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DIFFRACTION THEORY OF SHADOW KNIFE AND SLIT PHOTOMETRIC METHODS

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SUMMARY

In this paper the authors consider the diffraction theory of shadow knife and slit photometric methods. The basis of measurements conducted by shadow photometric methods lies in the assumption of linear dependence between the deflection angle of the ray and the plane illumination of shadow instrument's image. * This dependence follows from the geometrical scheme of the methods. The distribution of the illumination is found in the diffraction pattern and a dependence is established between the diffraction error and the conditions of measurements.

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The linear dependence between the deflection angle of the ray and the illumination of the plane of shadow instrument's image is the foundation of the method of measurements conducted by shadow photometric methods. Following from the geometric scheme itself, the illumination is assumed to be proportional the area of nondiaphragmated part of the image of the light source. The real distribution of illumination follows only to either extent from the linear dependence.

A standard measurement scheme is considered here, and is shown in Fig.1 It includes a collimator O_k with a slit light source S and a receiving part of the shadow instrument with Foucault knife H in the second focal plane of the optical system O_{Π}. The surface of the light wave is assumed to be either a paraboloid or a parabolic cylinder. The symmetry planes of the wave are parallel to coordinate planes xOz and yOz. In the plane of the object the wave is bounded by an opaque screen, i. e. the half-plane R.

In order to find the illumination of the plane of the image, the Huygen Fresnel principle is applied twice [2]: at the outset during the determination of light amplitude in the second focal plane of system O_{Π}, and then in the plane of the image $x'O'y'$. The illumination is

$$I_{\pm}(a, b) = \frac{1}{2\pi^2} \int_{-1}^{+1} \left| \int_{\eta a+\eta t-\eta\omega}^{\eta a+\eta t} \int_{-\infty}^{b\pm z} \exp[i(z^2 - y^2)]\, dz\, dy \right|^2 dt, \qquad (1)$$

where $a = \frac{m}{f'} \frac{1}{\alpha}$; m is the distance from the center of the elementary image of the slit S_A, corresponding to the considered point A, to edge of the Foucault knife; f' is the second focal distance of the optical system O_{Π}; 2α is the angular dimension of the slit light source in a direction perpendicular to knife edge;

$$b = \frac{h}{|\cos(\varphi - \psi)|} \sqrt{\frac{\pi |K|}{\lambda}};$$

h is the distance from the image A of the considered point A to the image of the edge of the opaque screen; ψ, ϕ are the angles between the slit and the screen and the symmetry plane of wave surface; λ is the light wavelength. K is the curvature of the cross-section of wave's surface by the plane perpendicular to the Foucault knife edge;

$$\eta = \alpha \sqrt{\pi/\lambda |K|}, \omega = \frac{L}{f} \frac{1}{\alpha};$$

L is the distance from the Foucault knife edge to the dge of the collimating diaphragm in the second focal plane, parallel to it.

There are two forms of illumination distribution (signs + and − in the upper limit of the interior integral). To them correspond diffraction patterns which can be made congruent with the diffraction patterns at a spherical wave, when the edges of the clear interval between the shadow from the knife and the image of the screen form a sharp (+) or blunt (−) angle.

Only the region of the penumbra ($-1 \leqslant a \leqslant 1$), where according to geometric optics th illumination is equal to $I_r(a) = (1 + a) / 2$, offers interest for measurements.

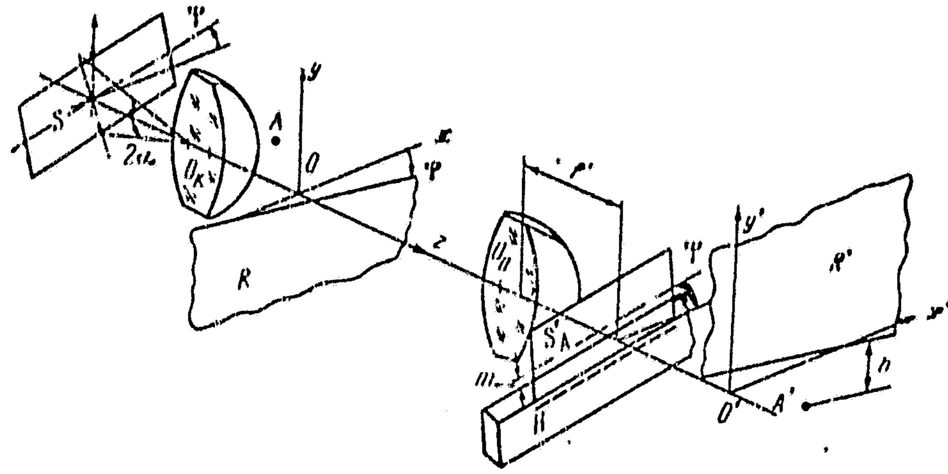
S
$2u_0$
O_K
A
y
O
x
φ
R
z
ρ
O_{II}
ψ
y'
R'
x'
S'_A
m
h
O'
A'
K

