Nonlinear Flutter of Curved Plates, II*

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ABSTRACT

In a series of papers by the author and his colleagues, nonlinear flutter analyses of plates have been made. Most recently in Part I curved plates were considered. In the present paper, Part II, numerical results are presented for three-dimensional curved plates of constant curvature and simply-supported on all edges. Quasi-steady supersonic aerodynamic theory is employed. These numerical results demonstrate some of the important qualitative and quantitative differences between three-dimensional plates and the two-dimensional ones discussed in detail in Part I.

INTRODUCTION

In Part I[1], a nonlinear analysis was made of two and three-dimensional plates of constant curvature within the framework of shallow shell theory (essentially Von Karman's approximation) and quasi-steady supersonic aerodynamics theory. Extensive numerical results were presented there for the two-dimensional case; in the present paper results are presented for the three-dimensional case and a discussion is given of some of the features characteristic of this case which have no counterpart in the two-dimensional problem.

GENERAL NATURE OF PROBLEM

The details of the mathematical analysis are contained in Part I. For the convenience of the reader we recapitulate the major assumptions and basic method of the analysis. Starting from the (nonlinear) shallow shell equation and quasi-steady aerodynamic theory, a modal solution is obtained via Galerkin's method for the spatial variables and numerical integration for the time variable. Formally the solution may be expressed as

$$w/h \equiv W \equiv W(\xi, \eta, \tau; \lambda, \mu/M, \Gamma_x, \Gamma_y, a/b, P) \tag{1}$$

where $w/h \equiv$ plate deflection/ plate thickness and the remaining non-dimensional variables are

ξ, η	spatial variables
τ	time

with non-dimensional parameters

λ	dynamic pressure
μ/M	mass ratio
Γ_x, Γ_y	(constant) curvature in x and y direction. Note $\Gamma_x \equiv 8H/h$ or $\Gamma_y \equiv 8(a/b)^2 \frac{H}{h}$ where H/h rise height/ thickness ratio.
a/b	length/ width ratio
P	static pressure loading

One might also include externally applied in-plane loads and the cavity effect with little additional difficulty. All of the results presented here will be for no cavity effect, zero in-plane applied loads and $P = 0$. The author has made calculations including these effects for certain practical cases but no systematic investigation has been made to date.

It should be noted that a relation similar to Equation (1) may be derived for stress. For practical calculations one ordinarily deals with stress as a design parameter. For insight into the physical problem, deflection is usually a parameter of greater interest. Hence although stresses have been computed for all cases presented here, we shall focus on deflection and omit the stresses.

RESULTS

We shall emphasize the following aspects of the problem:

(1) The effect of three-dimensionality

(2) The effect of curvature, streamwise, spanwise or both.

(3) The effect of in-plane boundary support conditions

To do this we shall consider a curved plate with square planform, $a/b = 1$. All results were obtained using six streamwise modes which insures good convergence for the parameter range studied.

In Fig. 1 we compare results for the two-dimensional plate, $a/b = 0$, and no spanwise bending to that for the square plate with streamwise curvature, $a/b = 1$, $\Gamma_x \neq 0$, and restrained at all edges. As expected λ_f is always larger for $a/b = 1$ than for $a/b = 0$. What is perhaps unexpected is the distinct peak for λ_f when $a/b = 1$ and $H/h \approx 2$. This appears to be associated with the static deflections of the plate under the aerodynamic loads due to the initial curvature of the plate. For $a/b = 1$ the static deflection is larger prior to flutter than for $a/b = 0$, particularly near $H/h = 2$. This large static deflection apparently gives the plate added stiffness. The source of the static deflection itself is apparently the greater spanwise bending for $a/b = 1$ than $a/b = 0$. (Recall $a/b = 0$ denotes no spanwise bending by hypothesis.) If this explanation is correct this suggests that three-dimensional effects may be somewhat more important for curved plates than flat ones. This is certainly true for the data shown in Fig. 1.

In Fig. 2 we compare several results for the square plate, $a/b = 1$, including streamwise curvature only, $\Gamma_x \neq 0$, $\Gamma_y = 0$; spanwise curvature only, $\Gamma_x \neq 0$, $\Gamma_y \neq 0$; and both streamwise and spanwise curvature, $\Gamma_x \neq 0$, $\Gamma_y \neq 0$. The results for $\Gamma_x \neq 0$, $\Gamma_y = 0$ have already been discussed. Consider by comparison the results for $\Gamma_x = 0$, $\Gamma_y \neq 0$. As can be seen there is a monotonic decrease in λ_f. Note that for $\Gamma_x = 0$, there is no static aerodynamic loading due to plate curvature within the quasi-steady aerodynamic approximation. Thus the explanation for the decrease in λ_f must be sought elsewhere. If one examines the natural frequencies of the first two streamwise modes, one finds that while both increase with H/h (or Γ_y) the first increases more rapidly than the second. Indeed in the present example for $H/h \approx 4$ the two frequencies coincide. Such frequency coincidence is known to have a detrimental effect on λ_f. More will be said of this interesting result later.

