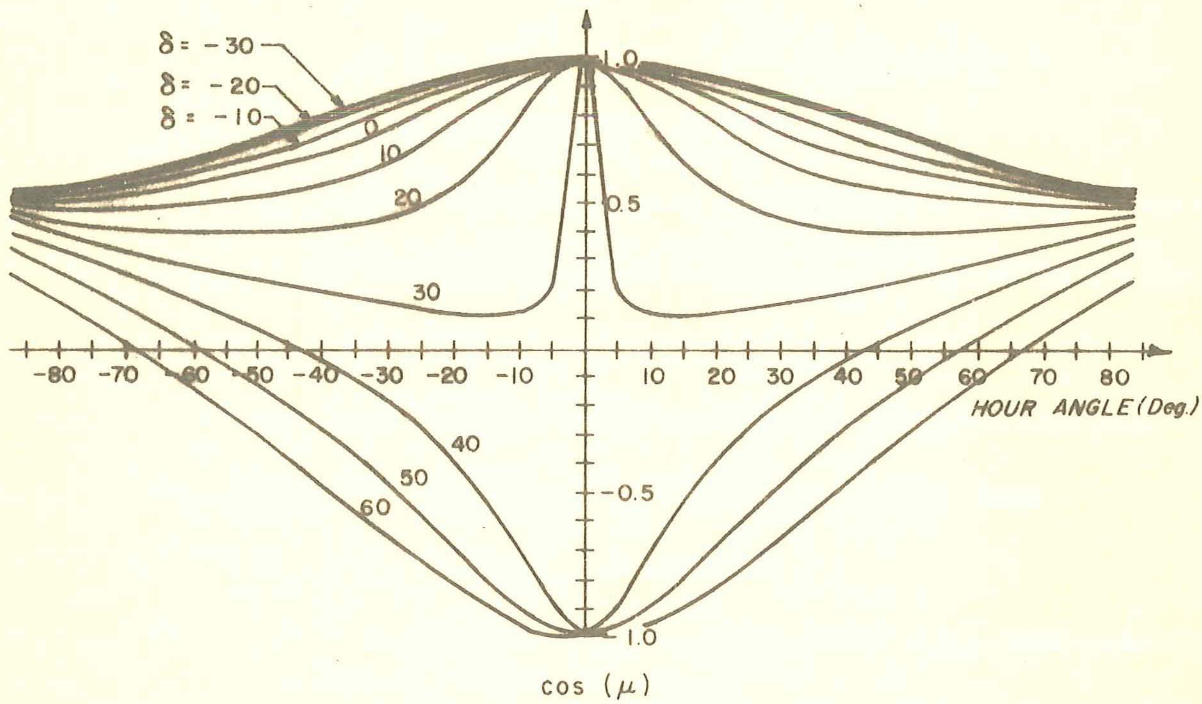

δ = -30
δ = -20
δ = -10
0
10
20
30
40
50
60
1.0
0.5
-0.5
1.0
-80 -70 -60 -50 -40 -30 -20 -10 10 20 30 40 50 60 70 80
HOUR ANGLE (Deg.)
cos (μ)

Fig. A2

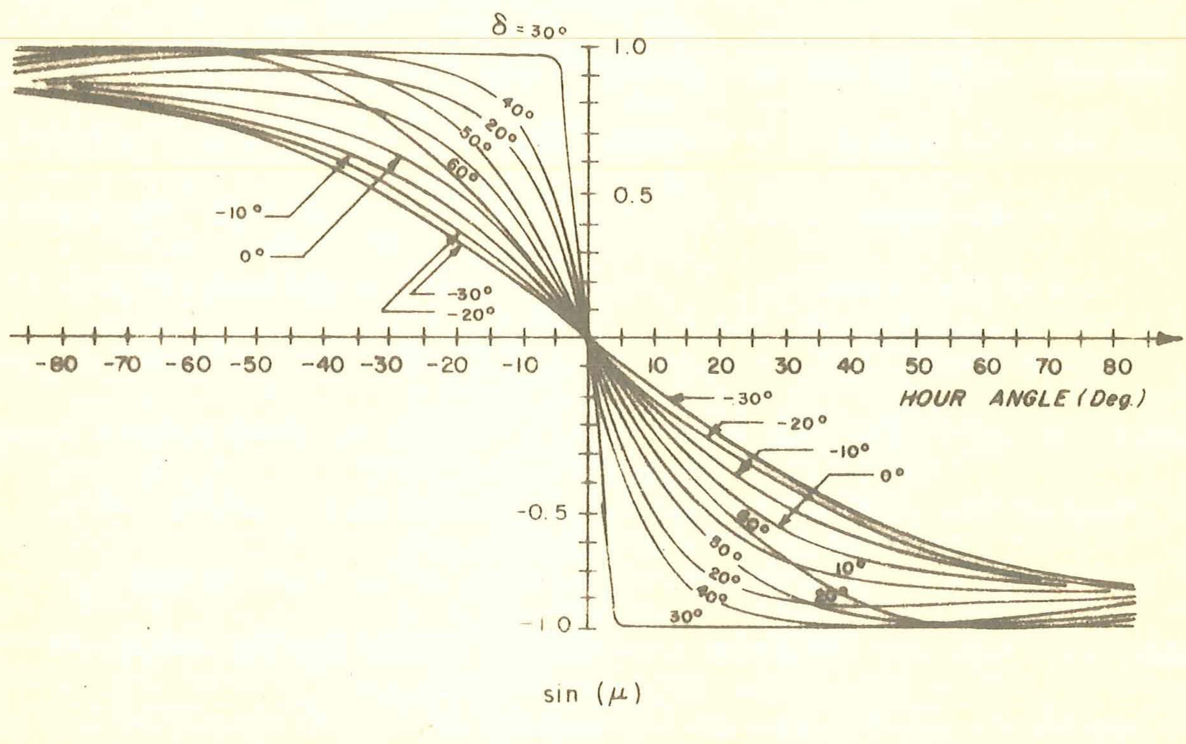

δ = 30°
1.0
0.5
-0.5
-1.0
40°
20°
50°
60°
-10°
0°
-30°
-20°
-80 -70 -60 -50 -40 -30 -20 -10 10 20 30 40 50 60 70 80
HOUR ANGLE (Deg.)
-30°
-20°
-10°
0°
60°
50°
10°
20°
40°
20°
30°
sin (μ)