Fig.1

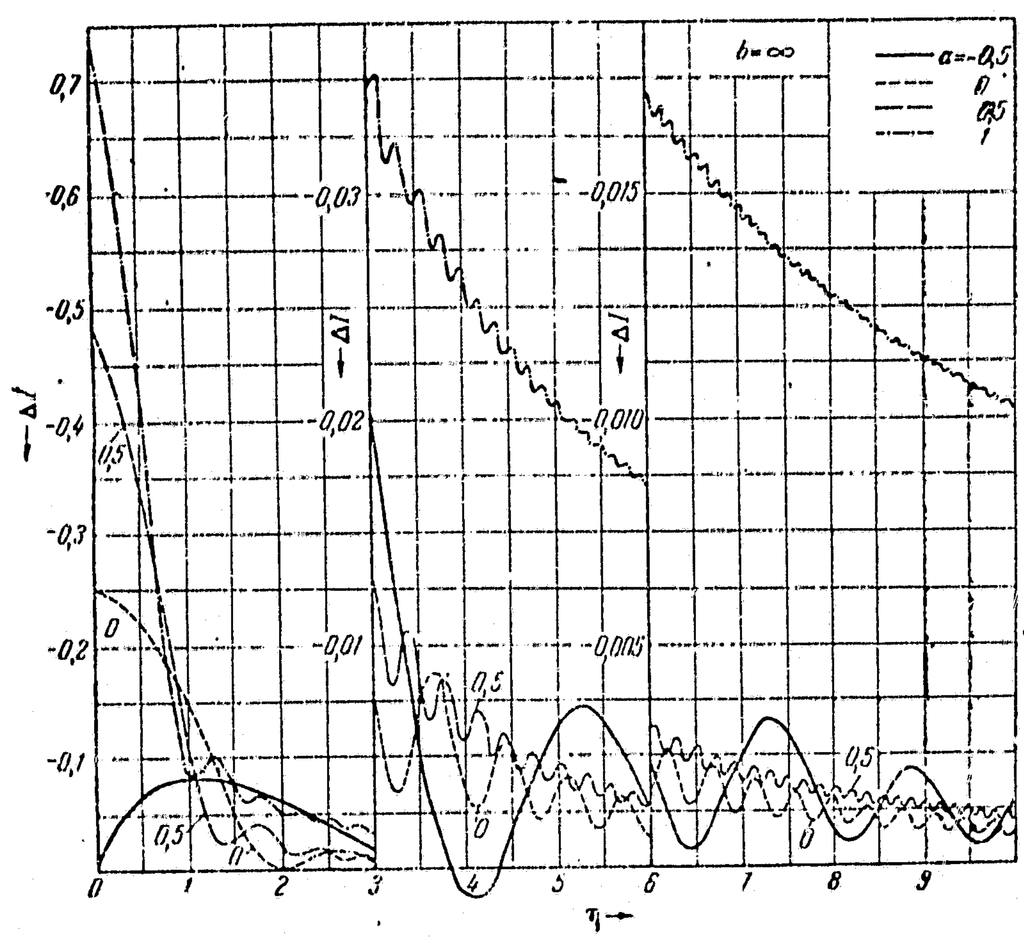
b=∞
a=-0,5
0
0,5
1
0,7
-0,6
-0,5
-0,4
-0,3
-0,2
-0,1
-0,03
-0,02
-0,01
-0,015
-0,010
-0,005
ΔI
η