Now let us turn to $\Gamma_x \neq 0$ and $\Gamma_y \neq 0$. Since $\Gamma_x \neq 0$, there is a static aerodynamic loading on the plate. Indeed it is found that typically prior to flutter the plate buckles! This unexpected buckling generally stiffens the plate with respect to flutter, and, as may be seen in Fig. 2, λ_f is higher for this case than any other when the curvature is sufficiently large, $H/h > 2$.

The flutter frequency is also of some interest and is shown in Fig. 3. Generally the frequency increases with increasing curvature, H/h, but note the drop-off when $\Gamma_x \neq 0$ for $H/h \approx 5$. This latter behavior is unexplained at present.

In Fig. 4 we plot w/h vs. λ for a typical case, $\Gamma_x = \Gamma_y \neq 0$, $H/h = 4$. Note the transition from unbuckled, $\lambda \leq 500$, to buckled, $\lambda = 625 \rightarrow 875$, to flutter, $\lambda = 1000$. For flutter the plate deflection oscillates over the range indicated. Other cases are not so unambiguous, for example, $\Gamma_x \neq 0$, $\Gamma_y = 0$ and $H/h = 2.0$. See Fig. 5. Here it is difficult if not impossible to make a distinction between unbuckled and buckled. The transition to flutter is clear.

Now let us return briefly to the interesting case $\Gamma_x = 0$, $\Gamma_y \neq 0$. The results shown in Fig. 3 directly contradict those found earlier by Voss[2] in his well known paper on cylindrical shells and curved plates, as well as various authors who have followed Voss. Here we find a decrease in λ_f with increase in spanwise curvature while other authors have determined spanwise curvature to be beneficial with respect to flutter. It first might be thought that these differences are due to nonlinearities, however this is not so. A linear analysis would give the same results for flutter boundary, i.e. at $\lambda = \lambda_f$, as long as $\Gamma_x = 0$. The difference in the present results and those due to Voss are entirely a result of satisfying different in-plane boundary conditions. Voss implicitly assumes zero in-plane stress on the boundaries, $x = 0$, a and $y = 0$, b. (In the present analysis this is equivalent to setting $\bar{R}_x = \bar{R}_y = 0$, see Part I.) Here we assume zero displacement. In Fig. 6 the two results are compared. The conclusion to be drawn from this comparison is not that one analysis or the other is right or wrong, but rather that the results for curved plates are sensitive to in-plane edge boundary conditions. Another, more trivial, example of this is the two-dimensional plate, $a/b \equiv 0$, with no spanwise bending but streamwise curvature. If the edges are completely restrained against in-plane displacement the results are those shown in Fig. 1. On the other hand if there is no restraint, i.e. zero in-plane stress at the edge, then there is no effect of curvature on λ_f and λ_f remains constant for all curvature, H/h. Thus for curved plates the flutter boundary, quite aside from the flutter motion per se, is dependent upon in-plane as well as out-of-plane edge restraints.

Three questions which deserve further study are the following:

One item is the question of the most critical spanwise mode. All of the results discussed so far were obtained using only the first spanwise mode. Based upon Voss[8] results one may expect that for some curved plates, particularly those with spanwise curvature and zero membrane stress at the edges, higher spanwise modes can be more critical for flutter. In Fig. 7 results are shown for the first three spanwise modes for $a/b = 1$, $\Gamma_x = 0$ and $\bar{R}_x = \bar{R}_y = 0$. As may be seen for this example the first spanwise mode is always the most critical although near $H/h = 3$ the second mode flutter boundary is very near that of the first. It should be emphasized that coupling between spanwise modes has not been accounted for in the present analysis. For the particular example there is no coupling between modes. However if $\bar{R}_y \neq 0$, the various spanwise modes will couple unless, of course, one reformulates the problem in terms of natural modes. In general it would seem reasonable to neglect spanwise coupling except when two modes which may couple have flutter boundaries in close proximity. However it would seem wise to investigate higher spanwise modes at least on an individual basis to assure that the most critical flutter mode has been determined.

A second item is the effect of spanwise variation of the aerodynamic loading on the pre-flutter deformation at the lower Mach numbers. A more refined aerodynamic theory will be required to study this effect, see Ref. 3.

Finally, a continuous variation of curvature should be studied so that the question of imperfections may be considered. This would be in the spirit of earlier work by Fung[4] and Kobayashi[5] but including three-dimensionality and sufficient modes to insure convergence.

CONCLUSIONS AND RECOMMENDATIONS

The major conclusions to be drawn from the present study are

(1) Three-dimensional curved plates with streamwise curvature will be more significantly affected by pre-flutter static deformation than two-dimensional ones. A non-linear structural theory is required to account for this deformation which may include in some instances buckling under the static aerodynamic loading.