Fig. A3

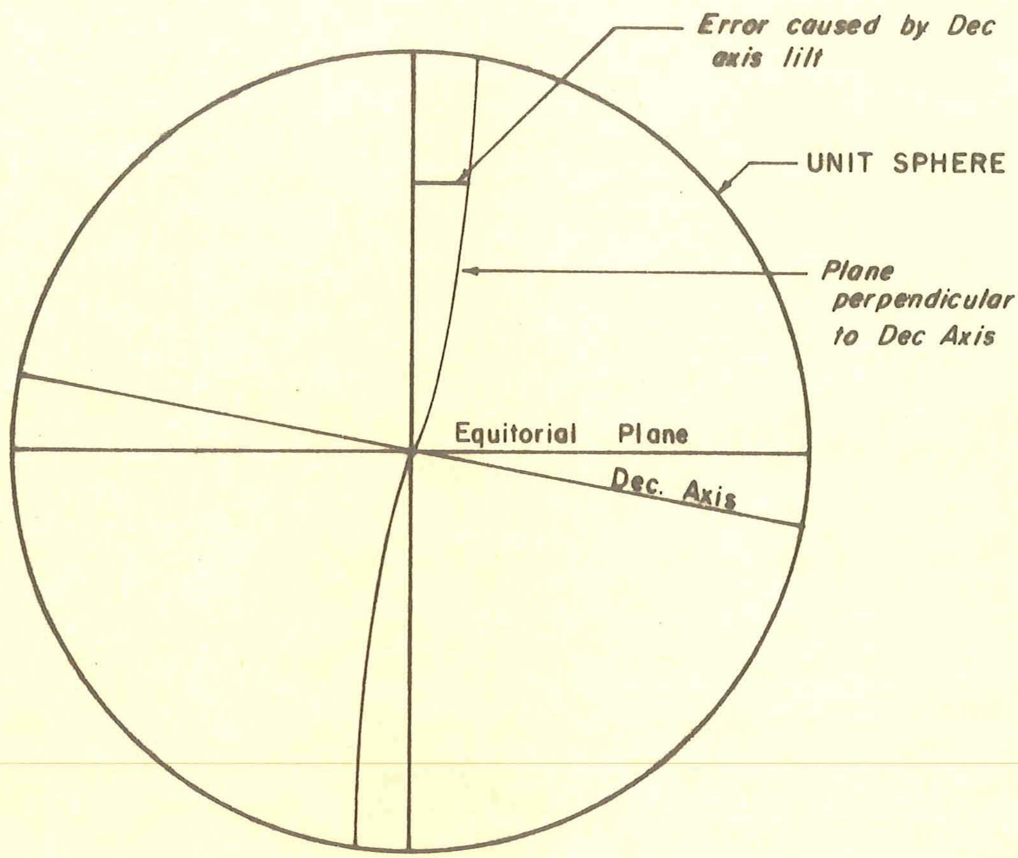

Error caused by Dec axis lilt
UNIT SPHERE
Plane perpendicular to Dec Axis
Equitorial Plane
Dec. Axis

Fig. A4

proportional to sin δ. But this arc angle must be converted to an hour angle by dividing by cos δ. Thus declination axis tilt gives an error in hour angle proportional to the tilt times tan δ. The tilt itself can be seen to arise from two causes. The first is declination axis non-orthogonality, the second is tilt caused by misalignment of the polar axis, which will vary with hour angle. The second term, $\{Z - R_o(E)\}$ sec δ is the standard correction for refraction and parallax. The third term L_o sec δ arises because the telescope direction may not be orthogonal to the declination axis. This gives an error of constant arc on the unit sphere which, when converted to hour angle, becomes L_o sec δ. The constant K_p is merely the encoder bias.

A question naturally arises about which parameters in (1a) and (1b) would change if the feed position were changed. A careful examination of Appendix AA reveals that the only two that change are K_{Rf} and L_{Rf}.

IV. Measurement Methods

All of the error parameters may be measured by simple experiments. The orthogonality errors and encoder biases may be measured by tracking stars with the optical telescopes. The sag and refraction, on the other hand, may be fit to a simple model by observing the sun as it passes through the beam of the telescope. Since the sag and refraction measurement is rather noisy a method is given to estimate the error in the measurement.

A. Star Tracking

The polar axis co-declination and hour angle may be measured by tracking a single star with the optical telescope over a wide range of hour angles. The declination readout as a function of hour angle, $D(t_i)$, is considered the data set for this experiment. Equation (2a) becomes

$$D_i = \delta + \epsilon \cos (t_i - \varphi) + R(E) \cos \mu_i + K_o,$$

where K_o, ϵ, and φ are taken to minimize the mean square error. Specifically,

$$\sum_{i=1}^{n} [\Omega_i - (K_o + \epsilon \cos (t_i - \varphi))]^2 \leq \sum_{i=1}^{n} [\Omega_i - (a + b \cos (t_i - c))]^2$$

for all a, b, and c, where

$$\Omega_i = D_i - \delta - R(E) \cos \mu_i.$$

B. Star Transits

The declination axis and telescope orthogonality errors as well as the polar encoder bias may be measured by an experiment which is complementary to the one above. The polar servo is set at P and the transit time of several stars is observed over a wide range of declinations. When the hour angle of the star agrees with t in Equation (2b), the star has transited the telescope. This will occur when

$$t_i = ST_i - \alpha_i$$

where ST_i is the siderial time of transit and α_i is the right ascension of the star. Substituting this into Equation (2b) yields:

$$P + \alpha_i - ST_i - \epsilon \sin (ST_i - \alpha_i - \varphi) \tan \delta_i - R(E) \sin \mu_i \sec (\delta_i) = -\Gamma \tan \delta_i$$

$$- L_o \sec \delta_i + K_p.$$

Just as in the case above, the constant, Γ, L_o, and K_p, are taken to be the best mean square fit to the data. If the polar servo is set at $P = \varphi$, then the term $\epsilon \sin (ST_i - RA_i - \varphi)$ will tend to drop out. Likewise if P is set to zero then the refraction term will tend to drop out. Therefore, if data is taken at both $P = \varphi$ and $P = 0$ then a check on both theory and experimental technique is provided.

C. Sag and Refraction Models

The theory of optical refraction is well understood. When an optical ray impinges on the earth's atmosphere it is bent in the plane determined by the vertical and the ray to the object. The amount of this bending is usually called refraction and is given to within a second of arc by Garfinkel's Theory. The refraction as given by this theory may be calculated on the digital computer and will be denoted by $R_o(E)$. We shall assume that the radio frequency refraction follows this same law buy may differ from it by a constant multiple $(1 + r)$, i.e.

$$R_{Rf}(E) = (1 + r)\, R_o(E).$$

We shall assume that the sag function is given by

$$S(E) = S \cos (E).$$

Thus, the problem of describing the sag and radio frequency refraction is reduced to finding the two constants S and r.

D. Polar Sun Scans

One way of relating these two parameters is to take scans of the sun. Suppose the polar servo is set and a transit of the sun is observed in both the radio frequency and optical telescopes. At transit the hour angle of the sun agrees with t in Equations (1b) and (2b). This will occur at a siderial time given by

$$ST = \alpha + t.$$

When this relation is substituted into Equation 3b, one obtains

$$P = ST_{Rf} - \alpha_{Rf} + [\epsilon \sin (ST_{Rf} - \alpha_{Rf} - \varphi) - \Gamma] \tan \delta$$

$$- [\{S(E) - R_{Rf}(E) + Z\} \sin \mu + L_{Rf}] \sec \delta + K_p,$$

and the appropriate equation for optical transit is

$$P = ST_o - \alpha_o + [\epsilon \sin (ST_o - \alpha_o - \varphi) - \Gamma] \tan \delta$$

$$- [\{Z - R_o(E)\} \sin \mu + L_o] \sec \delta + K_p.$$

The difference of these two equations is then

$$K (T_{Rf} - T_o) \cos \delta = \{S \cos (E) - r R_o(E)\} \sin \mu + L_{Rf} - L_o,$$

where K may be considered the mean solar rate and $(T_{Rf} - T_o)$ the difference in transit times. For each transit observed the difference in transit times may be considered a data point, so one can write

$$Kd_i \cos \delta_i = \{S \cos (E_i) - r R_o(E_i)\} \sin \mu_i + L_{Rf} - L_o \quad (3a)$$

where,

$$d_i = (T_{Rf} - T_o). \quad (3b)$$

Thus, if the sun is tracked over a wide range of hour angles, then the parameters S, r, and $L_{Rf} - L_o$ are determined with good accuracy by the best mean square fit of the data.