Fig.2

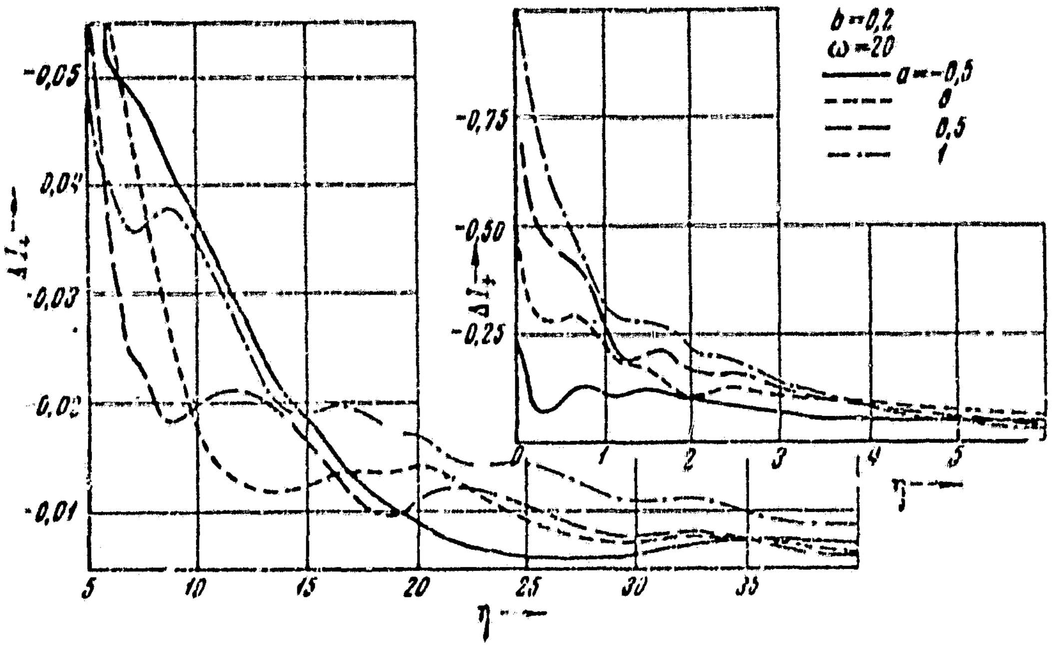

Fig.3

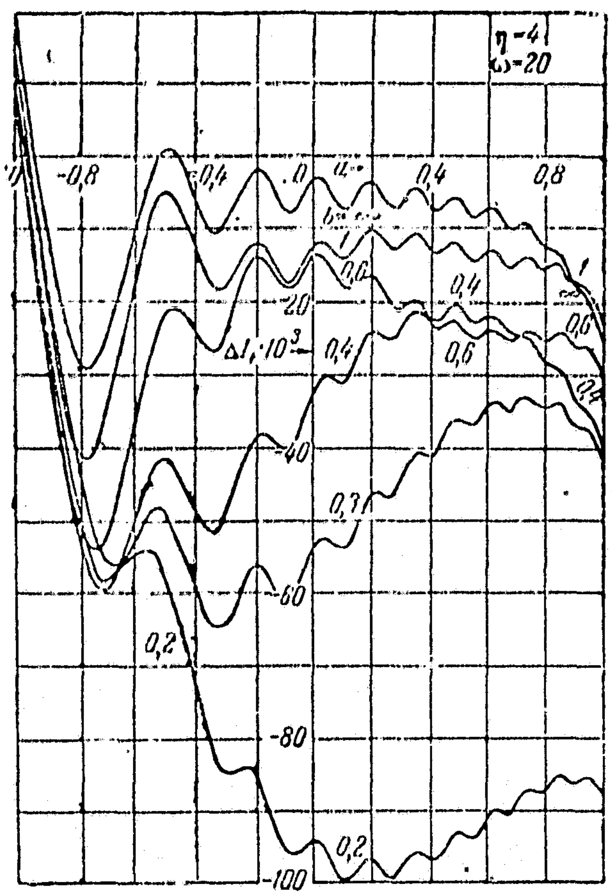

Fig.4

Three cases are considered within the interval of quantity b (0,∞) variation. For b >> 1, an approximate expression is obtained for the right hand part of (1): the greater b, the more precise it is. One may judge on the departure of the illumination from linear by the curves of Fig.2, corresponding to the limiting case (b = ∞), when collimating diaphragms are absent in the plane of the object.

For b > 1 the right-hand part of (1) is represented in the form of the sum of illumination at unlimited wave and the term taking into account the diffraction from the screen in the plane of the object.

The maximum diffraction error in the measurement of the angle of

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deflection of the light ray is

$$\varkappa = \left| \frac{\cos[b^2 + 2b\eta(a+1)]}{2\pi b[b + \eta(a+1)]} \right| + \left| \frac{dK/da}{2\pi\eta^2 K(a^2 - 1)} \right| + \left| \frac{dK/da}{4\pi\eta K[b + \eta(a+1)]} \right| . \qquad (2)$$

The first term in (2) represents the error taking place during the measurement of the rotation angle of the plane wave, and the second — of the unlimited wave with curvature varying in the process of the experiment.

For $0 \leqslant b \leqslant 1$, the expression (1) is transforemd into a form acceptable for computer operation; at the same time use is made of the representations of Fresnel integrals by power and asymptotic series. Examples of calculation of

$$\Delta I_{+}(a, b) = I_{+}(a, b) - I_{r}(a).$$

are brought out in Figures 4 and 3.

The authors consider it as their duty to extend their thanks to I. V. Obreyimov for his attention in the course of the present work.

*** T H E E N D ***

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