(2) Three-dimensional curved plates with spanwise curvature are sensitive to in-plane boundary support conditions even when linear theory may be used to determine the flutter boundary.

Topics recommended for additional study include

(3) The effect of coupling between spanwise modes.

(4) The effect of a more refined aerodynamic theory at lower Mach number.

(5) The effect of variable curvature with particular attention on plate imperfections.

(4) and (5) are thought to be more important than (3).

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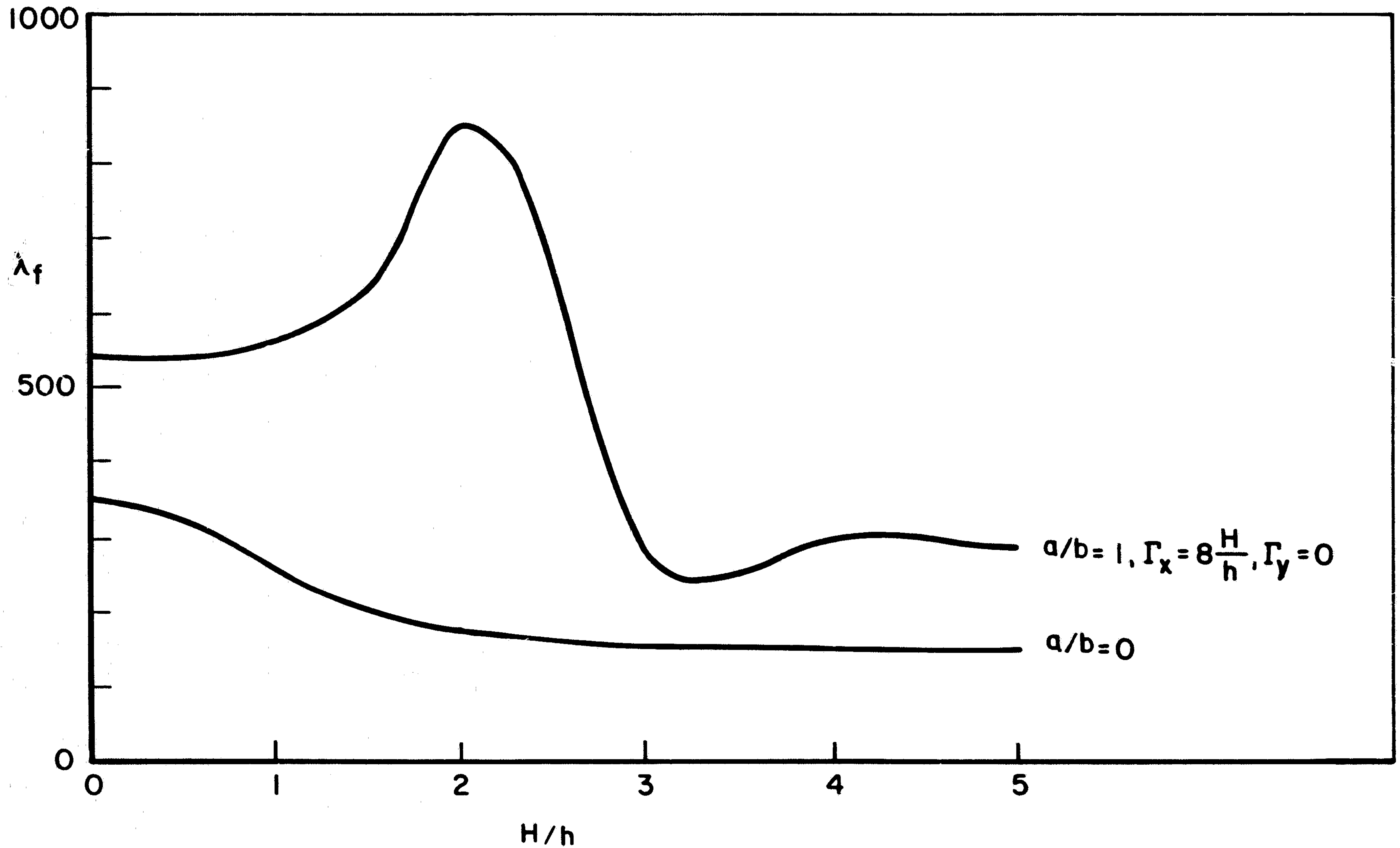
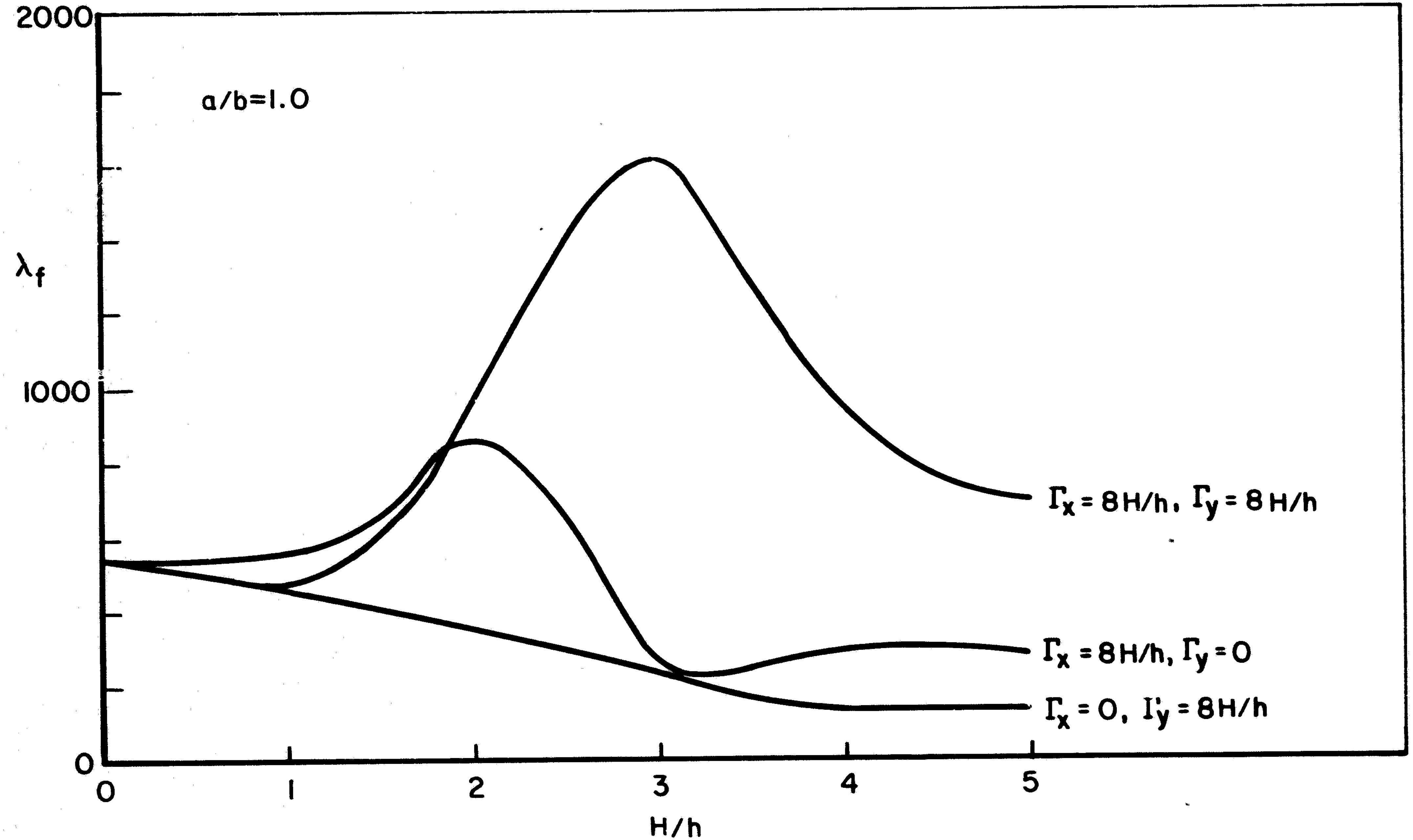
LIST OF FIGURES