An effort has been made to determine the expected errors in estimating the sag and refraction coefficients. Suppose one takes "n" data points in a day. Each data point consists of an estimate of $T_{Rf} - T_o$ along with the approximate hour angle and declination of the telescope during the scan. Suppose further that the estimate of $T_{Rf} - T_o$ can be modeled with statistics and that the estimate of the transit time difference may be written

$$d_i = T_{Rf} - T_o + X_i$$

where X_i is a sequence of identically distributed random variables with zero mean. Now the estimates of S and r are given by the random variables, S' and r', where S' and r' are the best mean square fit of the data, d_i, i.e.

$$\sum_{i=1}^{n} [d_i K \cos \delta_i - \{S' \cos E_i - r' R(E_i)\} \sin \mu_i - (L_{Rf} - L_o)]^2 \leq$$

$$\sum_{i=1}^{n} [d_i K \cos \delta_i + \{a \cos E_i + b R(E_i)\} \sin \mu_i + c]^2$$

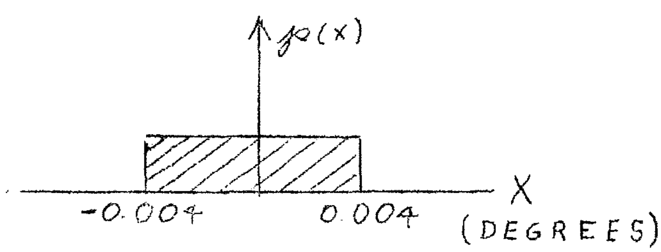
for all a, b, and c. It can be shown that the estimates S' and r' converge with probability one to S and r as n goes to infinity. This is the strongest type of statistical convergence and although the proof is simple it is rather tedious and will not be included here.

However this does not answer the basic question of what errors could be expected from a finite set of data. This can be done to some extent by estimating the variance of S' and r' on the digital computer. This is done by repeatedly calculating S' and r' where the X's are generated by a random noise generator. As might be expected the density functions of S' and r' were Gaussian with zero mean. Table I gives some of these results

Case	n	Elevation Range	Hour Angle Range	$2\sigma(S')$	$2\sigma(r')$
1	18	9.6° - 49°	273° - 320°	.015°	10%
2	40	20° - 75°	280° - 80°	.004°	20%
3	20	2° - 40°	284° - 30°	.003°	2%

Table I

where in each case the density function for X_i is given by



Study of this table shows that a wide range of hour angle and elevation angle is important in determining the sag coefficient and low elevation angle are particularly important in determining the refraction coefficient.

E. Determination of L_{Rf}

Note that L_{Rf} may be determined from $L_{Rf} - L_o$ if L_o is known, but L_o will be known only if the optical telescope has not been moved since the star transit experiment was performed. Since it is often necessary to move the optical telescope in practice, some other method of determining L_{Rf} is needed. This may be done by simply keeping time on the sun scan data. This method has the disadvantage that it requires the calculation of the right ascension of the sun for each data point.

For this experiment the time of transit as a function of polar servo setting, $T(P_i)$, is considered the data set.

Equation (1b), when solved for L_{Rf}, becomes

$$L_{Rf} = (P_i - ST_i + \alpha_i) \cos \delta_i - [\epsilon \sin (ST_i - \alpha_i - \varphi) - \Gamma] \sin \delta_i$$

$$+ [(S + \pi) \cos E_i - (1 + r) R_o(E_i)] \sin \mu_i - K_p \cos \delta_i \qquad (4)$$

where

ST_i = siderial time of transit

α_i = right ascension at transit

π = horizontal Parallax

One can now form an estimate of L_{Rf} by averaging the right hand side of Equation (4), which we shall denote by Q_i

$$L_{Rf} = \frac{1}{n} \sum_{i=1}^{n} Q_i .$$

F. Declination Sun Scans

The sag and refraction coefficients may be obtained from declination scans also. Suppose the antenna is scanned in declination at rate K, i.e,

$$D = D_o + K(T - T_s),$$

and the hour angle of the antenna is made equal to the hour angle of the sun. Then at time T_{Rf} the declination of the radio frequency beam will agree with the declination of the sun. From Equation (1a)

$$D_o + K(T_{Rf} - T_s) = \delta_{sun} + \epsilon \cos (t - \varphi) - \{S(E) + Z - R_{Rf}(E)\} \cos \mu + K_{Rf}.$$

Likewise when the declination of the optical telescope beam agrees with the declination of the sun, one has from Equation (2a)

$$D_o + K(T_o - T_s) = \delta_{sun} + \epsilon \cos (t - \varphi) - \{Z - R_o(E)\} \cos \mu + K_o .$$

Now, if one takes the difference of and substitutes the expression assumed for the sag and refraction function, he obtains

$$K(T_{Rf} - T_o) = \{r\, R_o(E) - S \cos E\} \cos \mu + (Krf - K_o).$$

Thus, the parameters r and S may be obtained by finding the best least square fit to the data where

$$d_i = (T_{rf} - T_{opt})$$

and

$$Kd_i = \{r\, R_o(E_i) - S \cos (E_i)\} \cos \mu_i + (K_{rf} - K_o).$$

G. Determination of K_{Rf}

The same considerations that apply to L_{Rf} also apply to K_{Rf}. Here again it is expedient to use a method that gives this parameter directly. This may be accomplished with Equation (1b), if the sag, refraction, and orthogonality coefficients are known. The servo declination is marked on a record while a declination sun scan is made. From this one determines the declination readout when the radio frequency beam is at the center of the sun. This readout will be denoted by D_i and Equation (1b) becomes

$$K_{Rf} = D_i - \delta_{sun} - \epsilon \cos (t - \varphi) + \{(S + \pi) \cos E - (1 + r) R_o(E)\} \cos \mu.$$

H. Summary of Measurement Methods

At this point it would be helpful to summarize the information given by each of these experiments and inquire into an orderly way to obtain all the parameters for the radio frequency beam. The parameters determined by each experiment, as well as the parameters that must be determined before the experiment can be performed, are summarized in Table II.

Information Given by Pointing Experiment

	ϵ	φ	Γ	L_{Rf}	L_o	K_p	K_o	K_{Rf}	S	r	other
Star Tracking	x	x					x				
Star Transit	o	o	x		x	x					
Polar Sun Scans									x	x	$L_{Rf} - L_o$
Dec Sun Scans									x	x	$K_{Rf} - K_o$
Determine L_{Rf}	o	o	o	x		o			o	o	
Determine K_{Rf}	o	o						x	o	o	

x - Information given by Experiment

o - Information required in data reduction

Table II

From Table II one can see that the experiments determined all the pointing parameters. Furthermore, it is clear that the experiments should be done in the order listed except that the polar and declination sun scans need not both be done, since they give the same information. Actually the parameter L_{Rf} may be determined from the polar sun scan data if time is marked on the record. It is clear that the five experiments; Star Tracking, Star Transits, Polar Sun Scans, Determination of L_{Rf}, and Determination of K_{Rf}, constitute an orderly way of determining all the pointing parameters.

Appendix AA

Derivation of Pointing Equations

It is the purpose of this appendix to derive the polar and declination servo readings from the geocentric coordinates considering the pointing errors discussed in Section II of the paper. The method for doing this will be to first resolve the point given by δ and t into rectangular coordinates. Then we consider a sequence of coordinate system transformations. Each transformation will suppress an angle from consideration. This angle may either be a desired angle generated by the servo-system or an error angle. After the final coordinate transformation, the point on the unit sphere, represented by δ and t, will be a simple basis element, i.e. a (001) vector. Once this exact relationship among the angles is derived, it is shown that the first order equations given in Section II satisfy this relationship.