Fig. No.	Caption
1	Flutter Dynamic Pressure vs. Curvature
2	Flutter Dynamic Pressure vs. Curvature
3	Flutter Frequency vs. Curvature
4	Plate Deflection vs. Dynamic Pressure
5	Plate Deflection vs. Dynamic Pressure
6	Flutter Dynamic Pressure vs. Curvature
7	Flutter Dynamic Pressure vs. Curvature



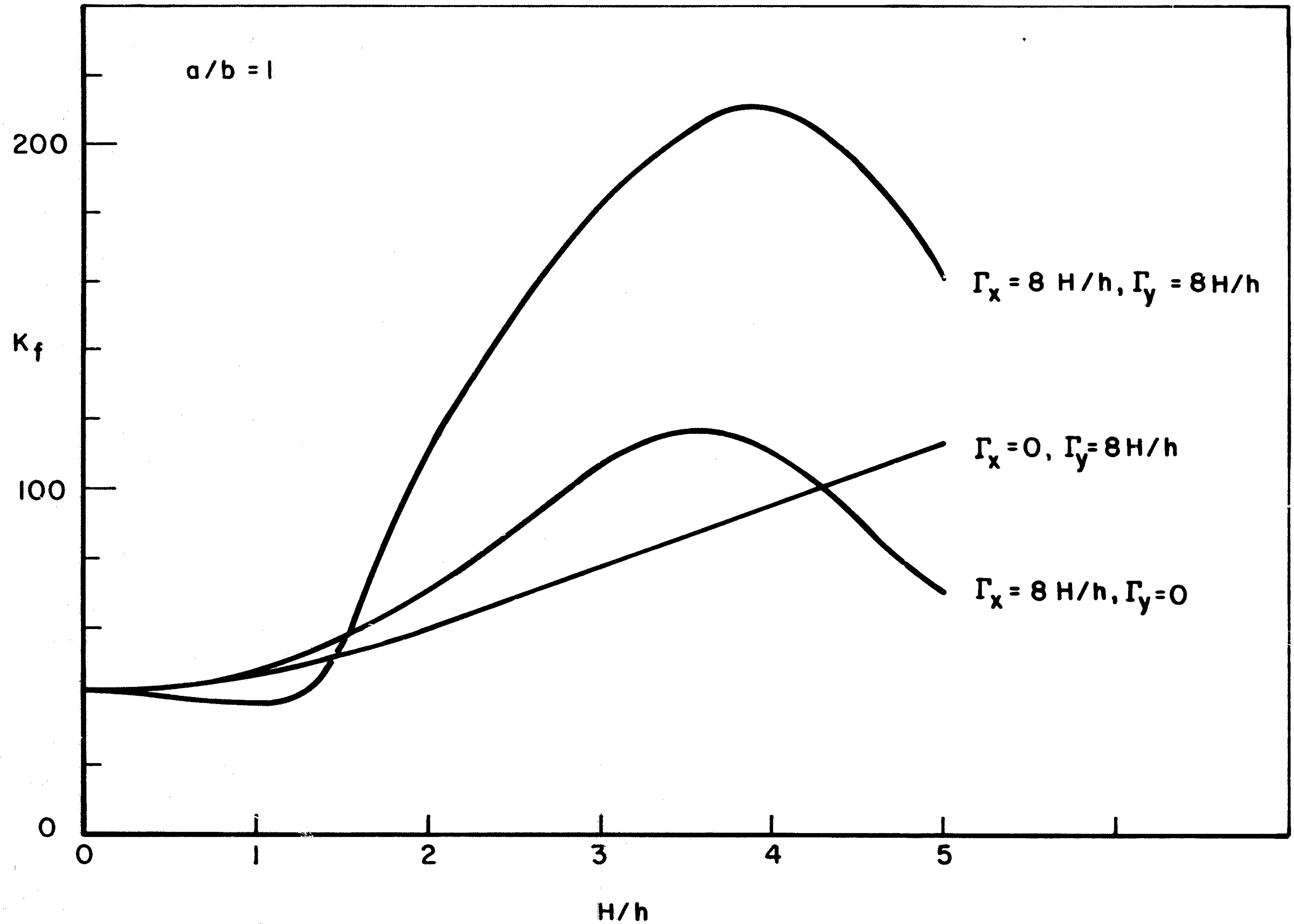
FLUTTER DYNAMIC PRESSURE VS CURVATURE

FIGURE 1



FLUTTER DYNAMIC PRESSURE VS CURVATURE

FIGURE 2