Suppose that the geocentric hour angle and declination of an object are given by t and δ. We wish to resolve the point on the unit sphere into the coordinate system given in Figure AA1. From simple trigonometry one can see that

$$A_1 = \sin\delta$$

$$A_2 = \cos\delta \cos t$$

$$A_3 = \cos\delta \sin t$$

Now following the method outlined above, the first angle that will be suppressed will be the hour angle of the polar axis. To do this we will transform to coordinates B_1, B_2, and B_3 as shown in Figure AA2. This coordinate transformation may be represented by the matrix equation

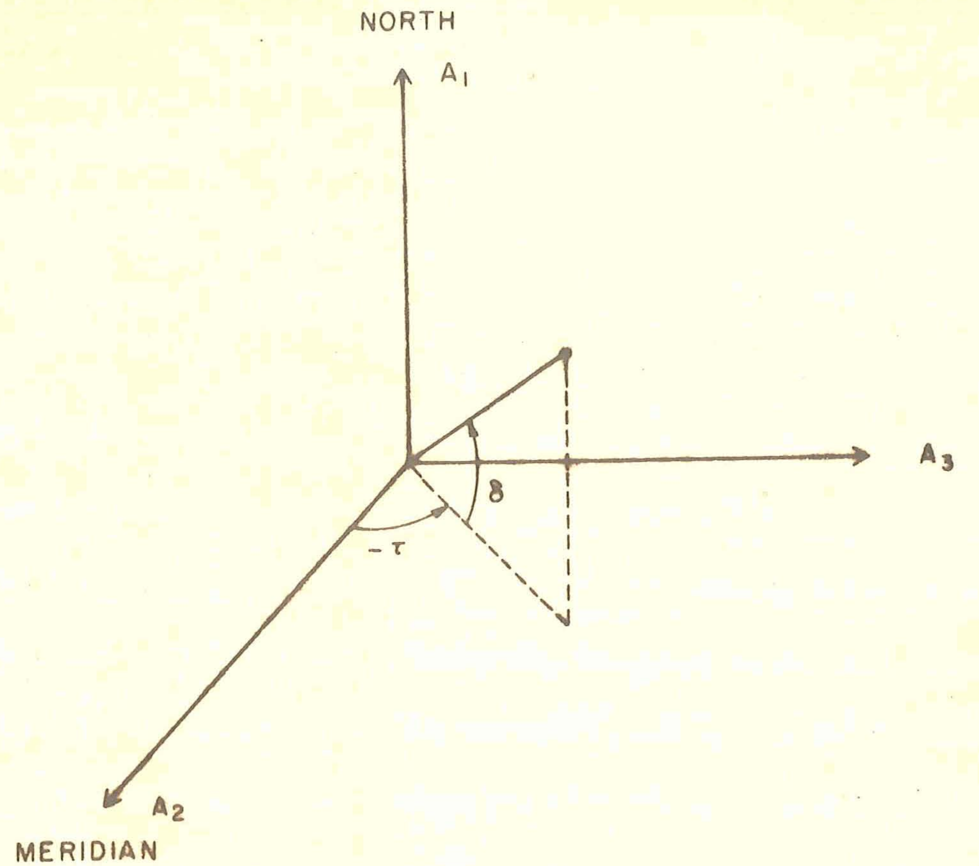


Fig. AA1.

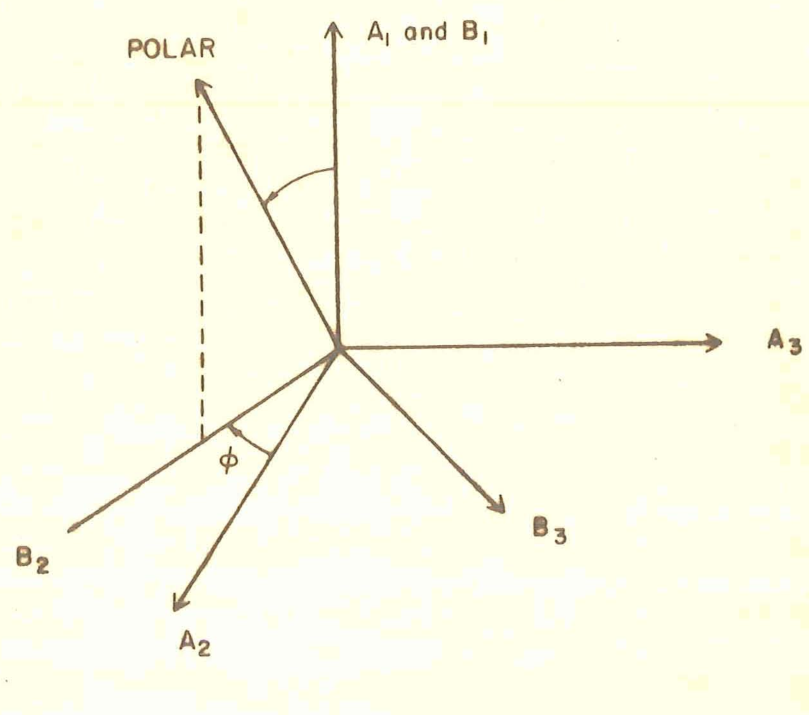


Fig. AA2.

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

The next angle to be suppressed is the polar axis co-declination ϵ. This is accomplished by transforming to C coordinates as shown in Figure AA3.

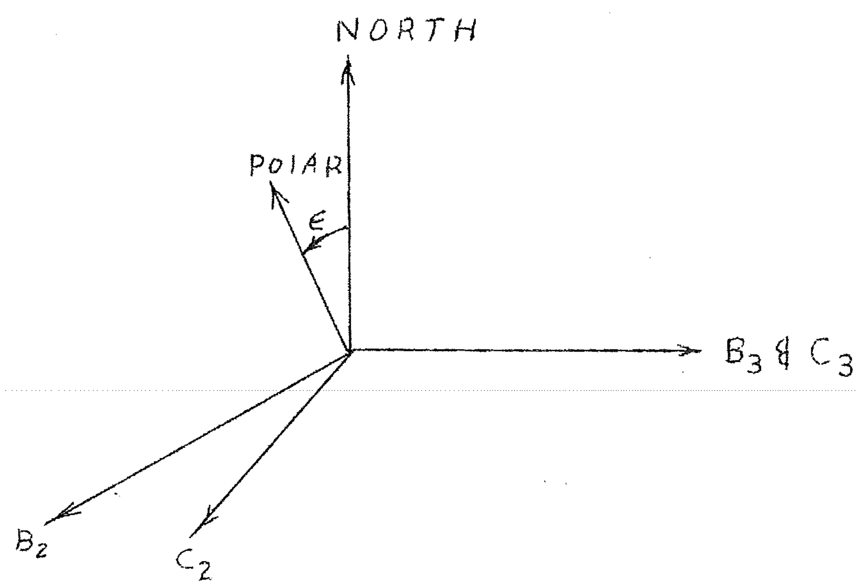


Fig. AA3

This may be represented by the matrix equation

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} \cos\epsilon & \sin\epsilon & 0 \\ -\sin\epsilon & \cos\epsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

Fig. AA4 shows the polar and declination in C coordinates.

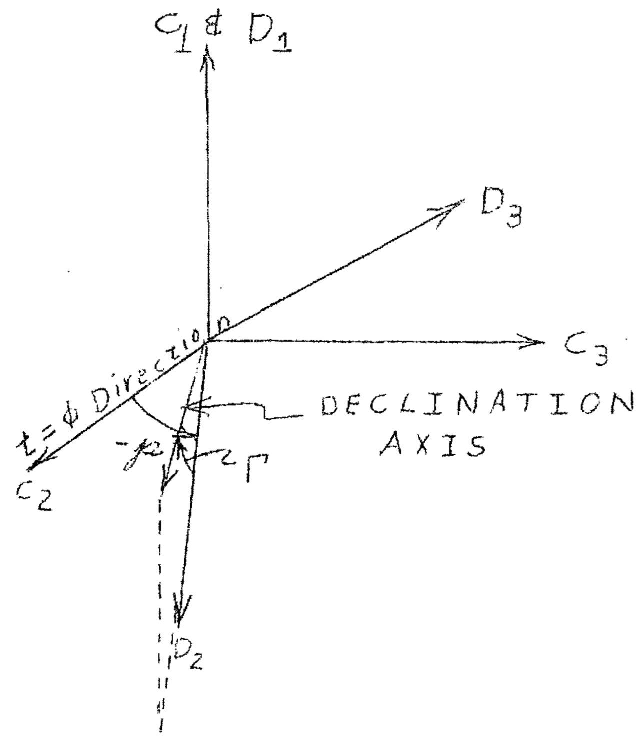


Fig. AA4

The polar axis is coincident with the C_1 direction but the declination axis is not in general in the (C_2, C_3) - plane. Suppose it makes an angle Γ with this plane. The function of the polar servo is to rotate the declination axis about the polar axis. This may be thought of as varying the angle p in Figure AA4.

Following our method let us suppress the angle p by transforming to the D-coordinates. The D-set is related to the C-set by the matrix equation

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-p) & \sin(-p) \\ 0 & -\sin(-p) & \cos(-p) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} .$$

which may be rewritten

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos p & -\sin p \\ 0 & \sin p & \cos p \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} .$$

Now the angle Γ may be suppressed by rotating about the D_3 axis as shown in Figure AA5.

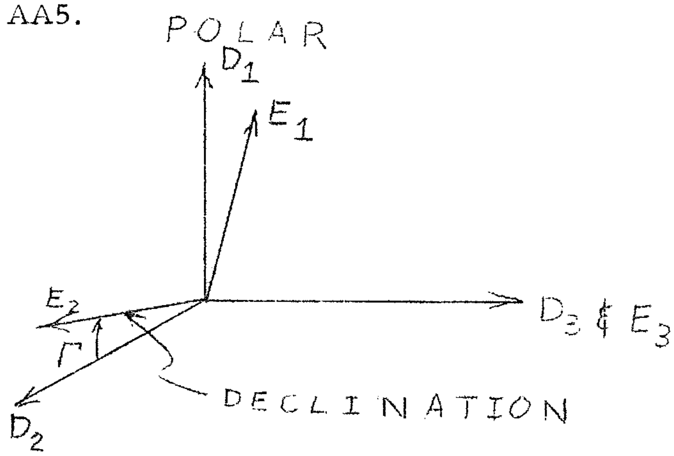


Fig. AA5

The E-coordinates are now related to the D-coordinates by the matrix equation

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \cos\Gamma & \sin\Gamma & 0 \\ -\sin\Gamma & \cos\Gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}.$$

The function of the declination servo system is to rotate the antenna backup structure about the declination axis. In order to speak quantitatively about the position of the backup structure one needs a reference direction for it, i.e. a direction that does not change with respect to the backup structure. This reference direction may be thought of as the direction of an optical telescope mounted on the side of the backup structure. This works well to describe the pointing of the optical telescopes, but it is more convenient to choose a different reference for the radio frequency axis because of the sag effect. However, the pointing equations for both the radio frequency axis and the telescope axes may be derived at the same time with a general reference direction keeping in mind that the constants that depend upon the reference directions are different in the two cases.

Let us denote the reference direction for the radio frequency axis by Q. Q is a direction imbedded in the backup structure that passes through the maximum of the antenna radiation pattern when the maximum is exactly vertical. Then it is assumed that the sag effect is described by saying that the radio frequency beam is rotated an amount S(E) toward the vertical in the plane determined by the vertical and Q. This definition serves to fix the sign of the sag function as well as the condition

$$S(\frac{\pi}{2}) = 0.$$

The direction Q is described by the angles d and L on the declination axis coordinates of Figure AA6. The angle d is, of course, within an additive constant of the angle readout by the declination servo encoder and L is the angle of Q direction makes with the plane perpendicular to the declination axis.

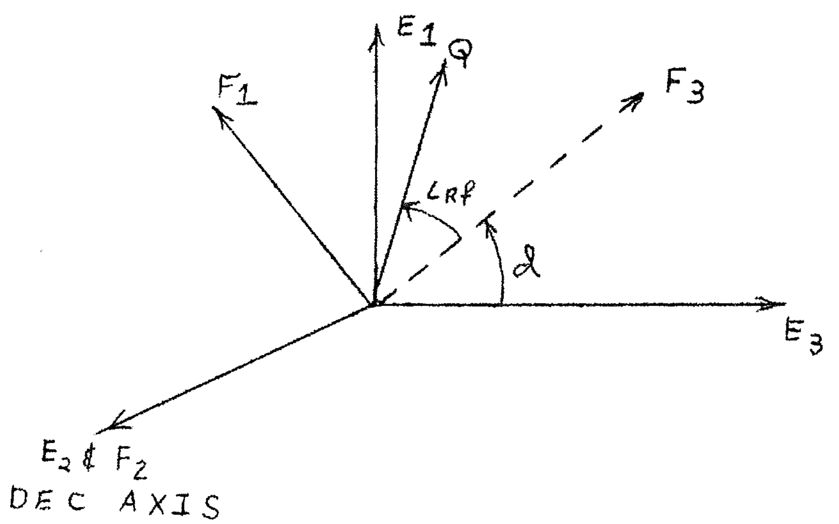


Fig. AA6

We can suppress the angle d by rotating about E_2 to the F coordinates. This may be represented by the matrix equation

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} \cos d & 0 & -\sin d \\ 0 & 1 & 0 \\ \sin d & 0 & \cos d \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

In a like manner we can suppress the angle L_{Rf} by rotating about the F_1 axis to the G coordinates. This rotation is shown in Figure AA7.

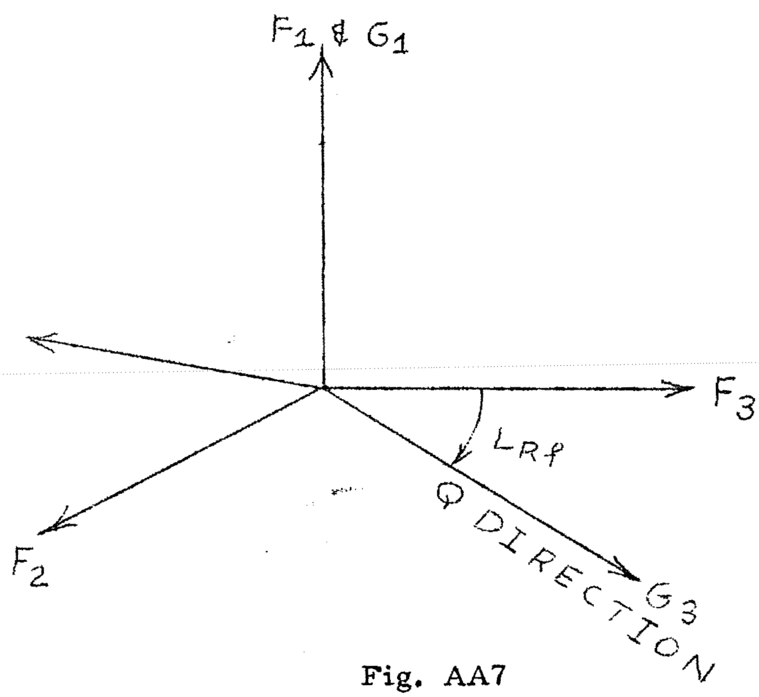


Fig. AA7

This transformation may be represented by the relationship

$$\begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos L_{Rf} & -\sin L_{Rf} \\ 0 & \sin L_{Rf} & \cos L_{Rf} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}.$$