FLUTTER FREQUENCY VS CURVATURE

FIGURE 3

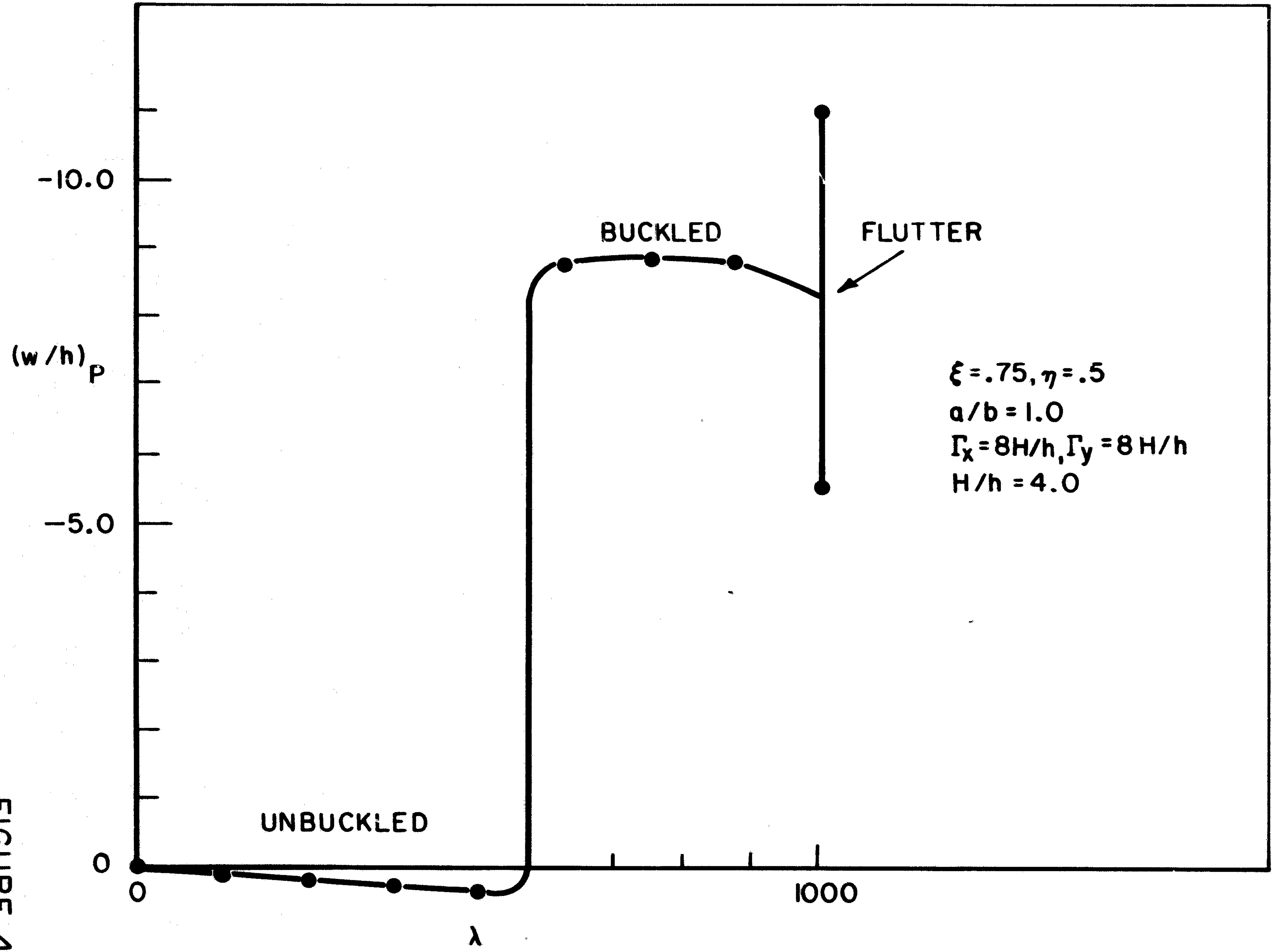


PLATE DEFLECTION VS DYNAMIC PRESSURE

FIGURE 4

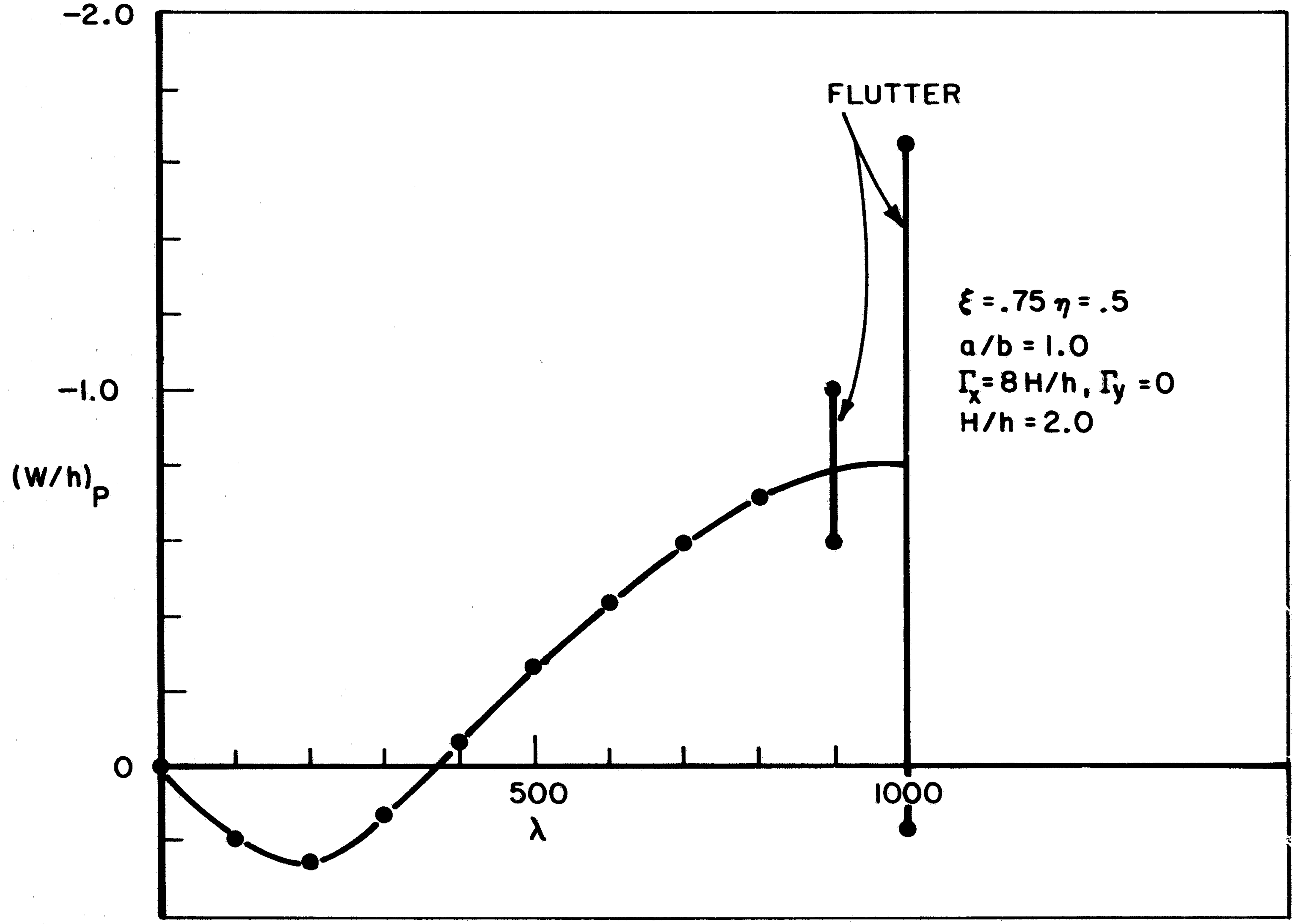
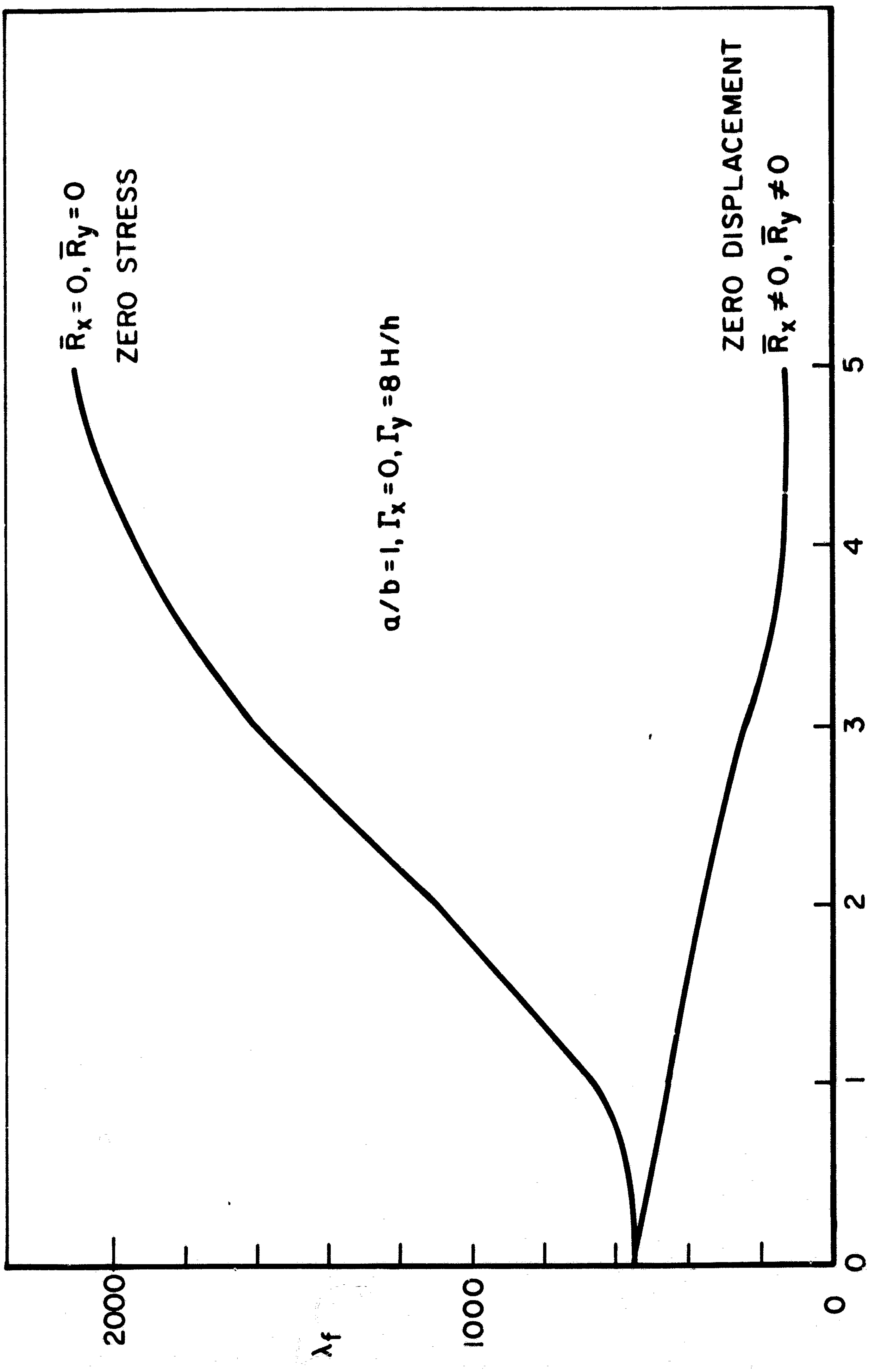