Now we need to know where the vertical is and rotate an amount S(E) toward it to find the R.f. beam. But finding the vertical in the y′ coordinates is no trivial matter until one realizes that we already have the tools available to find it. The vertical has an hour angle of zero and a declination angle equal to the latitude so it has A_1, A_2, A_3 coordinates

$$A_1 = \sin L$$

$$A_2 = \cos L$$

$$A_3 = 0$$

and it can be transformed to the G-coordinates by the equation.

$$\begin{bmatrix} G_1' \\ G_2' \\ G_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos L_{Rf} & -\sin L_{Rf} \\ 0 & +\sin L_{Rf} & \cos L_{Rf} \end{bmatrix} \begin{bmatrix} \cos d & 0 & -\sin d \\ 0 & 1 & 0 \\ +\sin d & 0 & \cos d \end{bmatrix}$$

$$\begin{bmatrix} \cos\Gamma & \sin\Gamma & 0 \\ -\sin\Gamma & \cos\Gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos p & -\sin p \\ 0 & \sin p & \cos p \end{bmatrix} \begin{bmatrix} \cos\epsilon & \sin\epsilon & 0 \\ -\sin\epsilon & \cos\epsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \sin L \\ \cos L \\ 0 \end{bmatrix}$$

Now let us define the angles μ and z by the equations

$$G_1' = \cos\mu \cos z$$

$$G_2' = \sin\mu \cos z$$

$$G_3' = \sin z$$

These angles are shown in Figure AA8. The angle z is the zenith angle of the unsagged beam and the angle μ can be shown to be the spherical angle at the object in the spherical triangle made by the object, the vertical, and the celestial pole.

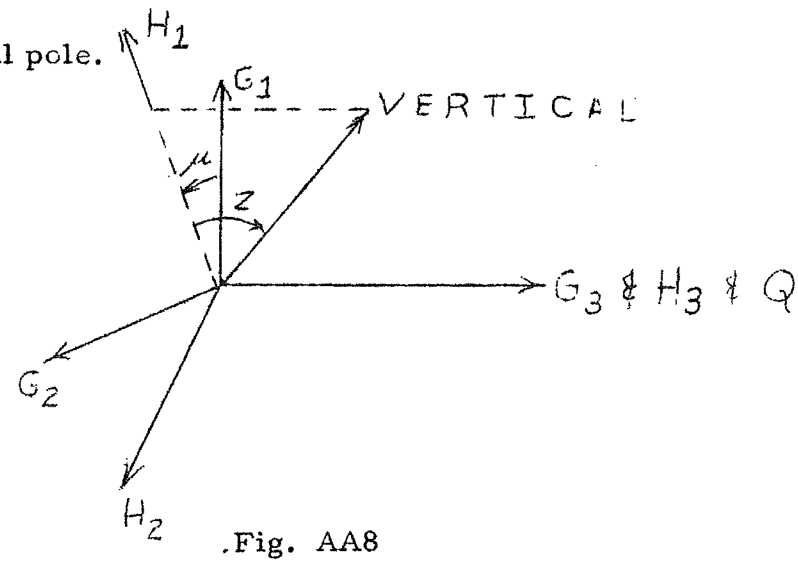


Fig. AA8

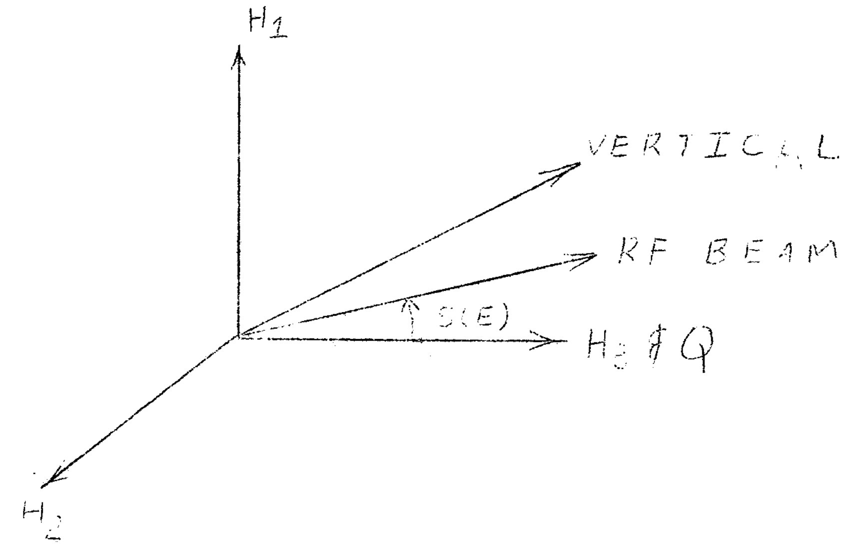


Fig. AA9

One can now rotate the (G_1, G_3)-plane into the (H_1, H_3) plane by rotating the angle μ about the G_3-H_3 axis. This may be represented by the matrix equation

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} \cos\mu & \sin\mu & 0 \\ -\sin\mu & \cos\mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}$$

Now the vertical and Q direction are in the (H_1, H_3)-plane and the radio frequency beam makes an angle S(E) with the H_3 axis as shown in Figure AA9. With one more change of coordinates we would obtain a system in which the radio frequency was a simple basis vector. However, it is clear from Figure AA9 that

$$H_1 = \sin S(E)$$

$$H_2 = 0$$

$$H_3 = \cos S(E).$$

Collecting all of these rotations, the radio frequency beam may be represented by

$$\begin{bmatrix} \sin\delta \\ \cos\delta \quad \cos(t-\varphi) \\ -\cos\delta \quad \sin(t-\varphi) \end{bmatrix} = J \begin{bmatrix} \sin S \\ 0 \\ \cos S \end{bmatrix} \tag{1}$$

where J is a rotation given by

$$J = \begin{bmatrix} \cos\epsilon & -\sin\epsilon & 0 \\ \sin\epsilon & \cos\epsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos p & \sin p \\ 0 & -\sin p & \cos p \end{bmatrix} \begin{bmatrix} \cos\Gamma & -\sin\Gamma & 0 \\ \sin\Gamma & \cos\Gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos d & 0 & \sin d \\ 0 & 1 & 0 \\ -\sin d & 0 & \cos d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos L_{Rf} & \sin L_{Rf} \\ 0 & -\sin L_{Rf} & \cos L_{Rf} \end{bmatrix} \begin{bmatrix} \cos\mu & -\sin\mu & 0 \\ \sin\mu & \cos\mu & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the angle μ is given by

$$\begin{bmatrix} \cos\mu \quad \cos z \\ \sin\mu \quad \cos z \\ \sin z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos L_{Rf} & -\sin L_{Rf} \\ 0 & \sin L_{Rf} & \cos L_{Rf} \end{bmatrix} \begin{bmatrix} \cos d & 0 & -\sin d \\ 0 & 1 & 0 \\ \sin d & 0 & \cos d \end{bmatrix}$$

$$\begin{bmatrix} \cos\Gamma & \sin\Gamma & 0 \\ -\sin\Gamma & \cos\Gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos p & -\sin p \\ 0 & \sin p & \cos p \end{bmatrix} \begin{bmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sin L \\ 0 \cos L & \cos\varphi \\ 1 \cos L & \sin\varphi \end{bmatrix}$$

This equation may be best regarded as an equation to be solved for the declination and hour angle given in polar and declination drive angles and all the errors.

The declination and hour angle, δ and t above, are topocentric coordinates, that is, they are the declination and hour angle referred to the site of the antenna. The astronomical coordinates that are usually used are geocentric coordinates, i.e. the coordinates of the object as seen from the center of the earth. There are two phenomena, refraction and parallax that make these two coordinate systems differ. Both may be considered rotations in the plane formed by the vertical and a ray to the object. This plane is also the plane formed by Q and the vertical in the discussion above. Refraction always rotates the ray toward the vertical and parallax always rotates it away from the vertical. Thus, the declination and hour angle in Equation (1) above may be changed from topocentric to geocentric coordinates by replacing

$$S(E) \text{ by } V = S(E) - R(E) + Z$$

where R(E) is the refraction and Z is the angle of parallax. One can now rewrite Equation (1) as

$$\begin{bmatrix} \sin \delta \\ \cos \delta \quad \cos (t - \varphi) \\ -\sin \delta \quad \sin (t - \varphi) \end{bmatrix} = J \begin{bmatrix} \sin V \\ 0 \\ \cos V \end{bmatrix} . \qquad (2)$$

This equation is rather complicated, but one can obtain an approximate solution by assuming that the errors are small. In this case an expression that solves Equation (2) to first order terms in small quantities is

$$\delta = d - \epsilon \cos (t - \varphi) + V \cos \mu$$

$$t = p + \varphi - \frac{\pi}{2} - [\epsilon \sin (t - \varphi) - \Gamma] \tan \delta + (V \sin \mu + L) \sec \delta .$$

Taking into account the servo encoder bias, the polar and declination servo readings for the radio frequency beam may be written

$$D = \delta + \epsilon \cos (t - \varphi) - V \cos \mu + K_{Rf} \qquad (3a)$$

$$P = t + [\epsilon \sin (t - \varphi) - \Gamma] \tan \delta - (V \sin \mu + L_{Rf}) \sec \delta + K_p , \qquad (3b)$$

and the appropriate equations for the optical telescope are

$$D = \delta + \epsilon \cos (t - \varphi) - \{Z - R(E)\} \cos \mu + K_o \qquad (4a)$$

$$P = t + [\epsilon \sin (t - \varphi) - \Gamma] \tan \delta - (\{Z - R(E)\} \sin \mu + L_o) \sec \delta + K_p, \qquad (4b)$$

where

D = Declination servo reading

P = Polar servo reading

δ = geocentric declination

t = geocentric hour angle

$R(E)$ = refraction correction

$S(E)$ = sag correction

Z = parallax correction

ϵ = co-declination of polar axis

φ = hour angle of polar axis

Γ = angle declination axis makes with plane perpendicular to polar axis

L_o = angle optical axis makes with plane perpendicular to declination axis

L_{Rf} = angle Q direction makes with plane perpendicular to declination axis

K_o = Declination additive constant for optical telescope

K_{Rf} = Declination additive constant for R.f. axis

K_p = Polar encoder additive bias.

The constant K_p is the same in both equations because it represents the encoder bias; however, the constants K_{Rf} and K_o are not the same in general. They differ by the difference in declination between the Q direction and the optical telescope direction.

Appendix BB

Measurements

This appendix presents the results of the sequence of measurements suggested in the main part of the paper. The results of these experiments are as follows:

$$\epsilon = 0° .078$$

$$\varphi = -.4823 \text{ radians}$$

$$\Gamma = -0.° .006$$

$$K_p = -0° .128$$

$$S = -0° .010$$

$$r = -20\%$$

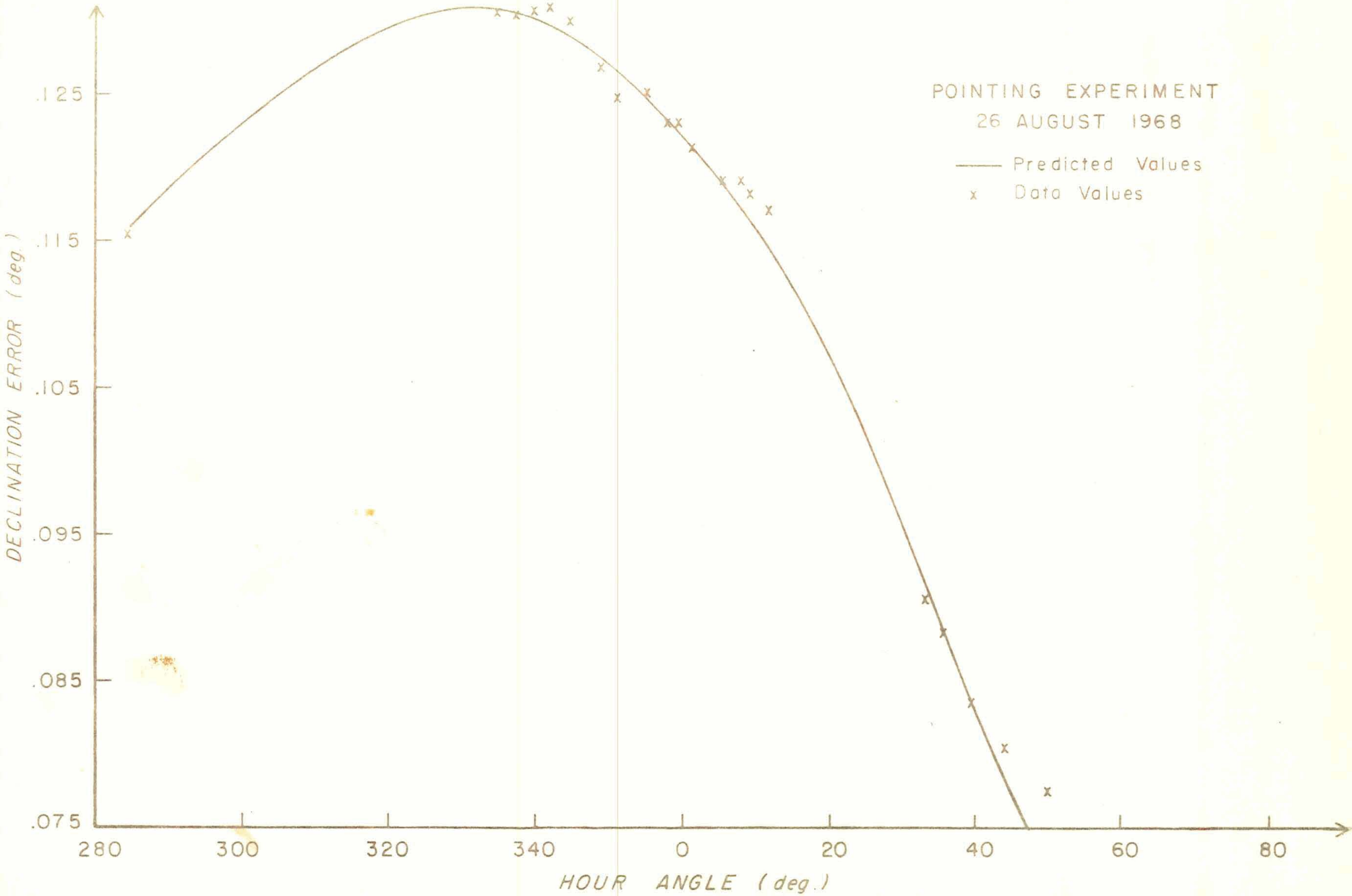
A star tracking experiment was performed August 26, 1968. This experiment gives the polar axis co-declination and hour angle, ϵ and φ as well as the constant K_o. Figure BB1 gives the experimental results in graphical form. The stars represent data points while the line is the best mean square fit.