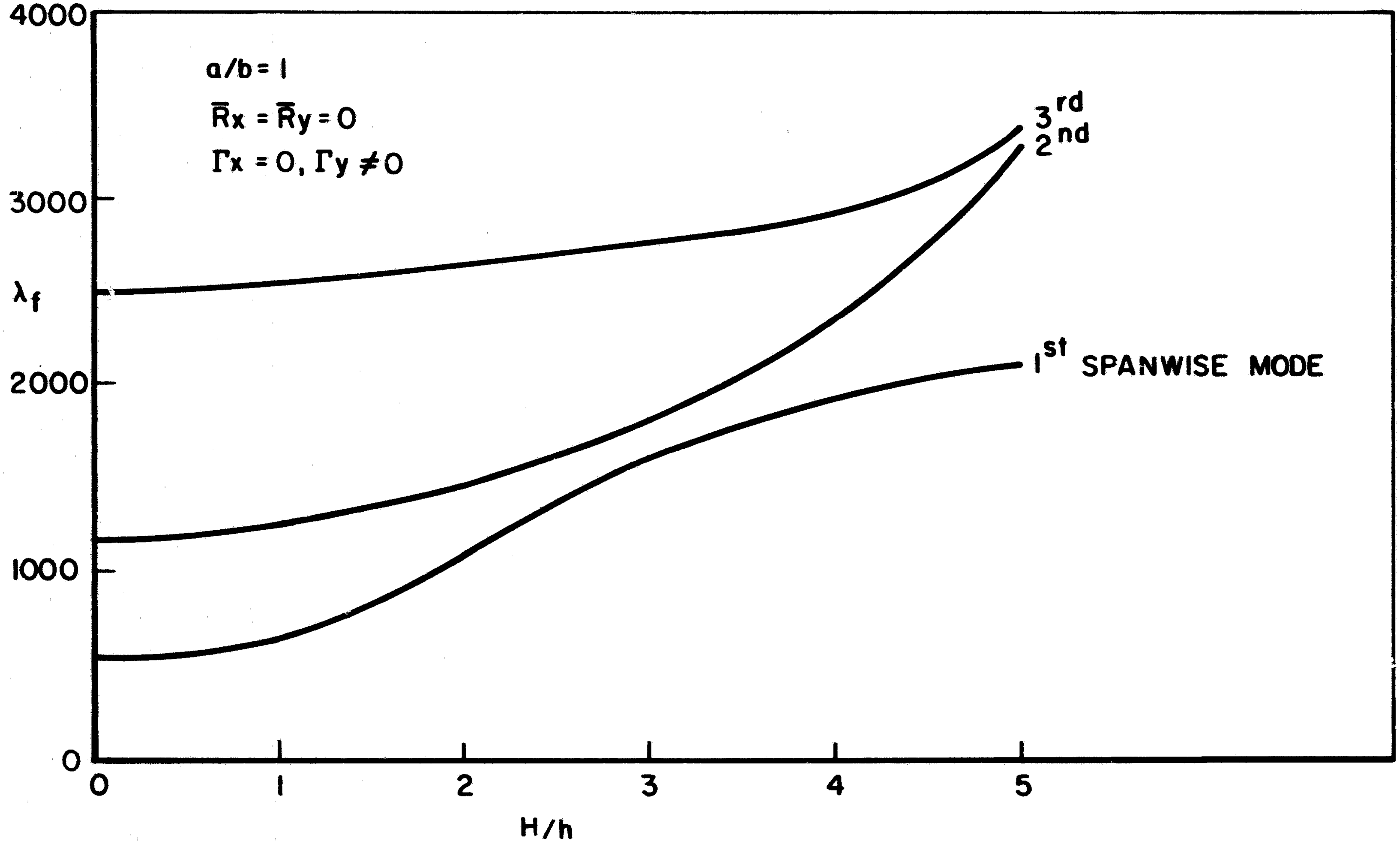
PLATE DEFLECTION VS DYNAMIC PRESSURE

FIGURE 5



FLUTTER DYNAMIC PRESSURE VS CURVATURE

FIGURE 6



FLUTTER DYNAMIC PRESSURE VS CURVATURE

FIGURE 7.