A star transit experiment was performed on May 11, 1969. This experiment gives the declination axis and telescope non-orthogonality as well as the polar encoder bias. Just as in the preceding example the results are shown in graphical form in Figure BB2.

Figure BB3 gives the results of a day of polar sun scans in graphical form. Although there are a large number of data points in this data set, the data set falls short of a good measurement because of the lack of data points at lower elevations. The sag and refraction coefficients are

$$S = .010 \pm .004$$

$$r = -.21 \pm .20.$$



DECLINATION POINTING ERRORS

Fig. BB1

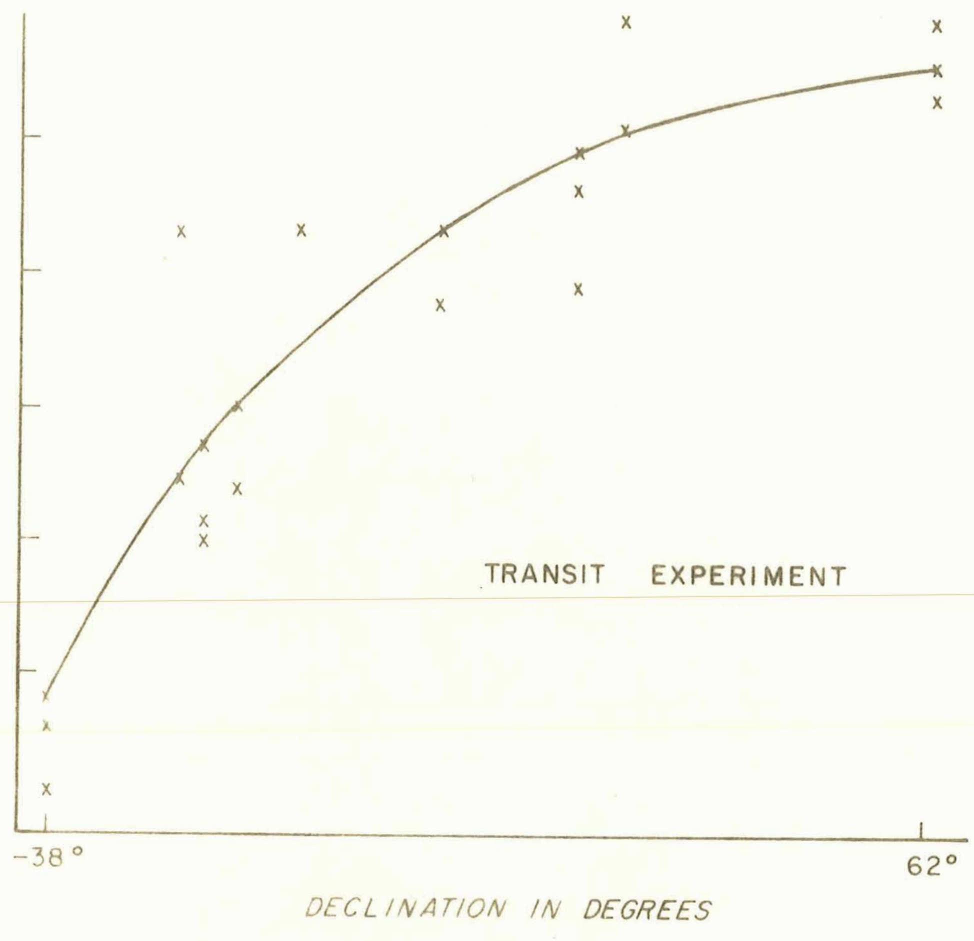
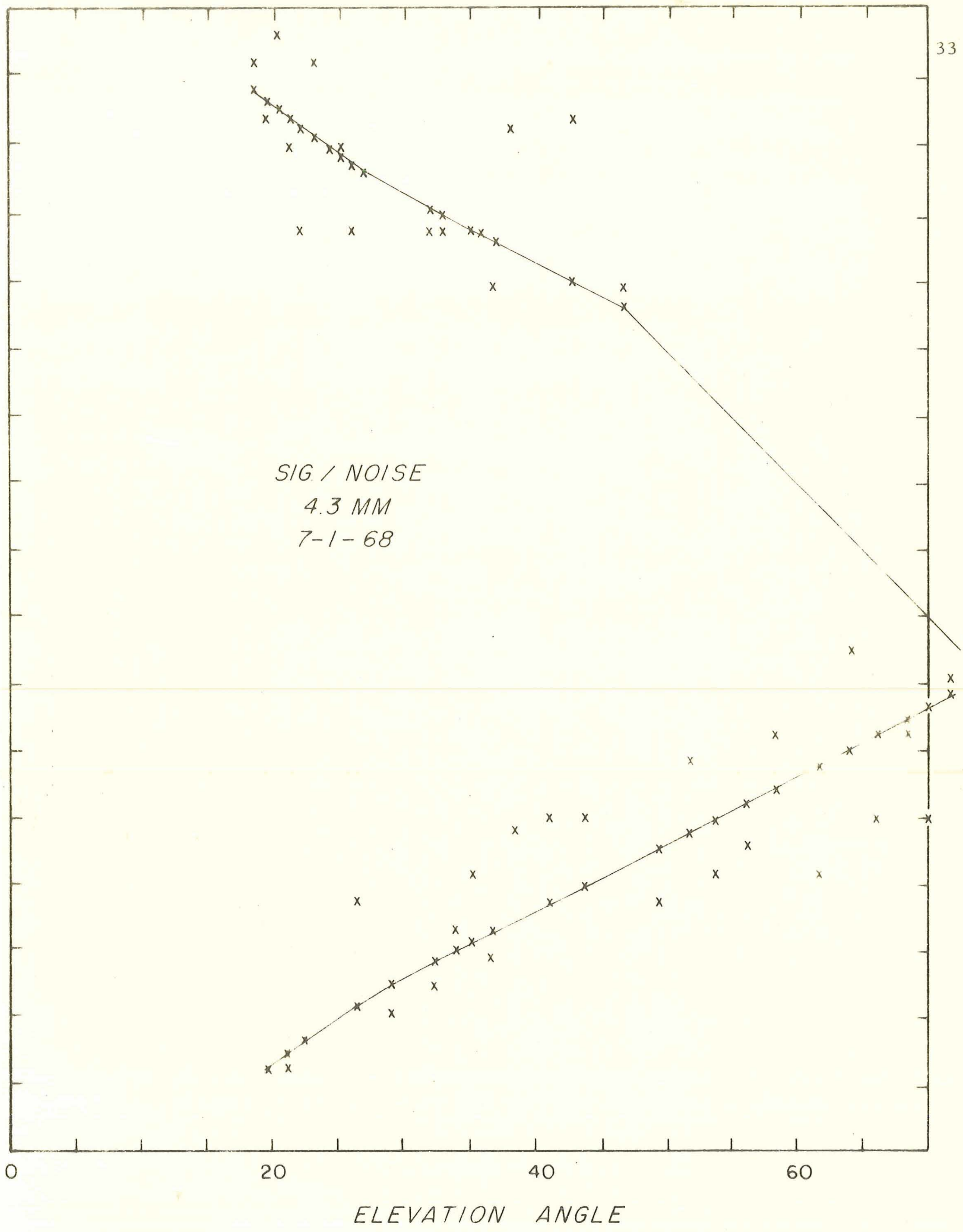


Fig. BB2


SIG. / NOISE
4.3 MM
7-1-68
0
20
40
60
ELEVATION ANGLE